



General introduction to frustrated magnetism

Leon Balents

HFM Paris, June 2022

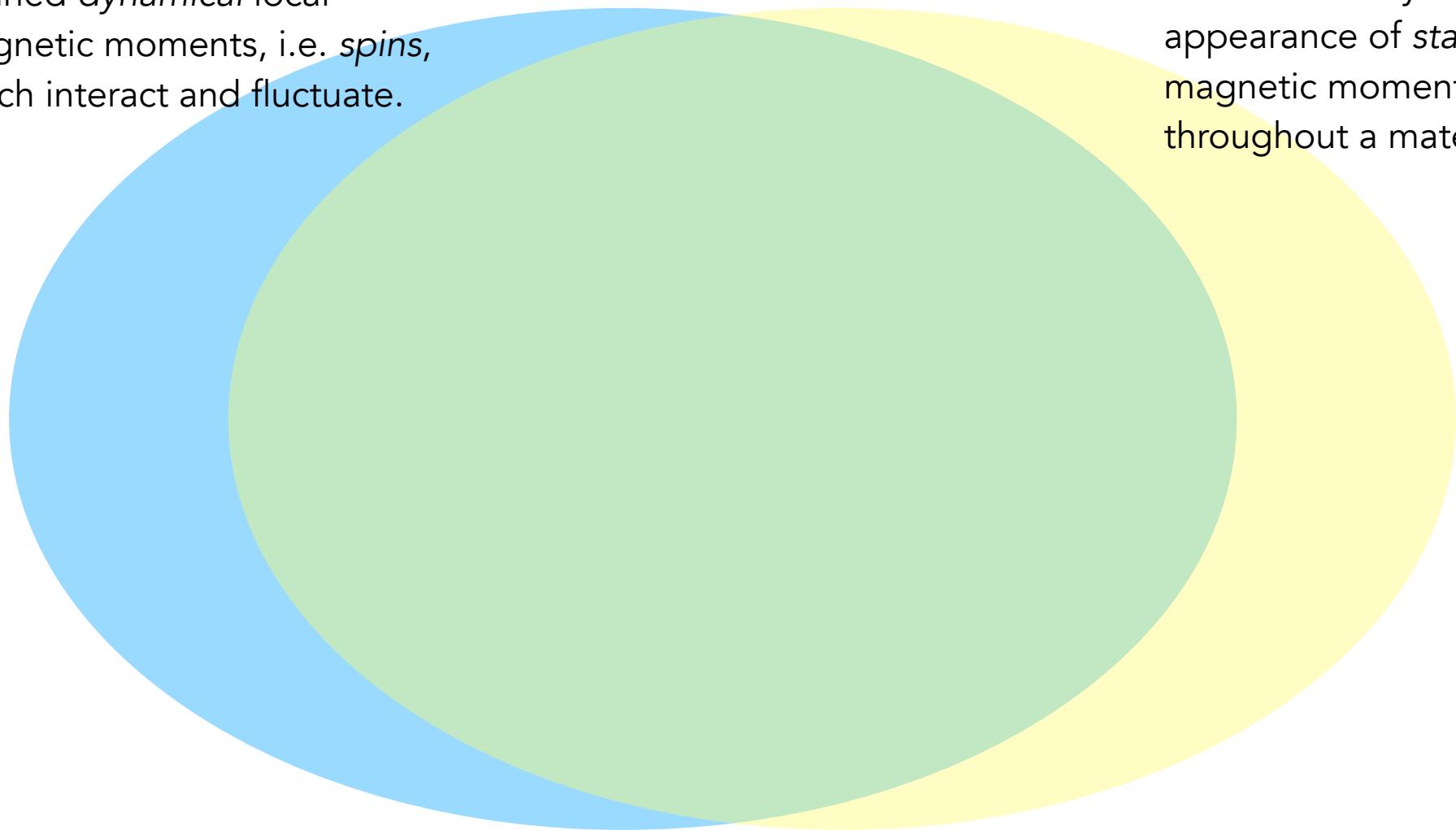
Magnetism

Magnetic degrees of freedom:

existence of well-defined *dynamical* local magnetic moments, i.e. *spins*, which interact and fluctuate.

Magnetic order:

spontaneous breaking of time-reversal symmetry, appearance of *static* magnetic moments throughout a material



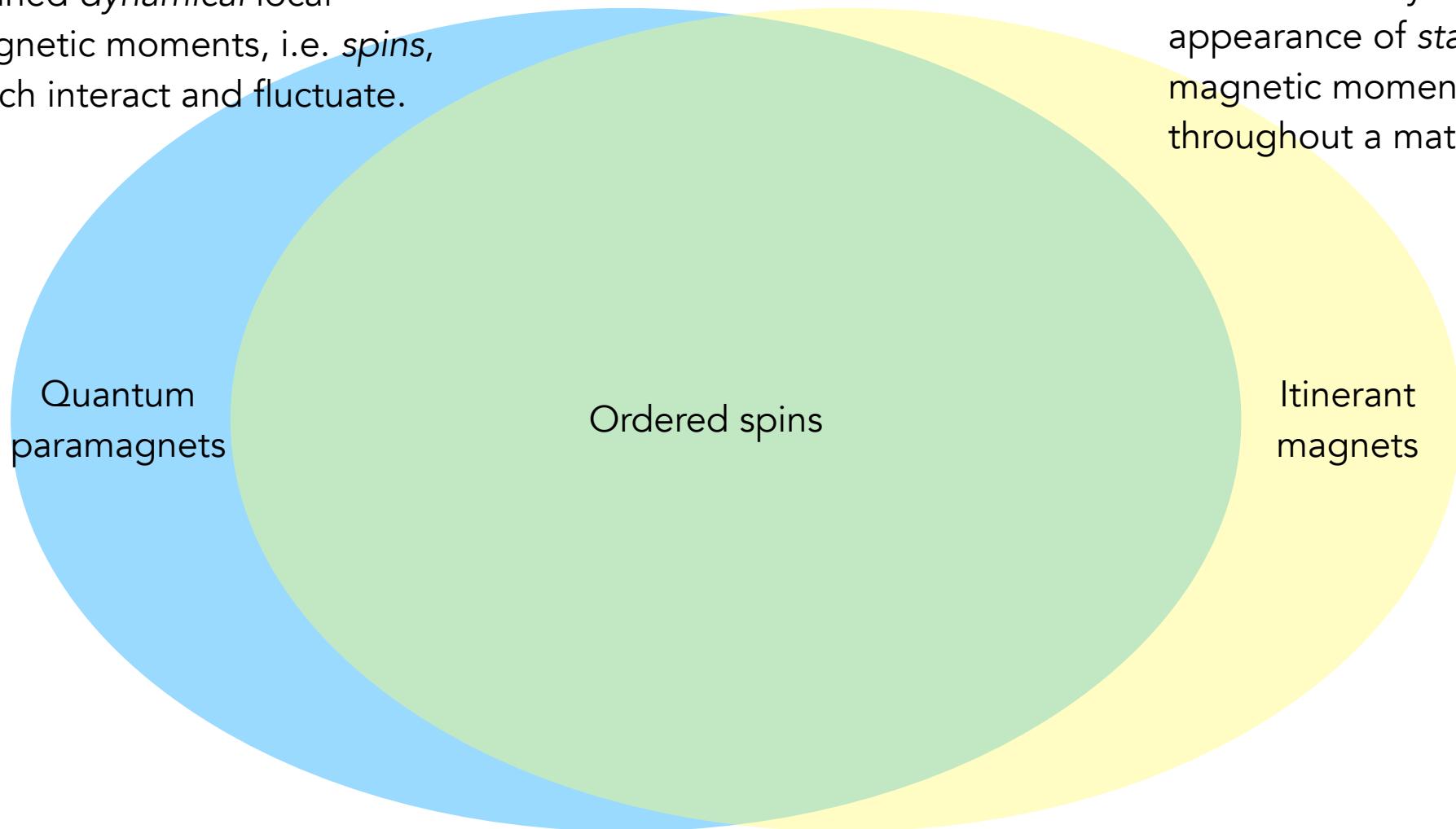
Magnetism

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Magnetic order:

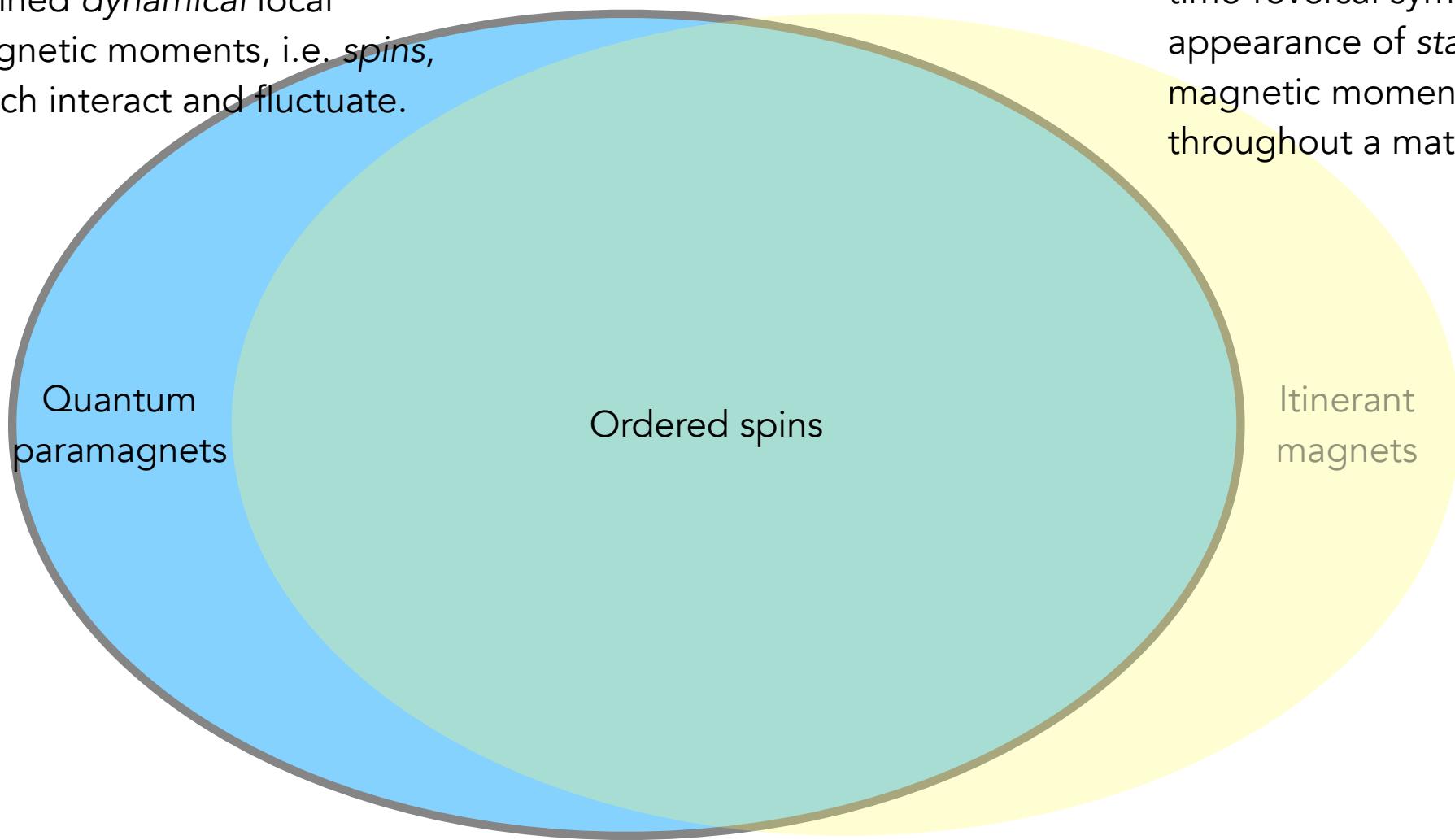
spontaneous breaking of time-reversal symmetry, appearance of *static* magnetic moments throughout a material



Magnetism

Magnetic degrees of freedom:

existence of well-defined *dynamical* local magnetic moments, i.e. *spins*, which interact and fluctuate.



Magnetic order:

spontaneous breaking of time-reversal symmetry, appearance of *static* magnetic moments throughout a material

Theorist's view

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad (+ \dots)$$



Ising = ± 1

Vectors

Quantum operators

Coupling to the lattice
Coupling to itinerant electrons
Multipolar, multi-spin interactions
Applied fields
...

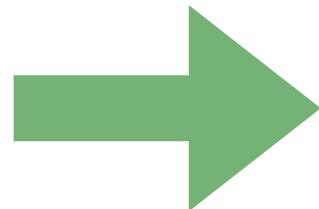
Materials design: find good choices of J_{ij} , S_i (+...)

Design

Input

J_{ij} , S_i
(+...)

Physics



Output

Emergent properties:

Ordering

e.g. FM, AF, spiral,
quadrupolar

Collective excitations

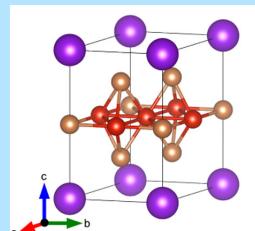
e.g. magnons,
skyrmions, spinons

Response functions

e.g. susceptibility,
Hall effect

Design

Input

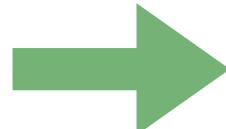


Physics

J_{ij} , S_i

(+...)

Physics



Output

Emergent properties:

Ordering

e.g. FM, AF, spiral,
quadrupolar

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e.g. magnons,
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e.g. susceptibility,
Hall effect

Design

Input



Physics

J_{ij} , S_i

(+...)

Output

What do we really control?

- Atoms and environment
- Structure and symmetry

Input properties:

• 1D, 2D, 3D
• AF, spiral,
• polar

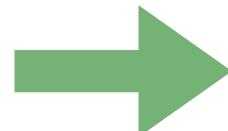
Collective excitations

e.g. magnons,
skyrmions, spinons

Response functions

e.g. susceptibility,
Hall effect

Physics



Design principle 1: get local moments

- Most magnetism in QMs comes from either 3d transition metal ions or 4f rare earths. These have relatively localized orbitals which don't overlap strongly with neighbors and have strong Coulomb repulsion, which localizes electrons best.

¹ H 1.00794	⁴ Be 9.012182	⁵ B 10.811	⁶ C 12.0107	⁷ N 14.00674	⁸ O 15.9994	⁹ F 18.9984032	¹⁰ Ne 20.1797												
³ Li 6.941	⁴ Be 9.012182	¹³ Al 26.581538	¹⁴ Si 28.0855	¹⁵ P 30.973761	¹⁶ S 32.066	¹⁷ Cl 35.4527	¹⁸ Ar 39.948												
¹¹ Na 22.989770	¹² Mg 24.3050	¹⁹ K 39.0983	²⁰ Ca 40.078	²¹ Sc 44.95591	²² Ti 47.867	²³ V 50.9415	²⁴ Cr 51.9961	²⁵ Mn 54.938049	²⁶ Fe 55.845	²⁷ Co 58.933200	²⁸ Ni 58.6534	²⁹ Cu 63.545	³⁰ Zn 65.39	³¹ Ga 69.723	³² Ge 72.61	³³ As 74.92160	³⁴ Se 78.96	³⁵ Br 79.504	³⁶ Kr 83.80
³⁷ Rb 85.4678	³⁸ Sr 87.62	³⁹ Y 88.90585	⁴⁰ Zr 91.224	⁴¹ Nb 92.90638	⁴² Mo 95.94	⁴³ Tc (98)	⁴⁴ Ru 101.07	⁴⁵ Rh 102.90550	⁴⁶ Pd 106.42	⁴⁷ Ag 106.42	⁴⁸ Cd 112.411	⁴⁹ In 114.818	⁵⁰ Sn 118.710	⁵¹ Sb 121.760	⁵² Te 127.60	⁵³ I 126.90447	⁵⁴ Xe 131.29		
⁵⁵ Cs 132.90545	⁵⁶ Ba 137.327	⁵⁷ La 138.9055	⁵⁸ Hf 178.49	⁵⁹ Ta 180.9479	⁶⁰ W 183.84	⁶¹ Re 186.207	⁶² Os 190.23	⁶³ Ir 192.217	⁶⁴ Pt 195.078	⁶⁵ Au 196.56655	⁶⁶ Hg 200.59	⁶⁷ Tl 204.3833	⁶⁸ Pb 207.2	⁶⁹ Bi 208.58038	⁷⁰ Po (209)	⁷¹ At (210)	⁷² Rn (222)		
⁸⁷ Fr (223)	⁸⁸ Ra (226)	⁸⁹ Ac (227)	¹⁰⁴ Rf (261)	¹⁰⁵ Db (262)	¹⁰⁶ Sg (263)	¹⁰⁷ Bh (262)	¹⁰⁸ Hs (265)	¹⁰⁹ Mt (266)	¹¹⁰ Dy (269)	¹¹¹ Ho (272)	¹¹² Tm (277)	¹¹⁴ (289)	¹¹⁶ (289)	¹¹⁷ (289)	¹¹⁸ (293)				

⁵⁸ Ce 140.116	⁵⁹ Pr 140.50765	⁶⁰ Nd 144.24	⁶¹ Pm (145)	⁶² Sm 150.36	⁶³ Eu 151.964	⁶⁴ Gd 157.25	⁶⁵ Tb 158.92534	⁶⁶ Dy 162.50	⁶⁷ Ho 164.93032	⁶⁸ Er 167.26	⁶⁹ Tm 168.93421	⁷⁰ Yb 173.04	⁷¹ Lu 174.967
⁹⁰ Th 232.0381	⁹¹ Pa 231.01588	⁹² U 238.0289	⁹³ Np (237)	⁹⁴ Pu (244)	⁹⁵ Am (243)	⁹⁶ Cm (247)	⁹⁷ Bk (247)	⁹⁸ Cf (251)	⁹⁹ Es (252)	¹⁰⁰ Fm (257)	¹⁰¹ Md (258)	¹⁰² No (259)	¹⁰³ Lr (262)

Local moments

- In 3d transition metals, usually magnetism is fairly isotropic, i.e. spins are “Heisenberg like”, because crystal fields split the d orbitals and spin-orbit coupling is relatively weak (Co is most common exception, when very localized). Exchange interactions between spins vary from quite strong (1000K) to quite weak (1K).

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¹⁹ K 39.0983	²⁰ Ca 40.078
²¹ Sc 44.95591	²² Ti 47.867
²³ V 50.9415	²⁴ Cr 51.9961
²⁵ Mn 54.938049	²⁶ Fe 55.845
²⁷ Co 58.933200	²⁸ Ni 58.6534
²⁹ Cu 63.545	³⁰ Zn 65.39
³¹ Ga 69.723	³² Ge 72.61
³³ As 74.92160	³⁴ Se 78.96
³⁵ Br 79.504	³⁶ Kr 83.80
³⁷ Rb 85.4678	³⁸ Sr 87.62
³⁹ Y 88.90585	⁴⁰ Zr 91.224
⁴¹ Nb 92.90638	⁴² Mo 95.94
⁴³ Tc (98)	⁴⁴ Ru 101.07
⁴⁵ Rh 102.90550	⁴⁶ Pd 106.42
⁴⁷ Ag 106.42	⁴⁸ Cd 112.411
⁴⁹ In 114.818	⁵⁰ Sn 118.710
⁵¹ Sb 121.760	⁵² Te 127.60
⁵³ I 126.90447	⁵⁴ Xe 131.29
⁵⁵ Cs 132.90545	⁵⁶ Ba 137.327
⁵⁷ La 138.9055	⁷² Hf 178.49
⁷³ Ta 180.9479	⁷⁴ W 183.84
⁷⁵ Re 186.207	⁷⁶ Os 190.23
⁷⁷ Ir 192.217	⁷⁸ Pt 195.078
⁷⁹ Au 196.56655	⁸⁰ Hg 200.59
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⁸³ Bi 208.58038	⁸⁴ Po (209)
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⁸⁷ Fr (223)	⁸⁸ Ra (226)
⁸⁹ Ac (227)	¹⁰⁴ Rf (261)
¹⁰⁵ Db (262)	¹⁰⁶ Sg (263)
¹⁰⁷ Bh (262)	¹⁰⁸ Hs (265)
¹⁰⁹ Mt (266)	¹¹⁰ Mt (269)
¹¹¹ (272)	¹¹¹ (277)
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Local moments

- In 4f lanthanides, spin-orbit coupling is dominant over crystal fields and so magnetic moments become large (incorporating orbital moment) and often very anisotropic (due to large SOC). They have complex multiplet structures, and weak exchange interactions.

