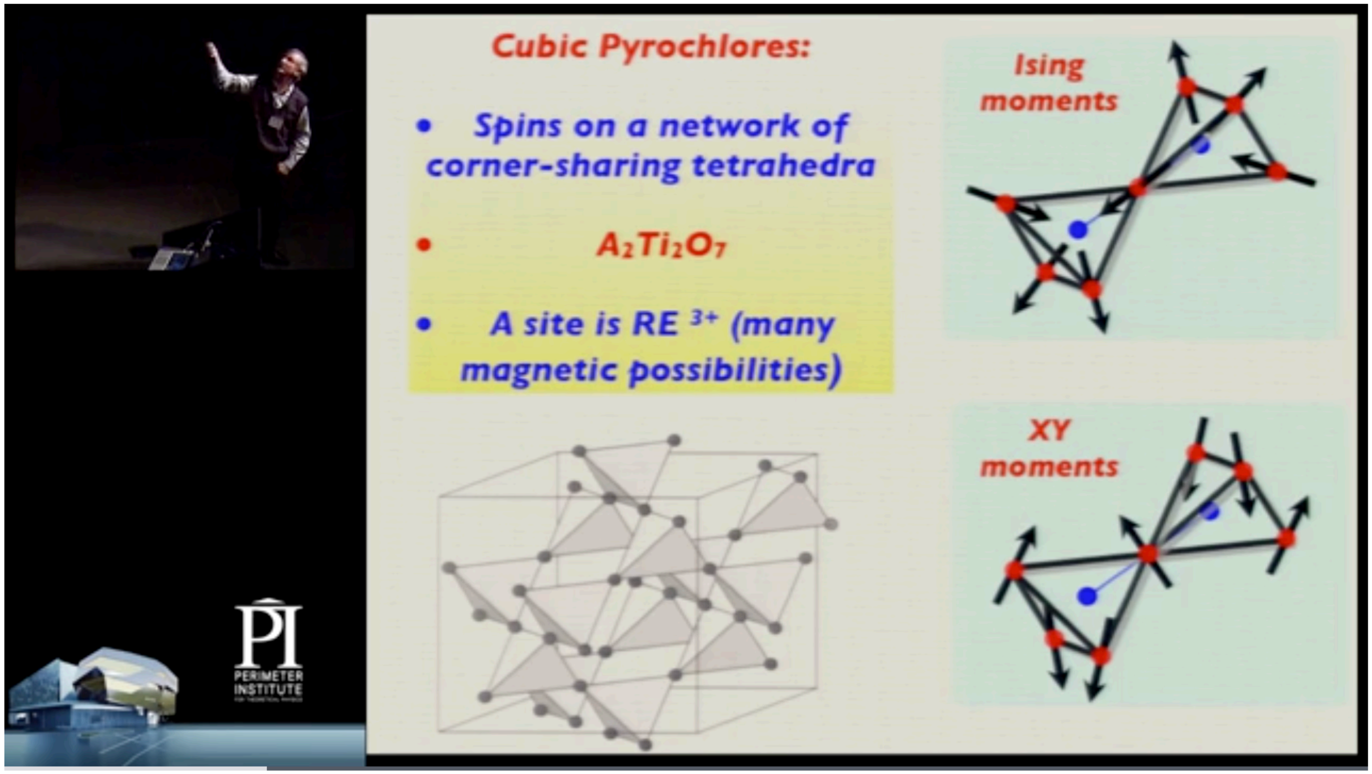
Leon Balents, “Spins and other liquids”, Hamilton, Ontario, May 2025

Signatures of an emergent gauge field in two dimensions

PIRSA:10040083



4-corners condensed matter symposium, PI, April 2010

Bruce teaches Leon about rare earth pyrochlores

Phase Transitions in Planar Pyrochlores

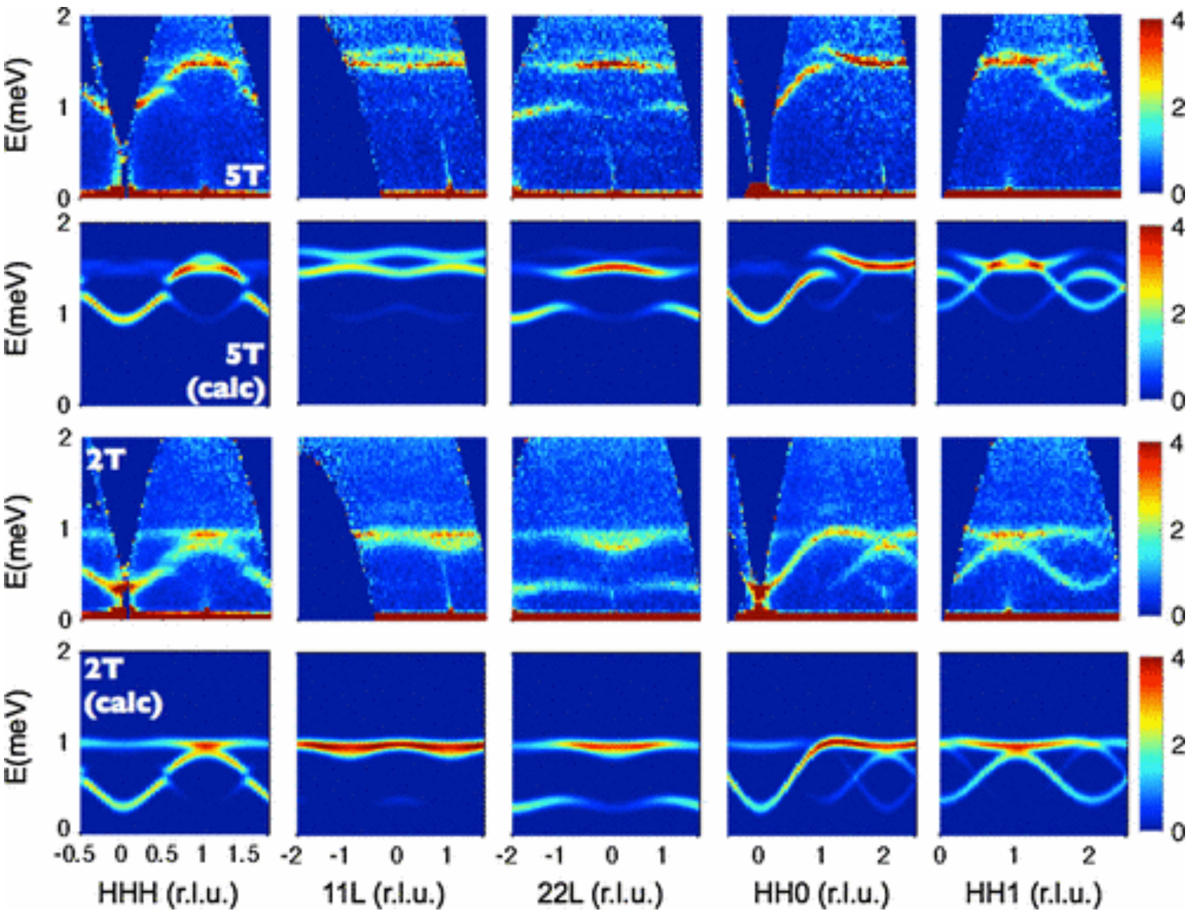
Cite Share

Bruce Gaulin Canadian Association of Physicists

April 22, 2010 | **Talk number:** PIRSA:10040083 | **DOI:** 10.48660/10040083

Collection: 4-Corner Southwest Ontario Condensed Matter Symposium 2010

Talk Type: Conference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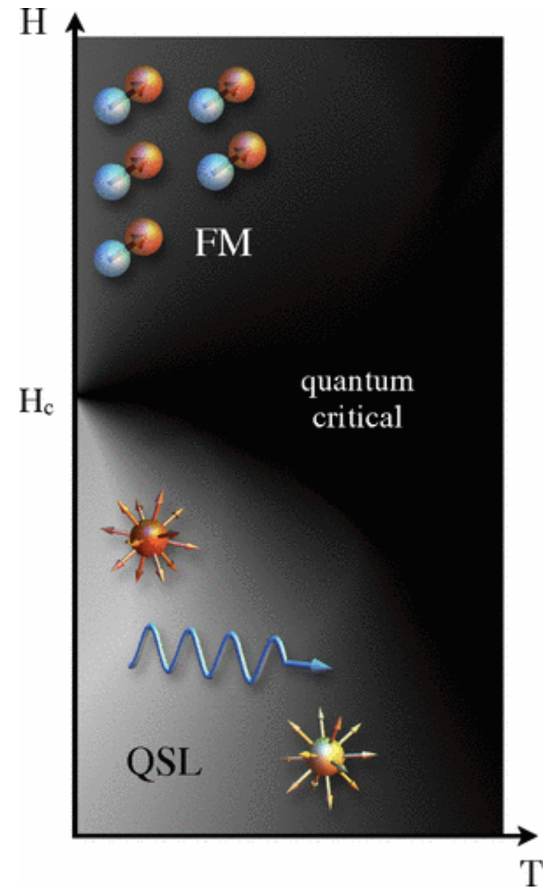
HH1 (r.l.u.)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PHYSICAL REVIEW X **1**, 021002 (2011)

Quantum Excitations in Quantum Spin Ice

Kate A. Ross,[1] Lucile Savary,[2] Bruce D. Gaulin,[1,3,4] and Leon Balents[5,*]
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(Received 22 July 2011; published 3 October 2011)

PRL **109,** 167201 (2012) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2012

Order by Quantum Disorder in $Er_2Ti_2O_7$

Lucile Savary,[1] Kate A. Ross,[2] Bruce D. Gaulin,[2,3,4] Jacob P. C. Ruff,[2,5] and Leon Balents[6]
[1]*Department of Physics, University of California, Santa Barbara, California 93106-9530, USA*
[2]*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada*
[3]*Canadian Institute for Advanced Research, Toronto, Ontario M5G 1Z8, Canada*
[4]*Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario L8S 4M1, Canada*
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(Received 5 April 2012; published 15 October 2012)

PRL **108,** 037202 (2012) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2012

Coulombic Quantum Liquids in Spin-1/2 Pyrochlores

Lucile Savary[1,2] and Leon Balents[3]
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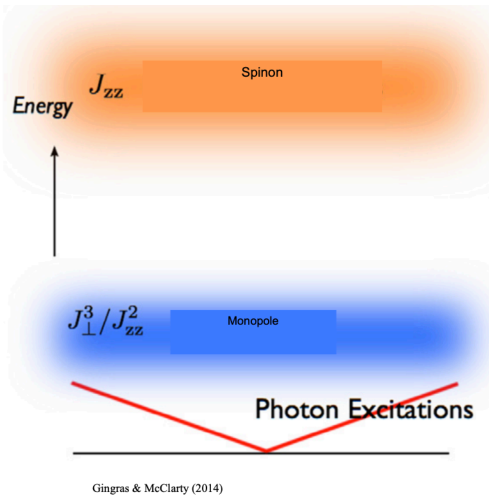
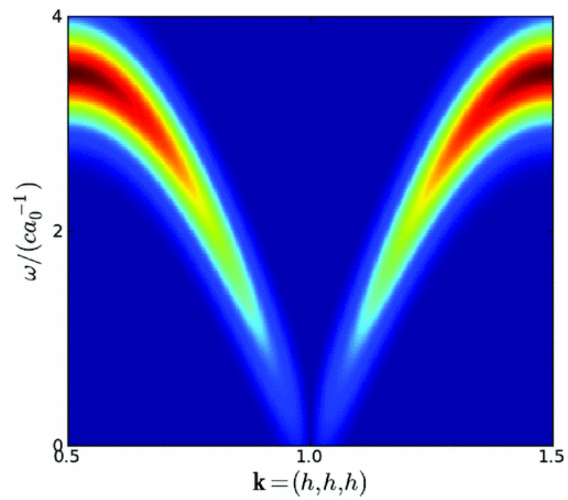
(Received 10 October 2011; published 19 January 2012)



