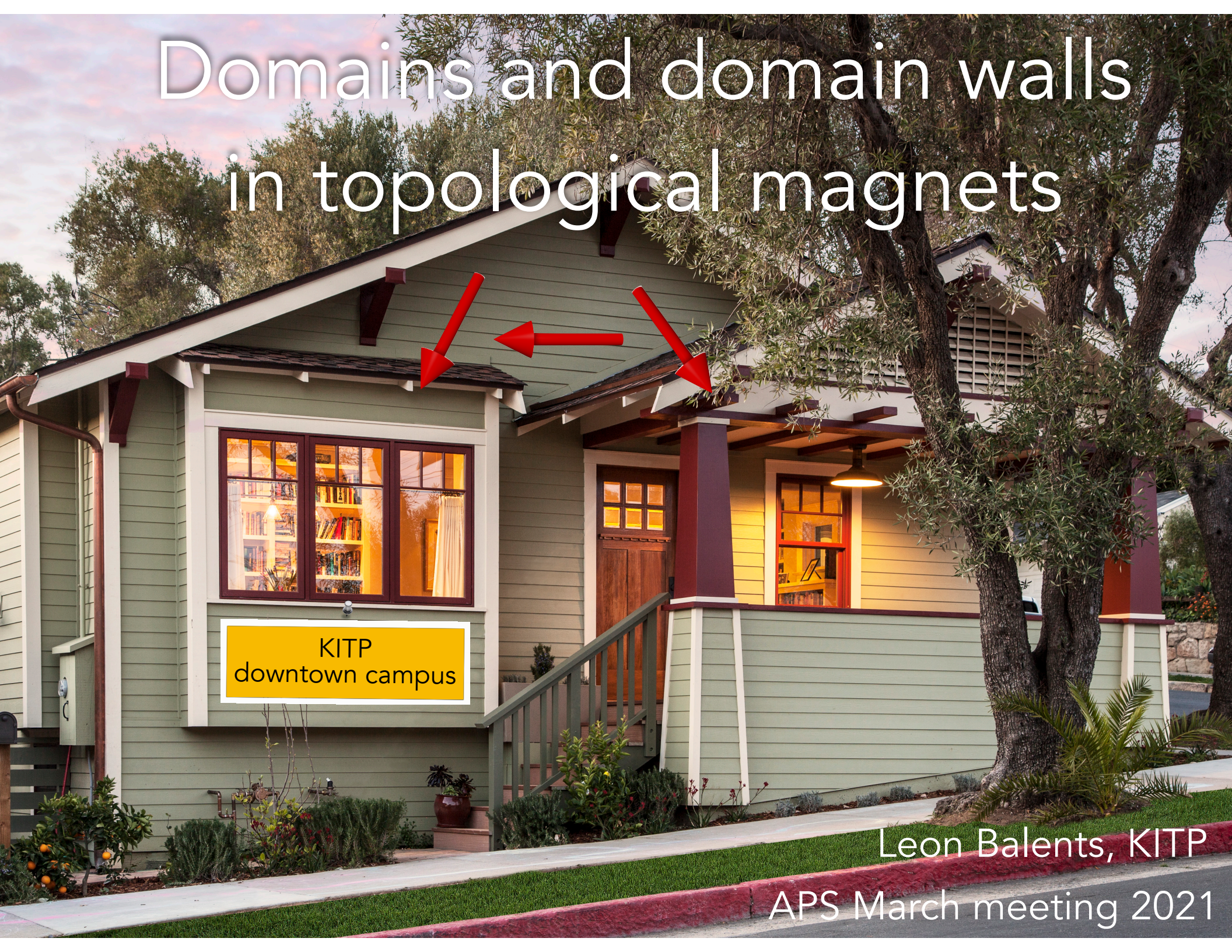


Domains and domain walls in topological magnets



KITP
downtown campus

Leon Balents, KITP
APS March meeting 2021

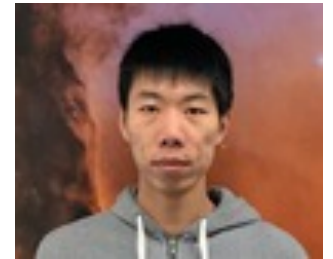
Collaborators



Kamran Behnia



Mengxing Ye



Xuzhe Ying

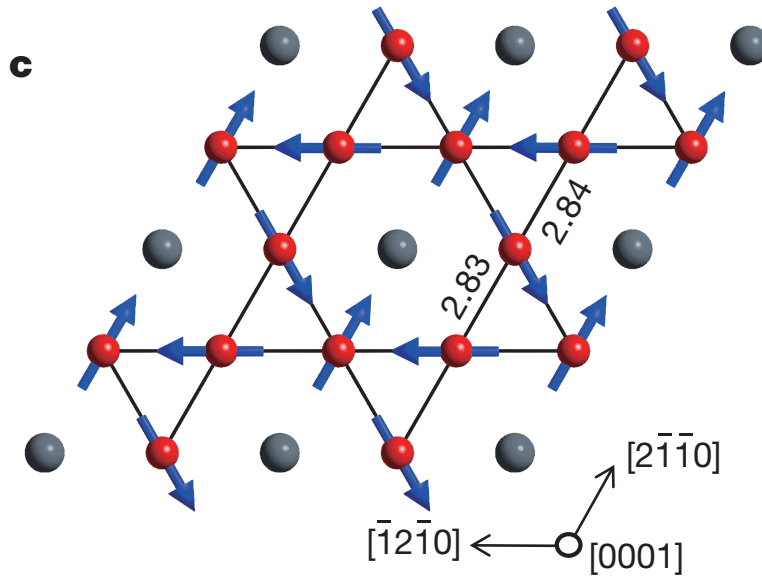
Outline

- This talk: Back in 2019 I had in mind some general discussion but now I decided to just discuss two aspects of anomalous Hall physics in topological magnetsMain classes
- Planar Hall effect in Mn_3Sn , what it tells us about domain walls, and what is still unclear
- Control of valley ferromagnetic domains in twisted bilayer graphene

Outline

- This talk: Back in 2019 I had in mind some general discussion but now I decided to just discuss two aspects of anomalous Hall physics in topological magnetsMain classes
- **Planar Hall effect in Mn_3Sn , what it tells us about domain walls, and what is still unclear**
- Control of valley ferromagnetic domains in twisted bilayer graphene

Mn₃Sn family



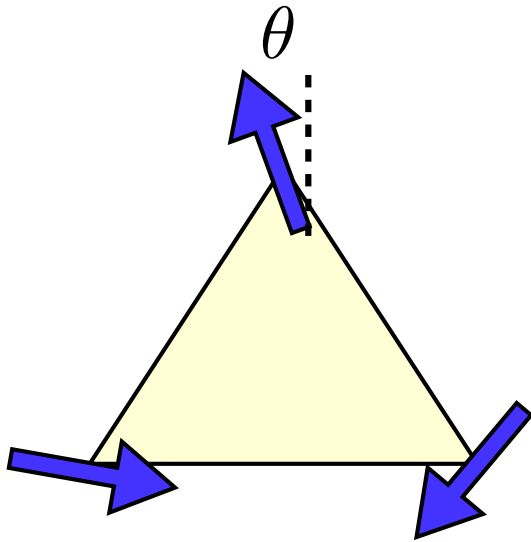
two kagomé layers of
Mn, related by inversion

large ordered
antiferromagnetic
moment
 $\sim 2 \mu_B / \text{Mn}$
tiny FM moment:
 $.002 \mu_B / \text{Mn}$

$$T_N \sim 420\text{K}$$

Nagamiya et al, 1982

Energetics: triangle



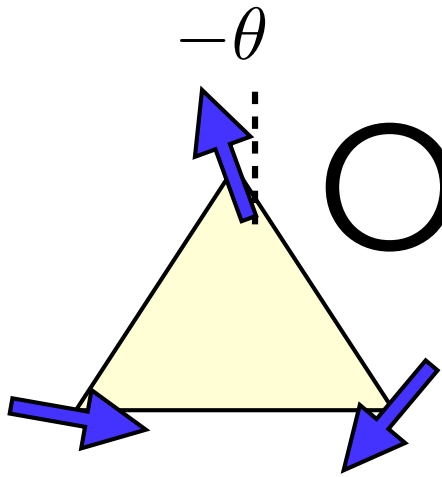
$$E = J (\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_1)$$

$$+ D \hat{\mathbf{z}} \cdot (\mathbf{S}_1 \times \mathbf{S}_2 + \mathbf{S}_2 \times \mathbf{S}_3 + \mathbf{S}_3 \times \mathbf{S}_1)$$

$$- K \sum_i (\hat{\mathbf{n}}_i \cdot \mathbf{S}_i)^2$$

$J \gg D \gg K$ **Hierarchy of interactions**

- J: spins at 120° angles and $M=0$
- D: spins are "anti-chiral" in XY plane
- K: weak canting toward easy axes creates tiny moment and fixes in-plane angle



Order parameter

$$\psi = |\psi|e^{i\theta}$$

J. Liu + L.B., 2017

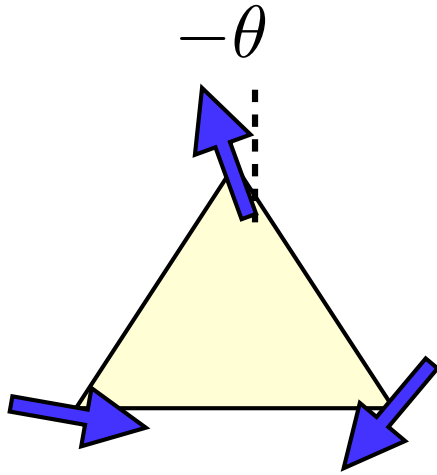
$$\mathbf{s}_a^{(0)} = m_s \begin{pmatrix} \cos(\theta - \frac{2\pi a}{3}) \\ \sin(-\theta - \frac{2\pi a}{3}) \\ 0 \end{pmatrix}$$

"octupole"

$$\mathbf{m} = \frac{K}{J} g m_s \begin{pmatrix} \cos \theta \\ \sin \theta \\ 0 \end{pmatrix}$$

"dipole"

For practical purposes, can view *direction* of magnetization as order parameter, even though it is small in magnitude.



