Two stories*

*that have nothing to do with one another

Leon Balents, KITP, UCSB

Oxford, March 2022

Outline

- Angular transition in triangle-based antiferromagnets

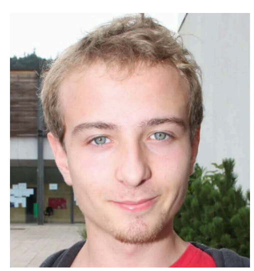
Kamran Behnia
ESPCI

Zengwei Zhu
Wuhan

Xiaokang Li
Wuhan

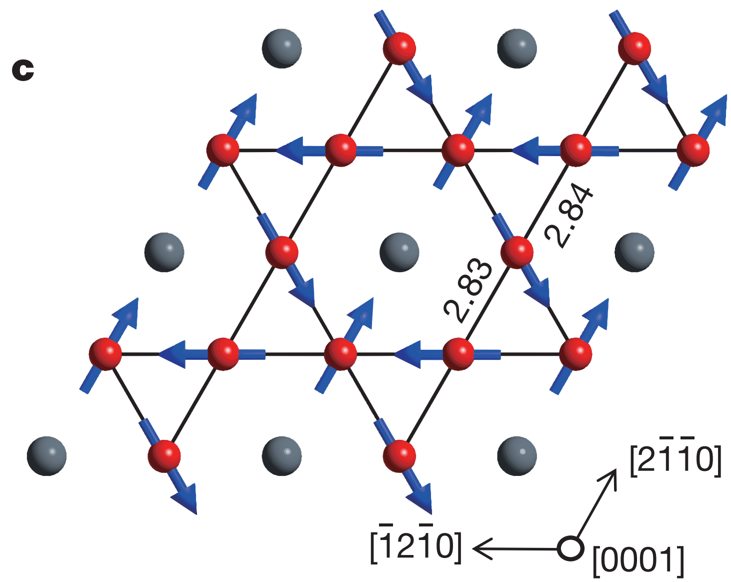
- Thermal Hall effect of phonons from intrinsic skew scattering

Lucile Savary
ENS Lyon



Léo Mangeolle
ENS Lyon

Mn_3Sn



two kagomé layers of
Mn, related by inversion

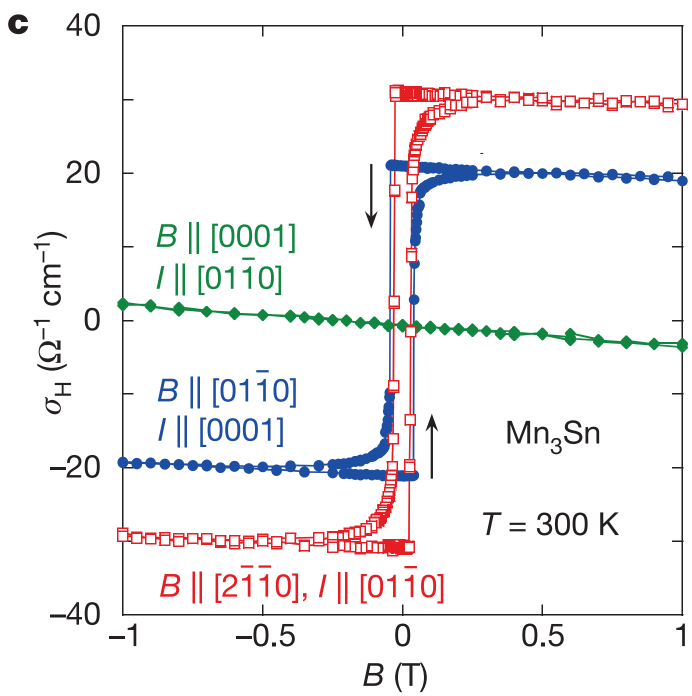
large ordered
antiferromagnetic
moment
~ 2 μ_B /Mn

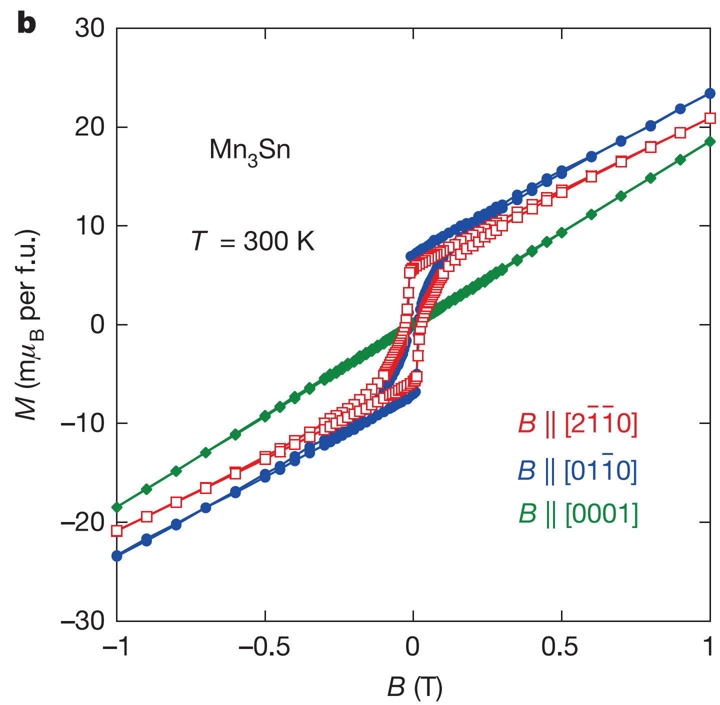
tiny FM moment:
.002 μ_B/Mn

T_N ~ 420K

Nagamiya *et al*, 1982

AHE



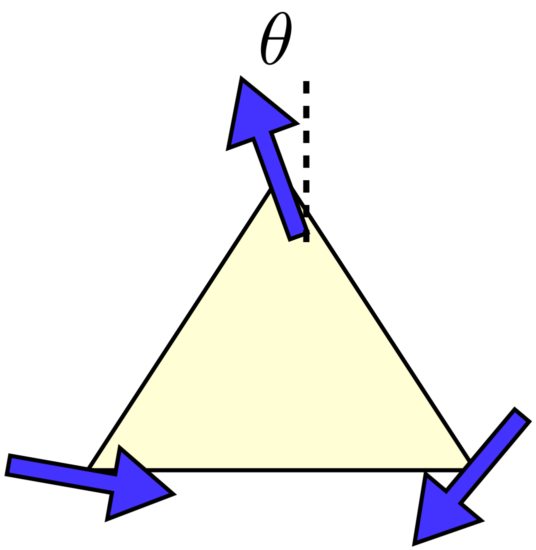


S. Nakatsuji *et al*, 2015

Why such a tiny moment?

Why such small coercive field?

Energetics: triangle



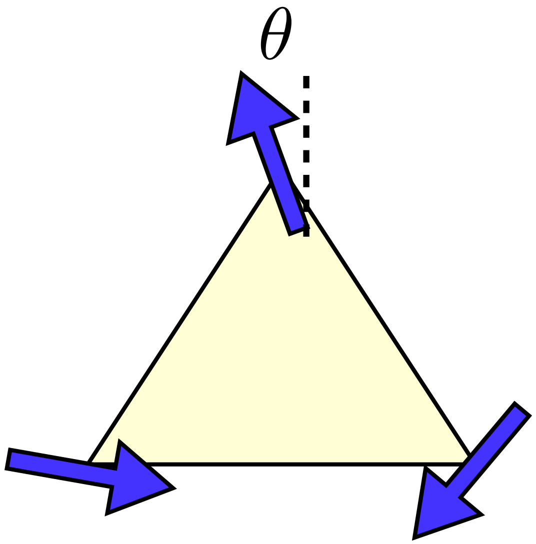
$$E = J\left(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + \boldsymbol{S}_2 \cdot \boldsymbol{S}_3 + \boldsymbol{S}_3 \cdot \boldsymbol{S}_1\right)$$
$$+D\hat{\boldsymbol{z}} \cdot \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2 + \boldsymbol{S}_2 \times \boldsymbol{S}_3 + \boldsymbol{S}_3 \times \boldsymbol{S}_1\right)$$
$$-K \sum_i \left(\hat{n}_i \cdot \boldsymbol{S}_i\right)^2$$

Jianpeng Liu + LB, 2017

$J \gg D \gg K$ **Hierarchy of interactions**

- J: spins at 120° angles and M=0
- D: spins are "anti-chiral" in XY plane
- K: weak canting toward easy axes creates tiny moment and fixes in-plane angle

Energetics: triangle



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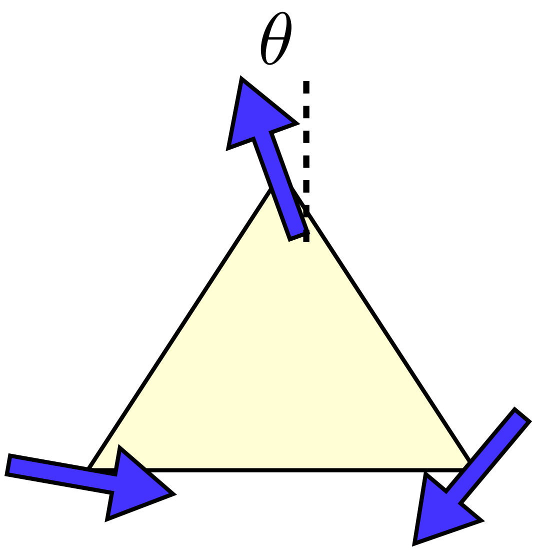
Global symmetries: spin-space group

magnetic space group

Energetics: triangle



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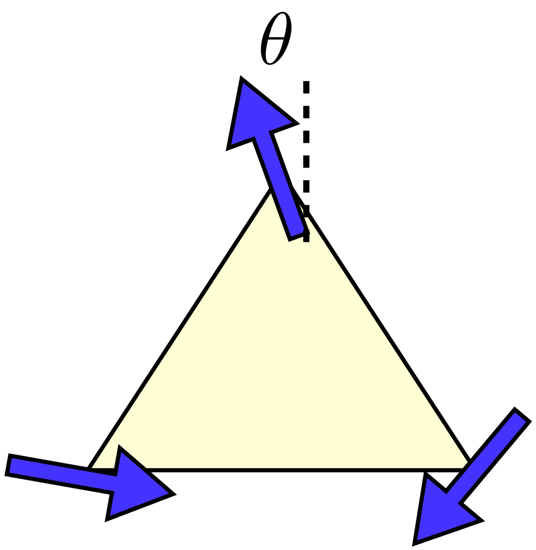

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$$-K \sum_i \left(\hat{n}_i \cdot \boldsymbol{S}_i\right)^2$$

Jianpeng Liu + LB, 2017

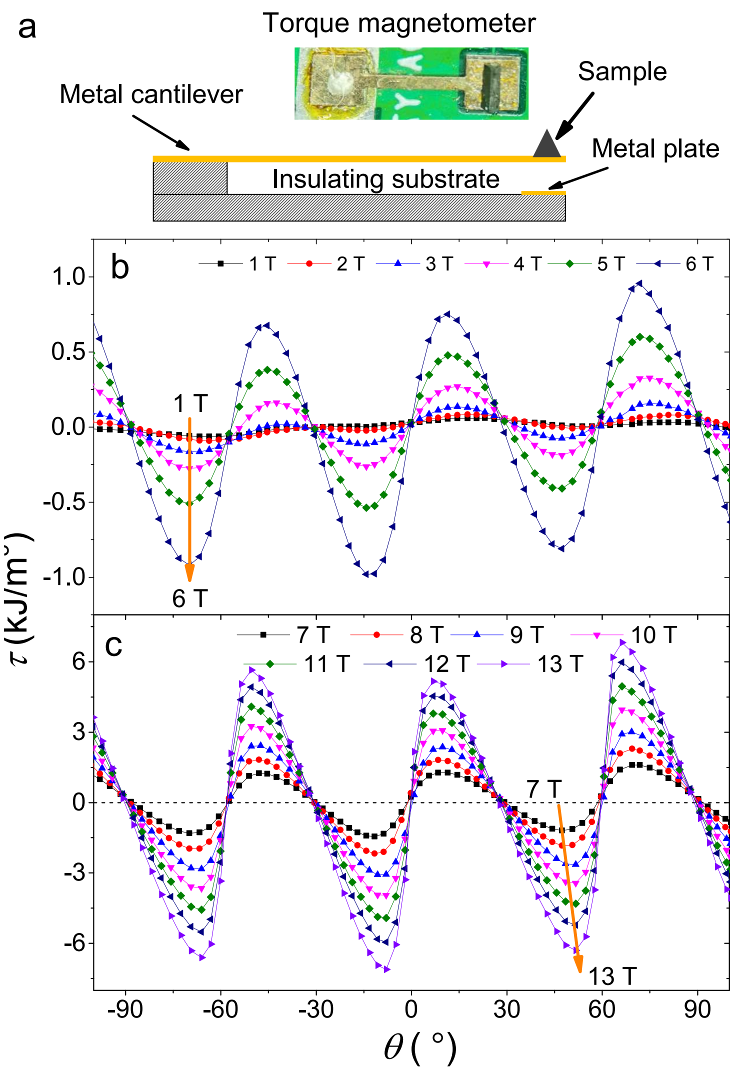
$J \gg D \gg K$ **Hierarchy of interactions**

$$m_0 = \frac{K}{J} m_s$$ Uniform moment

$$\lambda = \frac{K^3}{12J^2}$$ In-plane anisotropy

Low coercive field despite tiny Zeeman energy

Torque



The free energy of twisting spins in Mn_3Sn

Xiaokang Li[1,*], Shan Jiang[3,1], Qingkai Meng[1], Huakun Zuo[1], Zengwei Zhu[1,*], Leon Balents[2,4] and Kamran Behnia[3]

(1) Wuhan National High Magnetic Field Center and School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China

(2)Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA

(3) Laboratoire de Physique et d'Étude des Matériaux (ESPCI - CNRS - Sorbonne Université), PSL Research University, 75005 Paris, France

(4) Canadian Institute for Advanced Research, Toronto, Ontario, Canada

(Dated: February 25, 2022)