M. Hermele *et al*, 2004



O. Benton *et al*, 2012

Holy grail for quantum spin ice: the emergent photon

ANNUAL REVIEWS

Annual Review of Condensed Matter Physics

Experimental Insights into Quantum Spin Ice Physics in Dipole–Octupole Pyrochlore Magnets

Evan M. Smith,[1] Elsa Lhotel,[2] Sylvain Petit,[3] and Bruce D. Gaulin[1,4,5]

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[2] Institut Néel, CNRS, Université Grenoble Alpes, Grenoble, France; email: elsa.lhotel@neel.cnrs.fr

[3] LLB, CEA, CNRS, Université Paris-Saclay, CEA Saclay, Gif-sur-Yvette, France; email: sylvain.petit@cea.fr

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[5] Canadian Institute for Advanced Research, Toronto, Ontario, Canada

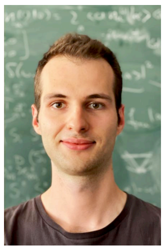
Annu. Rev. Condens. Matter Phys. 2025. 16:387–415

First published as a Review in Advance on December 3, 2024

Keywords

geometric frustration, pyrochlores, dipole–octupole pseudospins, quantum spin liquid, spin ice, neutron scattering

Collaborators



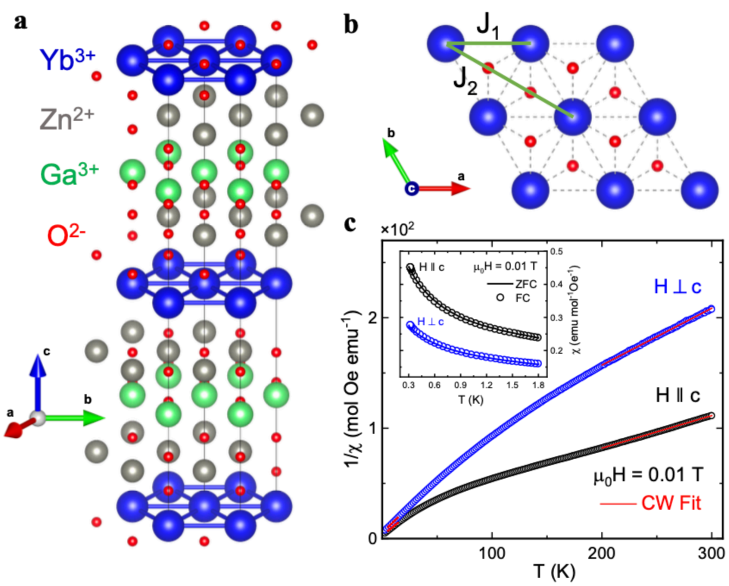
Urban Seifert
U. Köln

Oleg Starykh
U. Utah

Ziyang Meng
U. Hong Kong

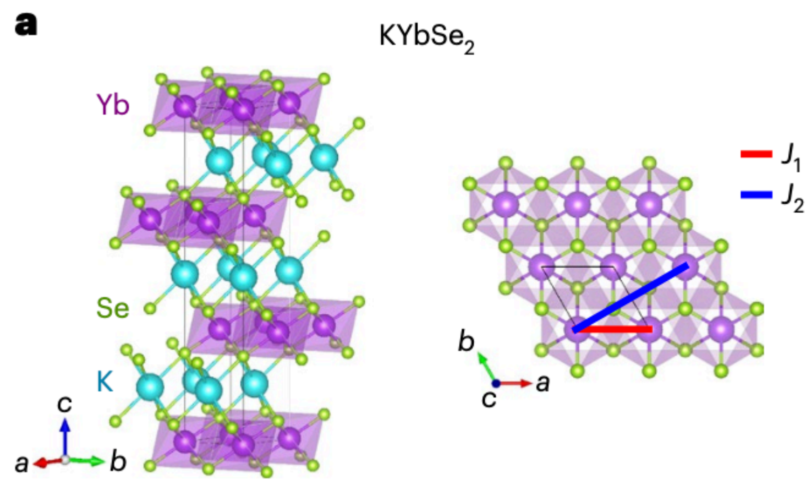
Wen Wang
KITP

Triangular lattice spin liquid



$YbZn_2GaO_5$

Haravifard group



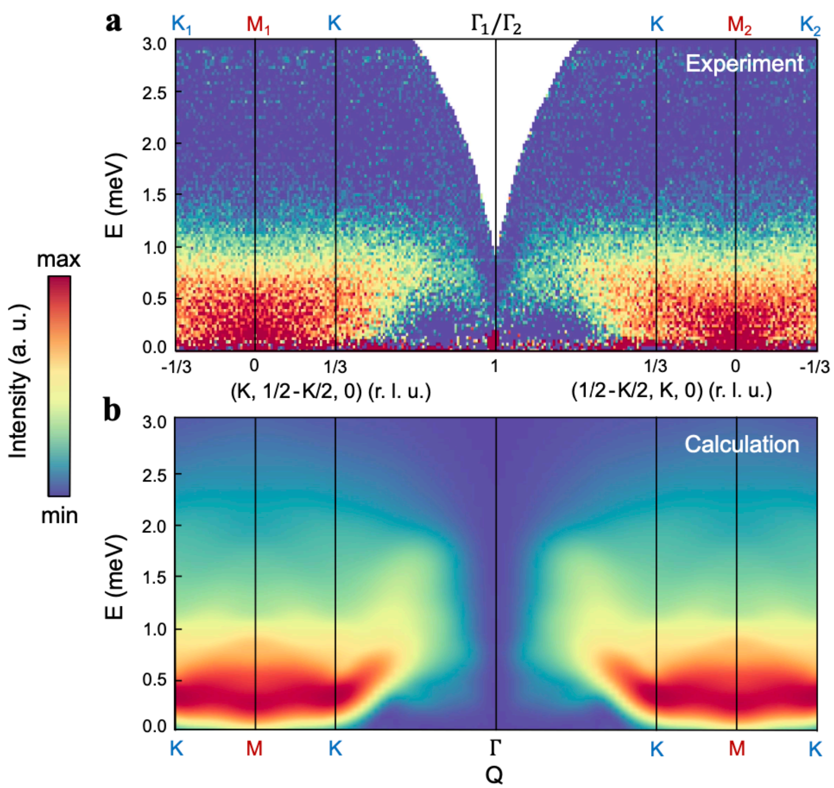
$KYbSe_2$

Tennant group

Triangular lattice spin liquid

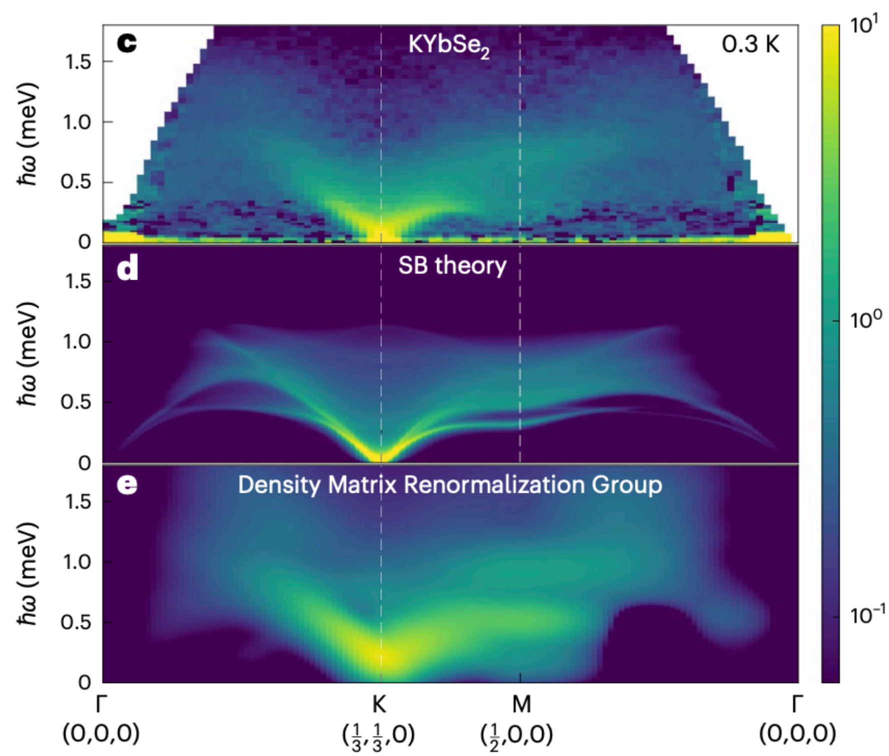
$YbZn_2GaO_5$

Haravifard group J_2/J_1=0.12

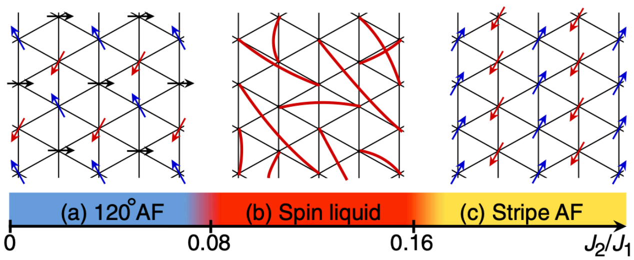


$KYbSe_2$

Tennant group J_2/J_1=0.05



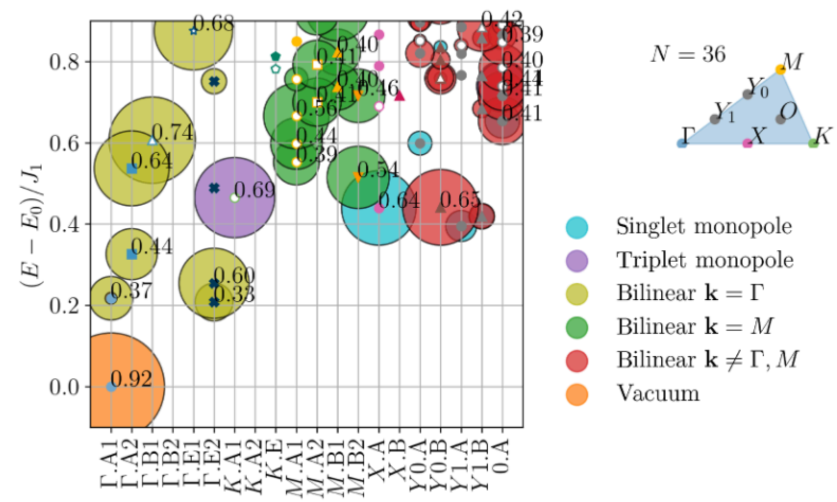
Triangular lattice spin liquid



$$\mathscr{L} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi + \cdots$$

Considerable support for U(1) Dirac spin liquid

- Y. Iqbal *et al*, VMC 2016
- S. Hu *et al*, DMRG 2019
- A. Wietek *et al*, ED **2024**



