



General introduction to frustrated magnetism

Leon Balents

HFM Paris, June 2022

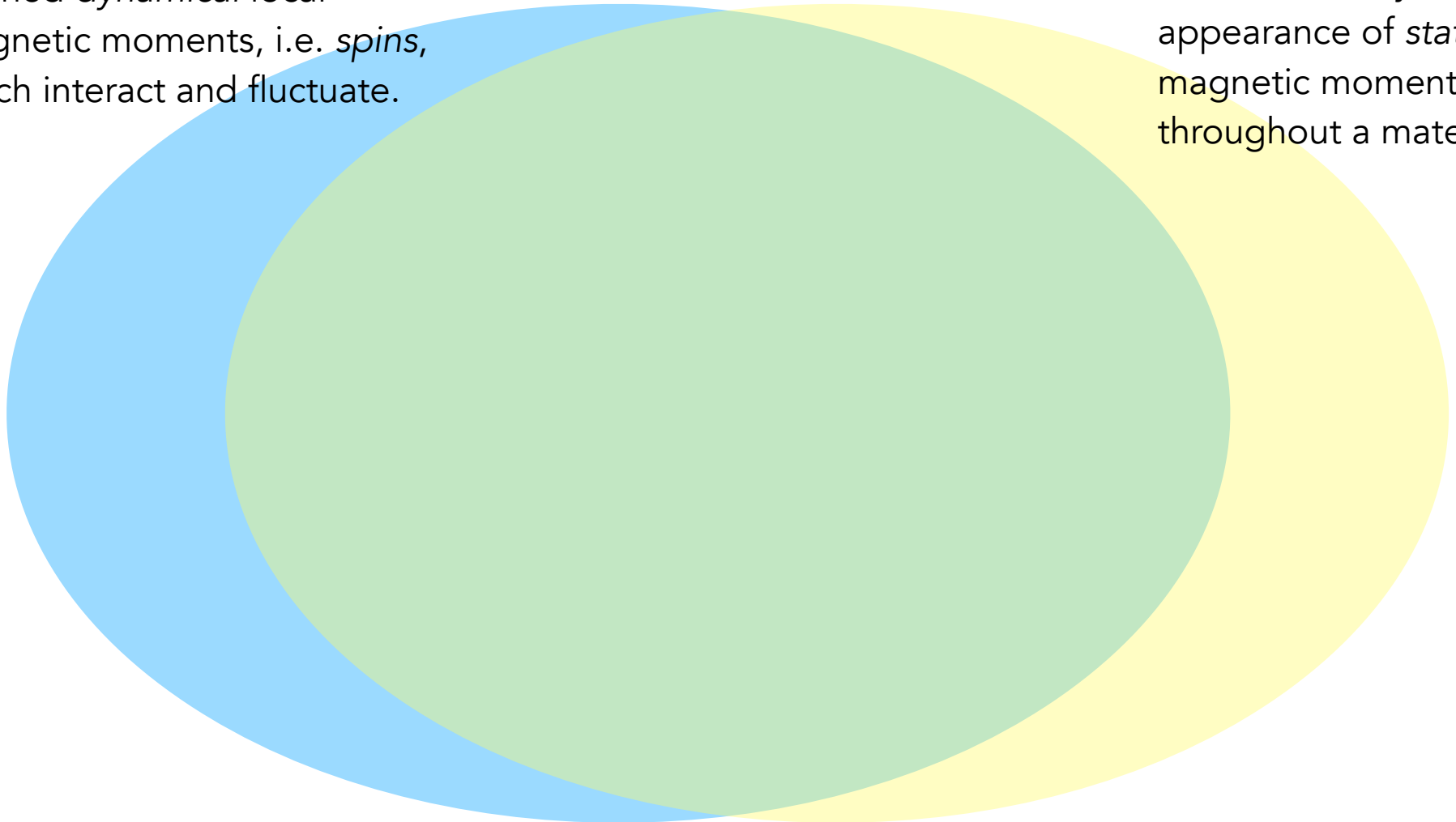
Magnetism

Magnetic degrees of

freedom: existence of well-defined *dynamical* local magnetic moments, i.e. *spins*, which interact and fluctuate.

Magnetic order:

spontaneous breaking of time-reversal symmetry, appearance of *static* magnetic moments throughout a material



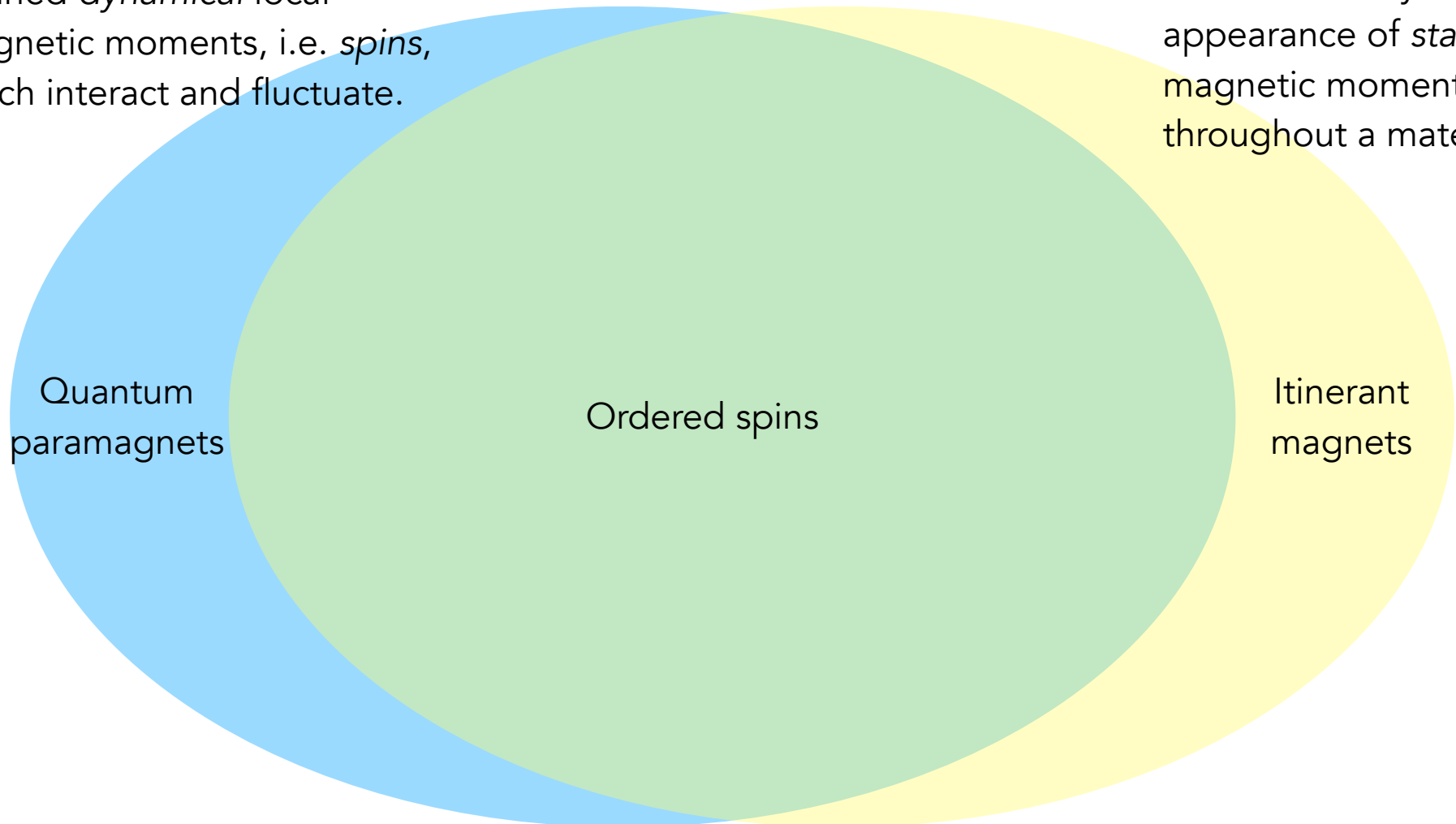
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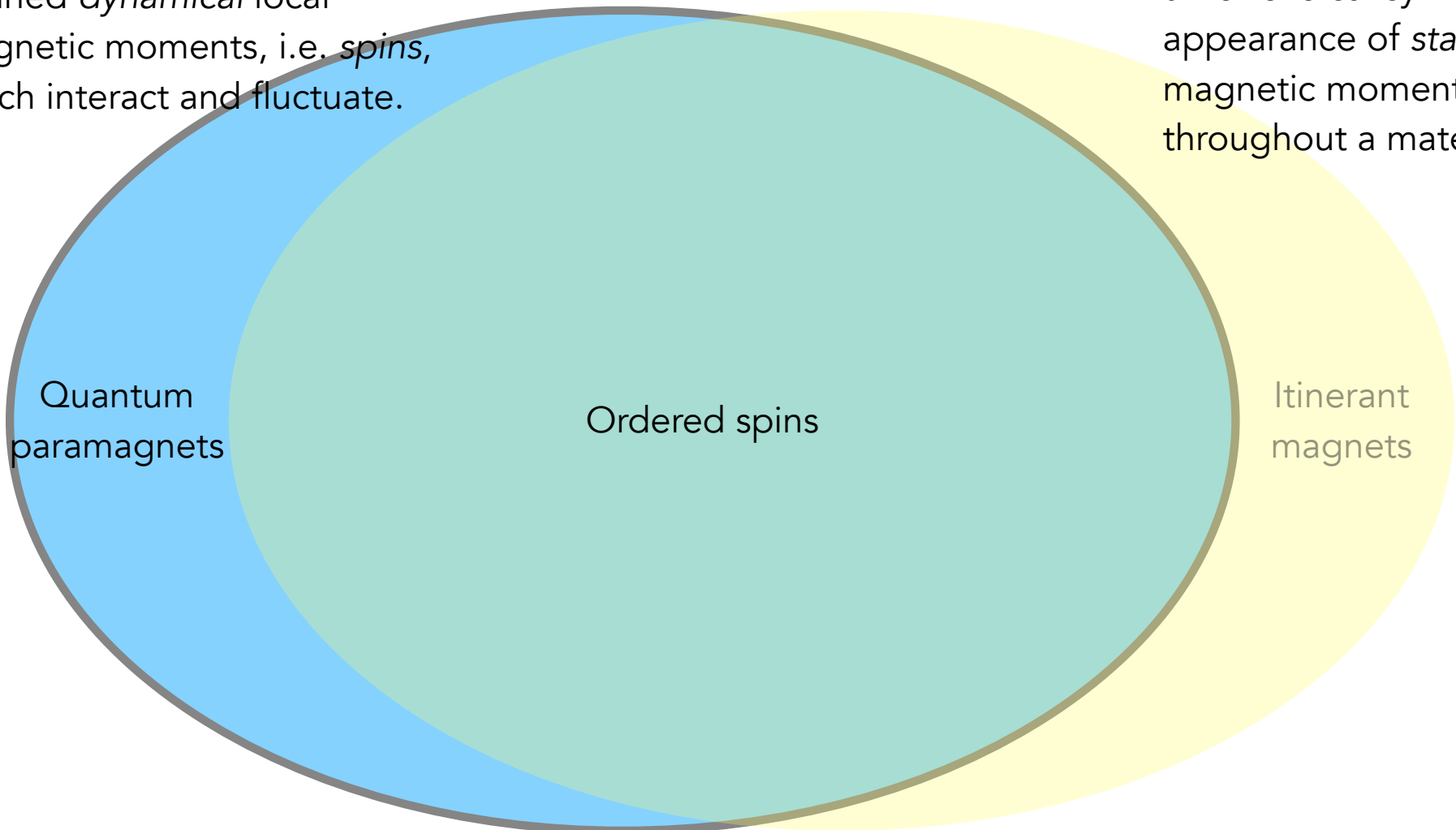
Magnetism

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Theorist's view

$$H = \sum_{i < j} J_{ij} \boxed{S_i} \cdot S_j \quad (+ \dots)$$

Ising = ± 1

Vectors

Quantum operators

Coupling to the lattice
Coupling to itinerant electrons
Multipolar, multi-spin interactions
Applied fields
...

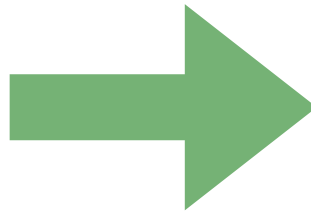
Materials design: find good choices of J_{ij} , S_i (+...)

Design

Input

J_{ij}, S_i
(+...)

Physics



Output

Emergent properties:

Ordering

e.g. FM, AF, spiral,
quadrupolar

Collective excitations

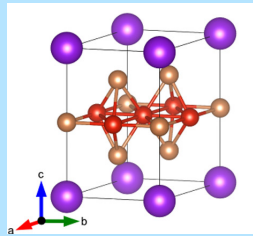
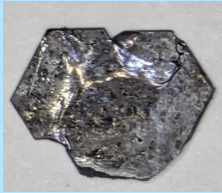
e.g. magnons,
skyrmions, spinons

Response functions

e.g. susceptibility,
Hall effect

Design

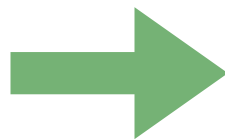
Input



Physics

J_{ij} , S_i
(+...)

Physics



Output

Emergent properties:

Ordering

e.g. FM, AF, spiral,
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Collective excitations

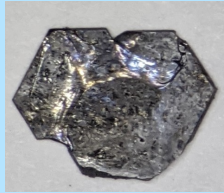
e.g. magnons,
skyrmions, spinons

Response functions

e.g. susceptibility,
Hall effect

Design

Input



What do we really control?

- Atoms and environment
- Structure and symmetry

Output

What properties:

AF, spiral,
polar

Collective excitations

e.g. magnons,
skyrmions, spinons

Response functions

e.g. susceptibility,
Hall effect

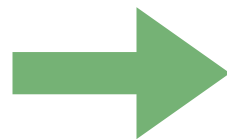


Physics

J_{ij} , S_i

(+...)

Physics



Design principle 1: get local moments

- Most magnetism in QMs comes from either 3d transition metal ions or 4f rare earths. These have relatively localized orbitals which don't overlap strongly with neighbors and have strong Coulomb repulsion, which localizes electrons best.

1 H 1.00794																	2 He 4.002602				
3 Li 6.941	4 Be 9.012182															5 B 10.811	6 C 12.0107	7 N 14.00674	8 O 15.9994	9 F 18.9984032	10 Ne 20.1797
11 Na 22.989770	12 Mg 24.3050															13 Al 26.981538	14 Si 28.0855	15 P 30.973761	16 S 32.066	17 Cl 35.4527	18 Ar 39.948
19 K 39.0983	20 Ca 40.078	21 Sc 44.955910	22 Ti 47.867	23 V 50.9415	24 Cr 51.9961	25 Mn 54.938049	26 Fe 55.845	27 Co 58.933200	28 Ni 58.6934	29 Cu 63.545	30 Zn 65.39	31 Ga 69.723	32 Ge 72.61	33 As 74.92160	34 Se 78.96	35 Br 79.904	36 Kr 83.80				
37 Rb 85.4678	38 Sr 87.62	39 Y 88.90585	40 Zr 91.224	41 Nb 92.90638	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.90550	46 Pd 106.42	47 Ag 107.8682	48 Cd 112.411	49 In 114.818	50 Sn 118.710	51 Sb 121.760	52 Te 127.60	53 I 126.90447	54 Xe 131.29				
55 Cs 132.90545	56 Ba 137.327	57 La 138.9055	72 Hf 178.49	73 Ta 180.9479	74 W 183.84	75 Re 186.207	76 Os 190.23	77 Ir 192.225	78 Pt 195.078	79 Au 196.96655	80 Hg 200.59	81 Tl 204.3833	82 Pb 207.2	83 Bi 208.98038	84 Po (209)	85 At (210)	86 Rn (222)				
87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 (269)	111 (272)	112 (277)					114 (289)	116 (289)	118 (293)			

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90 Th 232.0381	91 Pa 231.03588	92 U 238.0289	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)

Local moments

- In 3d transition metals, *usually* magnetism is fairly isotropic, i.e. spins are “Heisenberg like”, because crystal fields split the d orbitals and spin-orbit coupling is relatively weak (Co is most common exception, when very localized). Exchange interactions between spins vary from quite strong (1000K) to quite weak (1K).

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Local moments

- In 4f lanthanides, spin-orbit coupling is dominant over crystal fields and so magnetic moments become large (incorporating orbital moment) and often very anisotropic (due to large SOC). They have complex multiplet structures, and weak exchange interactions.

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QM Materials

- Quantum spin liquids and interesting insulating antiferromagnets
- $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$,
 $\alpha\text{-RuCl}_3$, $\text{Pr}_2\text{Zr}_2\text{O}_7$,
 Cs_2CuCl_4 , $\text{Yb}_2\text{Ti}_2\text{O}_7$

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QM Materials

- Orbital degeneracy/spin-orbit interaction
- $\text{RVO}_3, \text{RCoO}_3, \dots$
- $\text{Cd}_2\text{Os}_2\text{O}_7, \text{Sr}_2\text{IrO}_4, \text{Na}_2\text{IrO}_3, \text{a-RuCl}_3 \dots$
- $\text{NaYbO}_2, \text{TmMgGaO}_4, \dots$

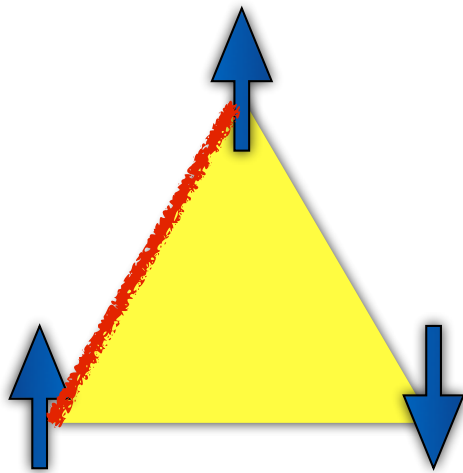
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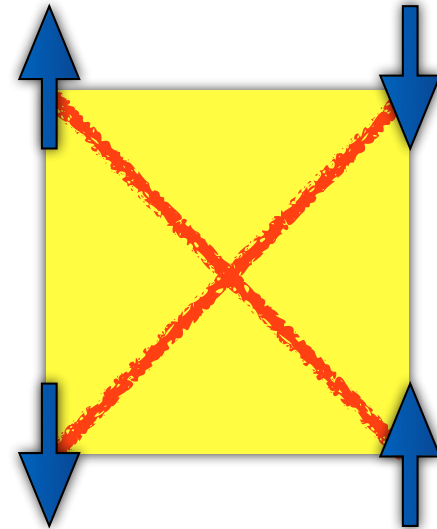
SOC increases with
atomic number

Design principle 2: Frustration

- What is it? Competing multiple interactions that cannot be simultaneously satisfied



geometric
frustration



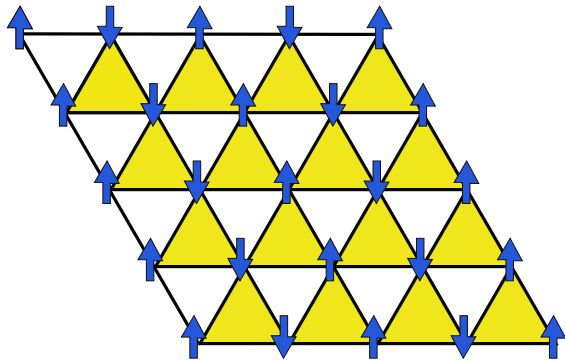
exchange
frustration

Why frustration?