Notice the evolution from sinusoidal to sawtooth

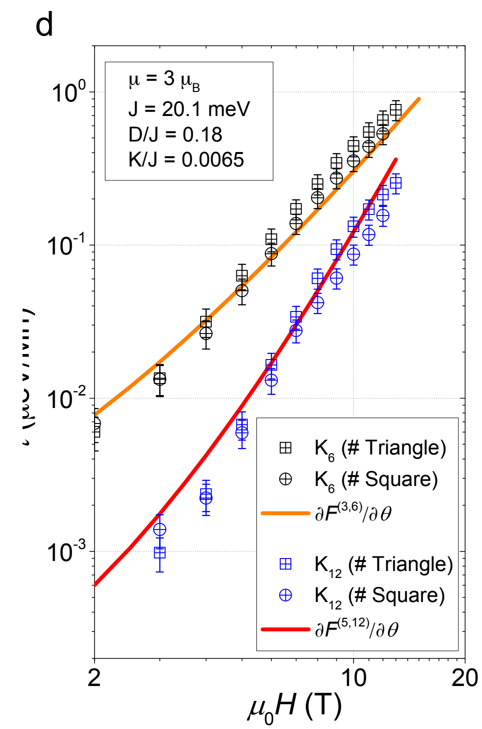
First explanation

Extension of our expansion from 2017

$$\phi_1 = \phi + \eta_1, \qquad \phi_2 = \phi - \frac{2\pi}{3} + \eta_2, \qquad \phi_3 = \phi - \frac{4\pi}{3} - \eta_1 - \eta_2.$$

$$\eta_i = \sum_{n=1}^{\infty} \eta_{i,n} r^n, \qquad E_{u.c.} = \sum_{n=0}^{\infty} E^{(n)}_{u.c.},$$

$$\begin{aligned}
E^{(0)}_{u.c.} &= -6J - 6\sqrt{3}D, \\
E^{(1)}_{u.c.} &= -3K, \\
E^{(2)}_{u.c.} &= -\frac{(\mu H)^2 + K^2 + 2\mu HK\cos(\theta+\phi)}{2\left(\sqrt{3}D + J\right)}, \\
E^{(3)}_{u.c.} &= -\frac{1}{36(J+\sqrt{3}D)^3}\Big[(3J+7\sqrt{3}D)K^3\cos(6\phi) + 6(J+3\sqrt{3}D)\mu HK^2\cos(5\phi-\theta) \\
&\quad + 3(J+5\sqrt{3}D)(\mu H)^2 K\cos(4\phi-2\theta) + 4\sqrt{3}D(\mu H)^3\cos(3\phi-3\theta)\Big].
\end{aligned}$$



Perturbatively solve

$\phi(\theta)$

$$E_{u.c.} = -6J - 6\sqrt{3}D - 3K - \frac{(\mu H + K)^2}{2(J+\sqrt{3}D)}\Bigg[1 + \frac{(3J+7\sqrt{3}D)K + 4\sqrt{3}D\mu H}{18(J+\sqrt{3}D)^2}\cos(6\theta) + \frac{\left((3J+7\sqrt{3}D)K^2 + 2(J+4\sqrt{3}D)\mu HK + 2\sqrt{3}D(\mu H)^2\right)^2}{36(J+\sqrt{3}D)^4\mu HK}\sin^2(6\theta)\Bigg].$$

Angular transitions

A little simpler picture

Heisenberg model

$$E_{\mathrm{tri}} = \frac{J}{2}\left(\boldsymbol{S}_1 + \boldsymbol{S}_2 + \boldsymbol{S}_3 - \frac{1}{J}\boldsymbol{h}\right)^2$$

"Order by disorder": thermal and quantum fluctuations favor coplanar states

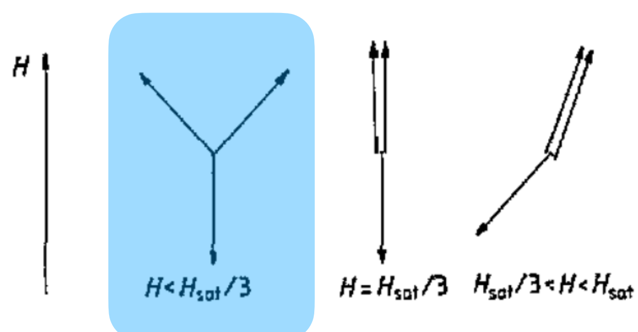
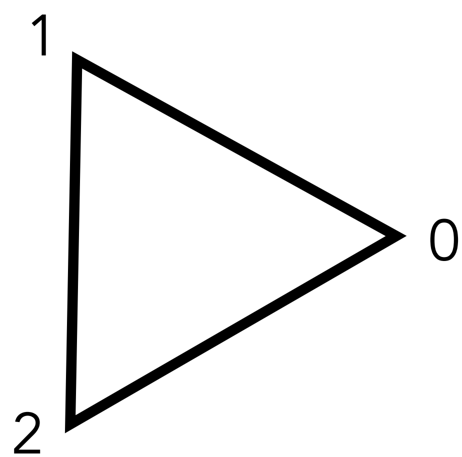


Figure 1. Reorientation process in the magnetic field in 2D Heisenberg AFM on a triangular lattice. Zero-point fluctuations stabilize the collinear phase in the finite region $H_1 < H < H_2$ in the vicinity of $H_{sat}/3$.

A. Chubukov and I. Golosov, 1991

Energy and symmetries: Heisenberg limit



$$\langle \boldsymbol{S}_n \rangle = \mathrm{Re}\left[\boldsymbol{d}\, e^{\frac{2\pi i n}{3}}\right]$$

$$\boldsymbol{d}\cdot\boldsymbol{d} = 0.$$

$$\boldsymbol{d} = \boldsymbol{u} + i\boldsymbol{v}$$

SO(3) $\boldsymbol{S}_n \to \mathsf{O}\boldsymbol{S}_n$

S_3 $\boldsymbol{S}_n \to \boldsymbol{S}_{P(n)}$

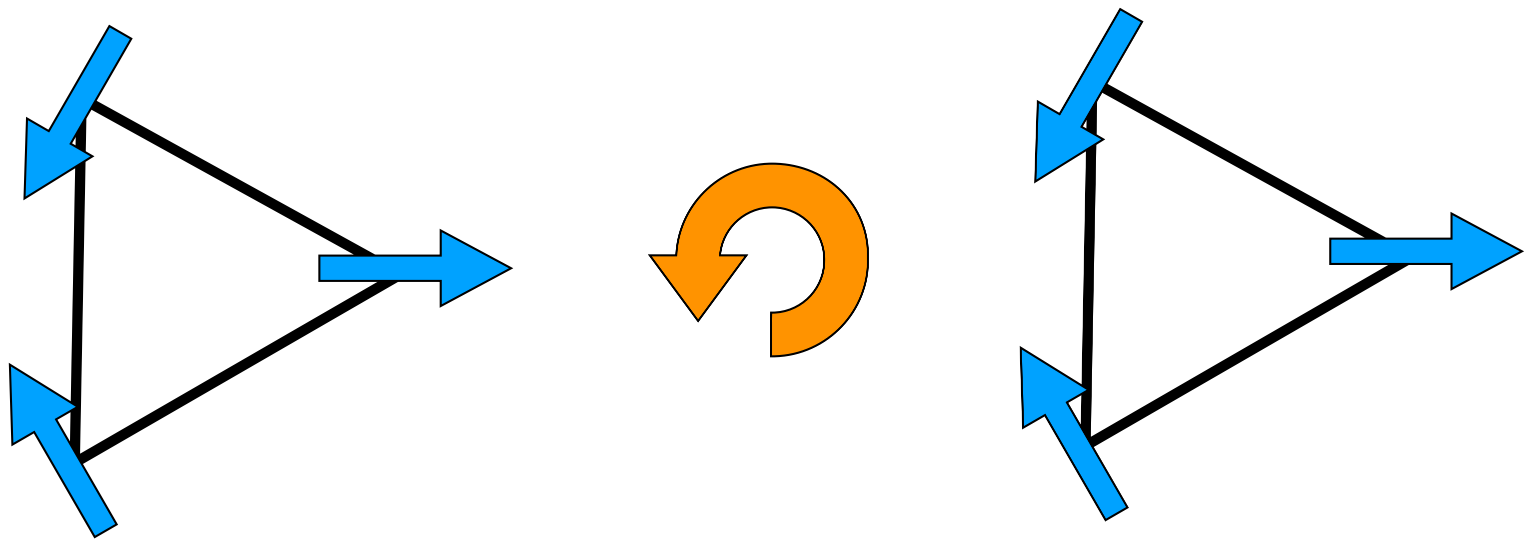
$$F_h^{\mathrm{iso}} = c_1|\boldsymbol{h}\cdot\boldsymbol{d}|^2 + c_2\mathrm{Re}\left[(\boldsymbol{h}\cdot\boldsymbol{d})^3\right] + O(h^4)$$

Selects plane — Selects angle in plane

$c_1 < 0$ — $c_2 > 0$

Anti-chiral state

Favored by D>0



counter-clockwise rigid rotation = clockwise spin rotation

$$d_\pm = d_x \pm i d_y \quad h_\pm = h_x \pm i h_y$$

Re[h_+ d_+] is an invariant

Full angular free energy

$$d_+ = ne^{i\phi}$$
$$d_- = d_z = 0$$

$$h_+ = he^{i\theta}$$

Zero field anisotropy
(Negligible)

$$f_+ = -w\cos 6\phi - uh\cos(\phi+\theta) - vh^3\cos 3(\phi-\theta)$$

Anti-chiral
magnetization

Heisenberg
response

$$u \sim \frac{K}{J}$$

Full angular free energy

$$d_+ = ne^{i\phi}$$
$$d_- = d_z = 0$$

$$h_+ = he^{i\theta}$$

$$f_+ = \quad - uh\cos(\phi+\theta) - vh^3\cos 3(\phi-\theta)$$

Anti-chiral magnetization

Heisenberg response

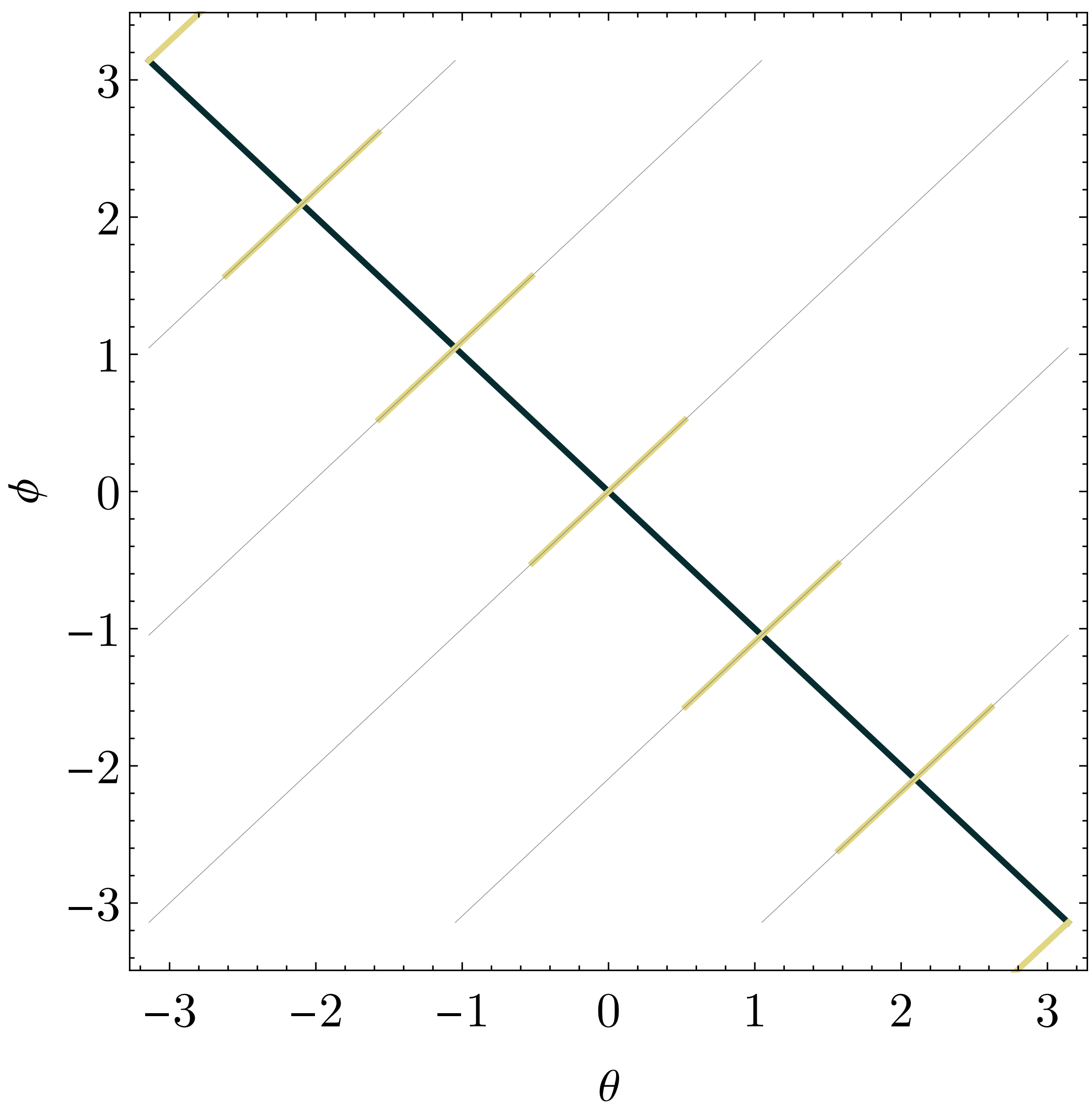
small h:

$$\phi \approx -\theta$$

small h:

$$\phi \approx \theta + \frac{2\pi k}{3}$$

Angular evolution



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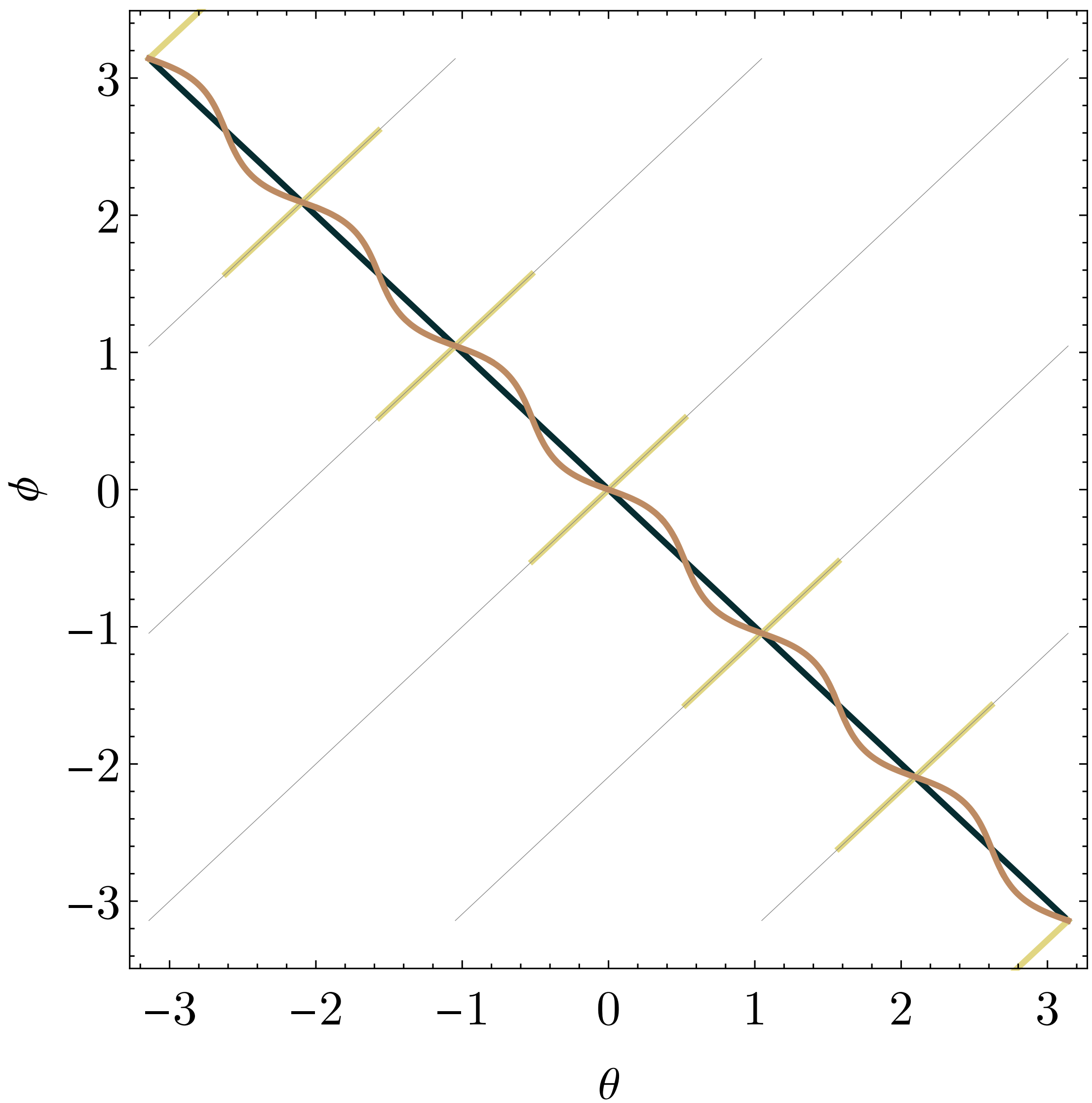

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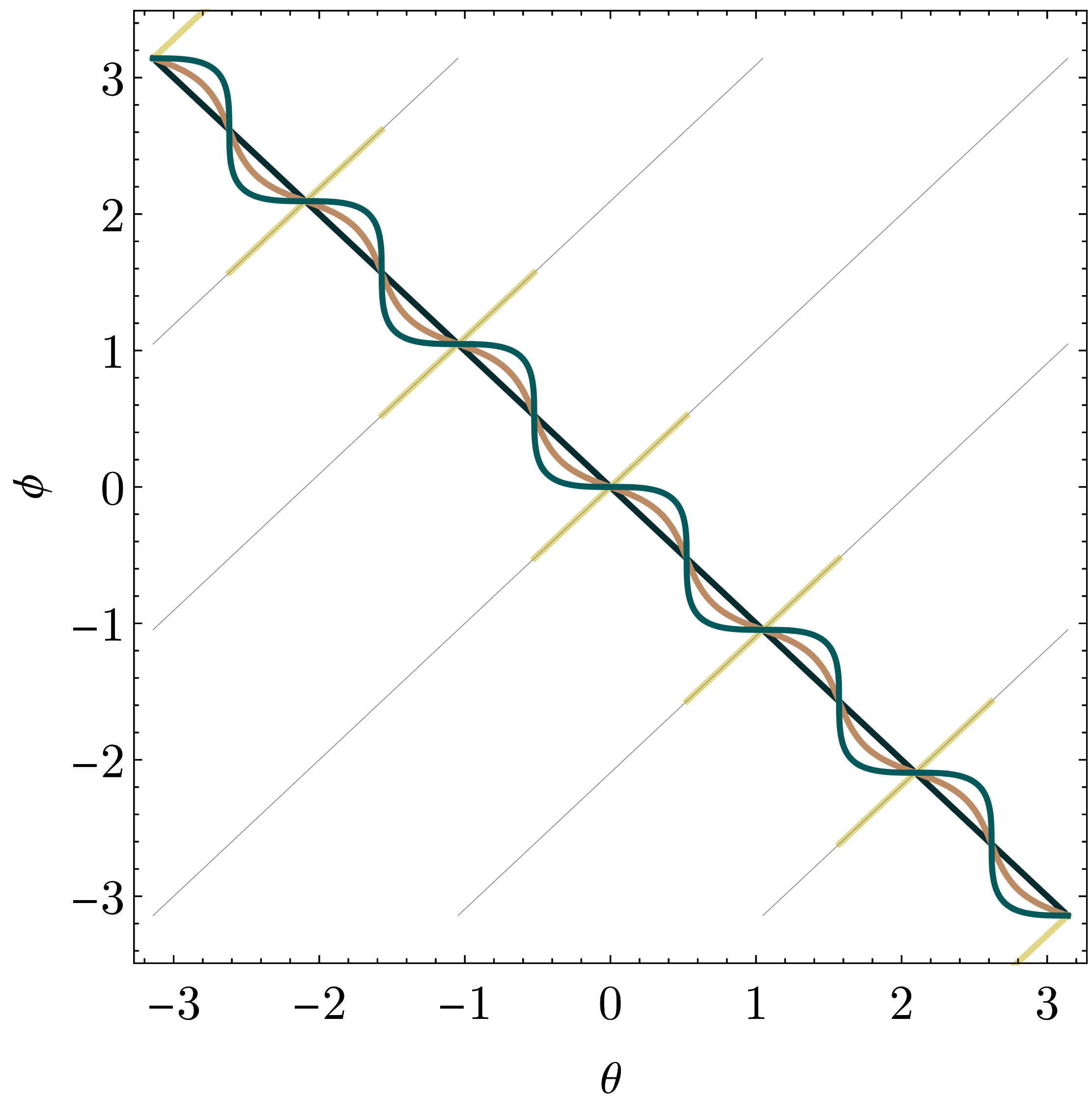
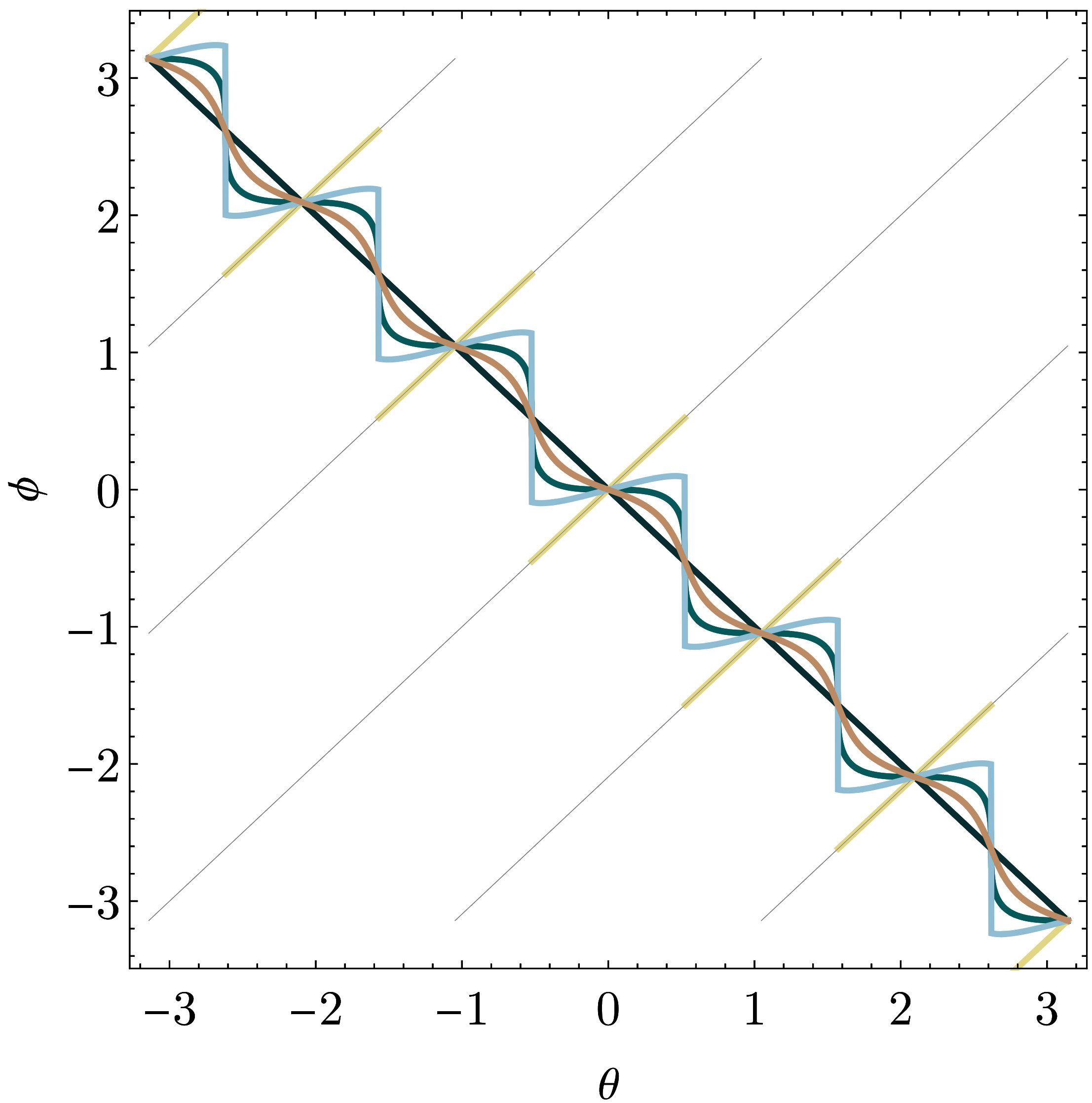


Angular evolution



$h > h_{crit}$

Jumps at

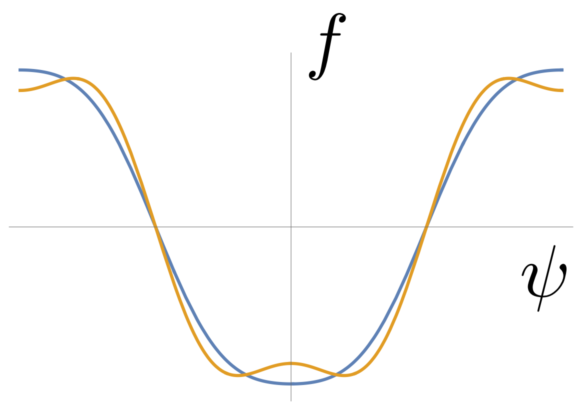
$$\theta = \frac{\pi}{6} + \frac{\pi m}{3}$$

$$\theta = \frac{\pi}{6} + \frac{\pi m}{3}$$

How do the jumps onset?

$$\psi = \phi + \theta, \qquad \theta = \pi/6 + \delta \qquad\qquad x = \sqrt{v/u}h^2$$

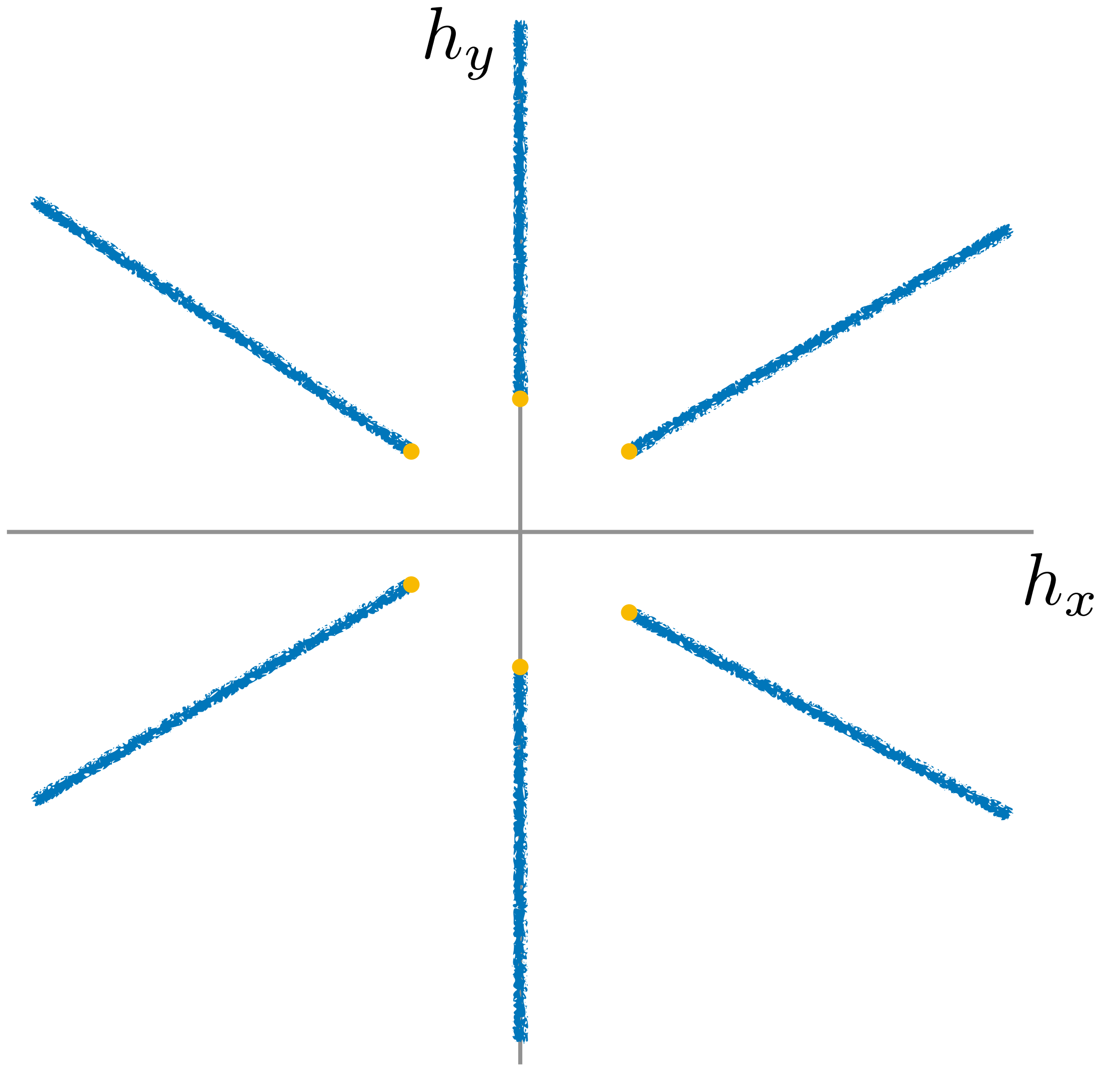
$$\frac{f}{uh} = -\cos\psi + x\cos 6\delta \cos 3\psi + x \sin 6\delta \sin 3\psi$$
$$= -\cos\psi + x\cos 3\psi \qquad (\delta = 0)$$



