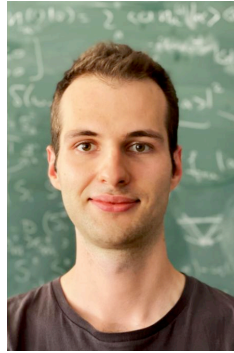


Leon Balents, Landau Week June 2025, Yerevan

Symmetry breaking patterns of 2+1d U(1)
gauge theory with an SU(N) chemical
potential and application to quantum
antiferromagnets



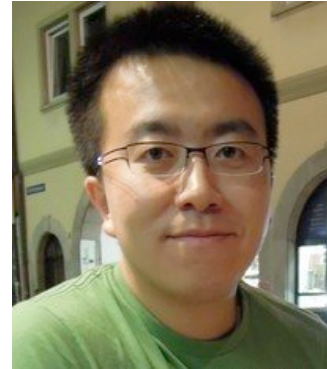
Collaborators



Urban Seifert
U. Köln



Oleg Starykh
U. Utah



Ziyang Meng
U. Hong Kong



Wen Wang
KITP

Quantum spin liquids

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \xrightarrow{\quad ? \quad} \text{non-ordered state with fractional excitations}$$

1941(!)

THE THERMAL CONDUCTIVITY OF THE PARAMAGNETIC DIELECTRICS AT
LOW TEMPERATURES

By I. POMERANCHUK

(Received October 25, 1940)

In the following we shall need the distribution function of the magnetic levels over the energies. When $T \gg \theta_k$ the spectrum is not degenerated and we can use the classical statistics (the Gibbs distribution) and expand the statistical sum in a power series of $1/T$ (cf. § 5). For $T \ll \theta_k$ it is necessary to know the statistics of the magnons. The experimental facts available suggest that the magnons are sub-

mitted to the Fermi statistics; namely, when $T \ll \theta_k$ the susceptibility tends to a constant limit, which is of the order of const/θ_k (*) [for $T > \theta_k$, $\chi = \text{const}/(T + \theta_k)$]. Evidently we have here to deal with the Pauli paramagnetism which can be directly obtained from the Fermi distribution. Therefore, we shall assume the Fermi statistics for the magnons*. Due to the Fermi

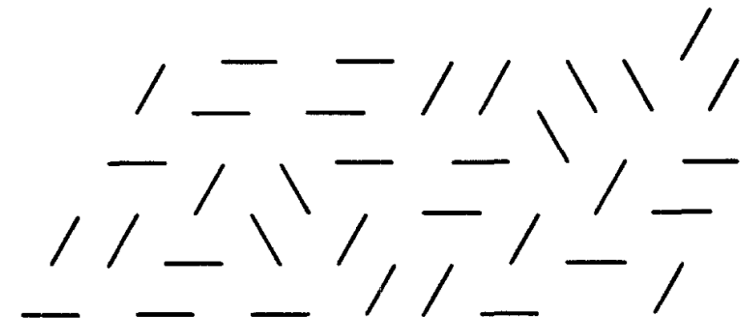
Quantum spin liquids

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \xrightarrow{\quad ? \quad} \text{non-ordered state with fractional excitations}$$

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

1973

P. W. Anderson
 Bell Laboratories, Murray Hill, New Jersey 07974
 and
 Cavendish Laboratory, Cambridge, England



Next excitations: In two dimensions, it is not at all clear that the two spins in a singlet necessarily ever separate by any appreciable distance, in which case there may be an energy gap to the lowest triplet excitation, so that the state need be only weakly paramagnetic if at all. But especially if it is a

Quantum spin liquids

Fermions and gauge fields

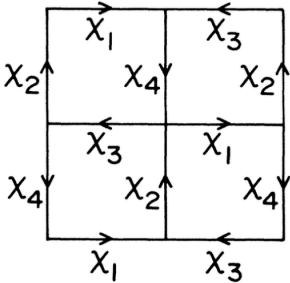
1988

PHYSICAL REVIEW B VOLUME 37, NUMBER 7 1 MARCH 1988

Large- n limit of the Heisenberg-Hubbard model: Implications for high- T_c superconductors

Ian Affleck
Physics Department, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada

J. Brad Marston
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(Received 13 October 1987)



$$E(\mathbf{k}) = \pm |2\chi| [\cos^2 k_x + \cos^2 k_y]^{1/2}.$$

1989

PHYSICAL REVIEW B VOLUME 39, NUMBER 13 15 JUNE 1989

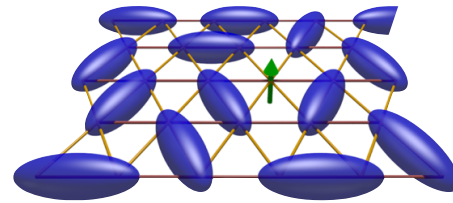
Gapless fermions and gauge fields in dielectrics

L. B. Ioffe and A. I. Larkin
Landau Institute for Theoretical Physics, 117940, Kosygina 2, Moscow V-334, U.S.S.R.
(Received 20 June 1988; revised manuscript received 28 November 1988)

$$\phi_{ij} = \psi_j - \psi_i + a_{ij}$$

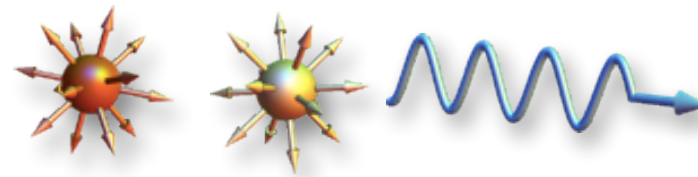
Quantum spin liquids

- Topological QSL



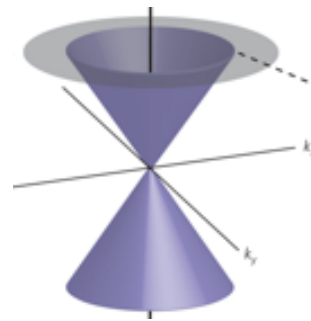
TQFT

- U(1) QSL



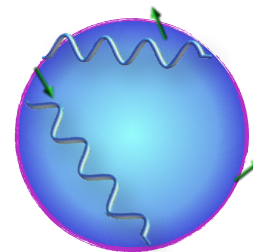
4d Maxwell

- Dirac QSLs



QED3

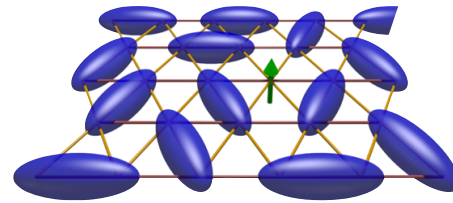
- Spinon Fermi surface



QED3 with
 $\mu \neq 0$

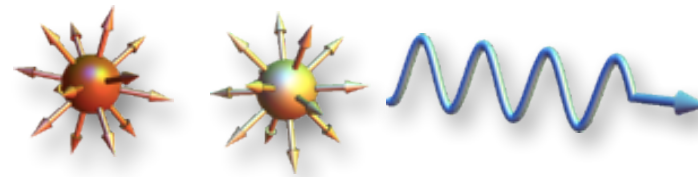
Quantum spin liquids

- Topological QSL



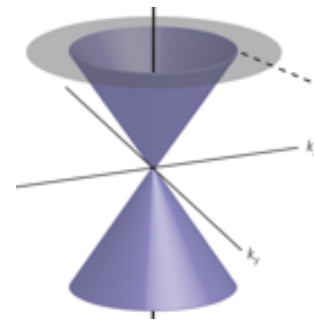
TQFT

- U(1) QSL



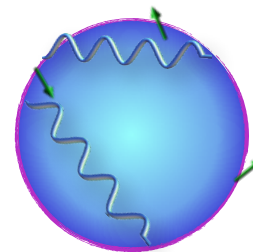
4d Maxwell

- Dirac QSLs



QED3

- Spinon Fermi surface



QED3 with
 $\mu \neq 0$

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site (S=0)

$$|\Psi_0\rangle = \prod_{k \in FS} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$

"partons"
"spinons"

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow & \updownarrow & \downarrow \\ \hline \downarrow & \downarrow & \updownarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \updownarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & & \downarrow \\ \hline \downarrow & & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \updownarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Gutzwiller Construction

- Project out any components with empty or doubly occupied sites

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

"partons"
"spinons"

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow\downarrow & \downarrow & \\ \hline \downarrow & \downarrow & \uparrow\downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow\downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow\downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Gutzwiller Construction

- Can build many QSL states by choosing different free fermion states

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

"partons"
"spinons"

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow\downarrow & \downarrow & \\ \hline \downarrow & \downarrow & \uparrow\downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow\downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow\downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Dirac spin liquid (1988)

Affleck+Marston flux phase

2007

Projected-Wave-Function Study of the Spin-1/2 Heisenberg Model on the Kagomé Lattice

Ying Ran, Michael Hermele, Patrick A. Lee, and Xiao-Gang Wen
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 21 November 2006; published 16 March 2007)

$$S = \int dx^3 \left[\frac{1}{g^2} (\varepsilon_{\lambda\mu\nu} \partial_\mu a_\nu)^2 + \sum_\sigma \bar{\psi}_{+\sigma} (\partial_\mu - i a_\mu) \tau_\mu \psi_{+\sigma} + \sum_\sigma \bar{\psi}_{-\sigma} (\partial_\mu - i a_\mu) \tau_\mu \psi_{-\sigma} \right] + \cdots, \quad (3)$$

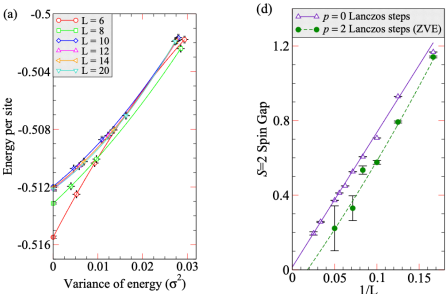
2016

PHYSICAL REVIEW B **93**, 144411 (2016)

Spin liquid nature in the Heisenberg J_1 - J_2 triangular antiferromagnet

Yasir Iqbal,^{1,*} Wen-Jun Hu,^{2,†} Ronny Thomale,^{1,‡} Didier Poilblanc,^{3,§} and Federico Becca^{4,||}

$$|\Psi_{\text{var}}\rangle = \mathcal{J}_z \mathcal{P}_G |\Phi_0\rangle$$
$$|\Psi_{p\text{-LS}}\rangle = \left(1 + \sum_{k=1}^p \alpha_k \mathcal{H}^k \right) |\Psi_{\text{var}}\rangle.$$

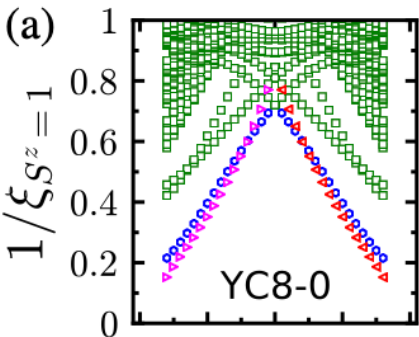


2019

Dirac Spin Liquid on the Spin-1/2 Triangular Heisenberg Antiferromagnet

Shijie Hu^{1,*}, W. Zhu,^{2,†} Sebastian Eggert,¹ and Yin-Chen He^{3,‡}
¹Department of Physics and Research Center Optimas, Technische Universität Kaiserslautern,
67663 Kaiserslautern, Germany

DMRG
w/ twist

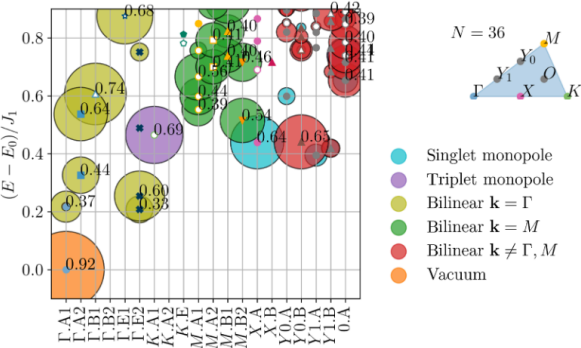


2024

Quantum Electrodynamics in 2+1 Dimensions as the Organizing Principle of a Triangular Lattice Antiferromagnet

Alexander Wietek^{1,2,*}, Sylvain Capponi³, and Andreas M. Läuchli^{4,5}

ED matched to
projected
wavefunctions



Dirac spin liquid

- Proposal: low energy effective field theory of 2+1-d Dirac fermions coupled to U(1) gauge field describes certain quantum antiferromagnets
- This field theory is *3 dimensional quantum electrodynamics*: QED3
- What is QED3 and how does it apply to physical systems?

QED3

- Lagrangian: N 2-component massless Dirac fermions w/ U(1) gauge field.

$$\mathcal{L} = \sum_{a=1}^N \bar{\psi}_a \gamma^\mu (\partial_\mu - i a_\mu) \psi_a + \frac{N}{4e^2} f_{\mu\nu}^2$$

- Symmetries:

SU(N)_f flavor :

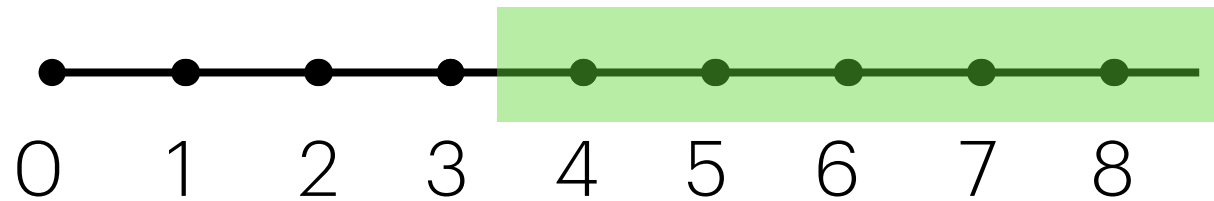
$$\psi_a \rightarrow U_{ab} \psi_b$$

U(1)_m magnetic/flux:

$$Q = \int d^2x f_{ij} \quad (j_\mu = \epsilon_{\mu\nu\lambda} f_{\nu\lambda})$$

QED3

- Analytical approach: $1/N$ expansion
- Conformal window: believed a CFT for $N > N_c$



- Gauge invariant/physical operators:
 - Conserved currents: $J_{ab}^{\mu} = \bar{\psi}_a \gamma^{\mu} \psi_b$
 - Fermion bilinear ("masses"): $M_{ab} = \bar{\psi}_a \psi_b$ (SU(N)_f scalar and adjoint)
 - Monopole operators: $\mathcal{M} \quad ??$

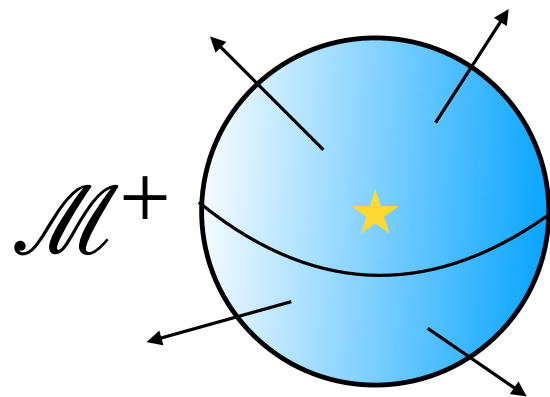
Monopole operators

Topological disorder operators in three-dimensional conformal field theory

Vadim Borokhov, Anton Kapustin and Xinkai Wu

2002

$$Q = \int d^2x f_{ij} \quad \text{Consider operators that are charged under } U(1)_m$$



$$Q\mathcal{M}^+ = \mathcal{M}^+(Q + 1)$$

After action of monopole, fermions experience one additional flux quantum

Monopole operators

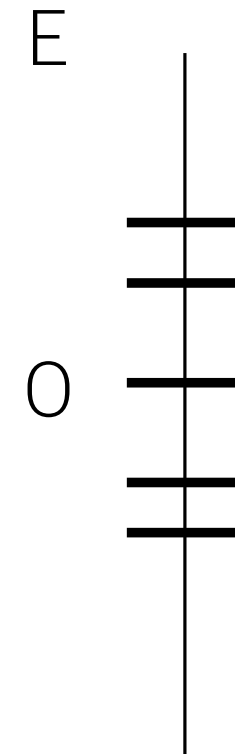
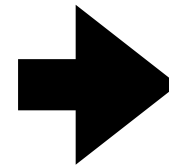
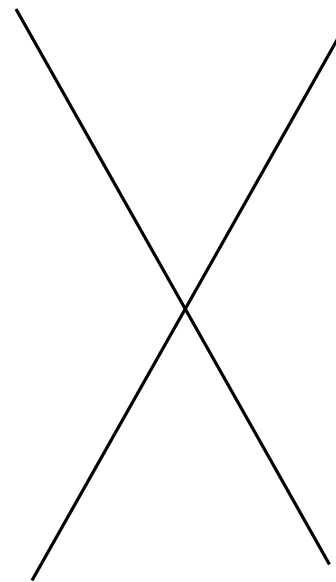
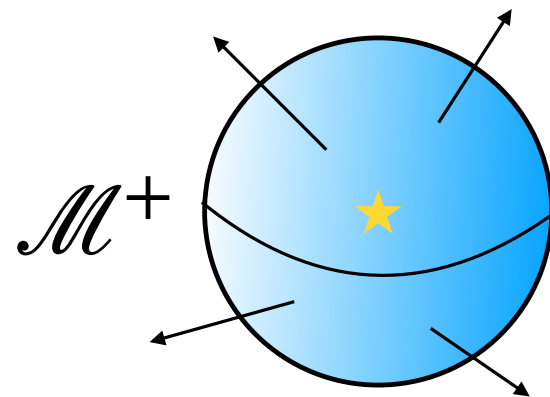
Topological disorder operators in three-dimensional conformal field theory

Vadim Borokhov, Anton Kapustin and Xinkai Wu

2002

$$Q = \int d^2x f_{ij}$$

Consider operators that are charged under $U(1)_m$



N^*q zero modes

Spectrum on the sphere

Monopole operators

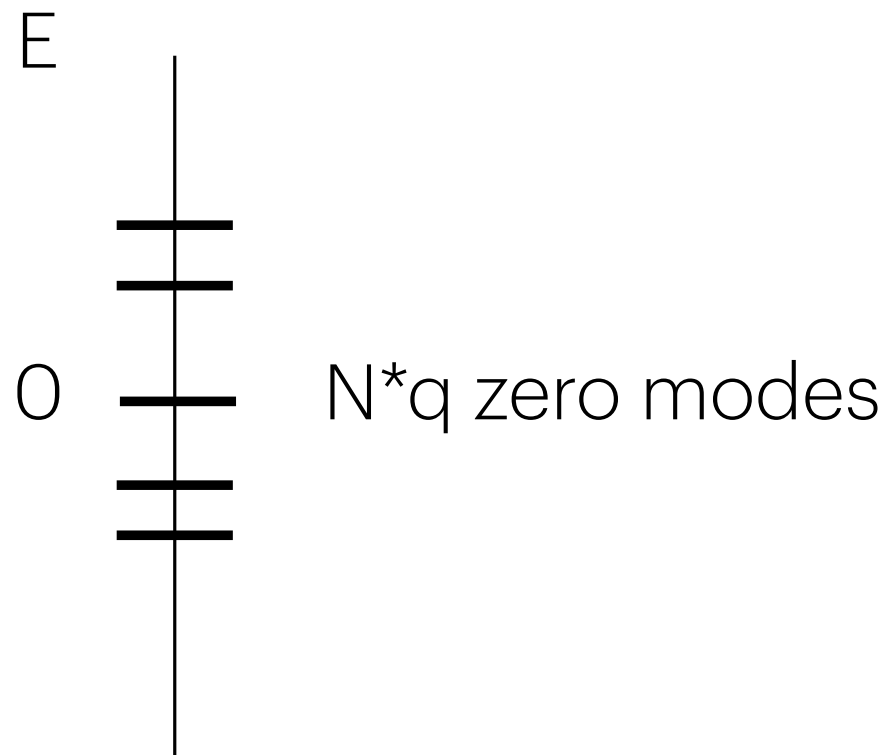
Topological disorder operators in three-dimensional conformal field theory

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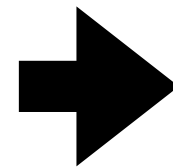
$$Q = \int d^2x f_{ij}$$

Consider operators that are charged under $U(1)_m$



Spectrum on the sphere

These modes must be half-filled to preserve gauge invariance



$$\binom{Nq}{Nq/2} \text{ Distinct states}$$

$q=1, N=4$



$$\mathcal{M}_{ab}^+ = - \mathcal{M}_{ba}^+$$

Anti-symmetric tensor

N=4 QED3

- Gauge invariant/physical operators:

- Conserved (flavor) currents: $J_{ab}^\mu = \bar{\psi}_a \gamma^\mu \psi_b$ $\Delta = 2$

- Scalar mass $M_s = \bar{\psi}_a \psi_a$ $\Delta \approx 2.3$

- Adjoint mass $M_{\text{adj}} = \bar{\psi}_a T_{ab}^i \psi_b$ $\Delta \approx 1.4$

- Monopole operators: $\mathcal{M}_A^\pm = \begin{pmatrix} \mathcal{M}_{12}^\pm \\ \mathcal{M}_{13}^\pm \\ \vdots \end{pmatrix} \in SO(6)_v$ $\Delta \approx 1$

Large N:
 $0.265 N - 0.0383 + O(1/N)$

Monopole operators

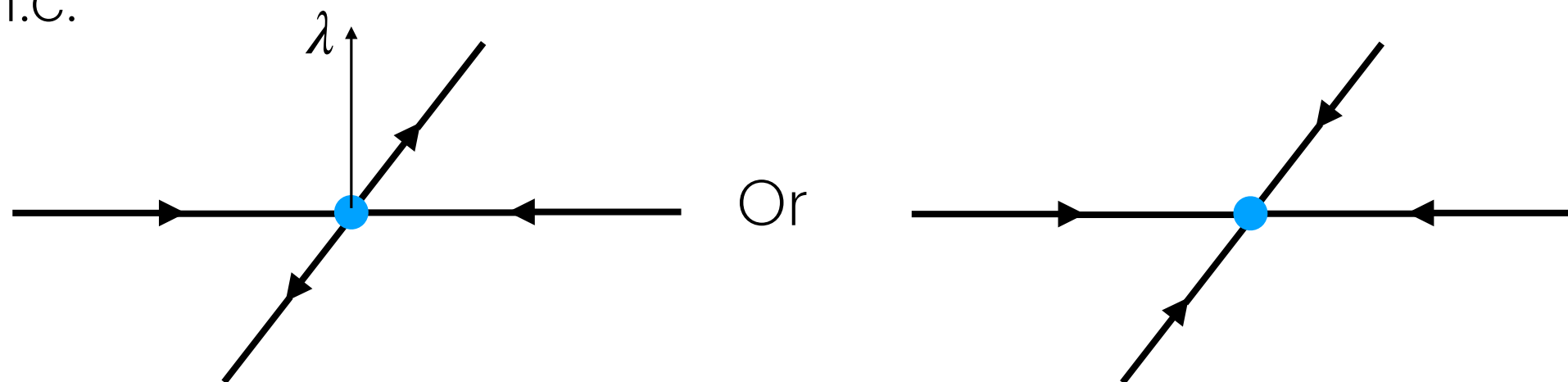
What are they good for?