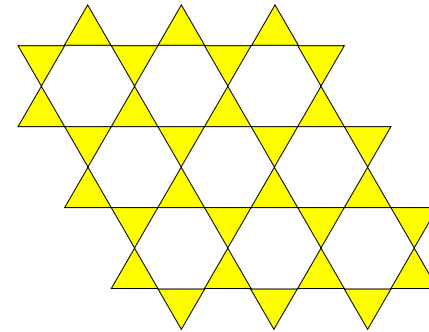
- Competition suppresses conventional ordered states: more exotic things are possible
- Fluctuating regimes
- Complex or quantum orders
- Spin liquids
- Unusual excitations

Examples

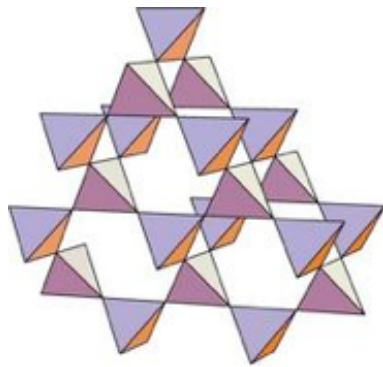
Triangle based lattices



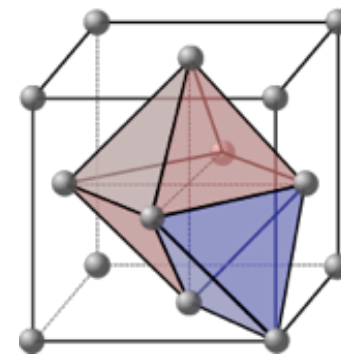
triangle



kagome



pyrochlore

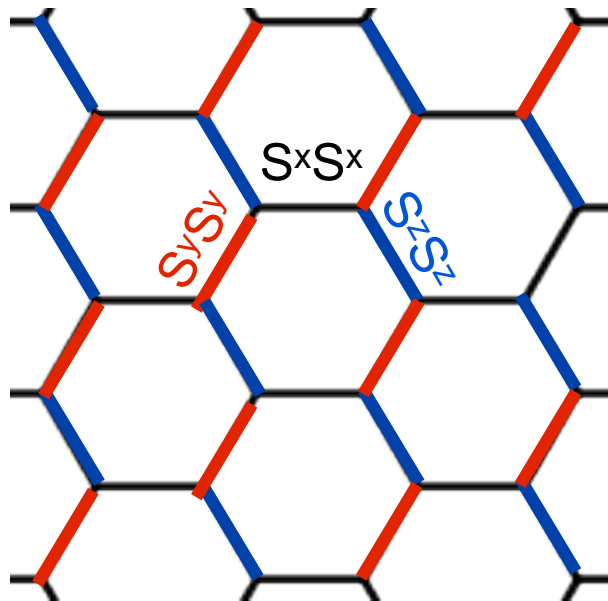


fcc

Other interactions

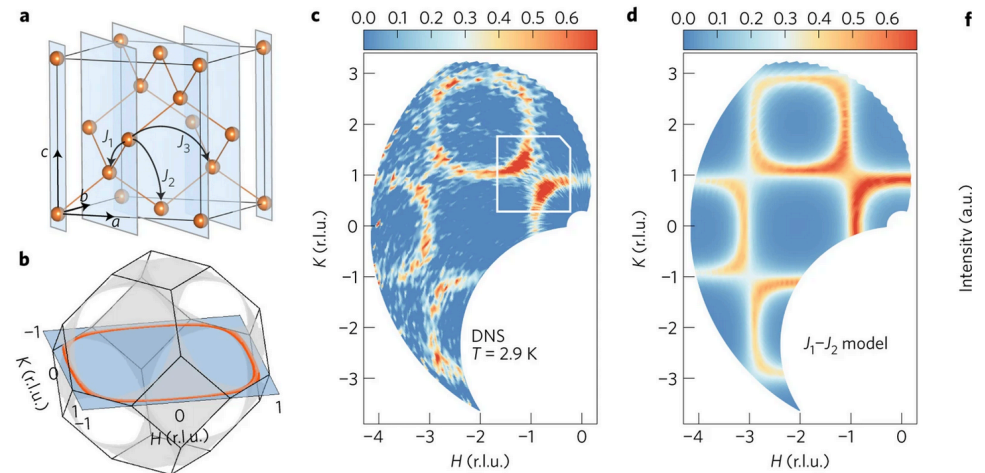
- With more structured interactions, even non-geometrically frustrated lattices can show frustration

“Kitaev” terms in Na_2IrO_3 , RuCl_3 ...



Jackeli+Khaliulin, 2009

Spiral spin liquid in MnSc_2S_4



S. Gao et al, 2017

Ising systems

- Ising models
$$H = \frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j \quad \sigma_i = \pm 1$$
- Physically occurs when single ion has a *doublet* ground state *and* only one component couples strongly

$$H_{\text{real}} = \frac{1}{2} \sum_{ij} J_{ij} S_i^z S_j^z + H' [S_i^x, S_i^y]$$

- Requires significant spin-orbit coupling: some rare earths, Co, ...

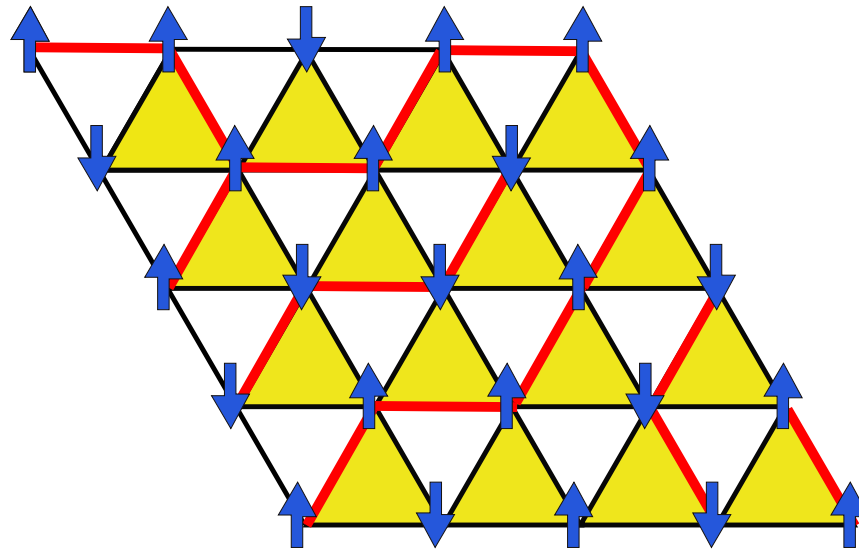
Wannier

- Triangular lattice Ising AF: macroscopic degeneracy (Wannier, 1950)

$$H = J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$
$$\sigma_i = \pm 1$$

$$\Omega = e^{S/k_B}$$

$$S \approx 0.34 N k_B$$



1 frustrated
bond per
triangle

Wannier

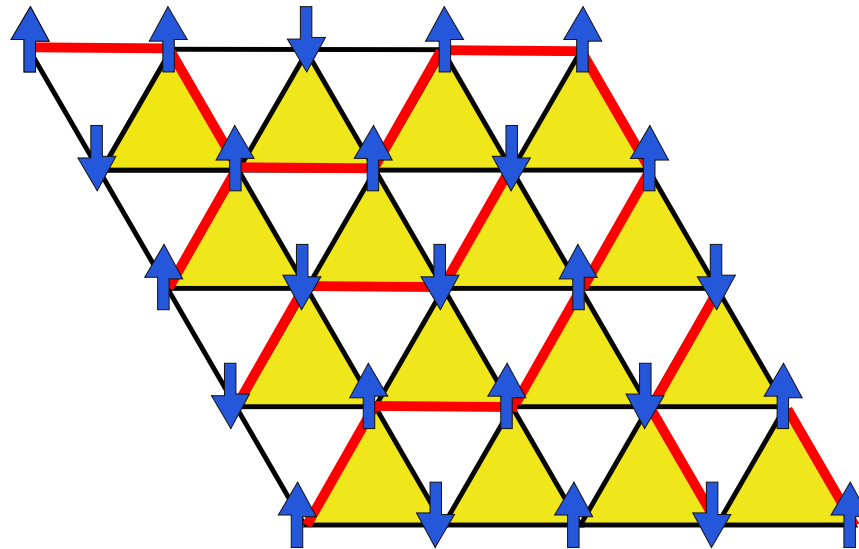
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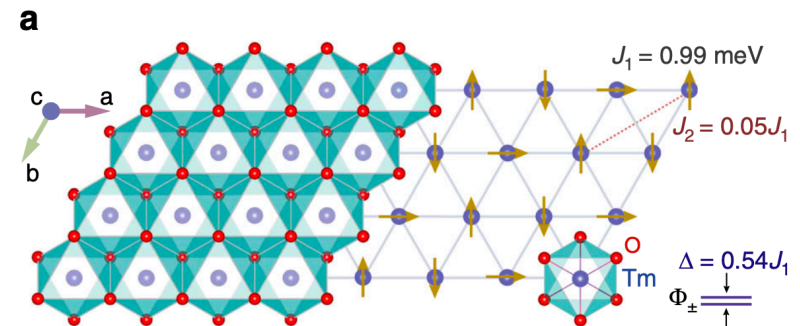


1 frustrated
bond per
triangle

Somewhat close realization?

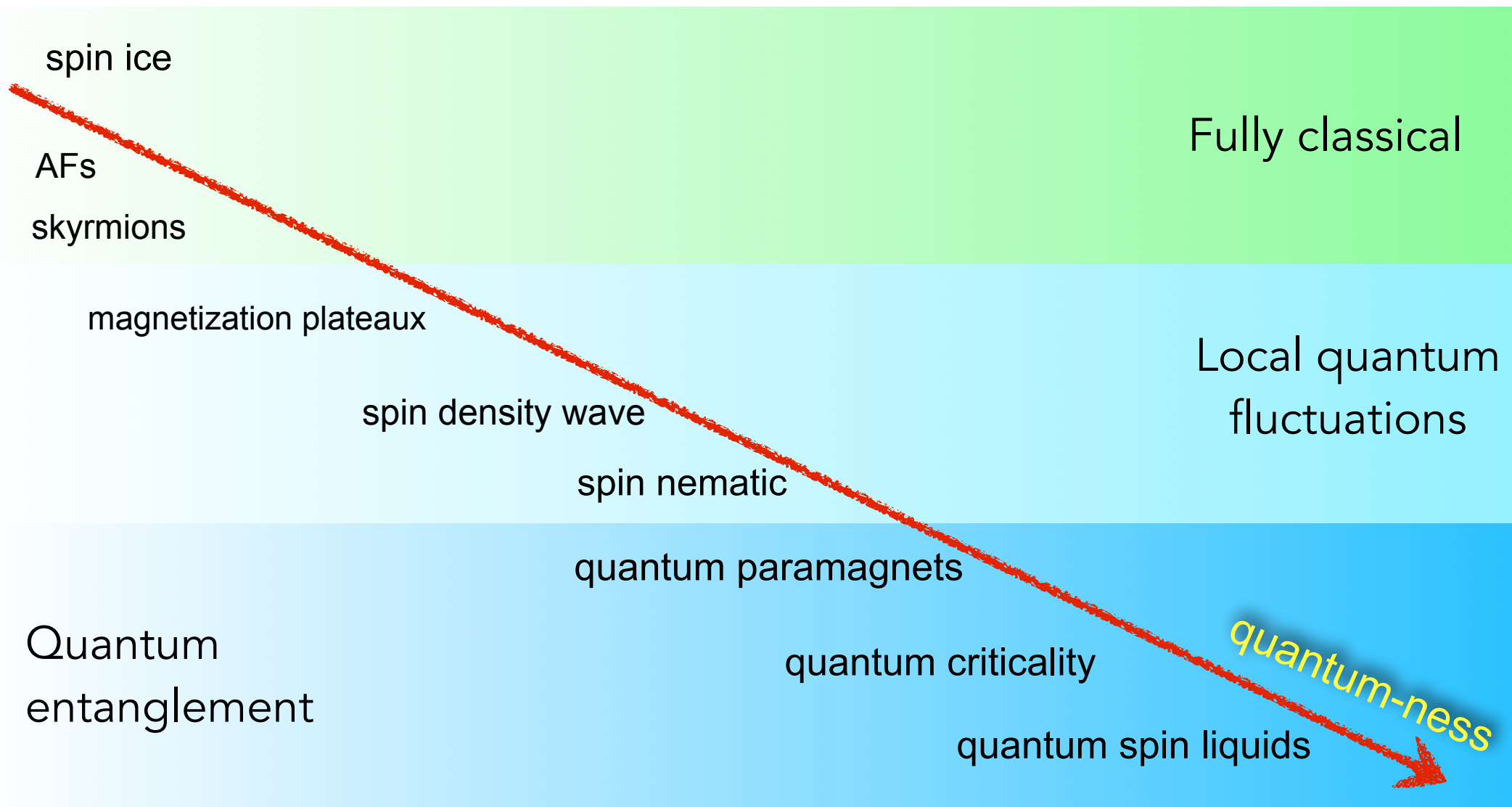
TmMgGeO4

Degeneracy-breaking perturbations determine the low energy physics



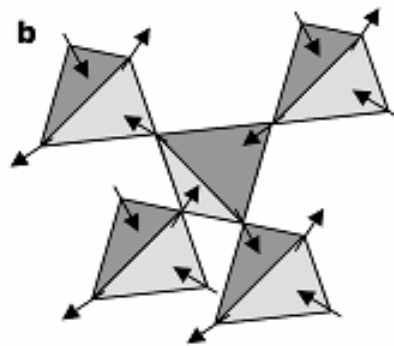
H. Li et al, 2020

Frustrated Magnetism



Spin ice

- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have Ising doublets with dominant NN coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$

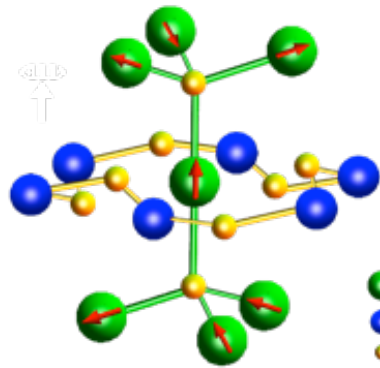


Spin ice

$$H \approx J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z$$

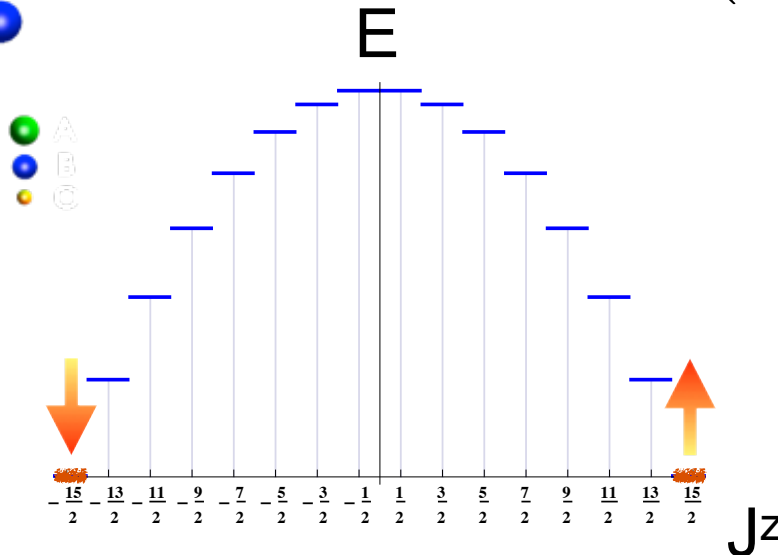
Spin ice

- rare earth pyrochlores $\text{Dy}_2\text{Ti}_2\text{O}_7$, $\text{Ho}_2\text{Ti}_2\text{O}_7$ with *Ising* doublet ground states



Local physics: $L+S = J$

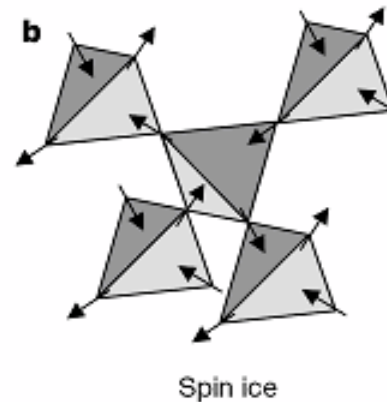
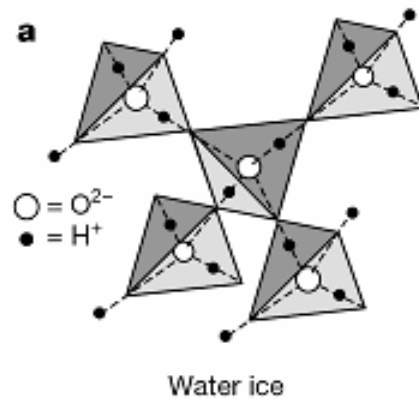
$$H_{\text{ion}} = -D \left(\vec{J}_i \cdot \hat{n}_i \right)^2$$



flips between $\pm J$
states difficult

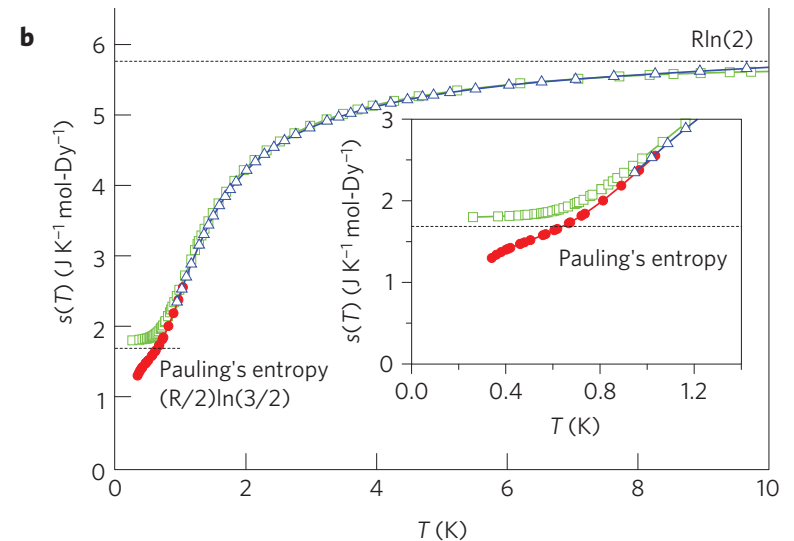
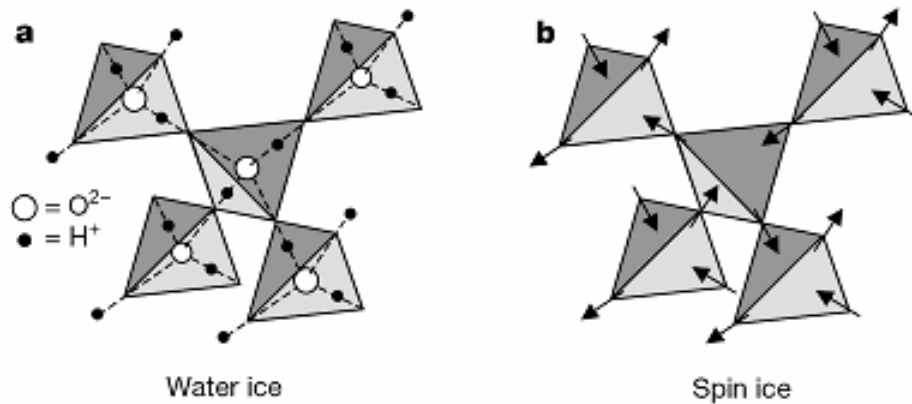
Spin ice

- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$



Spin ice

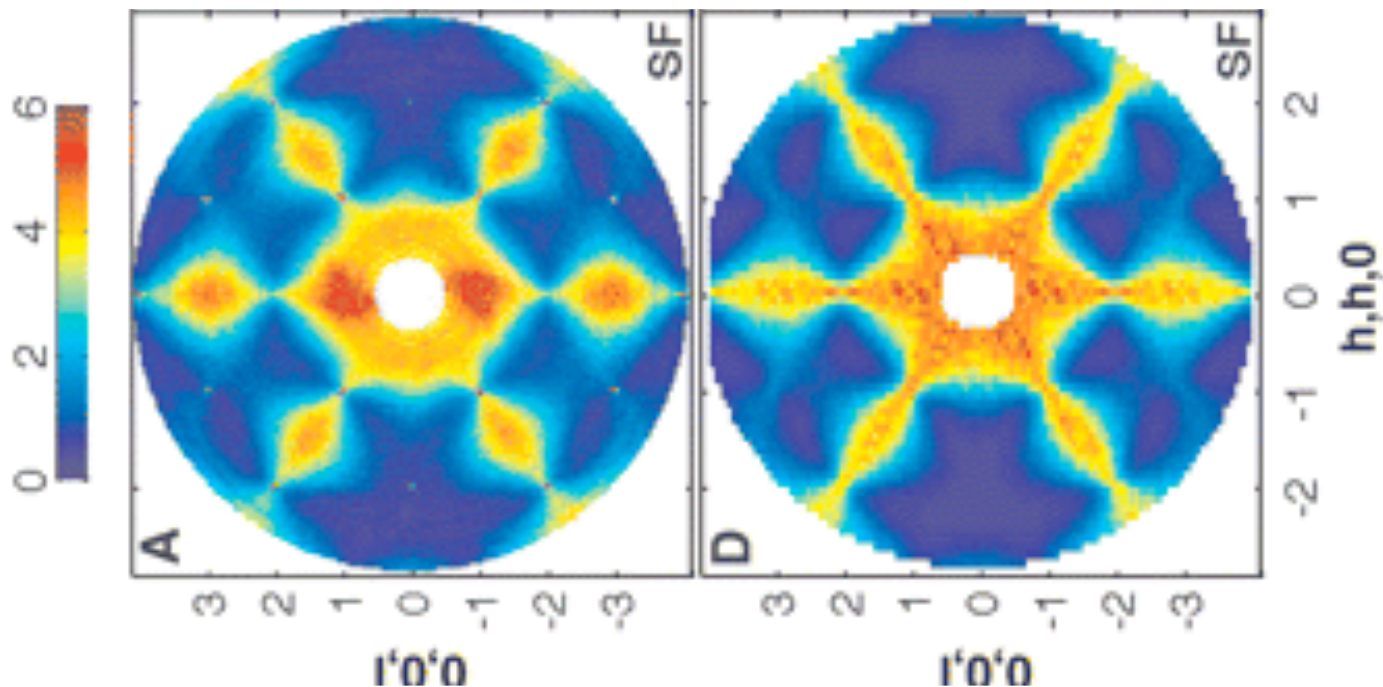
- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$



Pomaranski *et al*, $\text{Dy}_2\text{Ti}_2\text{O}_7$
(original expts. by Harris *et al*, 1999)

Ice correlations

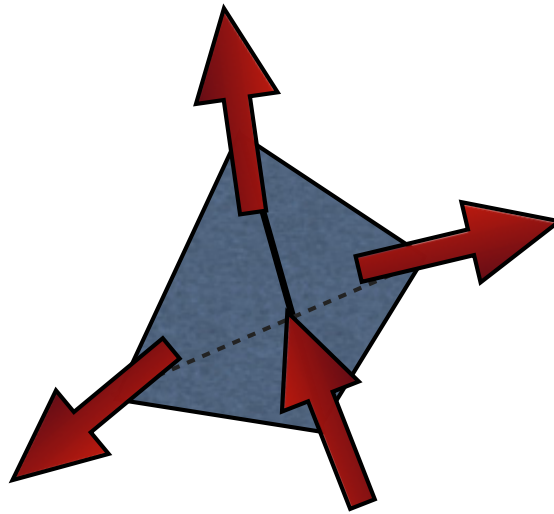
- “Pinch points” show that 2in-2out constraint holds



T. Fennell *et al*, 2009 experiment
 $\text{Ho}_2\text{Ti}_2\text{O}_7$

theory

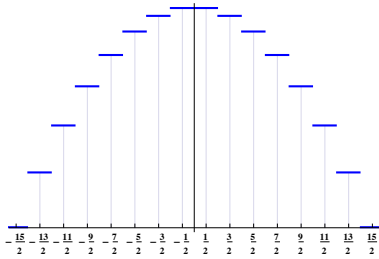
Monopoles



3in:1out defects act like monopoles,
and can move almost freely



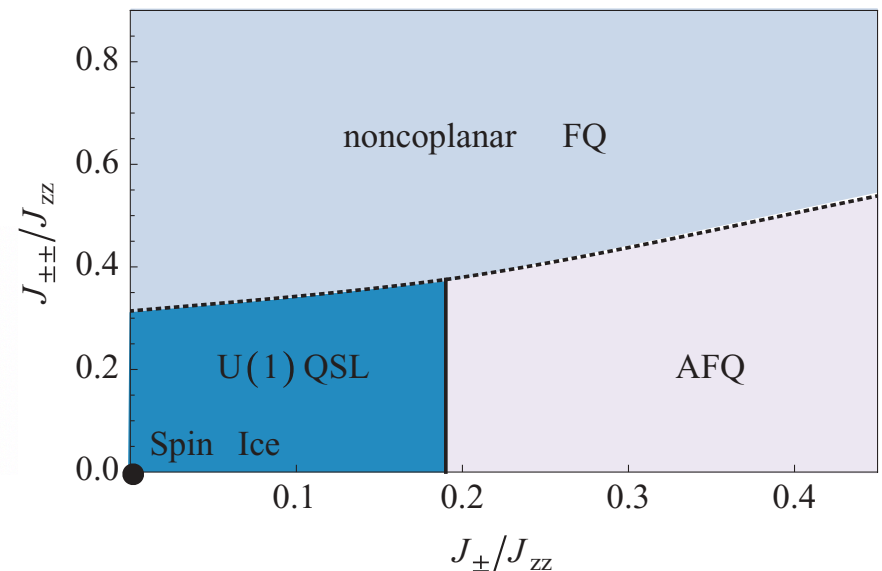
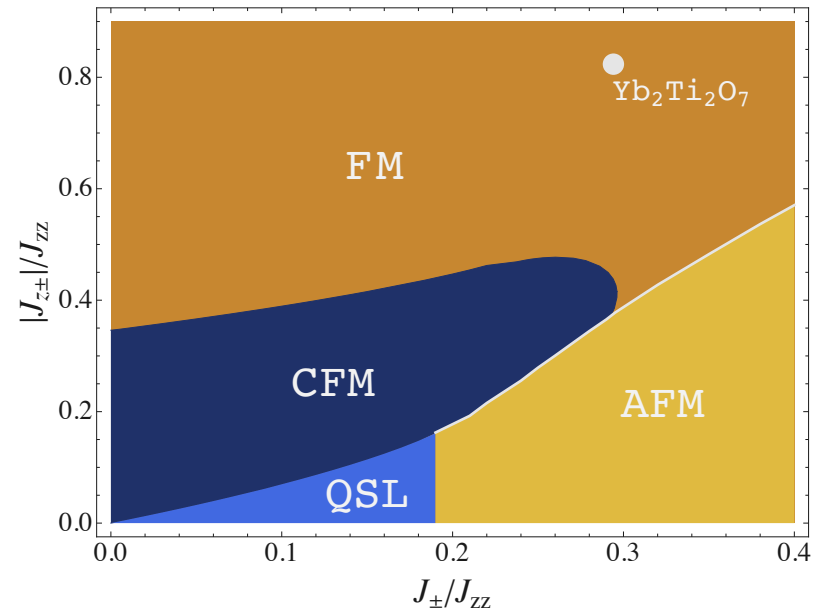
Quantum spin ice



M. Hermele, MPA Fisher, L.B., 2004;
 A. Banerjee et al, 2008
 L. Savary + LB, 2012
 S.B. Lee, S. Onoda + LB, 2012
 + Many subsequent numerical
 and analytical works

Still looking...

- **Invited - E. Smith** (McMaster University, Canada)
 "The case for a $U(1)_p$ Quantum Spin Liquid Ground State in the Dipole-Octupole Pyrochlore $Ce_2Zr_2O_7$ "
 ↓
- Y.B. Kim (University of Toronto, Canada)
 "Competing dipolar-octupolar quantum spin liquids on the pyrochlore lattice" ↓
- R. Sibille (Paul Scherrer Institut, Switzerland)
 "Octupolar correlations and spinon spectrum in $Ce_2Sn_2O_7$ quantum spin ice" ↓
- **Invited - R. Moessner** (Max Planck Institute for the Physics of Complex Systems, Germany)
 "Emergent QED in the quantum spin ices" ↓



Heisenberg systems

- Typically for *closed shells* - i.e. configurations w/o orbital degeneracy - of 3d transition metal ions, SOC effects are weak: good approximate spin-rotation symmetry

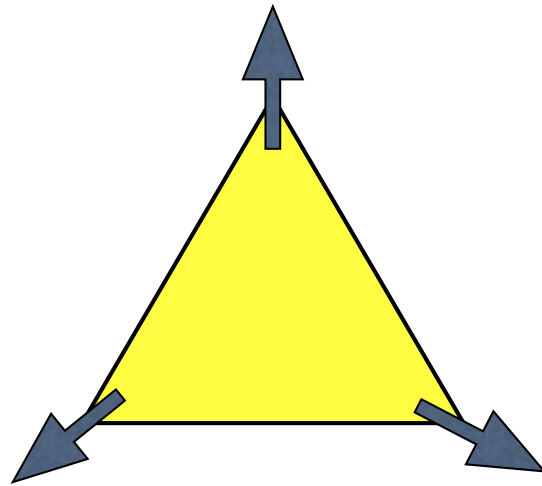
$$H = \frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