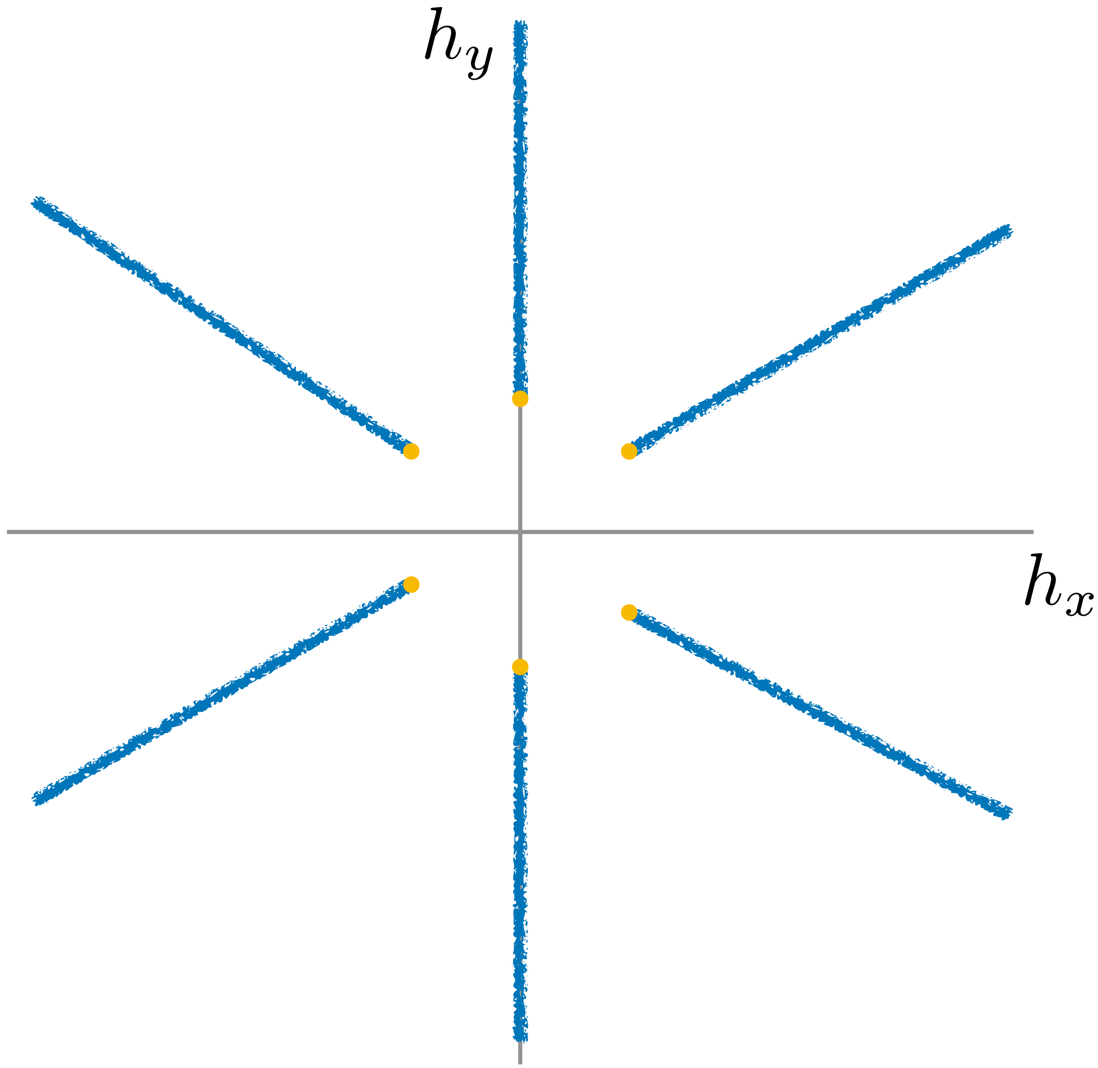
Ising transition
at x=1/9

δ acts as symmetry breaking field

Phase diagram

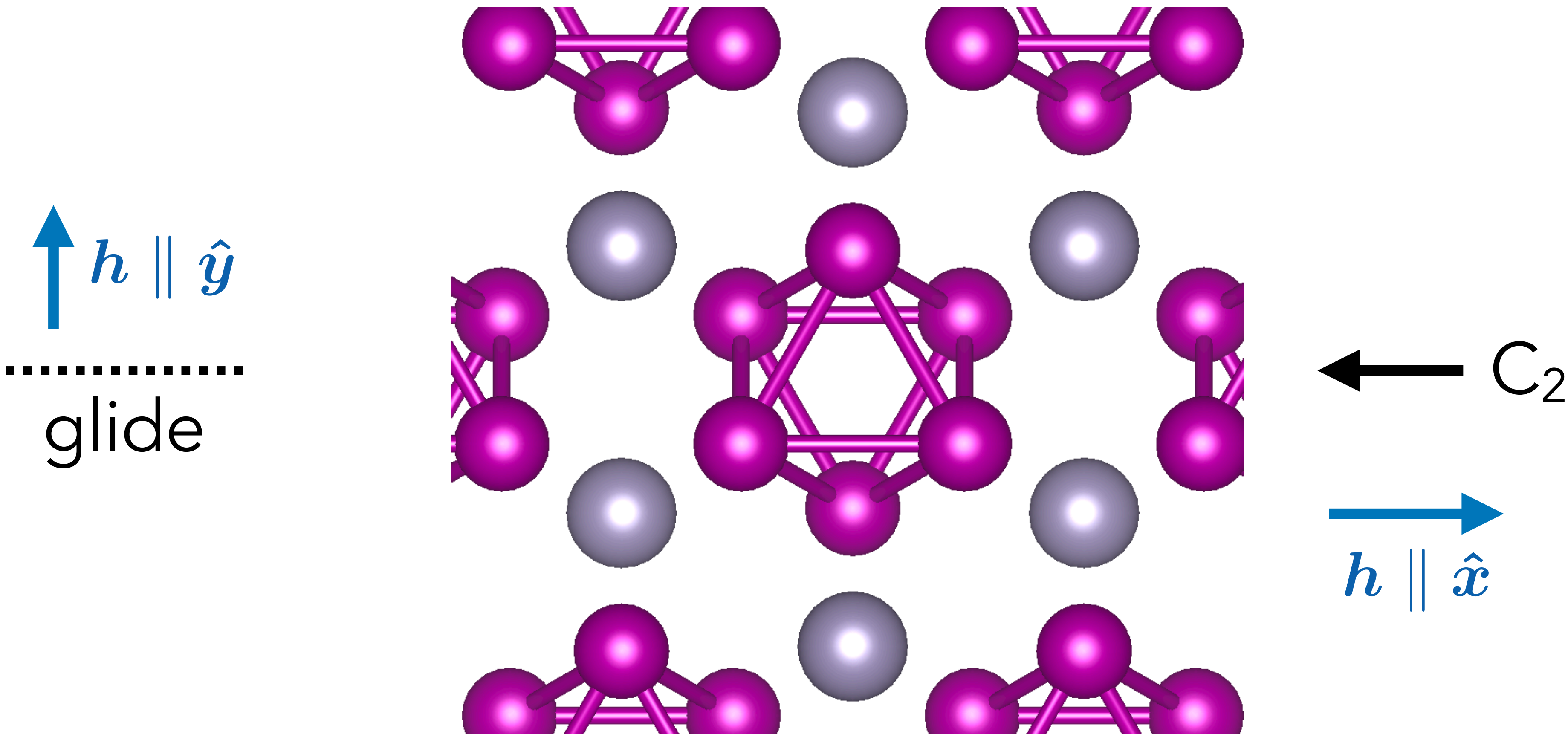


Phase diagram

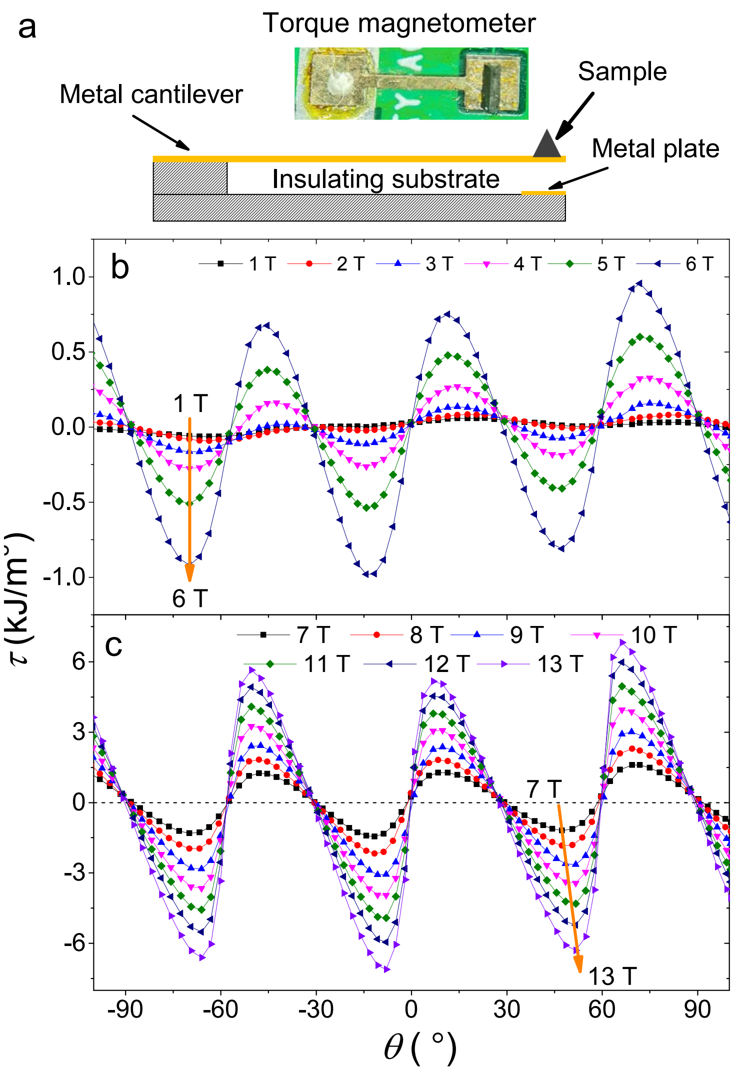


What symmetry is broken along the lines?