- As probes (way to measure magnetic symmetry)
- As physical perturbations for condensed matter systems

Flux conservation is not a microscopic symmetry, so operators violating it are generally present in the Lagrangian

$$\mathcal{L}' = \lambda(\mathcal{M}^+)^q + \text{h.c.}$$

Monopoles



Monopole operators

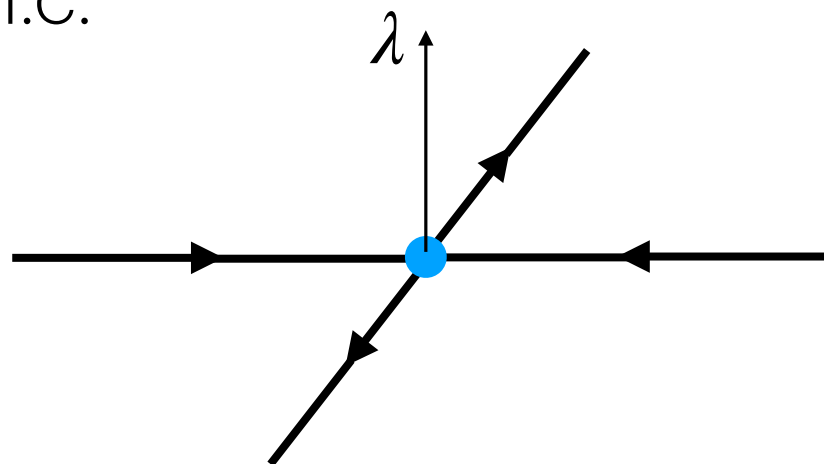
What are they good for?

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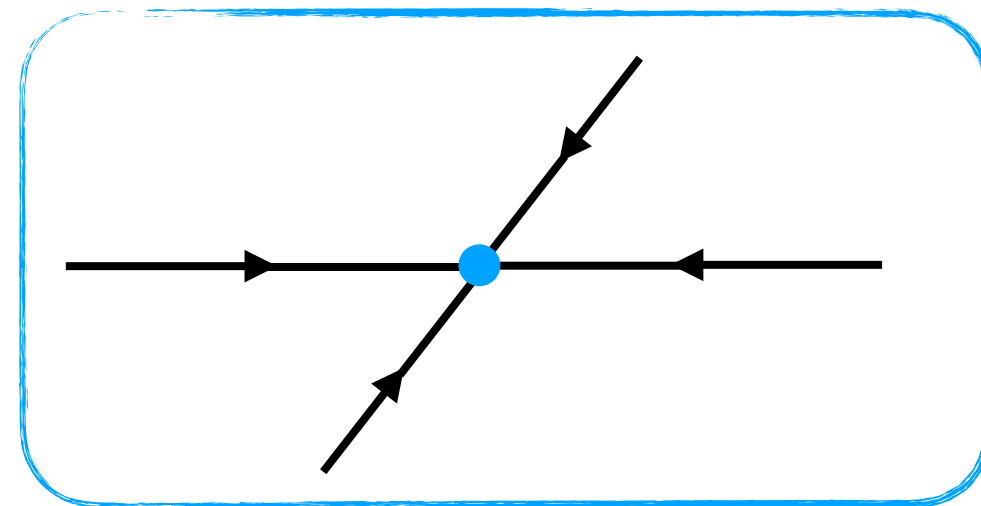
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Monopoles

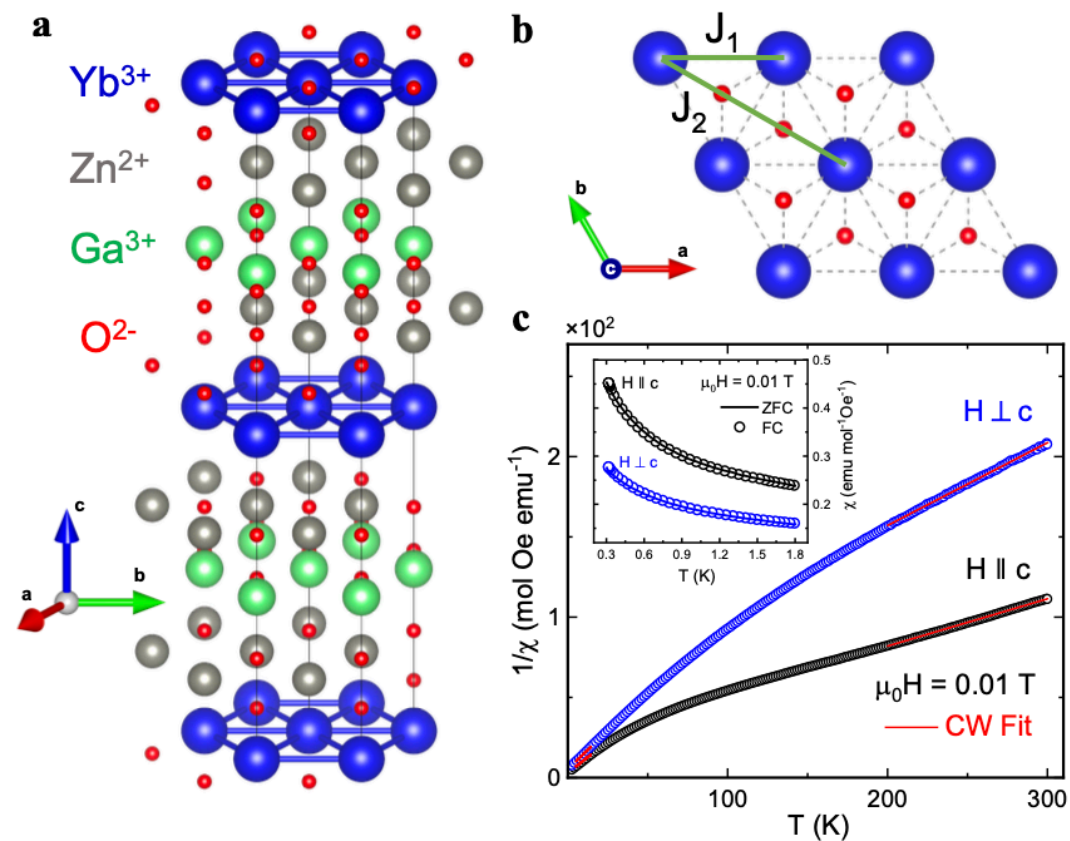


Or

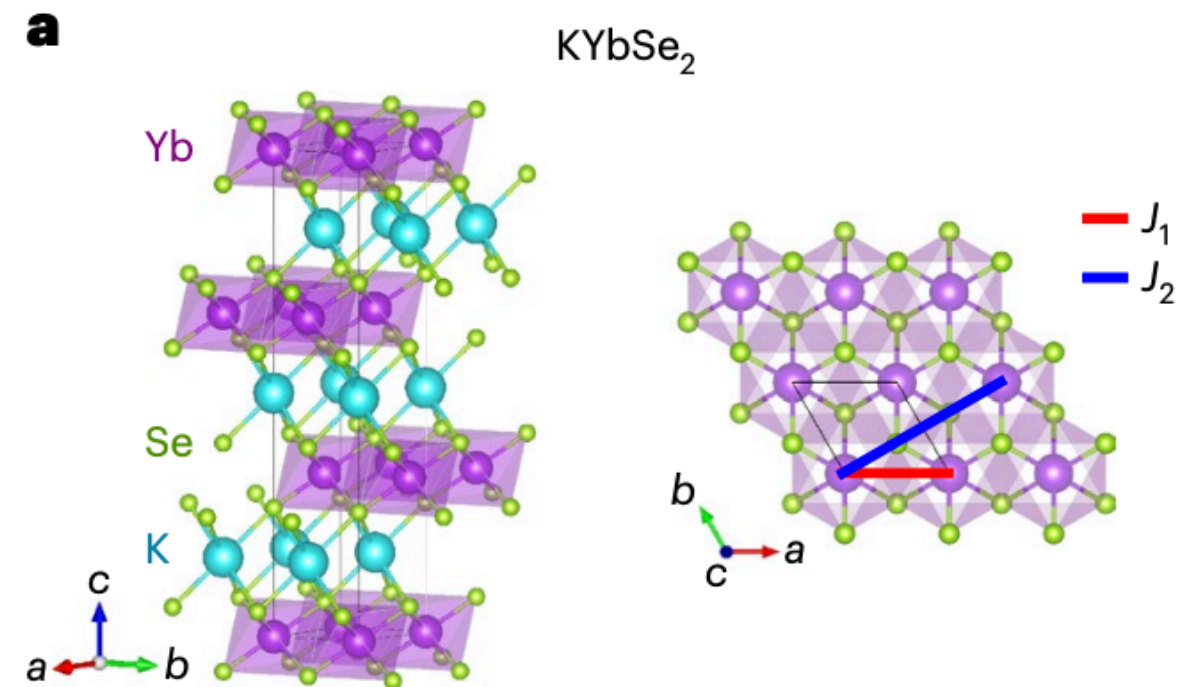


Triangular
DSL?

Triangular lattice spin liquid



YbZn₂GaO₅
Haravifard group

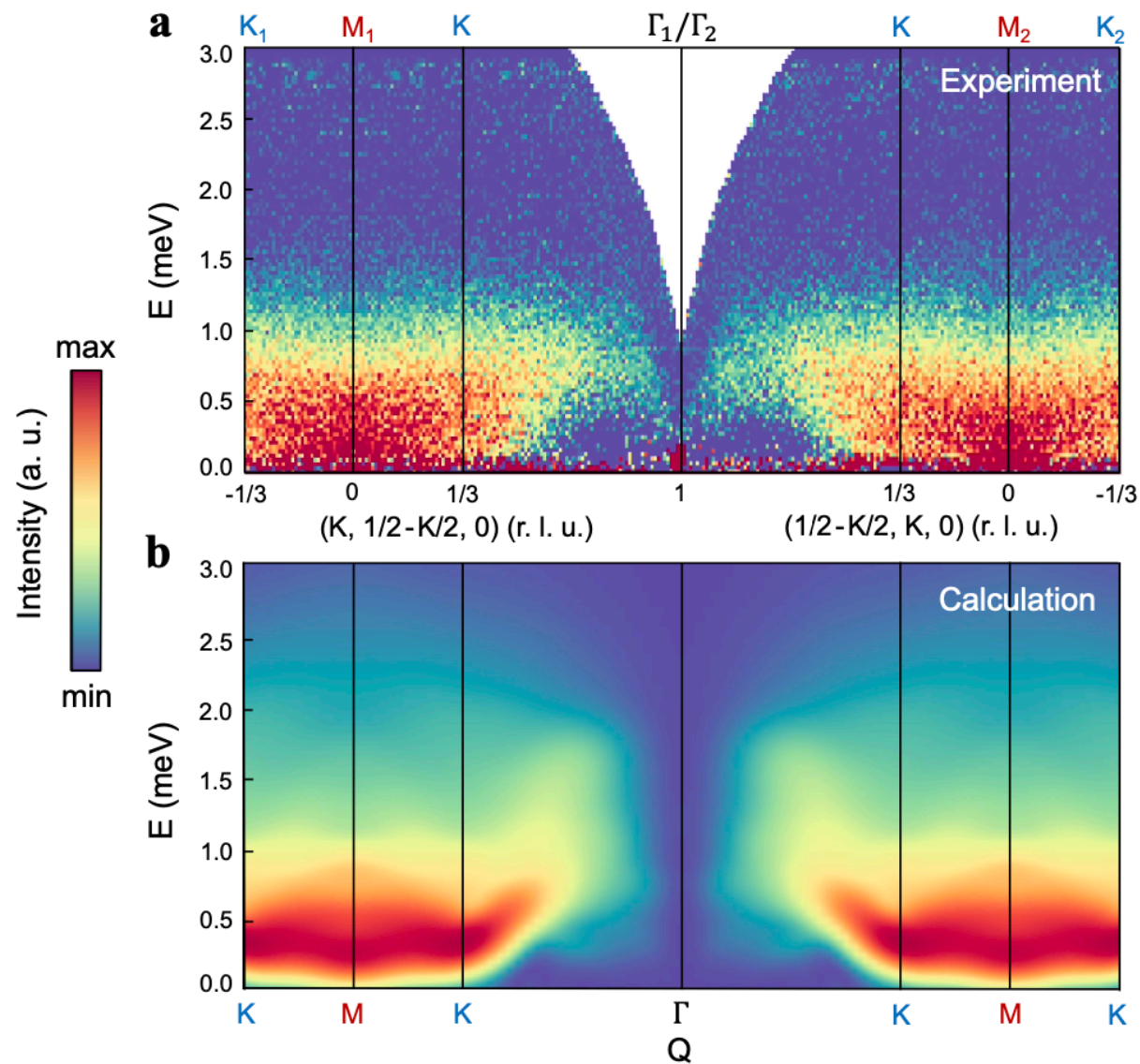


KYbSe₂
Tennant group

Triangular lattice spin liquid

$\text{YbZn}_2\text{GaO}_5$

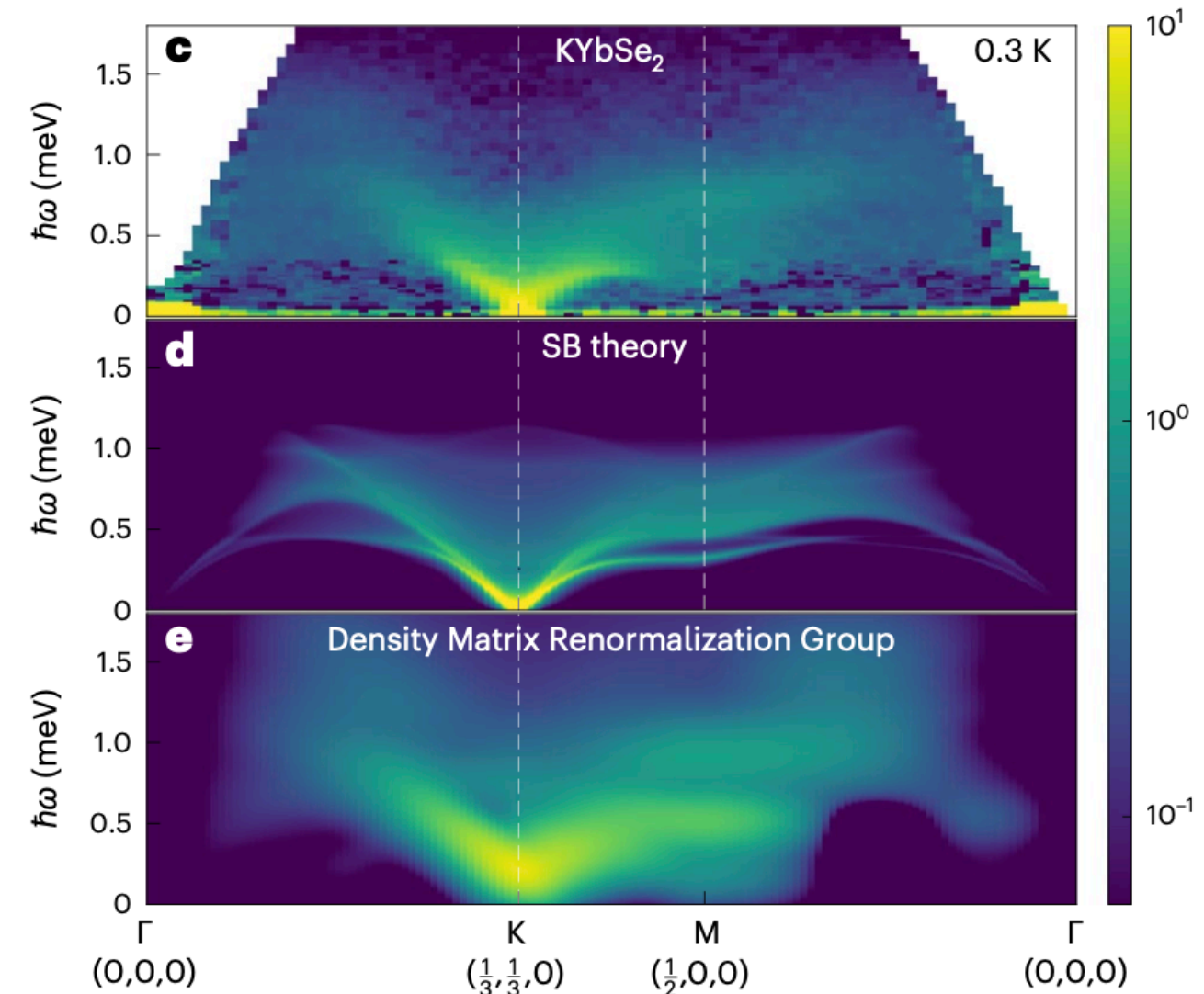
Haravifard group $J_2/J_1=0.12$



KYbSe_2

Tennant group

$J_2/J_1=0.05$



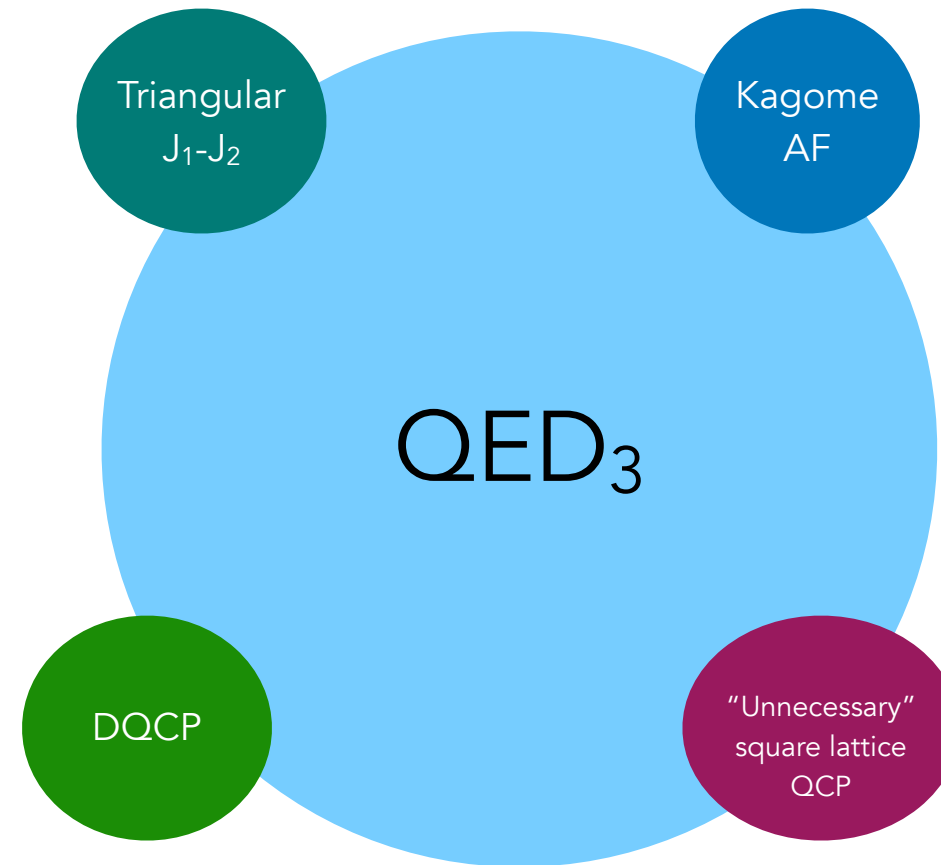
Spins and QED₃

Each system has its own:

- Microscopic (exact) symmetries
- Operator dictionary
- Perturbations to CFT

X.-Y. Song et al, 2019

$$\mathcal{L} = \sum_{a=1}^N \bar{\psi}_a \gamma^\mu (\partial_\mu - i a_\mu) \psi_a + \frac{N}{4e^2} f_{\mu\nu}^2$$



Spins and QED₃

$$\mathcal{L} = \sum_{a=1}^N \bar{\psi}_a \gamma^\mu (\partial_\mu - i a_\mu) \psi_a + \frac{N}{4e^2} f_{\mu\nu}^2$$

Each system has its own:

- Micro
- symm
- Oper
- Pertu

If we believe this is the right description,
what else can it predict?