single-ion and exchange
anisotropy, $\approx 10\%$ level

Spin "length" $S = 1/2$ is most quantum, $S \gg 1$ is semiclassical

Triangle

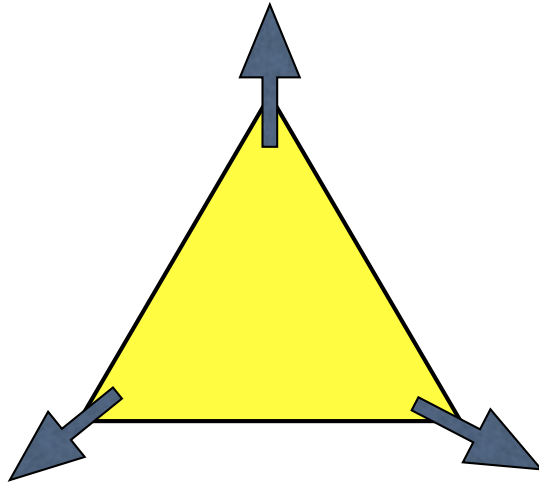
- Classically: spins must sum to zero



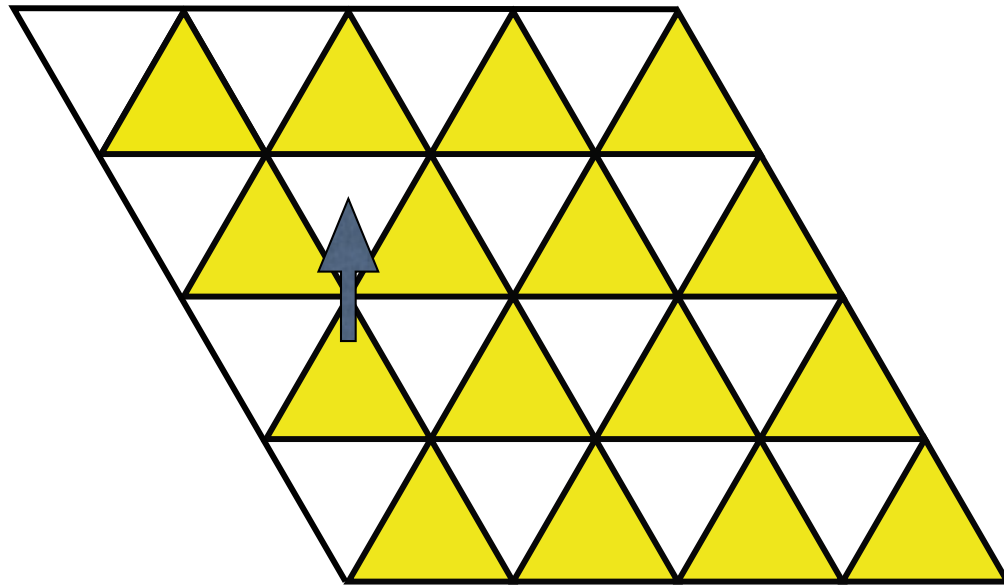
Tendency to non-collinear ordering

Triangular lattice

- Classically: spins must sum to zero

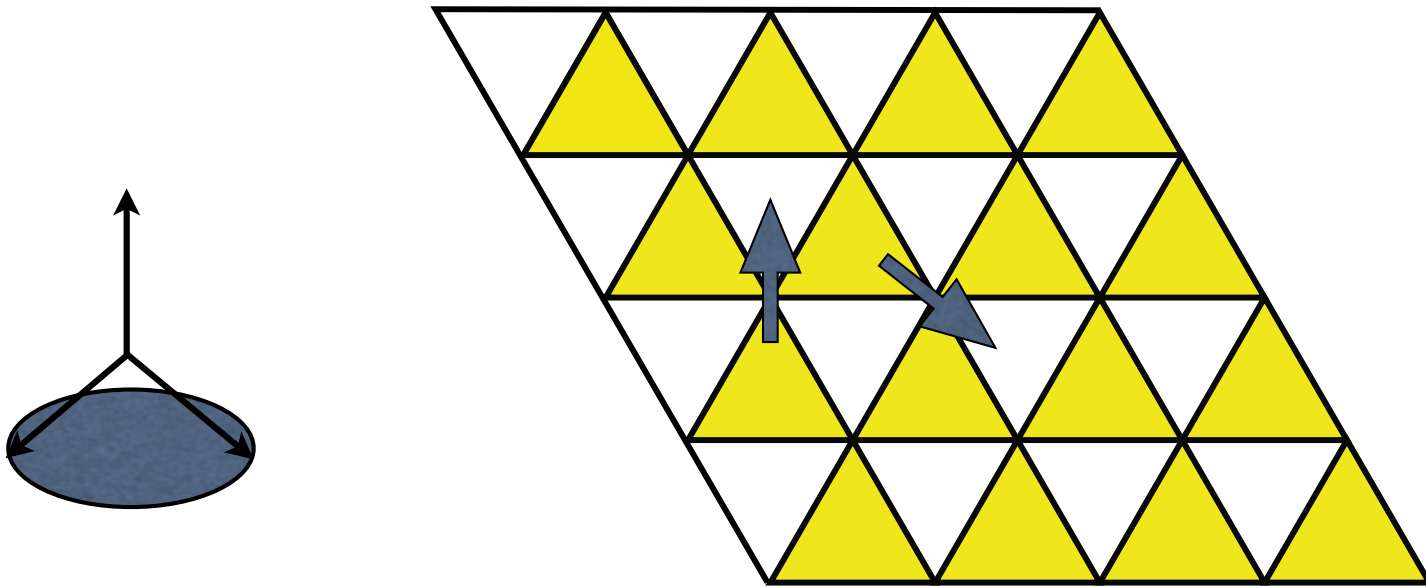


Triangular lattice



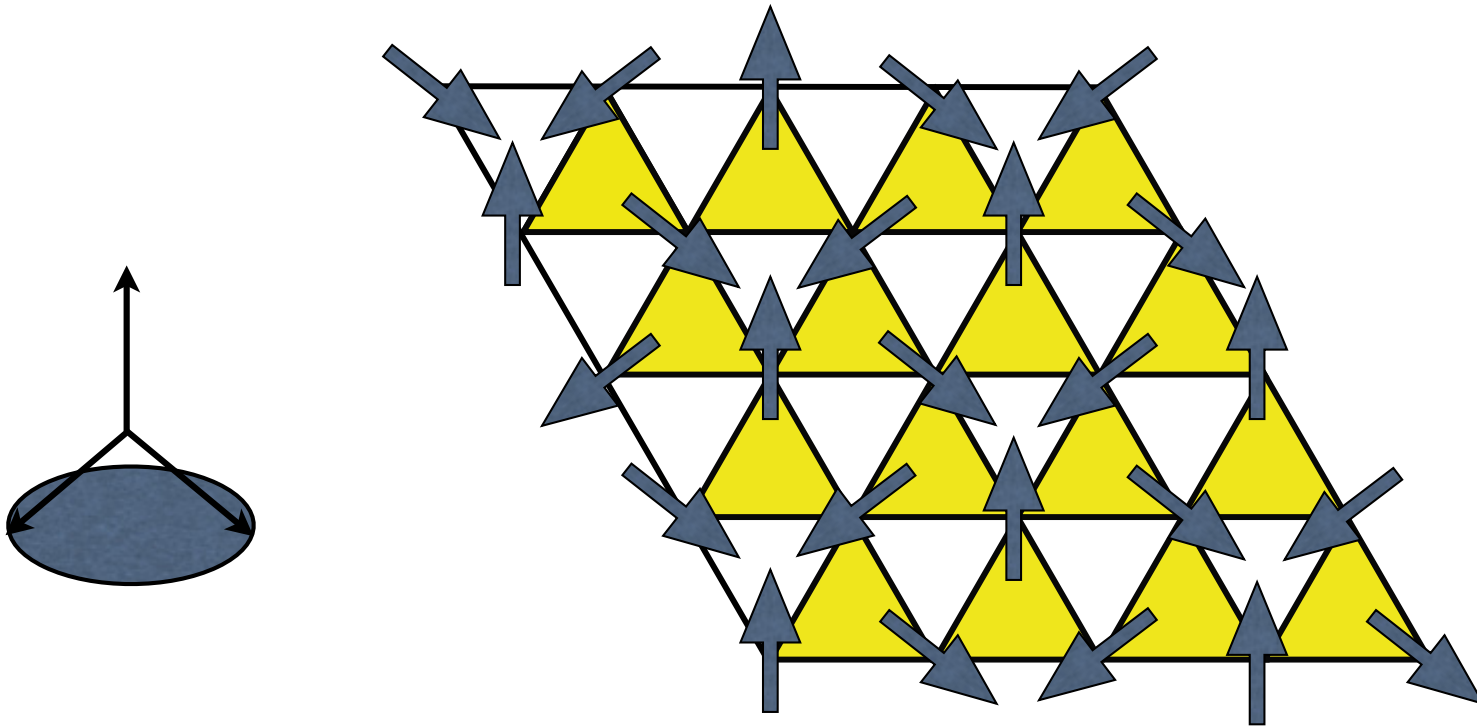
Degrees of freedom: 2

Triangular lattice



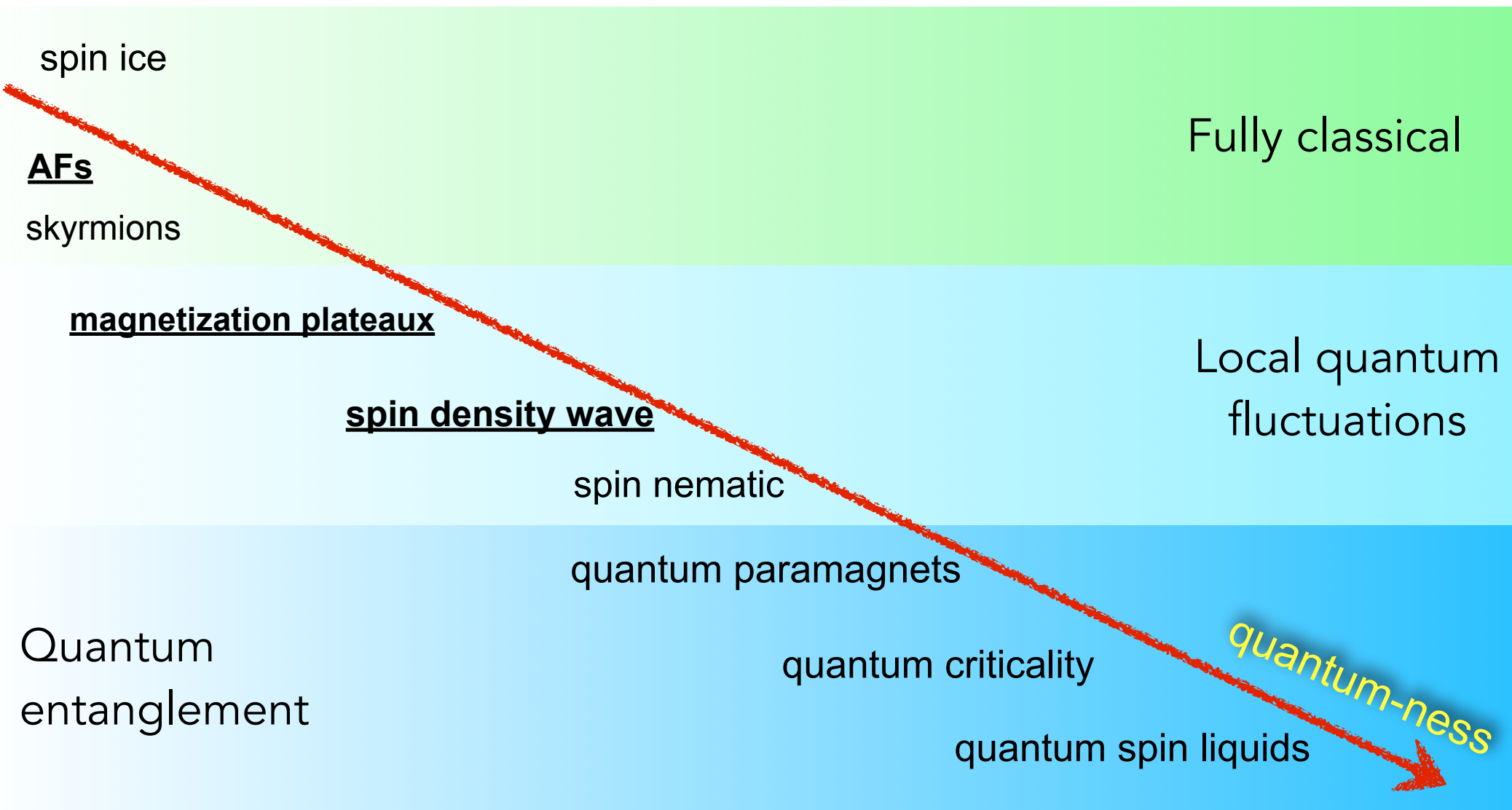
Degrees of freedom: $2+1$

Triangular lattice

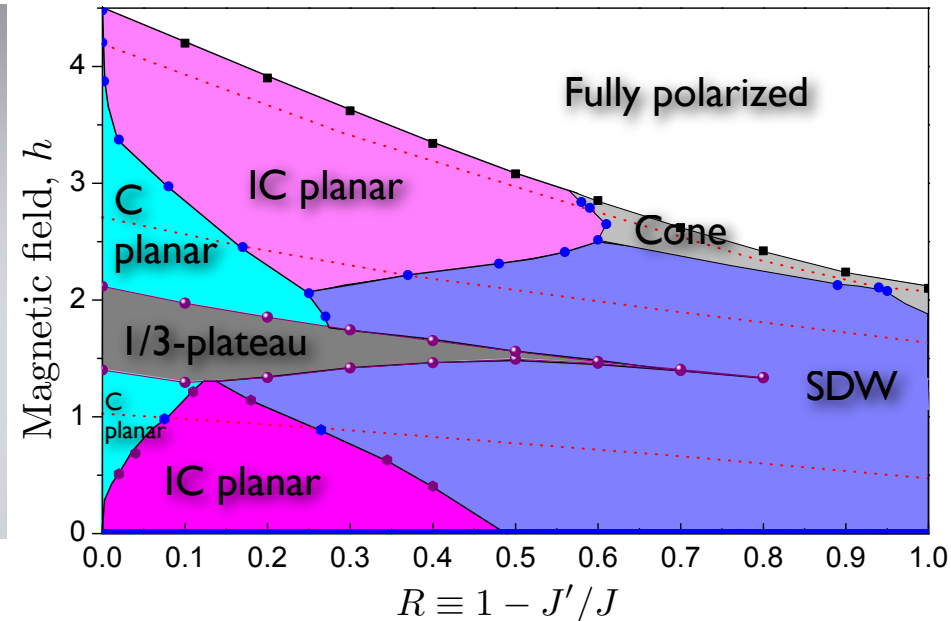
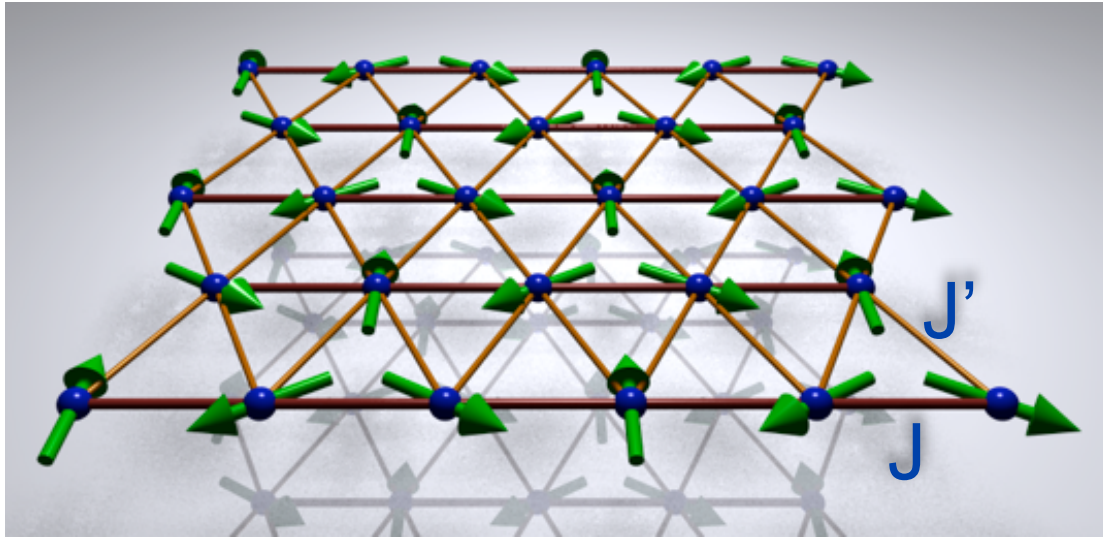


Degrees of freedom: $2+1$

Frustrated Magnetism

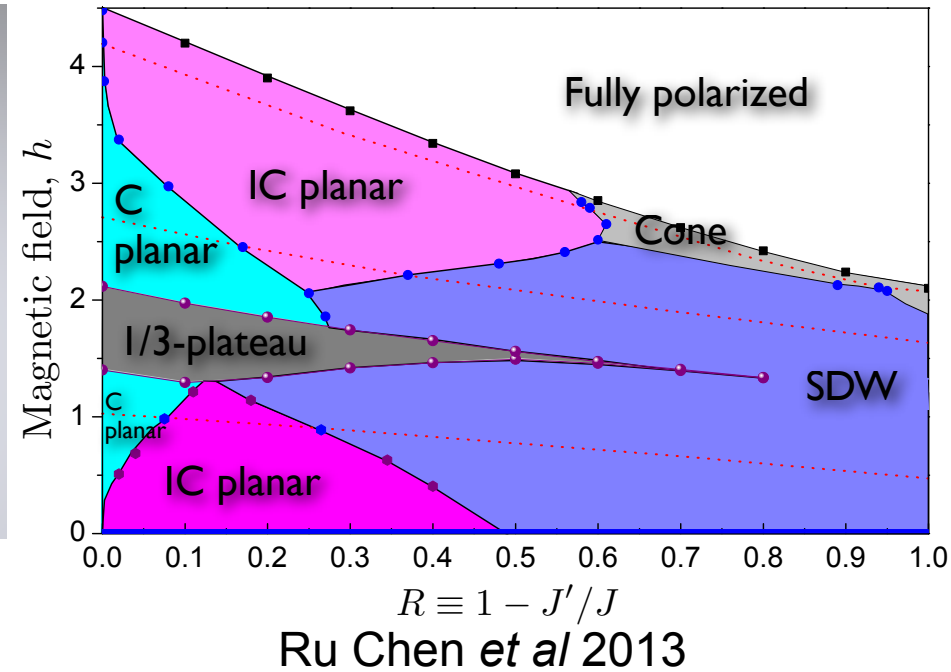
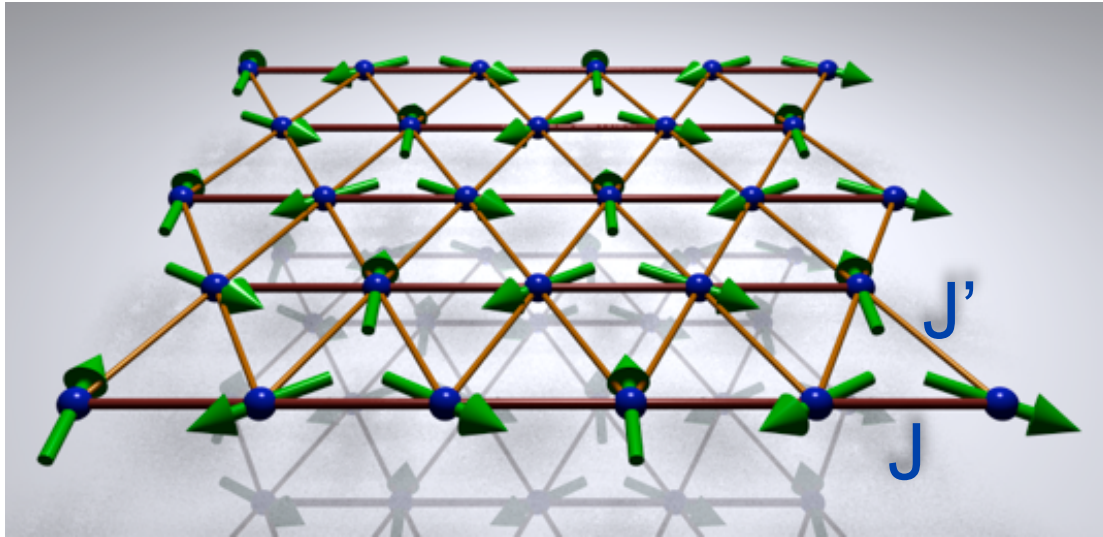


$S=1/2$ Triangular lattice



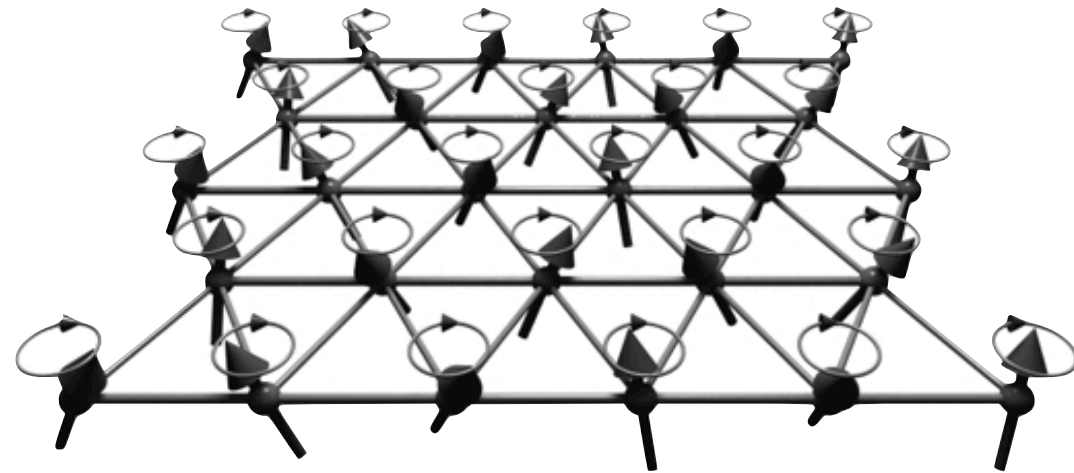
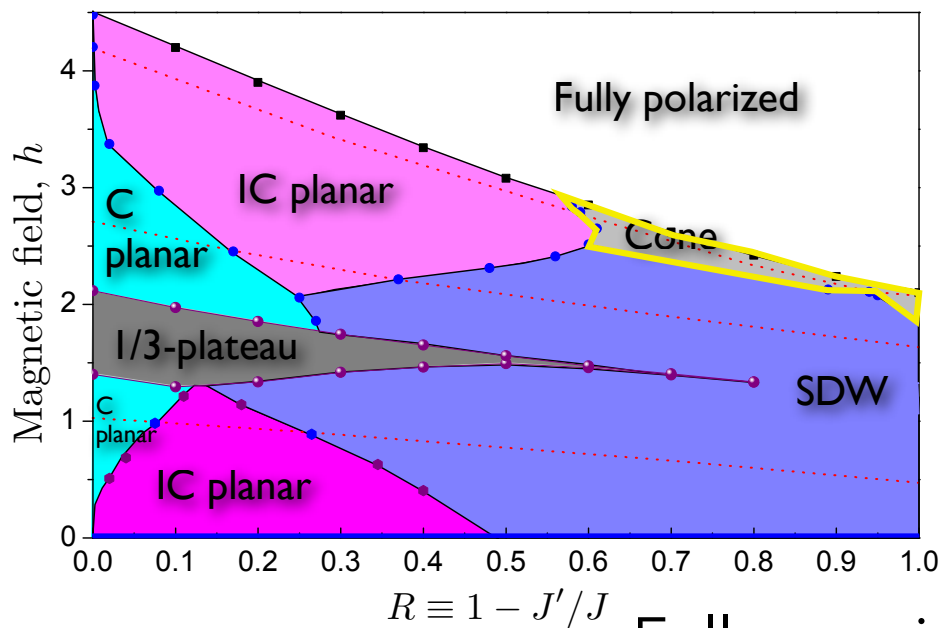
- Nice example with strong quantum renormalizations
 - All phases encountered are ordered, short-range entangled states
 - BUT most are different from those of the classical model
 - And excitations are highly renormalized from linear spin waves

$S=1/2$ Triangular lattice



Review: O. Starykh, RPP, 2015

Cone state

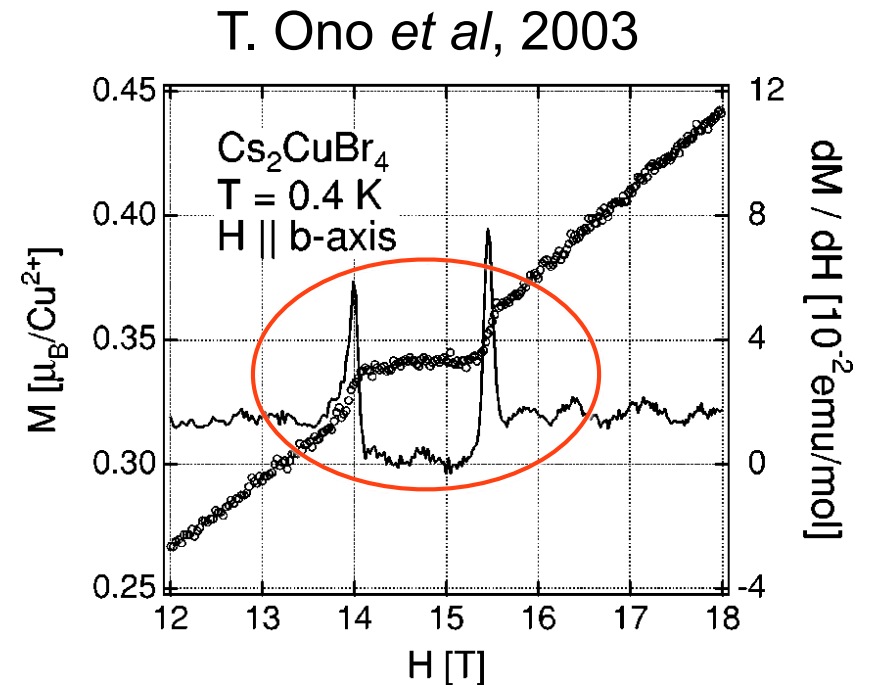
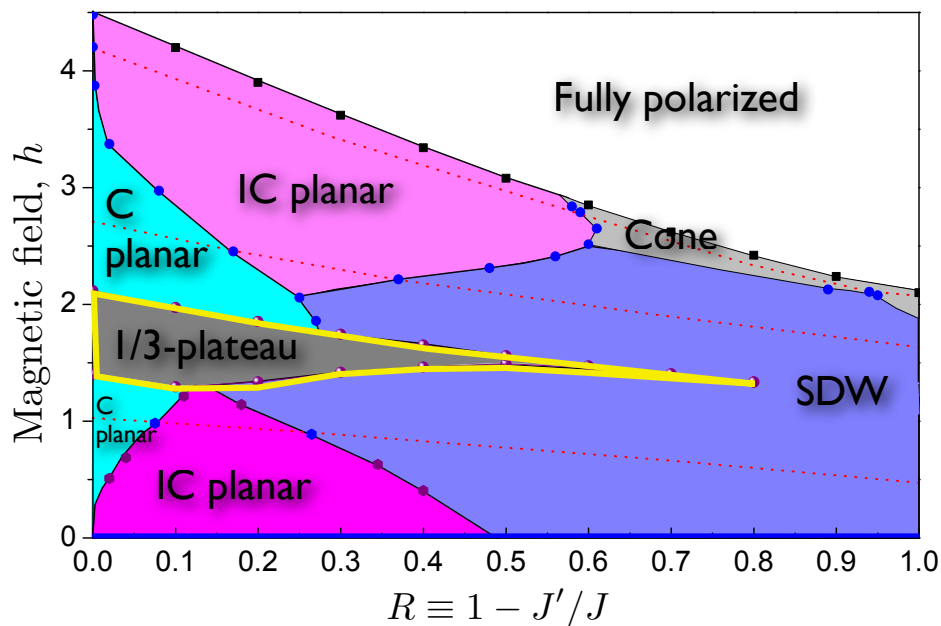