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²⁷ Co 58.933200	²⁸ Ni 58.6534
²⁹ Cu 63.545	³⁰ Zn 65.39
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³⁵ Kr 83.80	³⁶ Kr 83.80
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⁵¹ Sb 121.760	⁵² Te 127.60
⁵³ I 126.90447	⁵⁴ Xe 131.29
⁵⁵ La 138.9055	⁵⁷ Hf 178.49
⁵⁶ Ta 180.94.79	⁷² W 183.84
⁷⁴ Re 186.207	⁷⁵ Os 190.23
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QM Materials

- Quantum spin liquids and interesting insulating antiferromagnets

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- $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$,
 a-RuCl_3 , $\text{Pr}_2\text{Zr}_2\text{O}_7$,
 Cs_2CuCl_4 , $\text{Yb}_2\text{Ti}_2\text{O}_7$

QM Materials

- Orbital degeneracy/spin-orbit interaction
- $\text{RVO}_3, \text{RCoO}_3, \dots$
- $\text{Cd}_2\text{Os}_2\text{O}_7, \text{Sr}_2\text{IrO}_4, \text{Na}_2\text{IrO}_3, \text{a-RuCl}_3 \dots$
- $\text{NaYbO}_2, \text{TmMgGaO}_4, \dots$

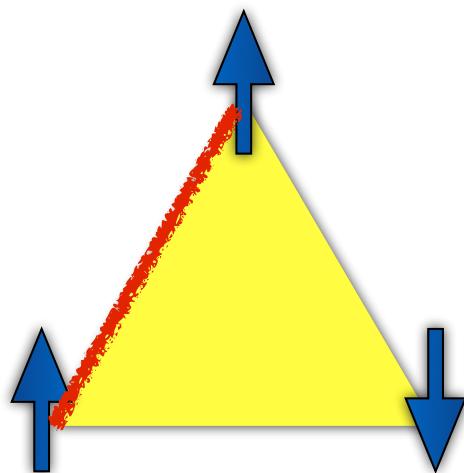
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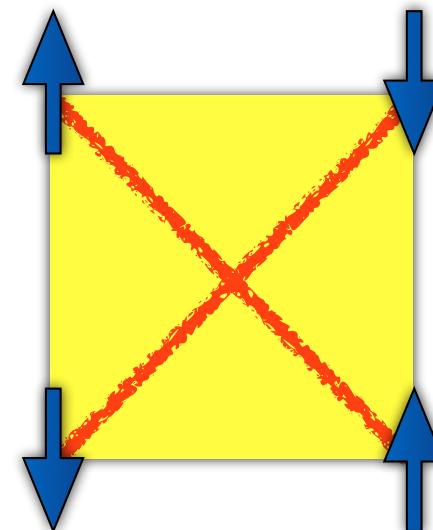
SOC increases with
atomic number

Design principle 2: Frustration

- What is it? Competing multiple interactions that cannot be simultaneously satisfied



geometric
frustration



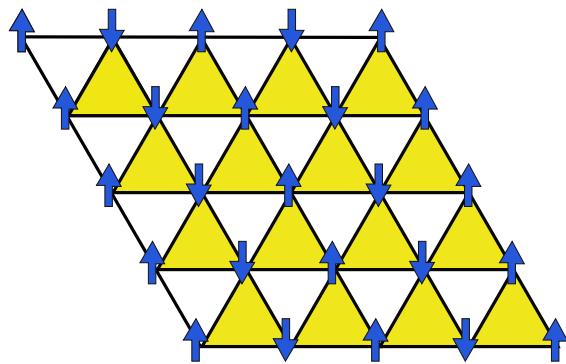
exchange
frustration

Why frustration?

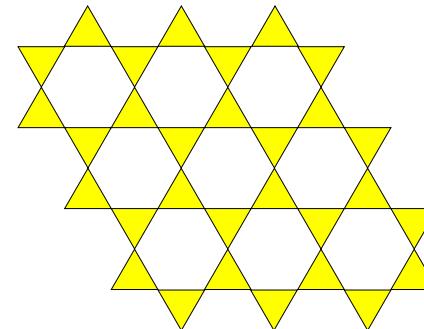
- Competition suppresses conventional ordered states: more exotic things are possible
- Fluctuating regimes
- Complex or quantum orders
- Spin liquids
- Unusual excitations

Examples

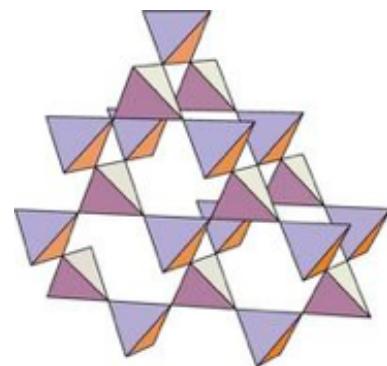
Triangle based lattices



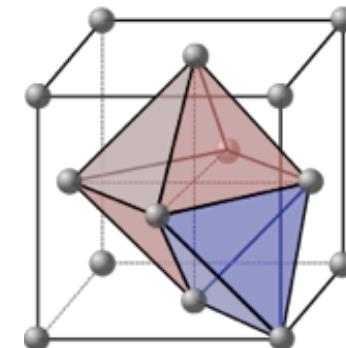
triangle



kagome



pyrochlore

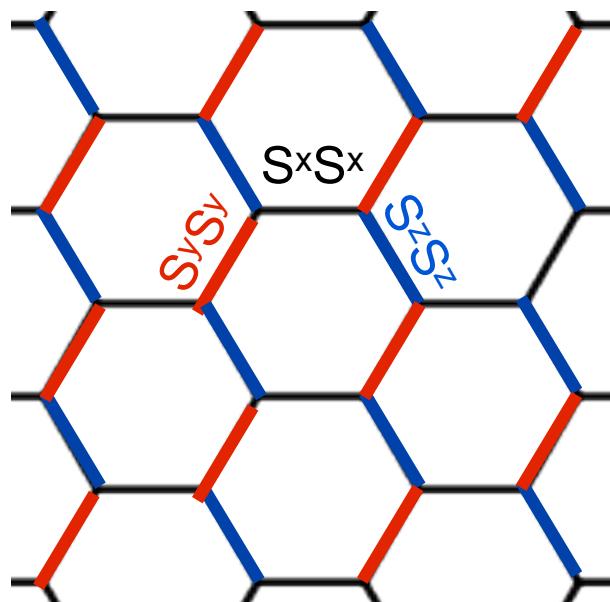


fcc

Other interactions

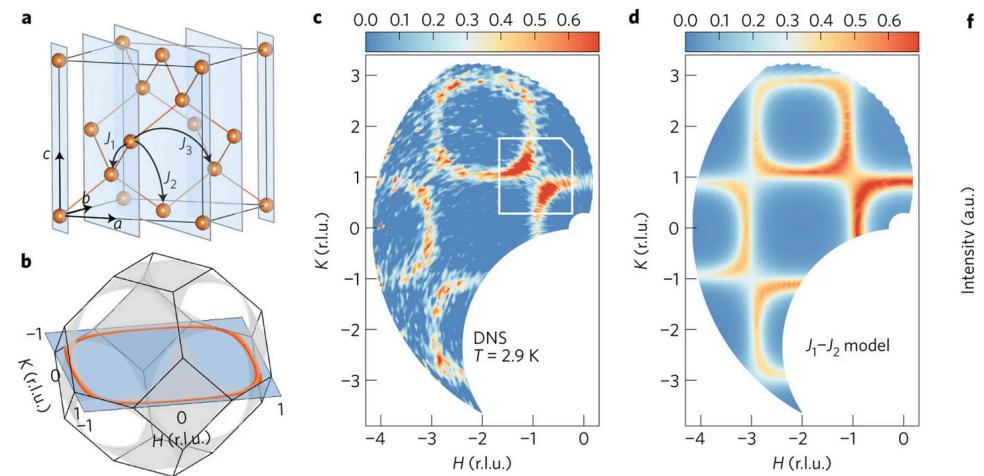
- With more structured interactions, even non-geometrically frustrated lattices can show frustration

“Kitaev” terms in Na_2IrO_3 , RuCl_3 ...



Jackeli+Khaliulin, 2009

Spiral spin liquid in MnSc_2S_4



S. Gao *et al*, 2017

Ising systems

- Ising models

$$H = \frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j \quad \sigma_i = \pm 1$$

- Physically occurs when single ion has a *doublet* ground state and only one component couples strongly

$$H_{\text{real}} = \frac{1}{2} \sum_{ij} J_{ij} S_i^z S_j^z + H' [S_i^x, S_i^y]$$

- Requires significant spin-orbit coupling: some rare earths, Co, ...

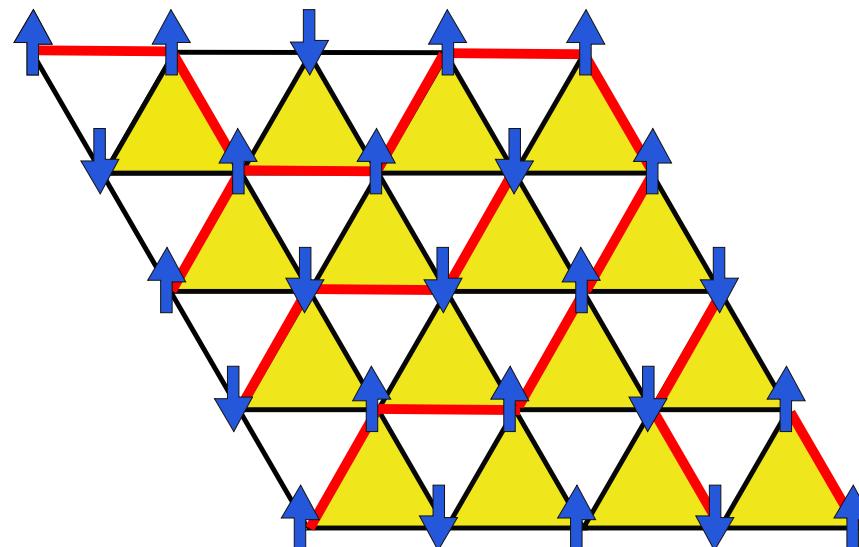
Wannier

- Triangular lattice Ising AF: macroscopic degeneracy (Wannier, 1950)

$$H = J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$
$$\sigma_i = \pm 1$$

$$\Omega = e^{S/k_B}$$

$$S \approx 0.34 N k_B$$



1 frustrated
bond per
triangle

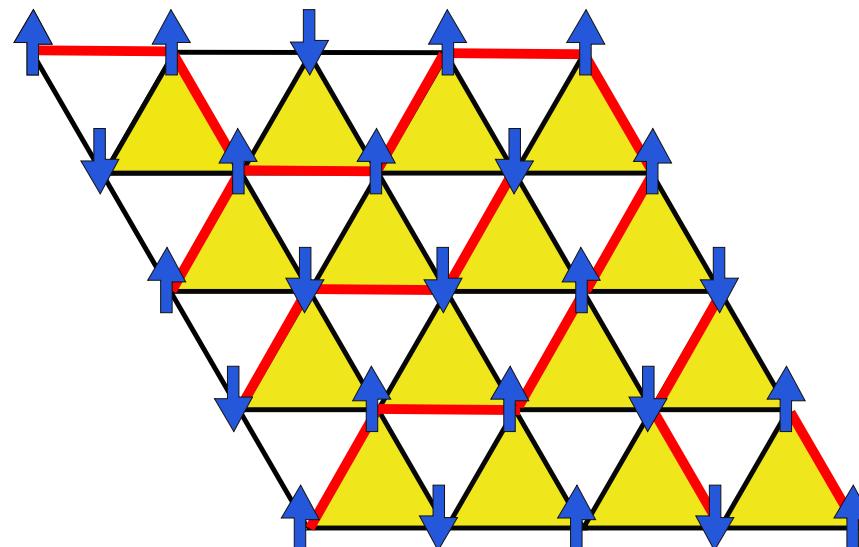
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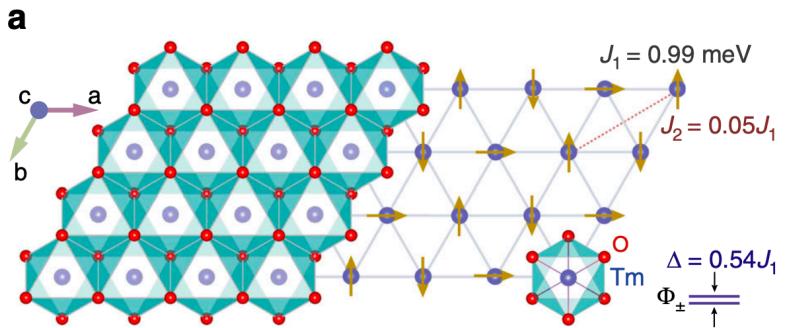
$$S \approx 0.34 N k_B$$



Somewhat close realization?

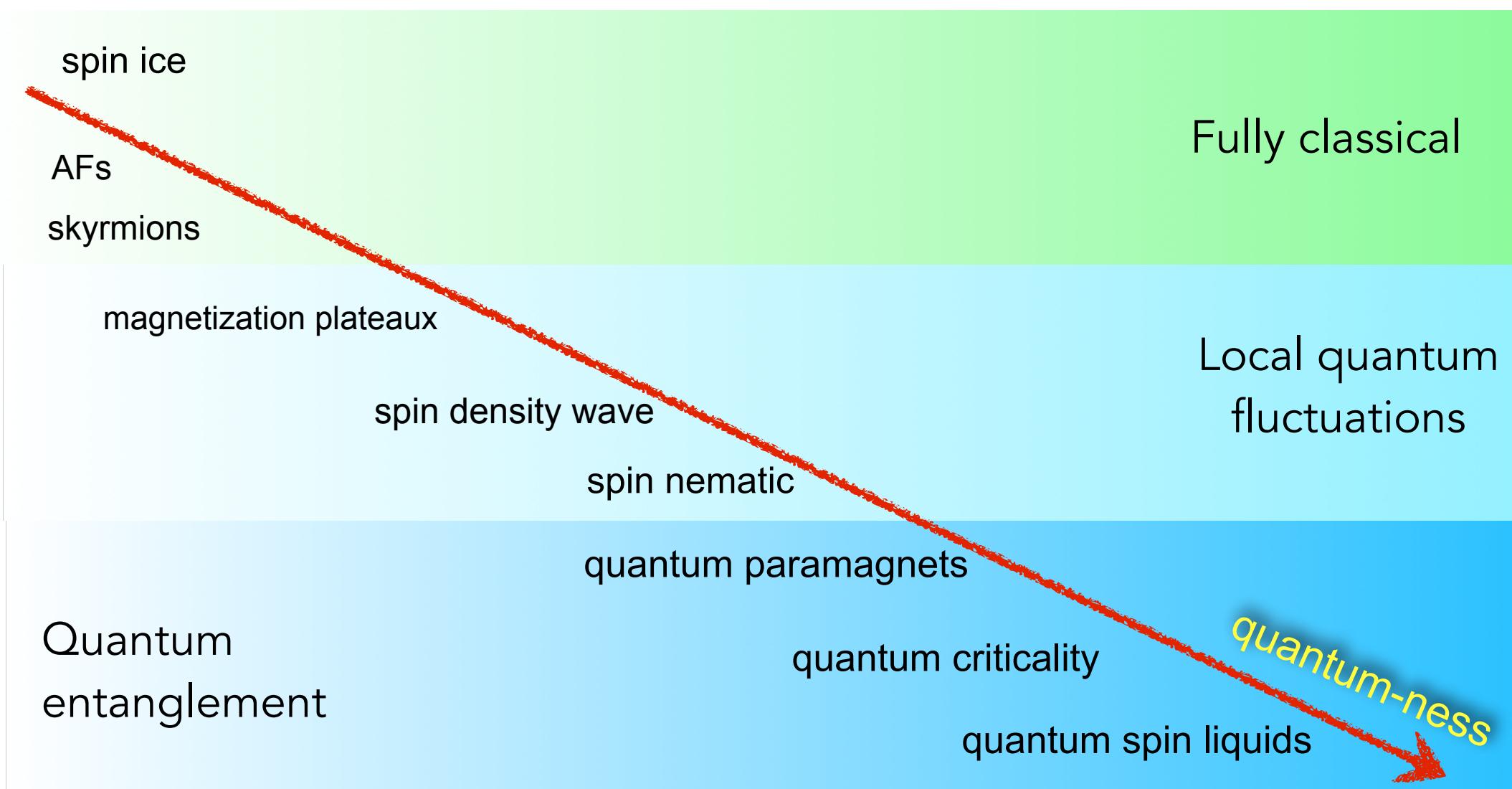
TmMgGeO_4

Degeneracy-breaking perturbations determine the low energy physics



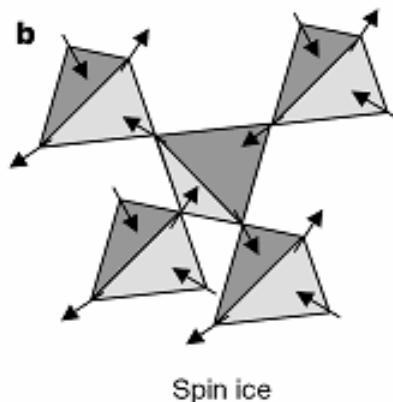
H. Li *et al*, 2020

Frustrated Magnetism



Spin ice

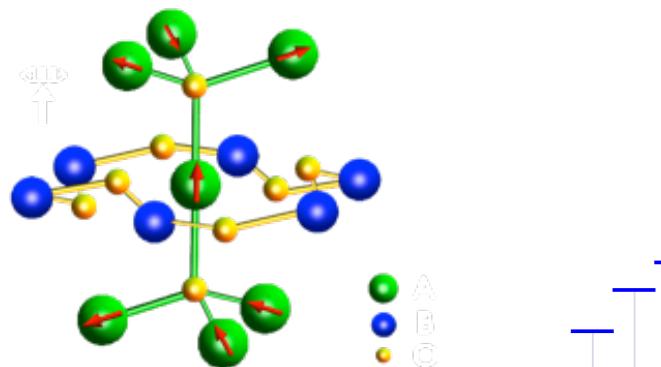
- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have Ising doublets with dominant NN coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$



$$H \approx J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z$$

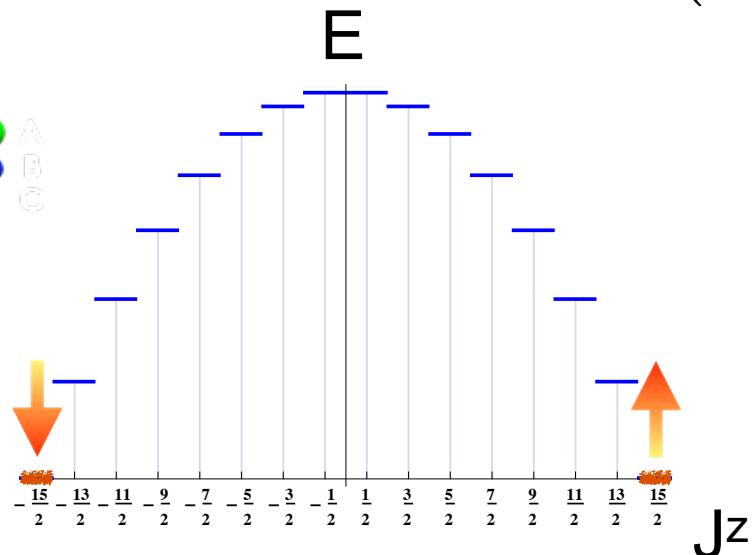
Spin ice

- rare earth pyrochlores $\text{Dy}_2\text{Ti}_2\text{O}_7$, $\text{Ho}_2\text{Ti}_2\text{O}_7$ with *Ising doublet* ground states