We will look for signatures in the
behavior under field.

Triangular
J₁-J₂

Kagome
AF

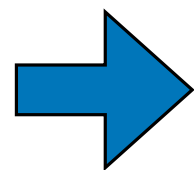
'Unnecessary'
square lattice
QCP

Applied field

$$\psi_a = \psi_{\underset{\text{valley}}{v}\underset{\text{spin}}{\alpha}}$$

$$\mathcal{L}' = -Bs^z = -B\bar{\psi}\gamma_0\sigma^z\psi$$

- $SU(4)_f$ broken to $SU(2)_v \times U(1)_s$
- Field couples to conserved $U(1)_s$ charge $M = S^z = \int d^2x s^z$
- Conformal QED3 also has emergent $U(1)_m$ flux conservation symmetry $\Phi = Q_m$



States labeled by M and Φ

$$E = E_0(M, \Phi) - BM$$

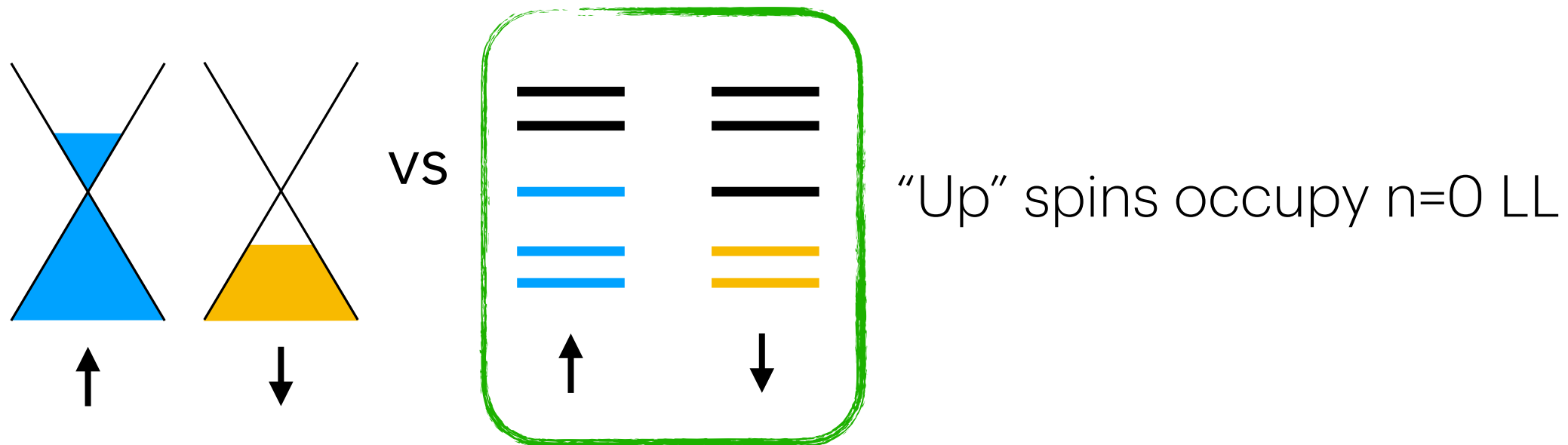
Applied field

$$E = E_0(M, \Phi) - BM$$

- Key question: what value of Φ minimizes E ?

[Y. Ran et al, 2009]: $\Phi = \pm \phi_0 M$

n.b. sign breaks TRS:
Spontaneous chiral order



(At $N=\infty$ can show the LL wins)

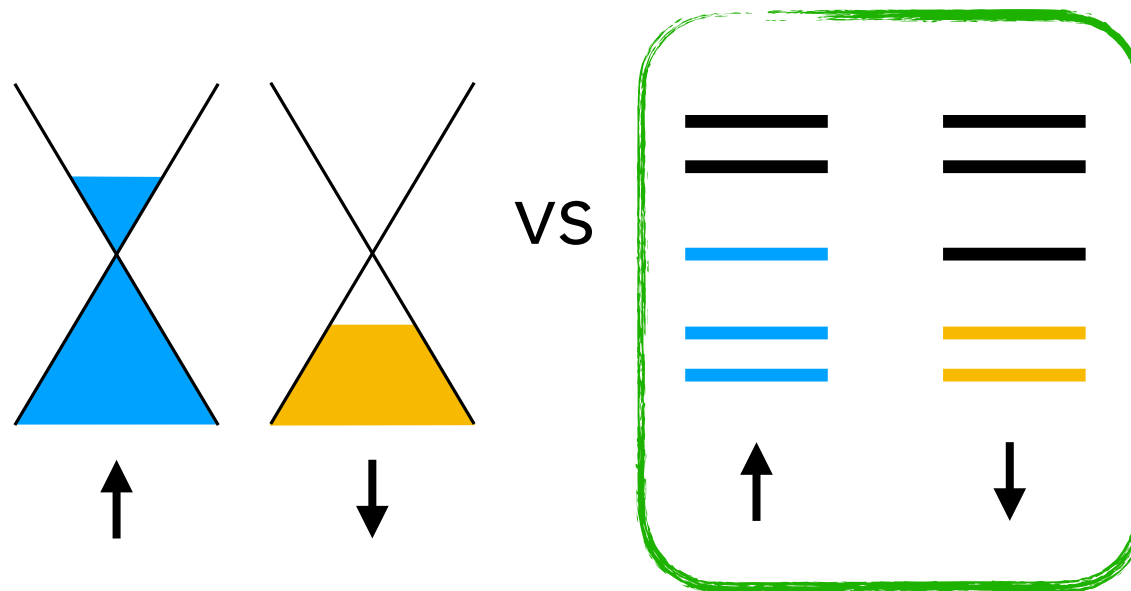
Applied field

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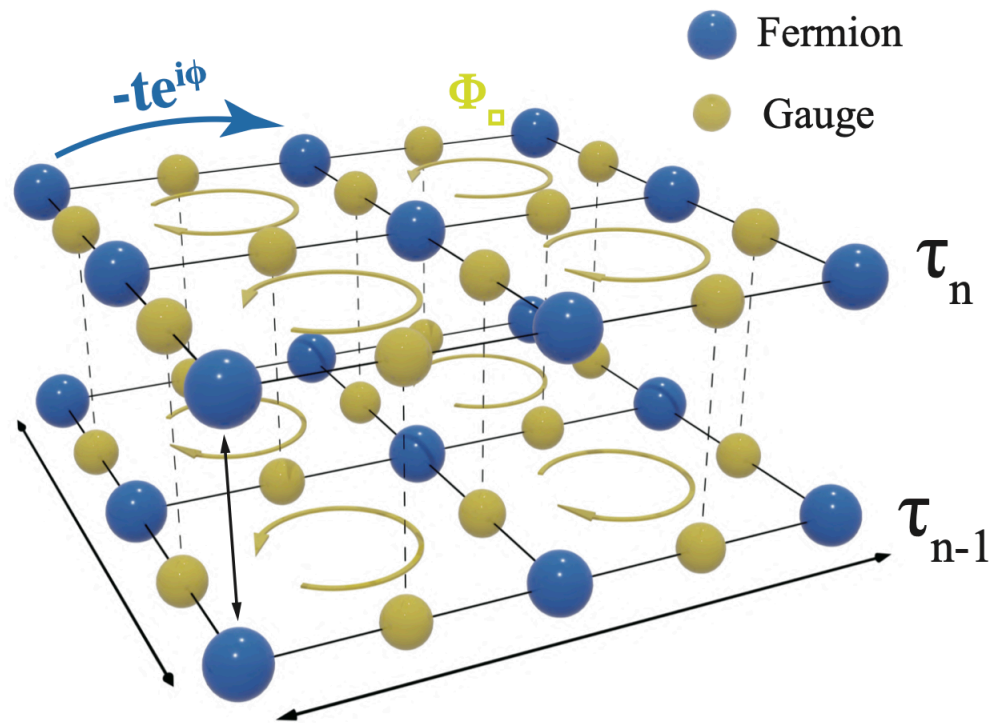
“Up” spins occupy $n=0$ LL

Appealing for observations because the effect is driven by the emergent flux

(At $N=\infty$ can show the LL wins)

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem

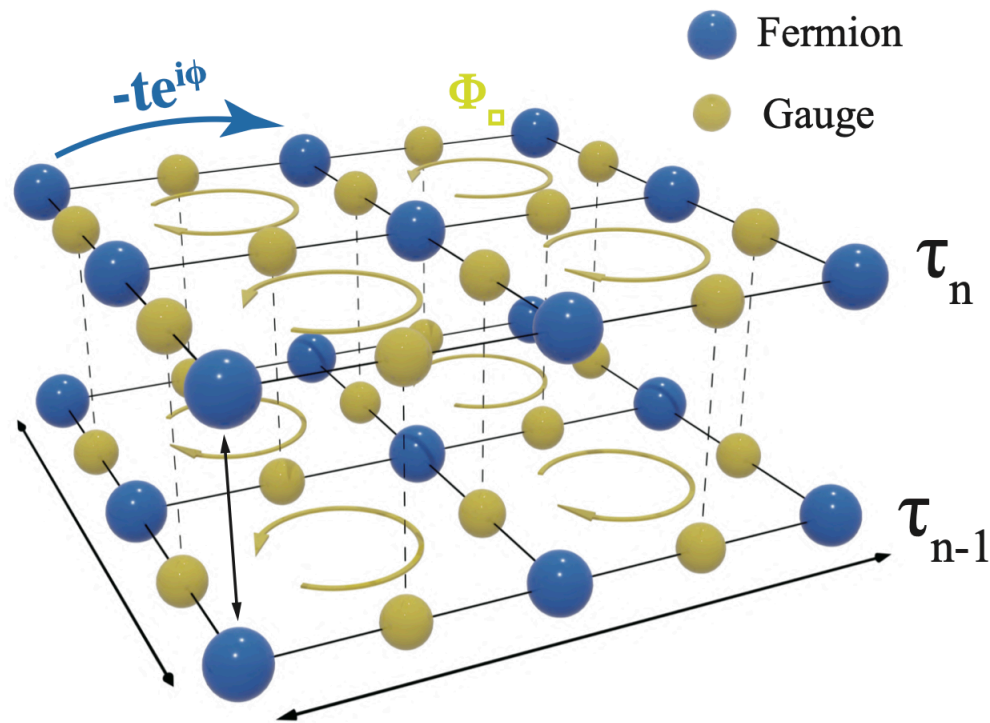


$$\begin{aligned}
 S = & \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
 & - t \sum_{\langle ij \rangle, n} \left[e^{ia_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \\
 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

n.b. square lattice -
avoids sign problem.

A model and quantum Monte Carlo

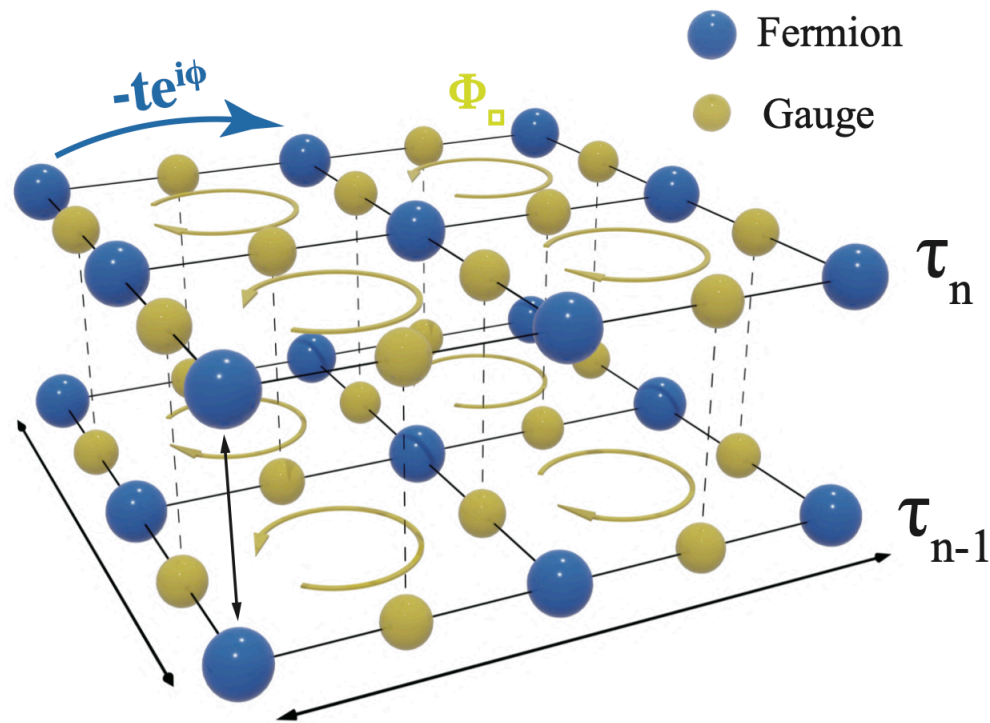
A lattice gauge theory — without a sign problem



$$\begin{aligned}
 S = & \sum_{i,n} \left[\overset{\text{time-derivative}}{\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1}))} - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
 & - t \sum_{\langle ij \rangle, n} \left[e^{ia_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \\
 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

A model and quantum Monte Carlo

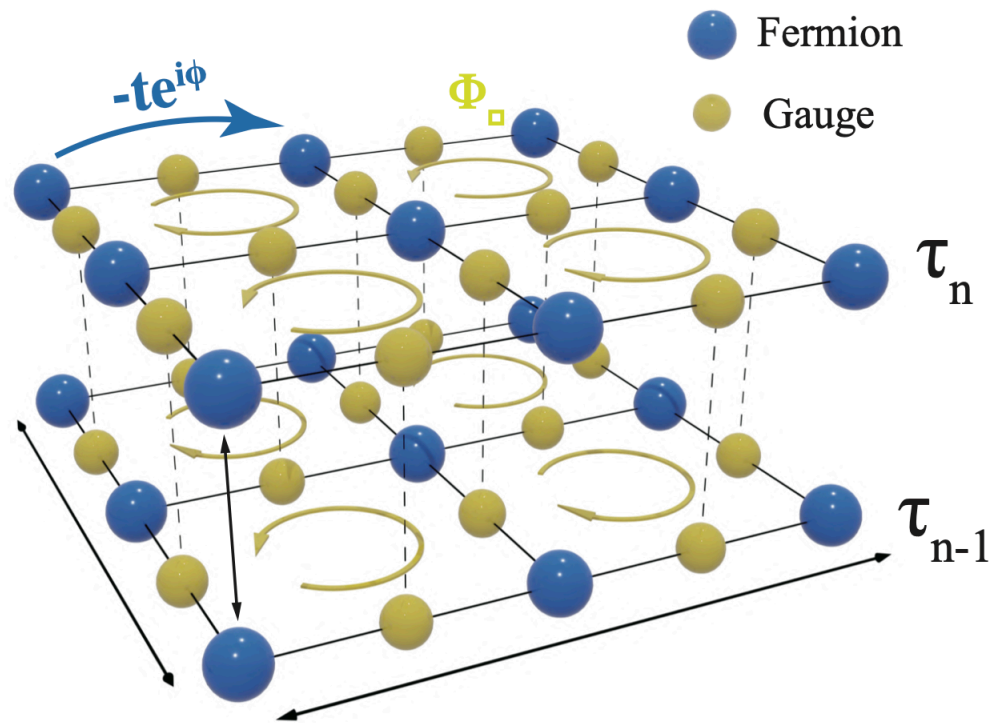
A lattice gauge theory — without a sign problem



$$\begin{aligned}
 S = & \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
 & - t \sum_{\langle ij \rangle, n} \left[e^{ia_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \quad \text{Hopping} \\
 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem



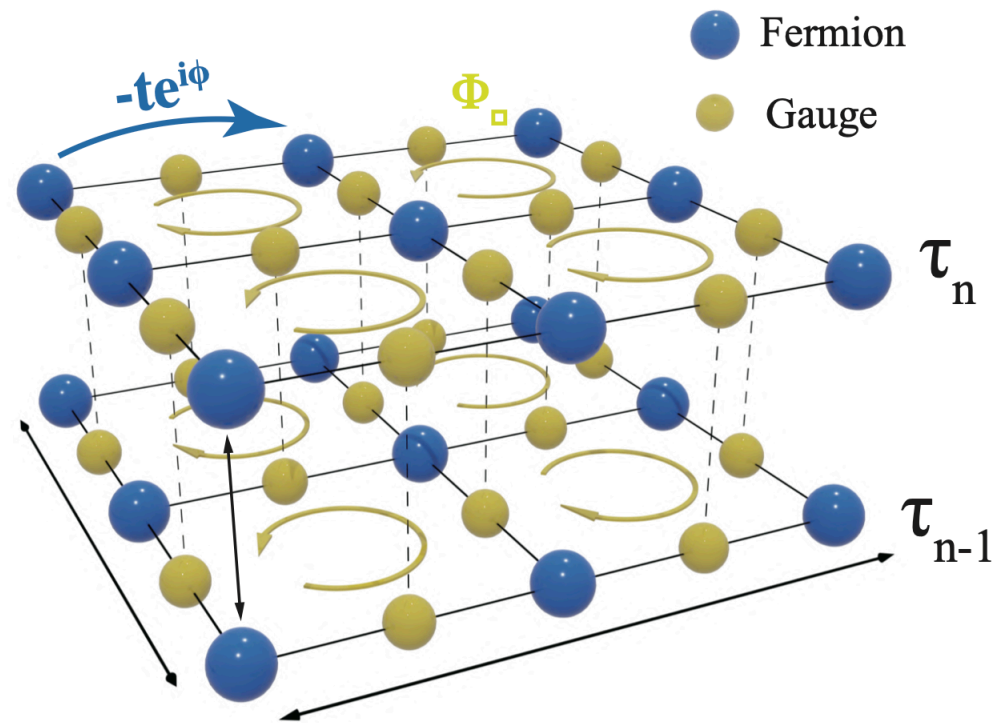
$$\begin{aligned}
 S = & \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
 & - t \sum_{\langle ij \rangle, n} \left[e^{i a_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \\
 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

“Maxwell” term: controls
gauge fluctuations

not periodic: “non-compact” theory.
Flux is exactly conserved

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem

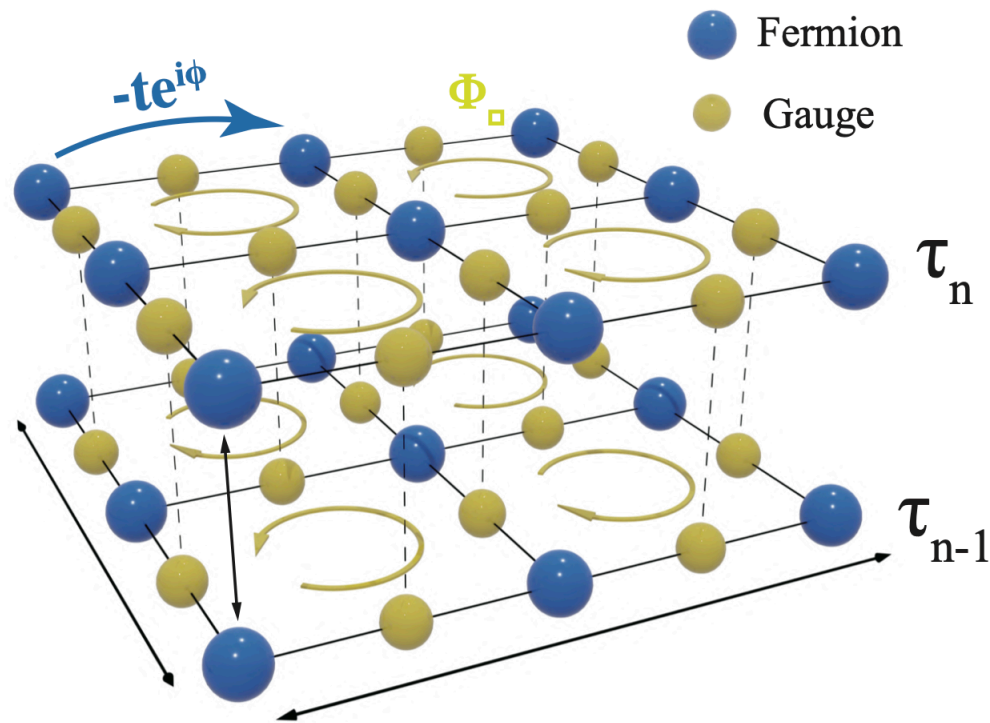


Zeeman field

$$\begin{aligned}
 S = & \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
 & - t \sum_{\langle ij \rangle, n} \left[e^{ia_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \\
 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