Matching of low-lying eigenstates with QED3 ones

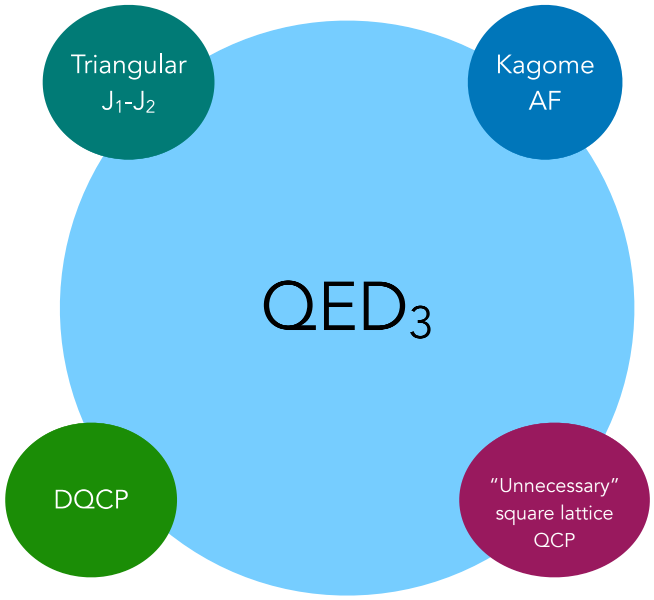
Spins and QED$_3$

$$\mathscr{L} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - i a_{\mu})\psi + \cdots$$

Each system has its own:

- Microscopic (exact) symmetries
- Operator dictionary
- Perturbations to CFT

X.-Y. Song et al, 2019

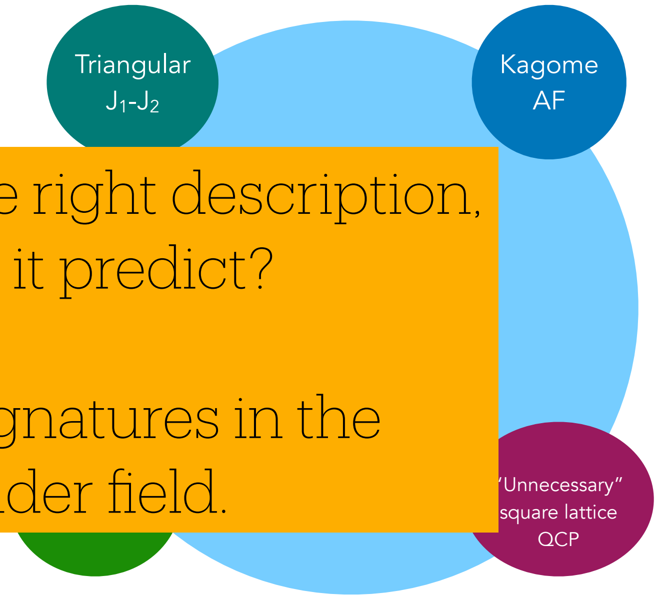


Spins and QED$_3$

$$\mathscr{L} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - i a_{\mu})\psi + \cdots$$

Each system has its own:

- Micro
  symm
- Oper
- Pertu

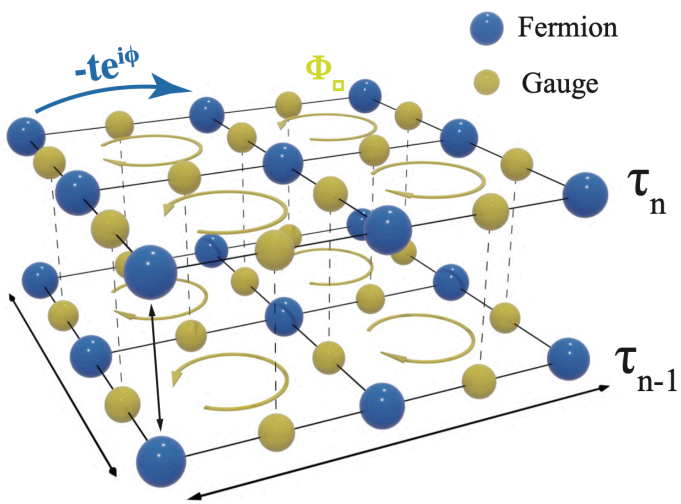


If we believe this is the right description, what else can it predict?

We will look for signatures in the behavior under field.

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem

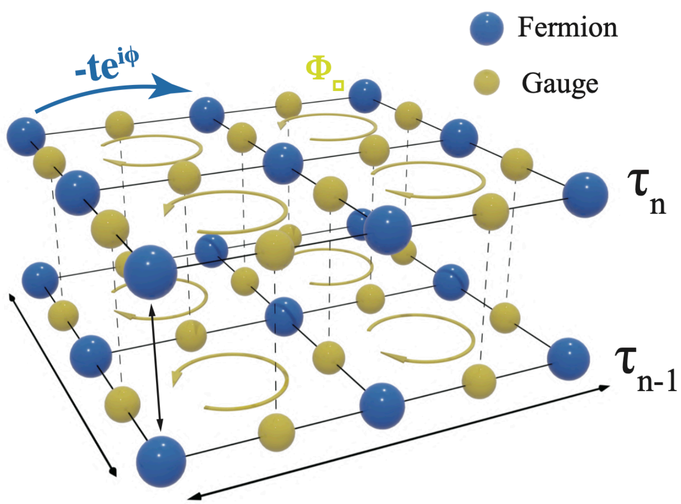


$$S = \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right]$$
$$- t \sum_{\langle ij \rangle, n} \left[e^{i a_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right]$$
$$+ \frac{1}{J} \sum_{\langle ij \rangle, n} \left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1}) \right]^2 \quad (1)$$

n.b. square lattice -
avoids sign problem.

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem

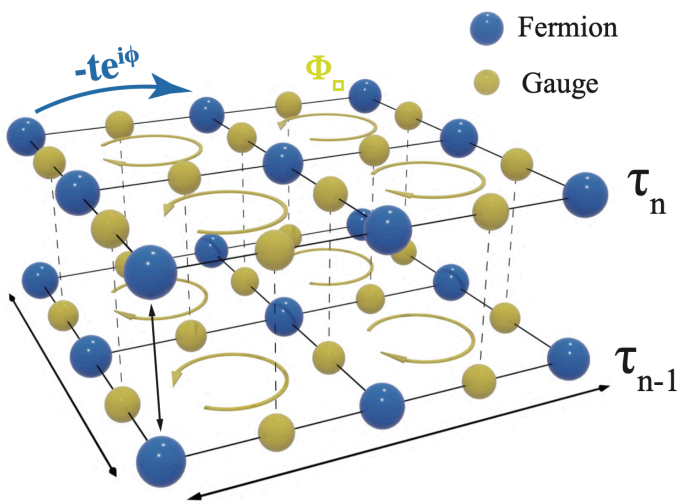


time-derivative

$$S = \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n)\sigma^z \psi_i(\tau_n) \right] - t \sum_{\langle ij \rangle, n} \left[e^{i a_{ij}(\tau_n)} \bar{\psi}_i(\tau_n)\psi_j(\tau_n) + \text{h.c.} \right] + \frac{1}{J} \sum_{\langle ij \rangle, n} \left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1}) \right]^2 \quad (1)$$

A model and quantum Monte Carlo

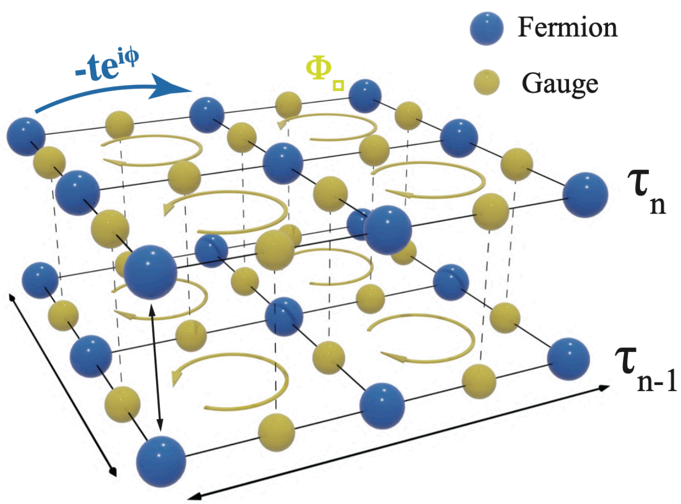
A lattice gauge theory — without a sign problem



$$S = \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right]$$
$$- t \sum_{\langle ij \rangle, n} \left[e^{i a_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \quad \text{Hopping}$$
$$+ \frac{1}{J} \sum_{\langle ij \rangle, n} \left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1}) \right]^2 \qquad (1)$$

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem

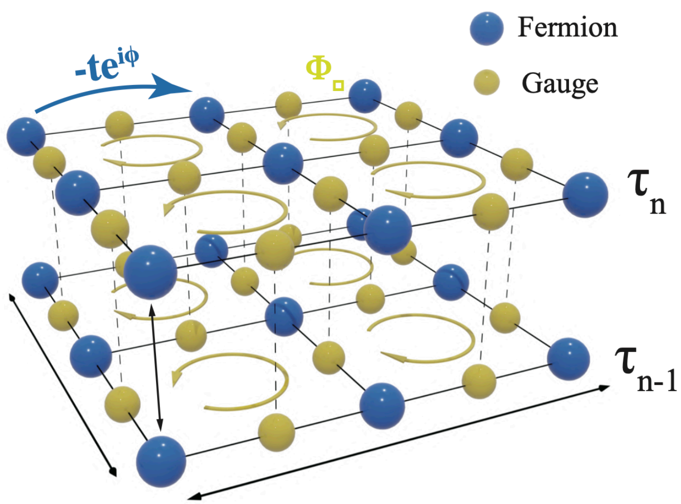


$$S = \sum_{i,n}\left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2}B\bar{\psi}_i(\tau_n)\sigma^z\psi_i(\tau_n)\right] - t\sum_{\langle ij\rangle,n}\left[e^{ia_{ij}(\tau_n)}\bar{\psi}_i(\tau_n)\psi_j(\tau_n) + \text{h.c.}\right] + \frac{1}{J}\sum_{\langle ij\rangle,n}\left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})\right]^2 \quad (1)$$