Fully consistent with rigid spins

This is the classical ground state *throughout* the phase space

Excitations are gapless spin waves - semiclassical quantization of small oscillations of spins

Magnetization plateau

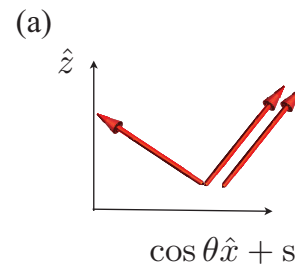
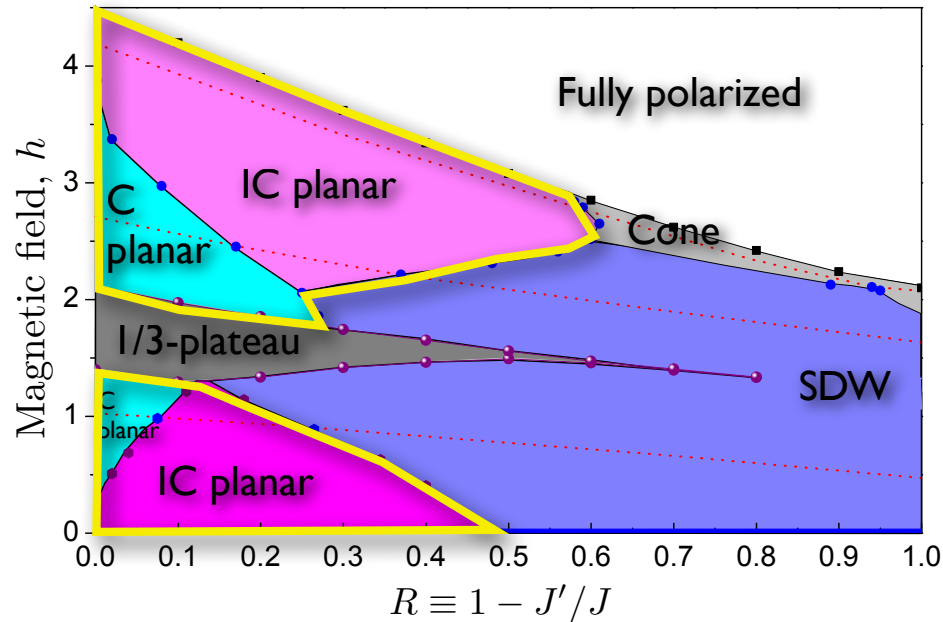


Spin gap stabilized by quantum zero point fluctuations

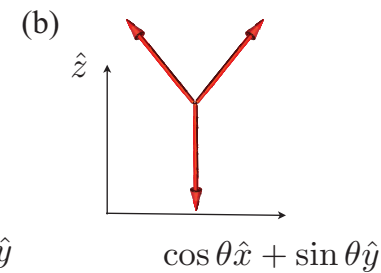
excitations are still spin waves but not Goldstone modes

c.f. Chubukov + Golosov, 91

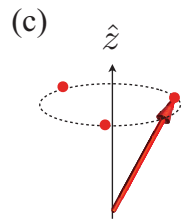
Planar orders



planar/fan

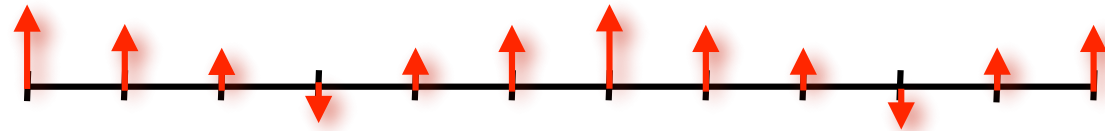
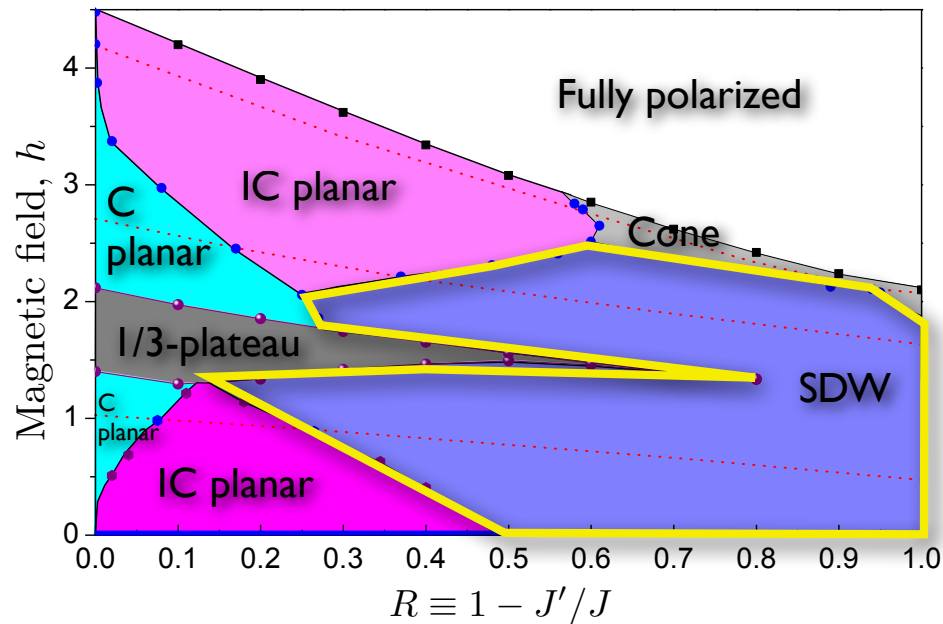


cone/umbrella



Classical ground state is *always* umbrella-like, but quantum fluctuations almost completely remove this

SDW

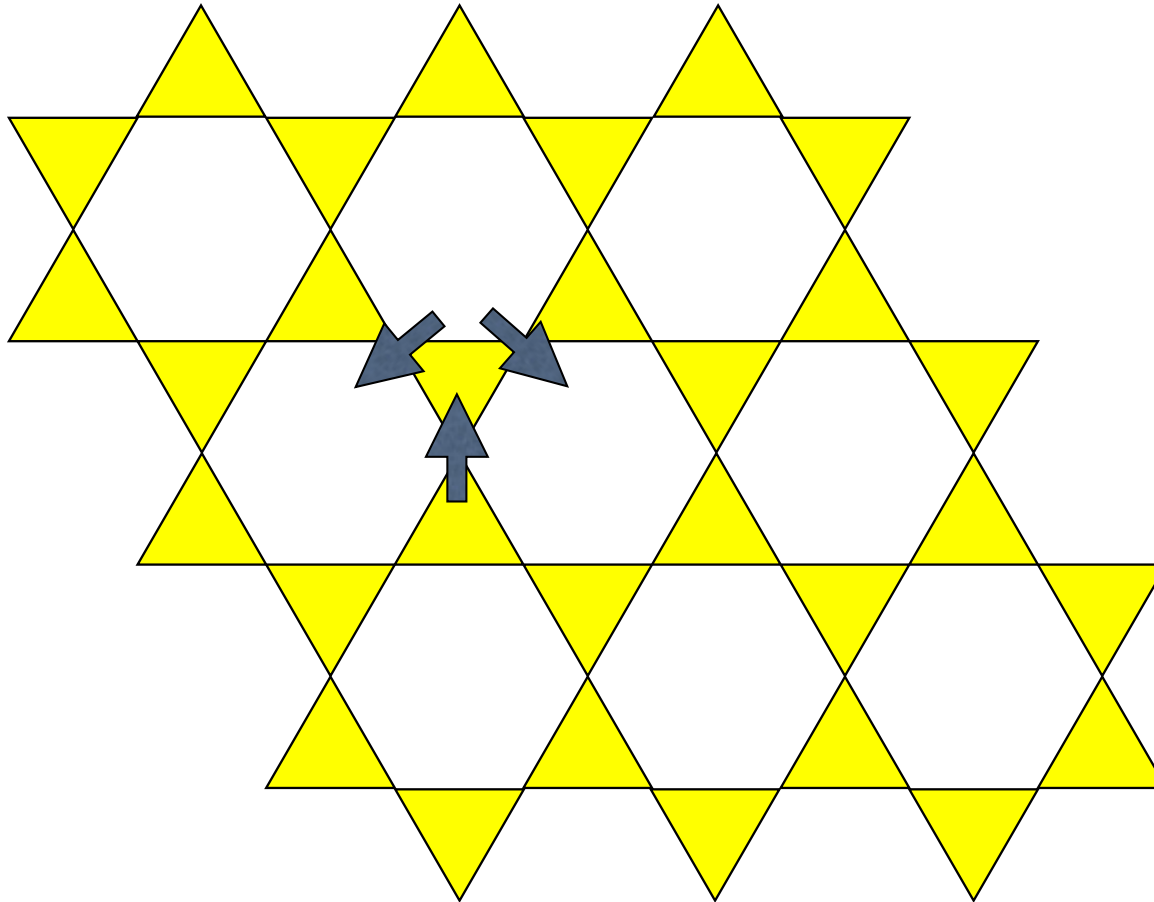


Non-uniform spin lengths
➡ non-classical!

SDW states can be considered soliton lattices, and can be understood based on the behavior of spin chains

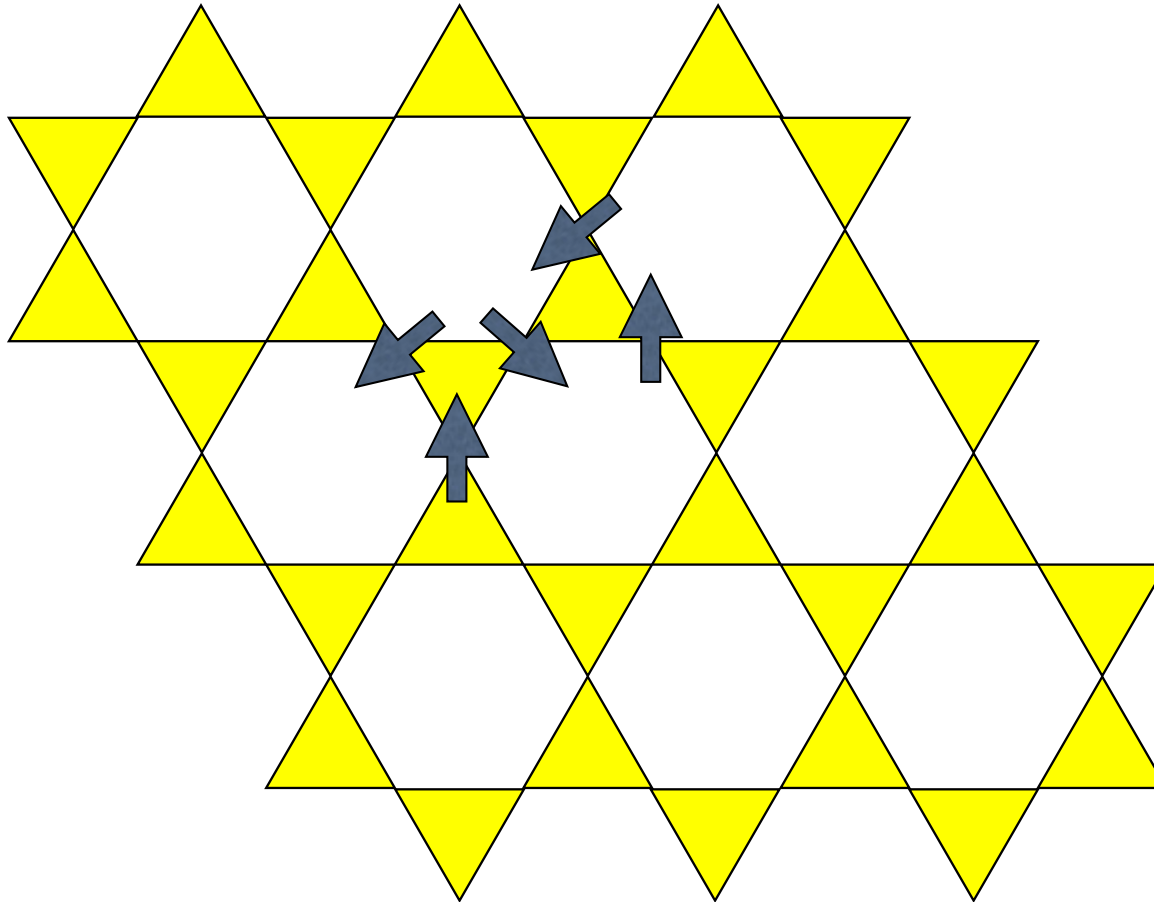
Large domain of SDW state means that quasi-1d nature is enhanced by quantum fluctuations

Kagome lattice



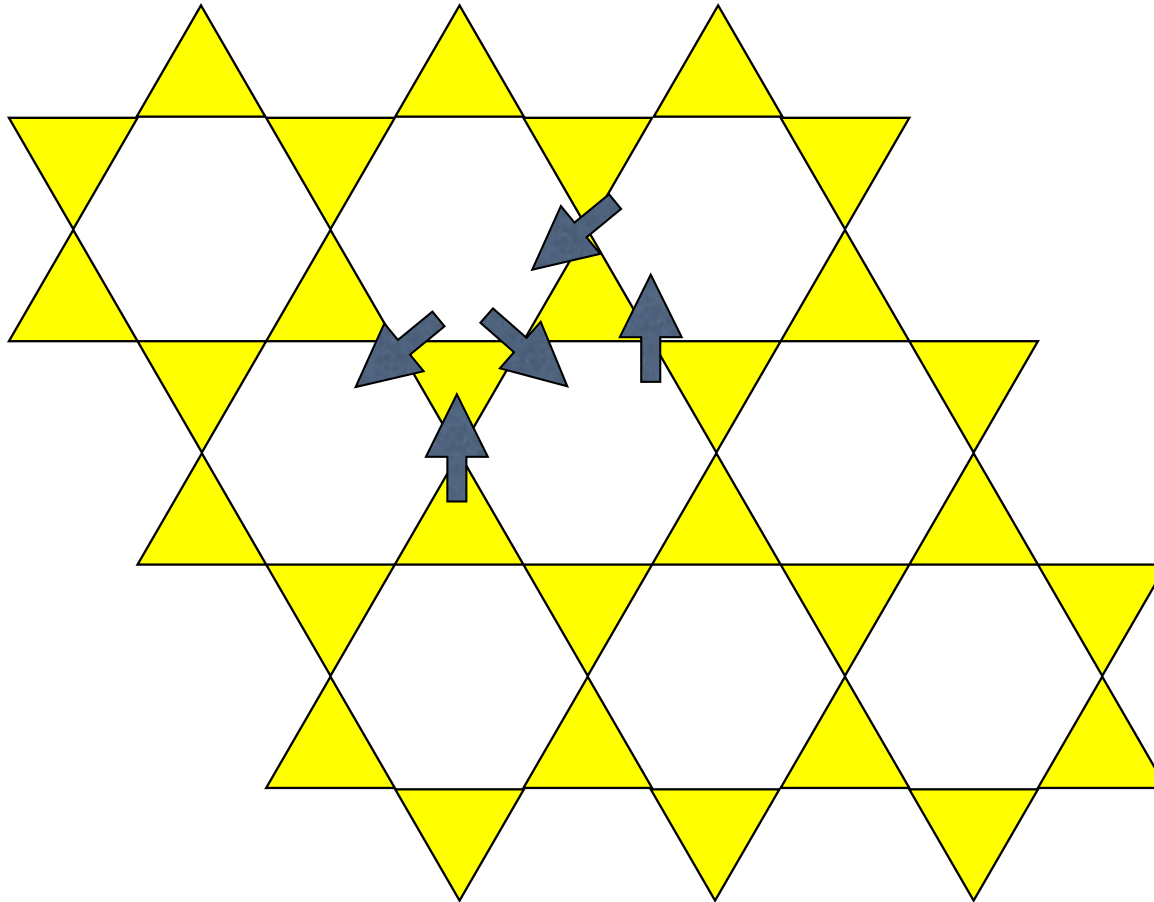
Degrees of freedom: 3

Kagome lattice



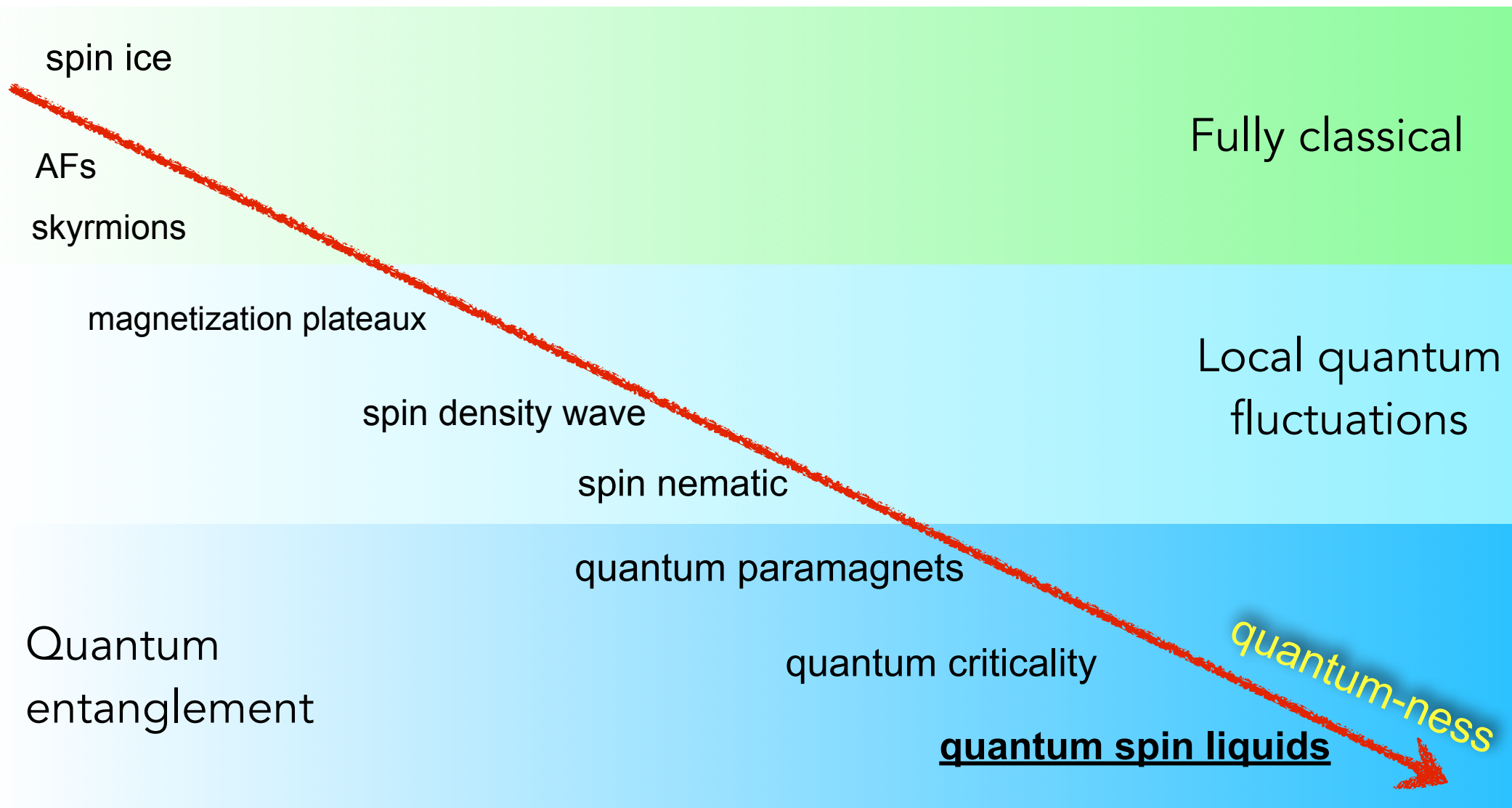
Degrees of freedom: $3+1+\dots$

Kagome lattice



- Degrees of freedom: $\sim N$. Much less likely to order.