Textures

J. Liu + L.B., 2017

$$\psi = |\psi|e^{i\theta}$$

$$F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla \theta)^2 - \lambda \cos 6\theta \right\}$$

sine-Gordon model with 6-fold anisotropy

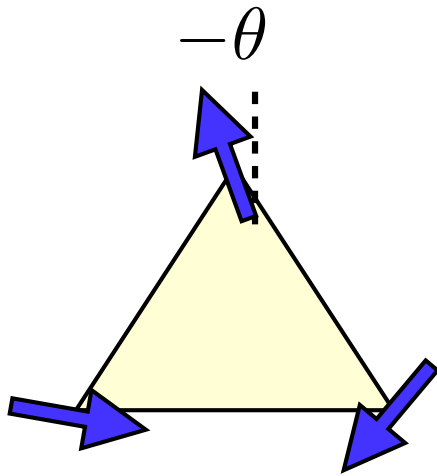
$$\rho \sim \frac{J}{a}$$

$$\lambda \sim \frac{K^3}{J^2 a^3}$$

n.b. reduced by
frustration

$$\frac{\sigma_{ij} - \sigma_{ji}}{2} = \sigma^H \epsilon_{ijk} \hat{m}_k$$

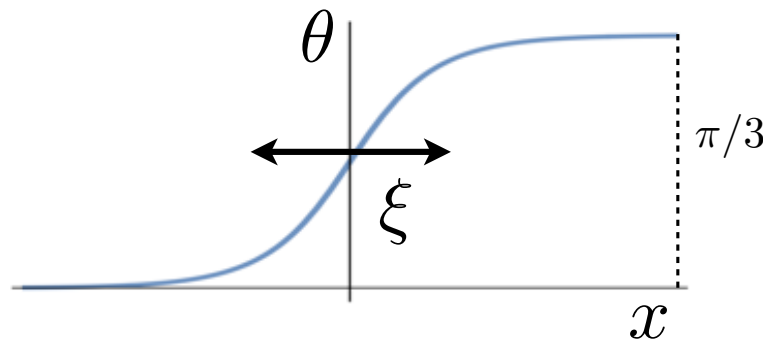
$$\hat{m} = (\cos \theta, \sin \theta, 0)$$



Textures

$$\psi = |\psi|e^{i\theta} \quad F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla \theta)^2 - \lambda \cos 6\theta \right\}$$

soliton = domain wall connecting
neighboring minima of cosine



$$\xi = \frac{1}{6} \sqrt{\frac{\rho}{\lambda}}$$

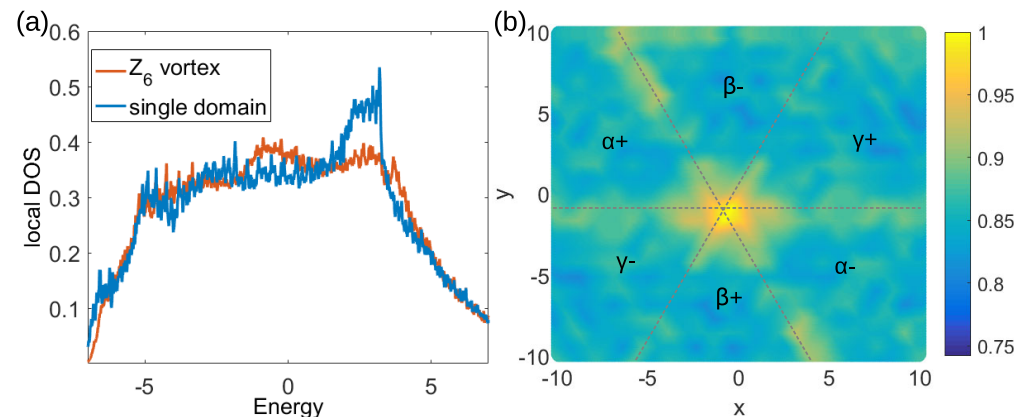
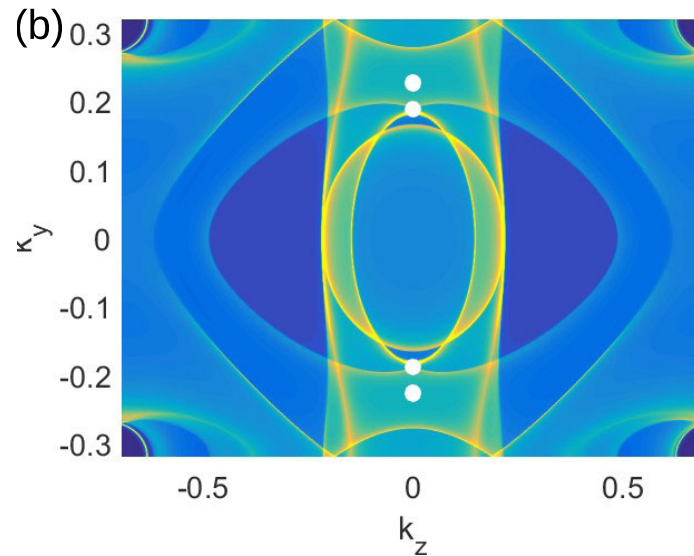
wide
DWs

$$\theta(x) = \frac{2}{3} \tan^{-1} \exp(x/\xi)$$

Electronic states at DWs

Theoretically: domain walls host bound/topological modes

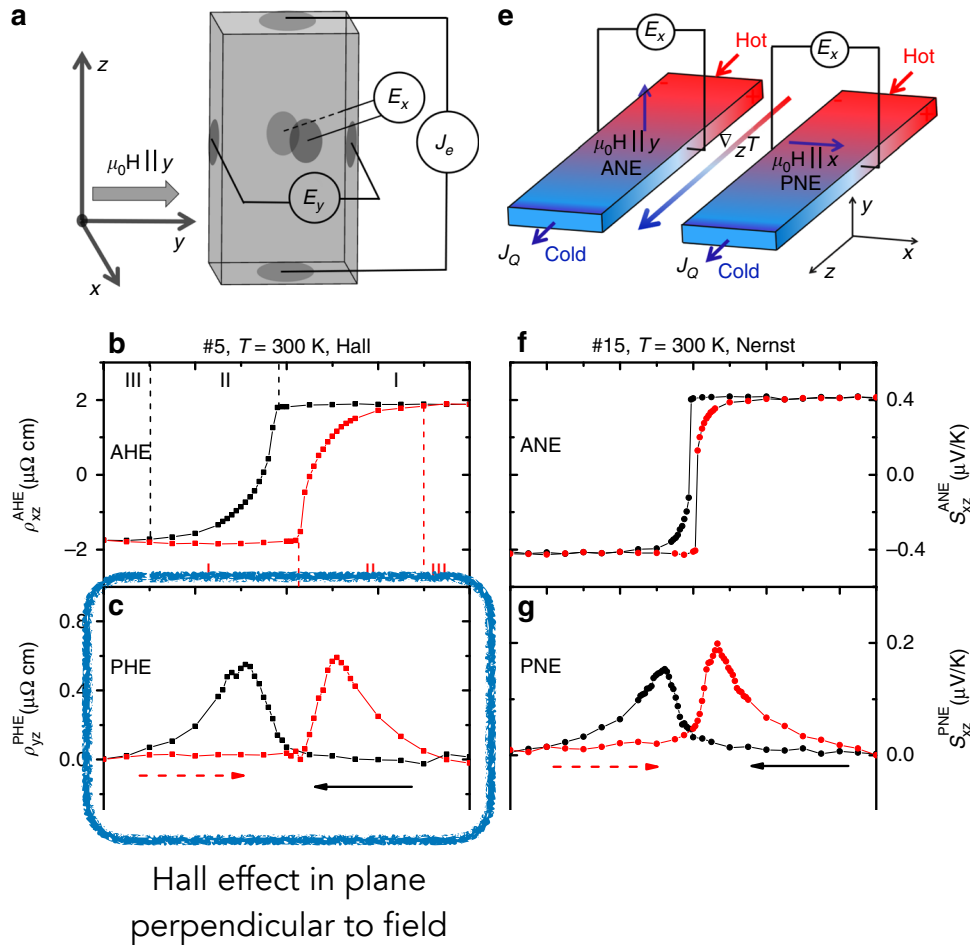
J. Liu + L.B., 2017



Many other calculations

Reality check?