Local physics: $\mathbf{L} + \mathbf{S} = \mathbf{J}$

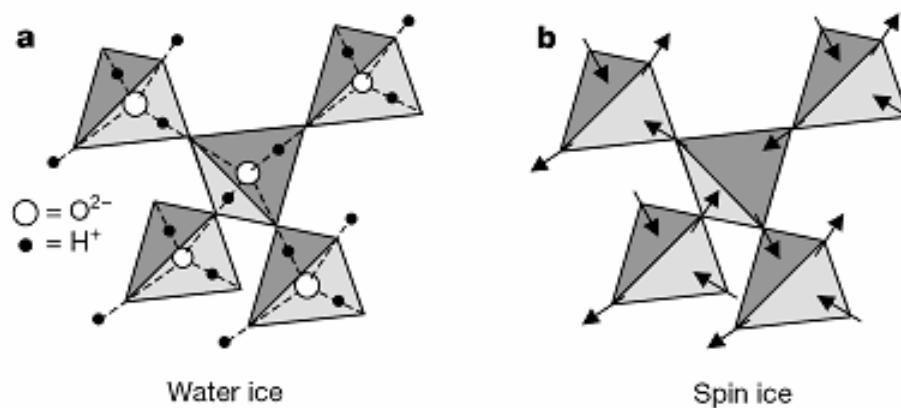
$$H_{\text{ion}} = -D \left(\vec{J}_i \cdot \hat{n}_i \right)^2$$



flips between $\pm J$
states difficult

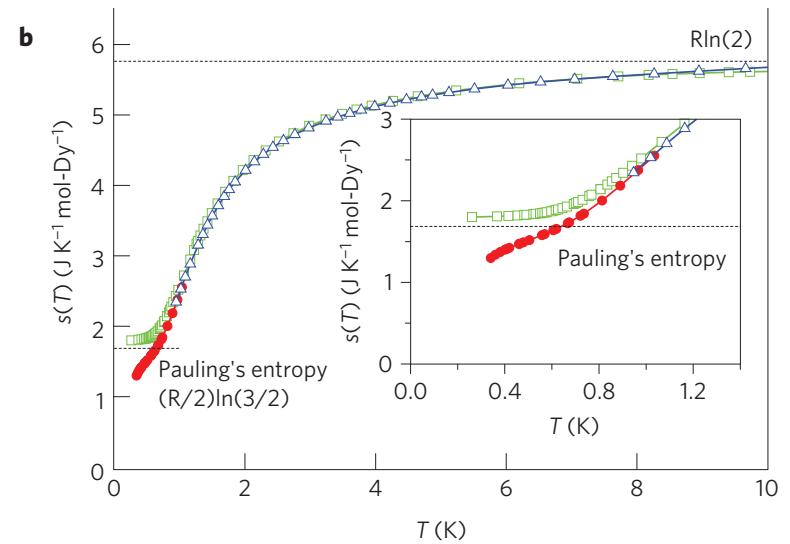
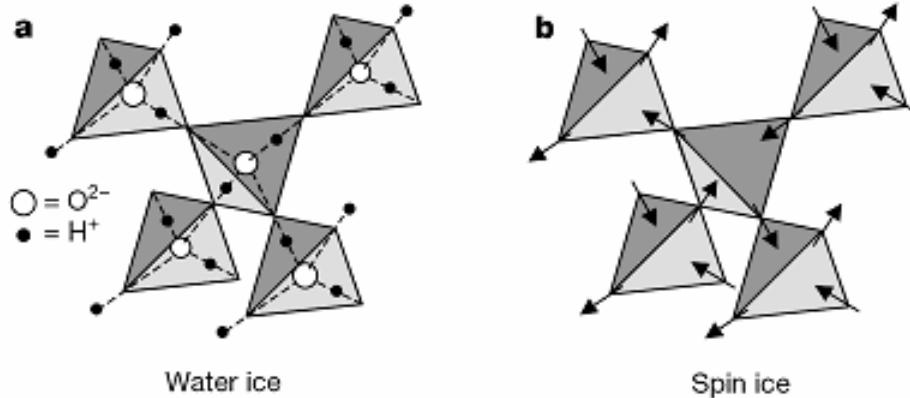
Spin ice

- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$



Spin ice

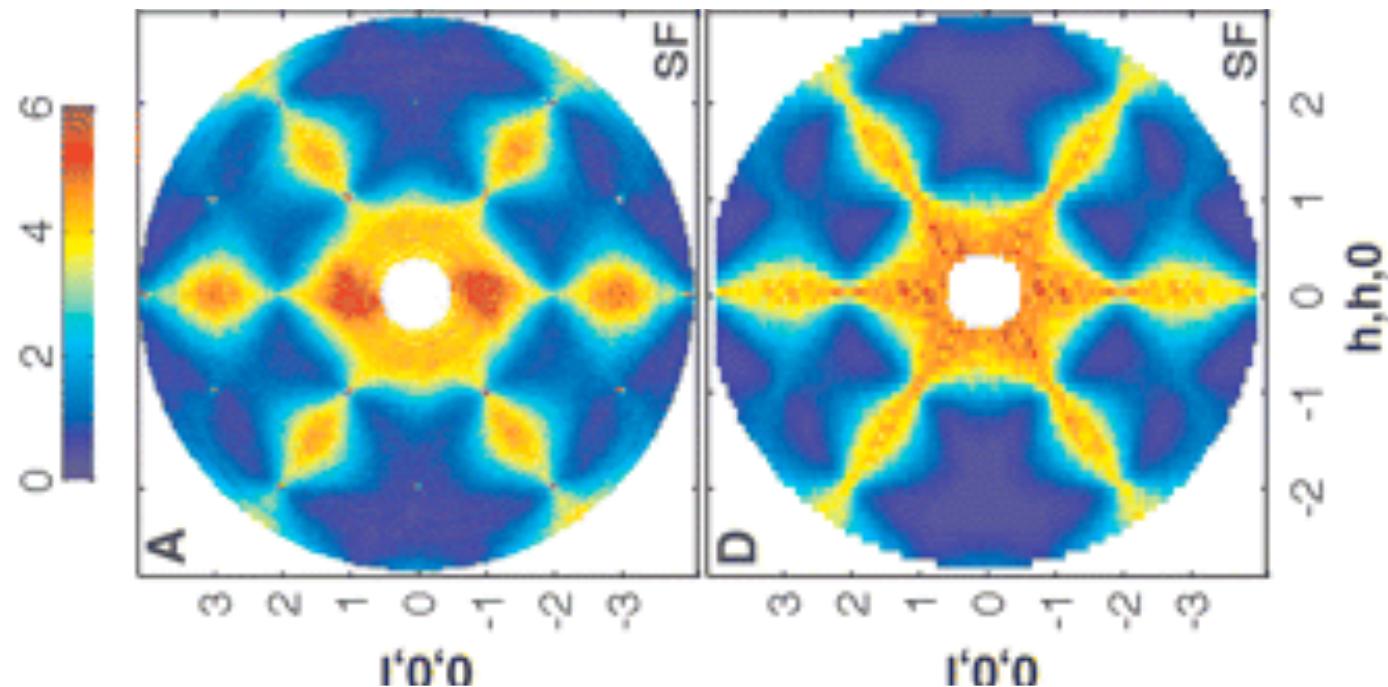
- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$



Pomaranski *et al*, $\text{Dy}_2\text{Ti}_2\text{O}_7$
(original expts. by Harris *et al*, 1999)

Ice correlations

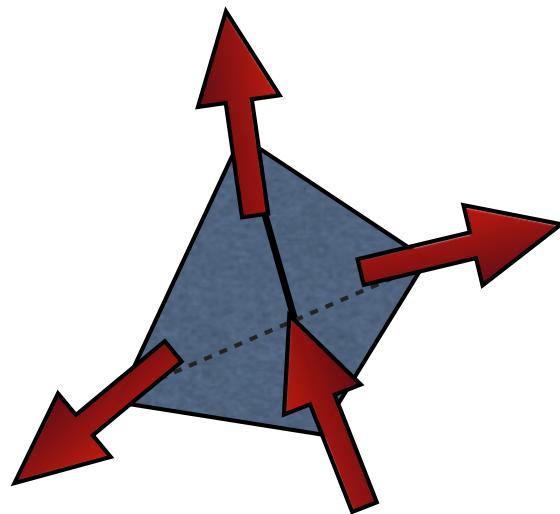
- “Pinch points” show that 2in-2out constraint holds



T. Fennell et al, 2009 experiment
 $\text{Ho}_2\text{Ti}_2\text{O}_7$

theory

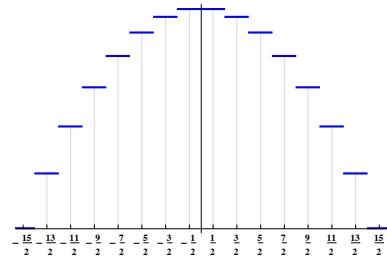
Monopoles



3in:1out defects act like monopoles,
and can move almost freely



Quantum spin ice



M. Hermele, MPA Fisher, L.B., 2004;
A. Banerjee et al, 2008
L. Savary + LB, 2012
S.B. Lee, S. Onoda + LB, 2012
+ Many subsequent numerical
and analytical works

Still looking...

- Invited - E. Smith (McMaster University, Canada)

"The case for a $U(1)_p$ Quantum Spin Liquid Ground State in the Dipole-Octupole Pyrochlore $Ce_2Zr_2O_7$ "



- Y.B. Kim (University of Toronto, Canada)

"Competing dipolar-octupolar quantum spin liquids on the pyrochlore lattice"



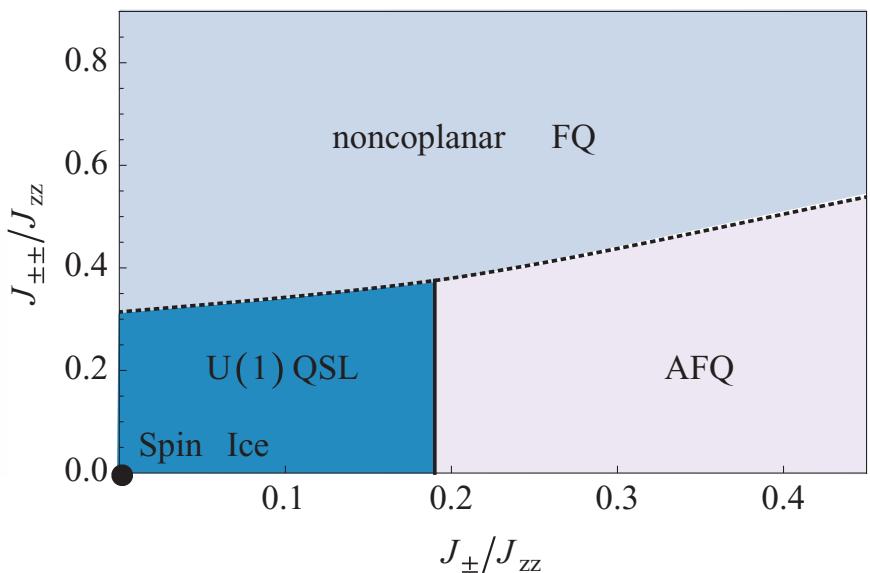
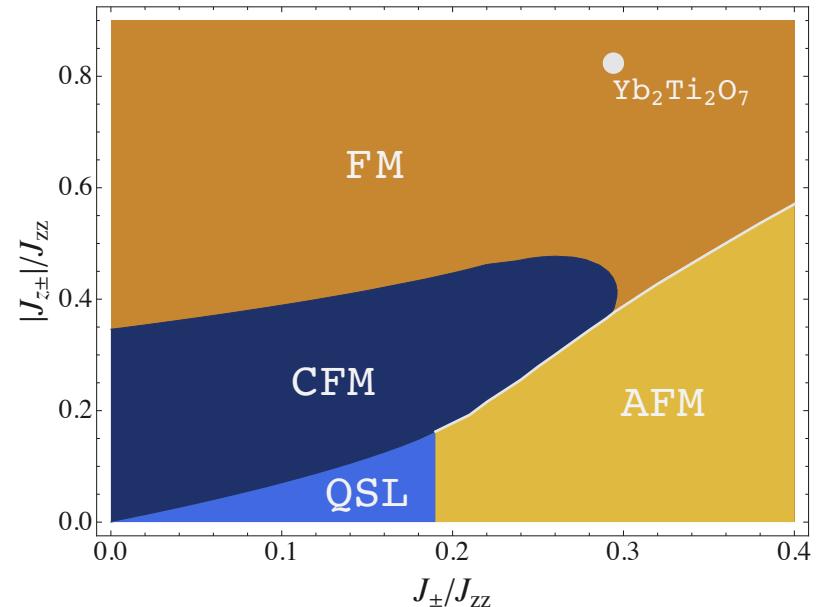
- R. Sibille (Paul Sherrer Institut, Switzerland)

"Octupolar correlations and spinon spectrum in $Ce_2Sn_2O_7$ quantum spin ice"



- Invited - R. Moessner (Max Planck Institute for the Physics of Complex Systems, Germany)

"Emergent QED in the quantum spin ices"



Heisenberg systems

- Typically for *closed shells* - i.e. configurations w/o orbital degeneracy - of 3d transition metal ions, SOC effects are weak: good approximate spin-rotation symmetry

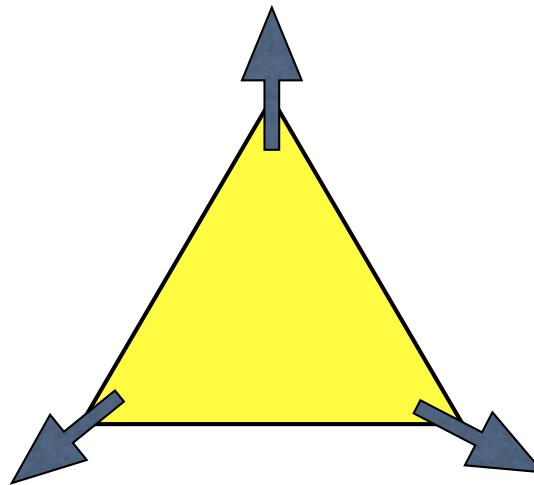
$$H = \frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

