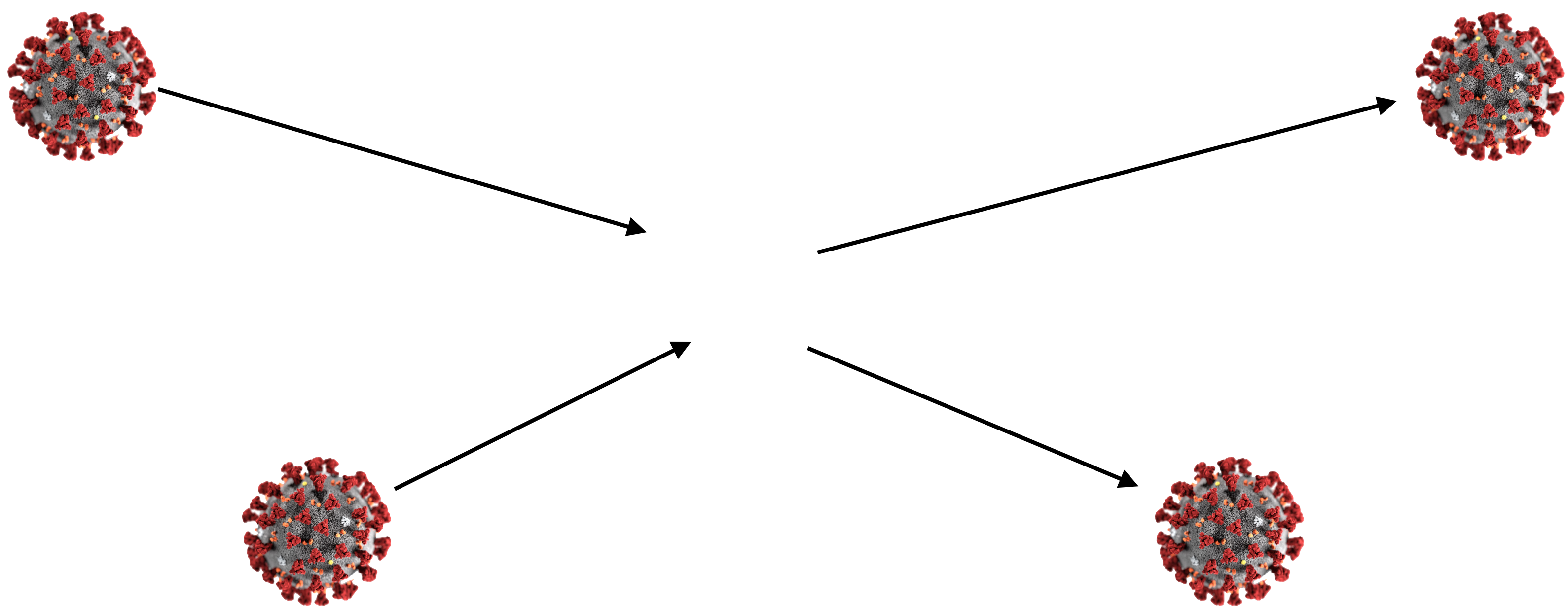
Spectral Signatures of Quasiparticle Interactions in Spin Liquids and Heisenberg Chains



Leon Balents, KITP

UQM Virtual meeting, June 2020

Collaborators

Oleg Starykh, U. Utah

Anna Keselman, KITP

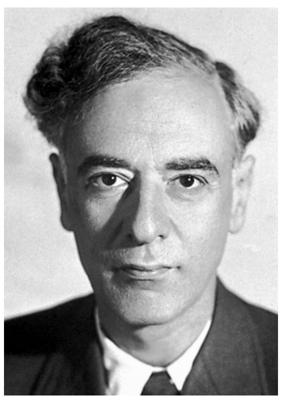
Outline

- A bit about quasiparticles and spin liquids
- Dynamical susceptibility of a spinon Fermi surface in a small Zeeman field
  - Interactions induce a gap between two “optical” modes
- Dynamical susceptibility of 1d spin chains
  - Similar effect at low fields, new effects at high fields

