

# Quantum excitations, real and simulated

Leon Balents,  
KITP

Recent Progress in Quantum Materials 2023





# Collaborators



Leonie Woodland  
Oxford



Radu Coldea  
Oxford



Rimika Jaiswal  
UCSB



Izabella Lovas  
EPiQS Moore Fellow  
KITP

# Outline

- Quantum Ising chain: flat bands and soliton interactions
- Computing excitations on a quantum computer

# Outline

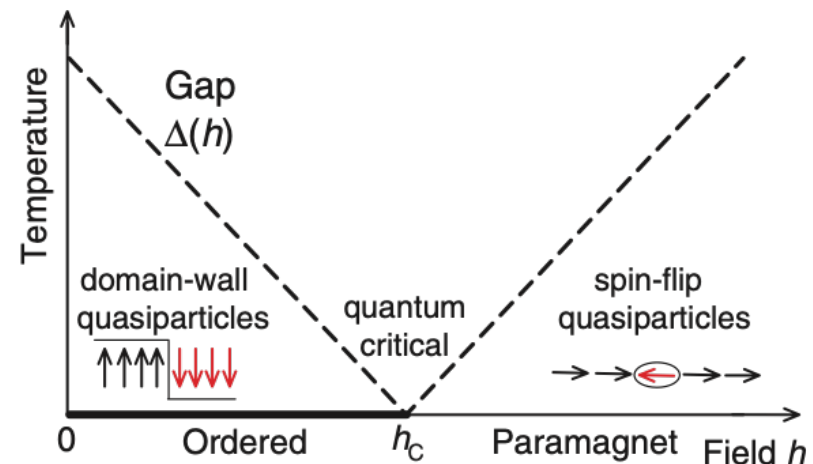
- Quantum Ising chain: flat bands and soliton interactions
- Computing excitations on a quantum computer



# Quantum Ising chain

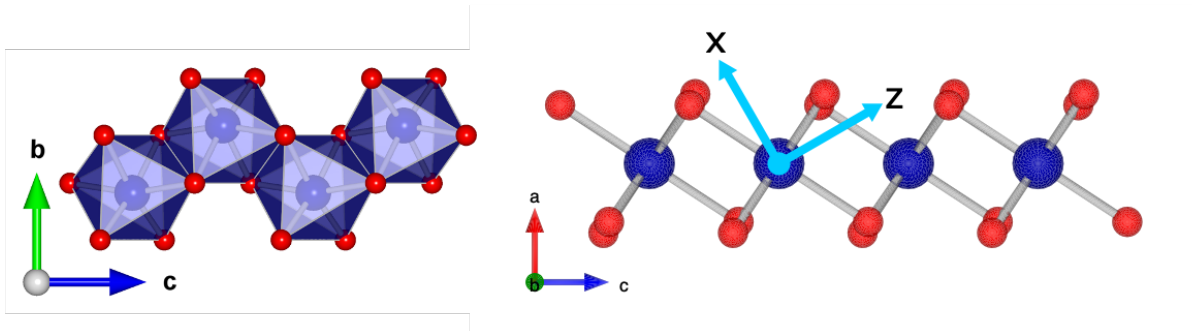
$$H = \sum_i -J_z S_i^z S_{i+1}^z - h_y S_i^y$$

Solitons = the simplest example  
of non-local excitations

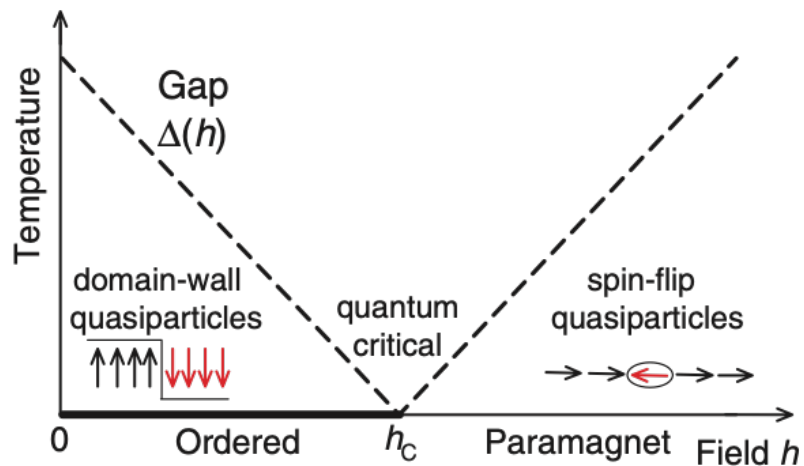


$S^z=1$  excitations are soliton pairs and form a continuum

# CoNb<sub>2</sub>O<sub>6</sub>

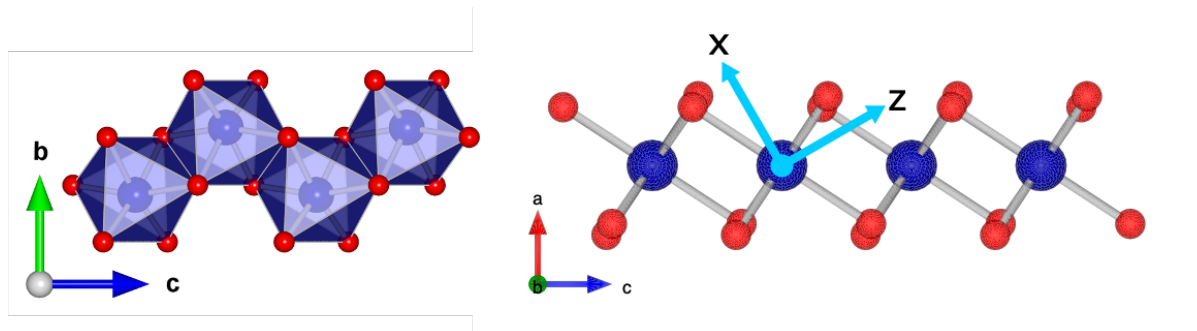


R. Coldea *et al*, Science **327**, 117 (2010)

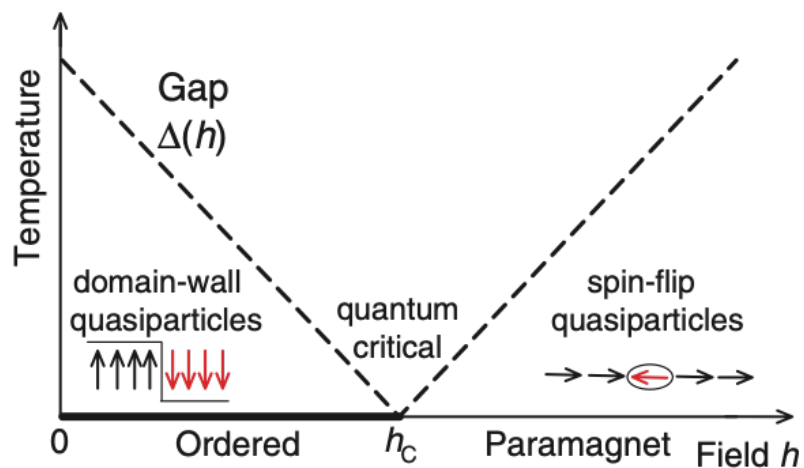


~5.5T

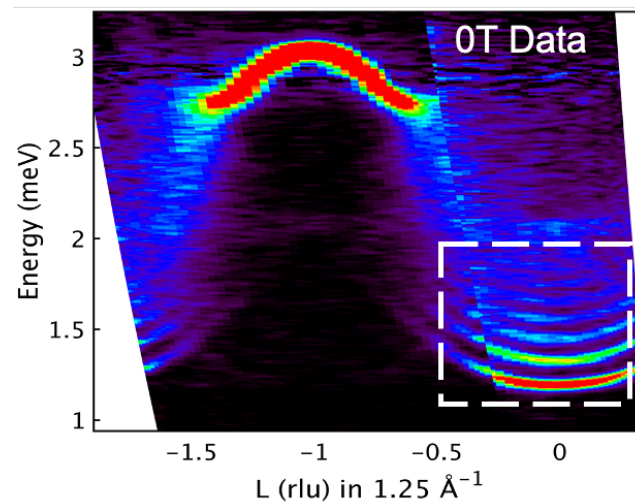
# CoNb<sub>2</sub>O<sub>6</sub>



R. Coldea *et al*, Science **327**, 117 (2010)

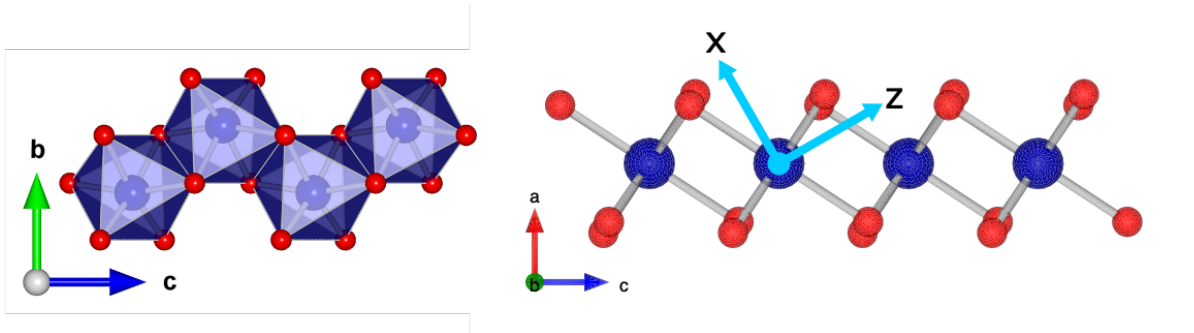


~5.5T

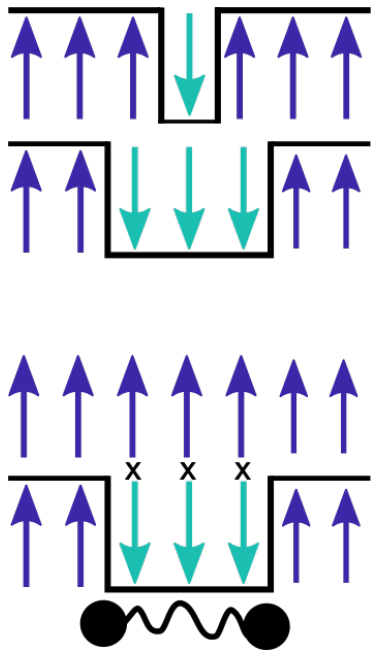




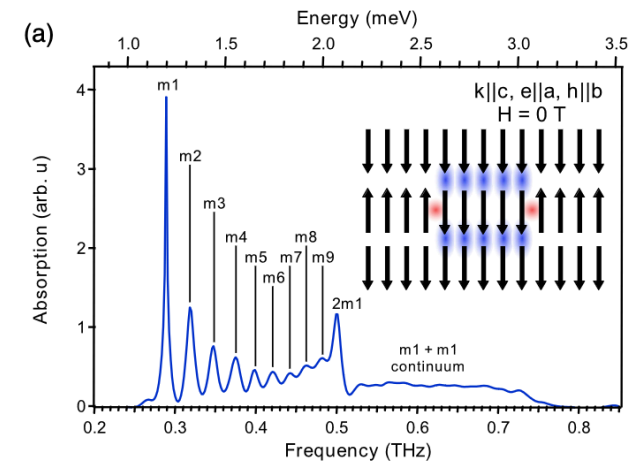
# CoNb<sub>2</sub>O<sub>6</sub>



R. Coldea *et al*, Science **327**, 117 (2010)

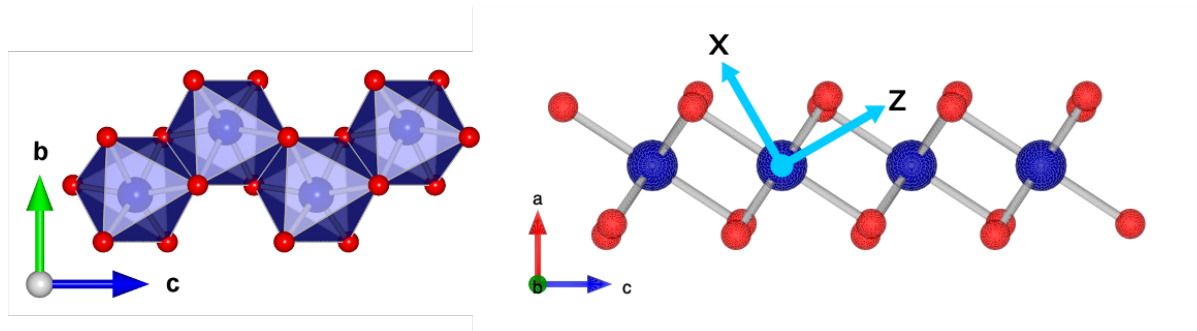


Continuum splits into tower of weakly bound states, due to weak *longitudinal* field from neighboring chains