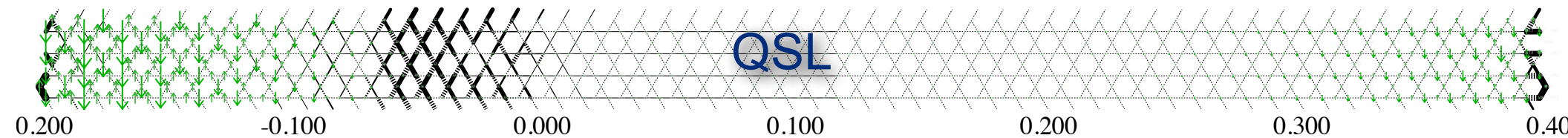
Frustrated Magnetism



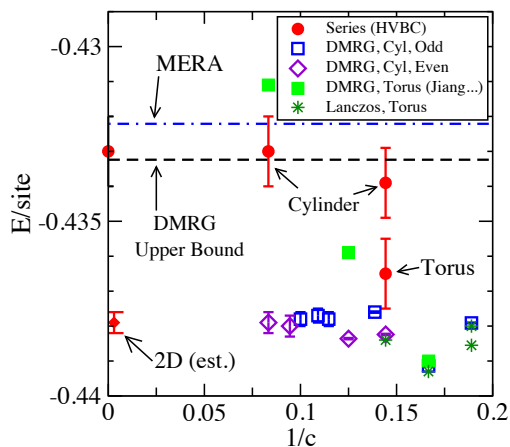
$S=1/2$ kagomé AF

- DMRG calculations give overwhelming evidence for QSL ground state

© Steve White



S. Yan *et al*, 2010



Theorists are still debating the nature of the QSL state.
Experimentalists are also debating the meaning of their observations.

Many kinds of QSLs

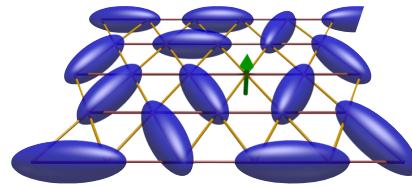
$$\Psi = \begin{array}{c} \uparrow \\ \# \end{array} \begin{array}{c} \text{Diagram 1} \end{array} + \begin{array}{c} \uparrow \\ \#' \end{array} \begin{array}{c} \text{Diagram 2} \end{array} + \dots$$

For ~ 500 spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.

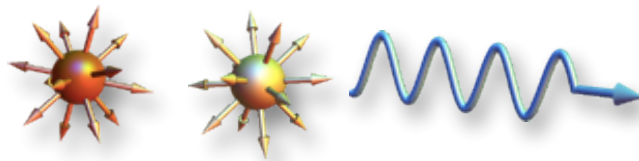
Classes of QSLs

- Topological QSLs



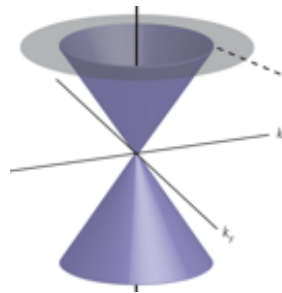
anyonic
spinons

- $U(1)$ QSL



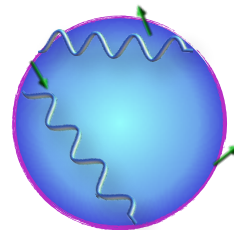
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

QSLs @ HFM2022

- **Invited - L. Clark** (University of Birmingham, UK)
"Unravelling Complexity in the Barlowite Family of $S=1/2$ Kagome Antiferromagnets" ↓
- M. Georgopoulou (Institut Laue Langevin, France & University College London, UK)
"Zn-claringbullite, $\text{ZnCu}_3(\text{OD})_6\text{FCl}$: a new quantum spin liquid candidate" ↓
- Q. Bartélemy (Université Paris-Saclay, France)
"Specific heat of the kagome antiferromagnet herbertsmithite in high magnetic fields" ↓
- G. Chen (University of Hong-Kong, China)
"Thermal Hall effects in spin liquids" ↓

- **Invited - P. Armitage** (John Hopkins University, USA)
"Recent results on Kitaev interactions in Co based magnets" ↓
- N.B. Perkins (University of Minnesota, USA)
"Footprints of the Kitaev spin liquid in the Fano lineshape of the Raman active optical phonons" ↓
- Y.-J. Kao (National Taiwan University, Taiwan)
"Excitation spectrum of spin-1 Kitaev spin liquids" ↓
- M. Udagawa (Gakushuin University, Japan)
"Manipulation of non-Abelian anyons in Kitaev's magnet" ↓

- **Invited - J. Nasu** (Tohoku University, Japan)
"Nonequilibrium dynamics and spin transport caused by fractional quasiparticles in Kitaev spin liquids" ↓
- S.C. Furuya (Ibaraki University, Japan & University of Tokyo, Japan)
"DC electric-field controls of Kitaev spin liquids and topological spin textures" ↓
- E. Lefrançois (Université de Sherbrooke, Canada)
"Evidence of a Phonon Hall Effect in the Kitaev Spin Liquid Candidate $\alpha\text{-RuCl}_3$ " ↓
- J. Bruin (Max Planck Institute for Solid State Research, Germany)
"Robustness of the thermal Hall effect close to half-quantization in $\alpha\text{-RuCl}_3$ " ↓

- F. Gruber (University of Augsburg, Germany)
"Comparative study of the triangular spin-liquid candidates NaYbO_2 , KYbO_2 and KYbS_2 " ↓
- S.H.Y. Huang (McMaster University, Canada)
"Dynamic and frozen quantum magnetism in the ground states of triangular lattice magnets ErMgGaO_4 and YbMgGaO_4 from inelastic neutron scattering" ↓
- S. Niu (CNRS & Université de Toulouse, France)
"A variational tensor network study of magnetic properties of Heisenberg model with Dzyaloshinskii-Moriya interaction on kagome lattice" ↓

- R.P. Nutakki (University of Munich, Germany & University of Augsburg, Germany)
"Proximate Spin Liquids in Metal-Azolate Frameworks" ↓

- L. Vanderstraeten (University of Ghent, Belgium)
"Tensor networks and the spectral function of 2-D quantum spin liquids" ↓

- S. Zhang (Max Planck Institute for the Physics of Complex Systems, Germany)
"Modeling a three-dimensional $S = 1$ spin liquid $\text{NaCaNi}_2\text{F}_7$ " ↓

4h-15h30 : Pyrochlore session

- **Invited - E. Smith** (McMaster University, Canada)
"The case for a $U(1)_\rho$ Quantum Spin Liquid Ground State in the Dipole-Octupole Pyrochlore $\text{Ce}_2\text{Zr}_2\text{O}_7$ " ↓
- Y.B. Kim (University of Toronto, Canada)
"Competing dipolar-octupolar quantum spin liquids on the pyrochlore lattice" ↓
- R. Sibille (Paul Scherrer Institut, Switzerland)
"Octupolar correlations and spinon spectrum in $\text{Ce}_2\text{Sn}_2\text{O}_7$ quantum spin ice" ↓

- **Invited - F. Pratt** (ISISs Neutron and Muon Source, UK)
"Probing triangular-lattice quantum spin liquids with $\text{LF-}\mu\text{SR}$ " ↓

Frustrated Magnetism

spin ice

This is all about the equilibrium phase/ground state.

We can also talk about excitations and response

spin density wave

fluctuations

spin nematic

quantum paramagnets

Quantum
entanglement

quantum criticality

quantum spin liquids

quantum-ness



Excitations in the usual case

Hamiltonian

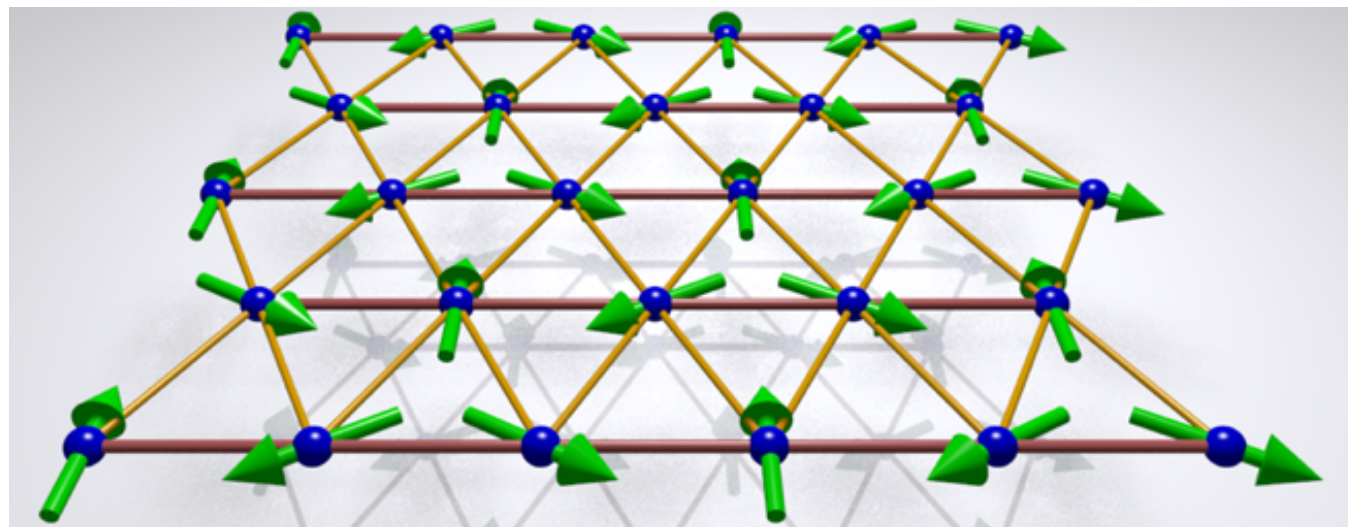
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-range: local

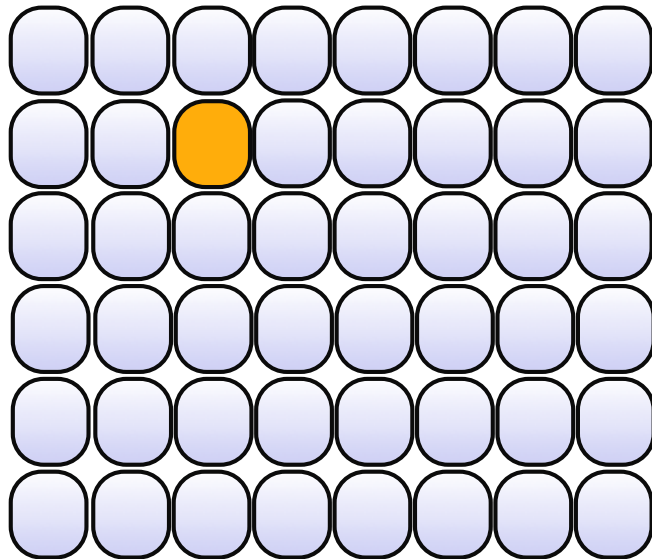
Wave
function

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{n}_i = +S\rangle$$

Product state



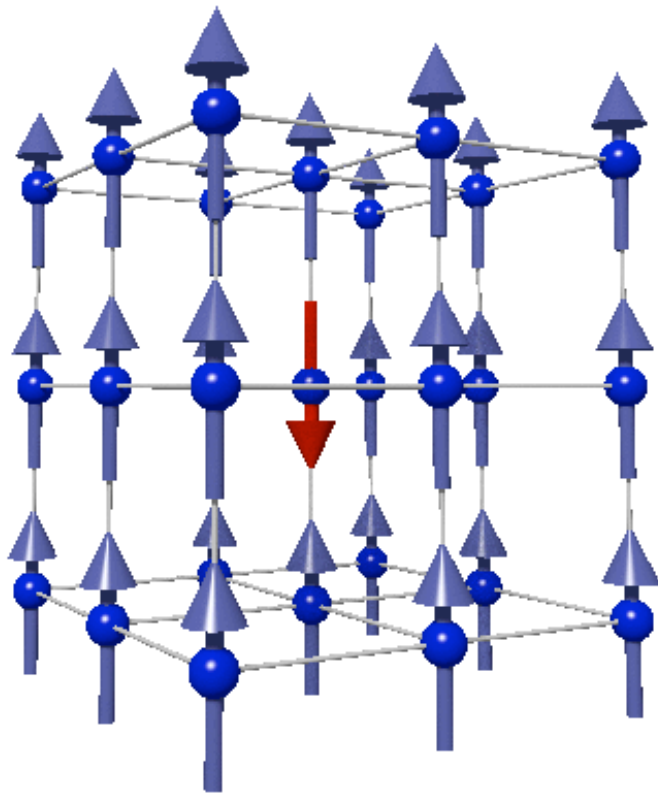
Quasiparticles



excited states \sim excited levels of one block

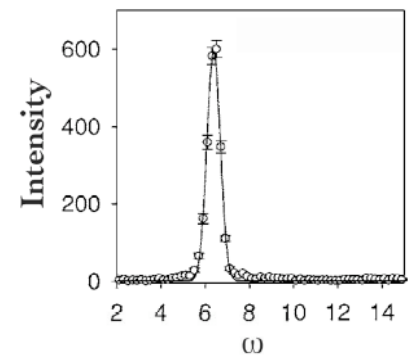
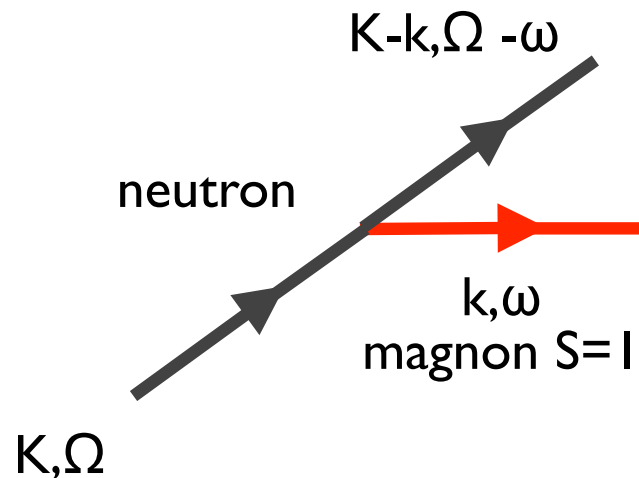
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

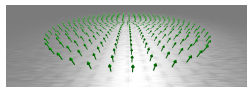
$$|f\rangle = S_k^+ |i\rangle$$



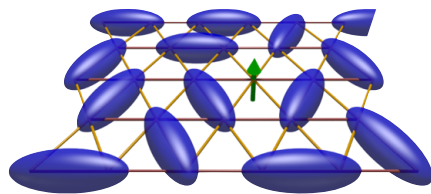
Line shape in Rb_2MnF_4

Emergent excitations

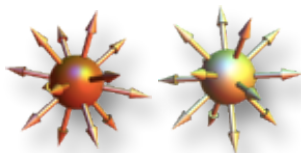
- Emergent excitations may be very different from spin flips
- May be created in multiples, or very hard to create at all with a neutron, or just have different properties