"Maxwell" term: controls gauge fluctuations

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem

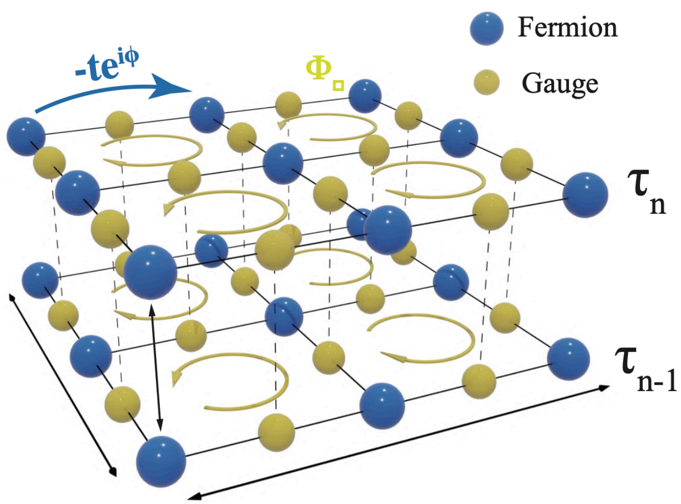


Zeeman field

$$S = \sum_{i,n}\left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2}B\bar{\psi}_i(\tau_n)\sigma^z\psi_i(\tau_n)\right]$$
$$- t\sum_{\langle ij\rangle,n}\left[e^{ia_{ij}(\tau_n)}\bar{\psi}_i(\tau_n)\psi_j(\tau_n) + \text{h.c.}\right]$$
$$+ \frac{1}{J}\sum_{\langle ij\rangle,n}\left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})\right]^2 \tag{1}$$

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem



$$S = \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right]$$
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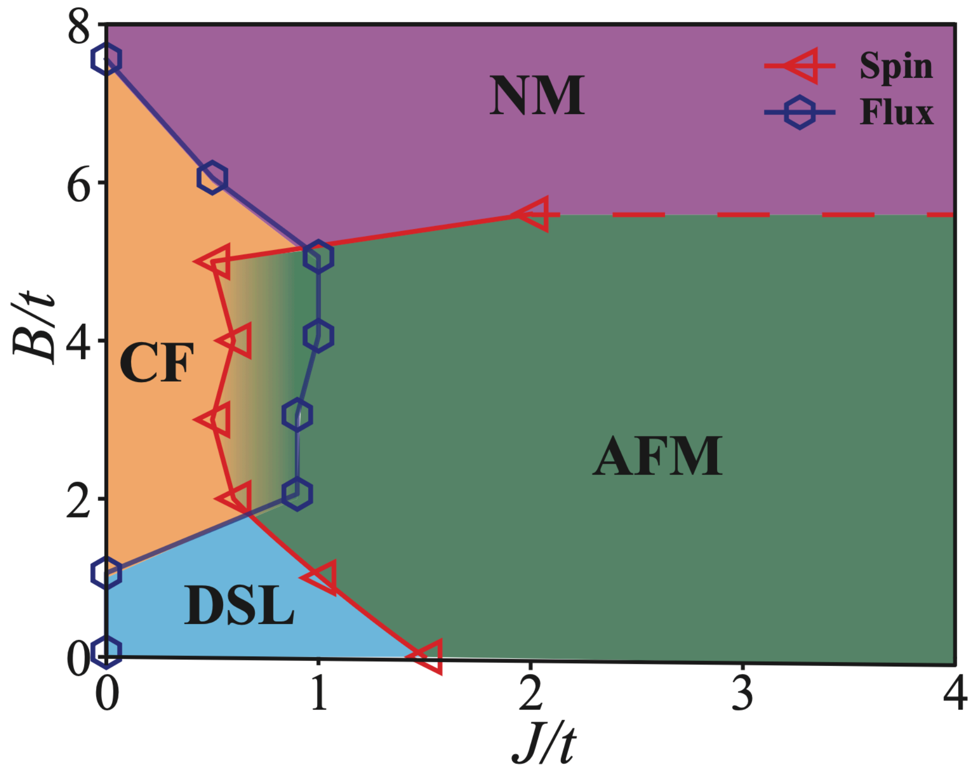
$$+ \frac{1}{J} \sum_{\langle ij \rangle, n} \left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1}) \right]^2 \quad (1)$$

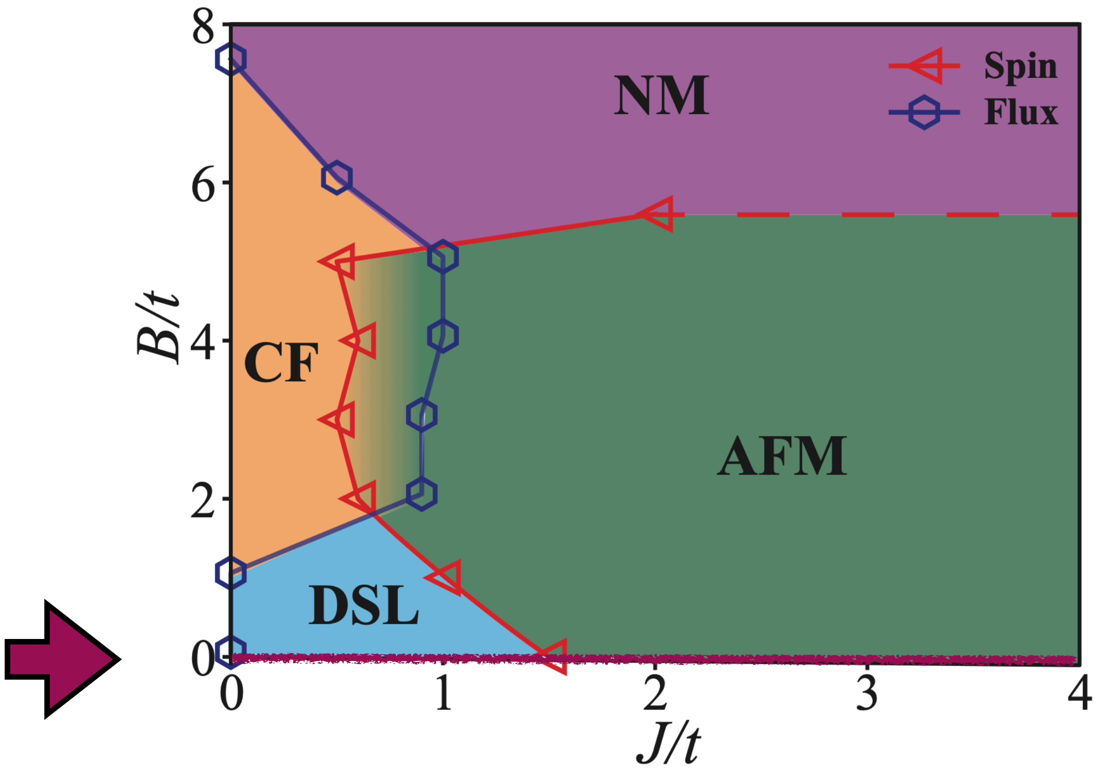
$J \rightarrow 0$, B=0: Lieb theorem guarantees π flux state, and hence Dirac fermions