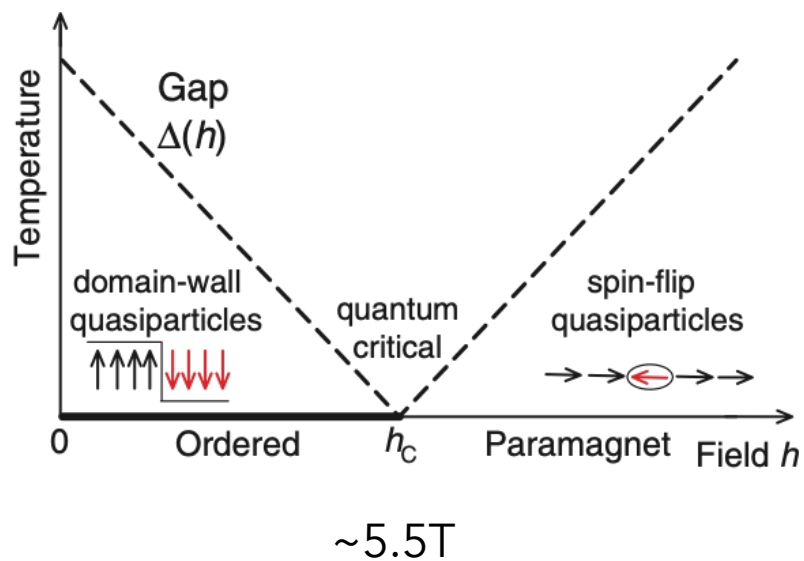


C. Morris *et al*, 2014

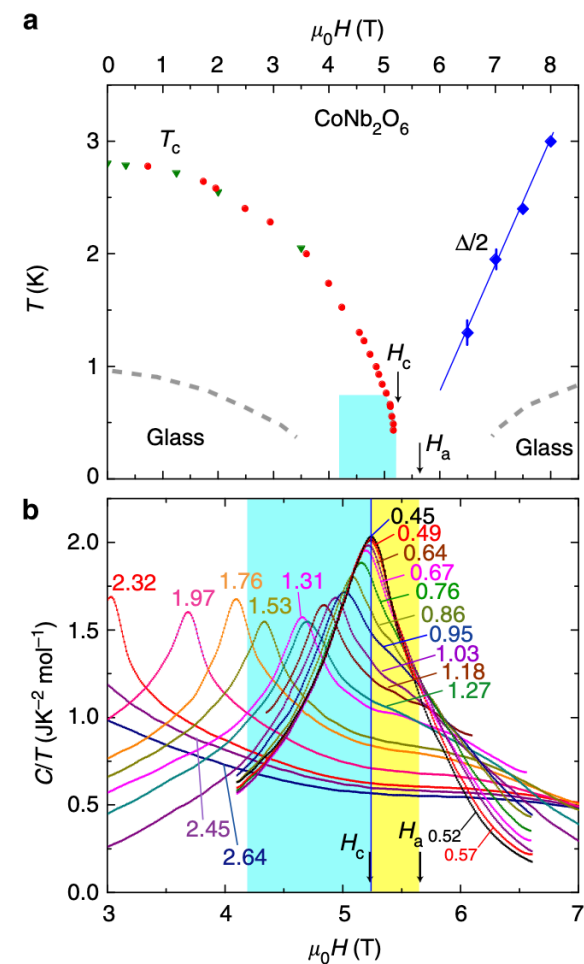
# CoNb<sub>2</sub>O<sub>6</sub>



R. Coldea *et al*, Science **327**, 117 (2010)

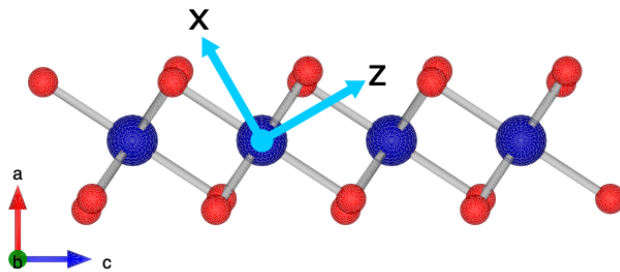
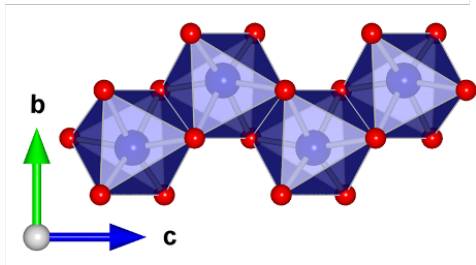


$\sim 5.5\text{T}$

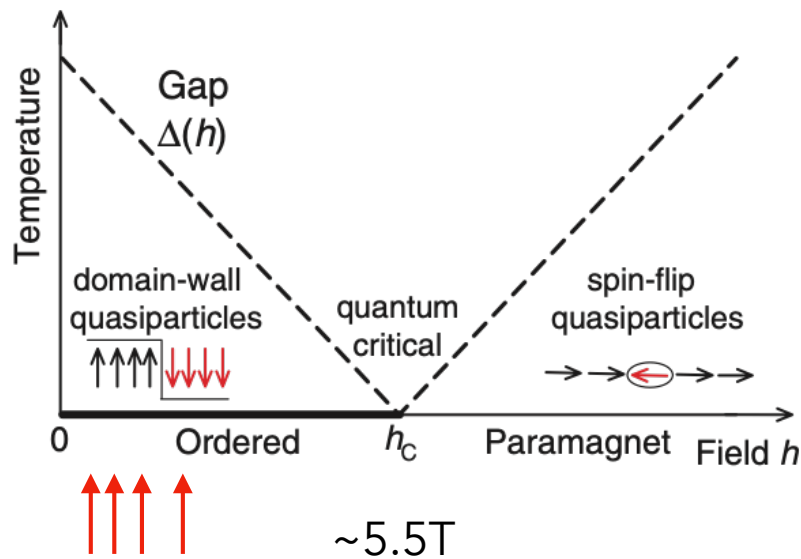


T. Liang *et al*, 2015

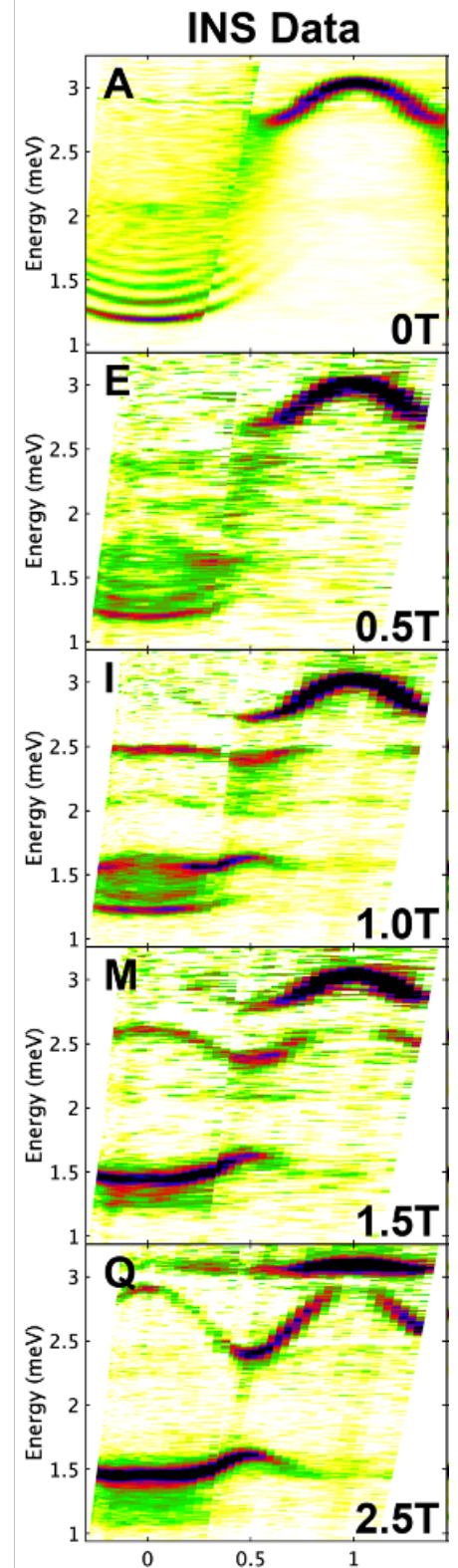
# CoNb<sub>2</sub>O<sub>6</sub>



L. Woodland et al, arXiv:2306.01948



Surprising complex behavior in between!





# A refined model

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3$$

where

$$\mathcal{H}_1 = J \sum_j \left[ -S_j^z S_{j+1}^z - \lambda_S (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) \right. \\ \left. + (-1)^j \lambda_{yz} (S_j^y S_{j+1}^z + S_j^z S_{j+1}^y) \right] + \sum_j h_y S_j^y,$$

XY

Staggered anisotropy

$$\mathcal{H}_2 = J \sum_j \left[ -\lambda_A (S_j^x S_{j+1}^x - S_j^y S_{j+1}^y) \right. \\ \left. + \lambda_{AF} S_j^z S_{j+2}^z + \lambda_{AF}^{xy} (S_j^x S_{j+2}^x + S_j^y S_{j+2}^y) \right],$$

M. Fava et al, 2020; L. Woodland et al, arXiv 2308.07699

# A refined model

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3$$

where

$$\mathcal{H}_1 = J \sum_j \left[ -S_j^z S_{j+1}^z - \lambda_S (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + (-1)^j \lambda_{yz} (S_j^y S_{j+1}^z + S_j^z S_{j+1}^y) \right] + \sum_j h_y S_j^y,$$

$$\mathcal{H}_2 = J \sum_j \left[ -\lambda_A (S_j^x S_{j+1}^x - S_j^y S_{j+1}^y) + \lambda_{AF} S_j^z S_{j+2}^z + \lambda_{AF}^{xy} (S_j^x S_{j+2}^x + S_j^y S_{j+2}^y) \right],$$

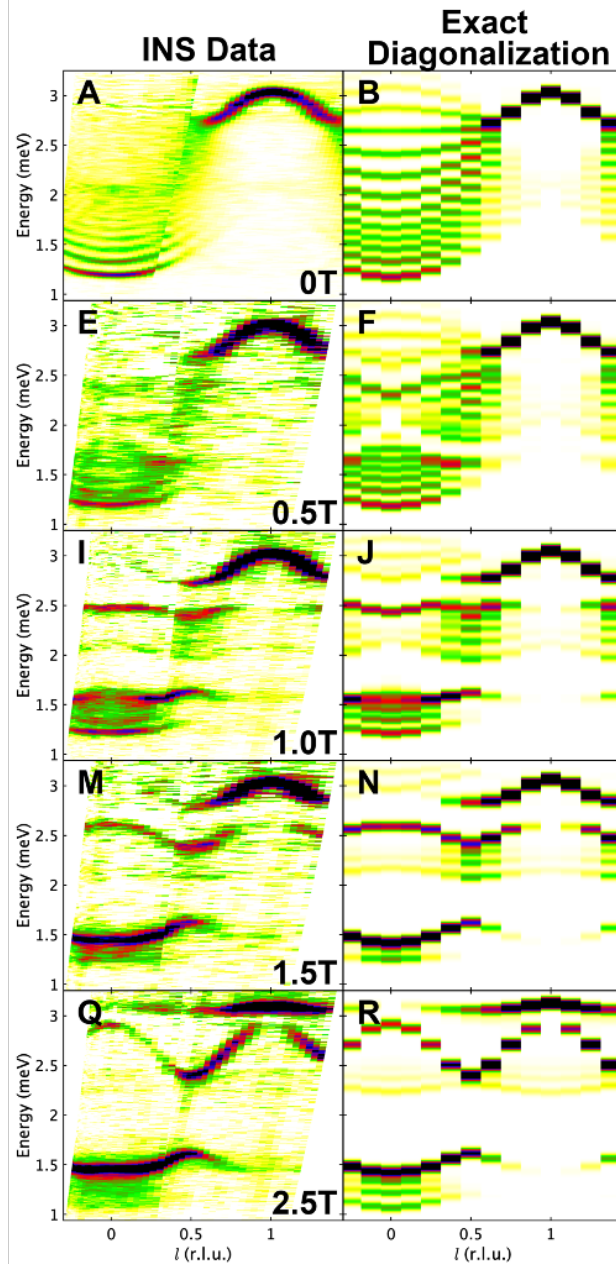
$J$	2.48(2) meV
$\lambda_S$	0.251(6)
$\lambda_{yz}$	0.226(3)
$g_y$	3.32(2)
$\lambda_A$	-0.021(1)
$\lambda_{AF}$	0.077(3)
$\lambda_{AF}^{xy}$	0.031(1)
$\lambda_{MF}$	0.0158(2)

M. Fava et al, 2020; L. Woodland et al, arXiv 2308.07699

(These are fit from different experiments)

Smallness of perturbations implies we should be able to understand this!