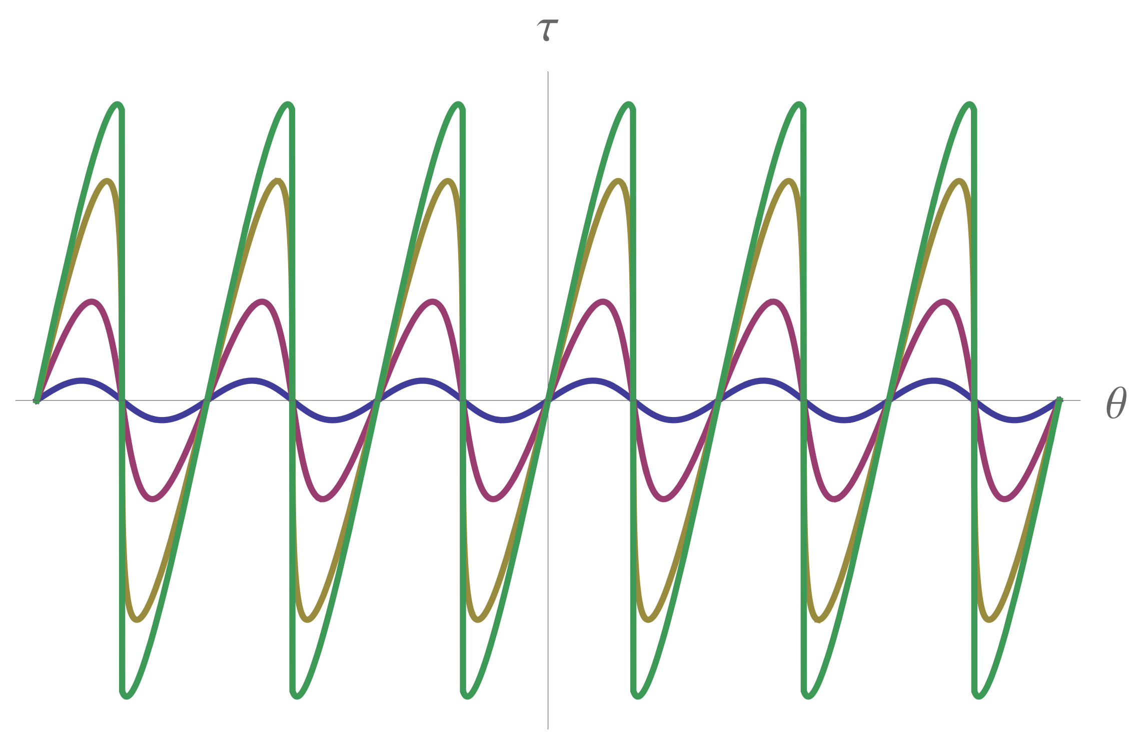
Mn_3Sn structure



Torque

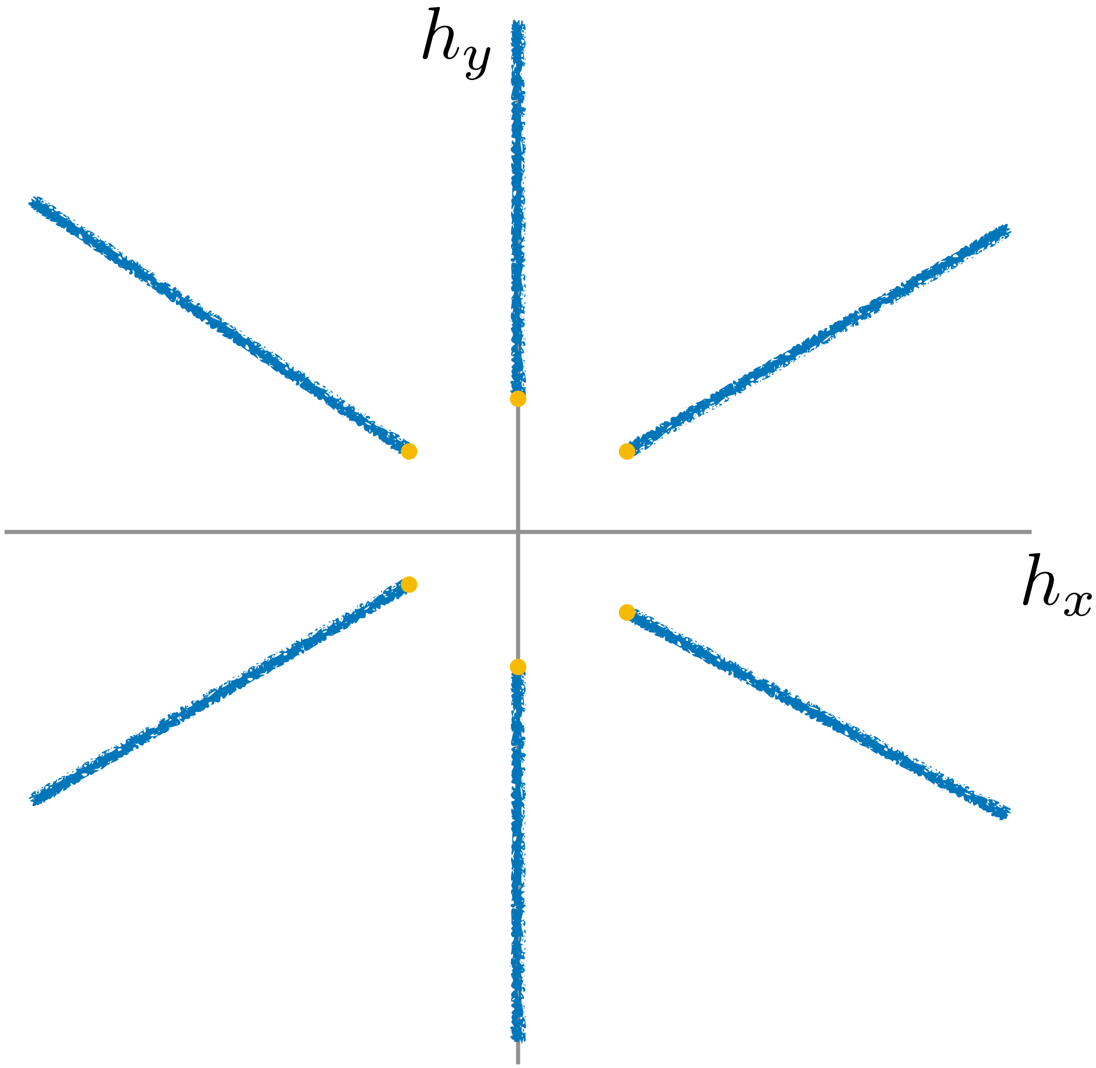


$$\tau = \frac{df}{d\theta}$$



Discontinuities for h>h_c

Estimate



Classical, T=0

$$H_c = \frac{J + \sqrt{3}D}{g\mu_B}\sqrt{\frac{K}{D}}$$

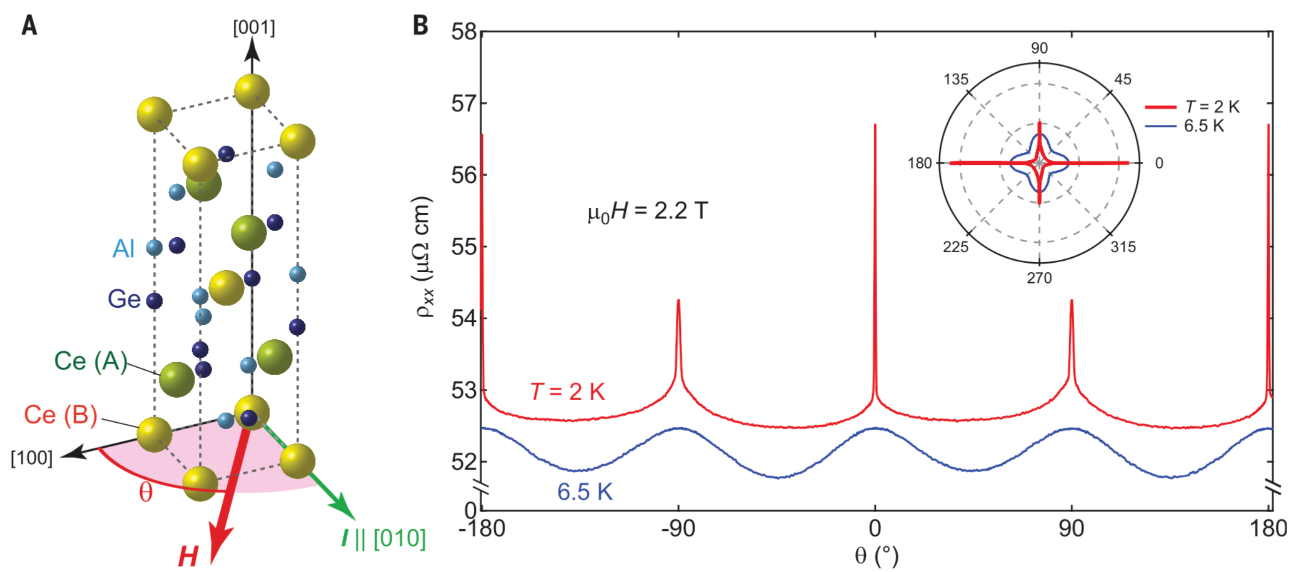
$$\approx 20\mathrm{T}$$

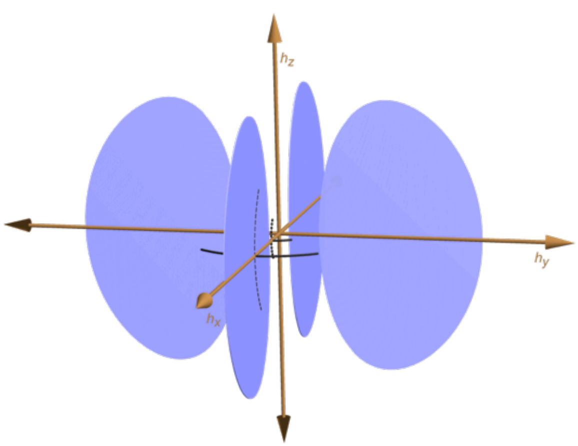
c.f.

MAGNETISM

Singular angular magnetoresistance in a magnetic nodal semimetal

T. Suzuki[1], L. Savary[1,2,3], J.-P. Liu[2,4], J. W. Lynn[5], L. Balents[2], J. G. Checkelsky[1]*

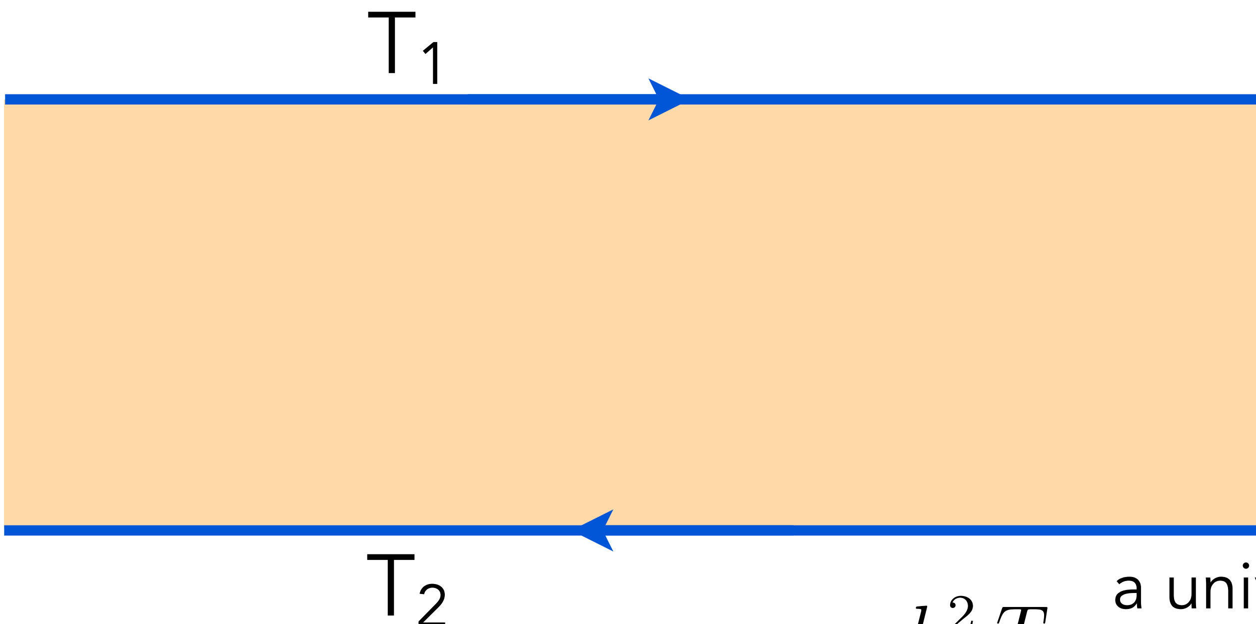




Would be interesting to search for transport signatures in Mn_3Sn

Thermal Hall effect

- Motivation: a probe of exotic phases. In insulators, “must” come from electrons

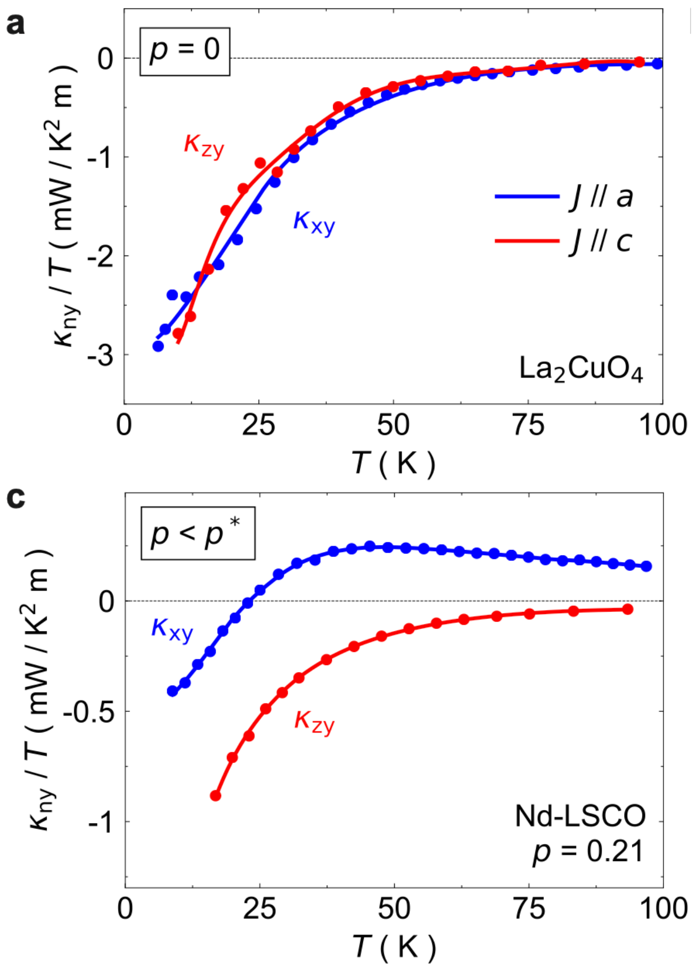


$$I_x = \kappa_H \Delta T_y$$

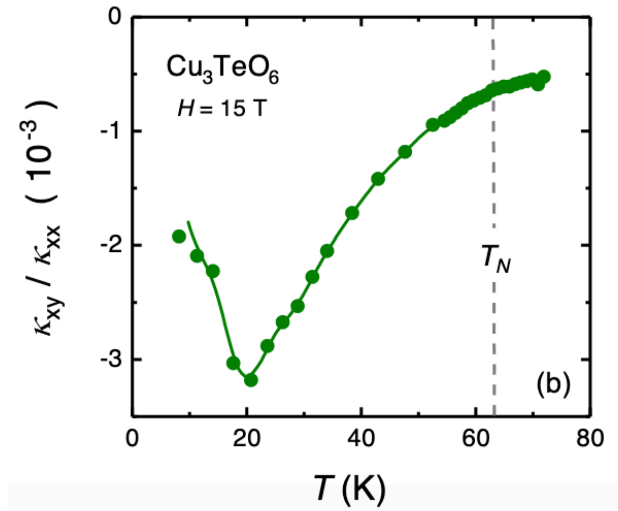
$$\kappa_H = \frac{\pi c k_B^2 T}{6\hbar}$$

a universal prediction for chiral “Ising anyon” phase: *agnostic to microscopic spin interactions*

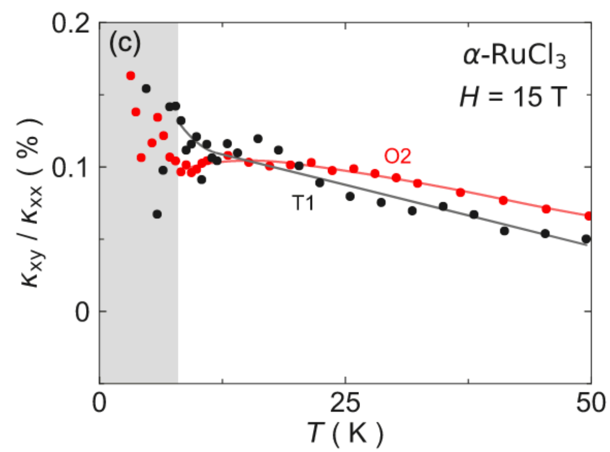
Phonons



Grissonanche *et al*, 2020



L. Chen *et al*, 2021



Evidence of a Phonon Hall Effect in the Kitaev Spin Liquid Candidate α-$RuCl_3$

É. Lefrançois,[1] G. Grissonnanche,[1] J. Baglo,[1] P. Lampen-Kelley,[2,3] J. Yan,[2] C. Balz,[4,*]
D. Mandrus,[2,3] S. E. Nagler,[4] S. Kim,[5] Young-June Kim,[5] N. Doiron-Leyraud,[1] and L. Taillefer[1,6]

Two types of effects

- Phonons are good quasiparticles

  - Non-dissipative effects: modifications of intrinsic dynamics of individual quasiparticles, e.g. Berry phase effects, etc.