Planar Hall effect



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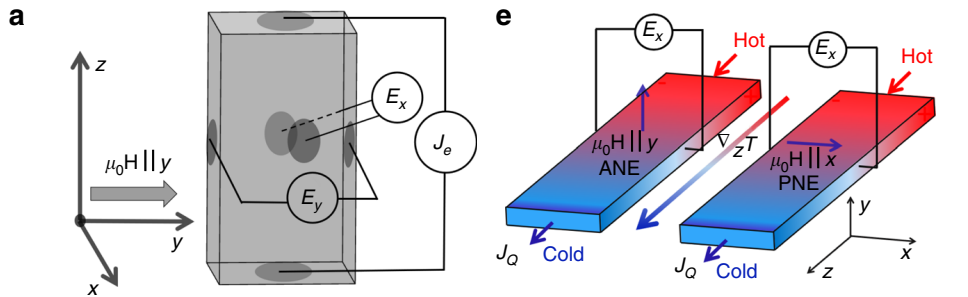
<https://doi.org/10.1038/s41467-019-10815-8>

OPEN

Chiral domain walls of Mn_3Sn and their memory

Xiaokang Li^{1,2}, Clément Collignon^{2,3}, Liangcai Xu¹, Huakun Zuo¹, Antonella Cavanna⁴, Ulf Gennser⁴, Dominique Mailly⁴, Benoît Fauqué³, Leon Balents⁵, Zengwei Zhu¹ & Kamran Behnia^{2,6}

Planar Hall effect



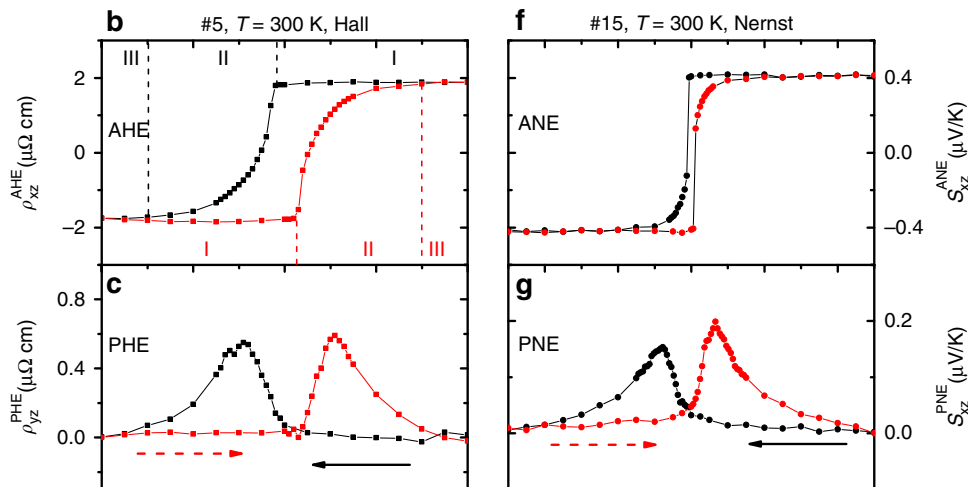
ARTICLE

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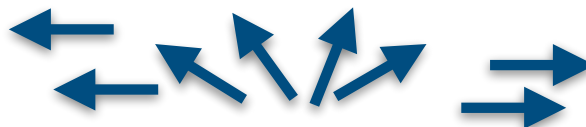


Explanation: order parameter rotates in the ab plane
(as predicted)

large -H



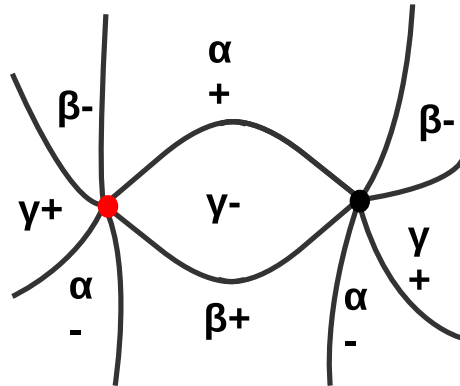
during reorientation



large +H



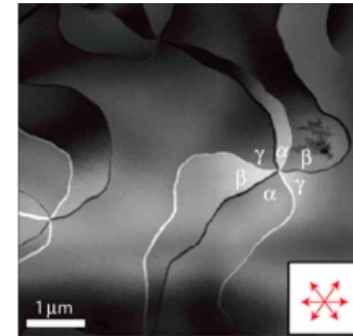
6 domains or 2 domains?



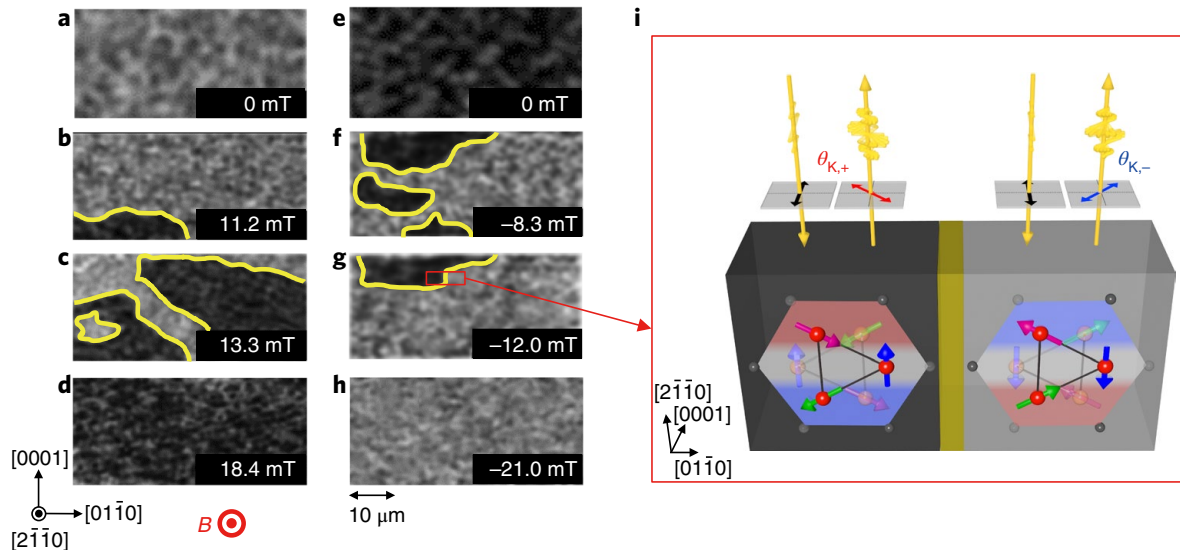
6-fold domains

Required by $C_3 + TR$ symmetries

J. Liu + L.B., 2017



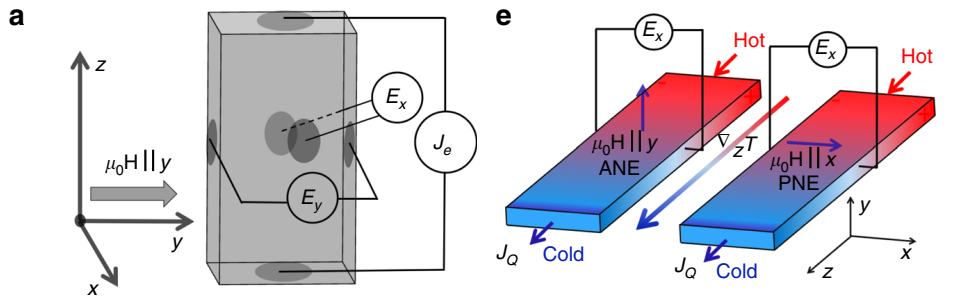
c.f. structural Z_6 vortex in $YMnO_3$
(T. Choi et al, 2010)



Large magneto-optical Kerr effect and imaging of magnetic octupole domains in an antiferromagnetic metal

Tomoya Higo^{1,2}, Huiyuan Man¹, Daniel B. Gopman³, Liang Wu^{4,5,6}, Takashi Koretsune^{2,7,8}, Olaf M. J. van 't Erve⁹, Yury P. Kabanov^{3,10}, Dylan Rees^{4,5}, Yufan Li¹¹, Michi-To Suzuki^{2,8}, Shreyas Patankar^{4,5}, Muhammad Ikhlās^{1,2}, C. L. Chien¹¹, Ryotaro Arita^{2,8}, Robert D. Shull³, Joseph Orenstein^{4,5} and Satoru Nakatsuji^{1,2*}