single-ion and exchange anisotropy, $\lesssim 10\%$ level

Spin “length” $S = 1/2$ is most quantum, $S \gg 1$ is semiclassical

Triangle

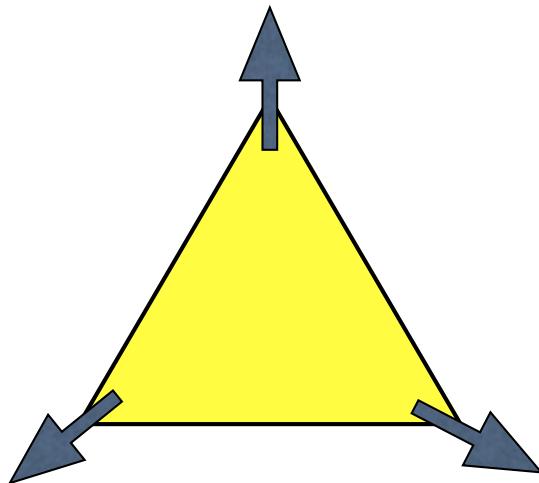
- Classically: spins must sum to zero



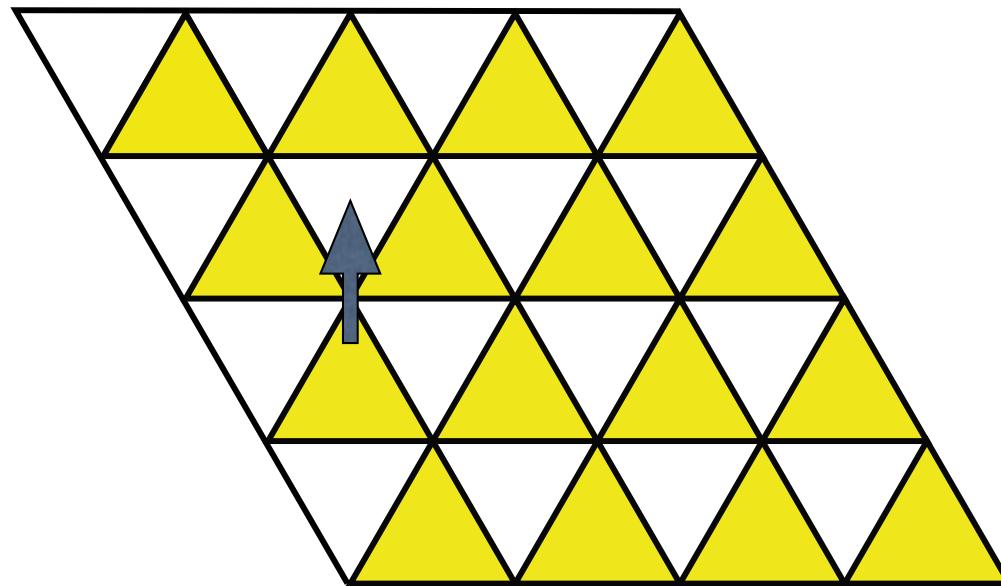
Tendency to non-collinear ordering

Triangular lattice

- Classically: spins must sum to zero

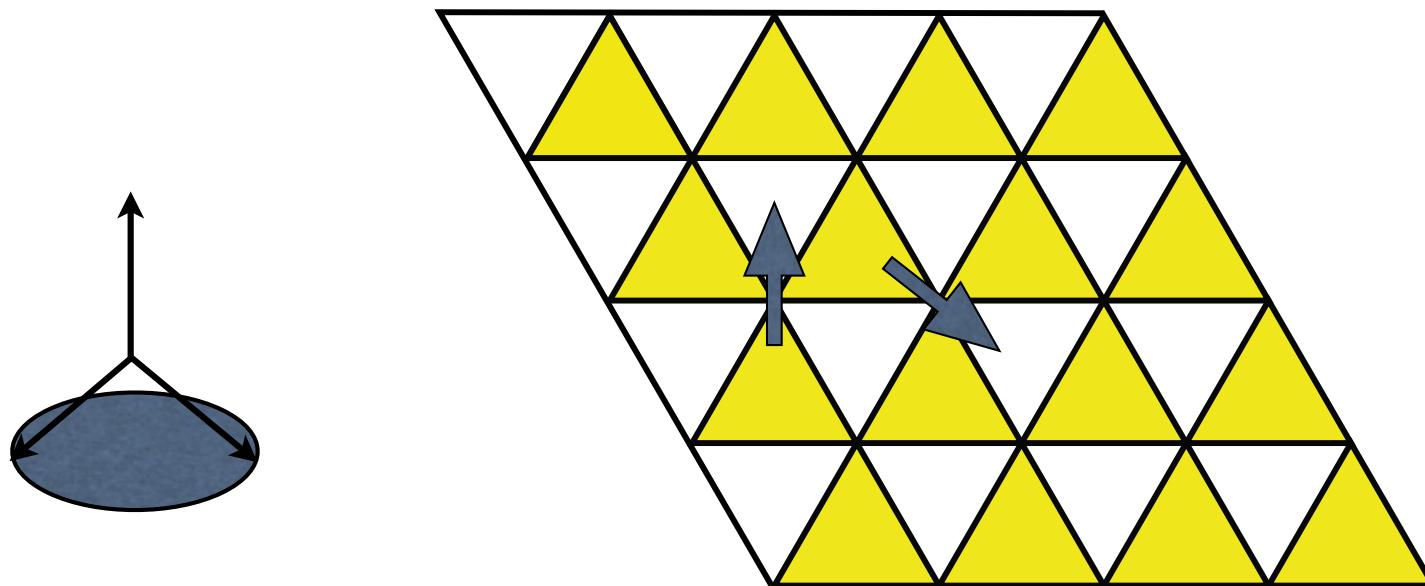


Triangular lattice



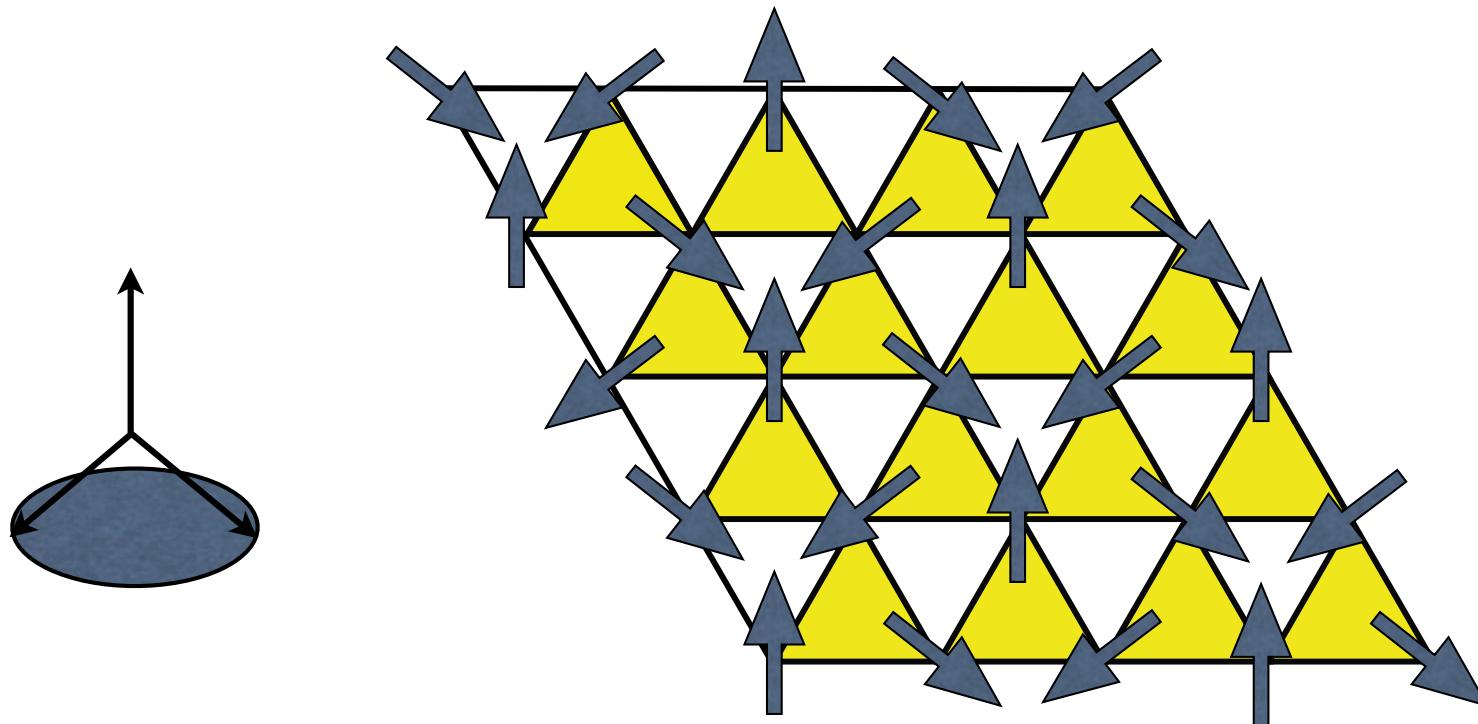
Degrees of freedom: 2

Triangular lattice



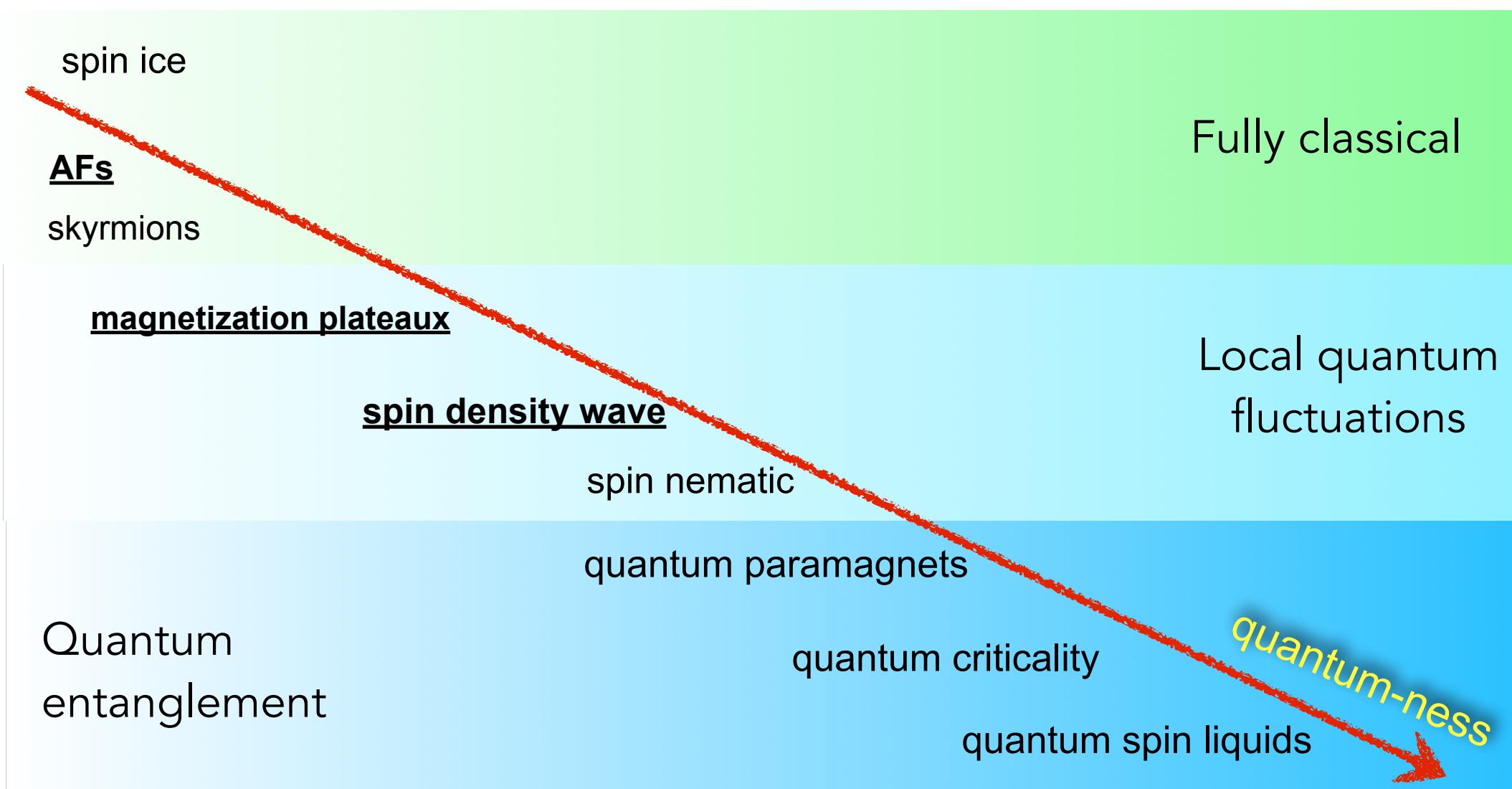
Degrees of freedom: 2+1

Triangular lattice

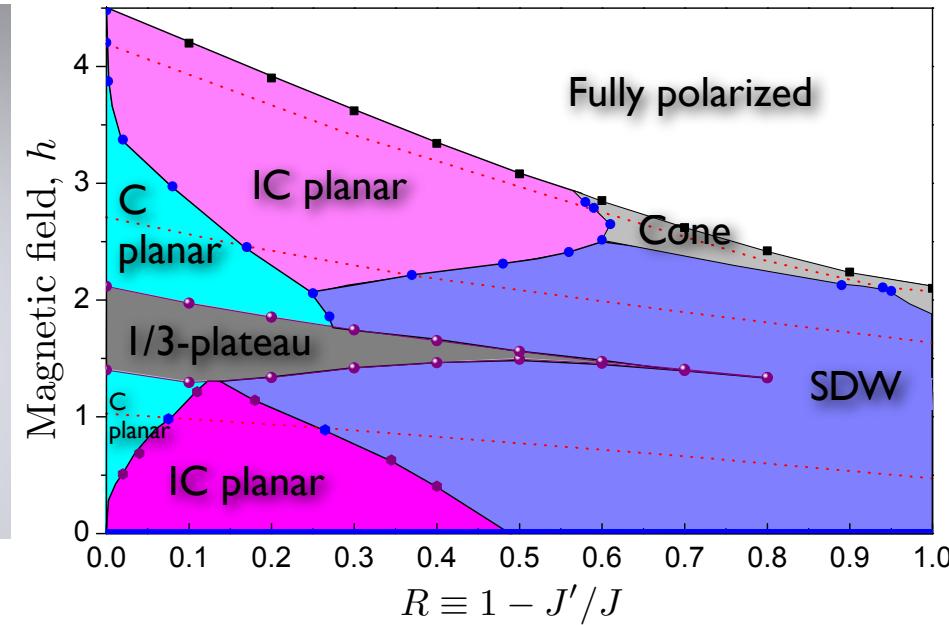
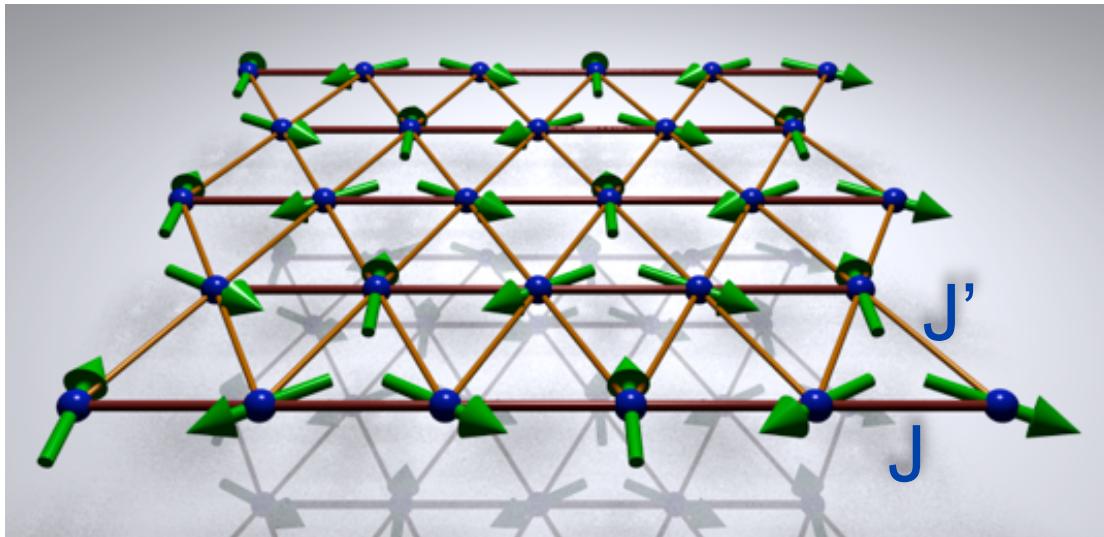


Degrees of freedom: 2+1

Frustrated Magnetism

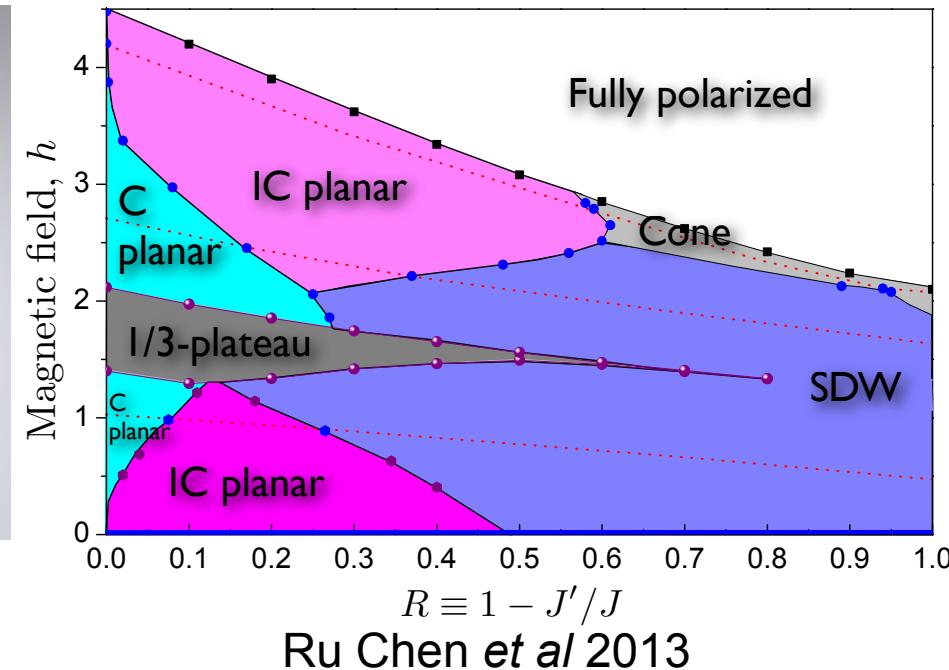
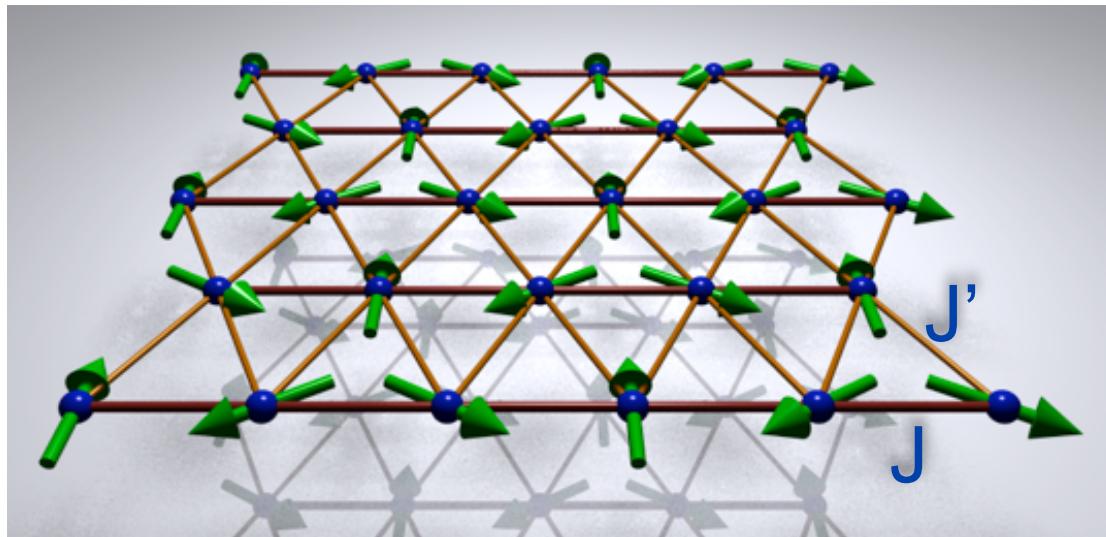


$S=1/2$ Triangular lattice



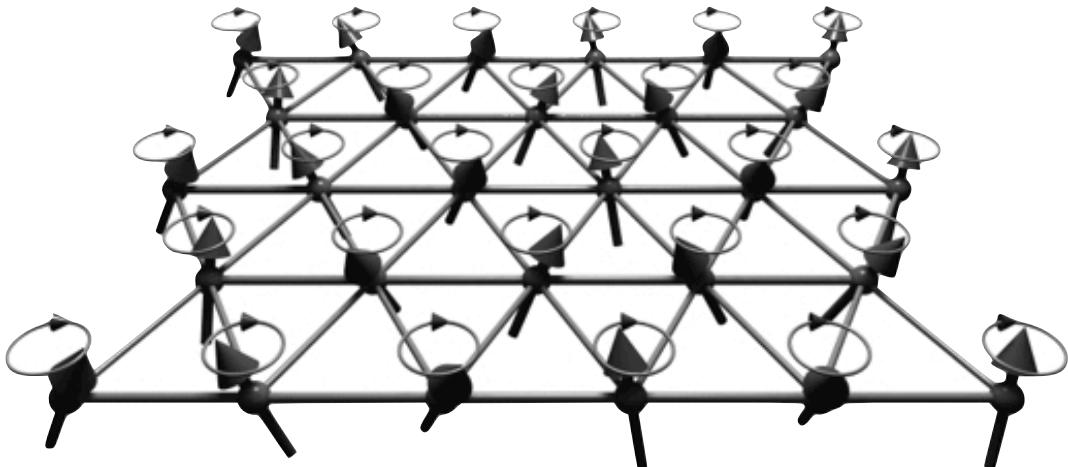
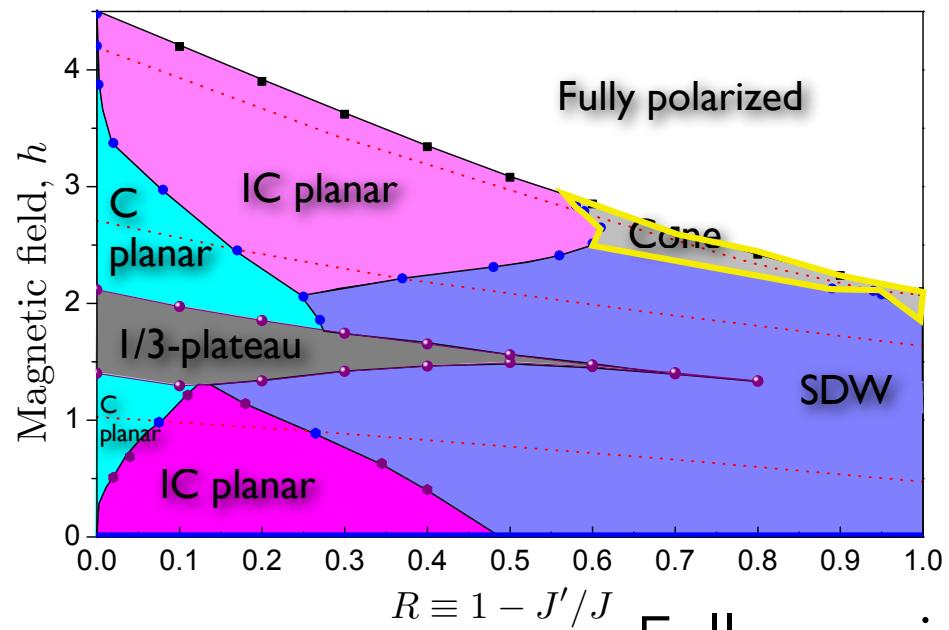
- Nice example with strong quantum renormalizations
 - All phases encountered are ordered, short-range entangled states
 - BUT most are different from those of the classical model
 - And excitations are highly renormalized from linear spin waves