  - Dissipative effects: modifications of scattering of quasiparticles

Phonon Boltzmann equation

Convective derivative. Dynamics.

$$D_t p = \Gamma[p]$$

Collision term

Dissipative effects

- Basically, this is "skew scattering" of phonons
- We ask how this arises through coupling to electronic degrees of freedom
- Transition matrix in *full many-body space of phonons+electrons:*

$$T_{\mathtt{i}\to\mathtt{f}} = T_{\mathtt{fi}} = \langle \mathtt{f}|H'|\mathtt{i}\rangle + \sum_{\mathtt{n}} \frac{\langle \mathtt{f}|H'|\mathtt{n}\rangle\langle \mathtt{n}|H'|\mathtt{i}\rangle}{E_{\mathtt{i}} - E_{\mathtt{n}} + i\eta} + \cdots$$

Important point: 1st order term is Hermitian, so 1st order T-matrix is effectively time-reversal invariant

- No Hall effect at leading order.

From T-matrix to collision term

"Spin"

- Coupling Hamiltonian $$H' = \sum_{n\boldsymbol{k}} \left(a^\dagger_{n\boldsymbol{k}} Q^\dagger_{n\boldsymbol{k}} + a_{n\boldsymbol{k}} Q_{n\boldsymbol{k}} \right)$$

Can be anything non-phononic, e.g. electronic

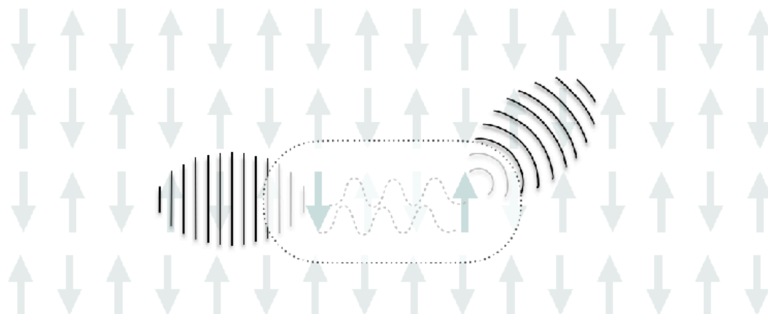
- Full transition rate $$\Gamma_{\mathbf{i}\to\mathbf{f}} = \frac{2\pi}{\hbar} |T_{\mathbf{i}\to\mathbf{f}}|^2 \delta(E_{\mathbf{i}} - E_{\mathbf{f}}).$$ $$p_{i_s} = \frac{1}{Z_s} e^{-\beta E_{i_s}}$$

- Phonon transition rate $$\tilde{\Gamma}_{i_p \to f_p} = \sum_{i_s f_s} \Gamma_{\mathbf{i}\to\mathbf{f}} \, p_{i_s},$$

- Master equation $$\mathcal{C}_{n\mathbf{k}} = \sum_{i_p, f_p} \tilde{\Gamma}_{i_p \to f_p} \left(N_{n\mathbf{k}}(f_p) - N_{n\mathbf{k}}(i_p) \right) p_{i_p}$$

In this way we can construct C_{nk} for any "spin" subsystem

Scattering rates



O(Q²)

$$D_{n\mathbf{k}} = -\frac{1}{\hbar^2}\int dt\, e^{-i\omega_{n\mathbf{k}}t}\left\langle [Q_{n\mathbf{k}}(t), Q^\dagger_{n\mathbf{k}}(0)]\right\rangle_\beta + \breve{D}_{n\mathbf{k}}$$

O(Q⁴)

$$\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = \frac{2N_{\mathrm{uc}}}{\hbar^4}\mathfrak{Re}\int_{t,t_1,t_2} e^{i[\Sigma^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}t+\Delta^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}(t_1+t_2)]}\mathrm{sign}(t_2)\left\langle\left[Q^{-q}_{n\mathbf{k}}(-t-t_2), Q^{-q'}_{n'\mathbf{k}'}(-t+t_2)\right]\left\{Q^{q'}_{n'\mathbf{k}'}(-t_1), Q^{q}_{n\mathbf{k}}(t_1)\right\}\right\rangle$$

commutator anti-commutator

Anti-detailed balance

$$\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = -\, e^{-\beta(q\omega_{n\mathbf{k}}+q'\omega_{n'\mathbf{k}'})}\, \mathfrak{W}^{\ominus,-q-q'}_{n\mathbf{k}n'\mathbf{k}'}$$

Thermal Hall effect

Anti-symmetric part

$$\kappa_H^{\mu\nu} = \frac{\hbar^2}{k_B T^2} \frac{1}{V} \sum_{n\mathbf{k}n'\mathbf{k}'} J_{n\mathbf{k}}^{\mu} \frac{e^{\beta\hbar\omega_{n\mathbf{k}}/2}}{2D_{n\mathbf{k}}} \left(\frac{1}{N_{\mathrm{uc}}} \sum_{q=\pm} \frac{\left(e^{\beta\hbar\omega_{n\mathbf{k}}} - e^{q\beta\hbar\omega_{n'\mathbf{k}'}}\right) \mathfrak{W}_{n\mathbf{k},n'\mathbf{k}'}^{\ominus,+,q}}{\sinh(\beta\hbar\omega_{n\mathbf{k}}/2)\sinh(\beta\hbar\omega_{n'\mathbf{k}'}/2)} \right) \frac{e^{\beta\hbar\omega_{n'\mathbf{k}'}/2}}{2D_{n'\mathbf{k}'}} J_{n'\mathbf{k}'}^{\nu}$$

$$J_{n\mathbf{k}}^{\mu} = N_{n\mathbf{k}}^{\mathrm{eq}}\, \omega_{n\mathbf{k}} v_{n\mathbf{k}}^{\mu}$$

Thermal Hall effect

Anti-symmetric part

$$\kappa_H^{\mu\nu} = \frac{\hbar^2}{k_B T^2} \frac{1}{V} \sum_{n\mathbf{k}n'\mathbf{k}'} J^{\mu}_{n\mathbf{k}} \frac{e^{\beta\hbar\omega_{n\mathbf{k}}/2}}{2D_{n\mathbf{k}}} \left(\frac{1}{N_{\mathrm{uc}}} \sum_{q=\pm} \frac{\left(e^{\beta\hbar\omega_{n\mathbf{k}}} - e^{q\beta\hbar\omega_{n'\mathbf{k}'}}\right) \mathfrak{W}^{\ominus,+,q}_{n\mathbf{k},n'\mathbf{k}'}}{\sinh(\beta\hbar\omega_{n\mathbf{k}}/2)\sinh(\beta\hbar\omega_{n'\mathbf{k}'}/2)} \right) \frac{e^{\beta\hbar\omega_{n'\mathbf{k}'}/2}}{2D_{n'\mathbf{k}'}} J^{\nu}_{n'\mathbf{k}'}$$

$$J^{\mu}_{n\mathbf{k}} = N^{\mathrm{eq}}_{n\mathbf{k}}\,\omega_{n\mathbf{k}} v^{\mu}_{n\mathbf{k}}$$

Basic idea

$$\#\nabla T = -\frac{1}{\tau}\delta n - \frac{1}{\tau_{\mathrm{skew}}}\delta n$$

$$\delta n = -\tau\#\nabla T - \frac{\tau}{\tau_{\mathrm{skew}}}\delta n$$

$$\approx -\tau\#\nabla T - \frac{\tau^2}{\tau_{\mathrm{skew}}}\#\nabla T$$

Thermal Hall effect

Conductivity versus resistivity

$$\kappa_H \sim \frac{\tau^2}{\tau_{\text{skew}}}$$

Sensitive to all ordinary scattering mechanisms. Very non-universal

$$\varrho_H \sim -\frac{\kappa_H}{\kappa^2} \sim \frac{1}{\tau_{\text{skew}}}$$

Only sensitive to skew scattering. A better quantity to study.

$$\varrho_H \sim \mathfrak{W}^{\ominus,\text{eff}}\,,$$

Indeed follows from our formulae

Many-body skew scattering

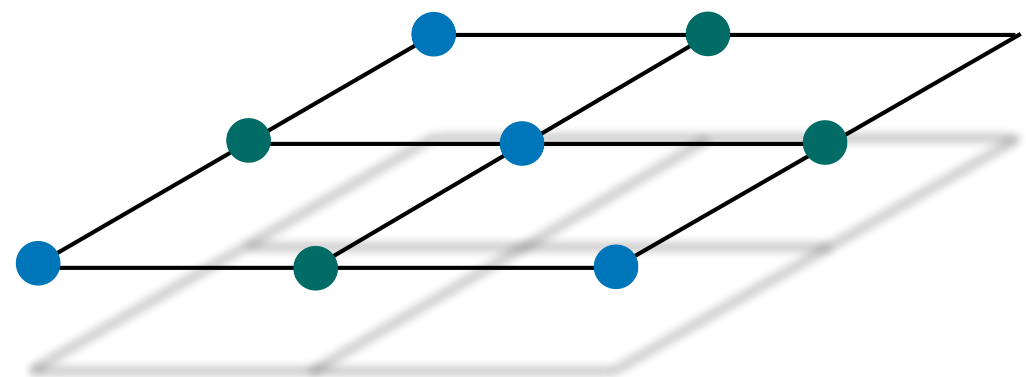
$$\mathfrak{W}^{\ominus,qq'}_{n\mathbf{k}n'\mathbf{k}'} = \frac{2N_{\rm uc}}{\hbar^4}\mathfrak{Re}\int_{t,t_1,t_2} e^{i[\Sigma^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}t+\Delta^{q,q'}_{n\mathbf{k}n'\mathbf{k}'}(t_1+t_2)]}\mathrm{sign}(t_2)\left\langle\left[Q^{-q}_{n\mathbf{k}}(-t-t_2),Q^{-q'}_{n'\mathbf{k}'}(-t+t_2)\right]\left\{Q^{q'}_{n'\mathbf{k}'}(-t_1),Q^{q}_{n\mathbf{k}}(t_1)\right\}\right\rangle$$

What good is it?

- In principle, this can be applied for any Q, could be e.g. quantum critical field etc.
- Can be used to analyze symmetries, *ala* Onsager
- That said, it is very hard to calculate such real-time correlation functions…maybe with a quantum simulator?

Application to an antiferromagnet

For concreteness,
2d, layered



Spin waves

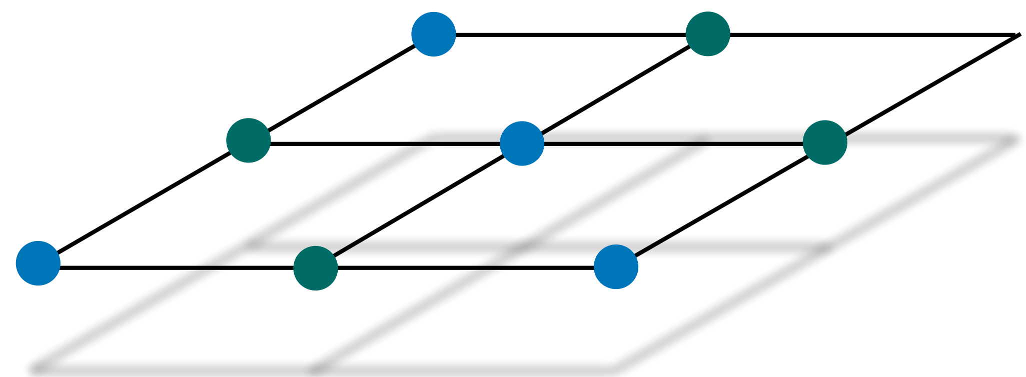
$$H_{\rm NLS} + H_{\rm field} = \sum_{\ell} \sum_{\mathbf{k}} \Omega_{\mathbf{k},\ell} b^{\dagger}_{\mathbf{k},\ell} b_{\mathbf{k},\ell}$$

Collective field

$$Q^{q}_{n\mathbf{k}} = \sum_{\ell, q_1, z} \mathcal{A}^{n,\ell|q_1 q}_{\mathbf{k}} \, e^{ik_z z} b^{q_1}_{\ell,\mathbf{k},z} \; + \frac{1}{\sqrt{N_{\rm uc}}} \sum_{\substack{\mathbf{p},\ell,\ell' \\ q_1, q_2, z}} \mathcal{B}^{n,\ell_1,\ell_2|q_1 q_2 q}_{\mathbf{k};\mathbf{p}} \; e^{ik_z z} b^{q_1}_{\ell_1,\mathbf{p}+\frac{q}{2}\mathbf{k},z} b^{q_2}_{\ell_2,-\mathbf{p}+\frac{q}{2}\mathbf{k},z}$$