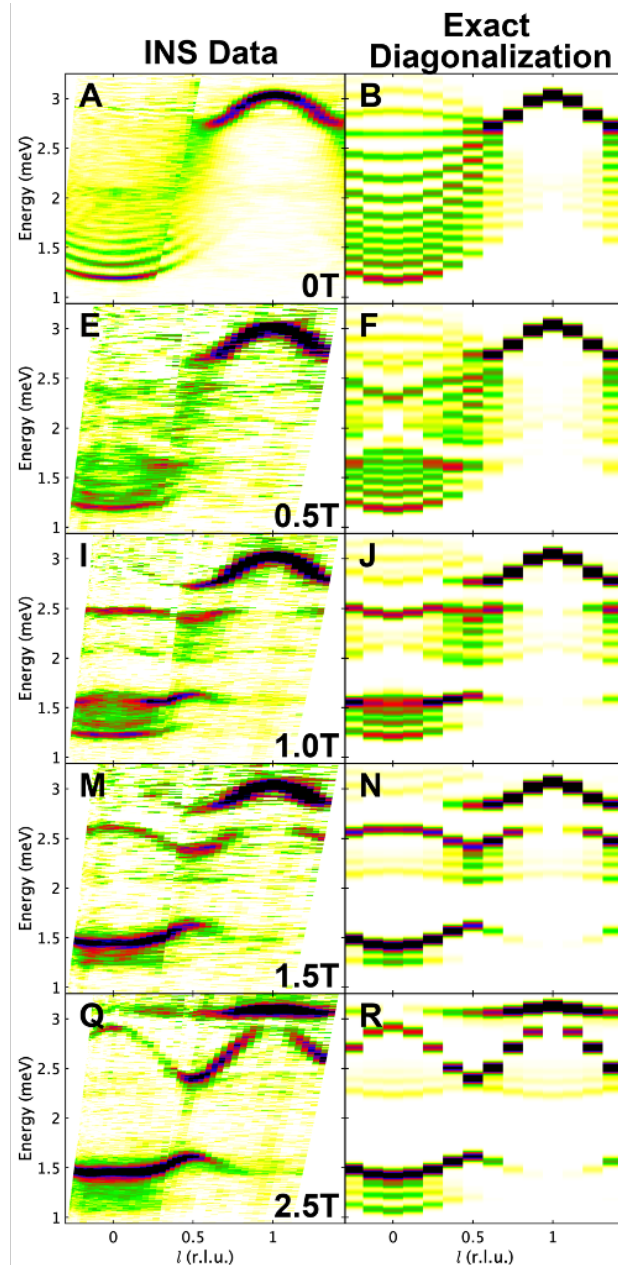
# The model works



I think this speaks  
for itself



# The model works

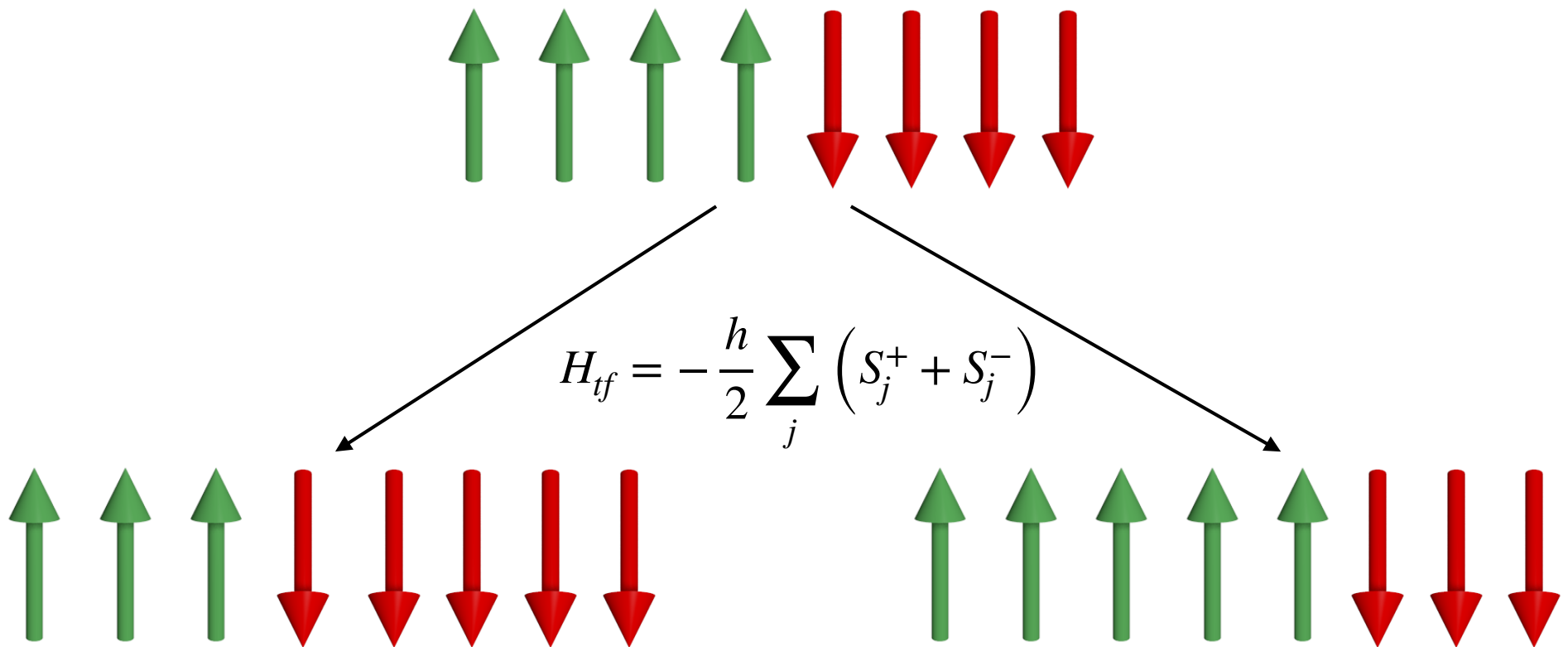


I think this speaks  
for itself

But let's try to  
understand it

# One soliton

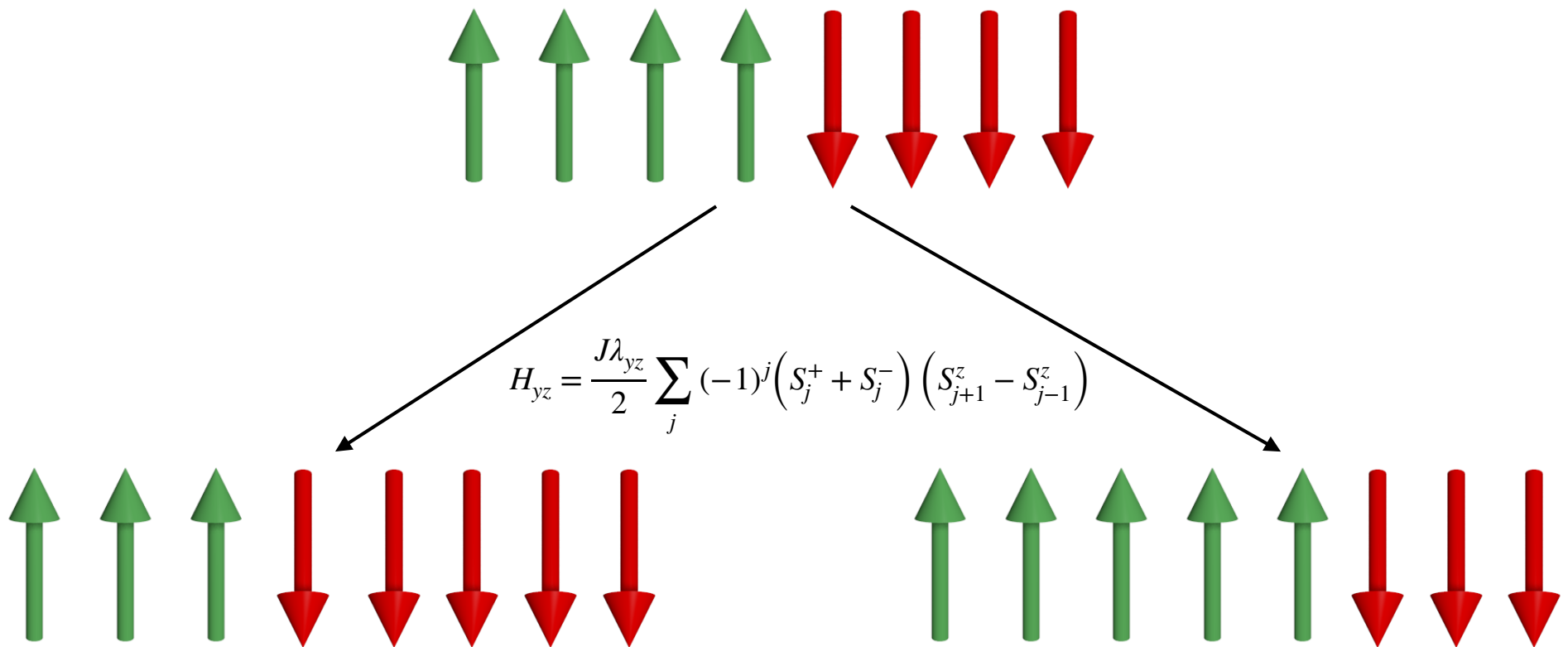
Start with one soliton:



Hopping amplitude  $h/2$

# One soliton

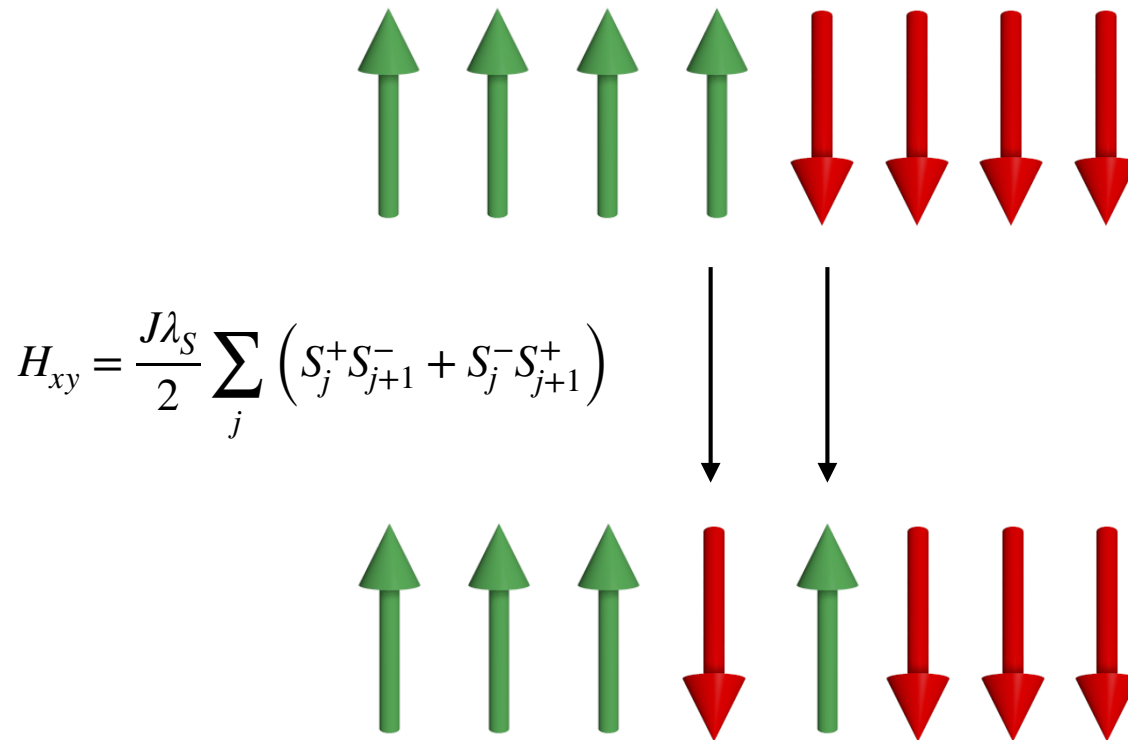
Start with one soliton:



Hopping amplitude  $\pm J\lambda_{yz}/2$

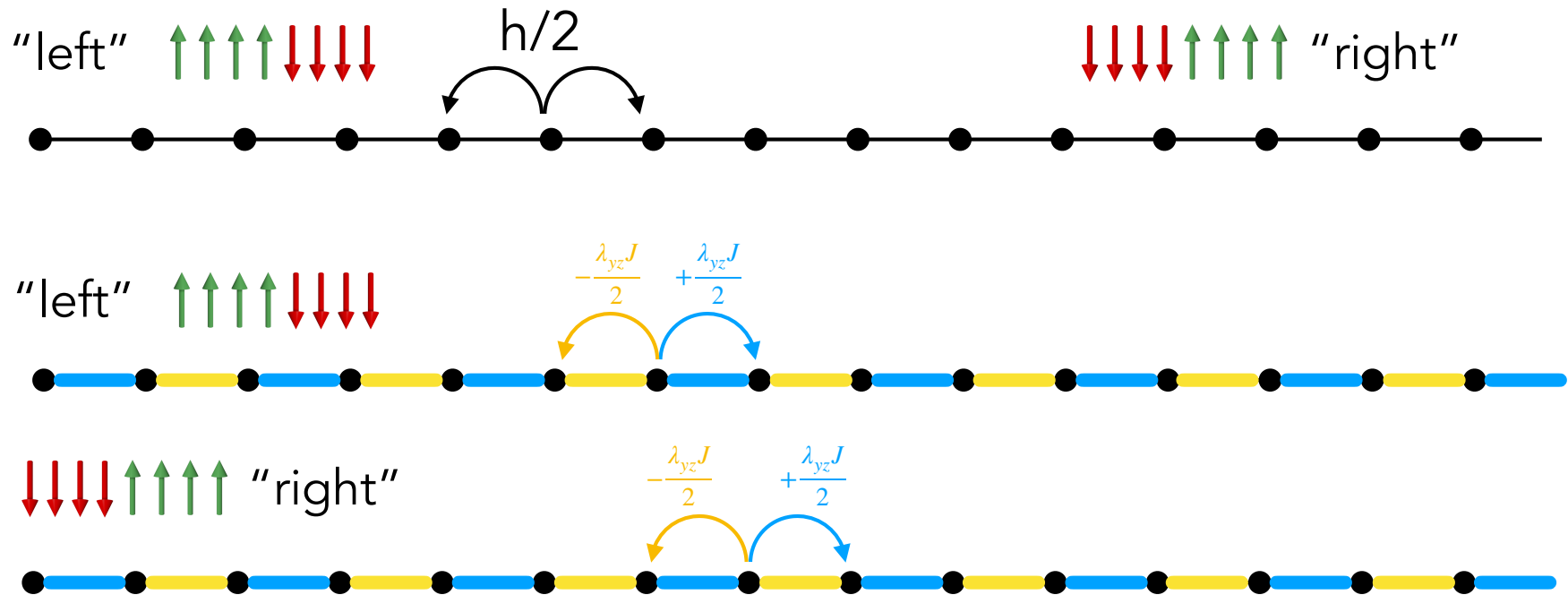
# One soliton

Start with one soliton:

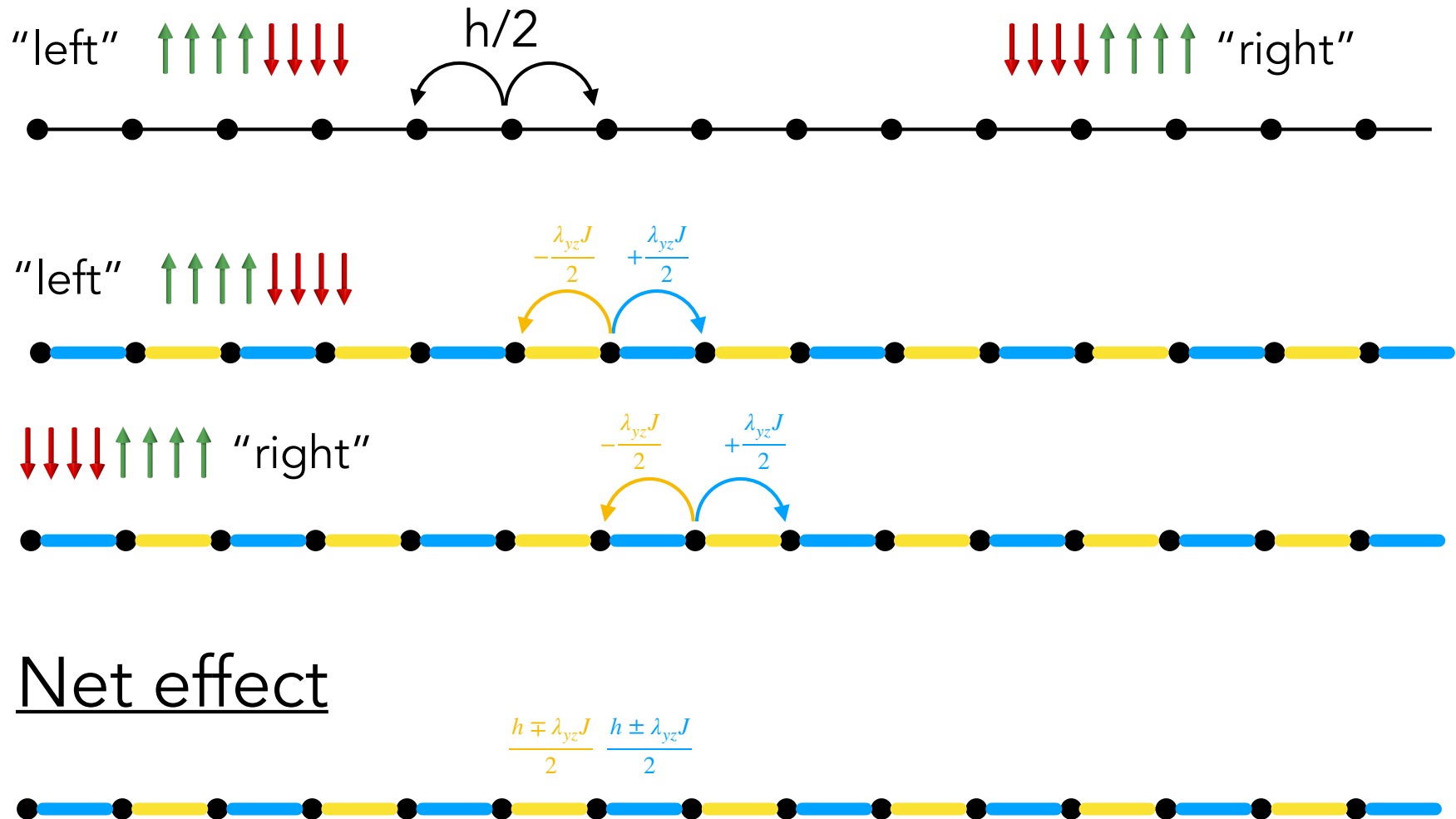


3 soliton state: high energy

# Hopping picture



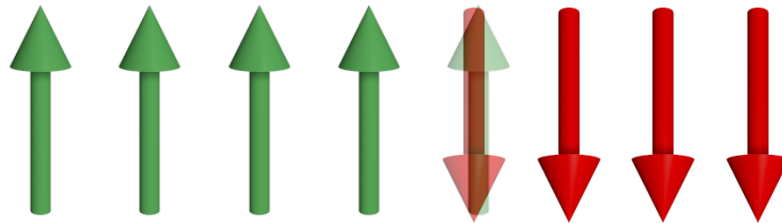
# Hopping picture



$$h = \lambda_{yz}J: \text{localized solitons!}$$

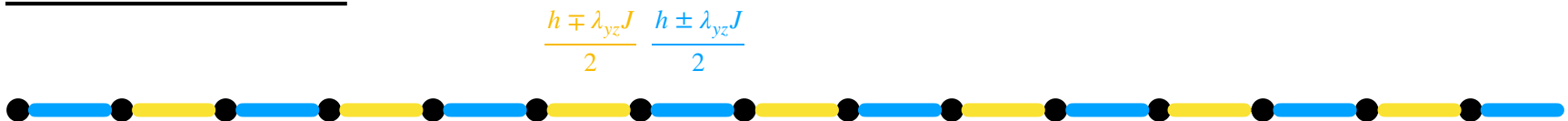


# Hopping picture



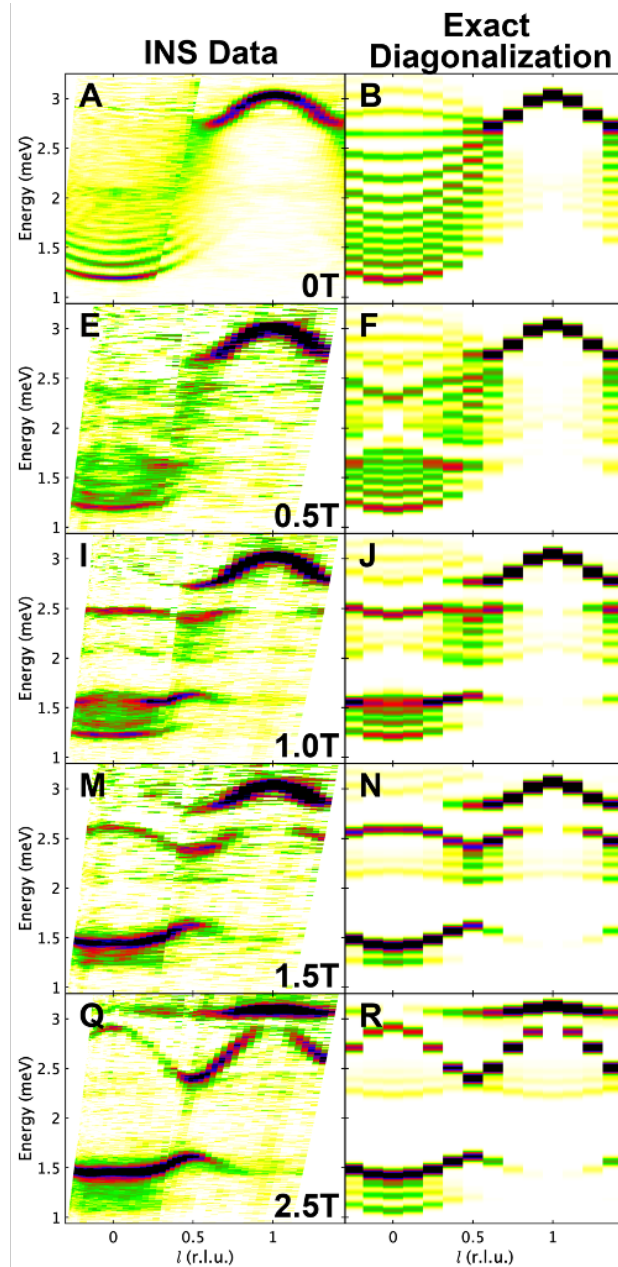
Another view of a localized soliton

Net effect



$h = \lambda_{yz}J$ : localized solitons!

# Back to experiment



← This is around the  
flat band condition

$$h = \lambda_{yz} J$$

# Two solitons

We expect that flat band solitons  
interact strongly when nearby.

$$|j_L, j_R\rangle = |\cdots \uparrow \uparrow_{j_L-1} \downarrow_{j_L} \cdots \downarrow_{j_R-1} \uparrow_{j_R} \uparrow \cdots\rangle$$

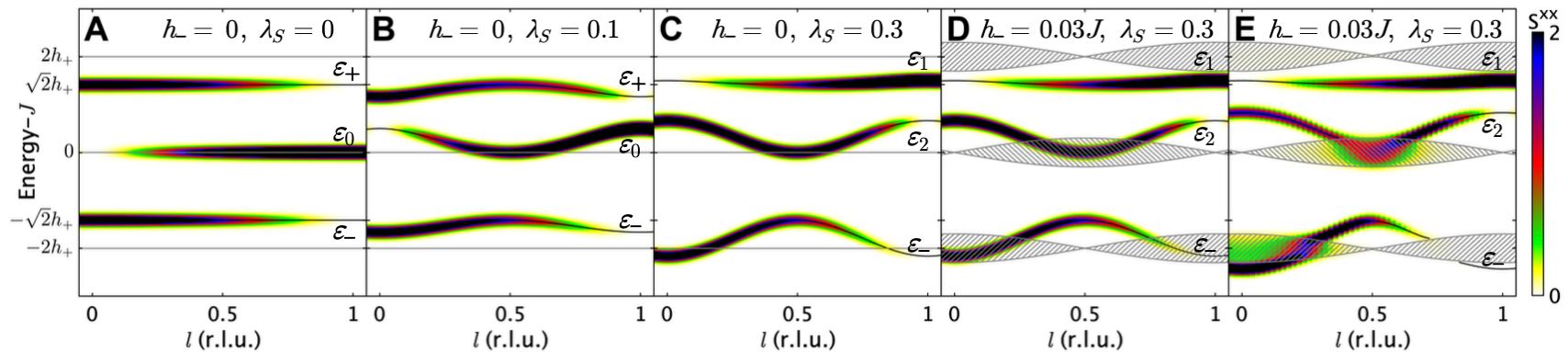
$$|\Psi\rangle = \sum_{j_L < j_R} \Psi(j_L, j_R) |j_L, j_R\rangle.$$

↑  
Obeys a 2-particle Schrödinger equation

# Two solitons

3 bound state modes

2 symmetric  
1 antisymmetric



Exactly  
flat

Small XY  
coupling

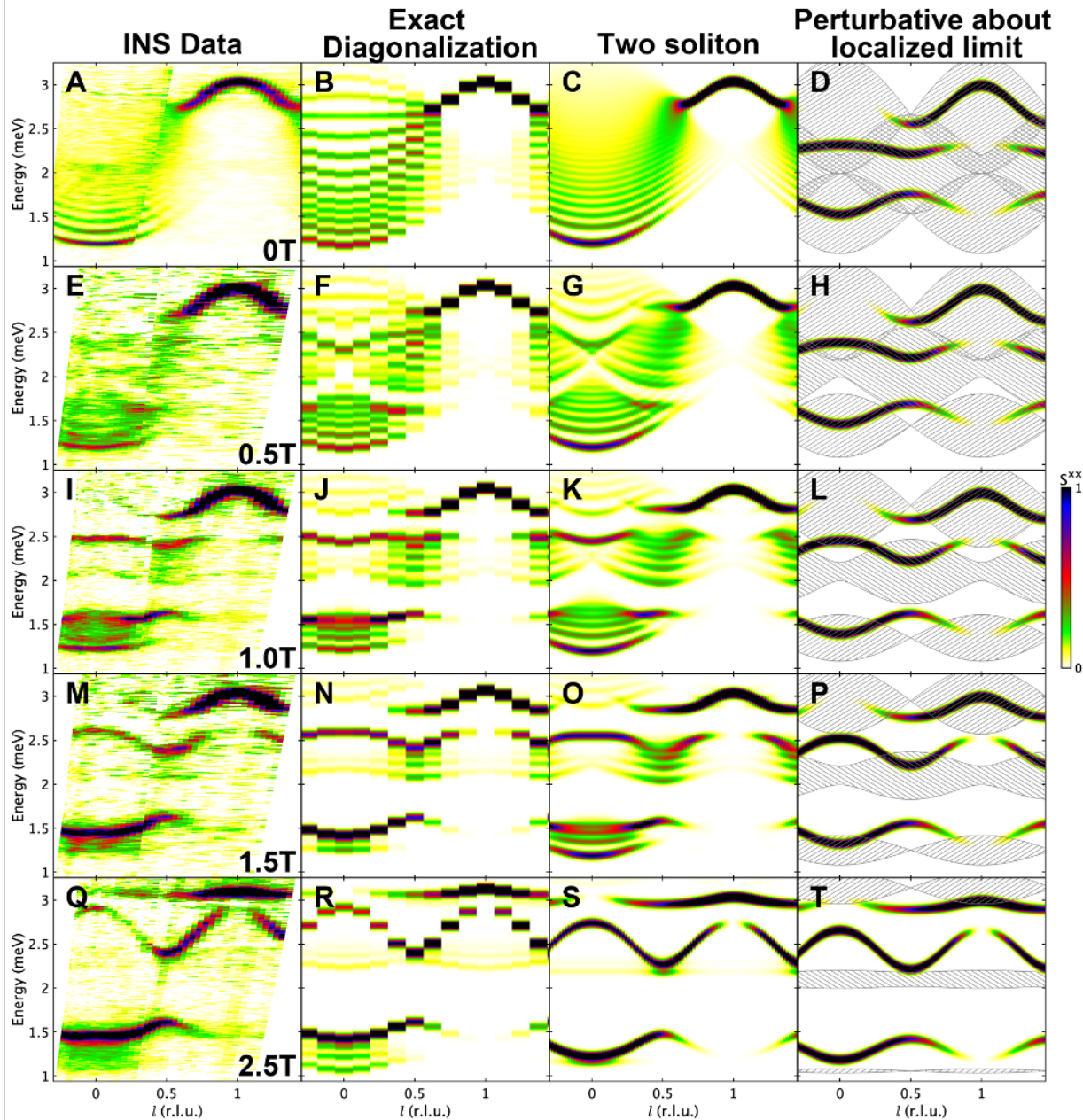
Experimental  
XY coupling

Tuning away  
(approx)