$S=1/2$ Triangular lattice



Review: O. Starykh, RPP, 2015

Cone state

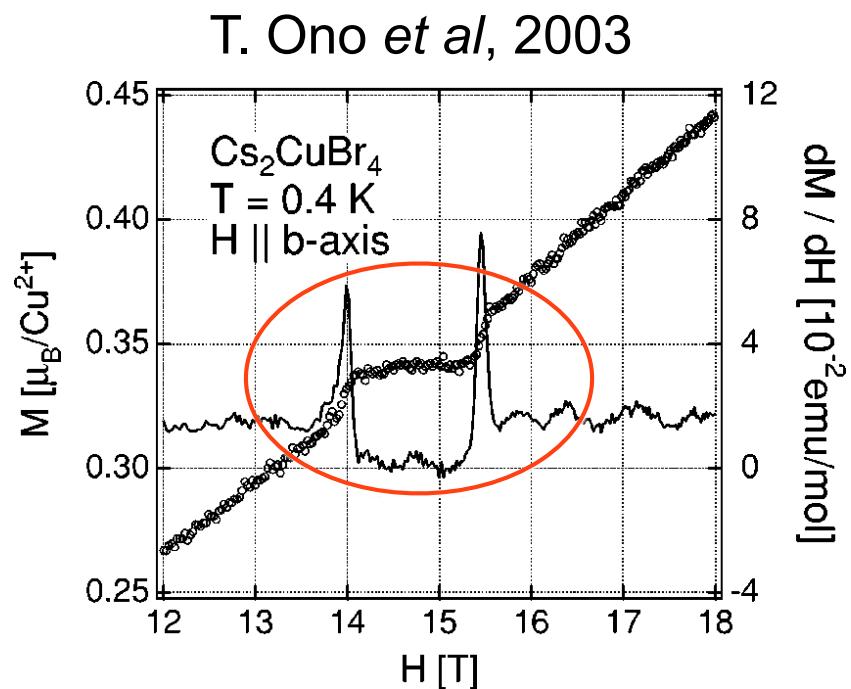
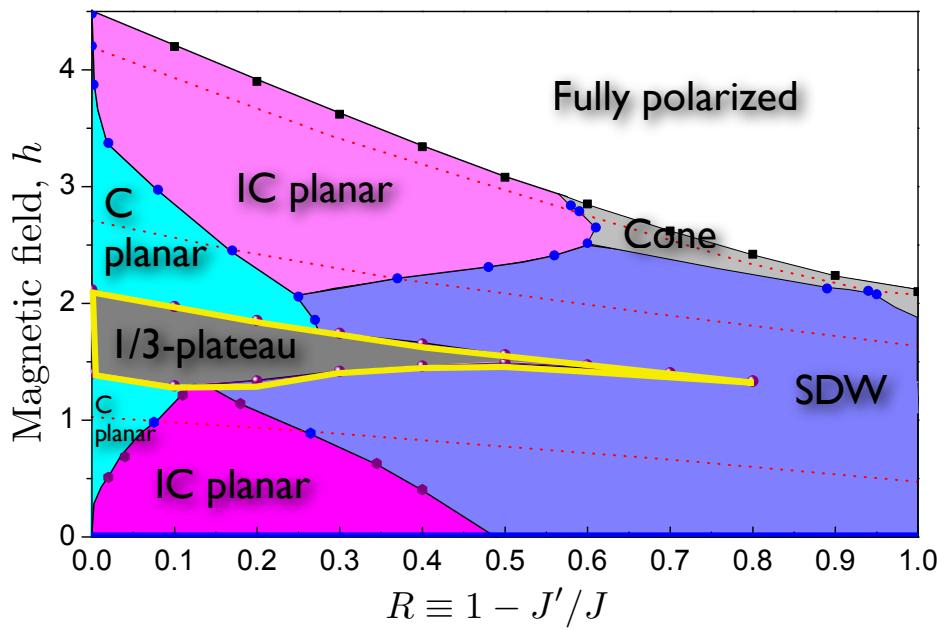


Fully consistent with rigid spins

This is the classical ground state *throughout* the phase space

Excitations are gapless spin waves - semiclassical quantization of small oscillations of spins

Magnetization plateau

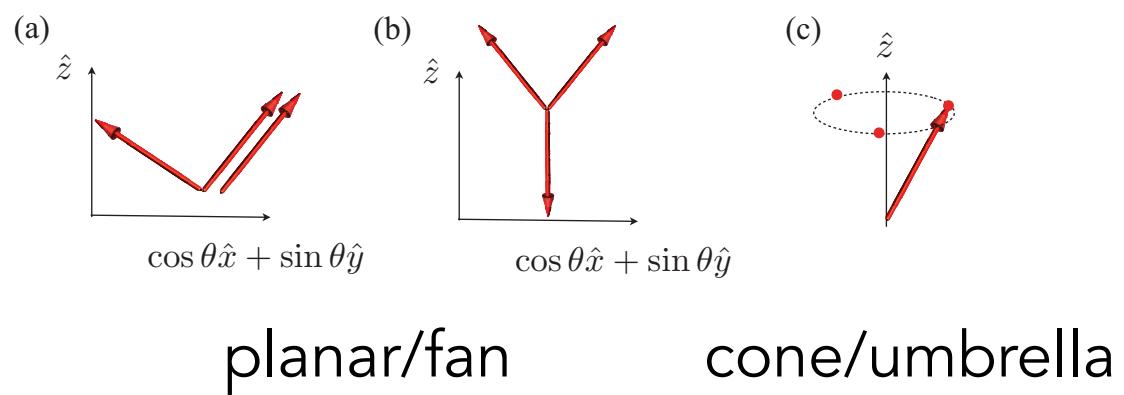
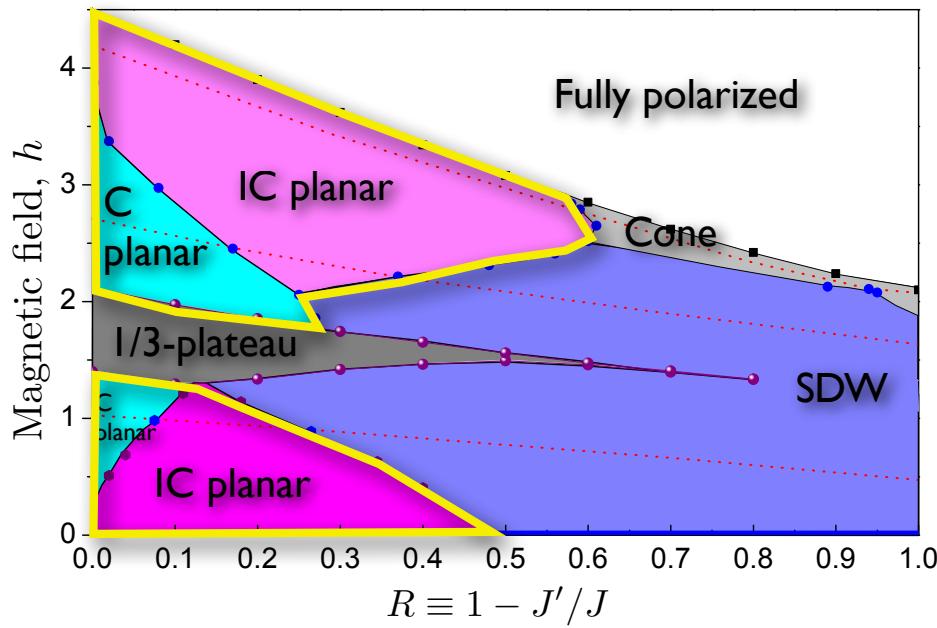


Spin gap stabilized by quantum zero point fluctuations

excitations are still spin waves but not Goldstone modes

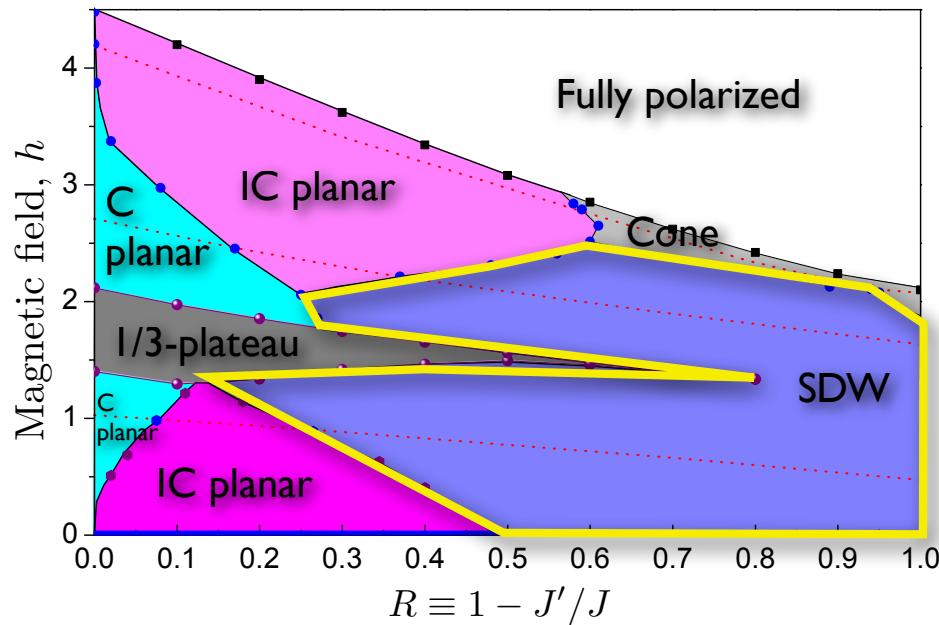
c.f. Chubukov + Golosov, 91

Planar orders



Classical ground state is *always* umbrella-like, but quantum fluctuations almost completely remove this

SDW

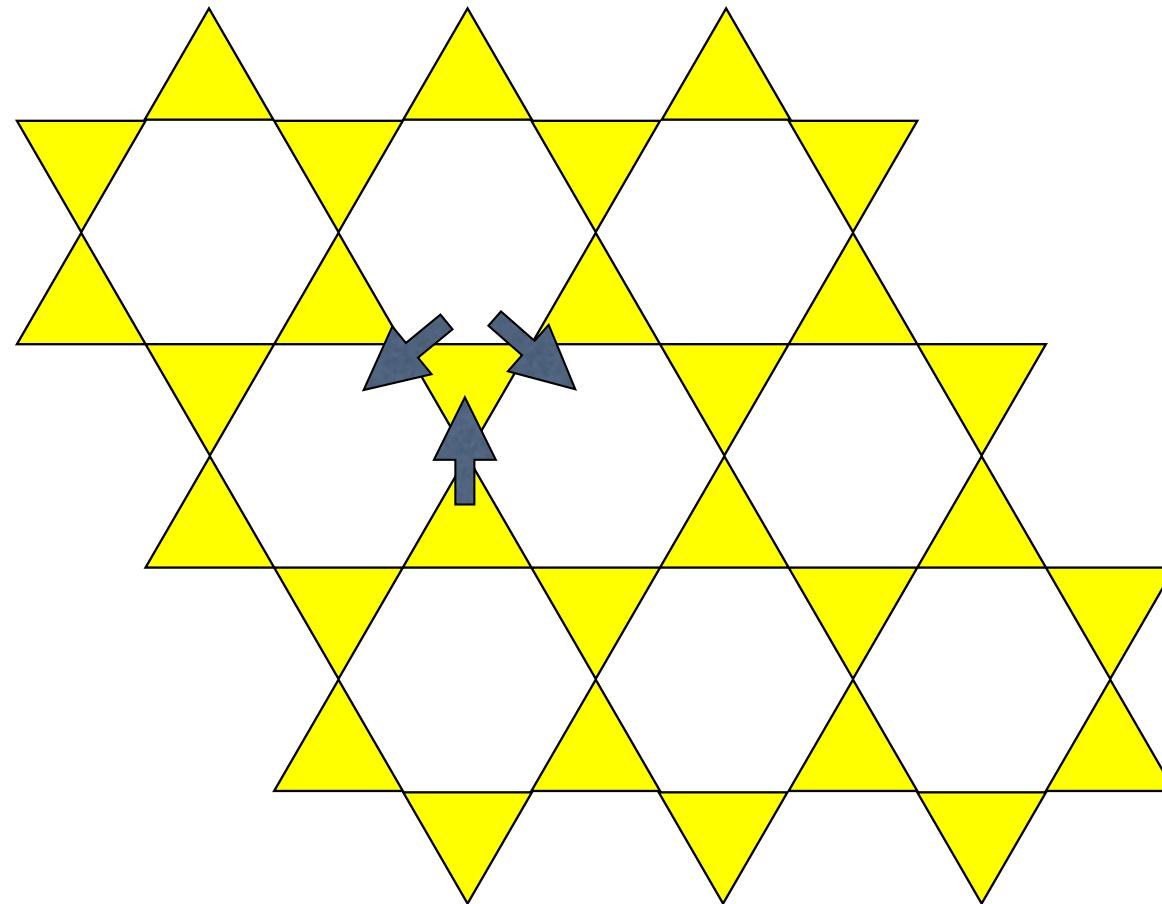


Non-uniform spin lengths
→ non-classical!

SDW states can be considered soliton lattices, and can be understood based on the behavior of spin chains

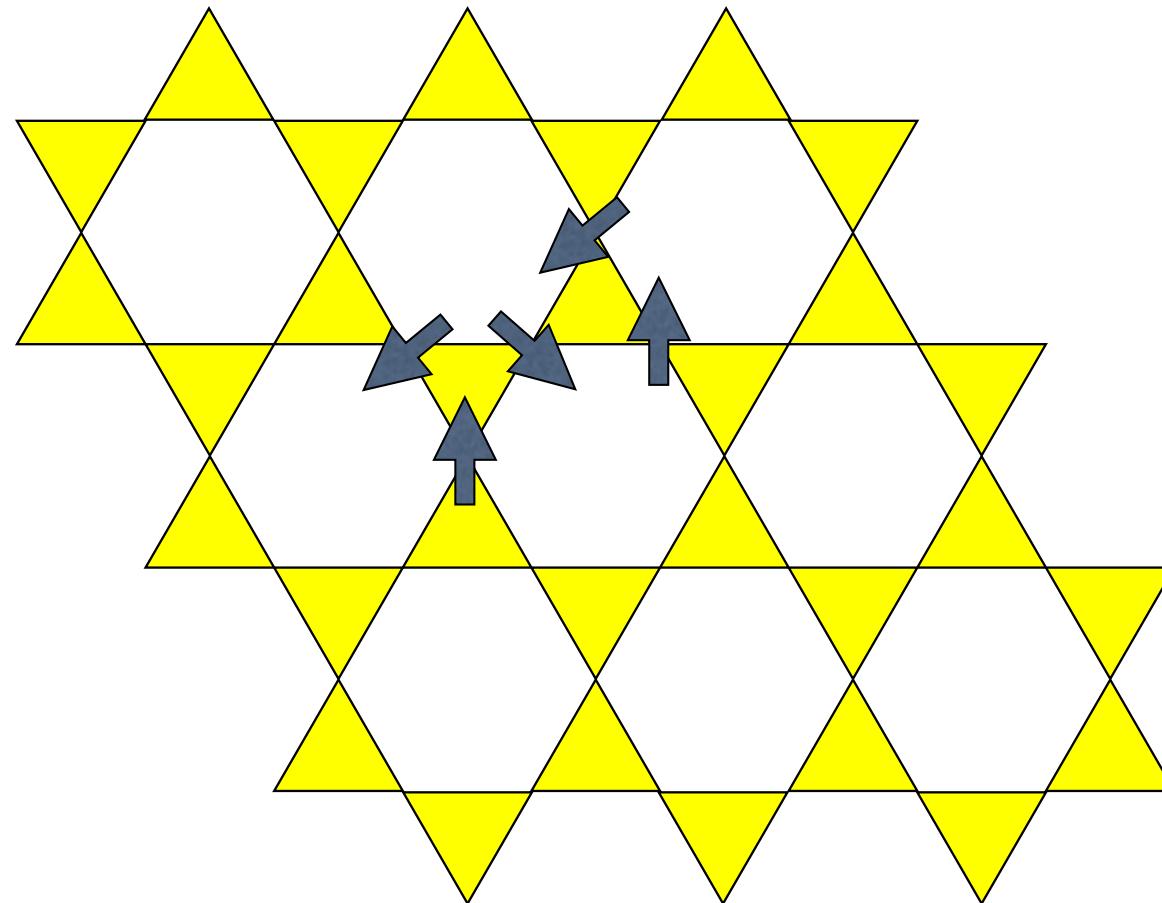
Large domain of SDW state means that quasi-1d nature
is enhanced by quantum fluctuations

Kagome lattice



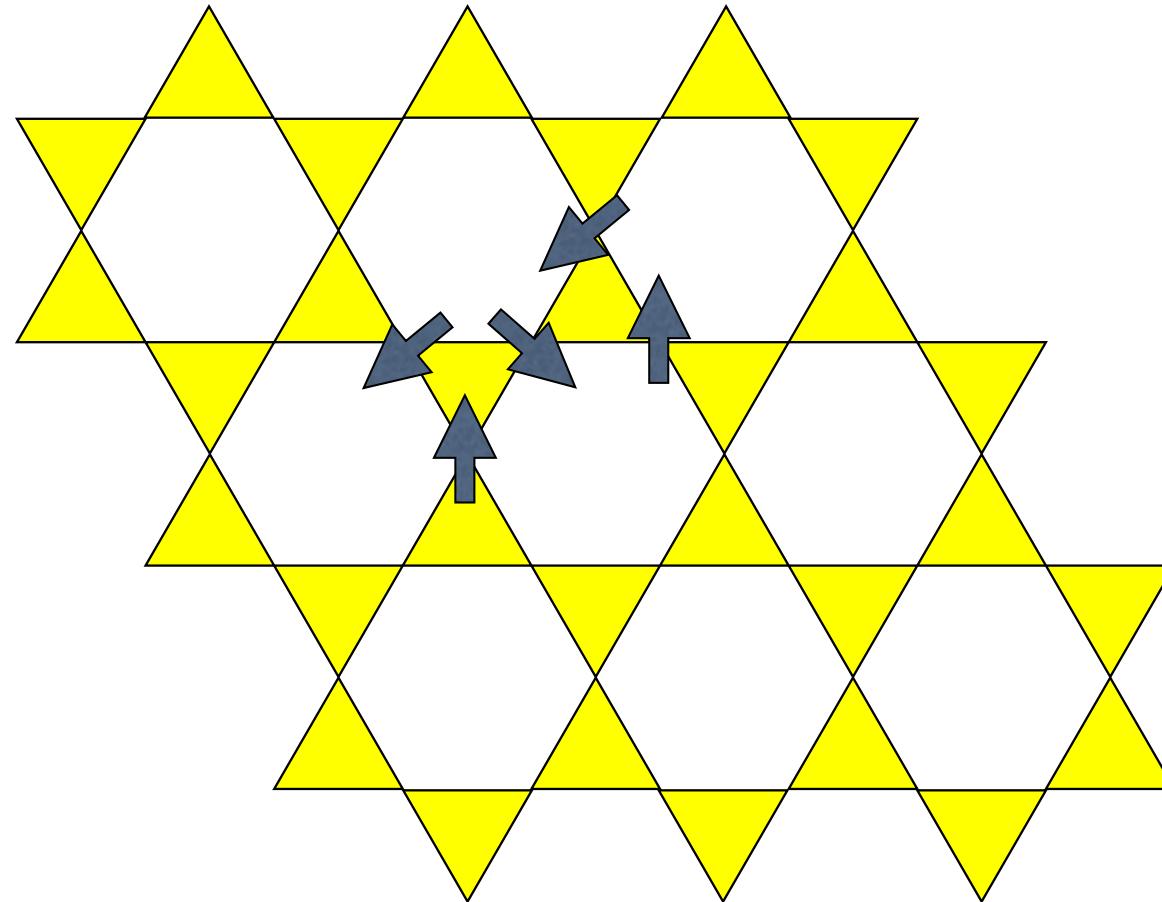
Degrees of freedom: 3

Kagome lattice



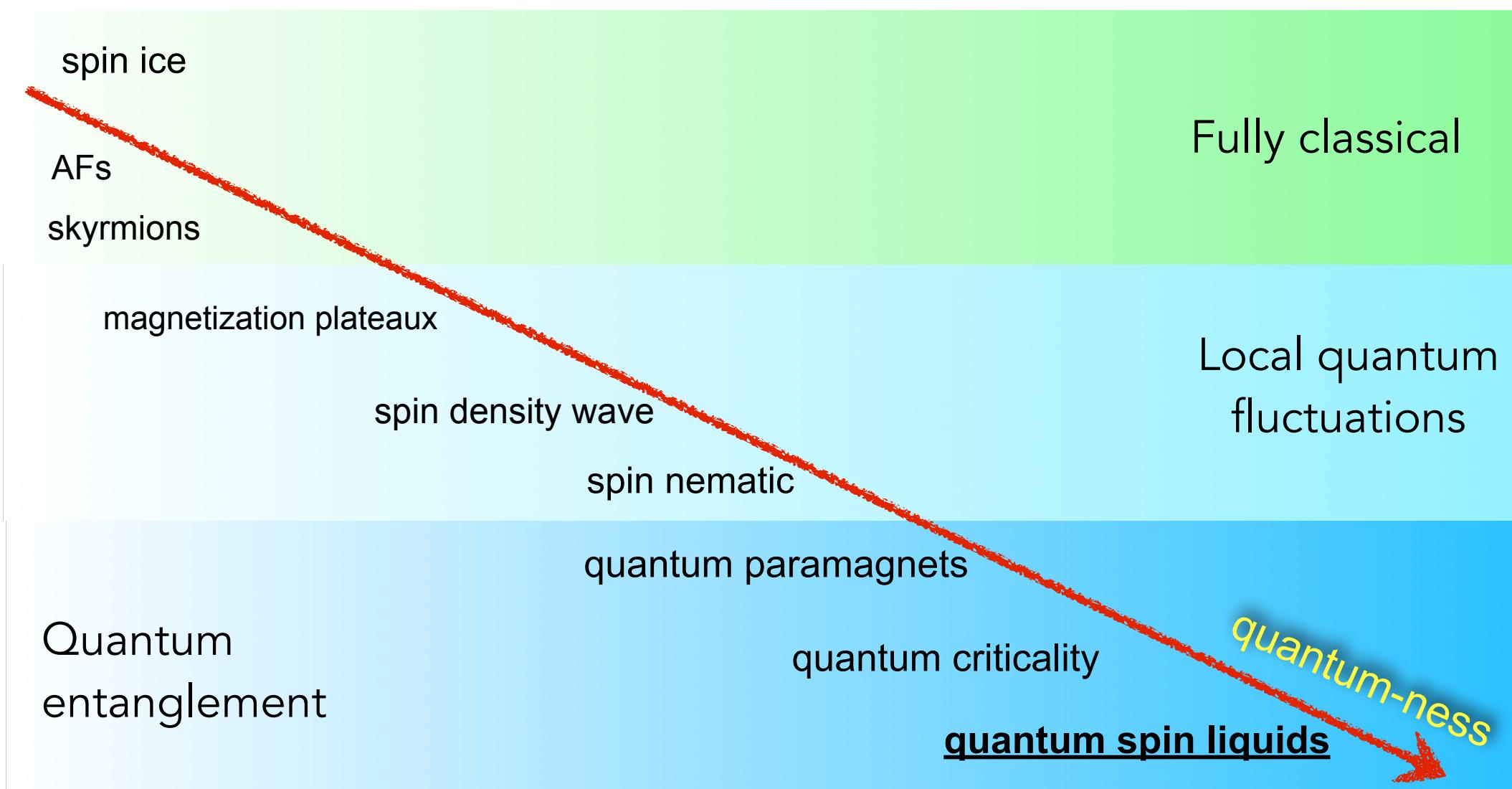
Degrees of freedom: 3+1+...