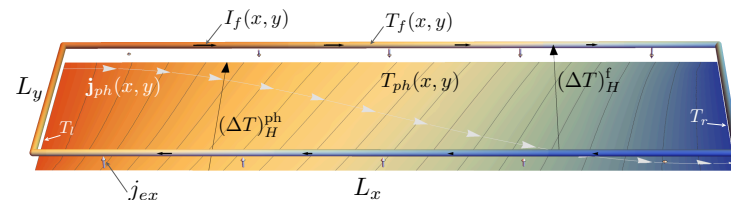
Skyrmion



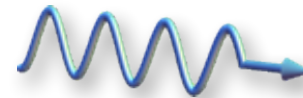
Spinon



Monopoles



Majorana

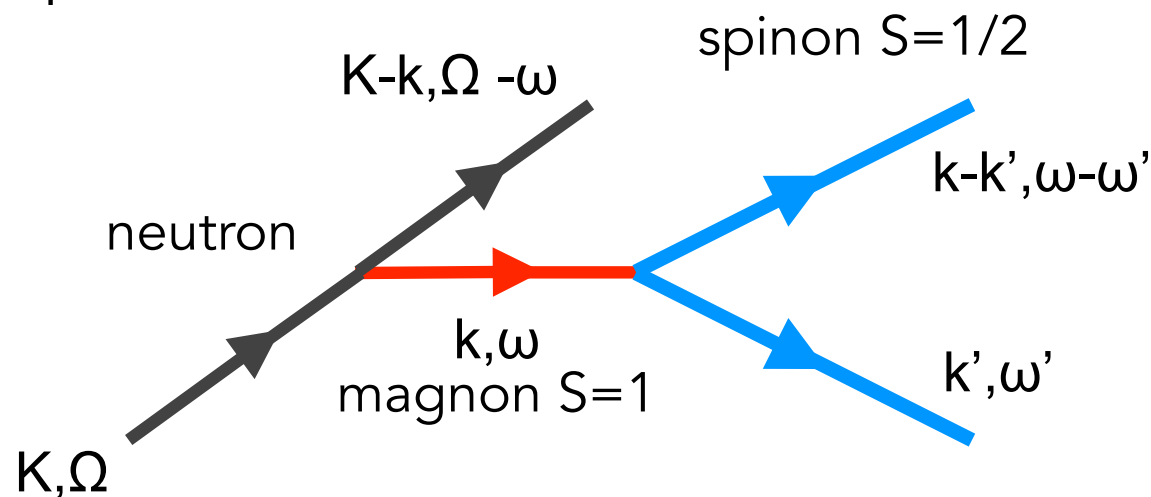


Emergent photon

Emergent excitations

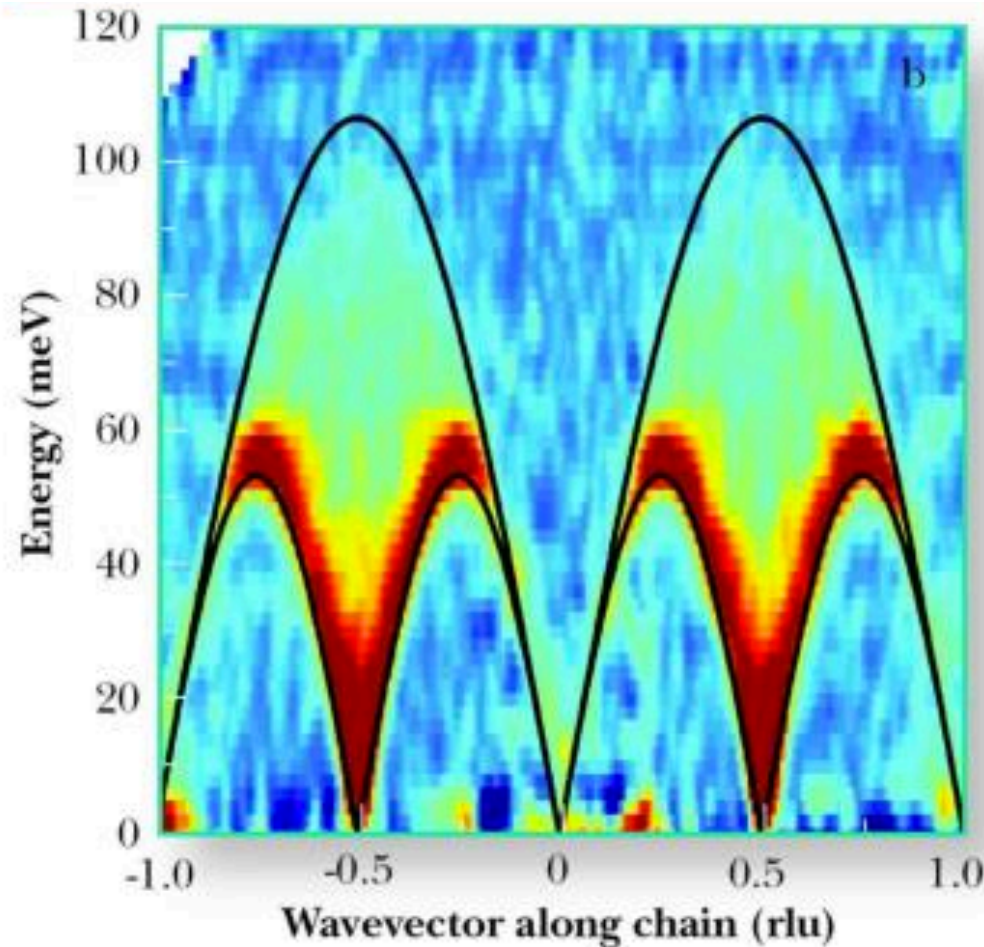
- Emergent excitations may be very different from spin flips
- May be created in multiples, or very hard to create at all with a neutron, or just have different properties

e.g. spinons



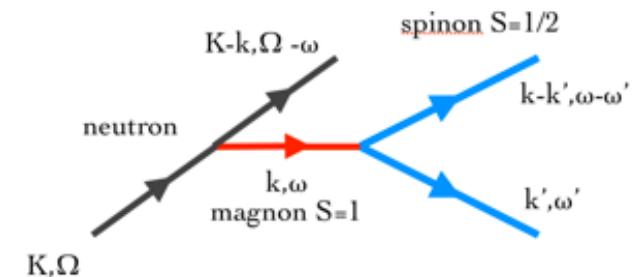
broad peak with
 $\omega = \varepsilon(k') + \varepsilon(k-k')$

c.f. One dimension



A. Tennant *et al*, 2001

KCuF_3



Understanding in $d > 1$ is much more limited

A rough guide to experiments on HFMs

Does it order?

- NMR line splitting
- μ SR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/x-ray

Delocalized excitations?

- thermal conductivity
- INS

Is there a gap?

- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of χ

Structure of excitations?

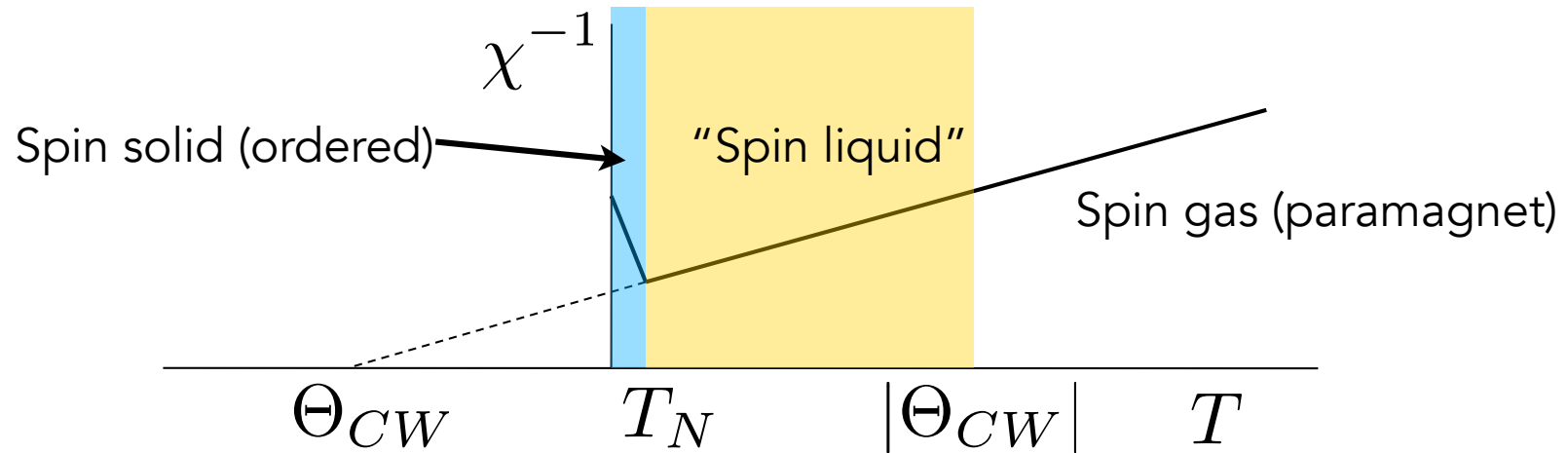
- $E(k)$ from INS, RIXS
- optics, Raman

Exotica

- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects



Ramirez Plot



- Local moments: Curie-Weiss law at high T

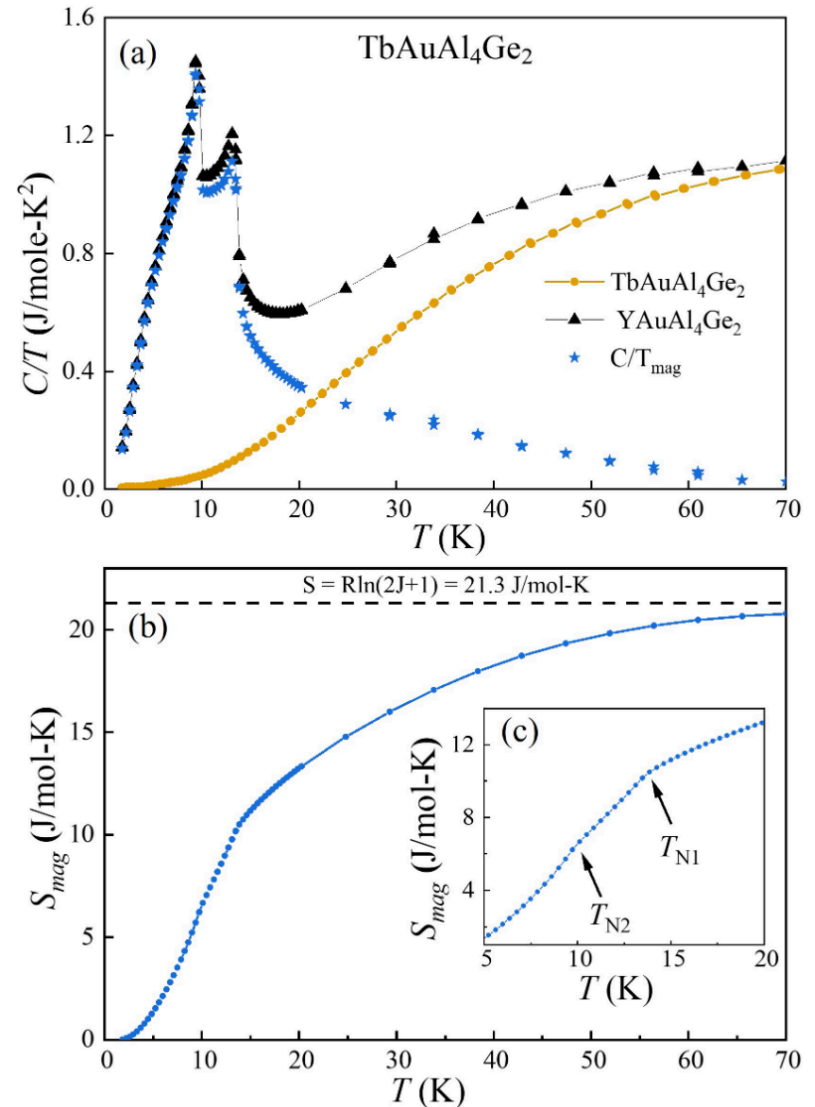
$$\chi \sim \frac{A}{T - \Theta_{CW}}$$

- Frustration parameter: $f = |\Theta_{CW}|/T_N$
- Larger $f \gg 1$ is more frustrated (or fluctuating)

Heat capacity

- Sensitive indicator of phase transitions
- Useful to assess entropy, e.g. confirm effective spin

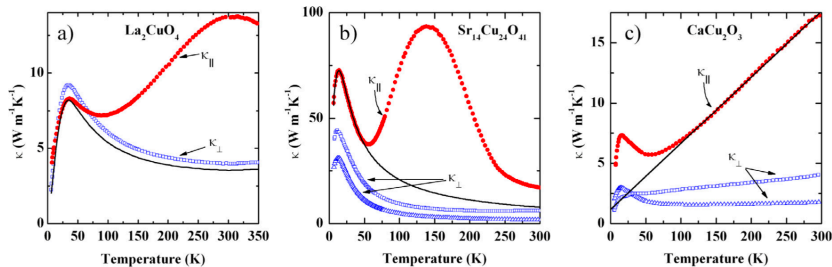
$$S(T) = \int_0^T dT' \frac{C(T')}{T'}$$



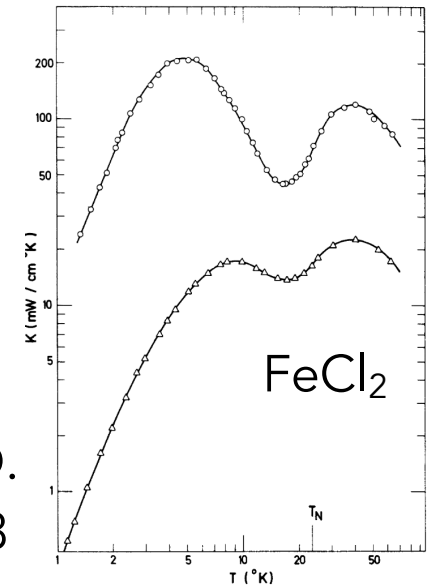
Thermal conductivity

Spins carry
heat

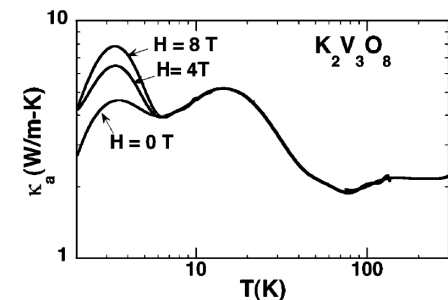
Phonons carry heat
but interact with spins



Review: C. Hess, 2019



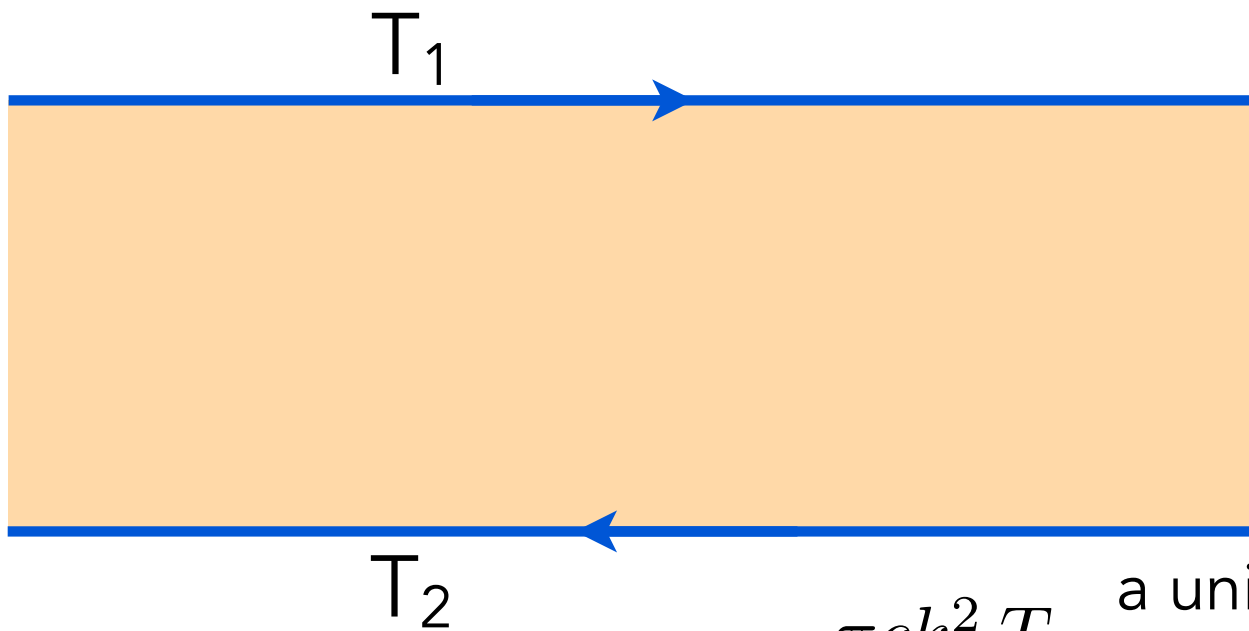
G. Laurence et D.
Petitgrand, 1973



B.C. Sales *et al*, 2002

Thermal Hall effect

- Motivation: electronic/spin contribution theoretically closed tied to topology



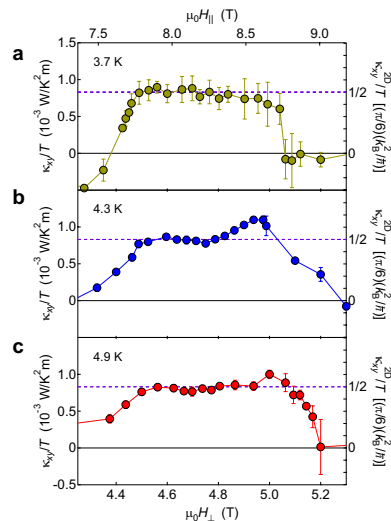
$$I_x = \kappa_H \Delta T_y$$

$$\kappa_H = \frac{\pi c k_B^2 T}{6\hbar}$$

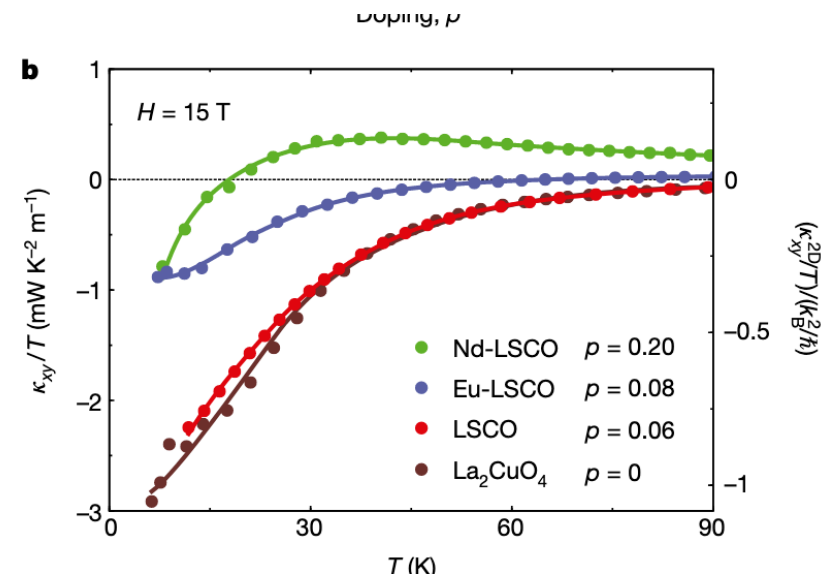
a universal prediction for chiral
"Ising anyon" phase: *agnostic to
microscopic spin interactions*

Thermal *Hall* conductivity

- Experimental situation very much under debate - electronic versus lattice transport, impurity versus intrinsic, Berry curvature versus scattering,...