Tuning away  
(full)

Band inversion

# Full comparison



- 2 soliton states highly accurate
- Localized soliton approximation quantitative near 2.5T as expected
- Visible features of band inversion

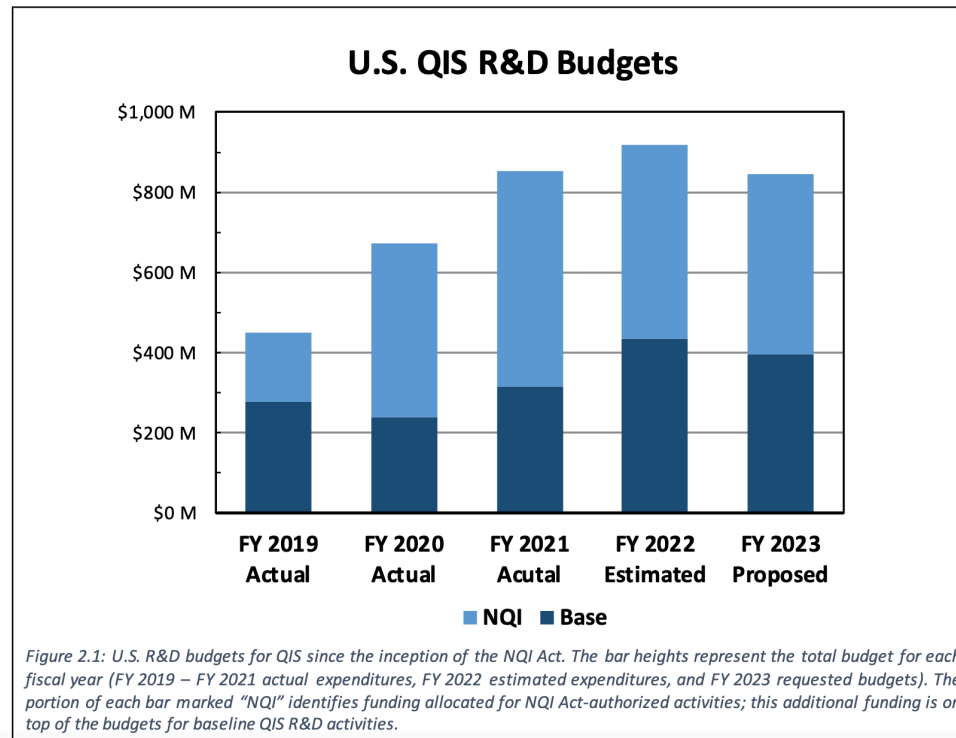
# Outline

- Quantum Ising chain: flat bands and soliton interactions
- Computing excitations on a quantum computer



# Quantum science

- \$\$:
- Microsoft Quantum: probably > \$300M per year.
- Many others!



# Quantum science

- People:
  - Compare arXiv “new” listings:
    - 172 CM vs 131 Quantum
- Experimentalists going to private sector
- Theorists mass movement to QI:
  - Let's look at UCSB faculty



1. [arXiv:2306.00058](#) [pdf, other] [quant-ph](#) cond-mat.stat-mech

### Universality of the cross entropy in $\mathbb{Z}_2$ symmetric monitored quantum circuits

**Authors:** Maria Tikhonovskaya, Ali Lavasani, Matthew P. A. Fisher, Sagar Vijay

Submitted 14 August, 2023; v1 submitted 31 May, 2023; **originally announced** June 2023.

Comments: 12+6 pages, 16 figures. V2: References added

2. [arXiv:2304.13198](#) [pdf, other] [quant-ph](#) cond-mat.stat-mech

### Continuous symmetry breaking in adaptive quantum dynamics

**Authors:** Jacob Hauser, Yaodong Li, Sagar Vijay, Matthew P. A. Fisher

Submitted 25 April, 2023; **originally announced** April 2023.

Comments: 17 pages, 10 figures

3. [arXiv:2303.01533](#) [pdf, other] [quant-ph](#) cond-mat.stat-mech

### Measurement-induced Floquet enriched topological order

**Authors:** DinhDuy Vu, Ali Lavasani, Jong Yeon Lee, Matthew P. A. Fisher

Submitted 2 March, 2023; **originally announced** March 2023.

Comments: 6+7 pages, 12 figures

4. [arXiv:2210.11547](#) [pdf, other] [quant-ph](#) cond-mat.dis-nn cond-mat.stat-mech

### Coherence requirements for quantum communication from hybrid circuit dynamics

**Authors:** Shane P. Kelly, Ulrich Poschinger, Ferdinand Schmidt-Kaler, Matthew P. A. Fisher, Jamir Marino

Submitted 23 May, 2023; v1 submitted 20 October, 2022; **originally announced** October 2022.

Comments: 19 pages, 12 figures

5. [arXiv:2209.00609](#) [pdf, other] [quant-ph](#) cond-mat.stat-mech [doi](#) 10.1103/PhysRevLett.130.220404

### Cross Entropy Benchmark for Measurement-Induced Phase Transitions

**Authors:** Yaodong Li, Yijian Zou, Paolo Glorio, Ehud Altman, Matthew P. A. Fisher

Submitted 7 June, 2023; v1 submitted 1 September, 2022; **originally announced** September 2022.

Comments: 7+8 pages, 6 figures. v2: 7+9 pages, 3+3 figures. Updated discussions on sample size (Fig. 2d, 2e), and new results from ra



1. [arXiv:2305.13240](#) [pdf, other] [cond-mat.str-el](#) cond-mat.stat-mech [quant-ph](#)

### Entanglement Spectrum as a diagnostic of chirality of Topological Spin Liquids: Analysis of an $SU(3)$ PEPs

**Authors:** Mark J. Arildsen, Ji-Yao Chen, Norbert Schuch, Andreas W. W. Ludwig

Submitted 22 May, 2023; **originally announced** May 2023.

Comments: 49 pages, 14 figures, 8 tables

2. [arXiv:2302.09094](#) [pdf, other] [cond-mat.stat-mech](#) cond-mat.dis-nn cond-mat.str-el [quant-ph](#)

### Measurement-induced entanglement transitions in quantum circuits of non-interacting fermions: Born-rule versus forced measurements

**Authors:** Chao-Ming Jian, Hassan Shapourian, Beile Bauer, Andreas W. W. Ludwig

Submitted 17 February, 2023; **originally announced** February 2023.

Comments: 16+5 pages, 6 figures

3. [arXiv:2207.03246](#) [pdf, other] [cond-mat.str-el](#) cond-mat.stat-mech [quant-ph](#)

### Entanglement spectra of non-chiral topological (2+1)-dimensional phases with strong time-reversal breaking, Li-Haldane state counting, and PEPs

**Authors:** Mark J. Arildsen, Norbert Schuch, Andreas W. W. Ludwig

Submitted 7 July, 2022; **originally announced** July 2022.

Comments: 45 pages, 9 figures, 5 tables

4. [arXiv:2110.02988](#) [pdf, other] [cond-mat.stat-mech](#) cond-mat.dis-nn cond-mat.str-el [quant-ph](#)

### Statistical Mechanics Model for Clifford Random Tensor Networks and Monitored Quantum Circuits

**Authors:** Yaodong Li, Romain Vasseur, Matthew P. A. Fisher, Andreas W. W. Ludwig

Submitted 6 October, 2021; **originally announced** October 2021.

Comments: 23 pages, 5 figures. Abstract shortened to meet arxiv requirements, see pdf for full abstract

5. [arXiv:2107.03393](#) [pdf, other] [cond-mat.dis-nn](#) cond-mat.stat-mech cond-mat.str-el [quant-ph](#) [doi](#) 10.1103/PhysRevLett.128.050602

### Operator scaling dimensions and multifractality at measurement-induced transitions

**Authors:** Aidan Zabalo, Michael J. Gillies, Justin H. Wilson, Romain Vasseur, Andreas W. W. Ludwig, Sarang Gopalakrishnan, David A. Huse, J. H. Pixley

Submitted 11 February, 2022; v1 submitted 7 July, 2021; **originally announced** July 2021.

Comments: (6 + 12) pages, (2 + 12) figures, (1 + 2) tables (Updated with published version)

Journal ref: Phys. Rev. Lett. 128, 050602 (2022)



1. [arXiv:2306.00058](#) [pdf, other] [quant-ph](#) cond-mat.stat-mech

### Universality of the cross entropy in $\mathbb{Z}_2$ symmetric monitored quantum circuits

**Authors:** Maria Tikhonovskaya, Ali Lavasani, Matthew P. A. Fisher, Sagar Vijay

Submitted 14 August, 2023; v1 submitted 31 May, 2023; **originally announced** June 2023.

Comments: 12+6 pages, 16 figures. V2: References added

2. [arXiv:2304.13198](#) [pdf, other] [quant-ph](#) cond-mat.stat-mech

### Continuous symmetry breaking in adaptive quantum dynamics

**Authors:** Jacob Hauser, Yaodong Li, Sagar Vijay, Matthew P. A. Fisher

Submitted 25 April, 2023; **originally announced** April 2023.

Comments: 17 pages, 10 figures

3. [arXiv:2304.02664](#) [pdf, other] [quant-ph](#) cond-mat.dis-nn cond-mat.stat-mech

### Quantum Coding Transitions in the Presence of Boundary Dissipation

**Authors:** Izabella Lovas, Utkarsh Agrawal, Sagar Vijay

Submitted 5 April, 2023; **originally announced** April 2023.

Comments: 21 pages, 14 figures

4. [arXiv:2303.15507](#) [pdf, other] [cond-mat.str-el](#) cond-mat.stat-mech [quant-ph](#) [doi](#) 10.1103/PRXQuantum

### Mixed-state long-range order and criticality from measurement and feedback

**Authors:** Tsung-Cheng Lu, Zhehao Zhang, Sagar Vijay, Timothy H. Hsieh

Submitted 13 September, 2023; v1 submitted 27 March, 2023; **originally announced** March 2023.

Comments: 25 pages, 11 figures; updated to the published version

Journal ref: PRX Quantum 4, 030318 (2023)

5. [arXiv:2211.05784](#) [pdf, other] [quant-ph](#) cond-mat.str-el

### The X-Cube Floquet Code

**Authors:** Zhehao Zhang, David Aasen, Sagar Vijay

Submitted 10 November, 2022; **originally announced** November 2022.

Comments: Main Text (6 pages, 5 figures), Appendices (4 pages, 5 figures)



1. [arXiv:2309.03946](#) [pdf, other] [cond-mat.str-el](#) [hep-th](#)

### Nonlinear Lifshitz Photon Theory in Condensed Matter Systems

**Authors:** Yi-Hsien Du, Cenke Xu, Dam Thanh Son

Submitted 7 September, 2023; **originally announced** September 2023.

2. [arXiv:2308.07380](#) [pdf, other] [cond-mat.str-el](#)

### Disorder Operator and Rényi Entanglement Entropy of Symmetric Mass Generation

**Authors:** Zi Hong Liu, Yuan Da Liao, Gaopei Pan, Menghan Song, Jiarui Zhao, Weilun Jiang, Chao-Ming Jian, Yi-Zhuang Yc Cenke Xu

Submitted 8 September, 2023; v1 submitted 14 August, 2023; **originally announced** August 2023.

Comments: 16 pages, 12 figures

3. [arXiv:2306.10105](#) [pdf, other] [cond-mat.str-el](#) [hep-th](#)

### A no-go result for implementing chiral symmetries by locality-preserving unitaries in a 3 dim model of fermions

**Authors:** Lukasz Fidkowski, Cenke Xu

Submitted 13 July, 2023; v1 submitted 16 June, 2023; **originally announced** June 2023.