Planar Hall effect



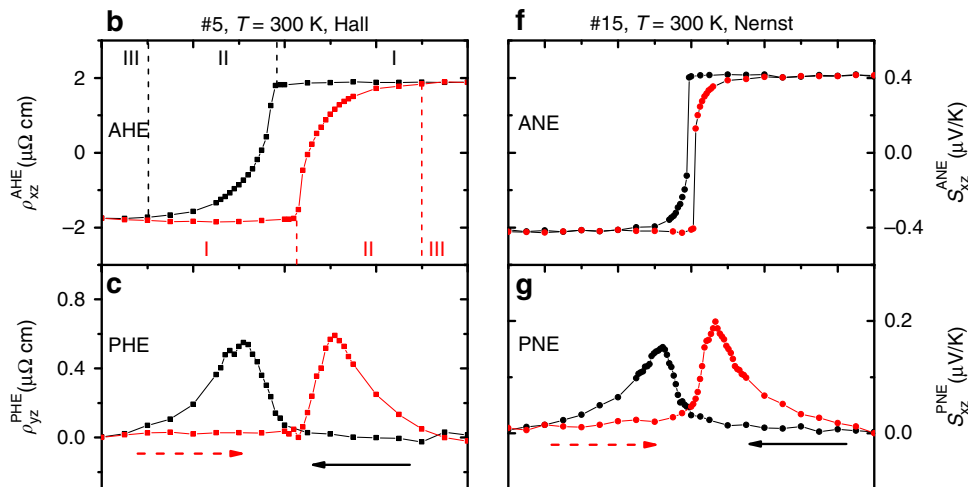
ARTICLE

<https://doi.org/10.1038/s41467-019-10815-8>

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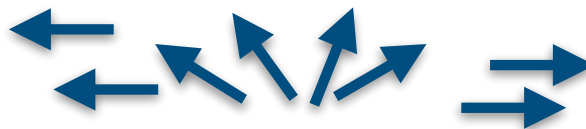


[Rotation inside DWs or in metastable domains?]

large -H



during reorientation

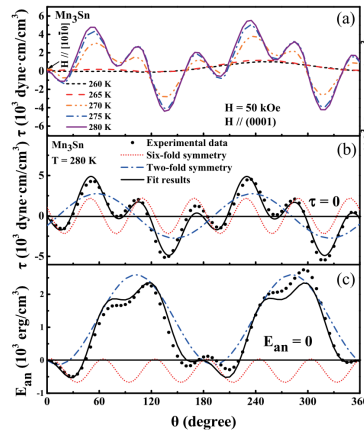


large +H



6 domains or 2 domains?

T. Duan et al *Appl. Phys. Lett.* 107, 082403 (2015)



Torque suggests some other source of two-fold anisotropy contributes.

Indicates breaking of lattice C_3 symmetry

Proposal: spin-lattice coupling

$$\mathcal{H}_{s-l} = -\alpha u_{ij} \hat{m}_i \hat{m}_j$$

$$\hat{m} = (\cos \theta, \sin \theta)$$

effective 2-fold anisotropy

magnitude of spin-lattice coupling can be measured by magnetostriction

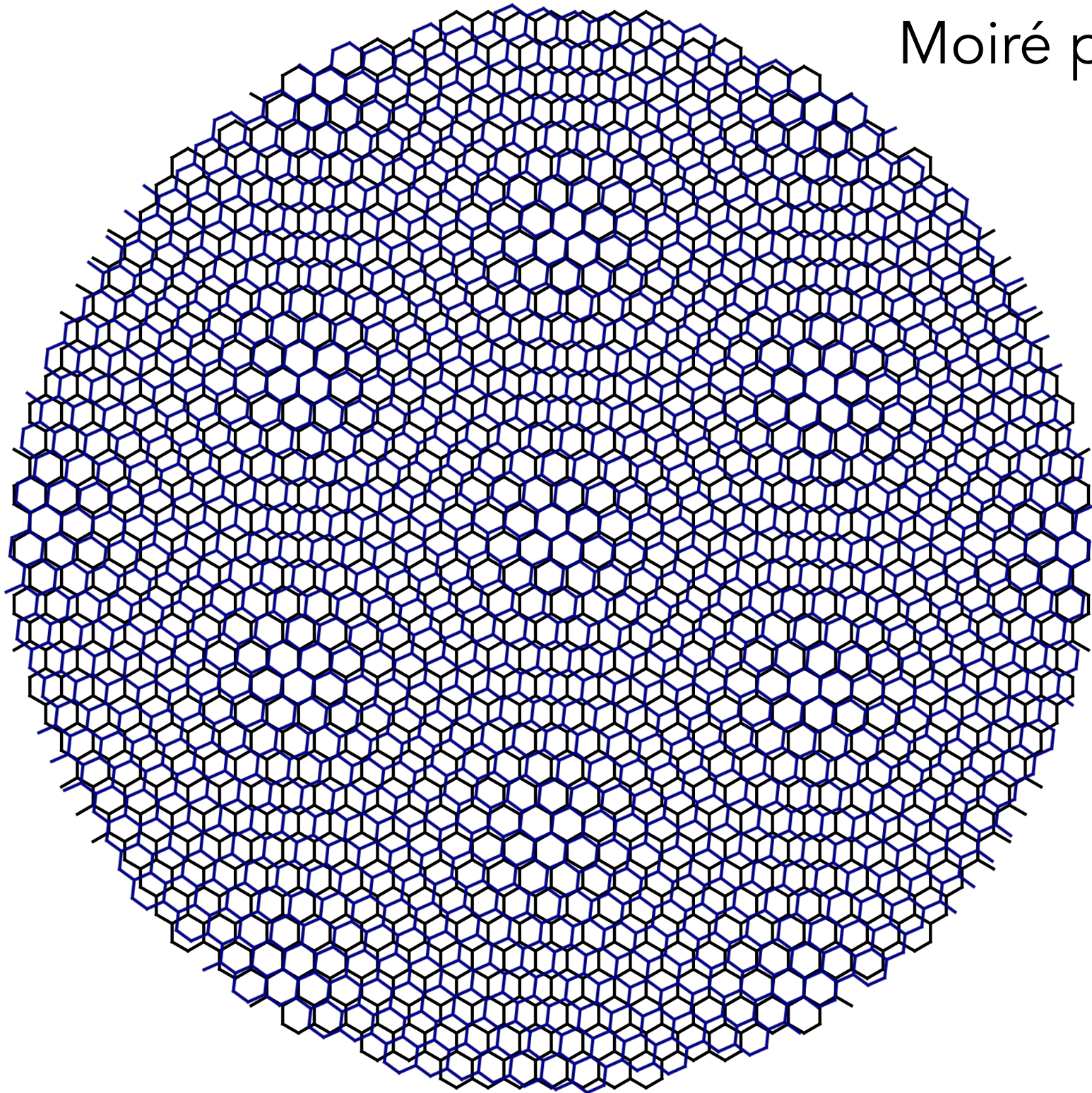
$$u_{xx} = -u_{yy} = \frac{\alpha}{2\mu} \cos 2\theta, \quad u_{xy} = \frac{\alpha}{2\mu} \sin 2\theta$$

Outline

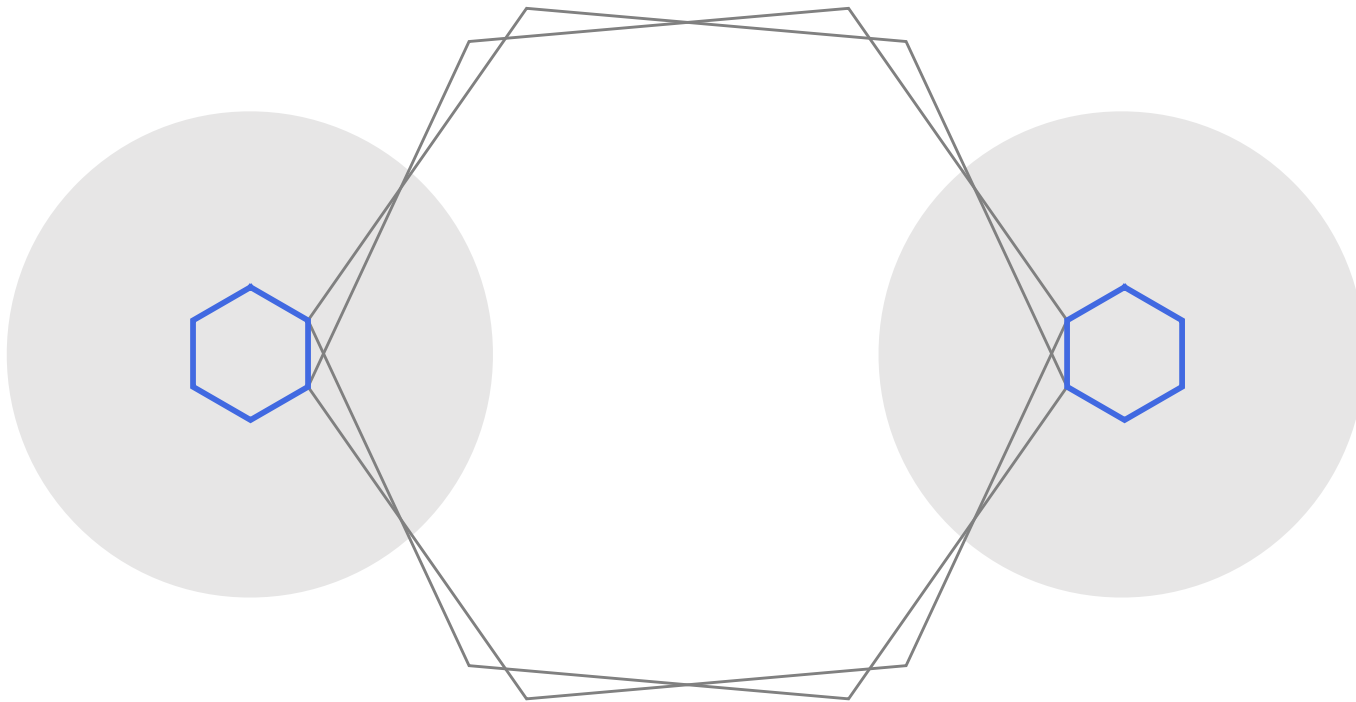
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Moiré pattern