A model and quantum Monte Carlo

A lattice gauge theory — without a sign problem



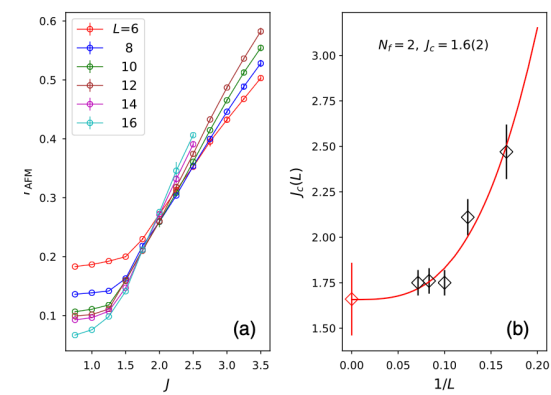
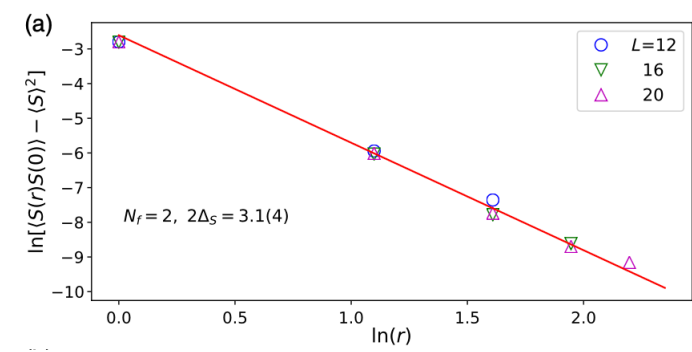
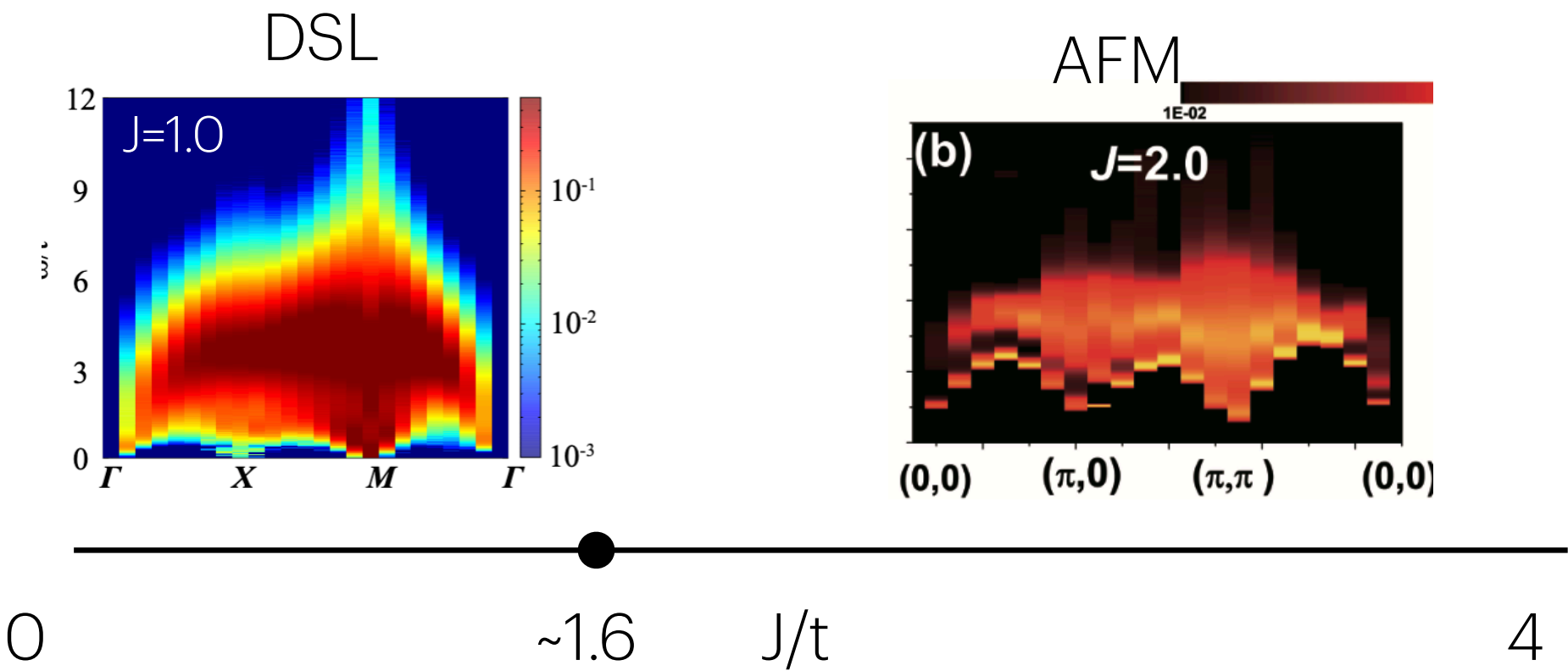
$$\begin{aligned}
 S = & \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
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 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

$J \rightarrow 0, B=0$: Lieb theorem guarantees π flux state, and hence Dirac fermions

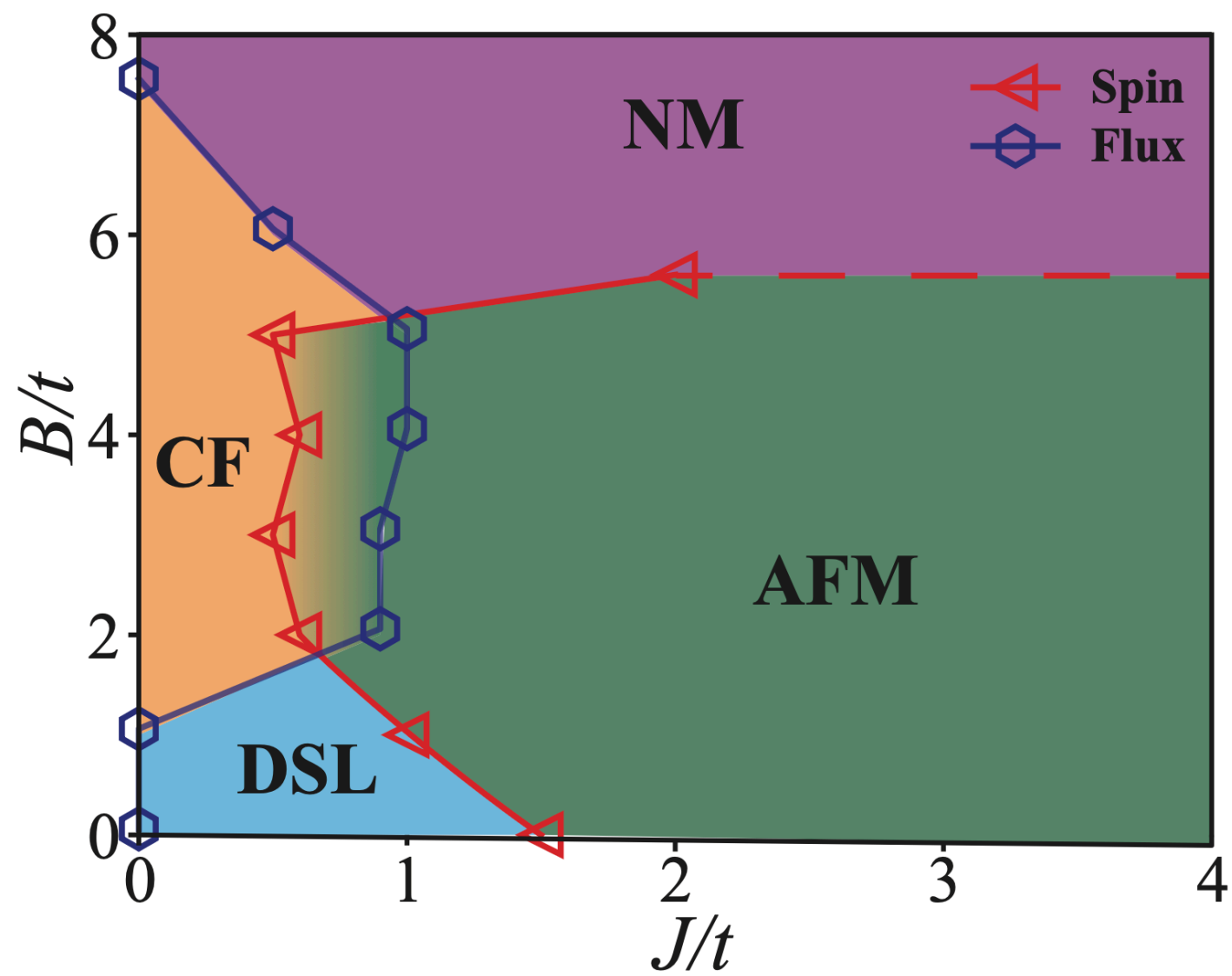
$$N_{\text{CFT}} = 2 N_{\text{lattice}}$$

$$B=0$$

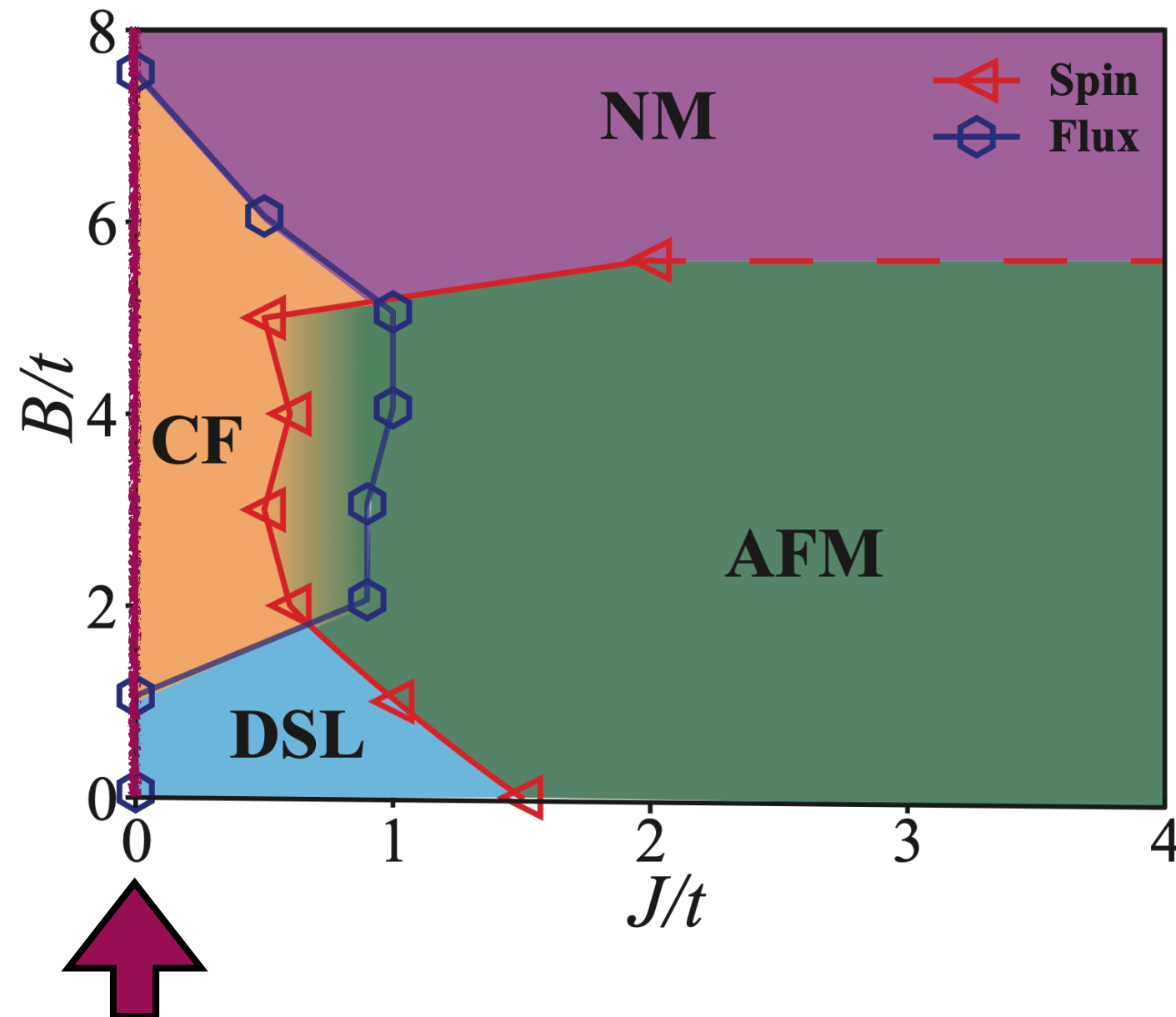
c.f X.-Y. Xu et al, 2019



Phase diagram



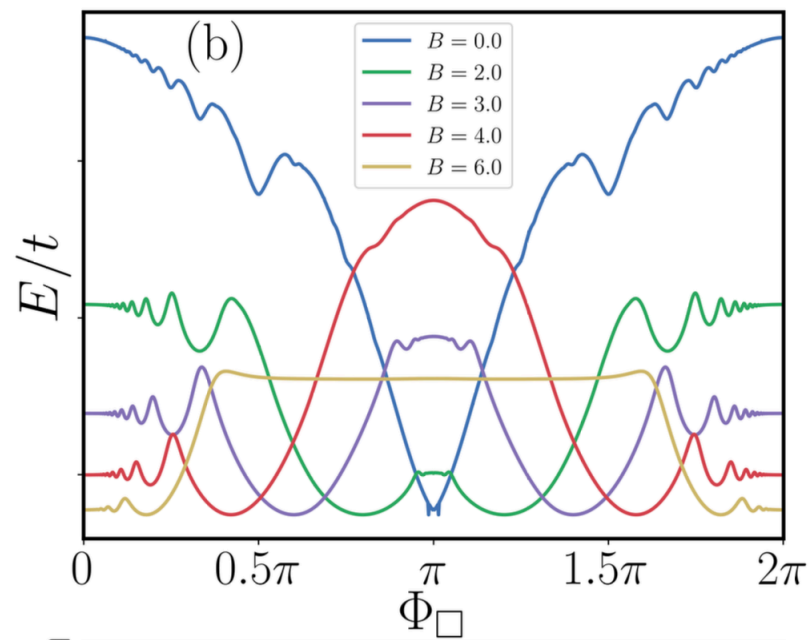
Phase diagram



Line of no gauge fluctuations: but there is an *average* gauge field

Energetics

- At $J=0$, the problem is equivalent to free fermions with a magnetic flux chosen to minimize the total energy

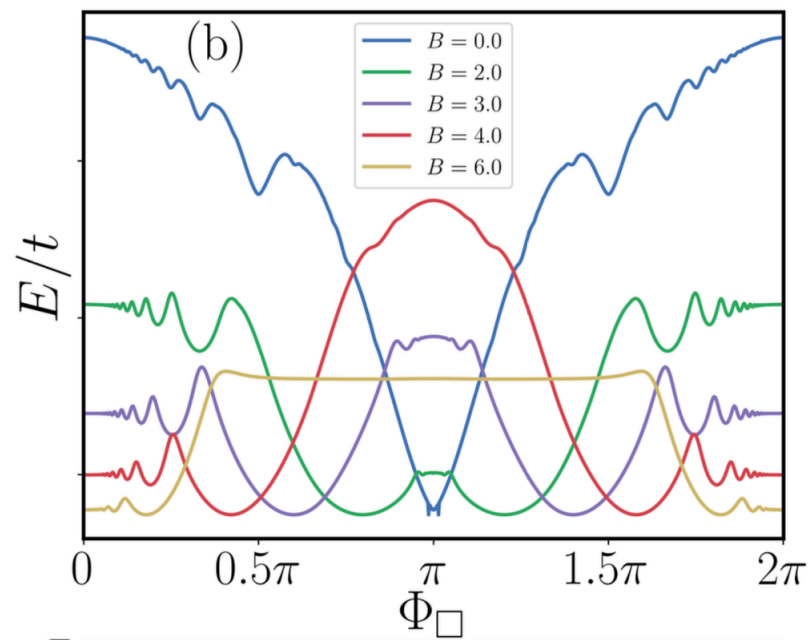


Optimal flux deviates
from π when $B > 0$

Double minimum: spontaneous chirality

Energetics

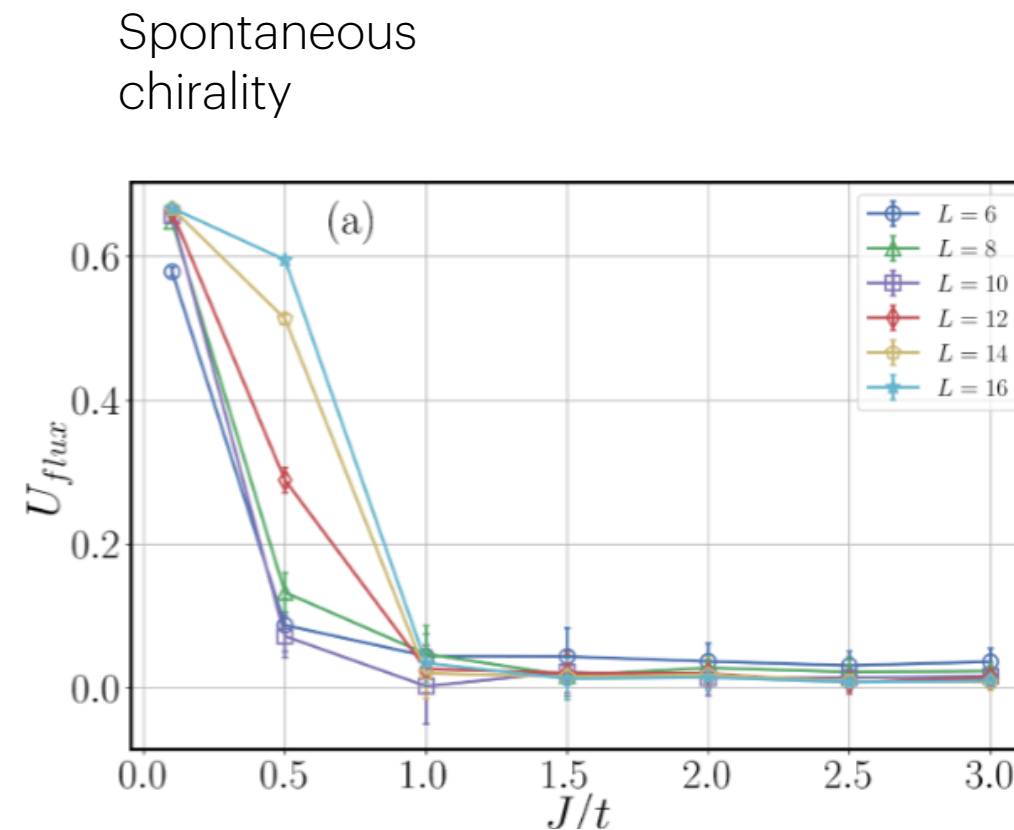
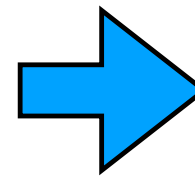
- The chiral flux persists for small J



Optimal flux deviates from π when $B > 0$

Double minimum: spontaneous chirality

$J > 0$

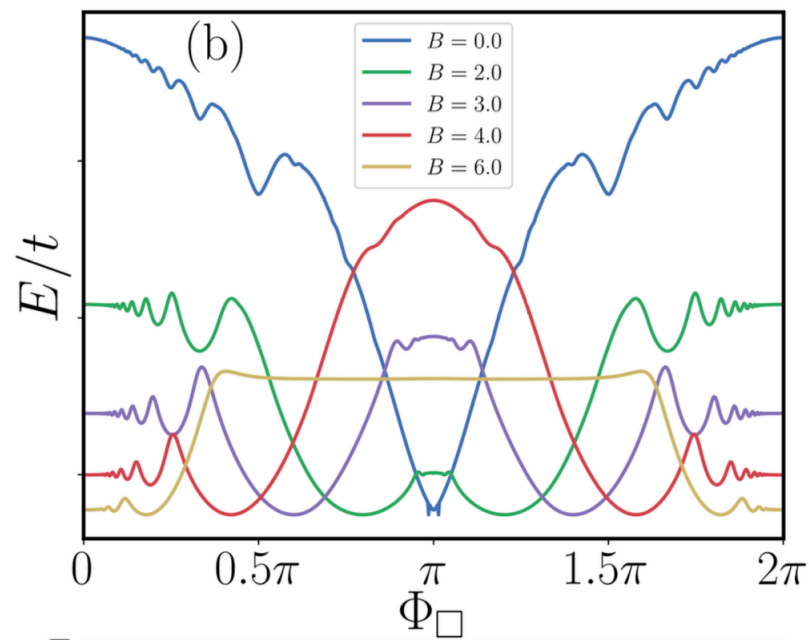


Binder cumulant ($B/t=2$)