Comments: 3 figures, v3 typo fixed

4. [arXiv:2305.13410](#) [pdf, other] [cond-mat.str-el](#) [hep-th](#) [quant-ph](#)

### Conformal Field Theories generated by Chern Insulators under Quantum Decoherence

**Authors:** Kaixiang Su, Nayan Myerson-Jain, Cenke Xu

Submitted 22 May, 2023; **originally announced** May 2023.

Comments: 8.5 pages, including references

5. [arXiv:2304.14433](#) [pdf, other] [cond-mat.str-el](#) [hep-th](#) [quant-ph](#)

### Higher-form Symmetries under Weak Measurement

**Authors:** Kaixiang Su, Nayan Myerson-Jain, Chong Wang, Chao-Ming Jian, Cenke Xu

Submitted 27 April, 2023; **originally announced** April 2023.

Comments: 9 pages, 1 figure

6. [arXiv:2301.05238](#) [pdf, other] [cond-mat.stat-mech](#) cond-mat.str-el [quant-ph](#) [doi](#) 10.1103/PRXQuantum.4.030

### Quantum criticality under decoherence or weak measurement

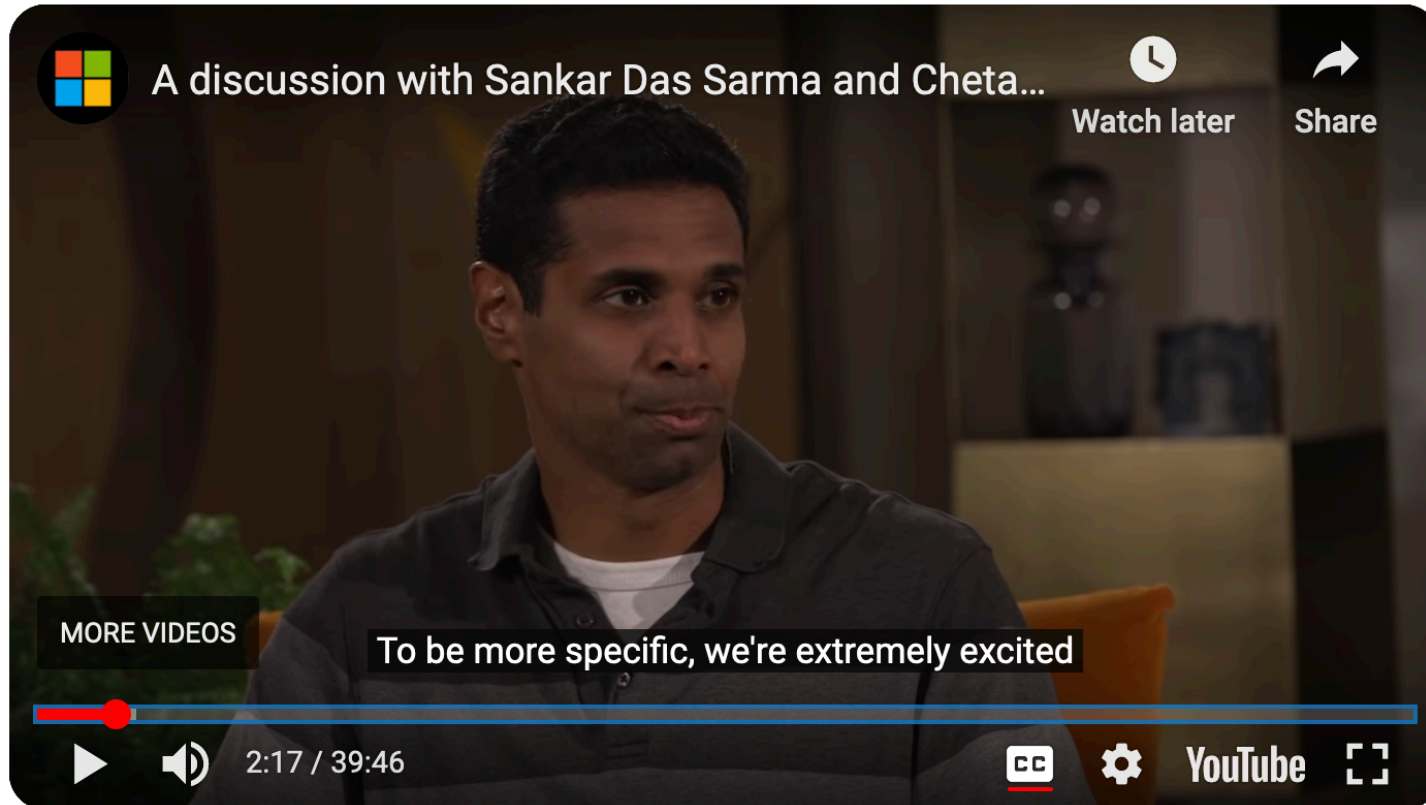
**Authors:** Jong Yeon Lee, Chao-Ming Jian, Cenke Xu

Submitted 26 July, 2023; v1 submitted 12 January, 2023; **originally announced** January 2023.

Comments: 18 pages, 5 figures (Accepted to PRX Quantum)

Not QI

# What is it good for?



# What is it good for?



# What is it good for?



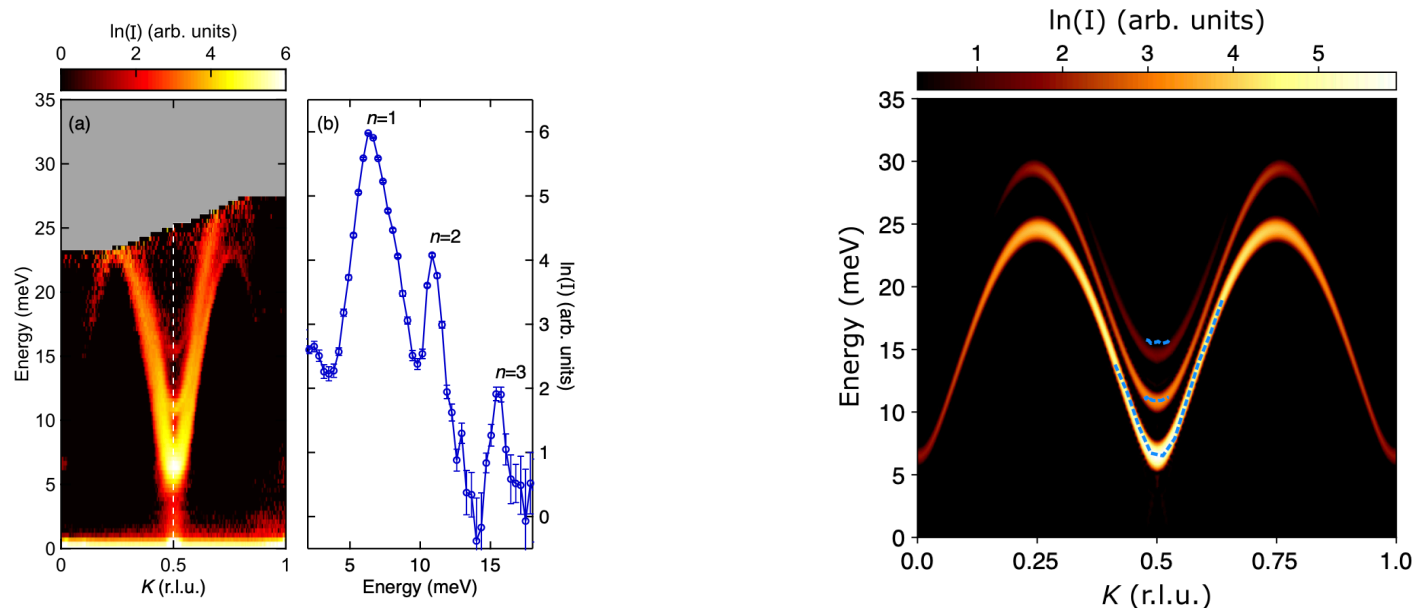


# What is it good for?



# Application to quantum materials?

- Try to *apply* quantum algorithms to actual quantum problems
- For example: how would we obtain  $S(k, \omega)$  on a quantum computer?



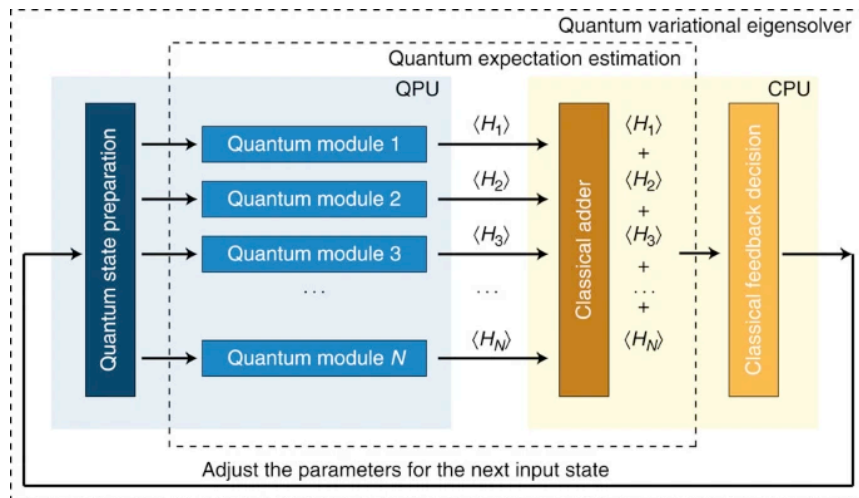
R. Dally et al, PRL (2020).

# Possible approaches

- Direct time evolution? Q: Isn't that what a quantum computer is good at?
  - A: Maybe a special purpose simulator, but a digital quantum computer like google machines can't. They apply controlled 1 and 2 qubit gates
  - You can Trotterize but this introduces substantial errors that can only be improved by scaling to many gates.
- Instead we will try to use a variational approach to obtain eigenstates.

# VQE

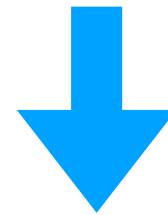
- Variational quantum eigensolver:  
Peruzzo et al, 2014



quantum  
circuit

$$|\Psi(\{\theta_i\})\rangle = U(\{\theta_i\})|\Psi_0\rangle$$

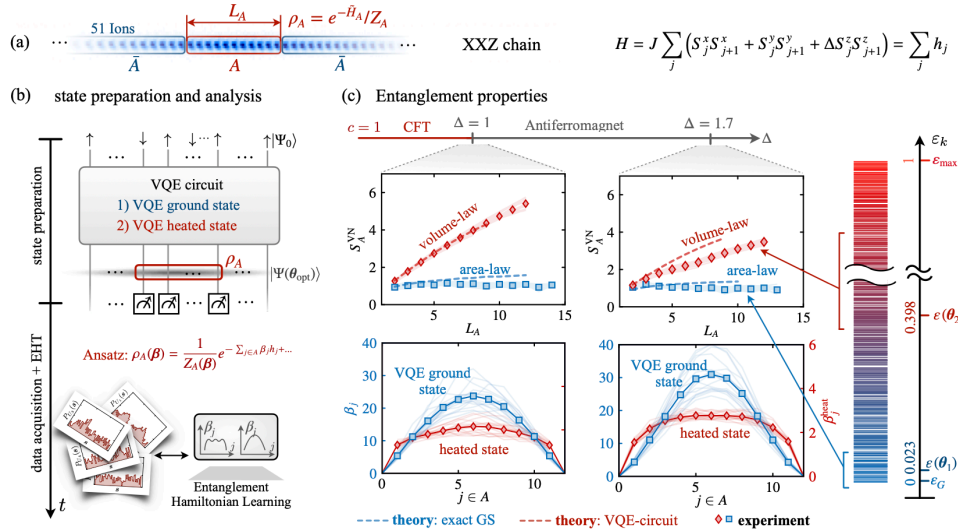
$$E_{\text{var}} = \underbrace{\langle \Psi | H | \Psi \rangle}_{\text{measure}} \geq E_0$$



Ground state

## Exploring Large-Scale Entanglement in Quantum Simulation

Manoj K. Joshi,<sup>1,2,\*</sup> Christian Kokail,<sup>1,3,\*</sup> Rick van Bijnen,<sup>1,3,\*</sup> Florian Kranzl,<sup>1,2</sup>  
Torsten V. Zache,<sup>1,3</sup> Rainer Blatt,<sup>1,2</sup> Christian F. Roos,<sup>1,2</sup> and Peter Zoller<sup>1,3</sup>



PHYSICAL REVIEW B **105**, 094409 (2022)

## Probing ground-state properties of the kagome antiferromagnetic Heisenberg model using the variational quantum eigensolver

Jan Lukas Bosse<sup>1,2,\*</sup> and Ashley Montanaro<sup>2,1,†</sup>

<sup>1</sup>School of Mathematics, University of Bristol, Bristol, BS8 1QU, United Kingdom

<sup>2</sup>Phasecraft Ltd, Bristol, BS1 5DD, United Kingdom

JAN LUKAS BOSSE AND ASHLEY MONTANARO

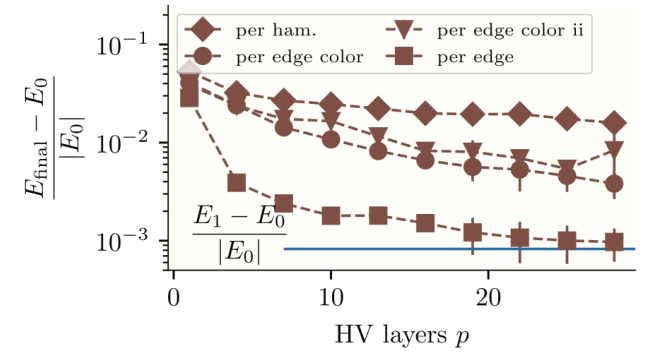


FIG. 12. Scaling of the relative energy error as a function of  $p$  for the  $3 \times 8$  lattice with different ansatz circuits. Results are shown for three runs per data point and with the initial parameters chosen uniformly random within  $[0, \frac{1}{p}]$ . The error bars reflect the standard deviation between the different runs.