Kagome lattice



- Degrees of freedom: $\sim N$. Much less likely to order.

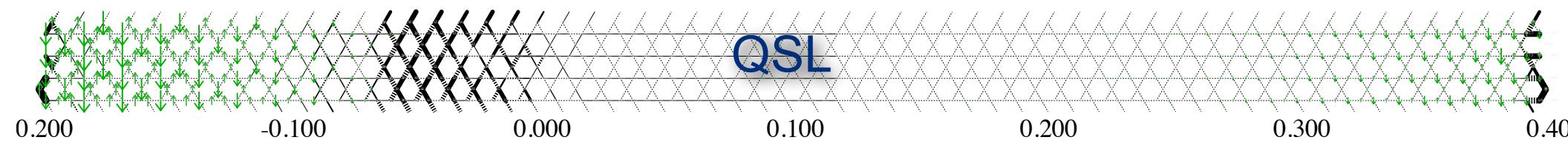
Frustrated Magnetism



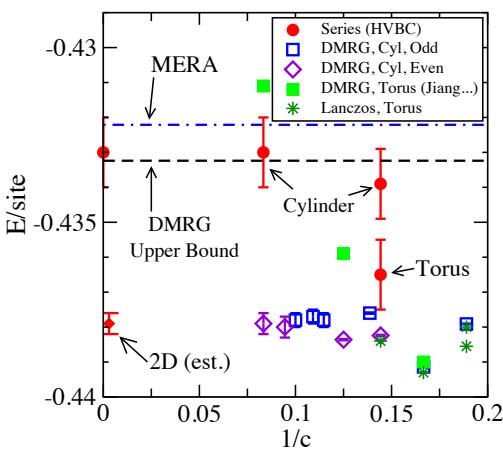
$S=1/2$ kagomé AF

- DMRG calculations give overwhelming evidence for QSL ground state

© Steve White



S. Yan *et al*, 2010



Theorists are still debating the nature of the QSL state.
Experimentalists are also debating the meaning of their observations.

Many kinds of QSLs

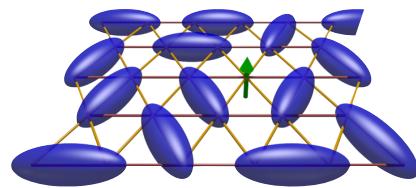
$$\Psi = \begin{array}{c} \text{Diagram of a triangular lattice with blue ovals representing spins, with a red arrow pointing to it labeled '#'} \\ + \end{array} \begin{array}{c} \text{Diagram of a triangular lattice with blue ovals representing spins, with a red arrow pointing to it labeled '#'} \\ + \dots \end{array}$$

For ~ 500 spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.

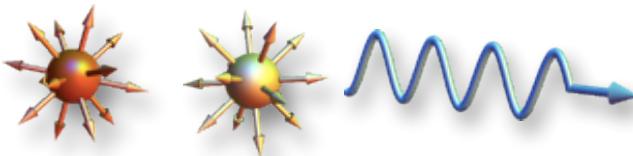
Classes of QSLs

- Topological QSLs



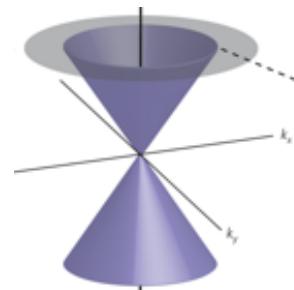
anyonic
spinons

- $U(1)$ QSL



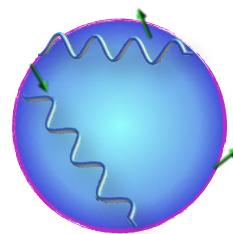
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

QSLs @ HFM2022

- **Invited** - L. Clark (University of Birmingham, UK)

"Unravelling Complexity in the Barlowite Family of $S=1/2$ Kagome Antiferromagnets" [↓](#)

- M. Georgopoulou (Institut Laue Langevin, France & University College London, UK)

"Zn-claringbullite, $ZnCu_3(OD)_6FCI$: a new quantum spin liquid candidate" [↓](#)

- Q. Bartélémy (Université Paris-Saclay, France)

"Specific heat of the kagome antiferromagnet herbertsmithite in high magnetic fields" [↓](#)

- G. Chen (University of Hong-Kong, China)

"Thermal Hall effects in spin liquids" [↓](#)

- **Invited** - P. Armitage (John Hopkins University, USA)

"Recent results on Kitaev interactions in Co based magnets" [↓](#)

- N.B. Perkins (University of Minnesota, USA)

"Footprints of the Kitaev spin liquid in the Fano lineshape of the Raman active optical phonons" [↓](#)

- Y-J. Kao (National Taiwan University, Taiwan)

"Excitation spectrum of spin-1 Kitaev spin liquids" [↓](#)

- M. Udagawa (Gakushuin University, Japan)

"Manipulation of non-Abelian anyons in Kitaev's magnet" [↓](#)

- **Invited** - J. Nasu (Tohoku University, Japan)

"Nonequilibrium dynamics and spin transport caused by fractional quasiparticles in Kitaev spin liquids" [↓](#)

- S.C. Furuya (Ibaraki University, Japan & University of Tokyo, Japan)

"DC electric-field controls of Kitaev spin liquids and topological spin textures" [↓](#)

- E. Lefrançois (Université de Sherbrooke, Canada)

"Evidence of a Phonon Hall Effect in the Kitaev Spin Liquid Candidate $a\text{-RuCl}_3$ " [↓](#)

- J. Bruin (Max Planck Institute for Solid State Research, Germany)

"Robustness of the thermal Hall effect close to half-quantization in $a\text{-RuCl}_3$ "

- R.P. Nutakki (University of Munich, Germany & University of Augsburg, Germany)

"Proximate Spin Liquids in Metal-Azolate Frameworks" [↓](#)

- S. Zhang (Max Planck Institute for the Physics of Complex Systems, Germany)

"Modeling a three-dimensional $S = 1$ spin liquid $NaCaNi_2F_7$ " [↓](#)

4h-15h30 : Pyrochlore session

- L. Vanderstraeten (University of Ghent, Belgium)

"Tensor networks and the spectral function of 2-D quantum spin liquids" [↓](#)

- **Invited** - E. Smith (McMaster University, Canada)

"The case for a $U(1)_p$ Quantum Spin Liquid Ground State in the Dipole-Octupole Pyrochlore $Ce_2Zr_2O_7$ " [↓](#)

- Y.B. Kim (University of Toronto, Canada)

"Competing dipolar-octupolar quantum spin liquids on the pyrochlore lattice" [↓](#)

- R. Sibille (Paul Scherrer Institut, Switzerland)

"Octupolar correlations and spinon spectrum in $Ce_2Sn_2O_7$ quantum spin ice" [↓](#)

- **Invited** - F. Pratt (ISIS Neutron and Muon Source, UK)

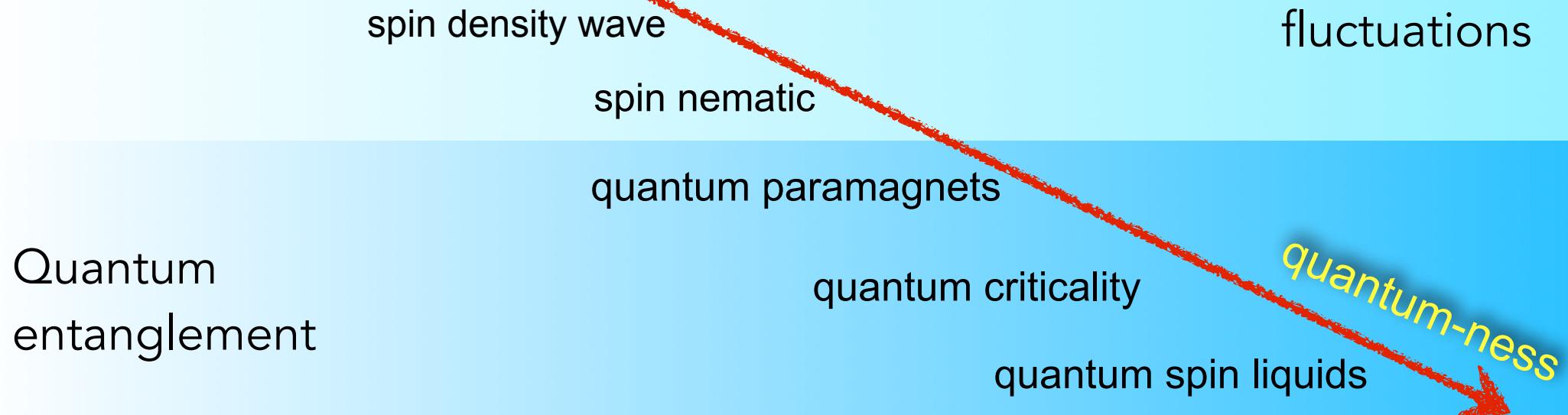
"Probing triangular-lattice quantum spin liquids with LF- μ SR" [↓](#)

Frustrated Magnetism

spin ice

This is all about the equilibrium phase/ground state.

We can also talk about excitations and response



Excitations in the usual case

Hamiltonian

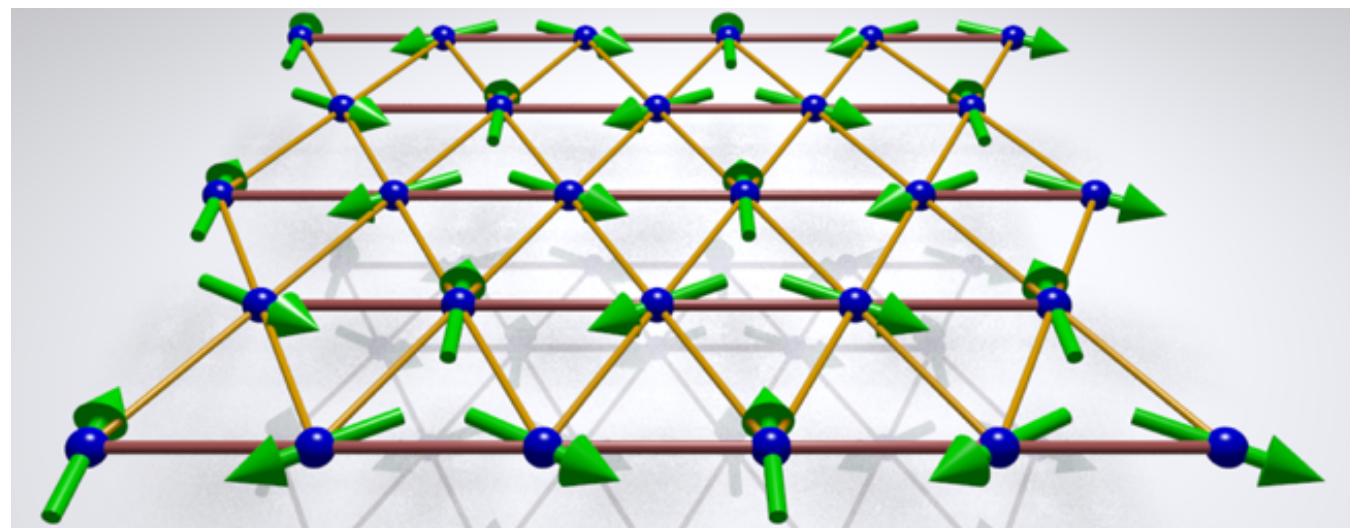
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-range: local

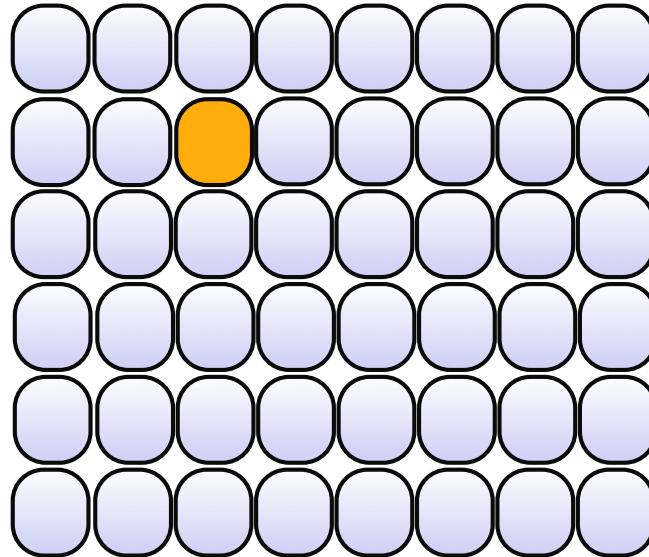
Wave function

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{\mathbf{n}}_i = +S\rangle$$

Product state



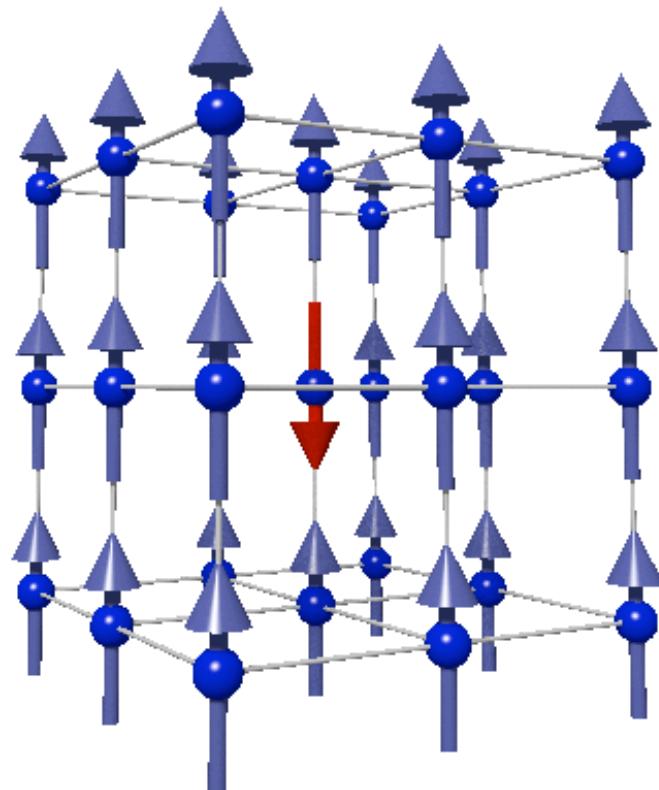
Quasiparticles



excited states \sim excited
levels of one block

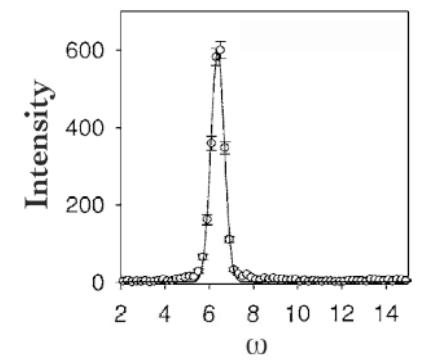
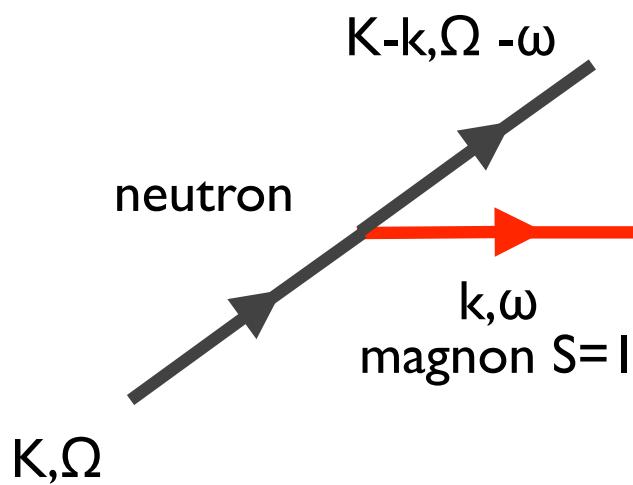
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

$$|f\rangle = S_k^+ |i\rangle$$



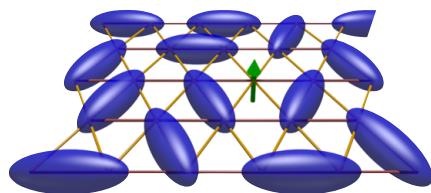
Line shape in Rb_2MnF_4

Emergent excitations

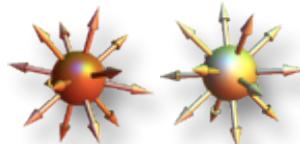
- Emergent excitations may be very different from spin flips
- May be created in multiples, or very hard to create at all with a neutron, or just have different properties