Phase diagram

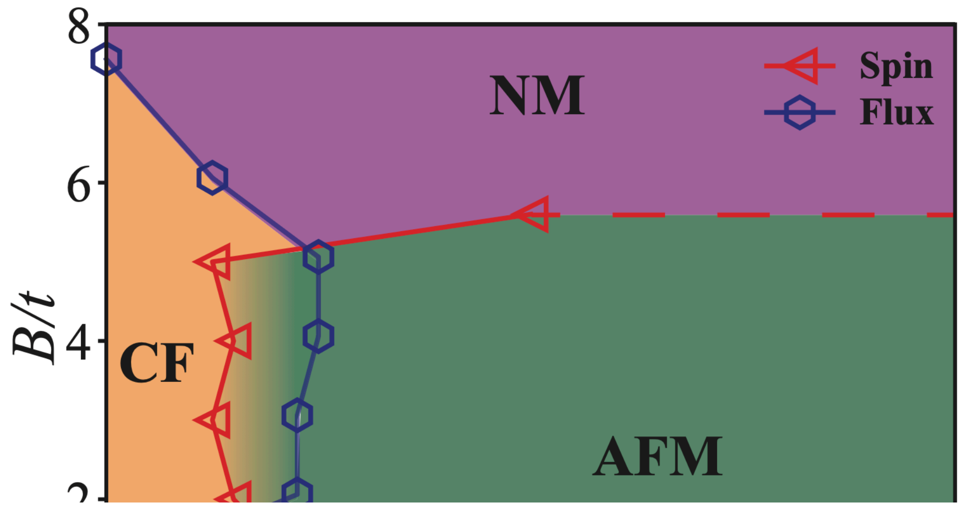


Phase diagram

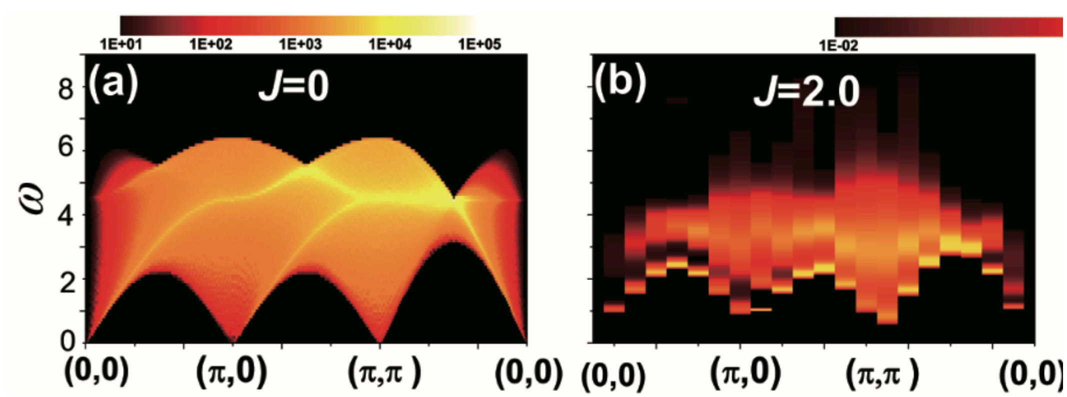
Studied earlier
X.-Y. Xu *et al*, 2019



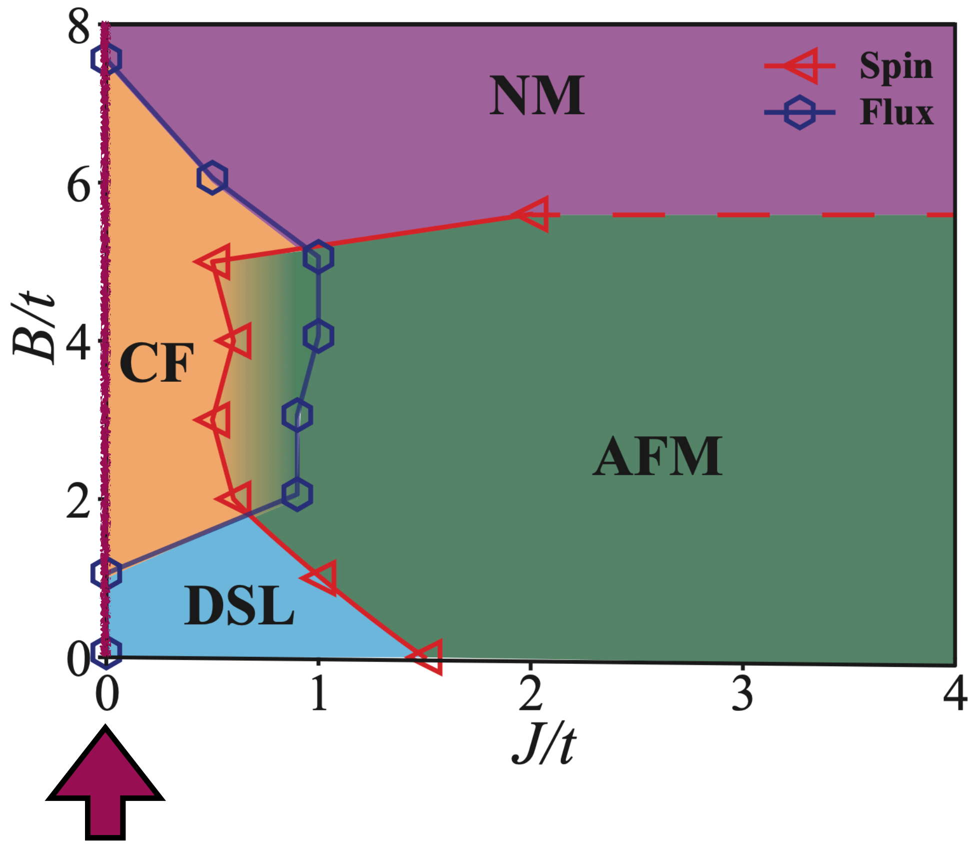
Phase diagram



W. Wang *et al*, 2019



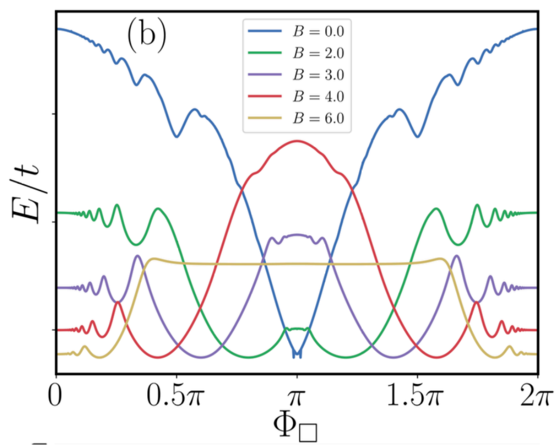
Phase diagram



Line of no gauge fluctuations: but there is an *average* gauge field

Energetics

- At J=0, the problem is equivalent to free fermions with a magnetic flux chosen to minimize the total energy

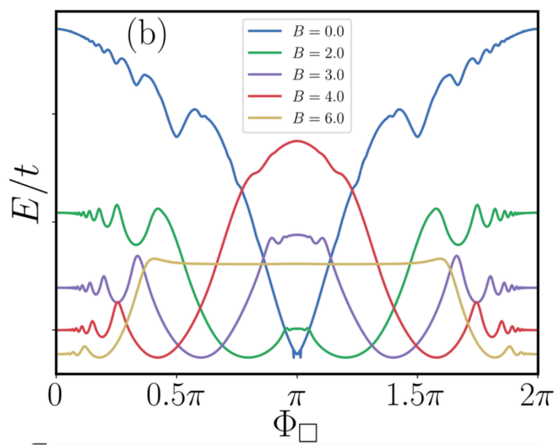


Optimal flux deviates from π when B>0

Double minimum: spontaneous chirality

Energetics

- The chiral flux persists for small J

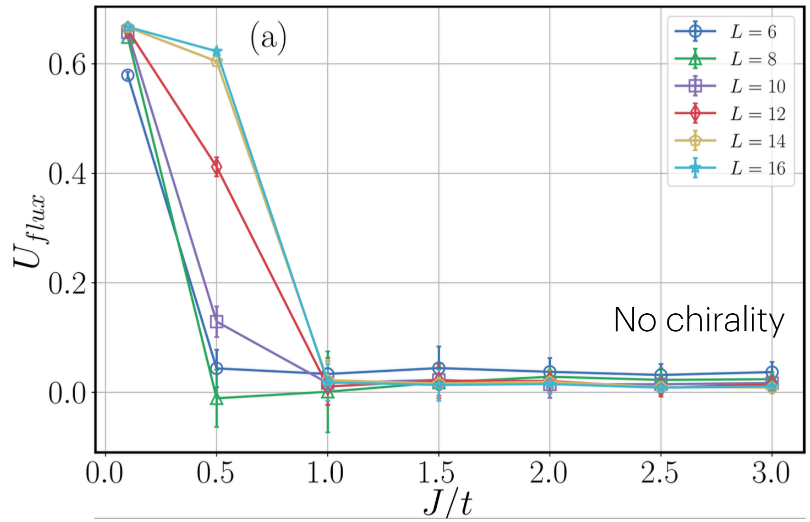


Optimal flux deviates from π when B>0

Double minimum: spontaneous chirality

J>0

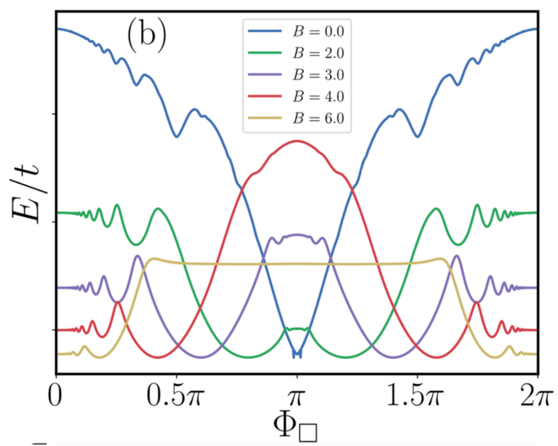
Spontaneous chirality



Binder cumulant

Energetics

- The chiral flux persists for small J



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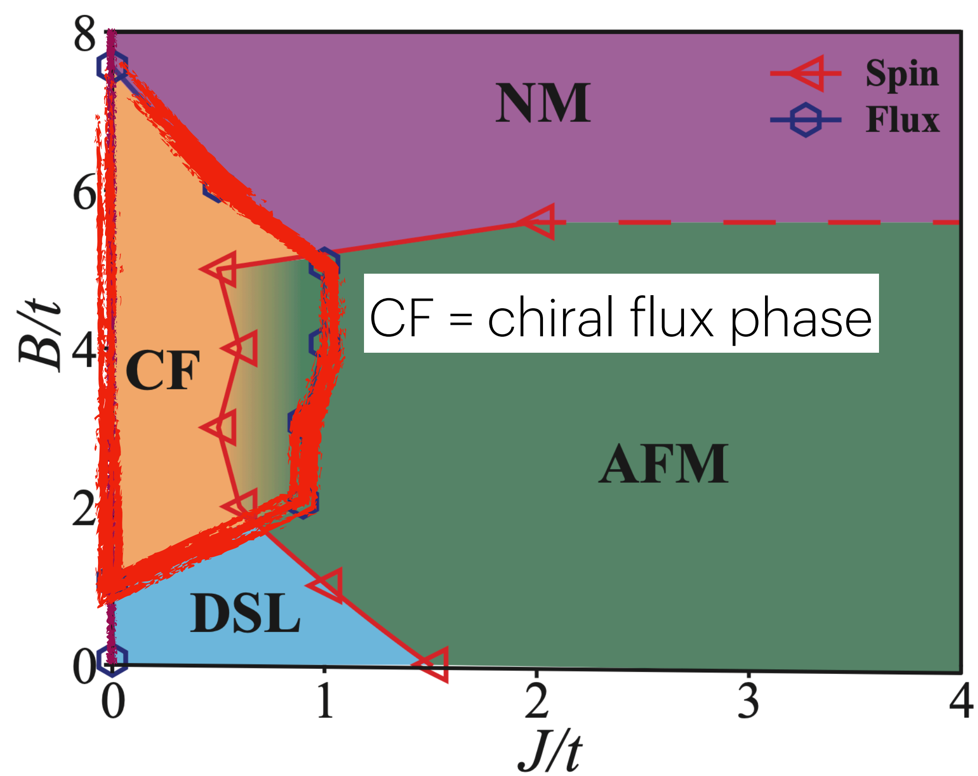
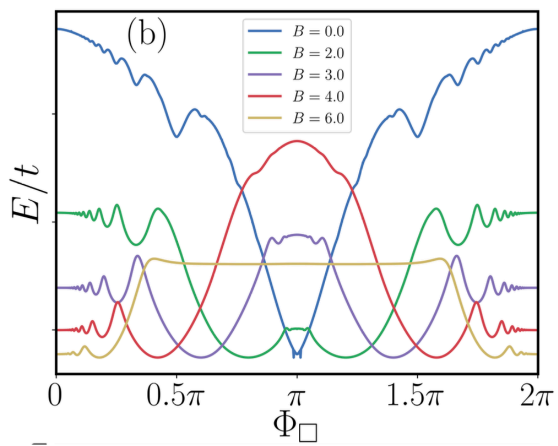


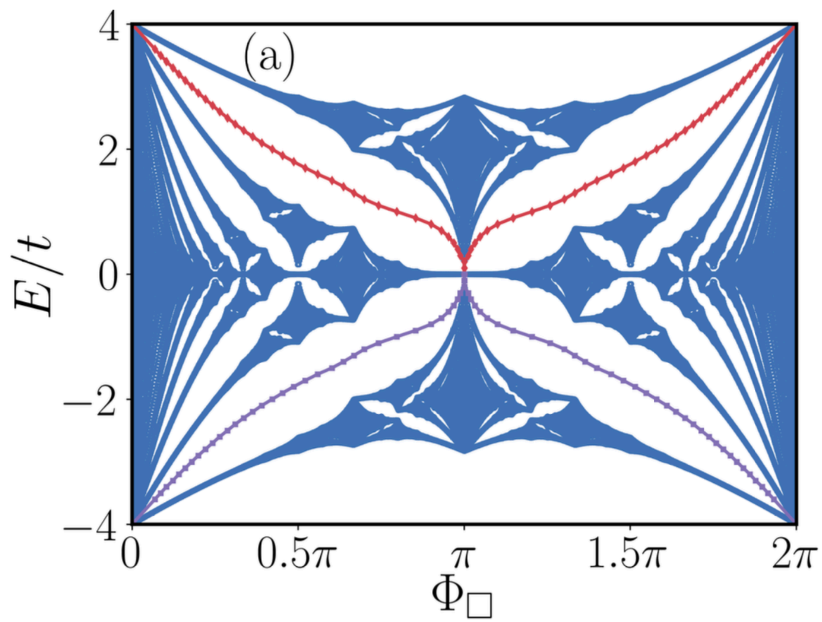
Fermion states

- The chiral flux induces a complex set of Hofstadter bands, similar to Landau levels