PHYSICAL REVIEW B **106**, 214429 (2022)

## Variational quantum eigensolver for the Heisenberg antiferromagnet on the kagome lattice

Joris Kattmölle<sup>1,2,3</sup> and Jasper van Wezel<sup>1,2</sup>

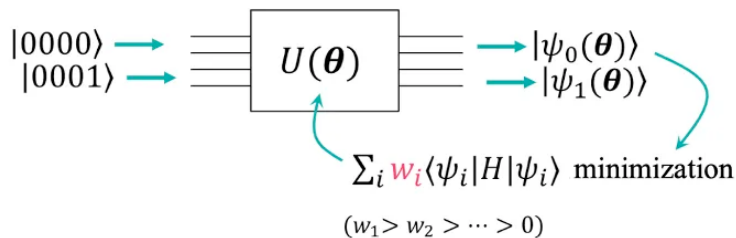
# SS VQE

- Subspace Search VQE: for excited states

K. Nakanishi *et al*, 2019

$$|\Psi_n(\{\theta_i\})\rangle = U(\{\theta_i\})|\Psi_{n,0}\rangle$$

Choose N  
orthogonal  
initial states



$$\langle \Psi_{n'} | \Psi_n \rangle = \langle \Psi_{n',0} | \Psi_{n,0} \rangle$$

$$E_{\text{var}} = \sum_n w_n \langle \Psi_n | H | \Psi_n \rangle$$

$$w_n > 0$$

Just repeat the VQE *with the same circuit* on N initial orthogonal states and minimize (weighted) energy sum.



# Elementary excitations

- Transverse field Ising chain

$$H_T = -J \sum_{i=1}^L Z_i Z_{i+1} - h \sum_{i=1}^L X_i$$

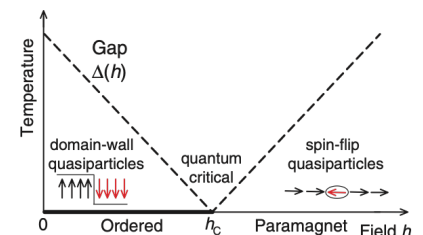
- Excitations at  $J \ll h$ :

$$|-_i\rangle = |++ \dots -_i + \dots +\rangle$$

$$|k\rangle = \frac{1}{\sqrt{N}} \sum_i e^{ikx_i} |-_i\rangle$$

$$\text{Exact energy } \epsilon_k = 2\sqrt{h^2 + J^2 - 2hJ \cos k}$$

? Can we get this from (SS) VQE?



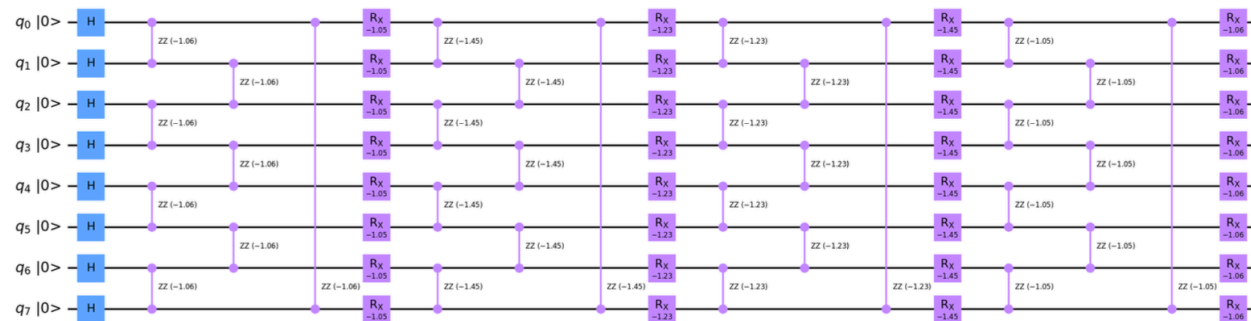
# VQE for Ising chain

- Natural circuit: preserve translational symmetry

$$H_T = \underbrace{-J \sum_{i=1}^L Z_i Z_{i+1}}_{H_1} - \underbrace{h \sum_{i=1}^L X_i}_{H_2}$$

$$|\psi_P(\gamma, \beta)\rangle = e^{-i\beta_p H_1} e^{-i\gamma_p H_2} \dots e^{-i\beta_1 H_1} e^{-i\gamma_1 H_2} |\psi_1\rangle$$

Example  
circuit



# VQE for Ising chain

## Efficient variational simulation of non-trivial quantum states

Wen Wei Ho<sup>1\*</sup> and Timothy H. Hsieh<sup>2,3</sup>

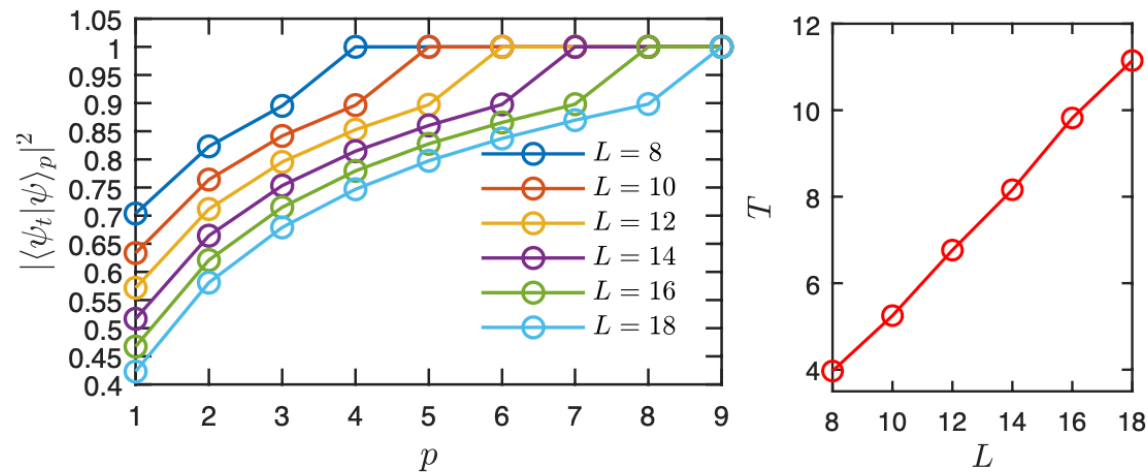


Figure 4: Preparation of critical state. (Left) Many-body overlap  $|\langle \psi_t | \psi \rangle_p|^2$  of the prepared state with the target ground state of (9) found by exact diagonalization. One sees perfect fidelity for  $p \geq L/2$ . (Right) Total minimum time  $T = \min_{(\gamma, \beta)} \left[ \sum_{i=1}^{p=L/2} (\gamma_i + \beta_i) \right]$  required for the VQCS to produce the critical state with perfect fidelity using  $\text{VQCS}_{p=L/2}$ . One sees a linear trend  $T \sim L$ .

# VQE for excited states?

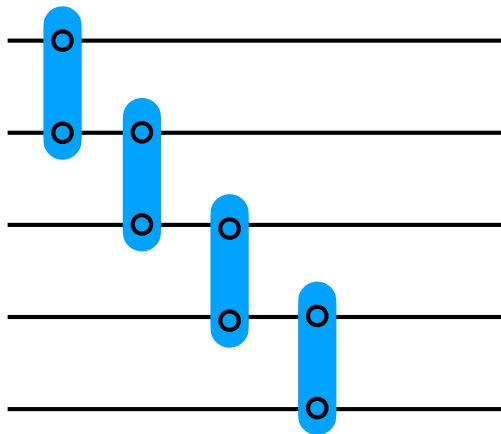
$$|-_i\rangle = |++\dots -_i + \dots +\rangle$$

$$|k\rangle = \frac{1}{\sqrt{N}} \sum_i e^{ikx_i} |-_i\rangle$$

For  $J/h \ll 1$

$$\epsilon_k = 2\sqrt{h^2 + J^2 - 2hJ \cos k}$$

Issue: *translation operator*  $T$  cannot be generated with a finite depth circuit (depth proportional to  $L$ ).



c.f D. Gross *et al*, 2012

Amount of translation is a “topological index” for 1d quantum cellular automata

# VQE Attempt 1

- Let's not worry about it and just initialize a momentum state.

$$|\psi_0(k)\rangle = \frac{1}{\sqrt{N}} \sum_i e^{ikx_i} |-_i\rangle$$

- Generate

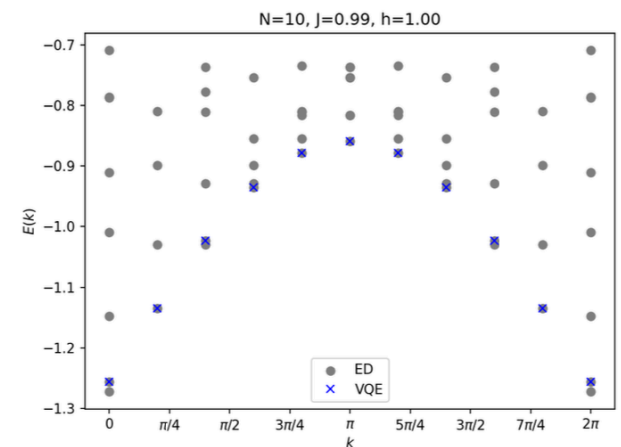
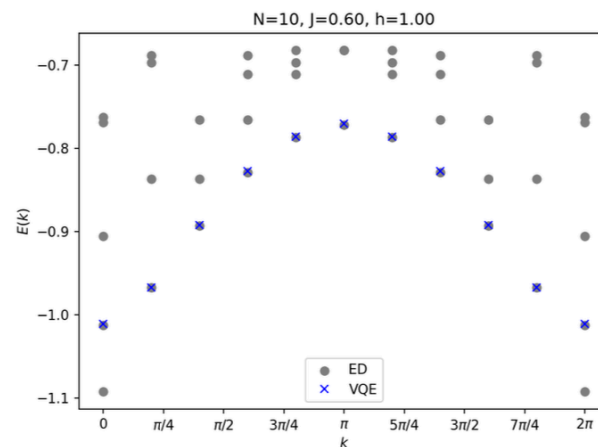
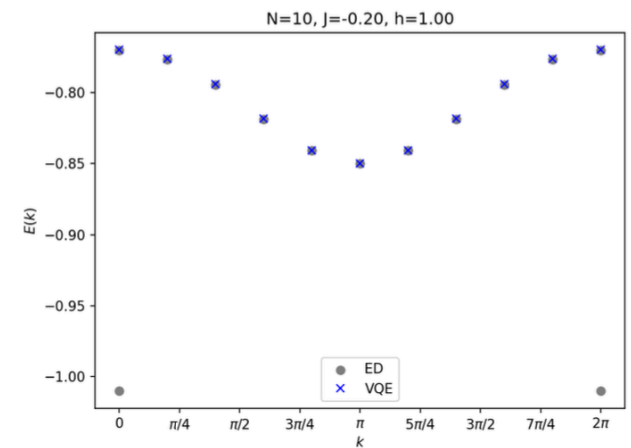
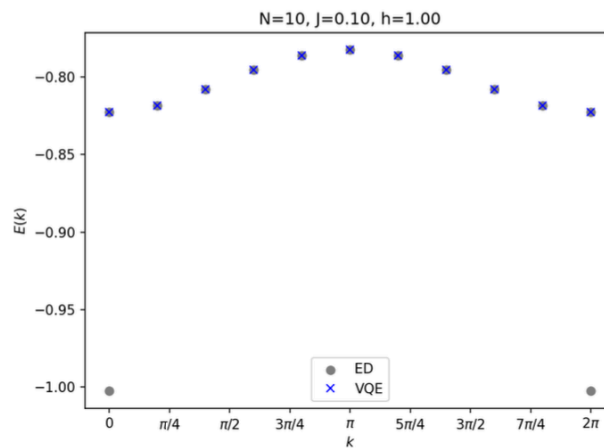
$$|\psi(k)\rangle = U(\{\beta_i\}) |\psi_0(k)\rangle$$

- Momentum conservation helps:  $k$  is conserved as is  $P = \otimes_i X_i$

# VQE Attempt 1

## Simulations with QISkit

Works!



# VQE attempt 2

- Make the system generate k state

- Trick 1: Parity conservation  $P = \bigotimes_i X_i$ .