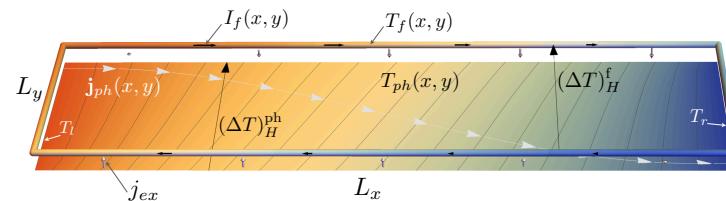
Skyrmion



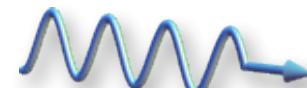
Spinon



Monopoles



Majorana

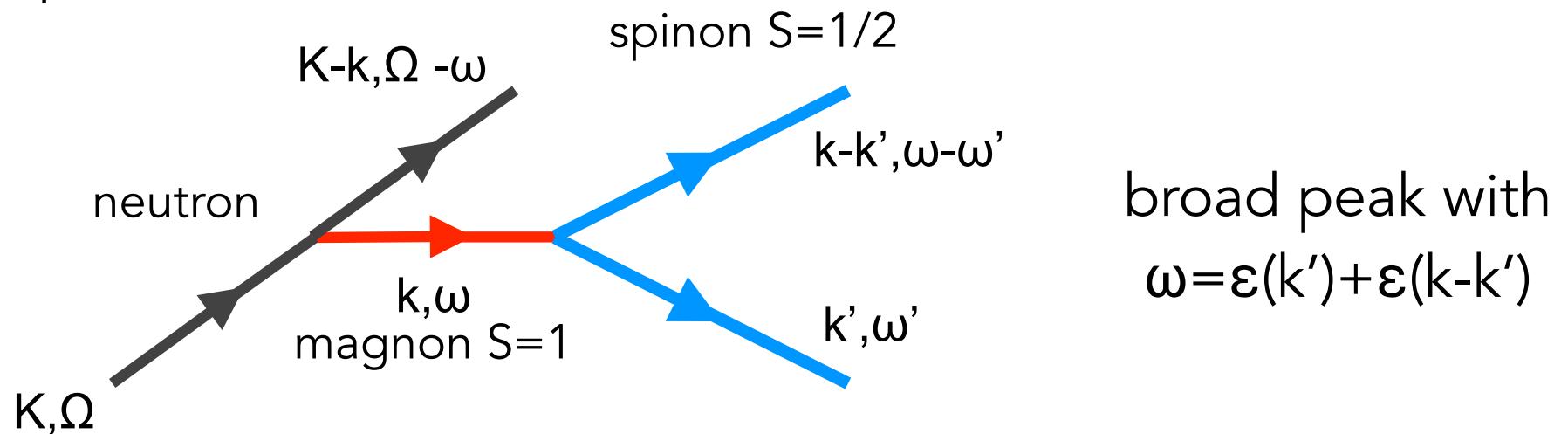


Emergent photon

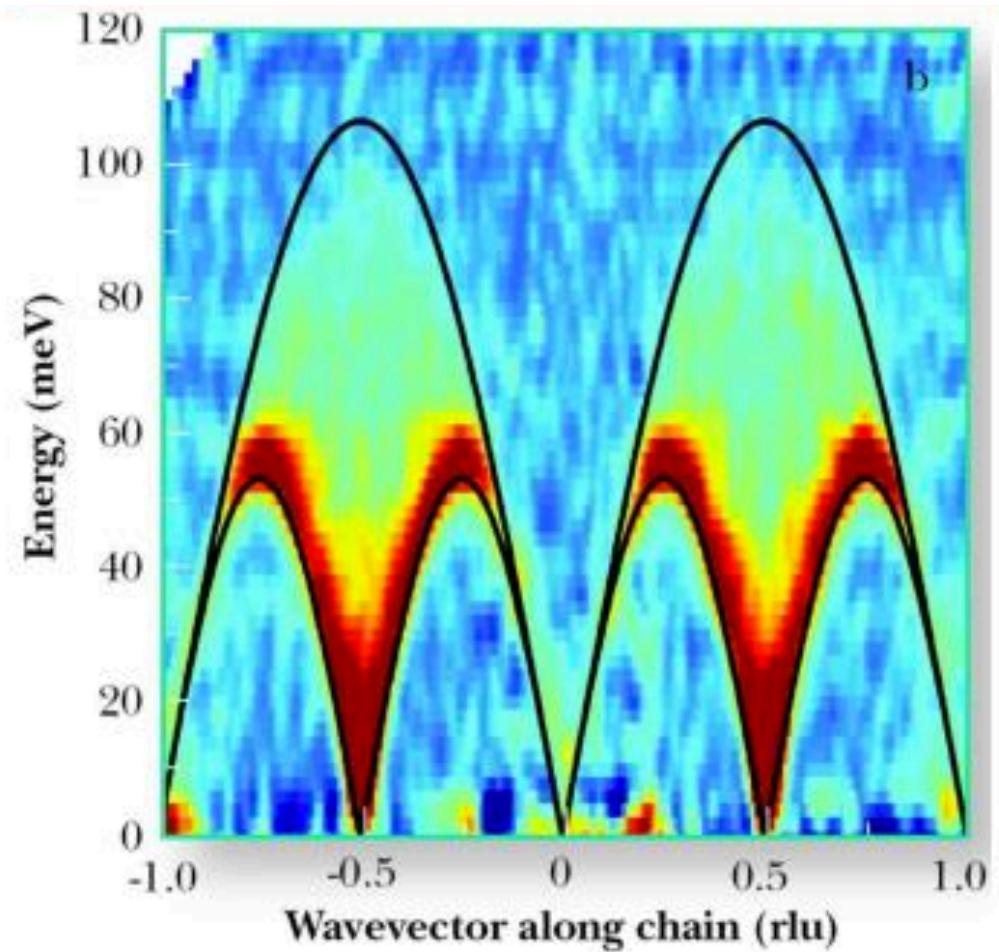
Emergent excitations

- Emergent excitations may be very different from spin flips
- May be created in multiples, or very hard to create at all with a neutron, or just have different properties

e.g. spinons

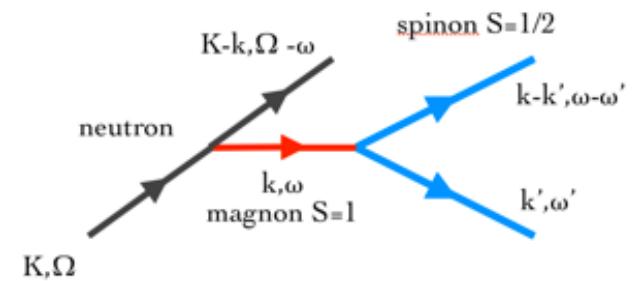


c.f. One dimension



A. Tennant *et al*, 2001

KCuF_3



Understanding in $d > 1$ is much more limited

A rough guide to experiments on HFM

Does it order?

- NMR line splitting
- muSR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/ x-ray

Delocalized excitations?

- thermal conductivity
- INS

Is there a gap?

- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of χ

Exotica

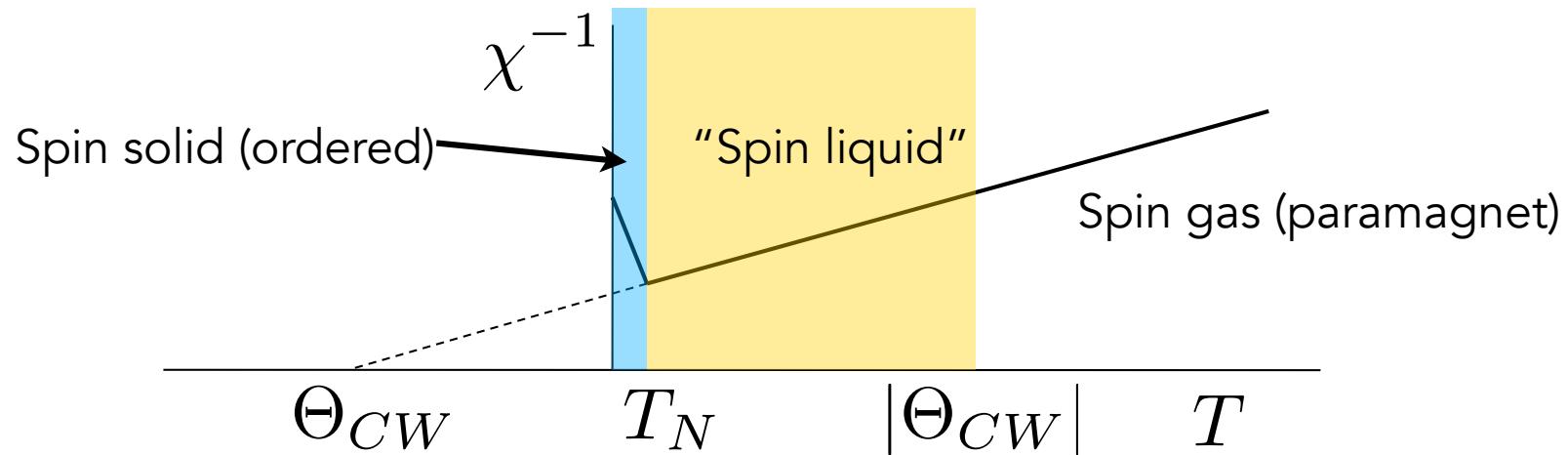
- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects

Structure of excitations?

- $E(k)$ from INS, RIXS
- optics, Raman



Ramirez Plot

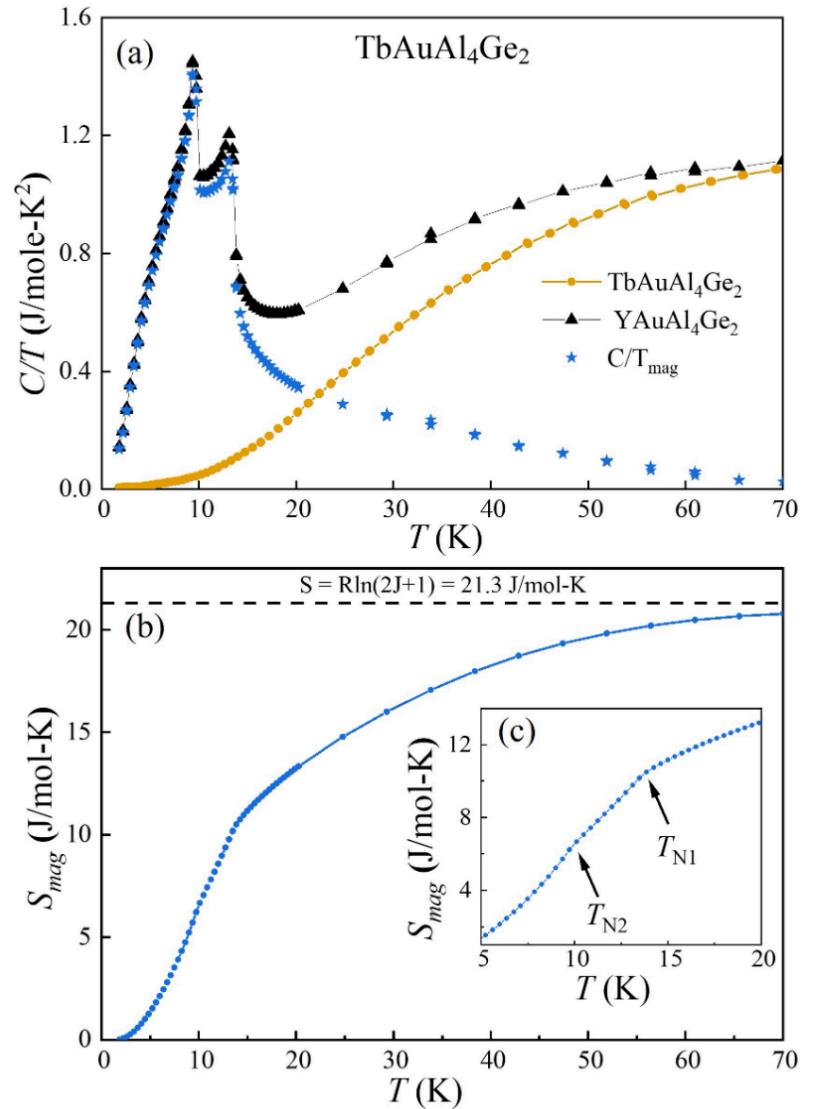


- Local moments: Curie-Weiss law at high T
$$\chi \sim \frac{A}{T - \Theta_{CW}}$$
- Frustration parameter: $f = |\Theta_{CW}|/T_N$
- Larger $f \gg 1$ is more frustrated (or fluctuating)

Heat capacity

- Sensitive indicator of phase transitions
- Useful to assess entropy, e.g. confirm effective spin

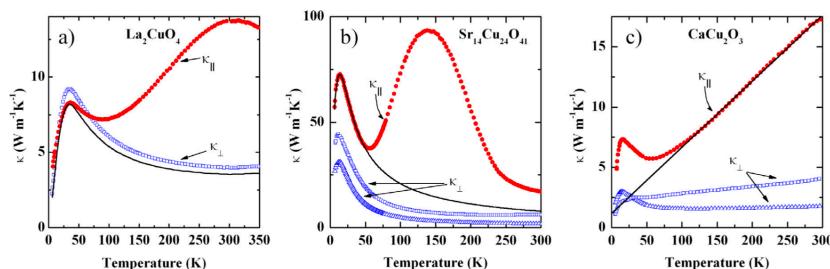
$$S(T) = \int_0^T dT' \frac{C(T')}{T'}$$



Thermal conductivity

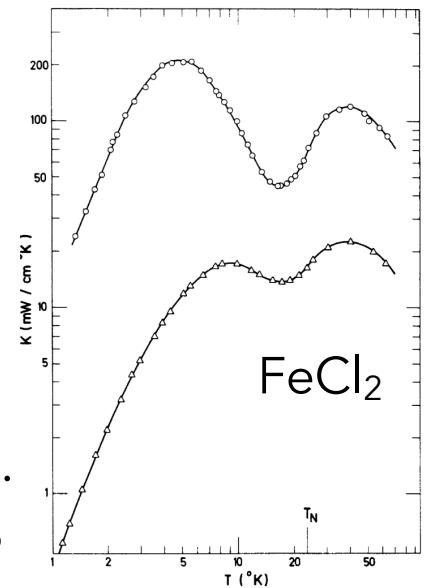
Spins carry
heat

Phonons carry heat
but interact with spins

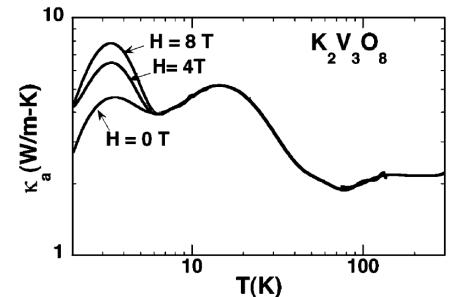


Review: C. Hess, 2019

G. Laurence et D.
Petitgrand, 1973

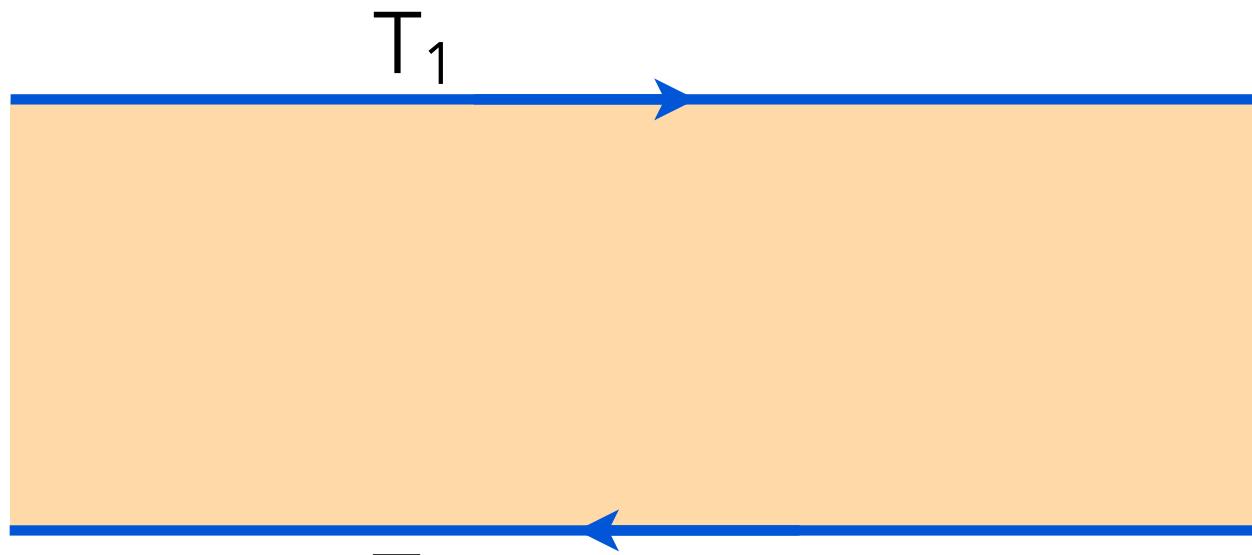


B.C. Sales *et al*, 2002



Thermal Hall effect

- Motivation: electronic/spin contribution theoretically closed tied to topology



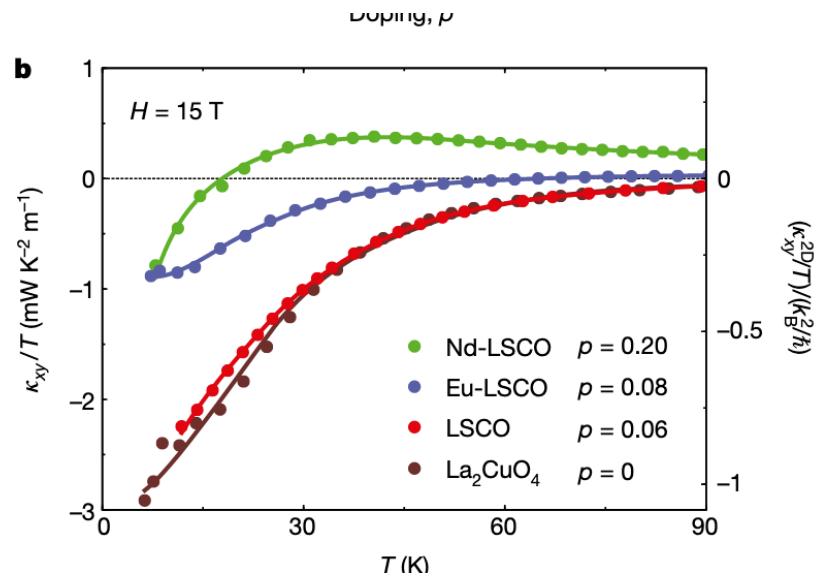
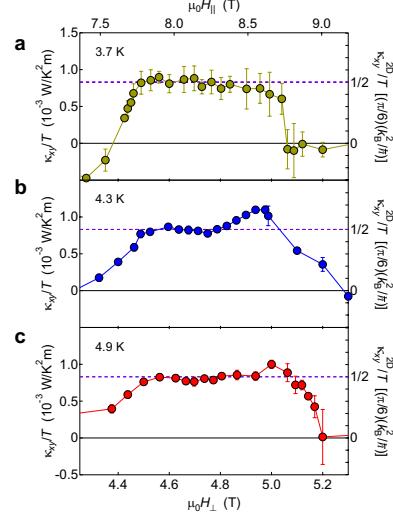
$$I_x = \kappa_H \Delta T_y$$

$$\kappa_H = \frac{\pi c k_B^2 T}{6\hbar}$$

a universal prediction for chiral
“Ising anyon” phase: agnostic to
microscopic spin interactions

Thermal Hall conductivity

- Experimental situation very much under debate - electronic versus lattice transport, impurity versus intrinsic, Berry curvature versus scattering,...