6°

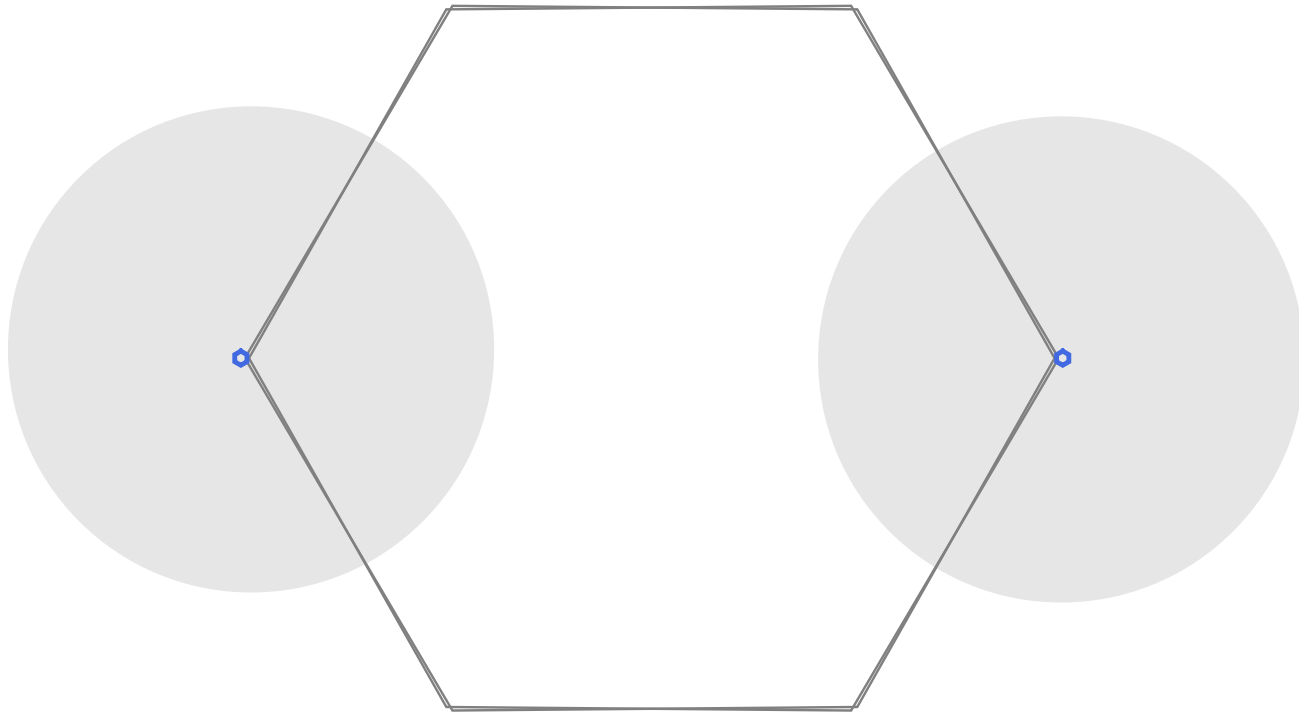


Continuum model



approximate single layer as Dirac cone
no mixing from one valley to the other

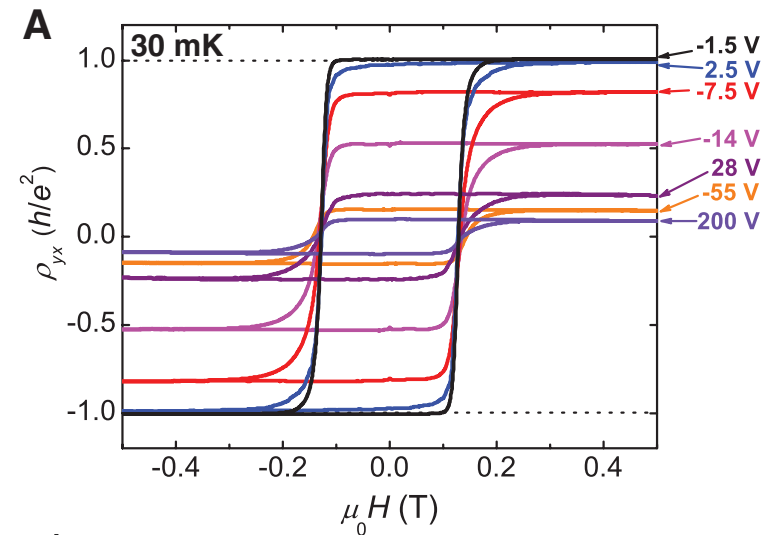
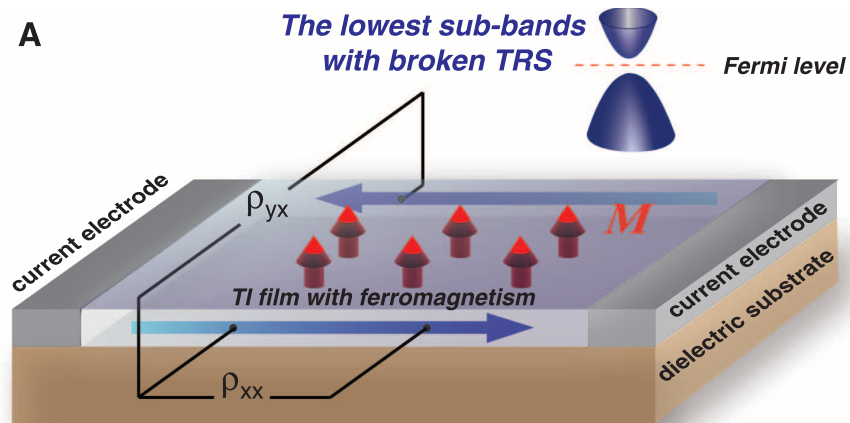
Continuum model



1°

Quantum Anomalous Hall Effect

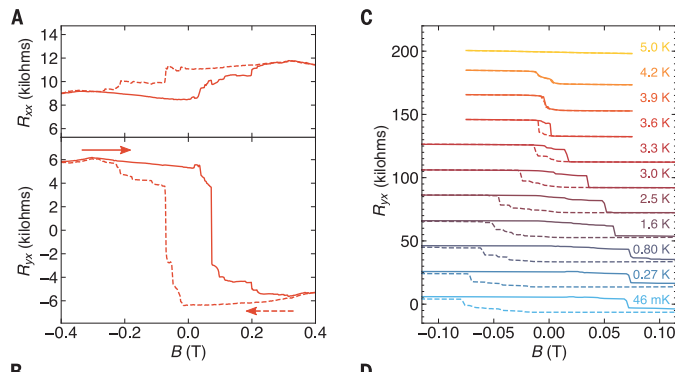
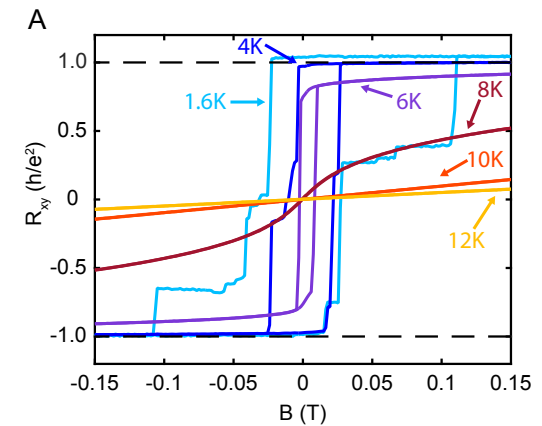
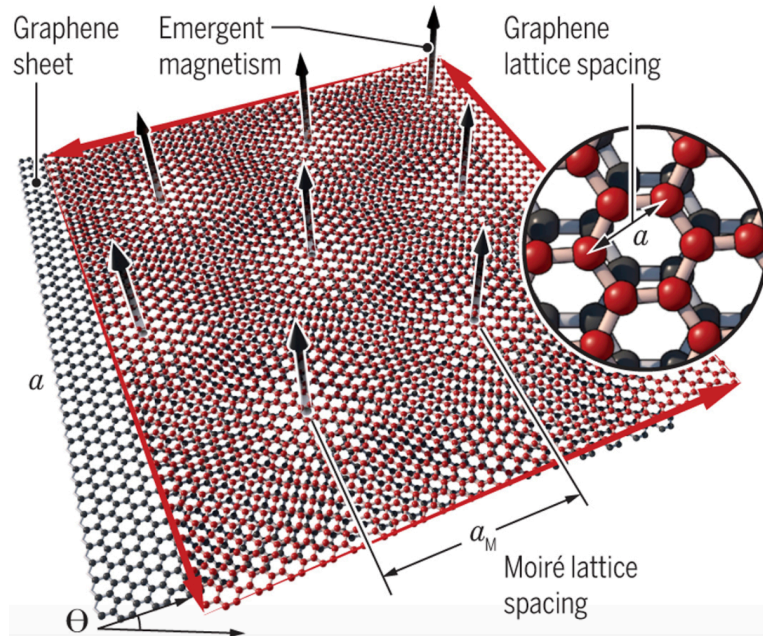
This is just the appearance of QHE in zero magnetic field by spontaneous breaking of time-reversal symmetry