Ground state  $P=+1, k=0$        $|GS\rangle = U_+ |++\cdots+\rangle$

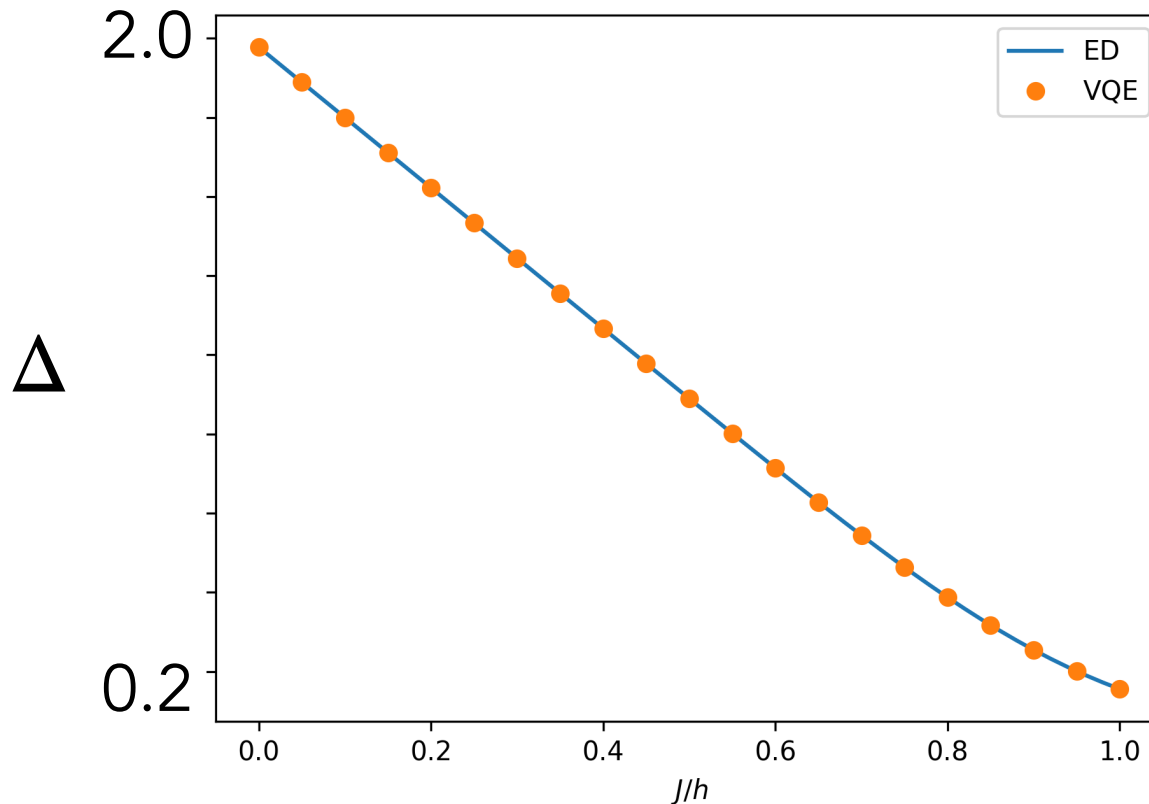
Excited state  $P=(-1)^N, k=0$        $|k=0\rangle = U_- |--\cdots-\rangle$

Generates quasiparticle state if N odd!

- In general this U depends on  $J/\hbar$  (and is non-trivial even for  $J/\hbar=0$ ).

# VQE attempt 2

- It works!  $|GS\rangle = U_+ |++ \dots +\rangle$   
 $|k=0\rangle = U_- |-- \dots -\rangle$





# VQE attempt 2

- Generate other  $k$  values?
- Trick 2: for *ideal* single spin-flip state, can *change*  $k$  via local unitary

$$|k\rangle_0 = U_k |k=0\rangle_0$$

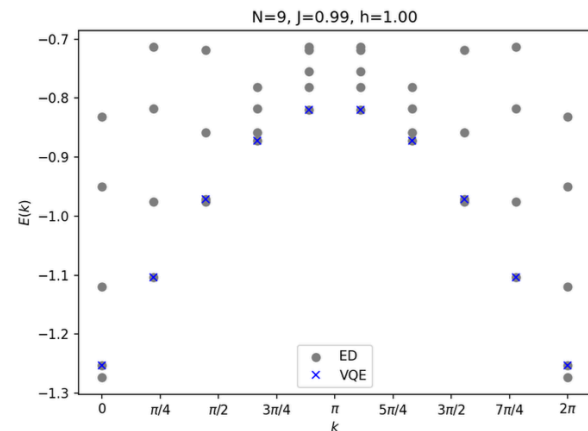
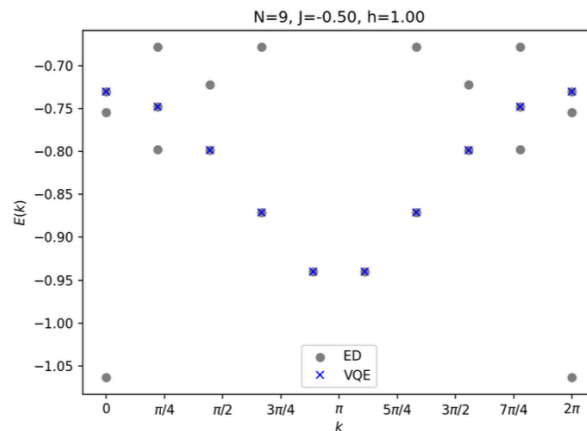
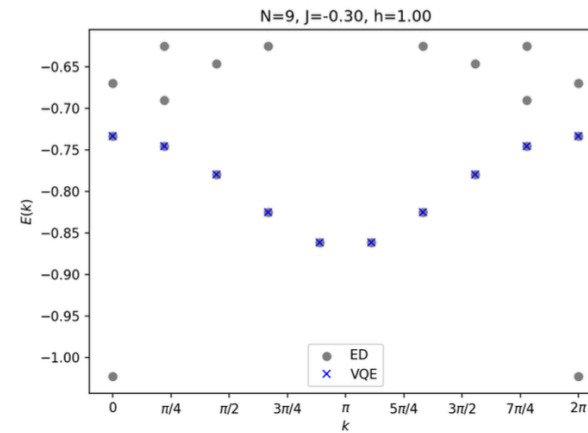
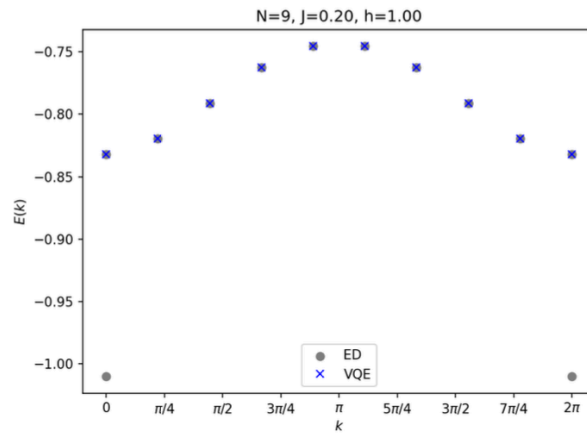
$$|k=0\rangle_0 = \frac{1}{\sqrt{N}} \sum_i | -_i \rangle$$

$$U_k = \prod_j e^{ikx_j(\frac{1}{2} - \frac{x_j}{2})}$$

- So we have a protocol

$$|k\rangle = U_{\text{int}} |k\rangle_0 = U_{\text{int}} U_k U_-^0 | - - \dots - \rangle$$

# VQE attempt 2



This also works!

# VQE attempt 3

- Can we work in *real* space instead of *k* space?
- What if we initialize to a *localized* excitation?

$$|x=0\rangle_0 = |++\cdots -_{x=0} + \cdots +\rangle = Z_0 \prod_i |+\rangle_i$$

- Evolved state

$$|x=0\rangle = U[\{\beta_i\}] |x=0\rangle_0$$

- Since  $U$  is translationally invariant and parity conserving, we have

$$|x=0\rangle = \frac{1}{\sqrt{N}} \sum_k U |k\rangle_0 = \frac{1}{\sqrt{N}} \sum_k |k\rangle$$

# VQE attempt 3

- Variational energy

$$\langle x = 0 | H | x = 0 \rangle = \frac{1}{N} \sum_{k,k'} \langle k' | H | k \rangle = \frac{1}{N} \sum_k \langle k | H | k \rangle$$

★ Minimum is reached only if it is reached for each k state individually!

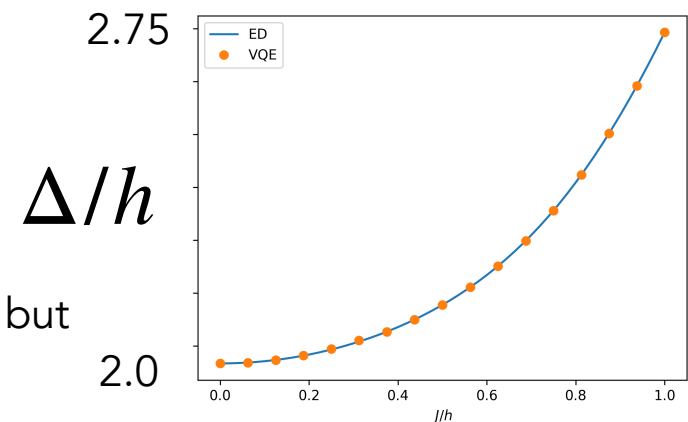
- Quantum parallelism! Just running VQE on this single state encodes the entire band of excited states!

# VQE attempt 3

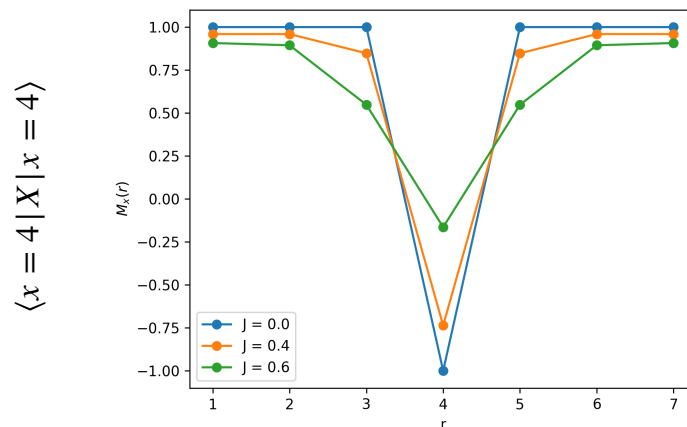
- Variational energy of this state gives the mean energy of the band

$$E_{x=0} - E_{GS} = \frac{1}{N} \sum_k \epsilon_k$$

With some work we can extract the entire band, but we're still trying to make it efficient



- We can also look at the state itself

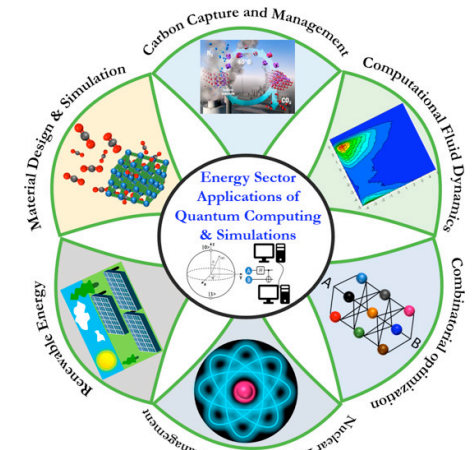
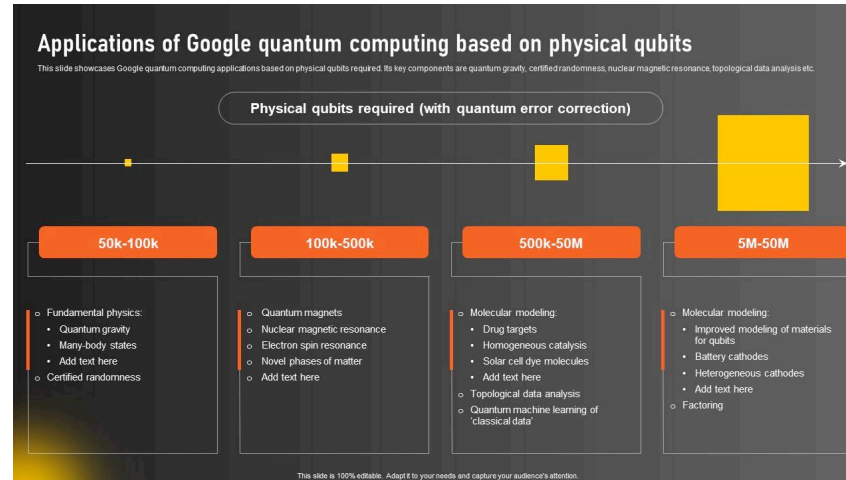


- Physically, we are generating the interacting analog of a Wannier state.

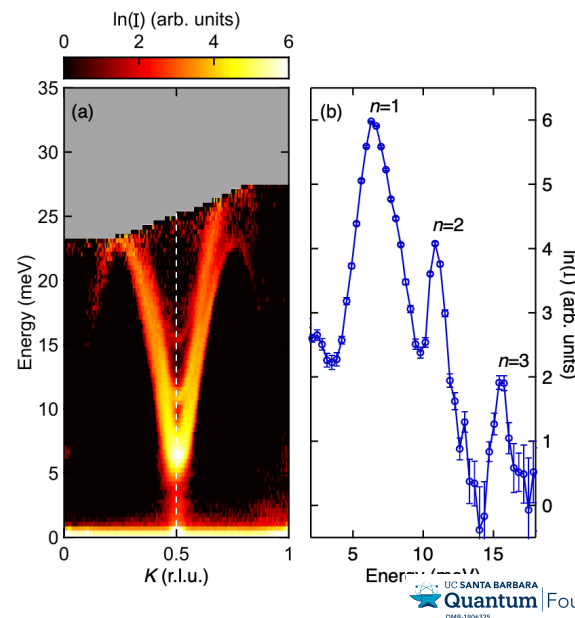
“The quasiparticle”

# Is a QC useful for us?

I'm still not sure about this



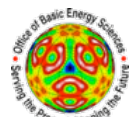
But maybe this.



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