Application to an antiferromagnet

For concreteness,
2d, layered



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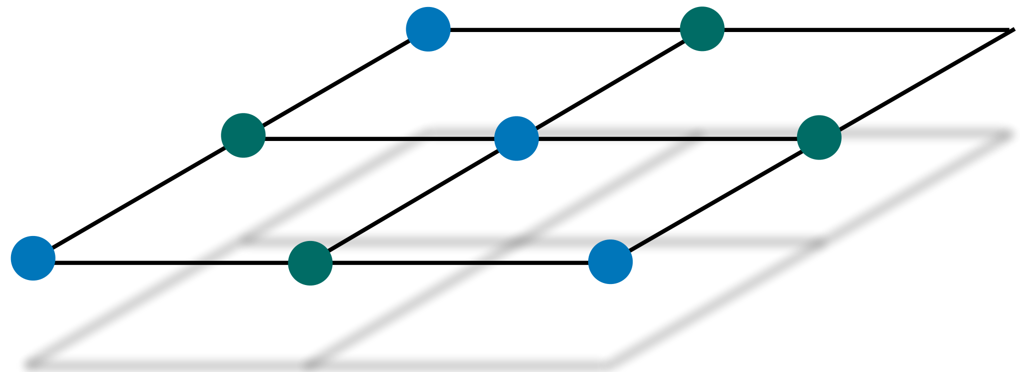
Negligible phase space

Application to an antiferromagnet

For concreteness,

2d, layered



Spin waves

$$H_{\mathrm{NLS}} + H_{\mathrm{field}} = \sum_{\ell} \sum_{\mathbf{k}} \Omega_{\mathbf{k},\ell} b^{\dagger}_{\mathbf{k},\ell} b_{\mathbf{k},\ell}$$

Collective field

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Negligible phase space

Structure hidden here

General result

- Diagonal scattering rate:

$$D_{n\mathbf{k}}^{(s)} = \frac{(3-s)\pi}{\hbar^2 N_{\mathrm{uc}}^{2\mathrm{d}}} \sum_{\mathbf{p}} \sum_{\ell,\ell'} \frac{\sinh(\frac{\beta}{2}\hbar\omega_{n\mathbf{k}})}{\sinh(\frac{\beta}{2}\hbar\Omega_{\ell,\mathbf{p}})\sinh(\frac{\beta}{2}\hbar\Omega_{\ell',\mathbf{p}-\mathbf{k}})} \quad \delta(\omega_{n\mathbf{k}} - \Omega_{\ell,\mathbf{p}} - s\Omega_{\ell',\mathbf{p}-\mathbf{k}}) \left| \mathcal{B}_{\mathbf{k};-\mathbf{p}+\frac{\mathbf{k}}{2}}^{n,\ell,\ell'|+s-} \right|^2$$

- Skew scattering rate:

$$\mathfrak{W}_{n\mathbf{k},n'\mathbf{k}'}^{\ominus,qq'} = \frac{64\pi^2}{\hbar^4} \frac{1}{N_{\mathrm{uc}}^{2d}} \sum_{\mathbf{p}} \sum_{\{\ell_i,q_i\}} \mathfrak{D}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}}^{nn'|q_1q_2q_3,\ell_1\ell_2\ell_3} \; \mathfrak{F}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}}^{q_1q_2q_4,\ell_1\ell_2\ell_3} \; \mathfrak{Im}\Big\{ \mathcal{B}_{\mathbf{k},\mathbf{p}+\frac{1}{2}q\mathbf{k}+q'\mathbf{k}'}^{n\ell_2\ell_3|q_2q_3q} \mathcal{B}_{\mathbf{k}',\mathbf{p}+\frac{1}{2}q'\mathbf{k}'}^{n'\ell_3\ell_1|-q_3q_1q'}$$
$$\times \mathrm{PP}\Big[\frac{\mathcal{B}_{\mathbf{k},\mathbf{p}+\frac{1}{2}q\mathbf{k}}^{n\ell_1\ell_4|-q_1q_4-q} \mathcal{B}_{\mathbf{k}',\mathbf{p}+q\mathbf{k}+\frac{1}{2}q'\mathbf{k}'}^{n'\ell_4\ell_2|-q_4-q_2-q'}}{\Delta_{n\mathbf{k}n'\mathbf{k}'}^{qq'} + q_1\Omega_{\ell_1,\mathbf{p}} - q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'} - 2q_4\Omega_{\ell_4,\mathbf{p}+q\mathbf{k}}} + \frac{\mathcal{B}_{\mathbf{k}',\mathbf{p}+\frac{1}{2}q'\mathbf{k}'}^{n'\ell_1\ell_4|-q_1-q_4-q'} \mathcal{B}_{\mathbf{k},\mathbf{p}+\frac{1}{2}q\mathbf{k}+q'\mathbf{k}'}^{n\ell_4\ell_2|q_4-q_2-q}}{\Delta_{n\mathbf{k}n'\mathbf{k}'}^{qq'} - q_1\Omega_{\ell_1,\mathbf{p}} + q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'} - 2q_4\Omega_{\ell_4,\mathbf{p}+q'\mathbf{k}'}} \Big] \Big\}$$

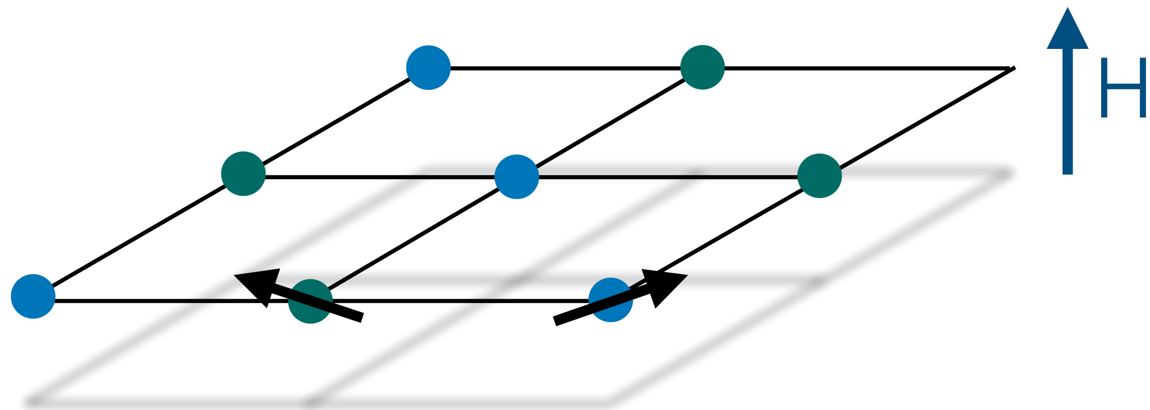
$$\mathfrak{D}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}}^{nn'|q_1q_2q_3,\ell_1\ell_2\ell_3} = \delta\left(\Sigma_{n\mathbf{k}n'\mathbf{k}'}^{qq'} + q_1\Omega_{\ell_1,\mathbf{p}} + q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'}\right) \delta\left(\Delta_{n\mathbf{k}n'\mathbf{k}'}^{qq'} + 2q_3\Omega_{\ell_3,\mathbf{p}+q'\mathbf{k}'} - q_1\Omega_{\ell_1,\mathbf{p}} + q_2\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'}\right),$$
$$\mathfrak{F}_{q\mathbf{k}q'\mathbf{k}',\mathbf{p}}^{q_1q_2q_4,\ell_1\ell_2\ell_3} = q_4\left(2n_{\mathrm{B}}(\Omega_{\ell_3,\mathbf{p}+q'\mathbf{k}'}) + 1\right)\left(2n_{\mathrm{B}}(\Omega_{\ell_1,\mathbf{p}}) + q_1 + 1\right)\left(2n_{\mathrm{B}}(\Omega_{\ell_2,\mathbf{p}+q\mathbf{k}+q'\mathbf{k}'}) + q_2 + 1\right).$$

Could be applied to any magnet

Continuum magnons

Hamiltonian

$$\mathcal{H}_{\mathrm{NLS}} = \frac{\rho}{2}\left(|\underline{\boldsymbol{\nabla}} n_y|^2 + |\underline{\boldsymbol{\nabla}} n_z|^2\right) + \frac{1}{2\chi}(m_y^2 + m_z^2) + \sum_{a,b=y,z} \frac{\Gamma_{ab}}{2} n_a n_b$$



Spin-lattice coupling

$$\mathcal{H}_{\mathrm{s-l}} = \sum_{\substack{\alpha,\beta \\ a,b=x,y,z}} \mathcal{E}_{\mathbf{r}}^{\alpha\beta} \left(\Lambda_{ab}^{(\mathrm{n}),\alpha\beta} \mathrm{n}_a \mathrm{n}_b + \frac{\Lambda_{ab}^{(\mathrm{m}),\alpha\beta}}{n_0^2} \mathrm{m}_a \mathrm{m}_b\right)\Bigg|_{\mathbf{x},z}$$

$$|\mathbf{n}|^2 + \frac{\mathfrak{a}^4}{\mu_0^2}|\mathbf{m}|^2 = 1, \qquad \mathbf{m}\cdot\mathbf{n} = 0.$$

Solve NLSM constraints, expand around canted state

$$\mathcal{H}_{\mathrm{s-l}} \approx \sum_{\alpha\beta} \mathcal{E}_{\mathbf{r}}^{\alpha\beta} \sum_{a,b=y,z} \sum_{\xi,\xi'=m,n} \lambda_{\xi_a,\xi'_b}^{\alpha\beta} n_0^{-\xi-\xi'} \xi_{a\mathbf{r}} \xi'_{b\mathbf{r}}$$

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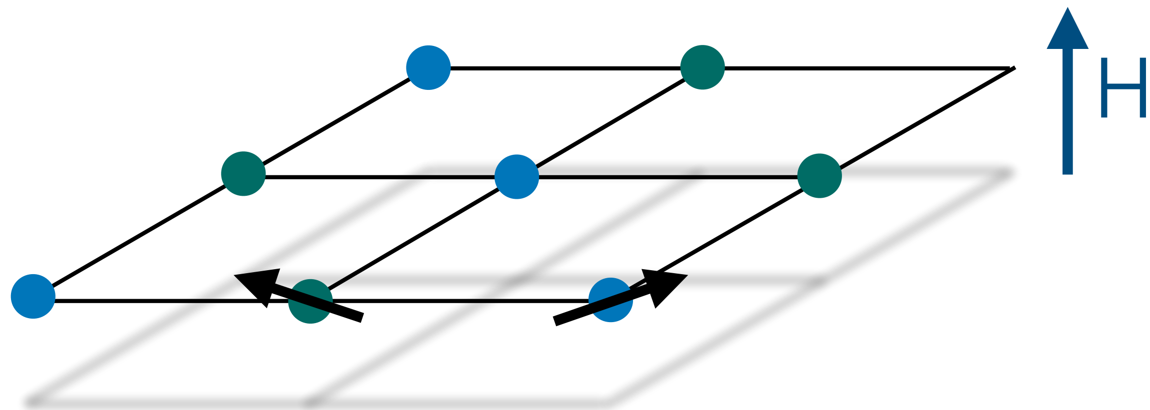

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n.b.

m m

n n

m n

Effective TRS breaking

Scaling

- B coefficients: $\Omega \sim \omega \sim v_{\mathrm{ph}} k \sim k_B T$

$$\mathcal{B} \sim \left(\frac{k_B T}{M v_{\mathrm{ph}}^2}\right)^{\frac{1}{2}} n_0^{-1} \left(\lambda_{mm} \frac{\chi k_B T}{n_0} + \lambda_{mn} + \lambda_{nn} \frac{n_0}{\chi k_B T}\right) \quad \sim T^{1/2+x}$$

smallness: ions are heavy.

Antiferromagnet: order-parameter (n) has strongest correlations

- Diagonal scattering rate: $D_{n\mathbf{k}}^{(s)} = \frac{(3-s)\pi}{\hbar^2 N_{\mathrm{uc}}^{2\mathrm{d}}} \sum_{\mathbf{p}} \sum_{\ell,\ell'} \frac{\sinh(\frac{\beta}{2}\hbar\omega_{n\mathbf{k}})}{\sinh(\frac{\beta}{2}\hbar\Omega_{\ell,\mathbf{p}}) \sinh(\frac{\beta}{2}\hbar\Omega_{\ell',\mathbf{p}-\mathbf{k}})} \quad \delta(\omega_{n\mathbf{k}} - \Omega_{\ell,\mathbf{p}} - s\Omega_{\ell',\mathbf{p}-\mathbf{k}}) \left|\mathcal{B}_{\mathbf{k};-\mathbf{p}+\frac{\mathbf{k}}{2}}^{n,\ell,\ell'|+s-}\right|^2$

$$\frac{1}{\tau} \sim T^{d-1} |\mathcal{B}|^2 \sim T^{d+2x}$$

$\sim T^{d+2}, T^{d}, T^{d-2}$?