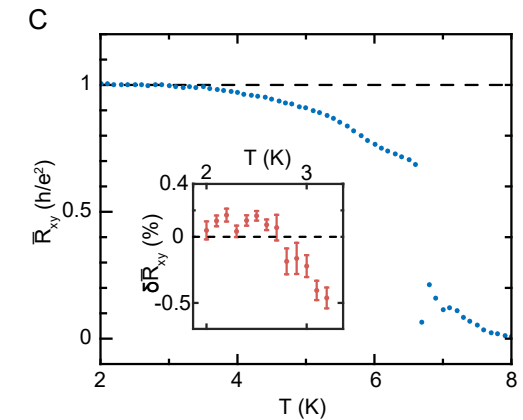
C.-Z. Zhang et al, 2013

Cr-doped $(\text{Bi/Sb})_2\text{Te}_3$

QAHE in graphene



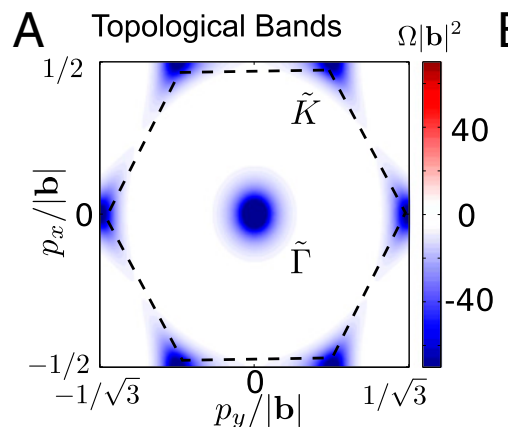
A. Sharpe et al, 2019



M. Serlin et al, 2019

Origin of QAHE?

- Underlying Dirac fermions of graphene have large incipient Berry curvature
- Curvature is realized by breaking C_2T symmetry



Topological Bloch bands in graphene superlattices

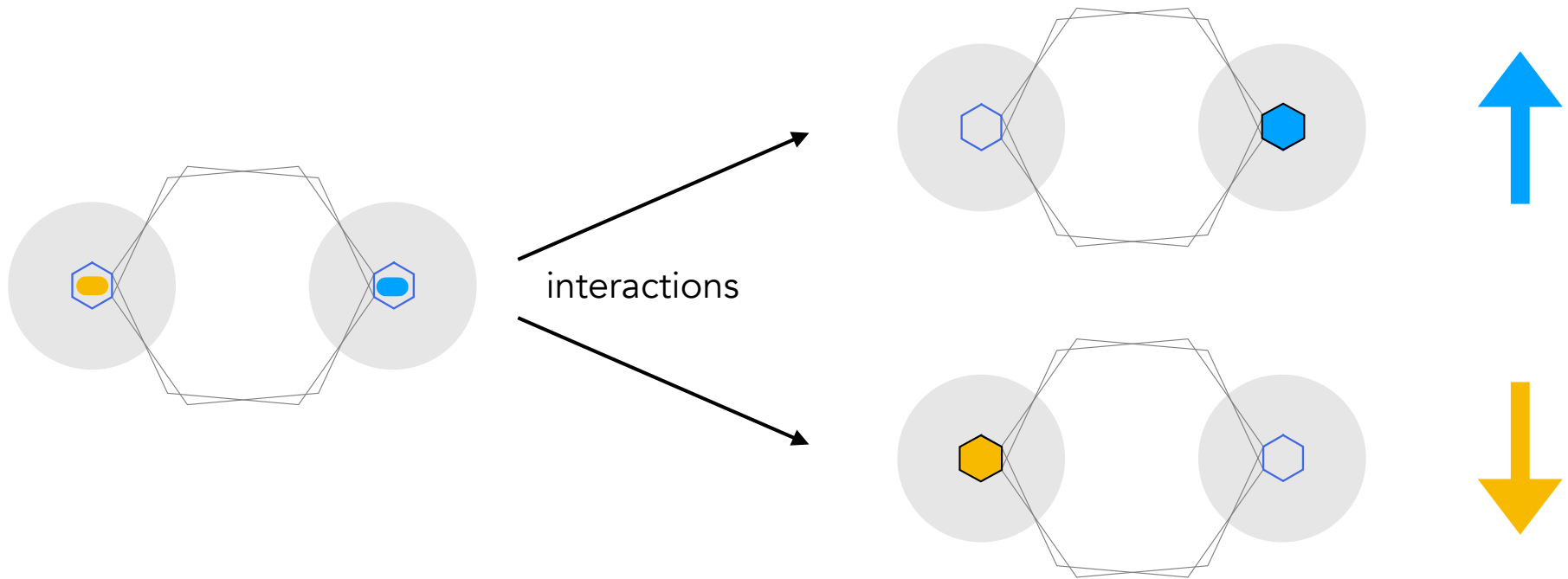
Justin C. W. Song^{a,b,c,1}, Polnop Samutgraphoot^c, and Leonid S. Levitov^{c,1}

^aWalter Burke Institute for Theoretical Physics, California Institute of Technology, CA 91125; ^bInstitute for Quantum Information and Matter, and Department of Physics, California Institute of Technology, CA 91125; and ^cDepartment of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

Edited by Subir Sachdev, Harvard University, Cambridge, MA, and approved June 24, 2015 (received for review December 30, 2014)

No interactions needed - just coupling to hBN - to generate Dirac mass and form valley Chern bands

Valley ferromagnetism



Valley polarization

$$\phi = n_K - n_{K'}$$

Orbital magnetization

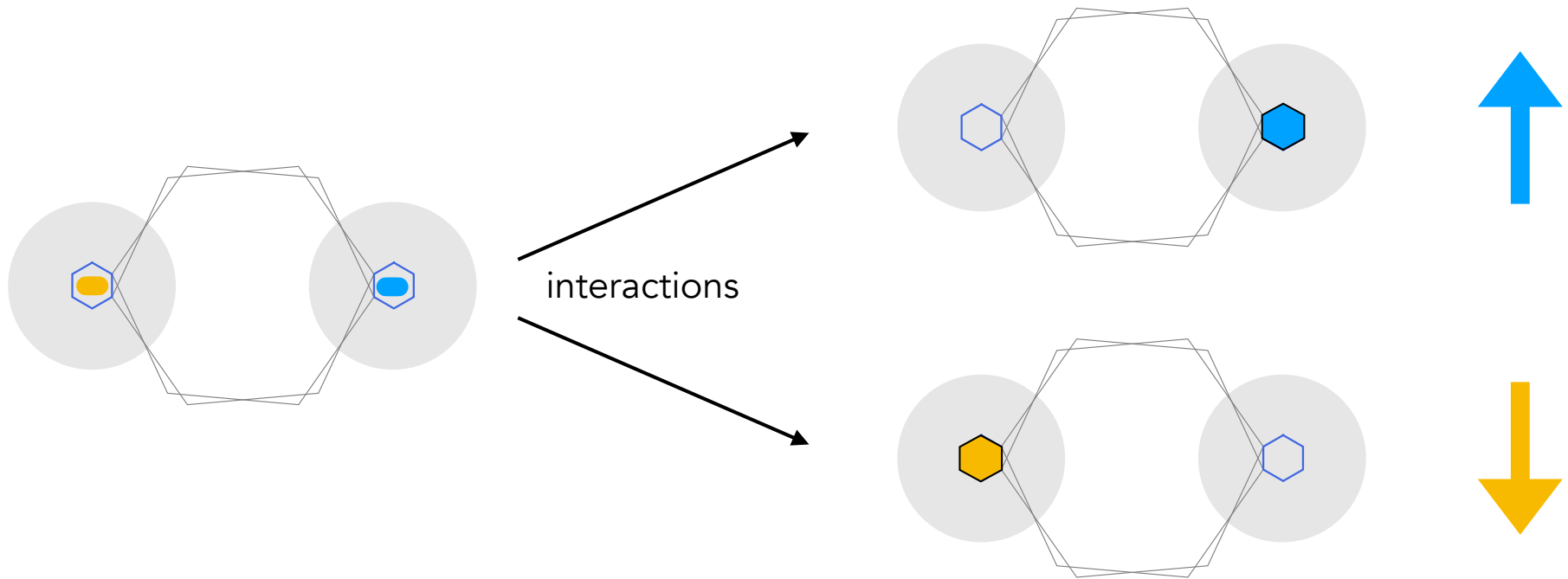
$$m^\mu(\mathbf{k}) = \frac{-ie}{2} \epsilon_{\mu\nu\lambda} \left\langle \frac{\partial u_{n\mathbf{k}}}{\partial k_\nu} \left| (\mathcal{H}_{\mathbf{k}} - \epsilon_{n\mathbf{k}}) \right| \frac{\partial u_{n\mathbf{k}}}{\partial k_\lambda} \right\rangle$$

Hall conductivity

$$\sigma_{xy} = \frac{e^2}{2\pi h} \sum_n \int d^2\mathbf{k} n_F(\epsilon_{n\mathbf{k}}) \Omega_n(\mathbf{k})$$