Quasiparticles

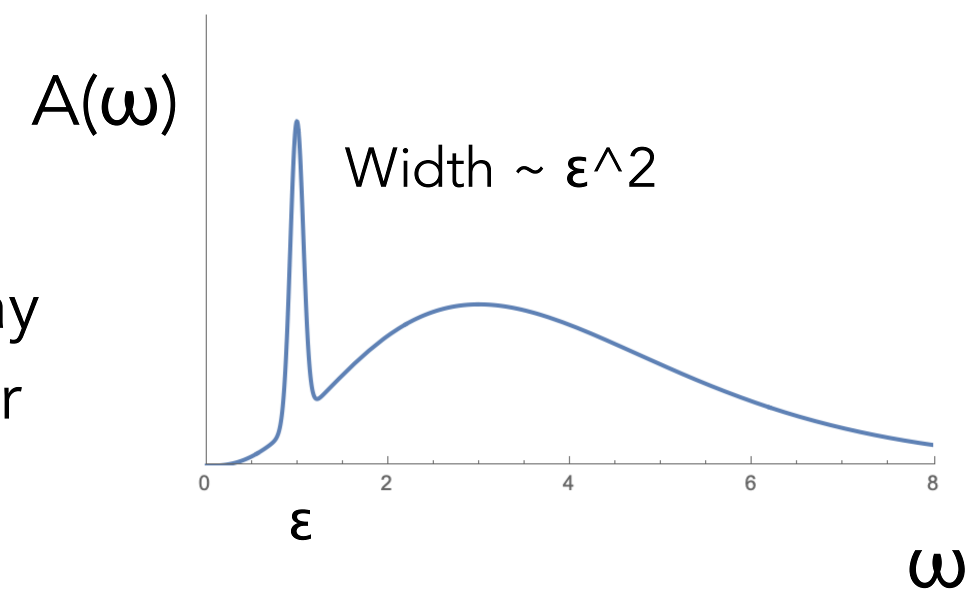
- Fundamental excitations of a many body ground state
- Behave like particles: single quasiparticle is long-lived
- Example: metal