Optimal flux deviates from $\boldsymbol{\pi}$ when B>0

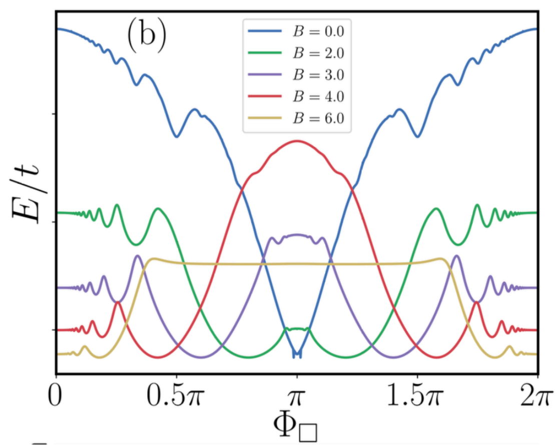
Double minimum: spontaneous chirality



Hofstadter butterfly

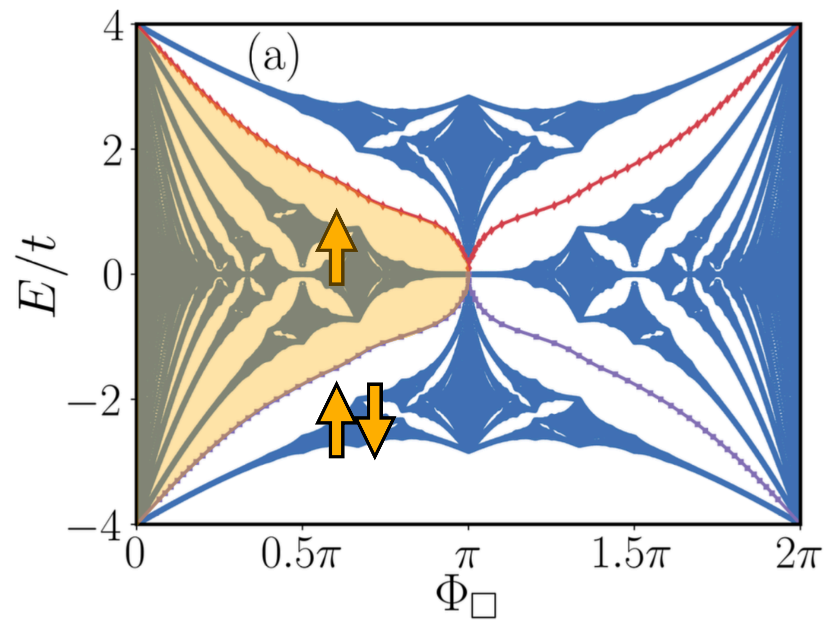
Fermion states

- The chiral flux induces a complex set of Hofstadter bands, similar to Landau levels