Y. Kasahara *et al*, 2018



G. Grissonanche *et al*, 2019

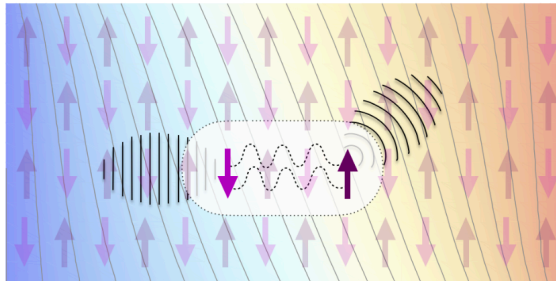
Thermal *Hall* conductivity

- Advertisement for some theory work

	zero field	$\hbar = \hbar\hat{z}$	
		lattice and spin	lattice effective
paramagnet	$G = P4/mmm1'$	$G(\mathbf{0}, \hbar\hat{z}) = P4/mm'm'$	$G^{\text{eff}}(\mathbf{0}, \hbar\hat{z}) = 4/mm'm'$
high sym. AFM	$G(\hat{x}, \mathbf{0}) = \langle i, TX, TY, C_{2x}, TC_{2z} \rangle$	$G(\hat{x}, \hbar\hat{z}) = \langle i, XY, TC_{2y}, XC_{2z} \rangle$	$G^{\text{eff}}(\hat{x}, \hbar\hat{z}) = \langle i, TC_{2y}, C_{2z} \rangle$
low sym. AFM	$G(\hat{e}, \mathbf{0}) = \langle i, TX, TY, TC_{2z} \rangle$	$G(\hat{e}, \hbar\hat{z}) = \langle i, XY, XC_{2z} \rangle$	$G^{\text{eff}}(\hat{e}, \hbar\hat{z}) = \langle i, C_{2z} \rangle$

arXiv:2103.04223

w/ Mengxing Ye+Lucile Savary



arXiv:2206.06183

arXiv:2202.10366

Poster W2 (Wed afternoon)

w/ **Léo Mangeolle**+Lucile Savary

$$\kappa_H^{\mu\nu} = \frac{\hbar^2}{k_B T^2} \frac{1}{V} \sum_{n\mathbf{k}n'\mathbf{k}'} J_{n\mathbf{k}}^\mu \frac{e^{\beta\hbar\omega_{n\mathbf{k}}/2}}{2D_{n\mathbf{k}}} \left(\frac{1}{N_{\text{uc}}} \sum_{q=\pm} \frac{(e^{\beta\hbar\omega_{n\mathbf{k}}} - e^{q\beta\hbar\omega_{n'\mathbf{k}'}})}{\sinh(\beta\hbar\omega_{n\mathbf{k}}/2) \sinh(\beta\hbar\omega_{n'\mathbf{k}'}/2)} \mathfrak{M}_{n\mathbf{k},n'\mathbf{k}'}^{\ominus,+q} \right) \frac{e^{\beta\hbar\omega_{n'\mathbf{k}'}/2}}{2D_{n'\mathbf{k}'}} J_{n'\mathbf{k}'}^\nu, \quad (9)$$

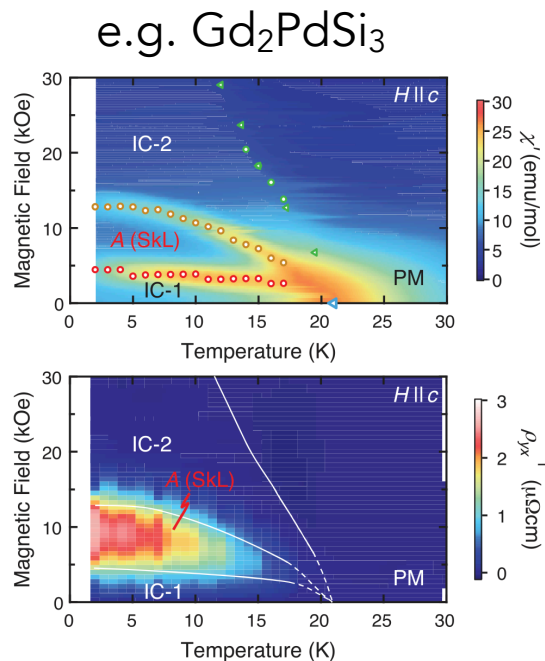
where

$$\mathfrak{M}_{n\mathbf{k}n'\mathbf{k}'}^{\ominus,q,q'} = \frac{2N_{\text{uc}}}{\hbar^4} \Re \int_{t_1, t_2} e^{i[\Sigma_{n\mathbf{k}n'\mathbf{k}'}^q(t_1+t_2) + \Delta_{n\mathbf{k}n'\mathbf{k}'}^{q,q'}(t_1+t_2)]} \text{sign}(t_2) \langle [Q_{n\mathbf{k}}^{-q}(-t-t_2), Q_{n'\mathbf{k}'}^{-q'}(-t+t_2)] \{Q_{n'\mathbf{k}'}^{q'}(-t_1), Q_{n\mathbf{k}}^q(t_1)\} \rangle, \quad (10)$$

Electrical conductivity

- Electron dynamics modified by magnet order

e.g. Hall conductivity due to skyrmions



Friday, June 24th

9h-10h30 : Skyrmions & multi-Q phases session

- **Invited - O. Zaharko** (Paul Scherrer Institut, Switzerland)

"Spin textures in frustrated magnetic materials" ↓

- K. Shimizu (University of Tokyo, Japan)

"Spin moiré engineering of topology and emergent electromagnetic fields in multiple-Q spin textures"

↓

- P. Pujol (CNRS & University of Toulouse, France)

"A skyrmion fluid and bimeron glass emerging from a chiral spin liquid" ↓

- K. Penc (Wigner Research Centre for Physics, Hungary)

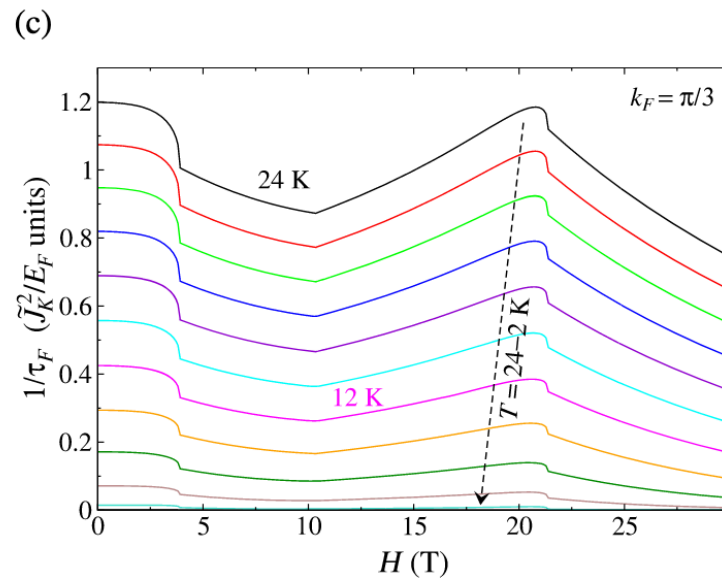
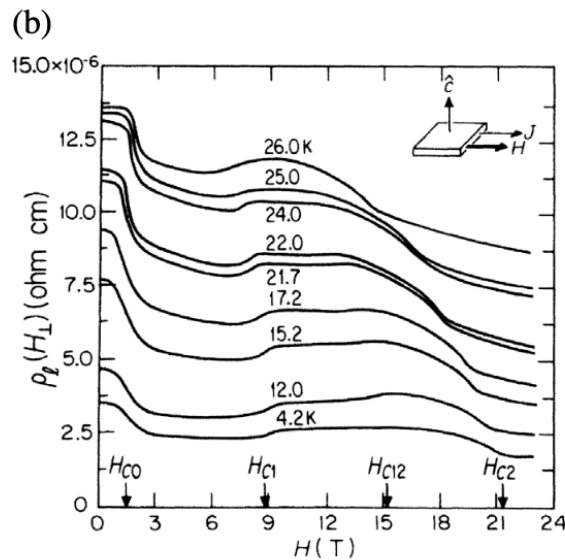
"Unified theory of the spiral spin-liquids on layered honeycomb, diamond, and fcc lattices" ↓

T. Kuramaji et al, 2019

Electrical conductivity

- Electrons scattered by magnetic excitations

e.g. "Roller coaster"



- A.L. Chernyshev (University of California, USA)

"Roller-Coaster in a Flatland: Magnetism of Eu-intercalated Graphite" ↓

More responses

- Diverse behaviors of HFMs demand a diverse set of probes:
 - NMR/muSR/ESR - **A. Zorko**
 - Scattering - neutron - **P. Deen** - Raman, X-ray
 - Optics, Kerr, Faraday, non-linear/ultrafast
 - Magnetostriction, ultrasound, ...

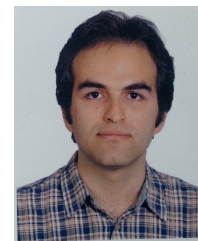
Moiré



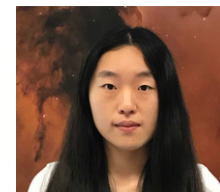
mohair

6°

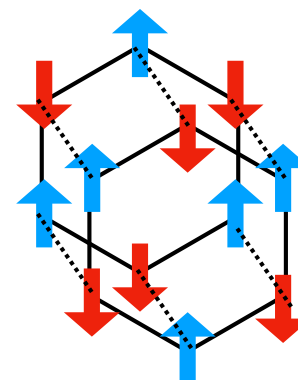
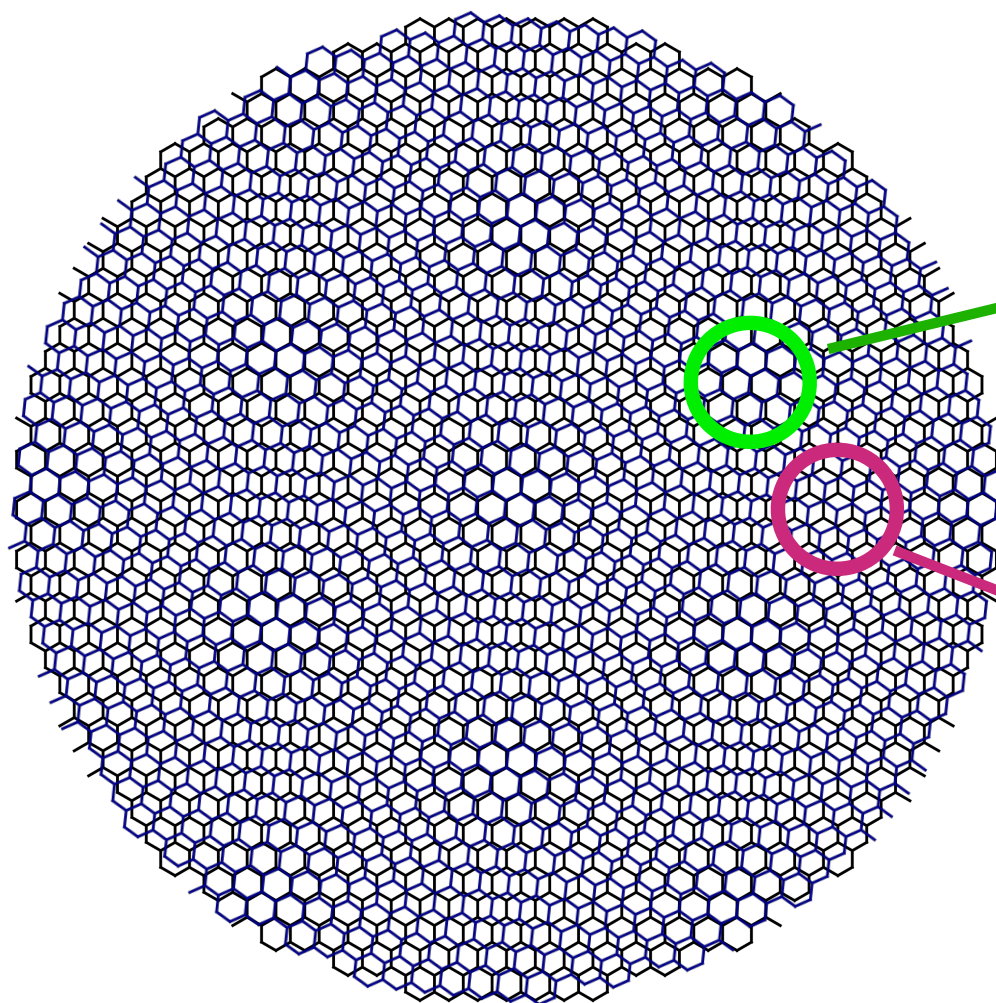
Twisted AF



Kasra Hejazi

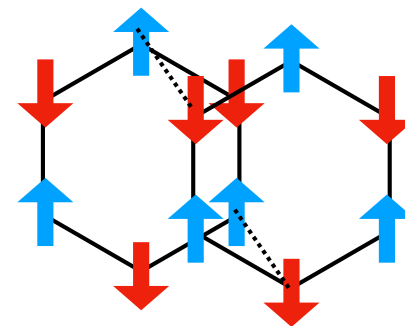


Zhu-Xi Luo



AA

$$N_1 = -N_2$$



AB

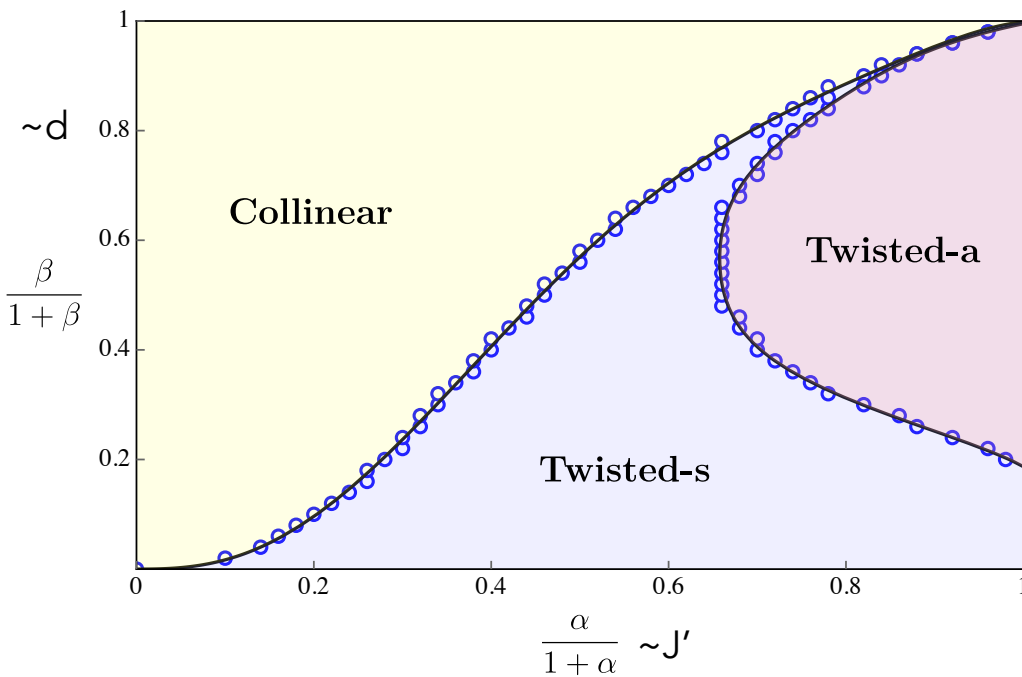
$$N_1 = N_2$$

Frustration: Néel vectors must rotate

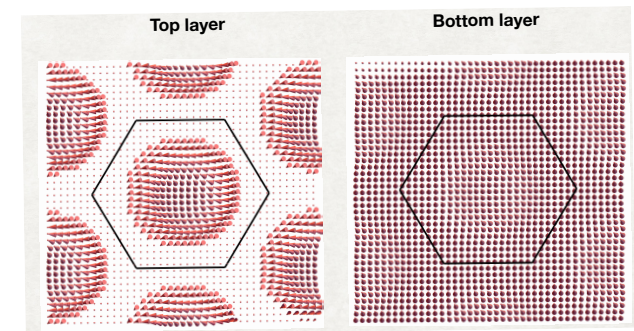
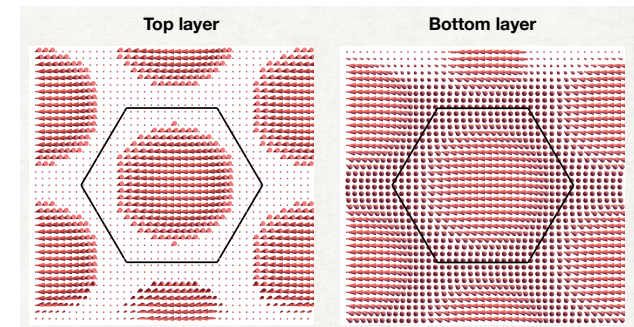
Twisted AF

$$\mathcal{H}_{\text{cl}} = \sum_l \left[\frac{\rho}{2} (\nabla \mathbf{N}_l)^2 - d (N_l^z)^2 \right] - J' \Phi(\mathbf{x}) \mathbf{N}_1 \cdot \mathbf{N}_2$$

Dimensionless parameter $\alpha = \frac{2J'}{\rho q_m^2} \sim \frac{J'}{J\theta^2}$



Coplanar spin textures



Transitions should be tunable by applied field



(A twisted *ferromagnet*)

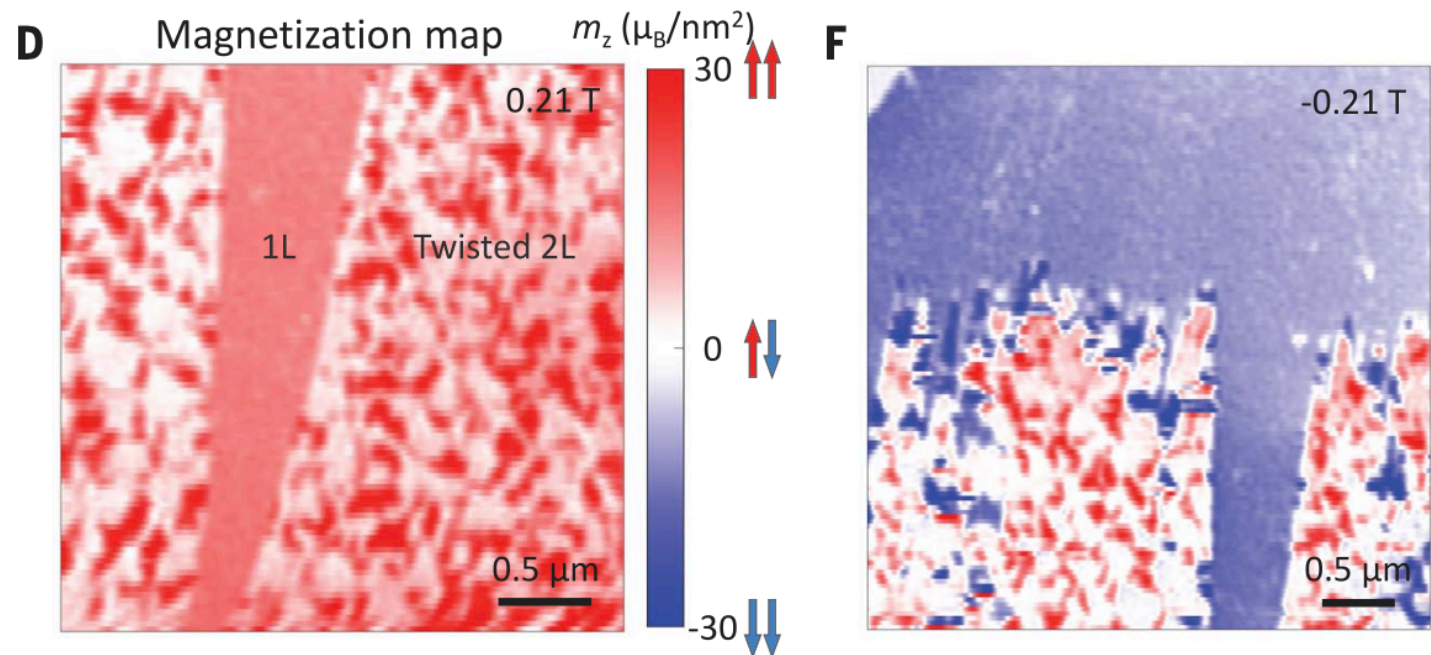
REPORT

MAGNETISM

Direct visualization of magnetic domains and moiré magnetism in twisted 2D magnets

Tiancheng Song^{1,†}, Qi-Chao Sun^{2,†}, Eric Anderson^{1,†}, Chong Wang³, Jimin Qian⁴, Takashi Taniguchi⁵, Kenji Watanabe⁶, Michael A. McGuire⁷, Rainer Stöhr^{2,8}, Di Xiao³, Ting Cao⁴, Jörg Wrachtrup^{2,9,*}, Xiaodong Xu^{1,4,*}

Scanning NV magnetometry



(twist disorder is evident)

Thank you

- Frustrated and quantum magnetism is an exciting place for theory and experiment to meet
- The basic point is frustration allows more unusual structures to emerge, be they atypical orders, unusual excitations, or unconventional responses
- We surely missed many things. That's why you need to go to the meeting!