No chirality

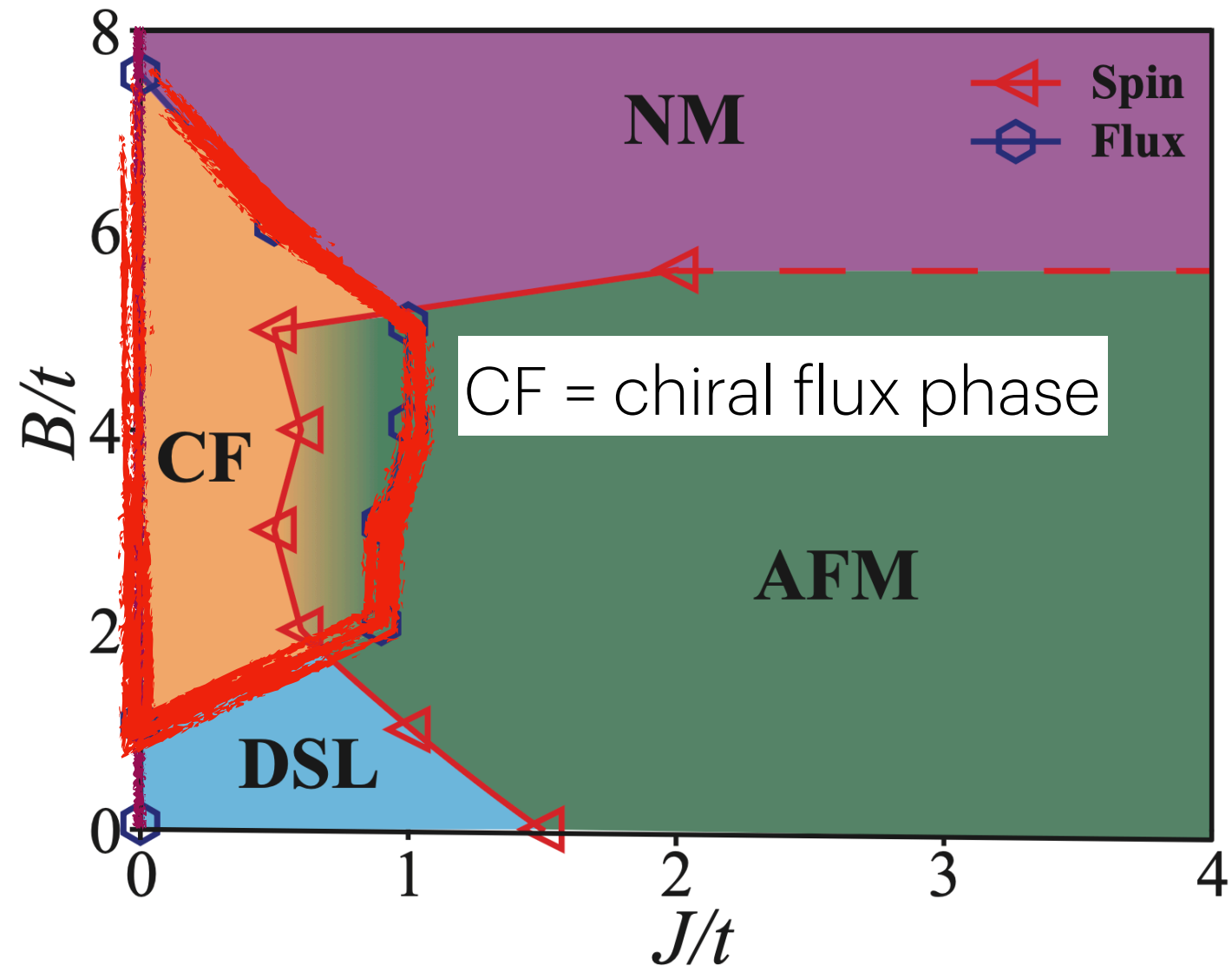
Energetics

- The chiral flux persists for small J



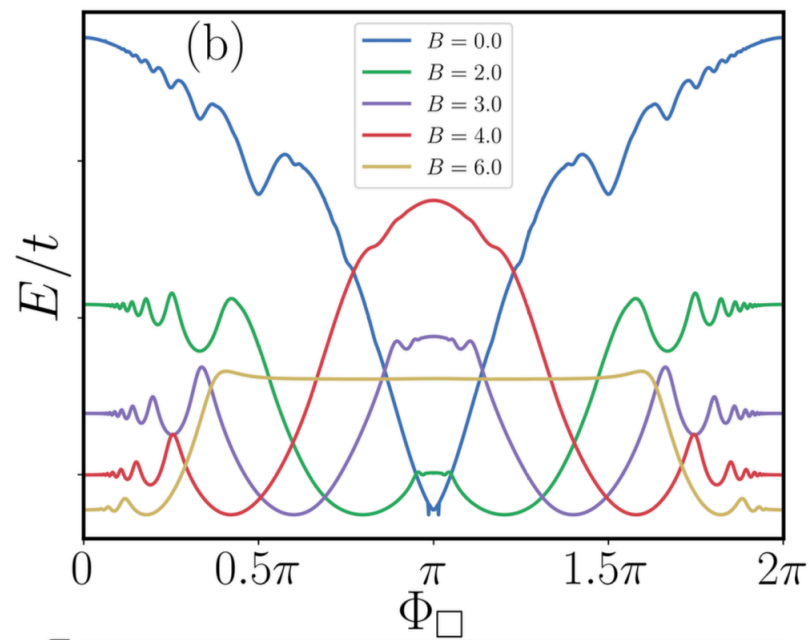
Optimal flux deviates from π when $B > 0$

Double minimum: spontaneous chirality



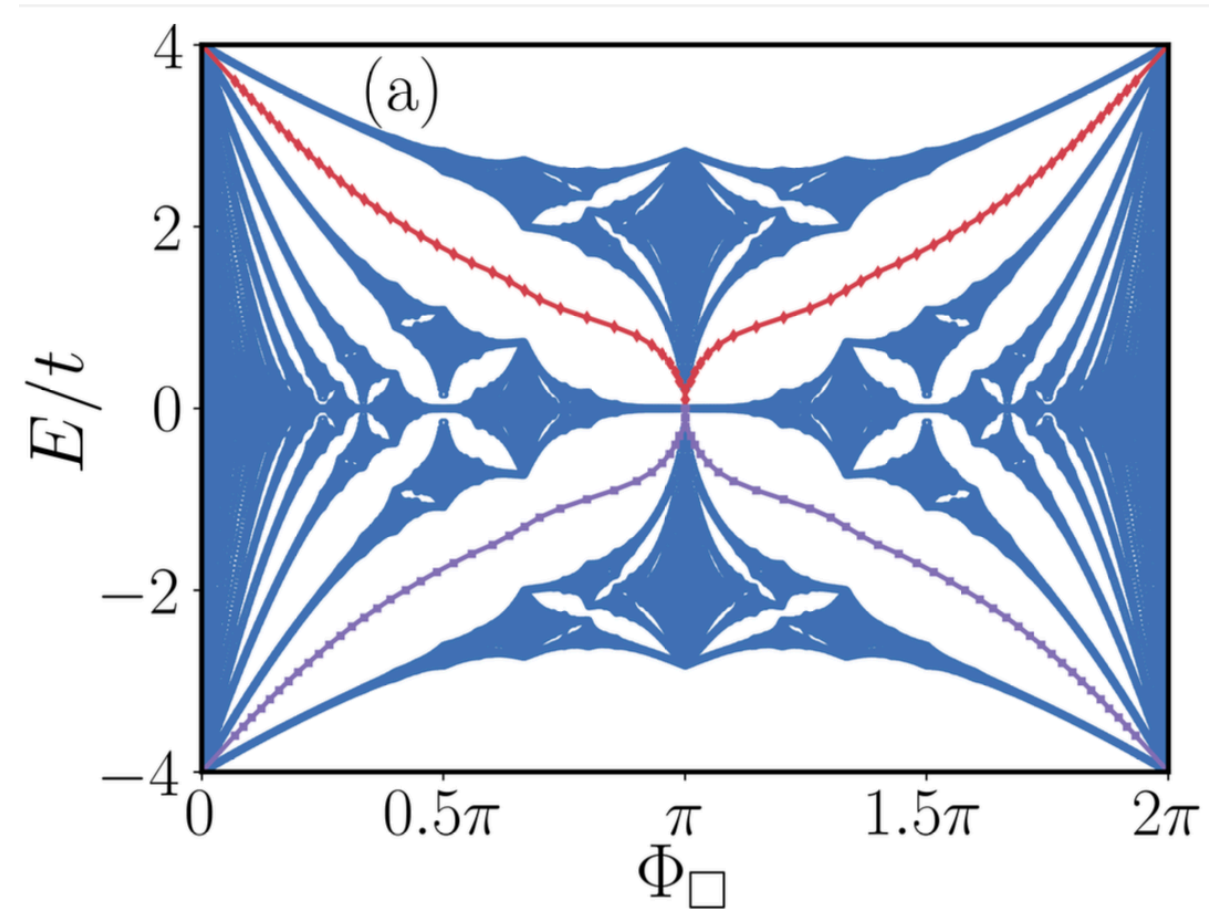
Fermion states

- The chiral flux induces a complex set of Hofstadter bands, similar to Landau levels



Optimal flux deviates
from π when $B > 0$

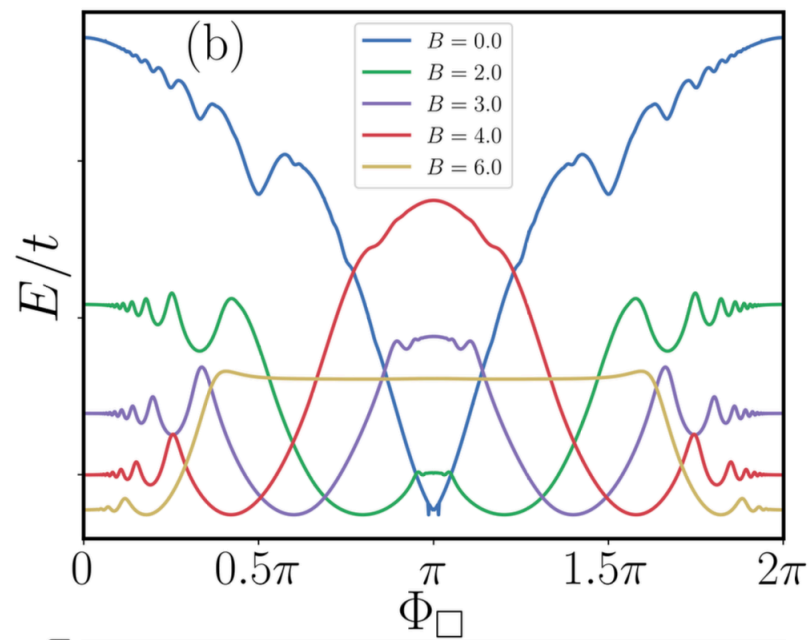
Double minimum: spontaneous chirality



Hofstadter butterfly

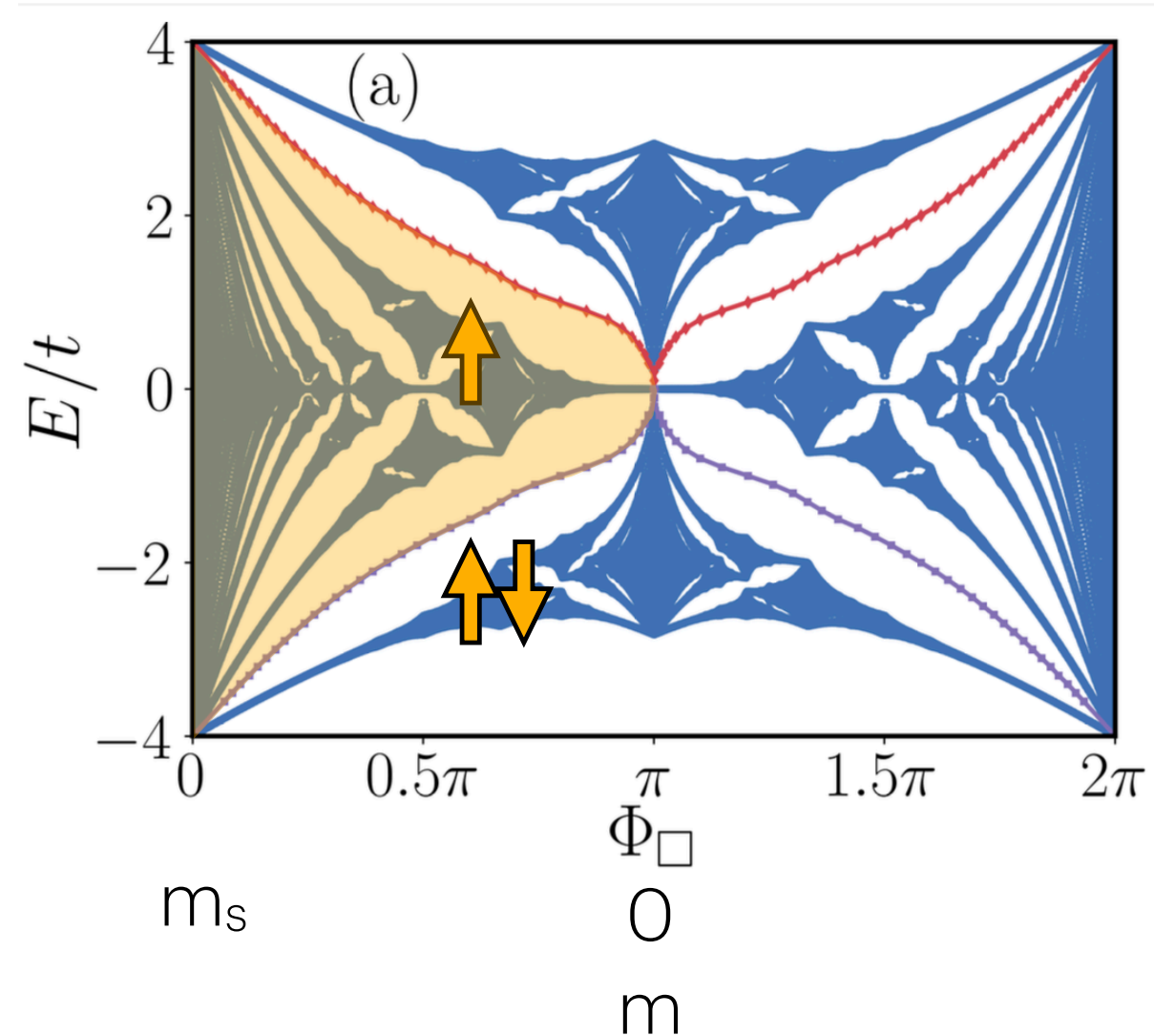
Fermion states

- The chiral flux induces a complex set of Hofstadter bands, similar to Landau levels



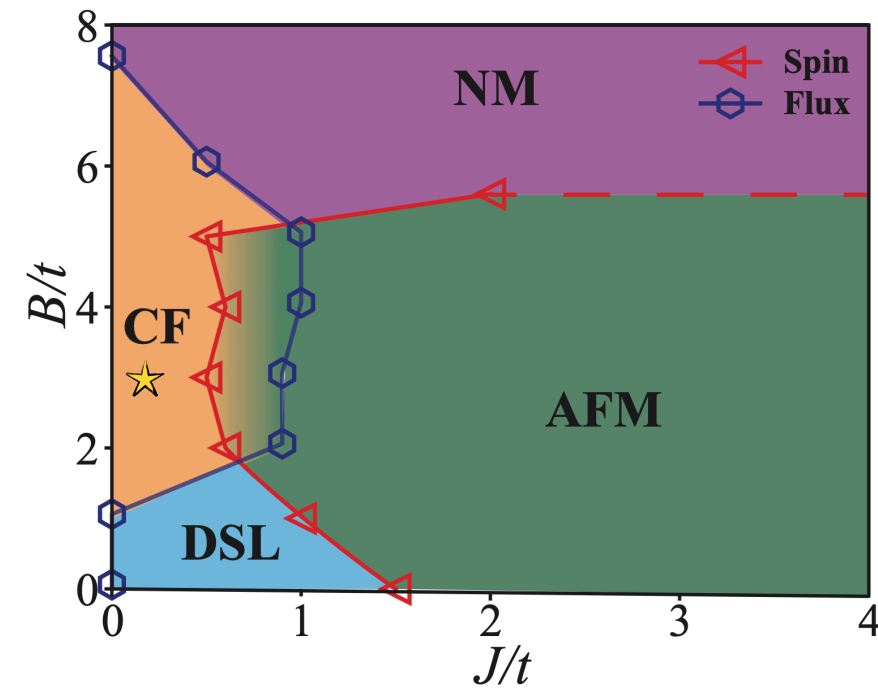
Optimal flux deviates
from π when $B > 0$