Y. Kasahara et al, 2018

G. Grissonanche et al, 2019

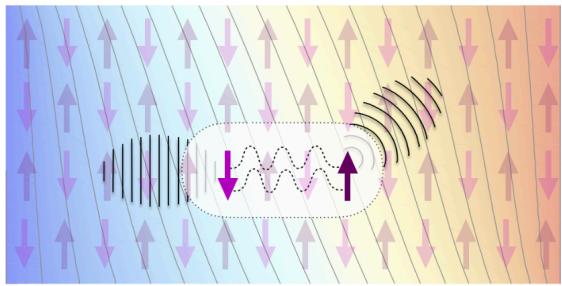
Thermal Hall conductivity

- Advertisement for some theory work

	zero field	$h = h\hat{z}$	
		lattice and spin	lattice effective
paramagnet	$G = P4/mmm1'$	$G(0, h\hat{z}) = P4/mm'm'$	$G^{\text{eff}}(0, h\hat{z}) = 4/mm'm'$
high sym. AFM	$G(\hat{x}, 0) = \langle i, TX, TY, C_{2x}, \mathcal{T}C_{2z} \rangle$	$G(\hat{x}, h\hat{z}) = \langle i, XY, \mathcal{T}C_{2y}, XC_{2z} \rangle$	$G^{\text{eff}}(\hat{x}, h\hat{z}) = \langle i, \mathcal{T}C_{2y}, C_{2z} \rangle$
low sym. AFM	$G(\hat{e}, 0) = \langle i, TX, TY, \mathcal{T}C_{2x} \rangle$	$G(\hat{e}, h\hat{z}) = \langle i, XY, XC_{2z} \rangle$	$G^{\text{eff}}(\hat{e}, h\hat{z}) = \langle i, C_{2z} \rangle$

arXiv:2103.04223

w/ Mengxing Ye+Lucile Savary



arXiv:2206.06183

arXiv:2202.10366

Poster W2 (Wed afternoon)

w/ Léo Mangeolle+Lucile Savary

where

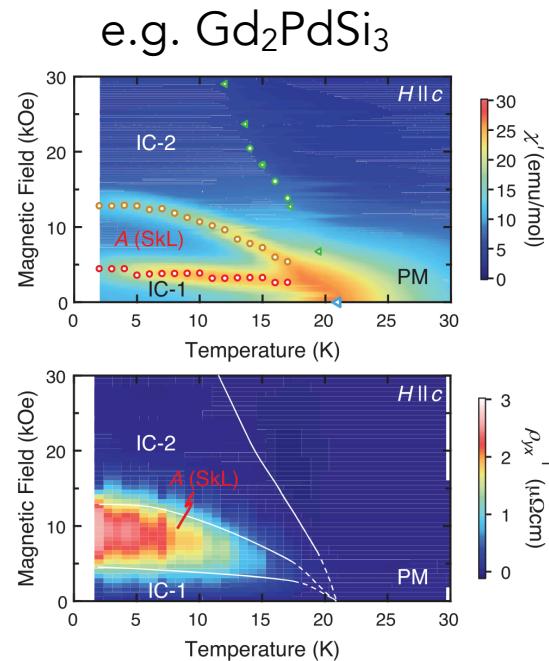
$$\kappa_H^{\mu\nu} = \frac{\hbar^2}{k_B T^2} \frac{1}{V} \sum_{n\mathbf{k}n'\mathbf{k}'} J_{n\mathbf{k}}^{\mu} \frac{e^{\beta\hbar\omega_{n\mathbf{k}}/2}}{2D_{n\mathbf{k}}} \left(\frac{1}{N_{\text{uc}}} \sum_{q=\pm} \frac{(e^{\beta\hbar\omega_{n\mathbf{k}}} - e^{q\beta\hbar\omega_{n'\mathbf{k}'}}) \mathfrak{W}_{n\mathbf{k},n'\mathbf{k}'}^{\ominus,+q}}{\sinh(\beta\hbar\omega_{n\mathbf{k}}/2) \sinh(\beta\hbar\omega_{n'\mathbf{k}'}/2)} \right) \frac{e^{\beta\hbar\omega_{n'\mathbf{k}'}/2}}{2D_{n'\mathbf{k}'}} J_{n'\mathbf{k}'}^{\nu}, \quad (9)$$

$$\mathfrak{W}_{n\mathbf{k}n'\mathbf{k}'}^{\ominus,qq'} = \frac{2N_{\text{uc}}}{\hbar^4} \Re \int_{t_1, t_2} e^{i[\Sigma_{n\mathbf{k}n'\mathbf{k}'}^{q,q'} t + \Delta_{n\mathbf{k}n'\mathbf{k}'}^{q,q'}(t_1 + t_2)]} \text{sign}(t_2) \left\langle \left[Q_{n\mathbf{k}}^{-q}(-t - t_2), Q_{n'\mathbf{k}'}^{-q'}(-t + t_2) \right] \left\{ Q_{n'\mathbf{k}'}^{q'}(-t_1), Q_{n\mathbf{k}}^q(t_1) \right\} \right\rangle, \quad (10)$$

Electrical conductivity

- Electron dynamics modified by magnet order

e.g. Hall conductivity due to skyrmions



Friday, June 24th

9h-10h30 : Skyrmions & multi-Q phases session

- Invited - **O. Zaharko** (Paul Scherrer Institut, Switzerland)

"Spin textures in frustrated magnetic materials" [↓](#)

- K. Shimizu (University of Tokyo, Japan)

"Spin moiré engineering of topology and emergent electromagnetic fields in multiple-Q spin textures" [↓](#)

- P. Pujol (CNRS & University of Toulouse, France)

"A skyrmion fluid and bimeron glass emerging from a chiral spin liquid" [↓](#)

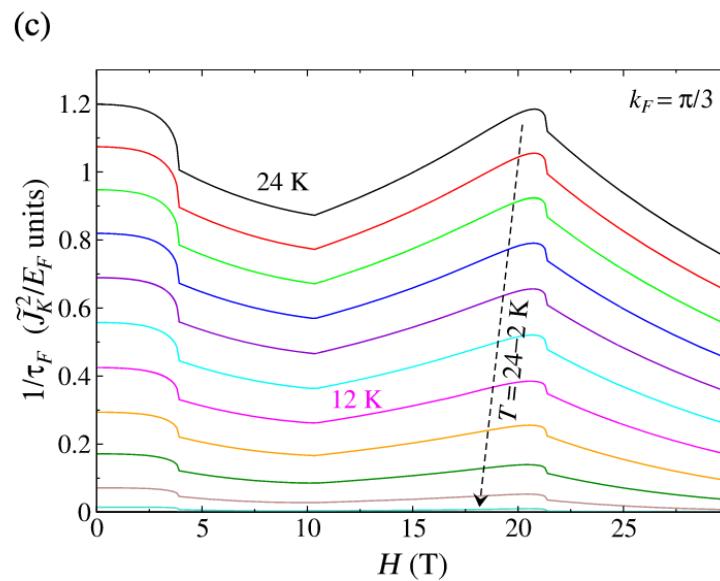
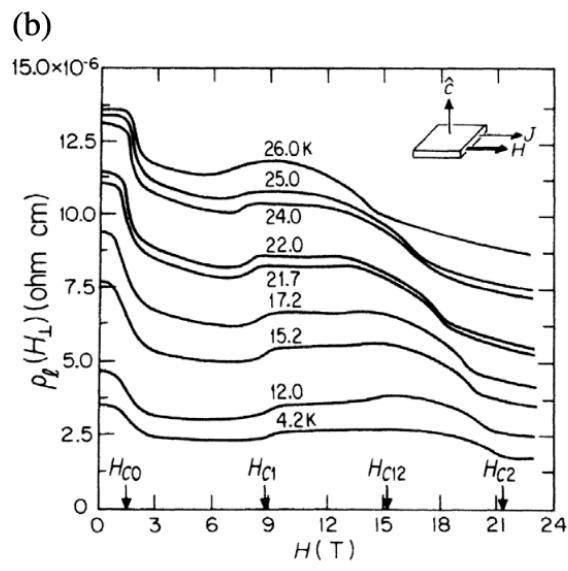
- K. Penc (Wigner Research Centre for Physics, Hungary)

"Unified theory of the spiral spin-liquids on layered honeycomb, diamond, and fcc lattices" [↓](#)

Electrical conductivity

- Electrons scattered by magnetic excitations

e.g. "Roller coaster"



- A.L. Chernyshev (University of California, USA)

"Roller-Coaster in a Flatland: Magnetism of Eu-intercalated Graphite"

More responses

- Diverse behaviors of HFM demand a diverse set of probes:
 - NMR/muSR/ESR - **A. Zorko**
 - Scattering - neutron - **P. Deen** - Raman, X-ray
 - Optics, Kerr, Faraday, non-linear/ultrafast
 - Magnetostriction, ultrasound, ...

Moiré



mohair

6°

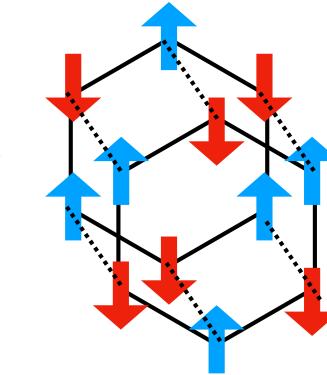
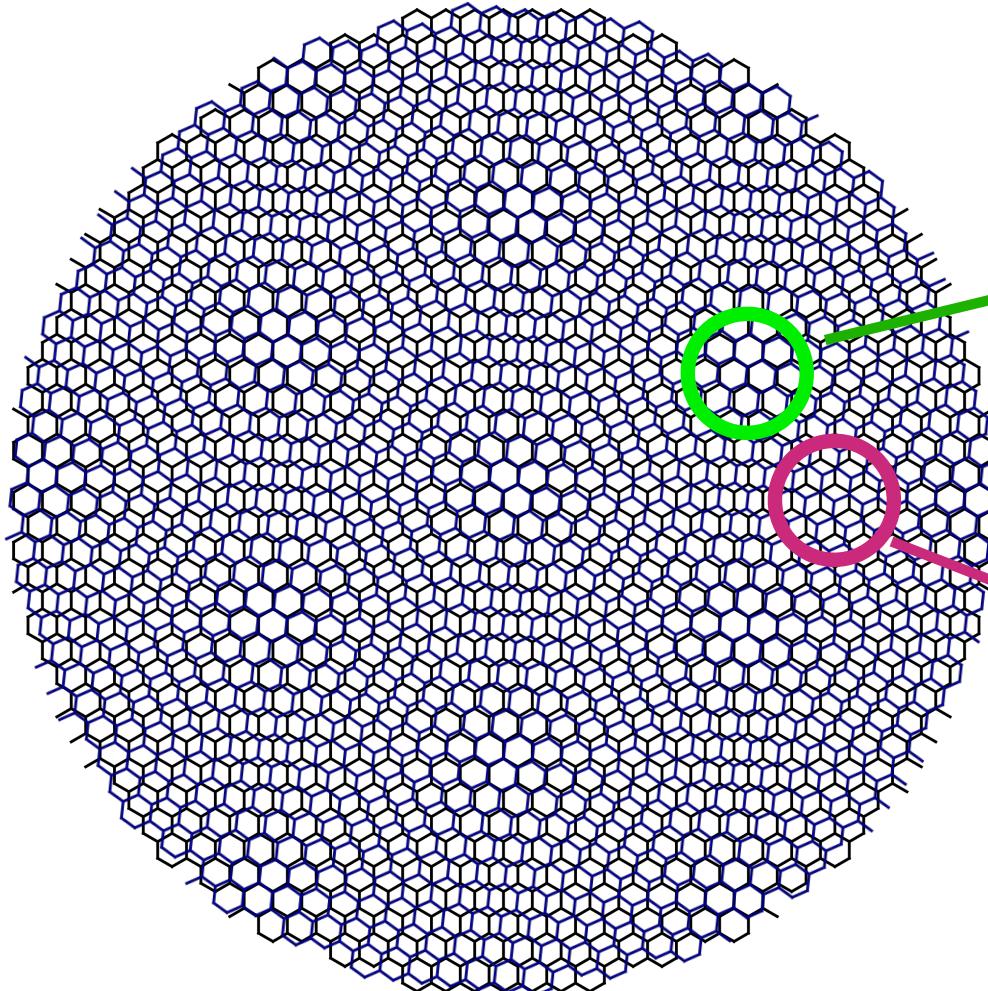
Twisted AF



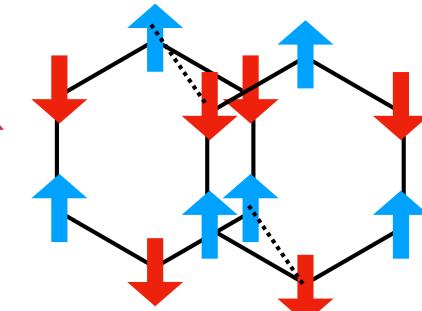
Kasra Hejazi



Zhu-Xi Luo



$$N_1 = -N_2$$



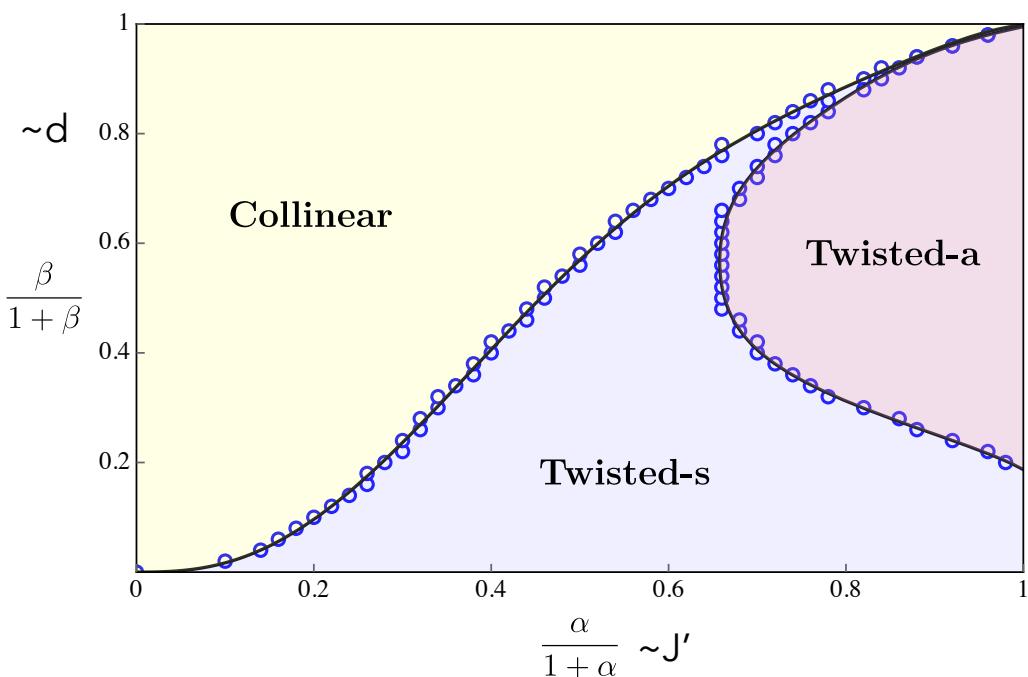
$$N_1 = N_2$$

Frustration: Neél vectors must rotate

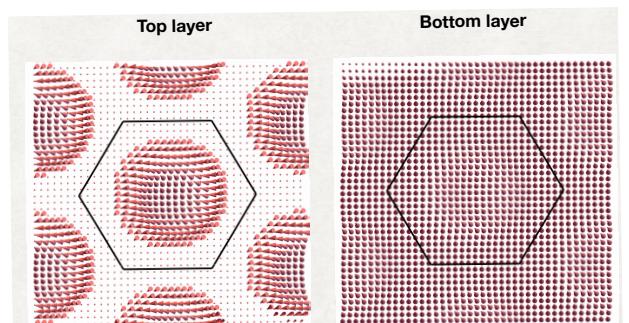
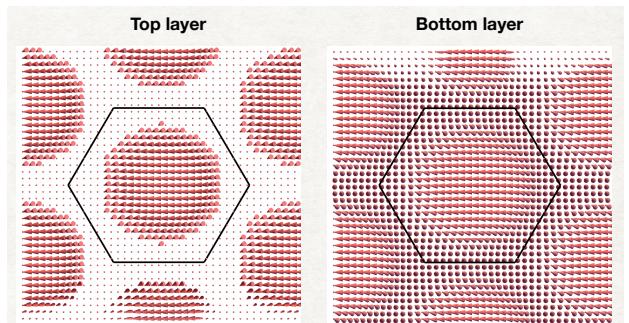
Twisted AF

$$\mathcal{H}_{\text{cl}} = \sum_l \left[\frac{\rho}{2} (\nabla \mathbf{N}_l)^2 - d (N_l^z)^2 \right] - J' \Phi(\mathbf{x}) \mathbf{N}_1 \cdot \mathbf{N}_2$$

Dimensionless parameter $\alpha = \frac{2J'}{\rho q_m^2} \sim \frac{J'}{J \theta^2}$



Coplanar spin textures



Transitions should be tunable by applied field

CrI₃

REPORT

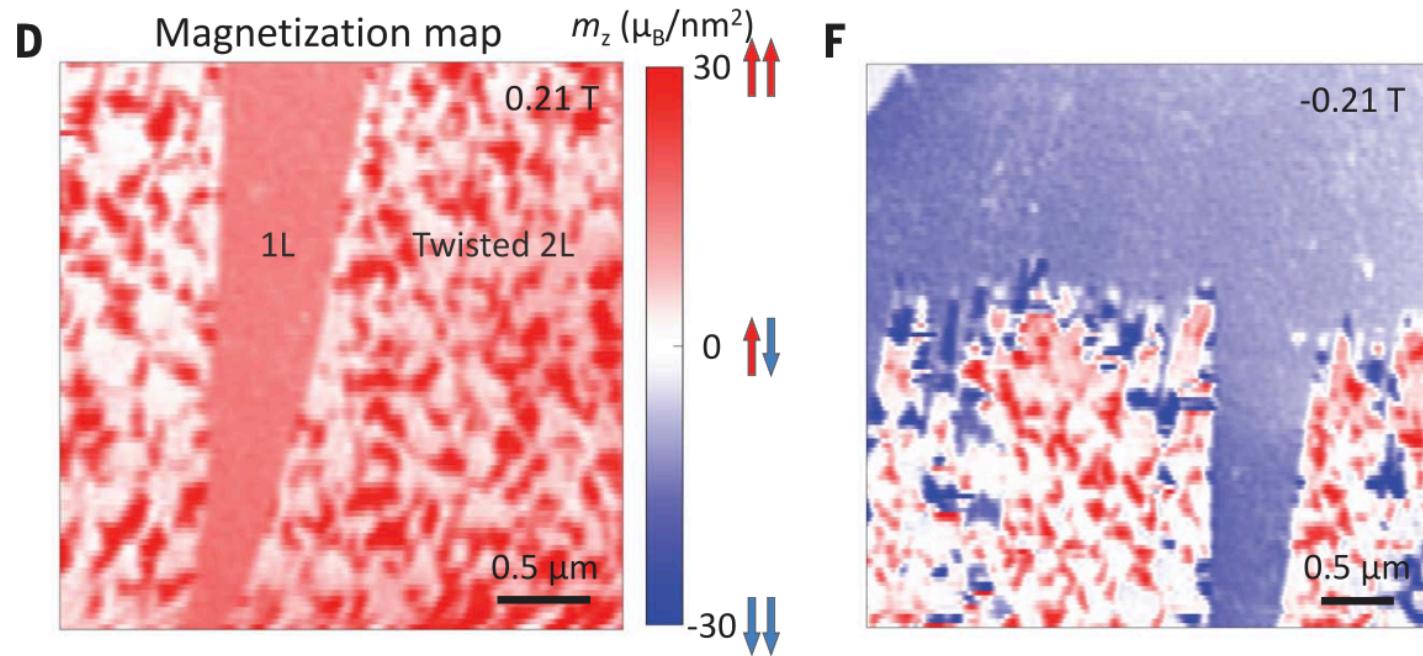
MAGNETISM

Direct visualization of magnetic domains and moiré magnetism in twisted 2D magnets

Tiancheng Song^{1†}, Qi-Chao Sun^{2†}, Eric Anderson^{1†}, Chong Wang³, Jimin Qian⁴, Takashi Taniguchi⁵, Kenji Watanabe⁶, Michael A. McGuire⁷, Rainer Stöhr^{2,8}, Di Xiao³, Ting Cao⁴, Jörg Wrachtrup^{2,9*}, Xiaodong Xu^{1,4*}

(A twisted ferromagnet)

Scanning NV magnetometry



(twist disorder is evident)

Thank you

- Frustrated and quantum magnetism is an exciting place for theory and experiment to meet
- The basic point is frustration allows more unusual structures to emerge, be they atypical orders, unusual excitations, or unconventional responses
- We surely missed many things. That's why you need to go to the meeting!