Optimal flux deviates from $\boldsymbol{\pi}$ when B>0

Double minimum: spontaneous chirality



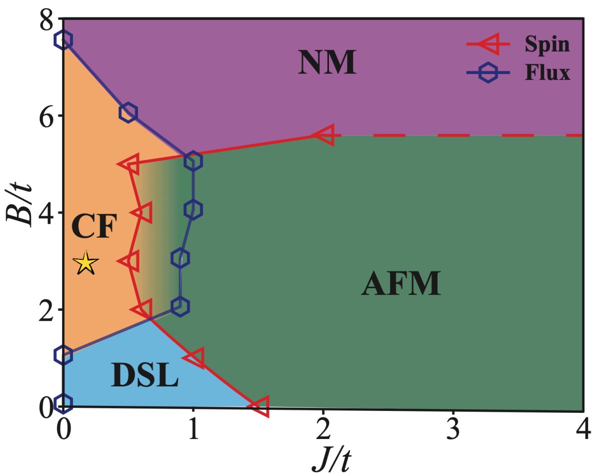
m_s

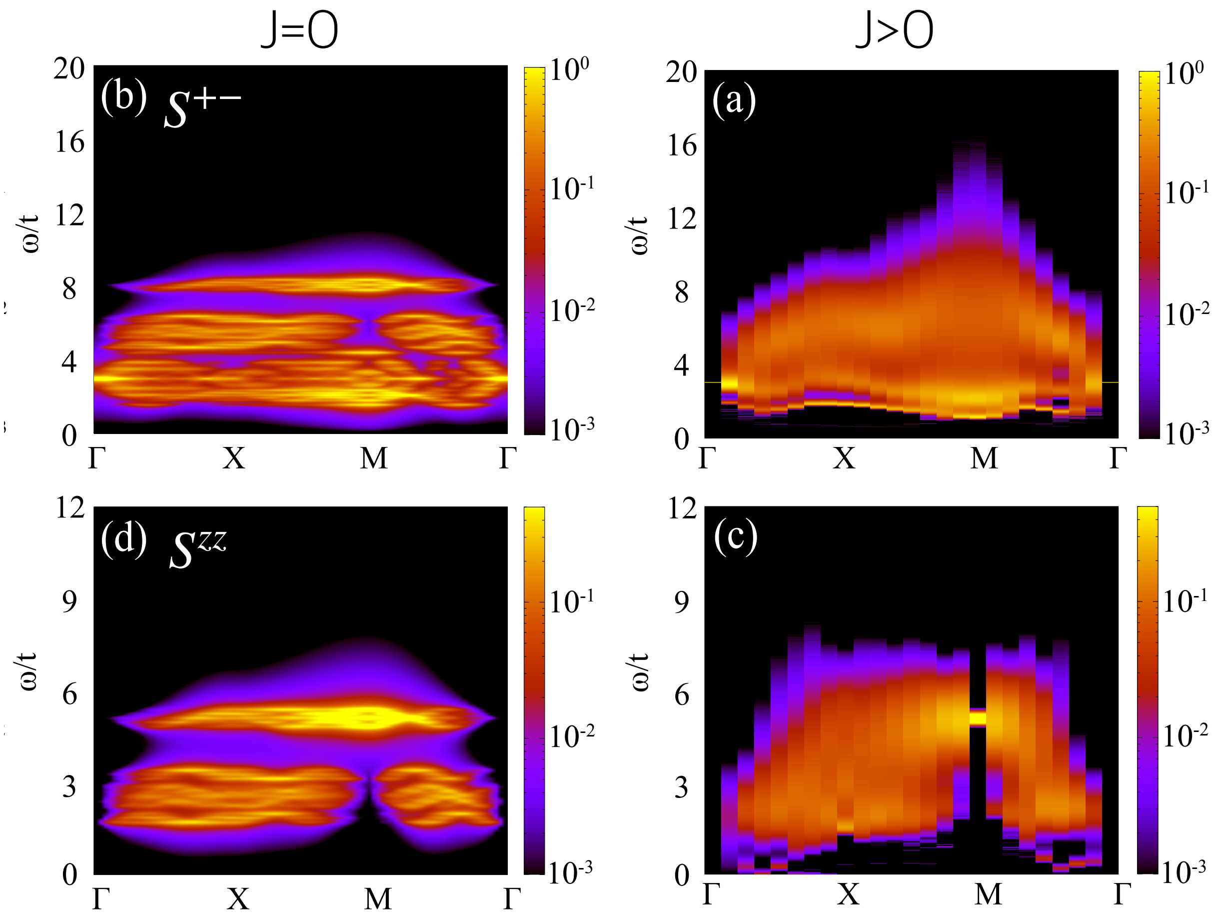
0

m

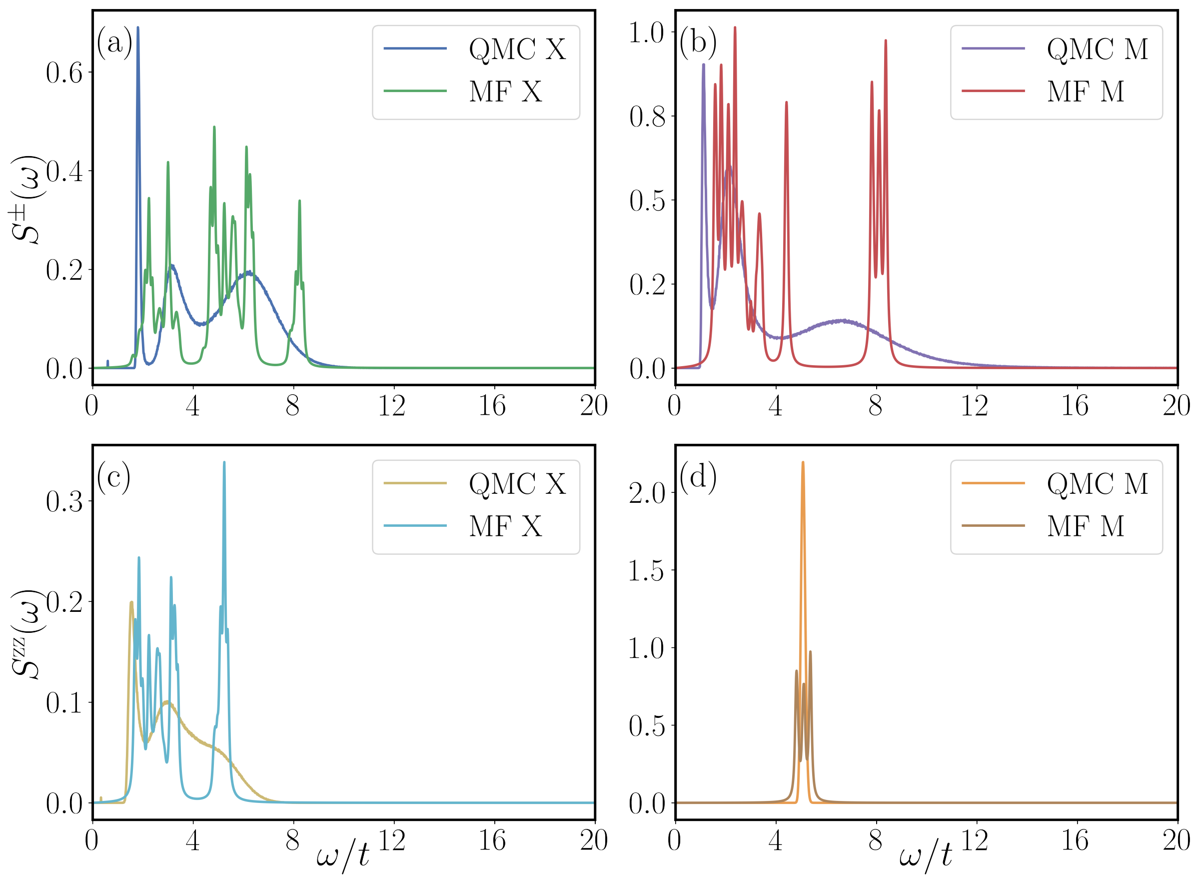
Dynamical correlations

"Landau level"-like features





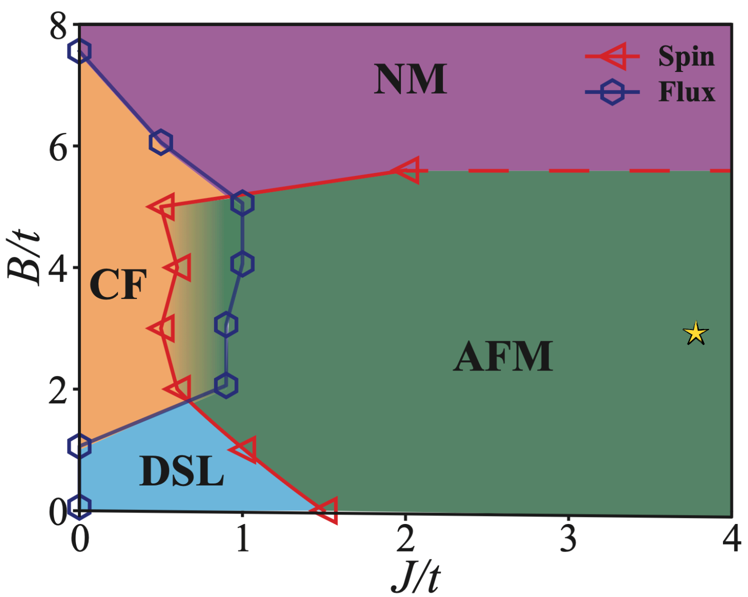
Dynamical correlations

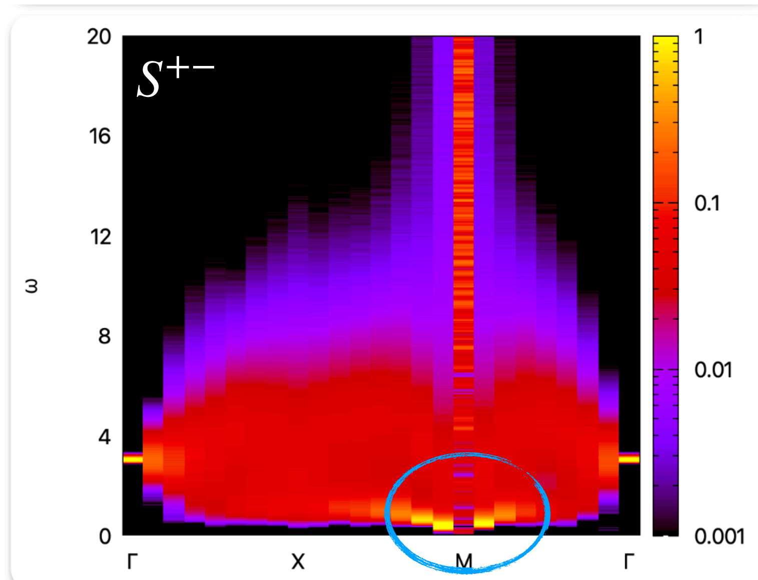


Peaks do not correspond to simple "spin wave" or "triplon" mode counting.

Hofstadter bands give a good guide to intensity even with gauge fluctuations

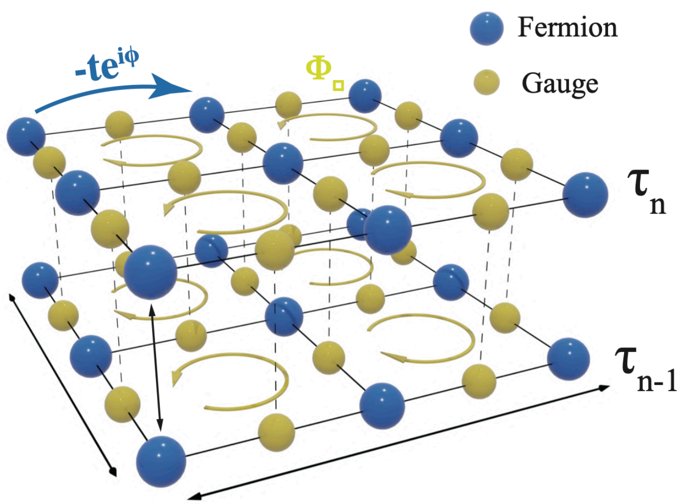
Comparison with AF phase





Emergent spin wave

Compact versus non-compact



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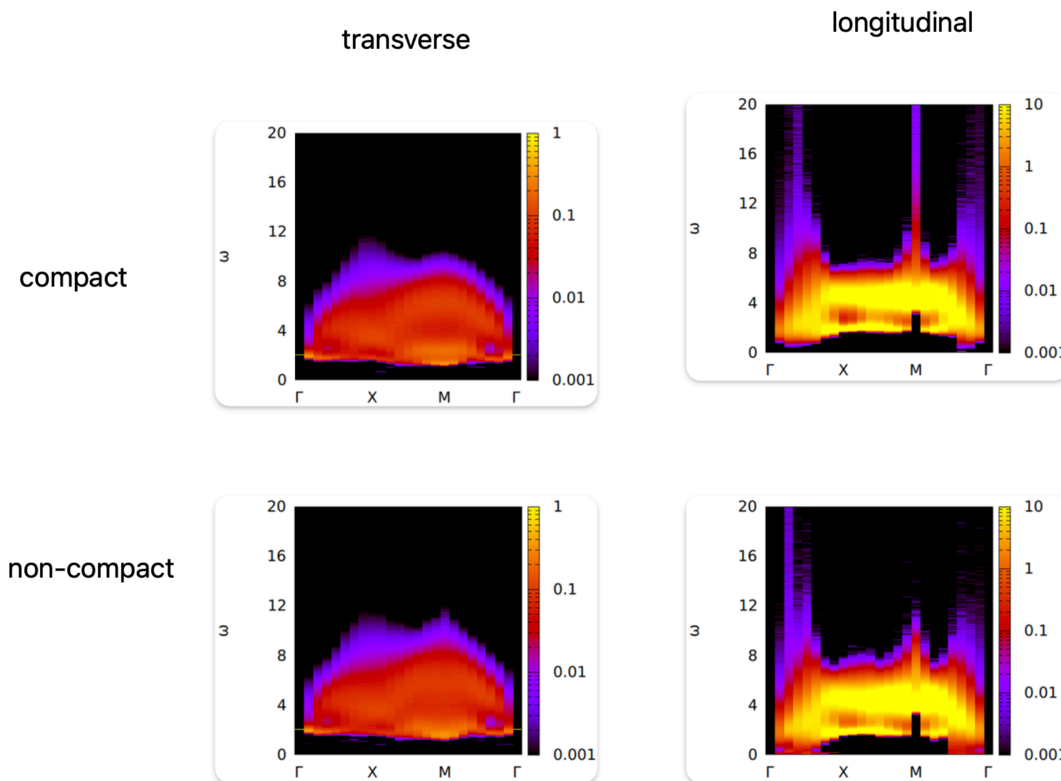
$$- t \sum_{\langle ij \rangle, n} \left[e^{i a_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right]$$
$$+ \frac{1}{J} \sum_{\langle ij \rangle, n} \left[a_{ij}(\tau_n) - a_{ij}(\tau_{n-1}) \right]^2 \quad (1)$$

"Non-compact" gauge field: prohibits "monopoles" in the simulation

Proper model is "compact": what are the corrections?

Compact versus non-compact

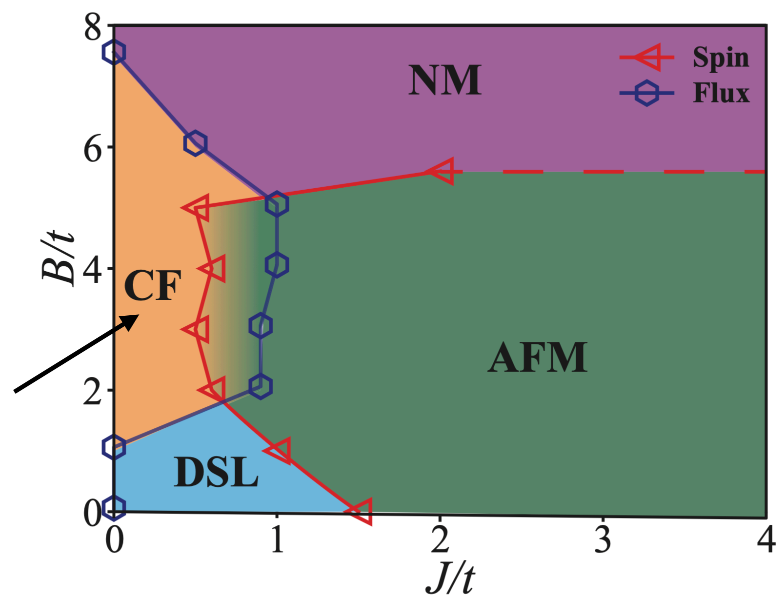
Difficult to see difference visually



Compact versus non-compact

Theoretical arguments:

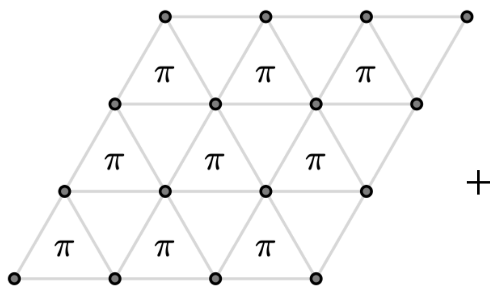
This region should be weakly magnetically ordered