In equilibrium $m_z \propto \sigma_{xy} \propto \phi$

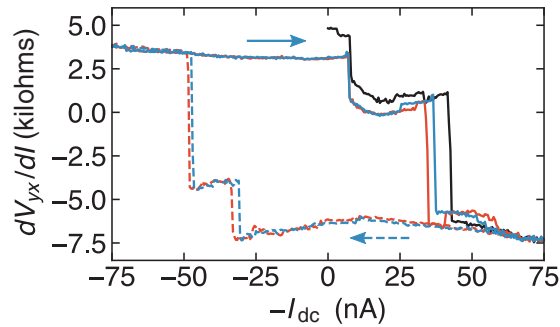
Valley ferromagnetism



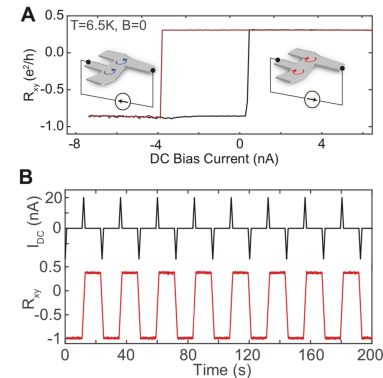
Differences from normal FMs:

- Two dimensional
- Dominant orbital moment
- Extreme Ising anisotropy
- Large Hall angle at low T

Current control

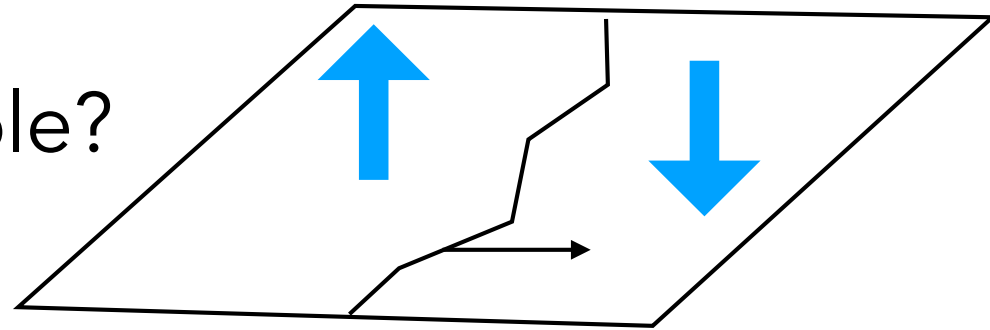


A. Sharpe et al, 2019



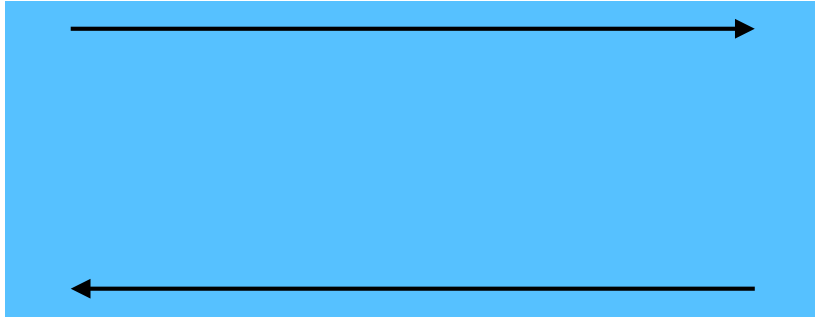
M. Serlin et al, 2019

How does the current couple?



Topological spintronics in a *pure* orbital ferromagnet

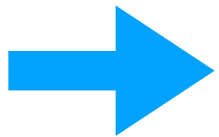
Quantized regime



$$\rho_{xx} \ll \rho_{xy}$$

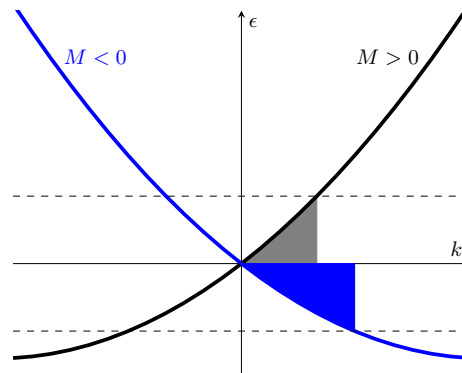
- no dissipation, only edge state transport
- Charge of each edge is separately conserved

✿ Can view current-carrying state as quasi-equilibrium ensemble where current determines edge occupation



Can formulate $F(I, M)$

$$\Delta F \sim \frac{\hbar^2}{me^3 v^3} L I^3$$



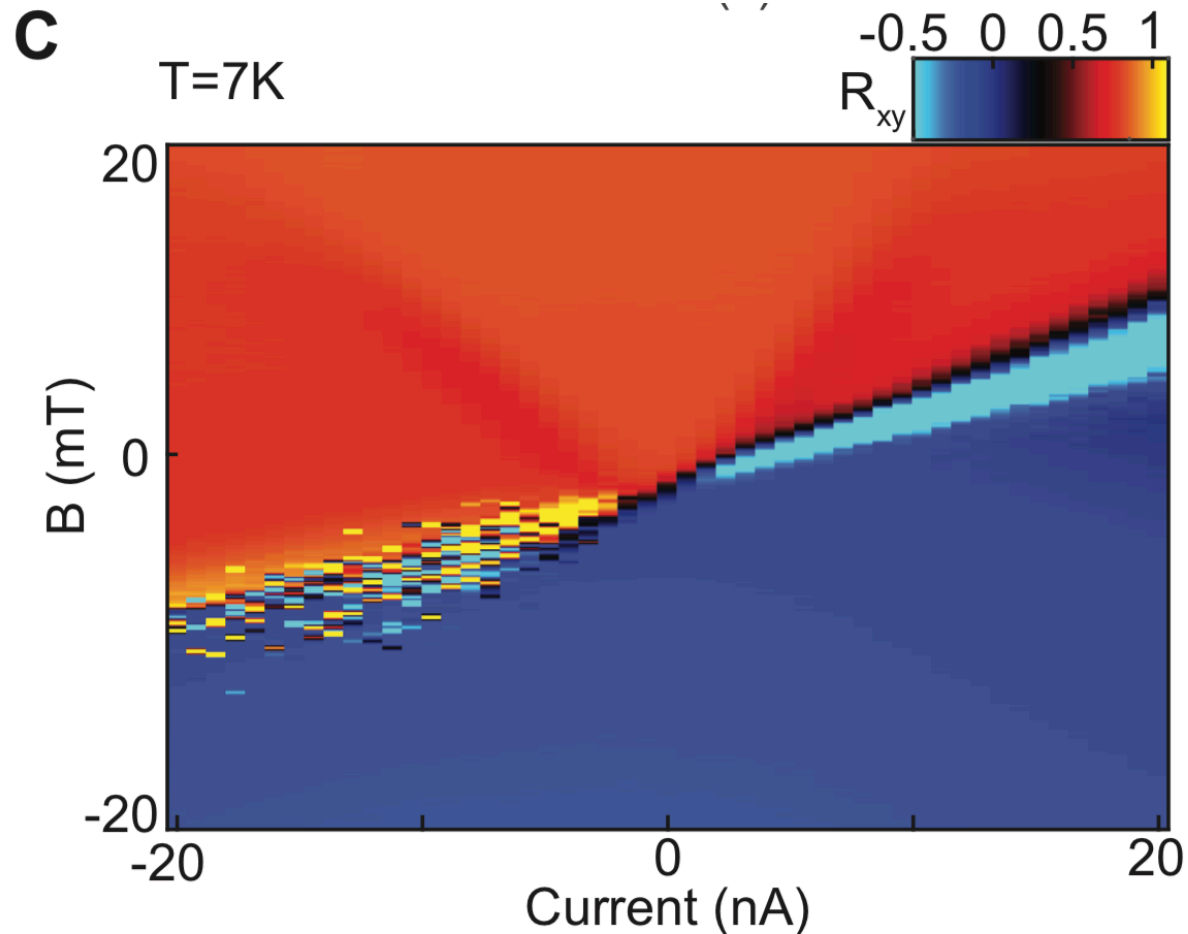
M. Serlin *et al*, 2019

another mechanism

Current Driven Magnetization Reversal in Orbital Chern Insulators

Chunli Huang, Nemin Wei, and Allan H. MacDonald
Department of Physics, University of Texas at Austin, Austin TX 78712
(Dated: September 22, 2020)

Dissipative Regime



A fully non-equilibrium problem, bulk 2d physics

Magneto-electric effect

ARTICLE

<https://doi.org/10.1038/s41467-020-15473-9>

OPEN

Giant orbital magnetoelectric effect and current-induced magnetization switching in twisted bilayer graphene

Wen-Yu He¹, David Goldhaber-Gordon^{2,3} & K. T. Law¹

$$M_i = \sum_{ij} \alpha_{ij} E_j,$$

$$\alpha_{ij} = -\tau \frac{e}{\hbar} \int_{\mathbf{q}} \sum_{s,\xi,\nu} M_{s,\xi,\nu}^i(\mathbf{q}) v_{s,\xi,\nu}^j(\mathbf{q}) f'(E_{s,\xi,\nu})$$