Scaling

- B coefficients: $\Omega \sim \omega \sim v_{\mathrm{ph}} k \sim k_B T$

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$$\frac{1}{\tau} \sim T^{d-1} |\mathcal{B}|^2 \sim T^{d+2x}$$

$\sim T^{d+2}$, T^{d}, T^{d-2}

Spin–phonon interactions in a Heisenberg antiferromagnet: II. The phonon spectrum and spin–lattice relaxation rate

M G Cottam
Department of Physics, University of Essex, Colchester CO4 3SQ, England

Received 11 March 1974

$$\frac{1}{\tau_{\mathrm{SL}}} \simeq \frac{b_1 S^2 (r^2 - 1)}{D^{10}} \left(\frac{5 T_{\mathrm{D}}^3}{12\pi^4} + \frac{\pi^2 D^3}{24 V}\right) Q_0^2 T^5$$

Scaling: Hall

From the formula:

$$\mathfrak{W}^{\ominus} \sim T^{d-3}\mathcal{B}^4$$

Effective TRS breaking: one factor of m-n coupling:

$$\sim T^{d-1}\lambda_{mn}\left(\lambda_{mm}T + \lambda_{nn}T^{-1}\right)^3 \sim T^{d-1+3x},$$

This gives Hall resistivity:

$$\varrho_H \sim \mathfrak{W}^{\ominus,\mathrm{eff}} \sim T^{d-1+3x}$$

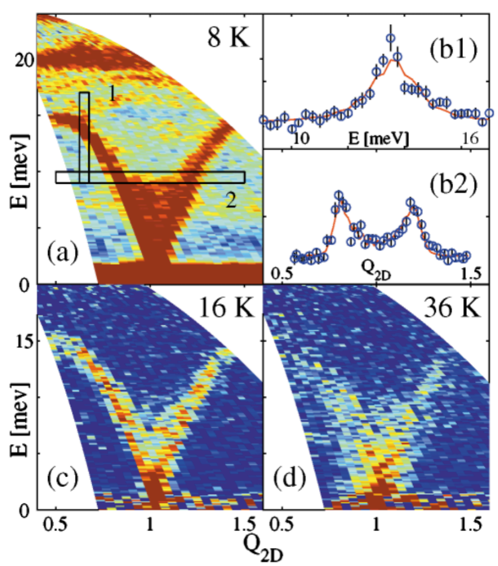
Check: numerical calculation

Many parameters: loosely inspired by Copper Deuteroformate Tetradeuterate (CFTD)

VOLUME 87, NUMBER 3 PHYSICAL REVIEW LETTERS 16 JULY 2001

Spin Dynamics of the 2D Spin $\frac{1}{2}$ Quantum Antiferromagnet Copper Deuteroformate Tetradeuterate (CFTD)

H. M. Rønnow,[1,2] D. F. McMorrow,[1] R. Coldea,[3,4] A. Harrison,[5] I. D. Youngson,[5] T. G. Perring,[4] G. Aeppli,[6] O. Syljuåsen,[7] K. Lefmann,[1] and C. Rischel[8]



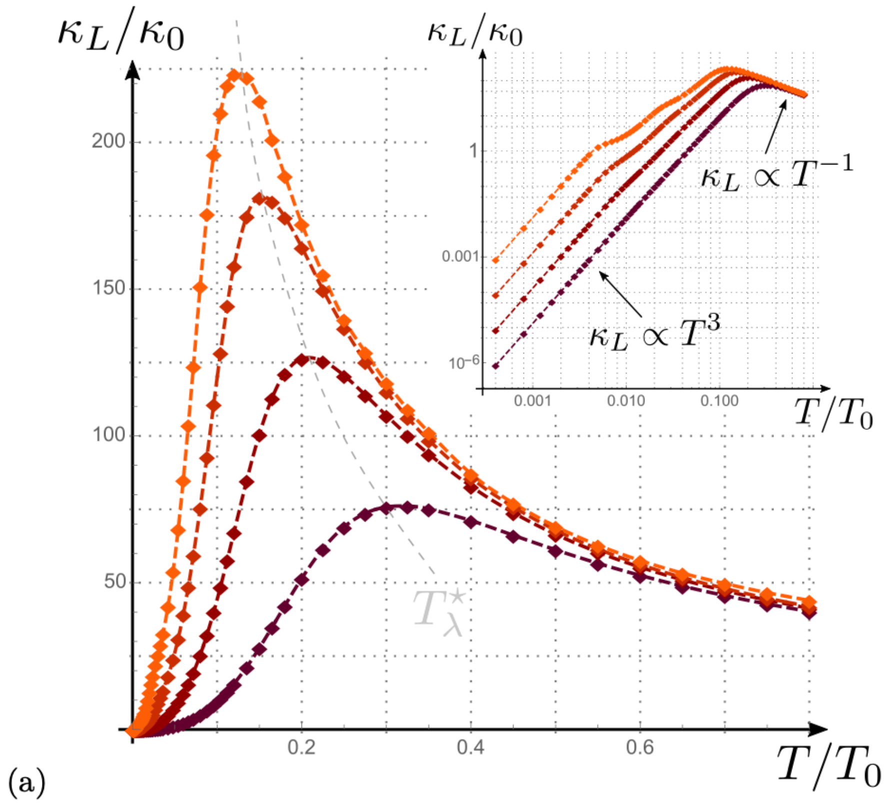
Good match of magnon and phonon phase space

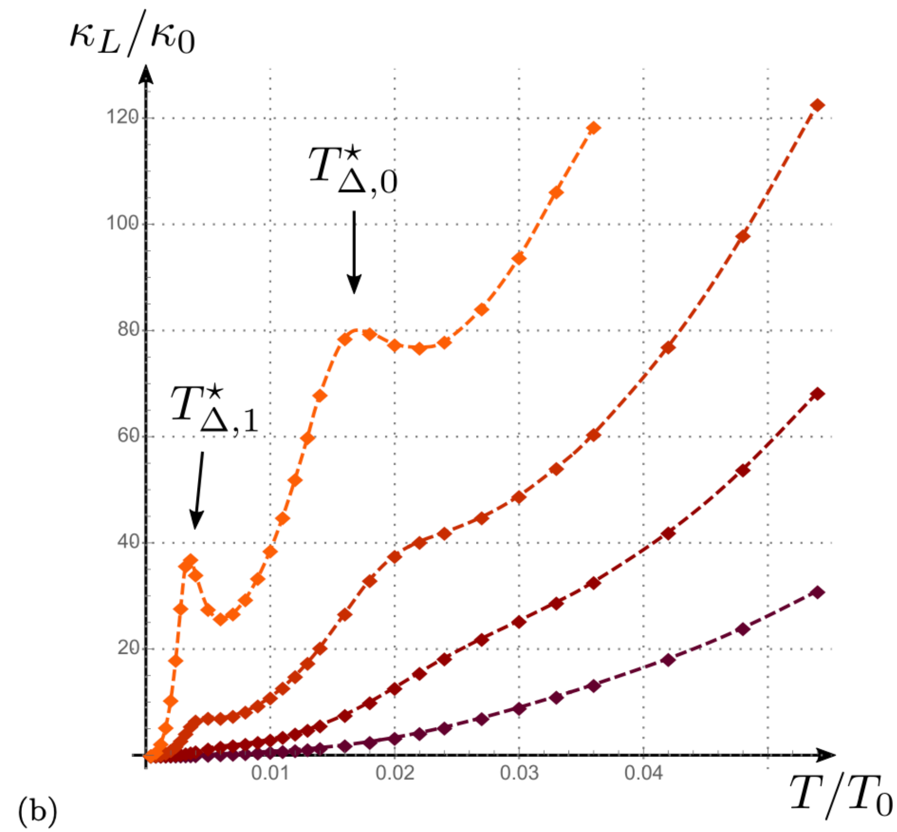
$\frac{v_m}{v_{ph}}$	$\chi\epsilon_0\mathfrak{a}^2$	n_0	$\frac{M_{uc}v_{ph}\mathfrak{a}}{\hbar}$	m_0^x	m_0^y	m_0^z	$\frac{\Delta_0}{\epsilon_0}$	$\frac{\Delta_1}{\epsilon_0}$
2.5	0.19	1/2	$8\cdot10^3$	0	0.0 0.05	0.05 0.0	0.2	0.04

ξ	$\Lambda_1^{(\xi)}$	$\Lambda_2^{(\xi)}$	$\Lambda_3^{(\xi)}$	$\Lambda_4^{(\xi)}$	$\Lambda_5^{(\xi)}$	$\Lambda_6^{(\xi)}$	$\Lambda_7^{(\xi)}$
n = 0	12.0	10.0	14.0	10.0	12.0	0.6	0.8
m = 1	−10.0	−12.0	−14.0	−12.0	−10.0	−0.8	−0.6

TABLE I: Numerical values of the fixed dimensionless parameters used in all numerical evaluations. The upper and lower entries for m_0^y and m_0^z correspond to the two cases for calculating ϱ_H^{xy} and ϱ_H^{xz}, respectively. The couplings $\Lambda_i^{(\xi)}$ are given in units of $\epsilon_0/\mathfrak{a}$.

Diagonal conductivity

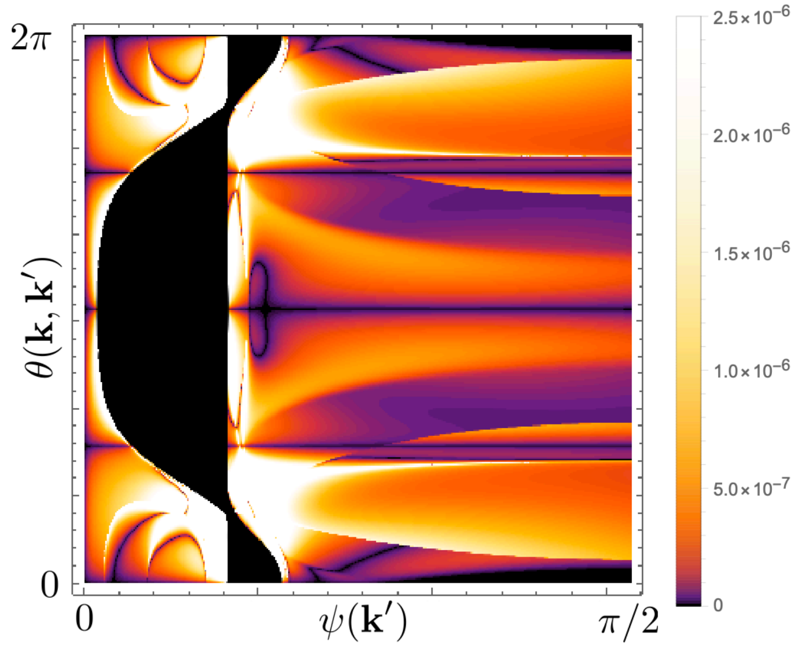




One can see Heisenberg regimes,
anisotropic regime, extrinsic regime

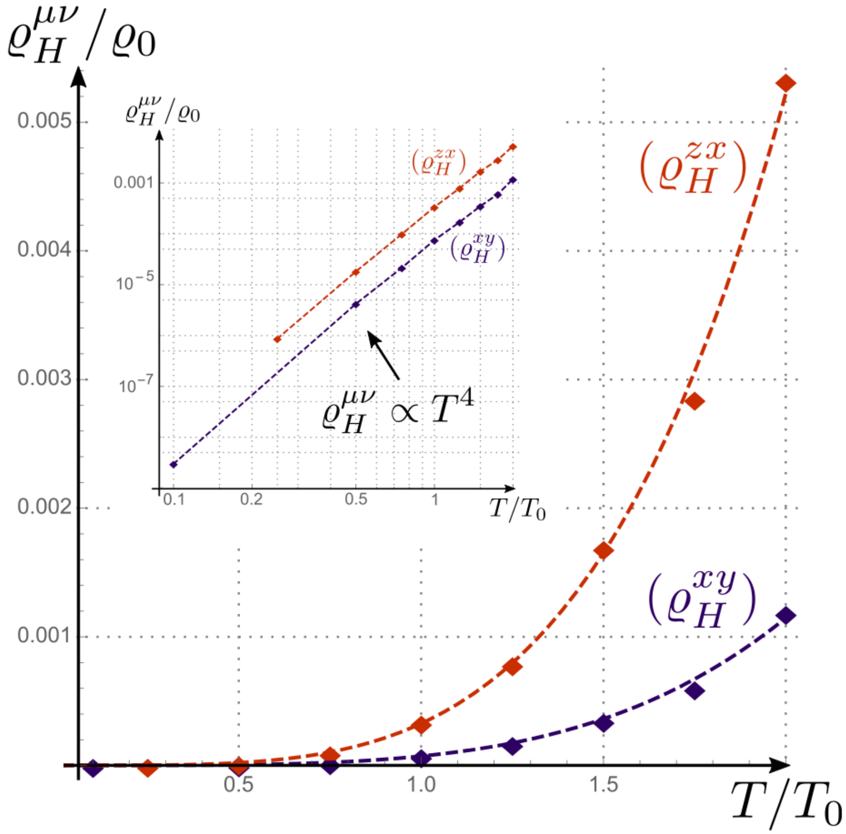
Skew scattering

Cut through the skew scattering rate:



A very complex object, lots of phase space features

Thermal Hall resistivity



Observe T4 behavior
(Heisenberg regime)

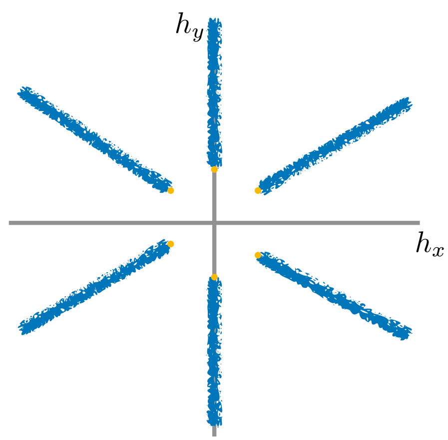
Larger effect with current perpendicular to plane, even though we took the magnetism *strictly* 2d (magnons do not propagate in z direction)

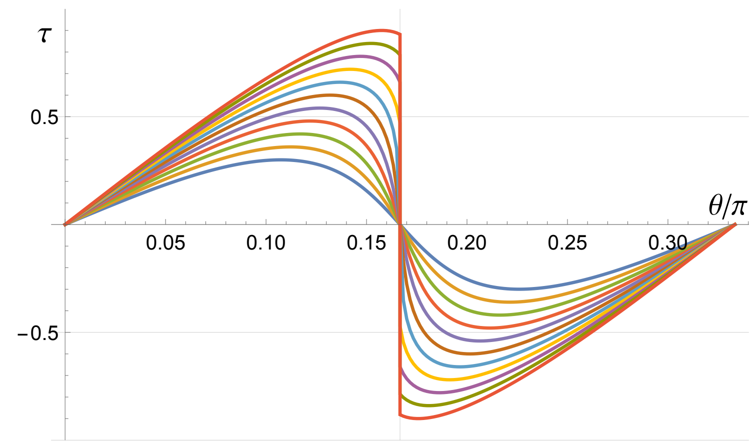
$$\kappa_0 = k_B v_{\rm ph}/\mathfrak{a}^2$$

$$\varrho_0 = \kappa_0^{-1}$$

$$\kappa_0^{\rm CFTD} = 0.17\ {\rm W{\cdot}K^{-1}{\cdot}m^{-1}}$$

Thanks





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