Double minimum: spontaneous chirality



Dynamical correlations

“Landau level”-like features

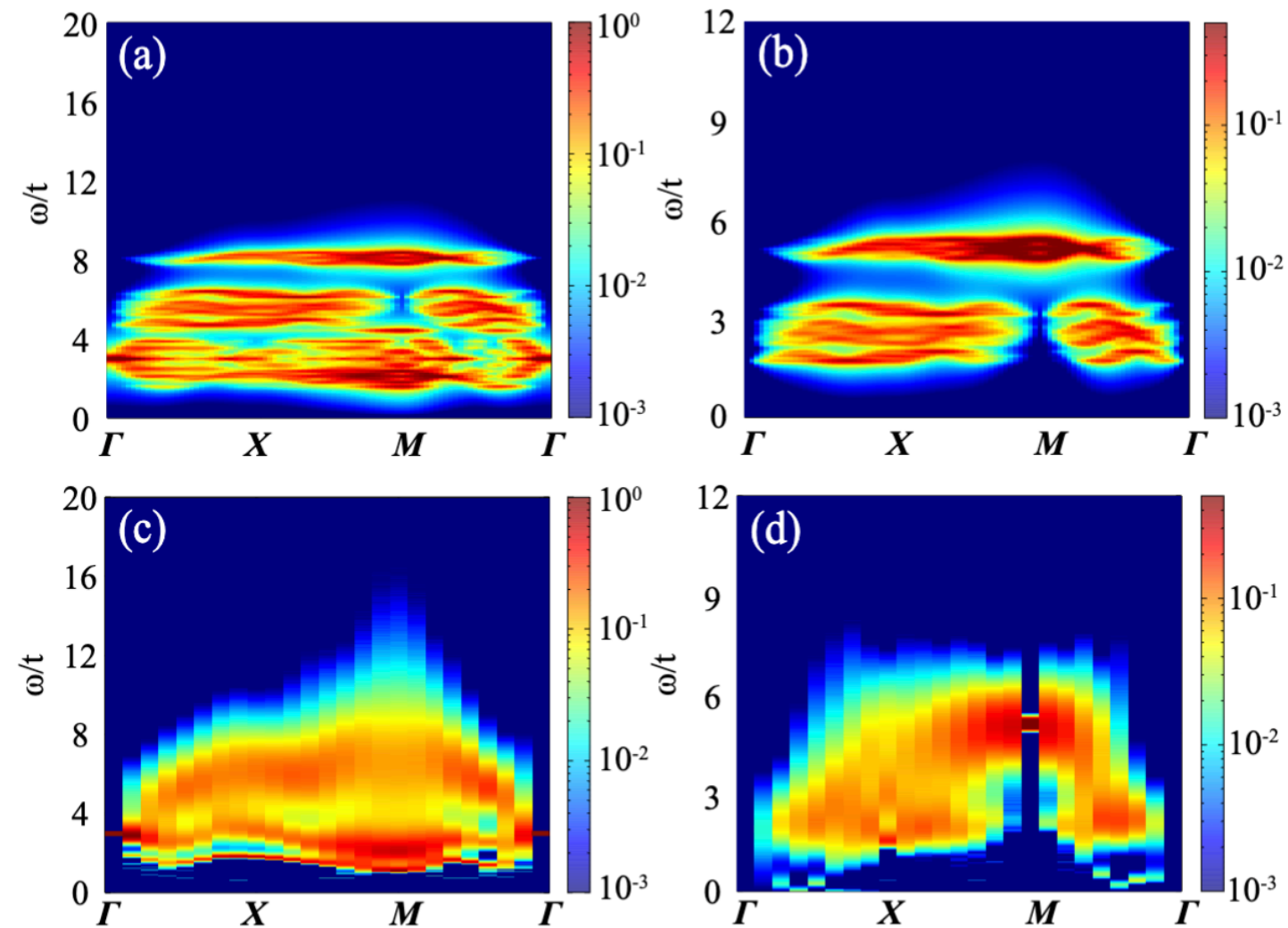


$J=0$

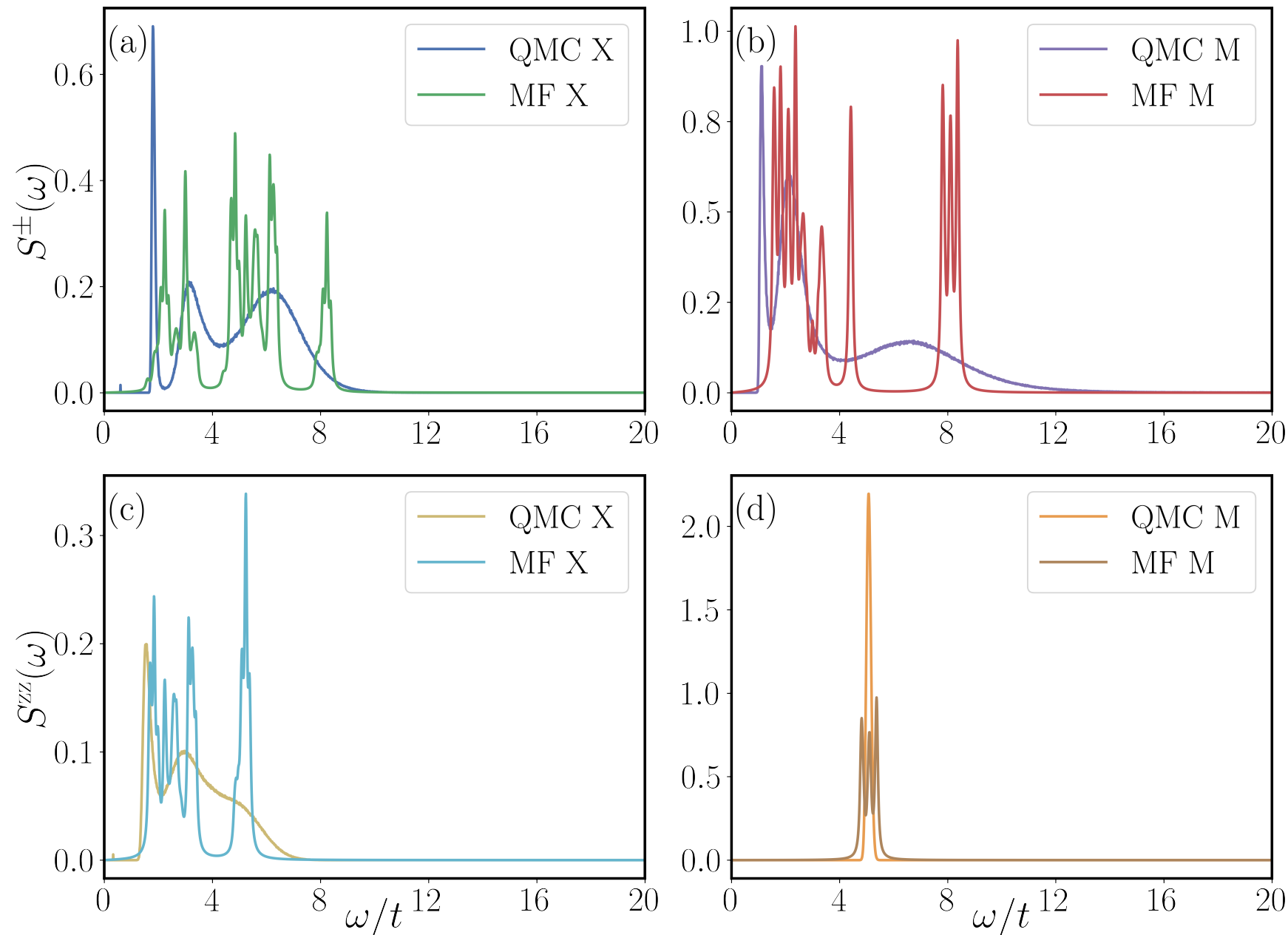
$J>0$

Transverse

Longitudinal



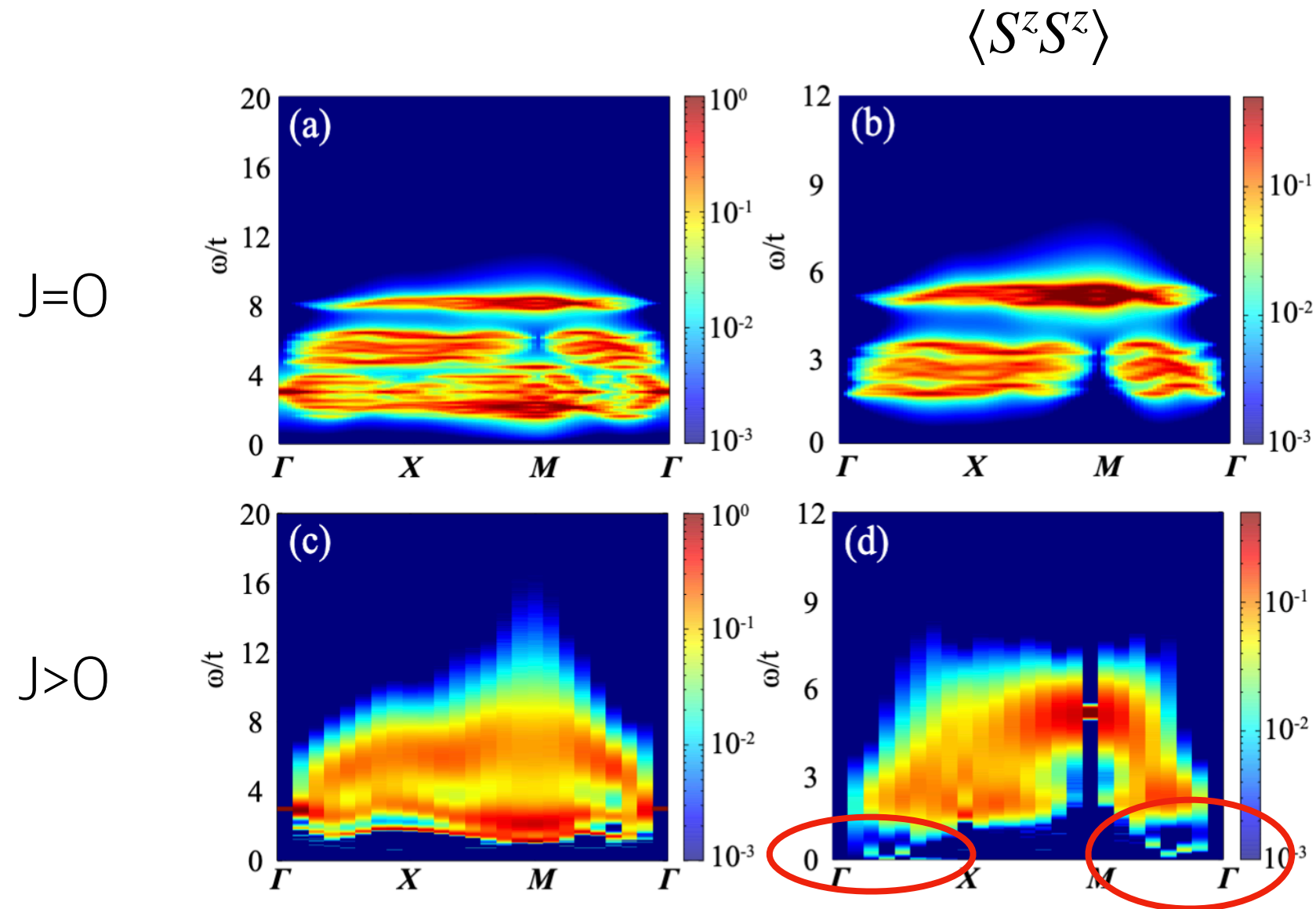
Dynamical correlations



Peaks do not correspond to simple “spin wave” or “triplon” mode counting.

Hofstadter bands give a good guide to intensity even with gauge fluctuations

Low energy?



Low energy weight induced by gauge fluctuations!

Effective field theory

- Include “probe” gauge field A_s coupled to spin:

$$\mathcal{L} = (dA)^2 + \mathcal{L}_{\uparrow}(A + A_s, \epsilon_F > 0) + \mathcal{L}_{\downarrow}(A - A_s, \epsilon_F < 0)$$

Dirac LLs
2-fold degenerate

- Integrate out fermions

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= (dA)^2 + \frac{1}{4\pi}(A + A_s) \wedge d(A + A_s) - \frac{1}{4\pi}(A - A_s) \wedge d(A - A_s) \\ &= (dA)^2 + \frac{1}{\pi}A \wedge dA_s\end{aligned}$$

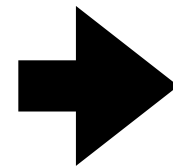
- Mixed Chern-Simons term.

Mixed CS term

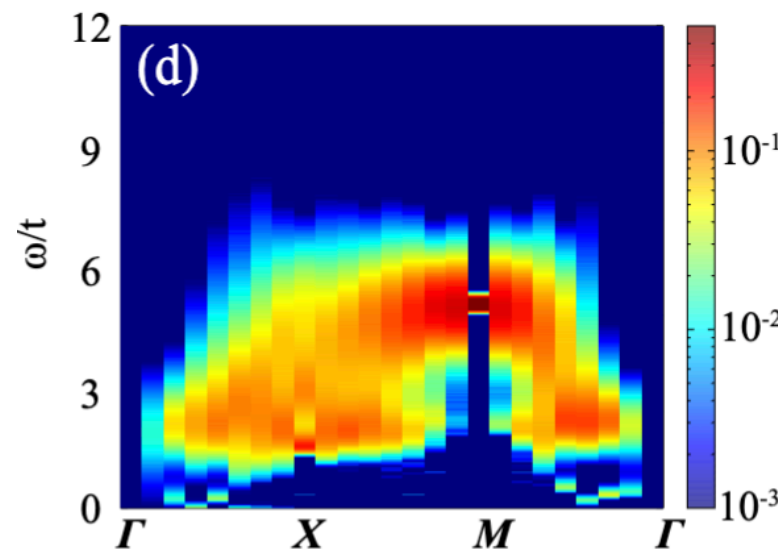
$$\mathcal{L}_{\text{eff}} = (dA)^2 + \frac{1}{\pi} A_s \wedge dA$$

- If we set the probe to zero, we see that the gauge field remains gapless
- Consider time component A_s^0 — source to generate S^z correlations

$$S^z \sim \frac{1}{\pi} \epsilon_{ij} \partial_i A_j$$



$$\langle S^z S^z \rangle \sim q \delta(\omega - vq)$$



Low energy weight = photon

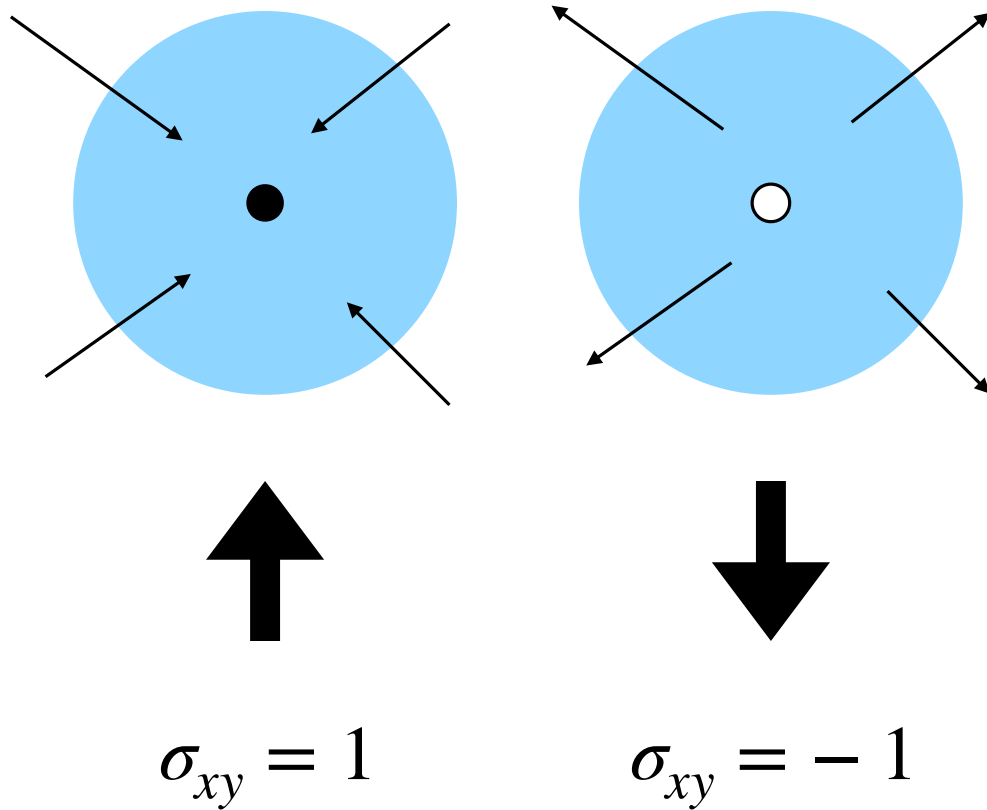
Mixed CS term

$$\mathcal{L}_{\text{eff}} = (dA)^2 + \frac{1}{\pi} A_s \wedge dA$$

- Mixed Chern-Simons term: insertion of flux creates spin.
- Further integration of A:

$$\mathcal{L}_{\text{eff}} = (dA_s)^2 \quad ? \text{ Spin symmetry is broken ?}$$

Flux insertion



Adiabatic insertion of one flux creates one up fermion and removes one down fermion.

The new ground state has therefore $\Delta S^z = +1$

Monopole

$$\mathcal{L}_{\text{eff}} = (dA)^2 + \frac{1}{\pi} A_s \wedge dA$$

- Polyakov: for free U(1) gauge theory, monopole has finite action.
- Here: monopole creation must be accompanied by change of spin.
- Hence: true condensate mixes these two

$$\mathcal{M}_{\text{ord}} = \mathcal{M}^+ S^+ \quad \text{Both } U(1)_m \text{ and } U(1)_s \text{ broken}$$

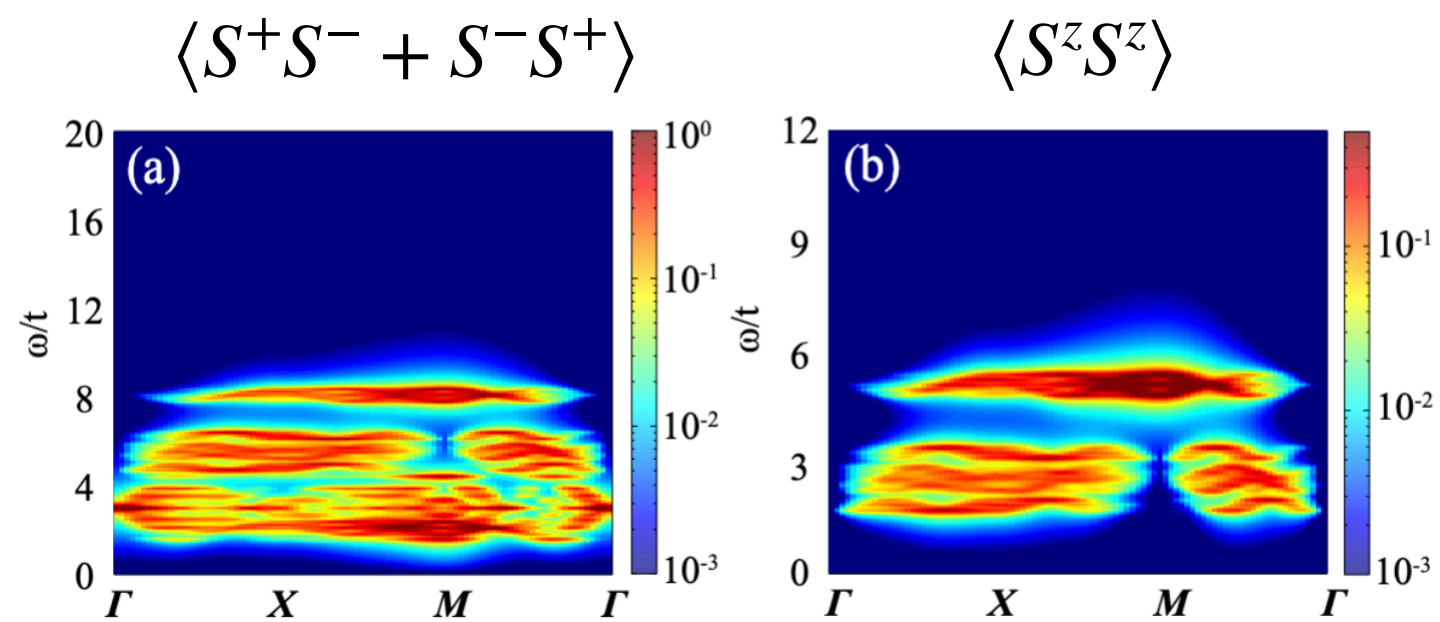
$$\langle \mathcal{M}_{\text{ord}} \rangle \neq 0 \quad \text{Residual } U(1)_{m-s} \text{ preserved}$$

$$n_{GM} = \dim \frac{U(1) \times U(1)}{U(1)} = 1$$

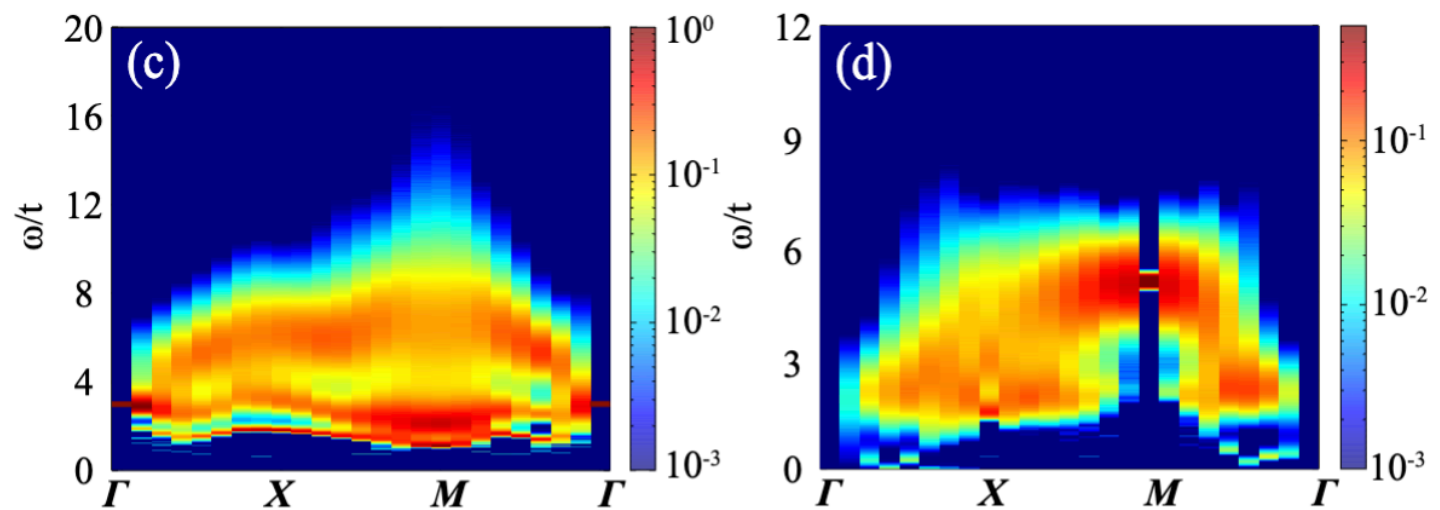
$$\langle S^+ \rangle = \langle \mathcal{M} \rangle = 0$$