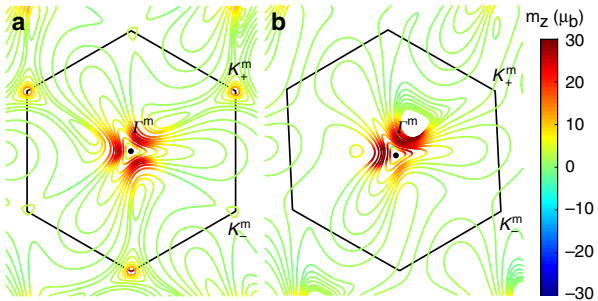
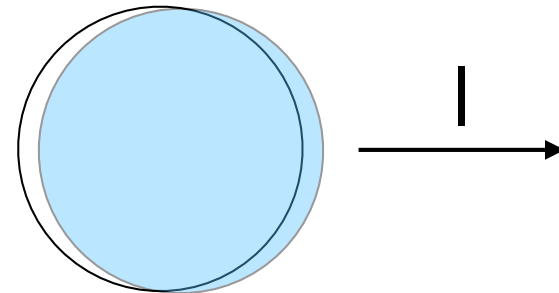


Fig. 2 The orbital magnetic moments of the Bloch electrons. a The orbital

Orbital moment induced by local change to electron distribution



Current Control

ARTICLE

<https://doi.org/10.1038/s41467-020-15473-9>

OPEN

Giant orbital magnetoelectric effect and current-induced magnetization switching in twisted bilayer graphene

Wen-Yu He¹, David Goldhaber-Gordon^{2,3} & K. T. Law¹

How to connect to I-induced force?

Use ME effect in free energy?

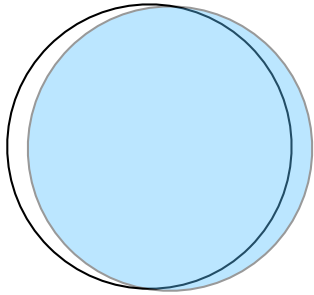
$$\begin{aligned} F &= -a_0(M_z + \delta M_z)^2 + b_0(M_z + \delta M_z)^4 \\ &\approx -a_0M_z^2 + b_0M_z^4 - 2a_0M_z(\alpha_{zx}E_x + \alpha_{zy}E_y) \end{aligned}$$

c.f.

$$\Delta E = -\mathbf{B} \cdot \mathbf{M}$$

Should be cautious: not in equilibrium

Liouville Theorem



Semi-classical dynamics preserves phase space volume:
Valley polarization is **not** induced by this mechanism

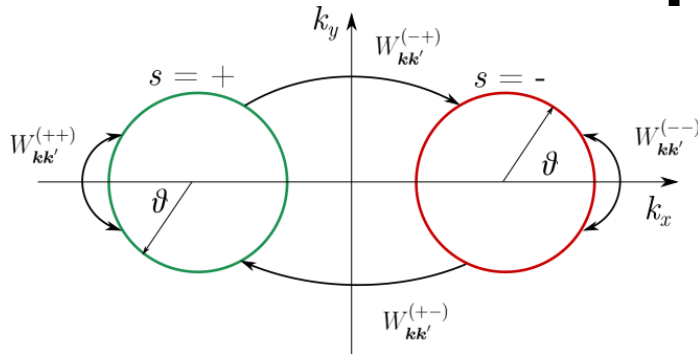
$$\partial_t f_{\mathbf{k}}^{(s)} + \mathbf{v}_{\mathbf{k}}^{(s)} \cdot \partial_{\mathbf{x}} f_{\mathbf{k}}^{(s)} + e \mathbf{E} \cdot \partial_{\mathbf{k}} f_{\mathbf{k}}^{(s)} = \sum_{s'=\pm} \int d\Gamma' W_{\mathbf{k}\mathbf{k}'}^{(ss')} \left(f_{\mathbf{k}'}^{(s')} - f_{\mathbf{k}}^{(s)} \right) \delta(\epsilon_{\mathbf{k}'}^{(s')} - \epsilon_{\mathbf{k}}^{(s)})$$

In relaxation time approximation:

$$\mathbf{M} = \alpha \mathbf{E} \sim \tau \mathbf{E} \sim \mathbf{j}$$

$\phi = 0$ Valley polarization is primary order parameter, but not induced by ME mechanism (not in equilibrium)

Anisotropic scattering

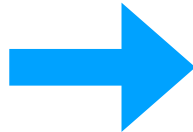


anisotropic $W_{\mathbf{k}\mathbf{k}'}^{(+ -)}$ needed

$$\partial_t f_{\mathbf{k}}^{(s)} + \mathbf{v}_{\mathbf{k}}^{(s)} \cdot \partial_{\mathbf{x}} f_{\mathbf{k}}^{(s)} + e\mathbf{E} \cdot \partial_{\mathbf{k}} f_{\mathbf{k}}^{(s)} = \sum_{s'=\pm} \int d\Gamma' W_{\mathbf{k}\mathbf{k}'}^{(ss')} \left(f_{\mathbf{k}'}^{(s')} - f_{\mathbf{k}}^{(s)} \right) \delta(\epsilon_{\mathbf{k}'}^{(s')} - \epsilon_{\mathbf{k}}^{(s)})$$

e.g.

$$W_{\mathbf{k}\mathbf{k}'}^{(-+)} = W_{\mathbf{k}'\mathbf{k}}^{(+-)} = \frac{1}{\nu\tau'} (1 + a_1 \cos \theta_{\mathbf{k}} + b_1 \sin \theta_{\mathbf{k}} + a'_1 \cos \theta_{\mathbf{k}'} + b'_1 \sin \theta_{\mathbf{k}'})$$



$$\Delta n_0 = n^{(+)} - n^{(-)} = \frac{1}{2} \nu v_F \tau [eE_x (a_1 + a'_1) + eE_y (b_1 + b'_1)] \propto \frac{\tau}{\tau'}$$

$$\Delta n_0 = \frac{1}{2h\nu_F} \frac{h}{e^2} [ej_x (a_1 + a'_1) + ej_y (b_1 + b'_1)]$$

Result

Rough estimate

$$\Delta n_0 \simeq \frac{1}{ev_F} \mathbf{j} \cdot \boldsymbol{\delta}_\epsilon \propto \frac{\epsilon}{\theta_w} \frac{1}{\hbar v_F} \frac{\hbar}{e^2} e j$$

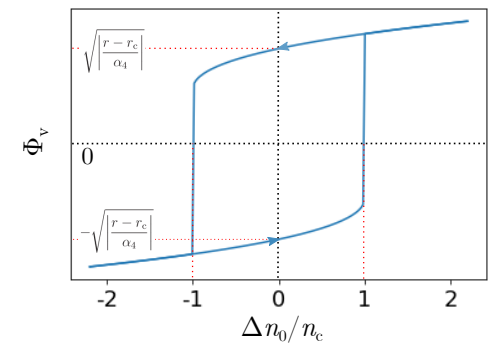
This is the “bare” response just from quasiparticle physics. Should be included in a TDGL-like formulation as a force, to take into account both QP physics and interactions.

Keldysh result:

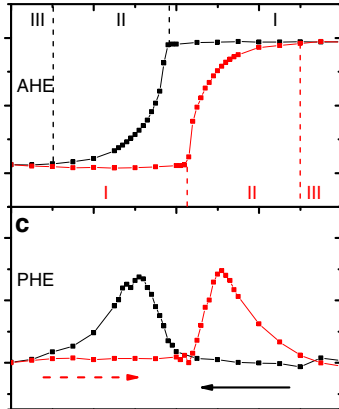
$$(r - r_c) \Phi_v - \alpha_4 \Phi_v^3 = \Delta n_0$$

RHS acts as force on valley order parameter

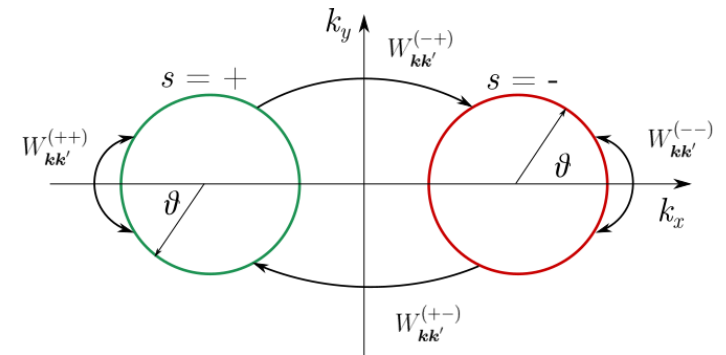
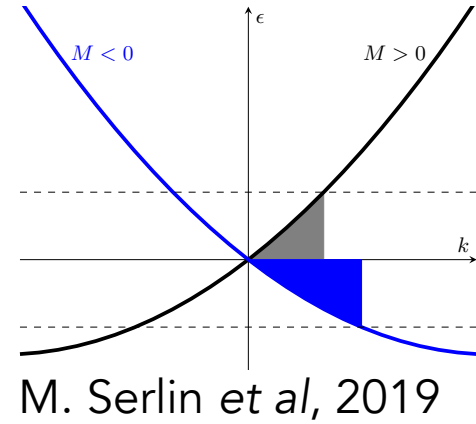
n.b. result is parametrically larger than ME one near T_c



Thanks



X. Li *et al*, Nat. Comm. (2019)



X. Ying, M. Ye, LB, arXiv:2101.01790