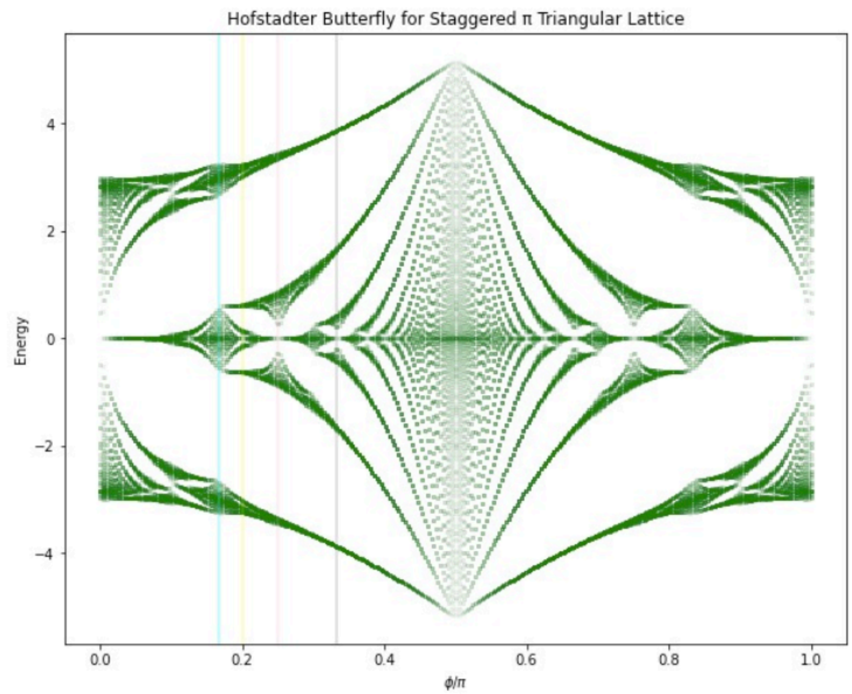
Wavefunction study

Not restricted by sign problem

triangular lattice

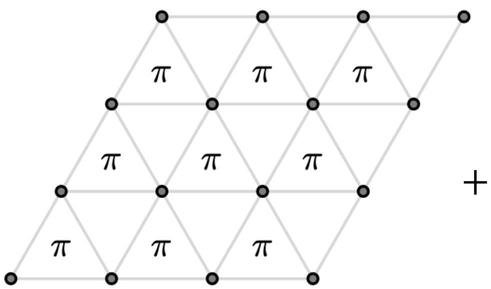


+ uniform flux



Wavefunction study

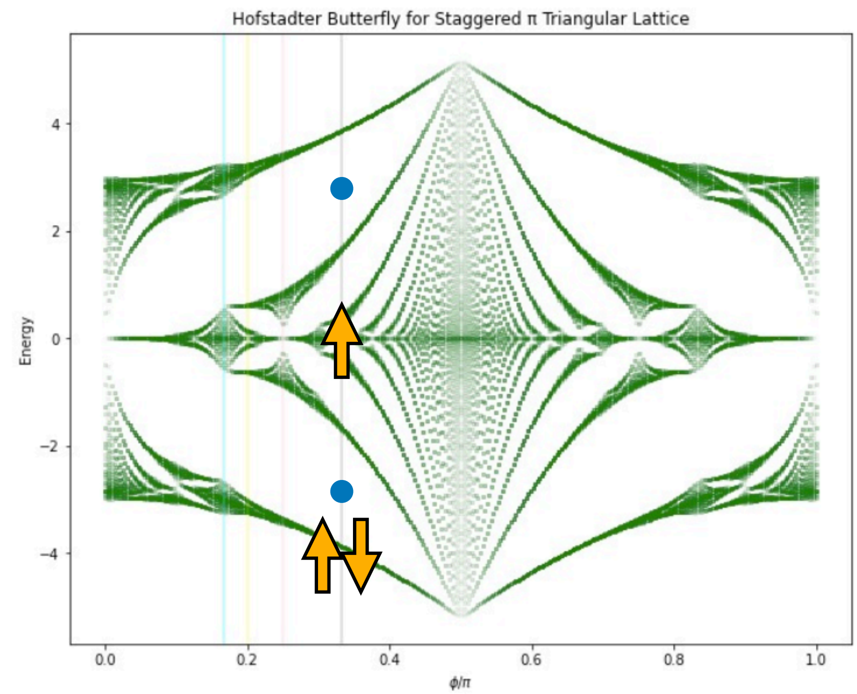
triangular lattice



+ uniform flux

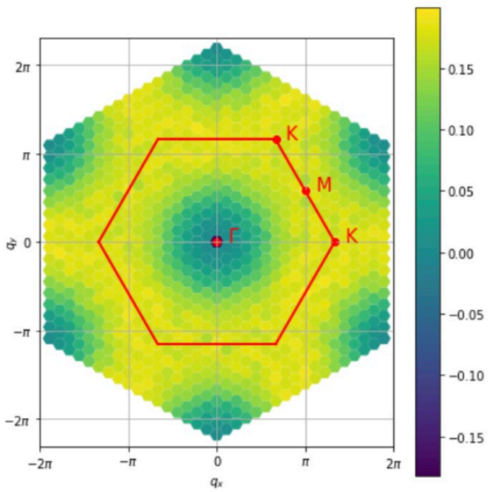
Gutzwiller projection

$M = 2/3\ M_S$

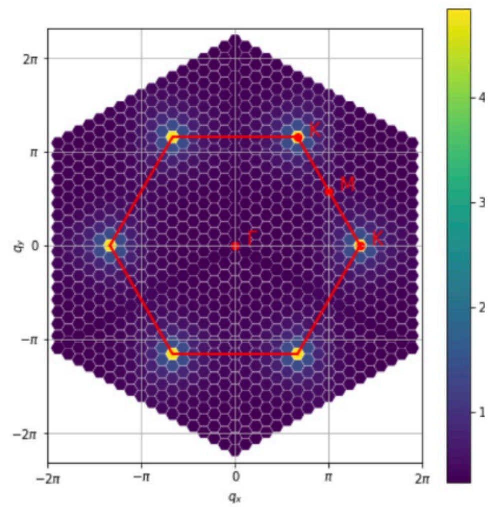


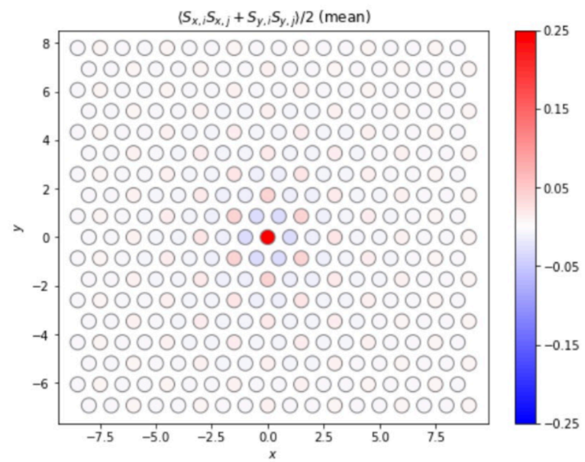
Wavefunction study

S^{zz}



S^{+-}

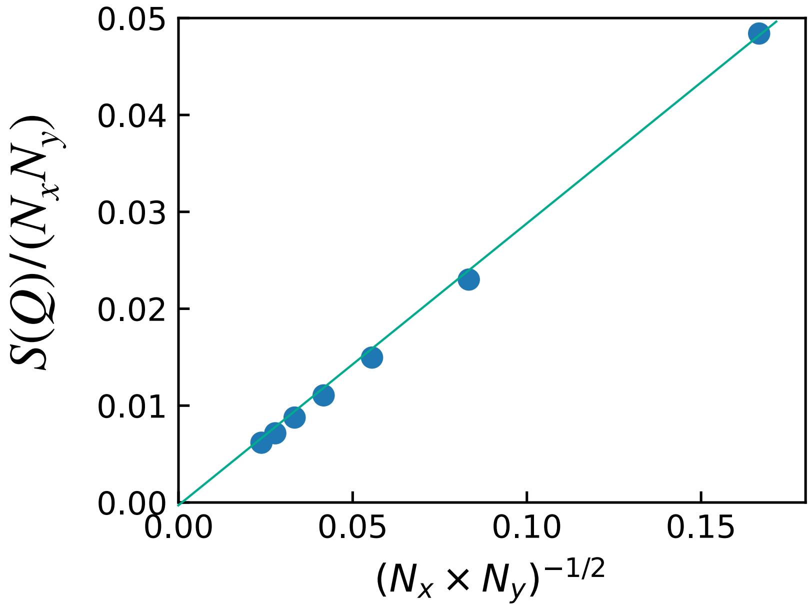




Ordered?

Wavefunction study

Néel order parameter



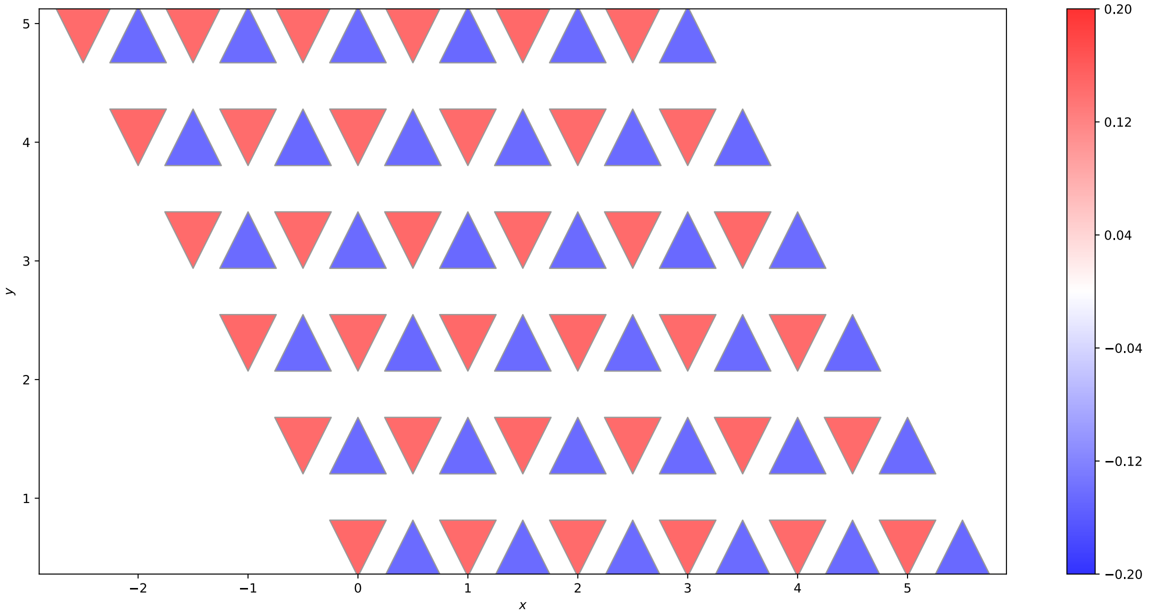
$$\langle S_i^+ S_j^- \rangle \sim \frac{e^{i\mathbf{K}\cdot(x_i - x_j)}}{|x_i - x_j|}$$

Power-law order

Stay tuned!

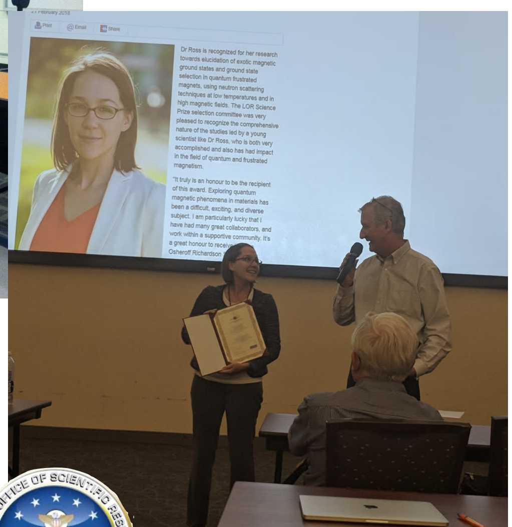
Wavefunction study

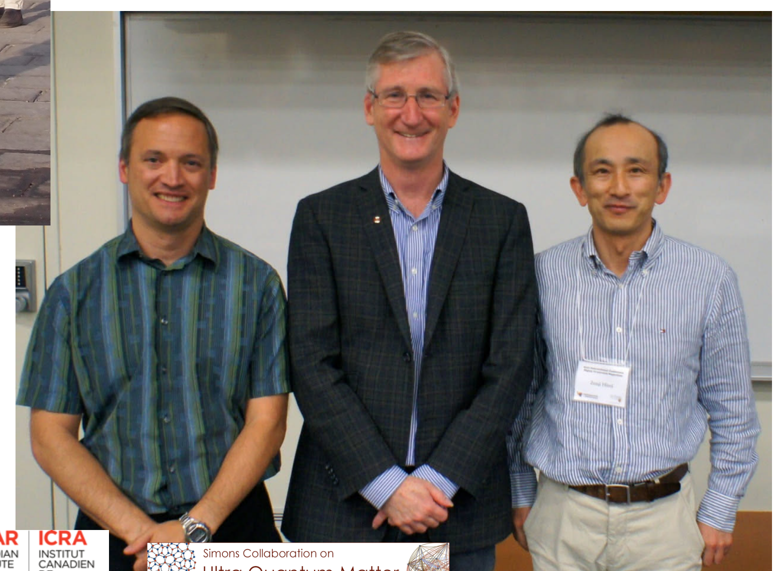
$$\langle S_i \cdot S_j \times S_k \rangle$$



Long-range chiral order

Thanks for letting me celebrate with you





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Simons Collaboration on Ultra-Quantum Matter