Low energy

$J=0$



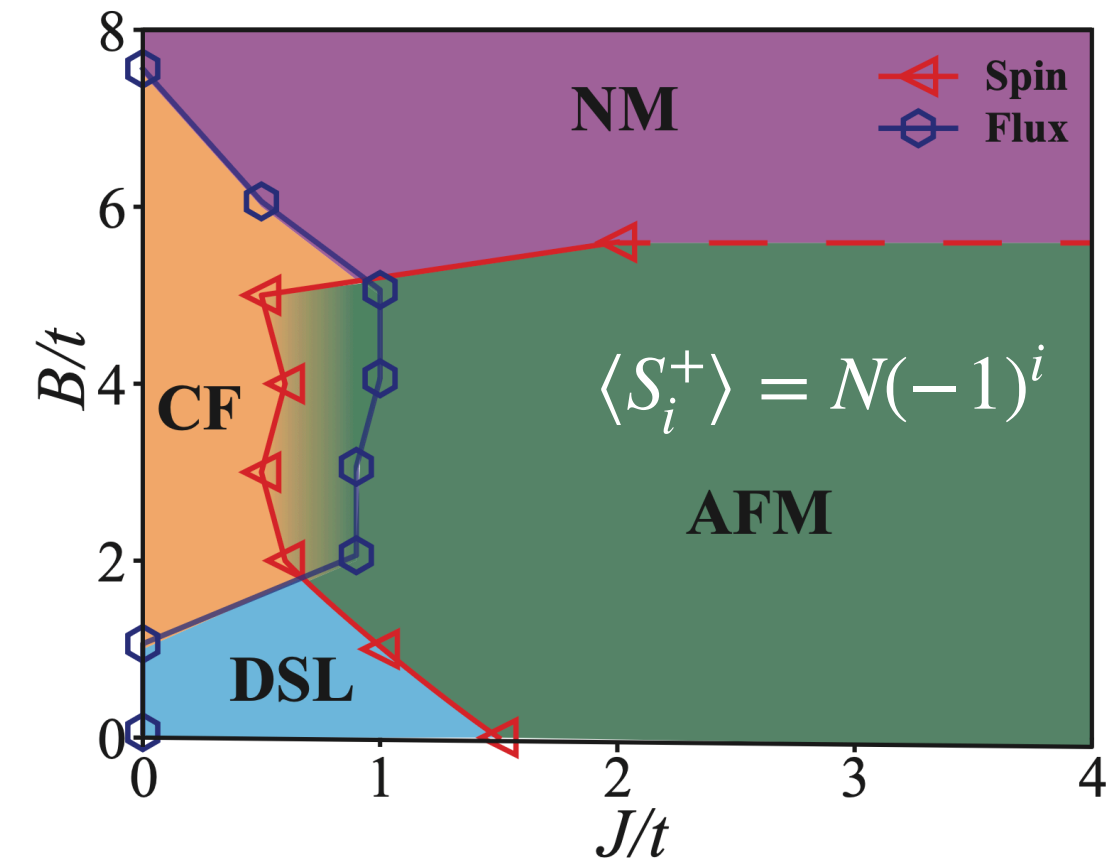
$J>0$



Gap

$$\langle S^+ \rangle = 0$$

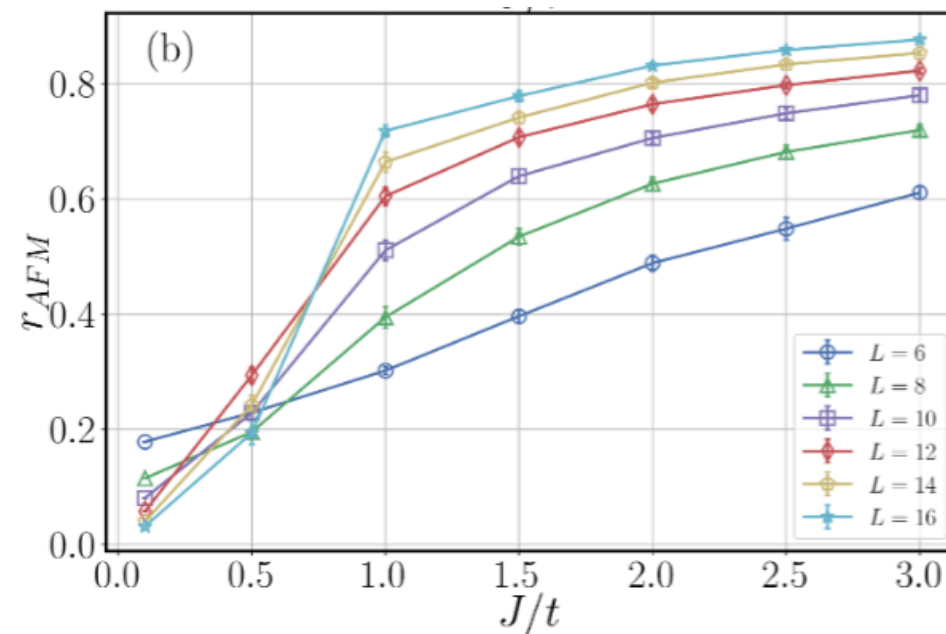
Comparison with AF phase



$$S^\pm(\mathbf{q}) = \frac{1}{N} \sum_{ij} \langle S_i^+ S_j^- + h.c. \rangle e^{-i\mathbf{q} \cdot \mathbf{r}_{ij}}$$

$$r_{\text{AFM}} = 1 - \frac{S^\pm(\mathbf{q} + \delta\mathbf{q})}{S^\pm(\mathbf{q})}$$

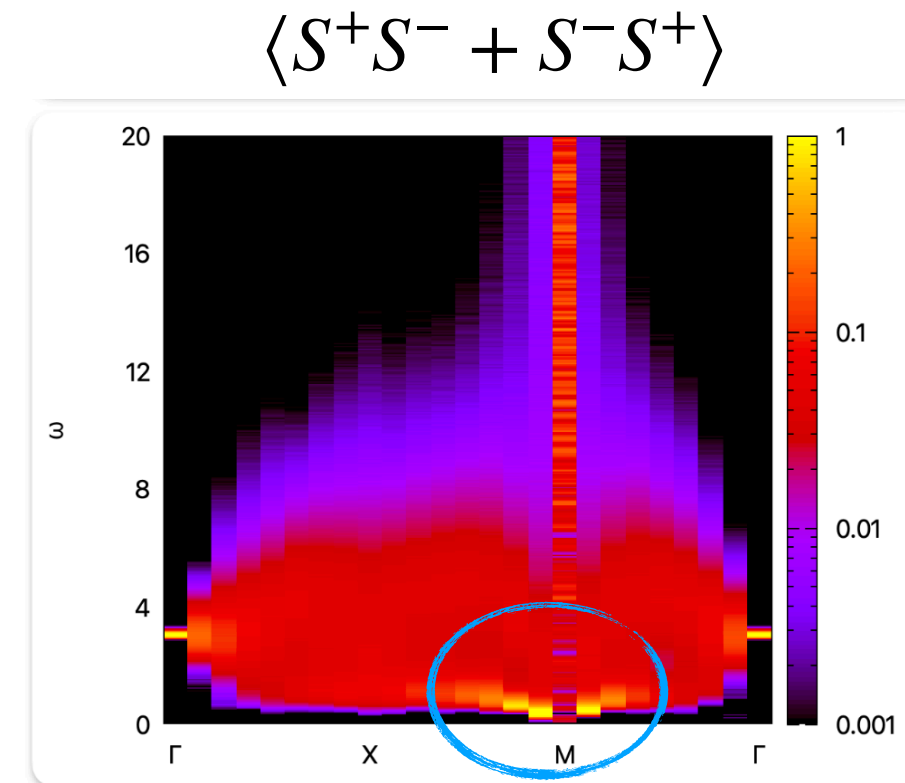
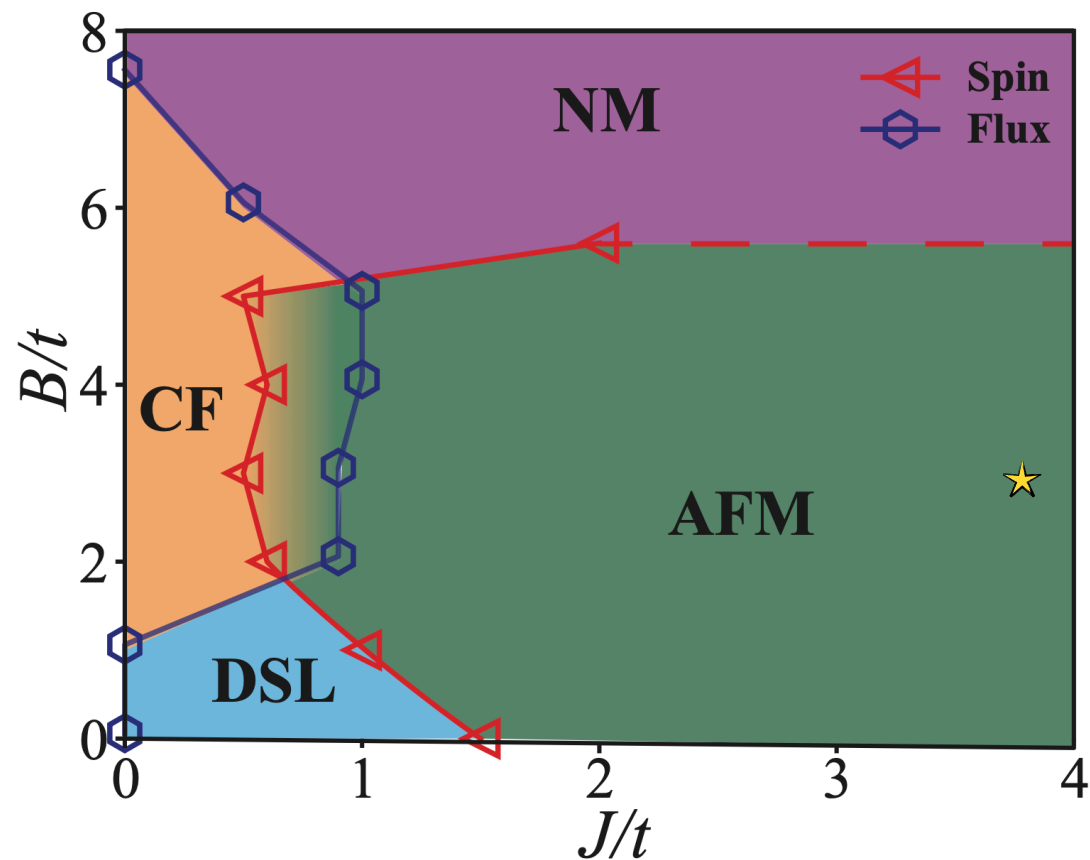
Correlation ratio



$B/t=2$

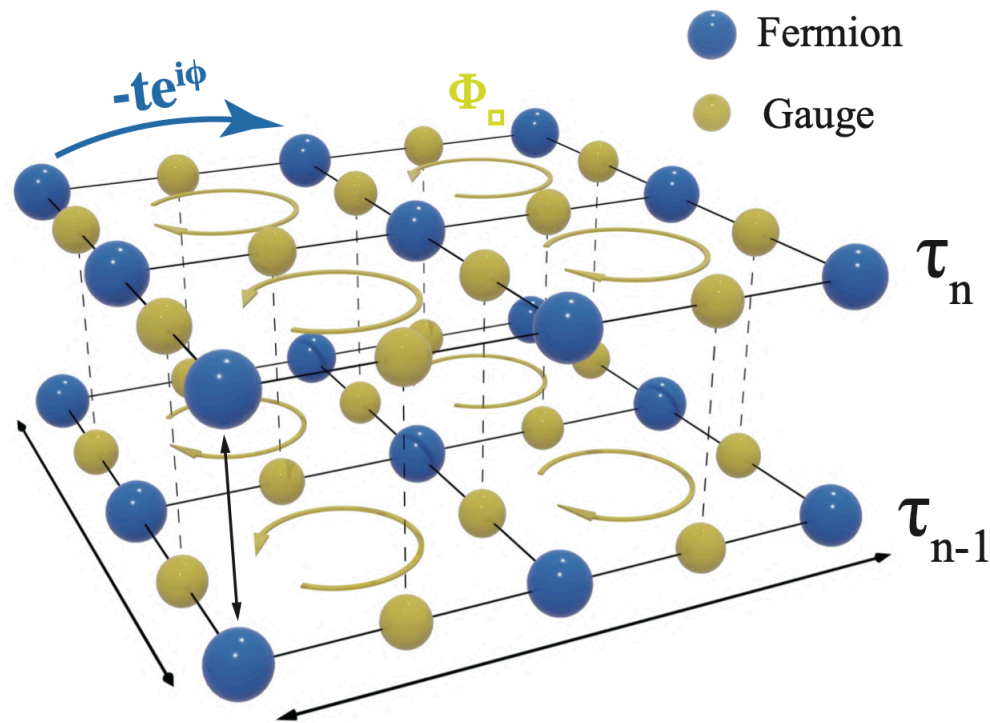
Fermions and spins

- AFM order creates a *mass* for the fermions
- Polyakov argument implies monopoles are condensed
- Here $U(1)_s$ and $U(1)_m$ separately broken: 2 Goldstone modes.



Emergent spin wave

Compact versus non-compact



$$\begin{aligned}
 S = & \sum_{i,n} \left[\bar{\psi}_i(\tau_n)(\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \bar{\psi}_i(\tau_n) \sigma^z \psi_i(\tau_n) \right] \\
 & - t \sum_{\langle ij \rangle, n} \left[e^{ia_{ij}(\tau_n)} \bar{\psi}_i(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right] \\
 & + \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2
 \end{aligned} \tag{1}$$

“Non-compact” gauge field: prohibits “monopoles” in the simulation

Proper model is “compact”: what are the corrections?

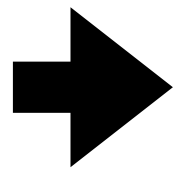
Compact versus non-compact

- Lagrangian should be supplemented by “monopole fugacity” terms

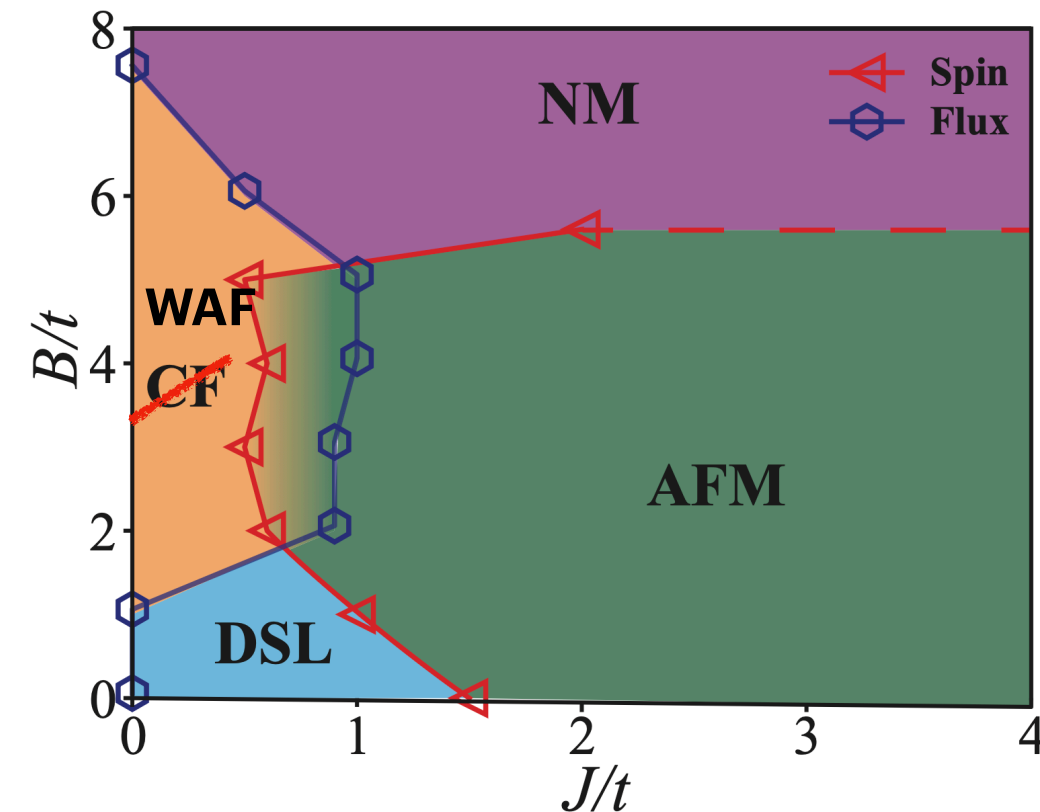
$$\mathcal{L}' = \lambda \mathcal{M}^+ e^{iQ \cdot x} + \text{h.c.} + \dots$$

- This mixes original condensate into pure spin

$$\langle S^+ \rangle = \lambda \langle S^+ \mathcal{M}^+ \rangle_0 e^{iQ \cdot x} = \lambda \langle \mathcal{M}_{\text{ord}}^+ \rangle e^{iQ \cdot x}$$

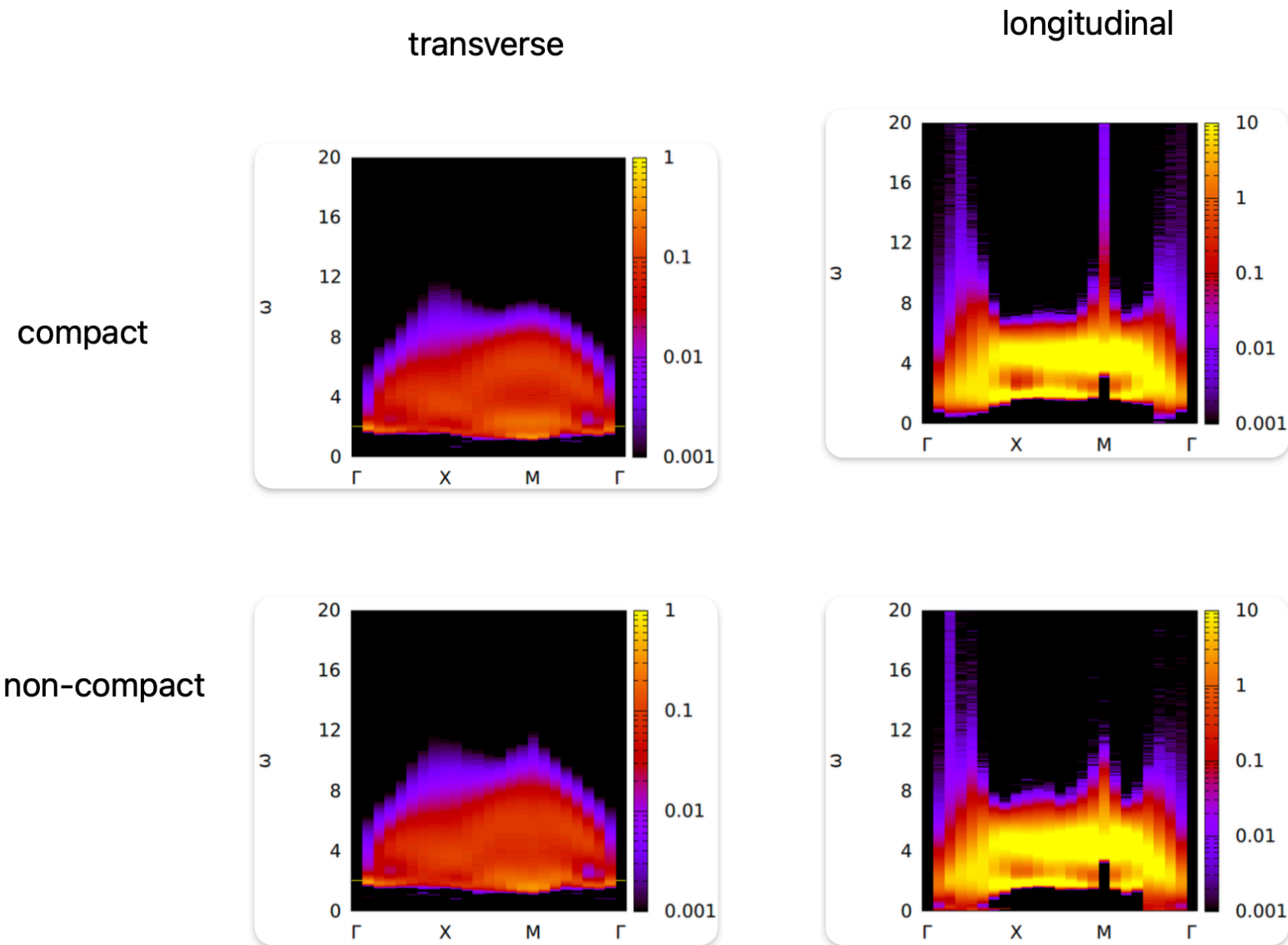


CF phase becomes weakly ordered.
Precise order depends on how
monopoles are added to the model.



Compact versus non-compact

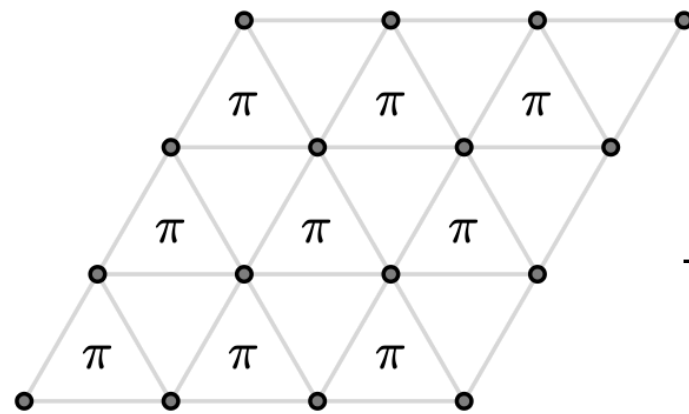
In our current simulation, it is difficult to see the difference visually. We are working to improve numerics of compact model.



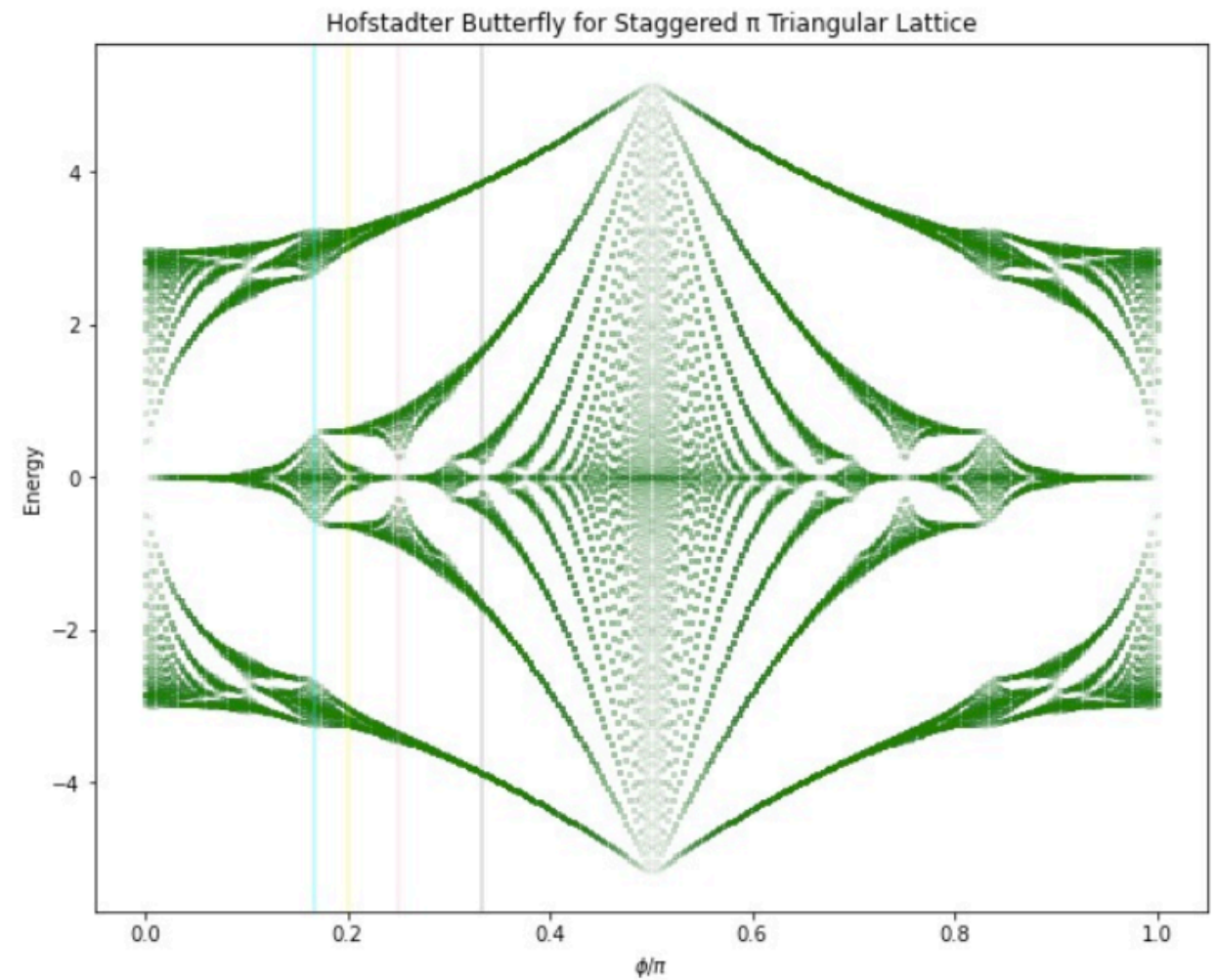
Wavefunction study

Not restricted by sign problem

triangular lattice

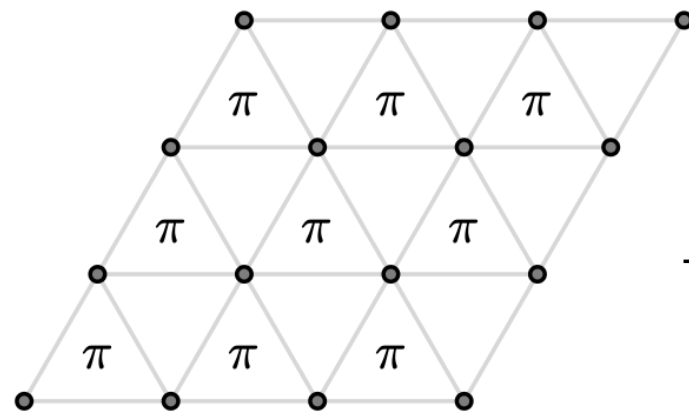


+ uniform flux

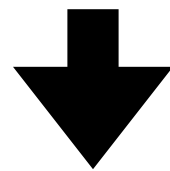


Wavefunction study

triangular lattice

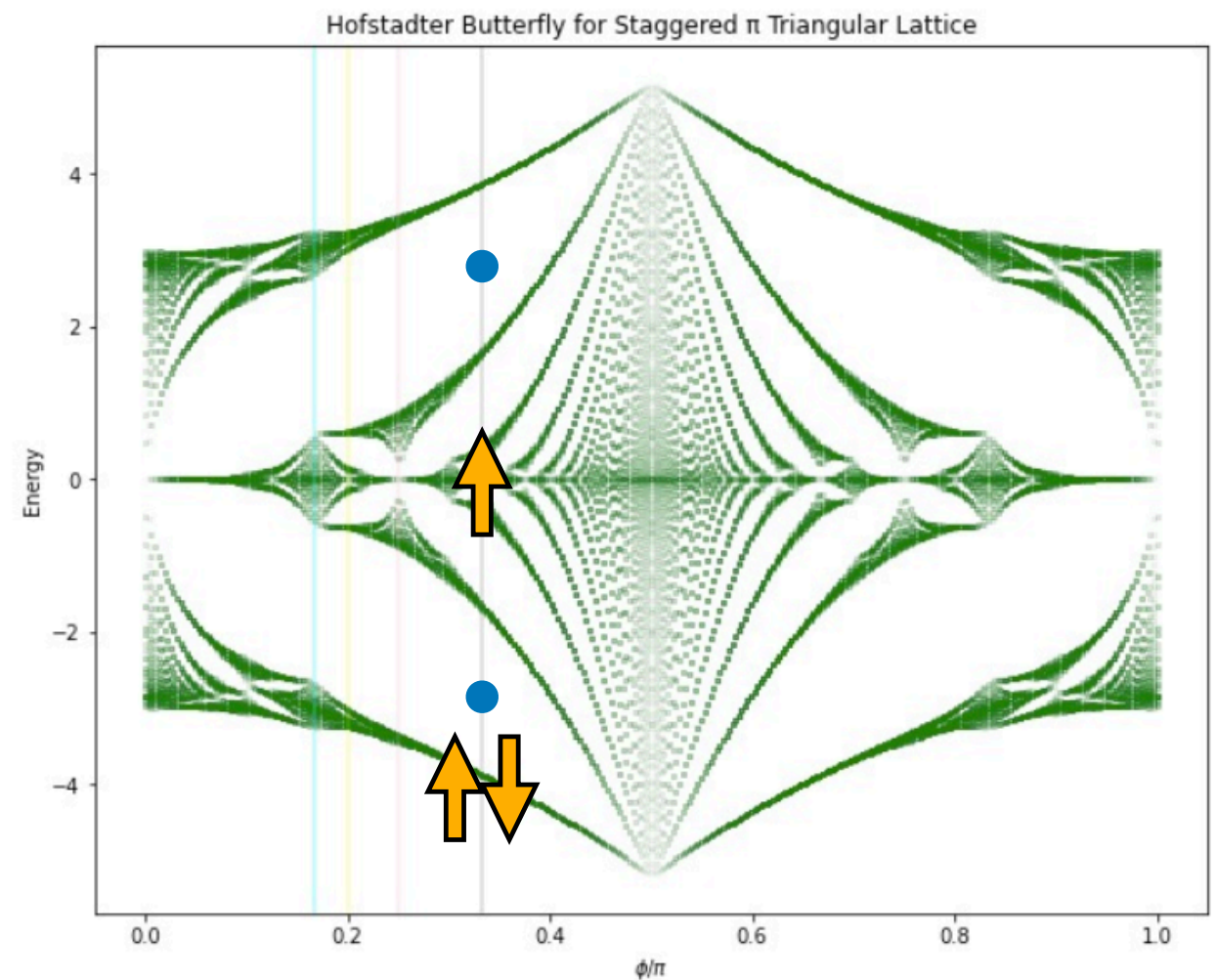


+ uniform flux



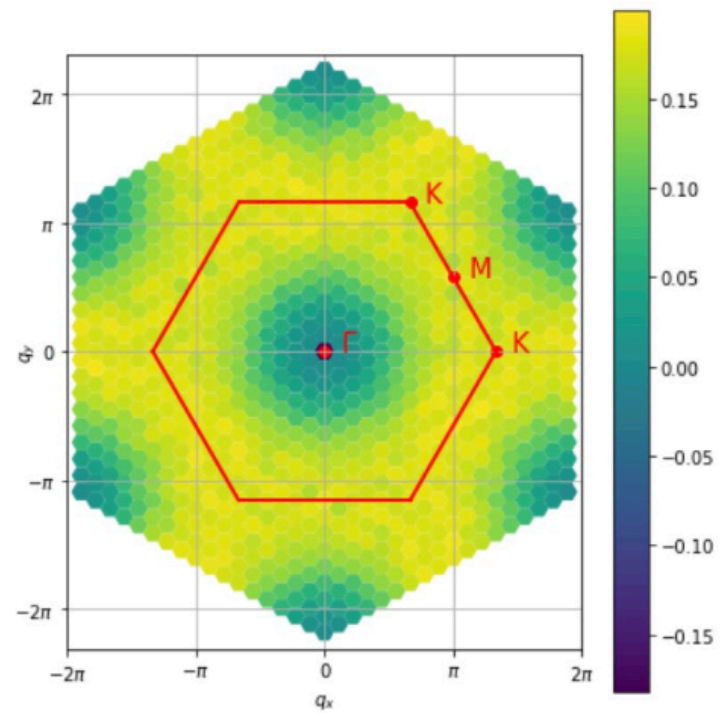
Gutzwiller projection

$$M = \frac{2}{3} M_s$$

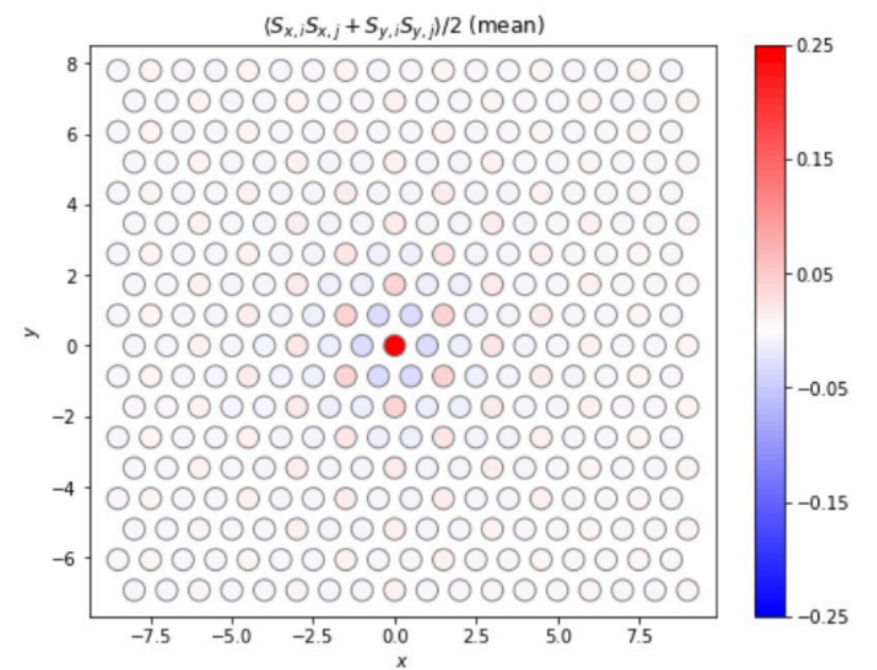
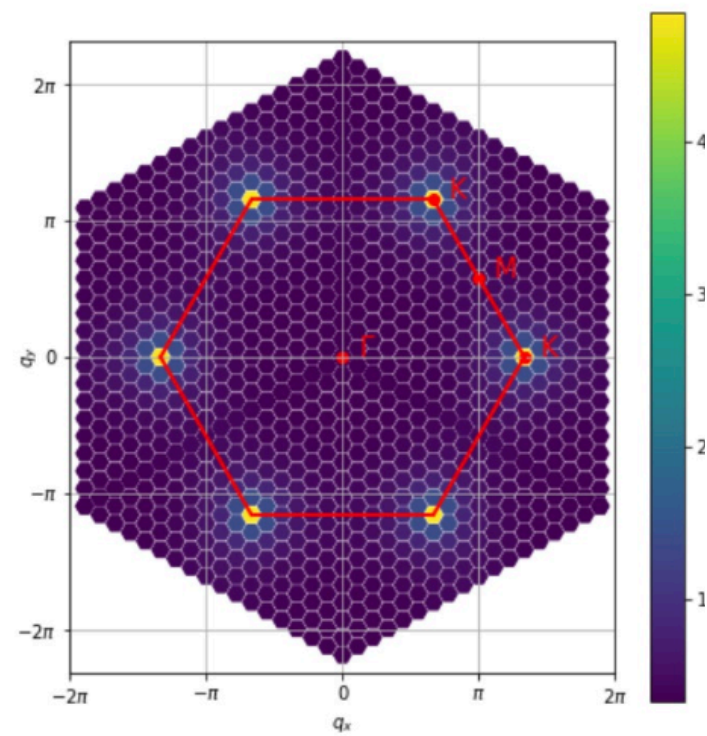


Wavefunction study

S_{zz}



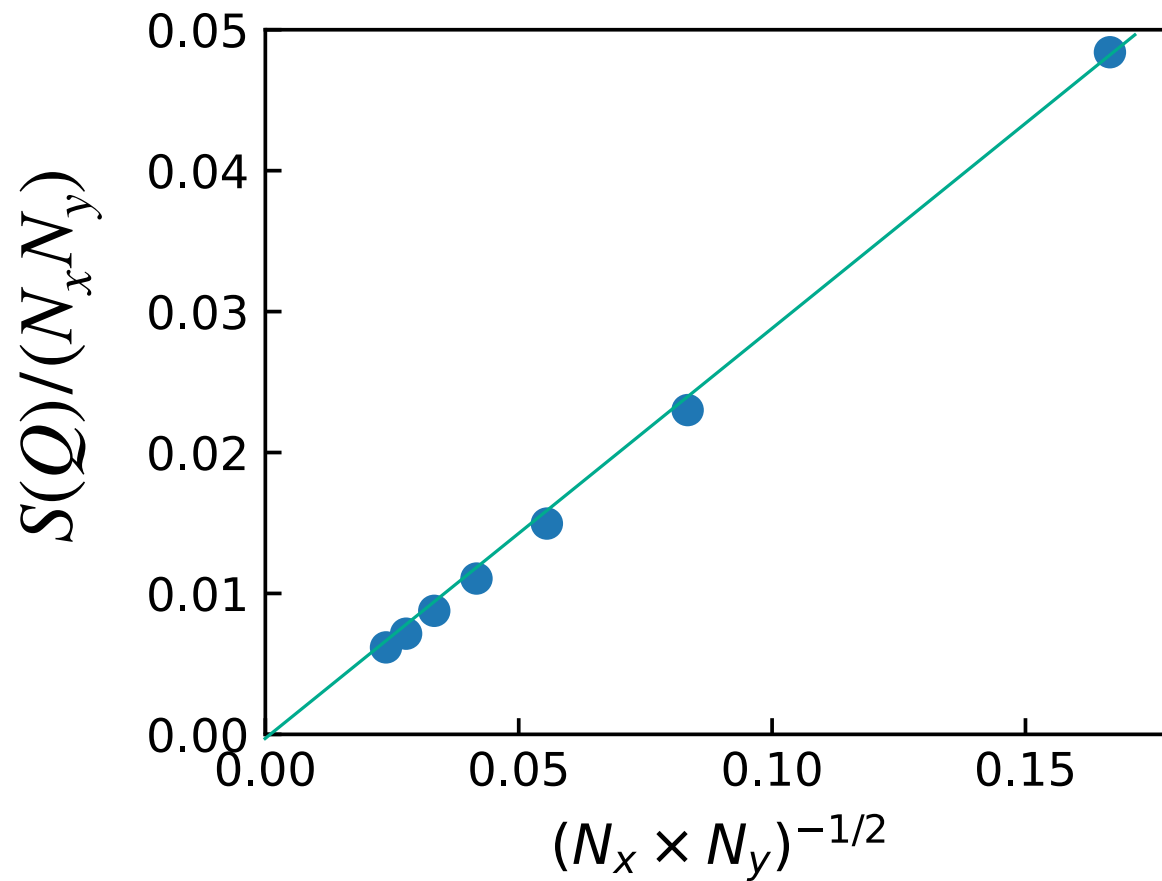
S^{+-}



Ordered?

Wavefunction study

Néel order
parameter



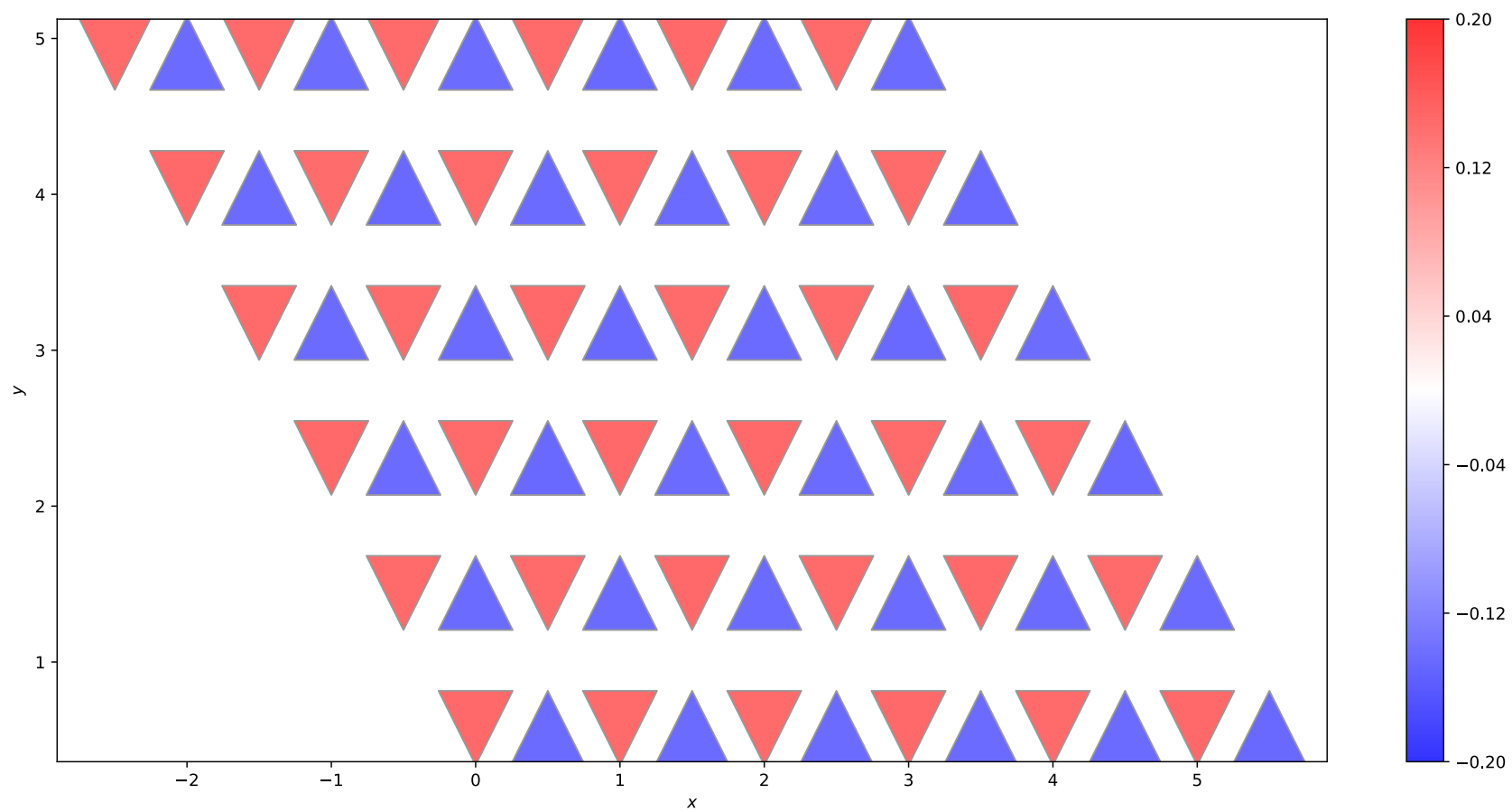
$$\langle S_i^+ S_j^- \rangle \sim \frac{e^{i\mathbf{K} \cdot (x_i - x_j)}}{|x_i - x_j|}$$

Power-law order

If you have a
general argument
for this I would be
interested.

Wavefunction study

$$\langle S_i \cdot S_j \times S_k \rangle$$



Long-range chiral order

Ամենակարճաձույն շնորհակալությանը
Спасибо



Charles Aznavour (Shahnur Vaghinak Aznavourian)
Musée Grévin, Paris