1-e spectral function



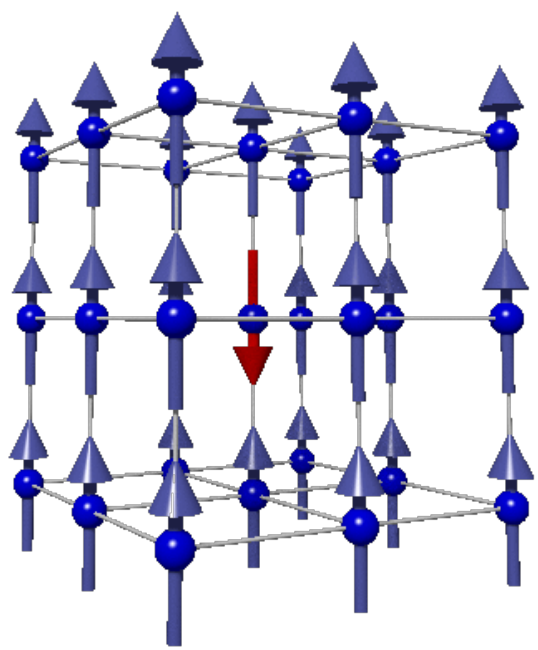
<u>Fermi Liquid</u>

Quasi electron decay rate is much smaller than its energy



Quasiparticles

- Spin wave: bosonic quasiparticle in a magnet

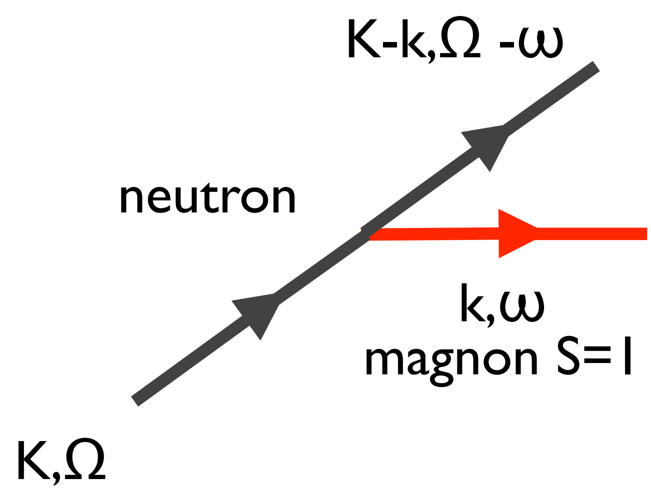


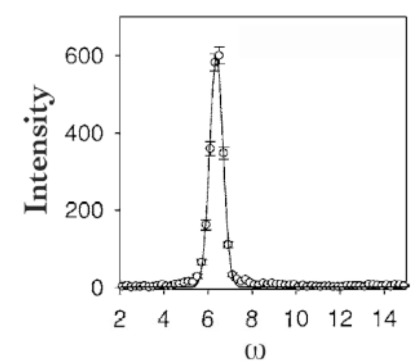
$$|f\rangle = S_k^+|i\rangle$$

$$\omega(k) \approx \Delta - 2t\cos k_x a - \cdots$$

Inelastic neutron scattering:
dynamical susceptibility

$$S(k,\omega) = FT\left[\langle \vec{S}(r,t)\cdot\vec{S}(0,0)\rangle\right]$$



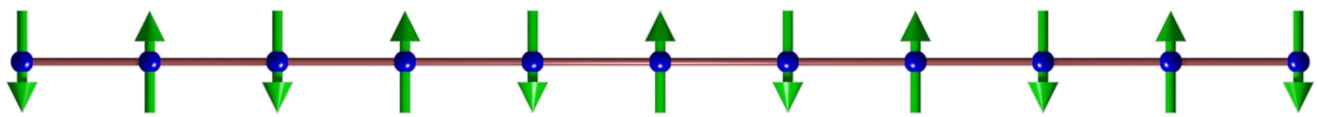


Line shape in Rb_2MnF_4

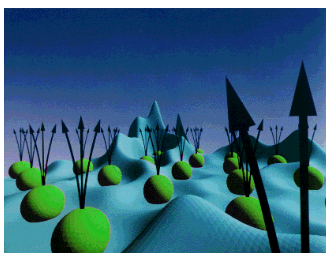
Exotic quasiparticles

c.f. Subir: Fractionalization

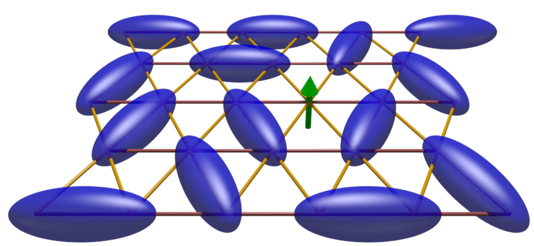
- **1d solitons (spin chains, SSH model)**



- Laughlin QPs



- **Spinon in 2d spin liquid**



Parton MFT

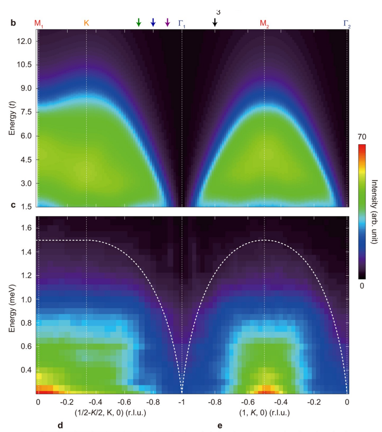
- Spins $\vec{S}_r = \frac{1}{2} c_r^\dagger \vec{\sigma} c_r$ two-particle continuum?

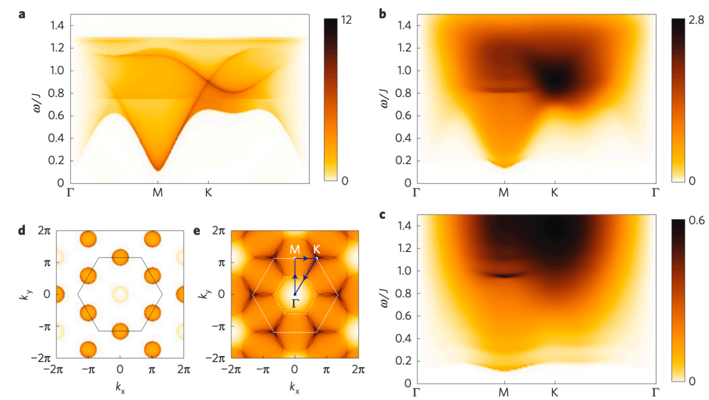
nature

Letter | Published: 05 December 2016

Evidence for a spinon Fermi surface in a triangular-lattice quantum-spin-liquid candidate

Yao Shen, Yao-Dong Li, Hongliang Wo, Yuesheng Li, Shoudong Shen, Bingying Pan, Qisi Wang, H. C. Walker, P. Steffens, M. Boehm, Yiqing Hao, D. L. Quintero-Castro, L. W. Harriger, M. D. Frontzek, Lijie Hao, Siqin Meng, Qingming Zhang, Gang Chen & Jun Zhao

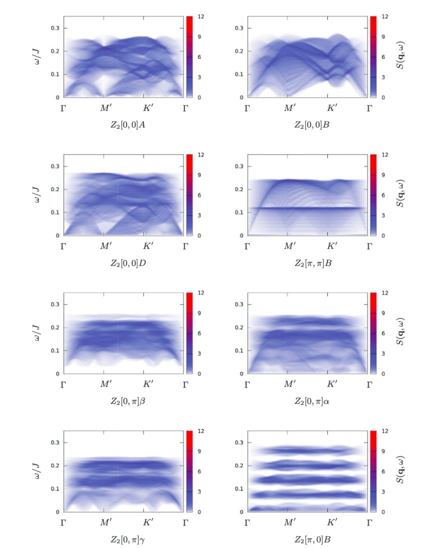




nature physics LETTERS
PUBLISHED ONLINE: 9 MARCH 2014 | DOI: 10.1038/NPHYS2887

Topological excitations and the dynamic structure factor of spin liquids on the kagome lattice

Matthias Punk[1,2], Debanjan Chowdhury[1] and Subir Sachdev[1]*



Spinon Fermi surface spin liquid in a triangular lattice antiferromagnet $NaYbSe_2$

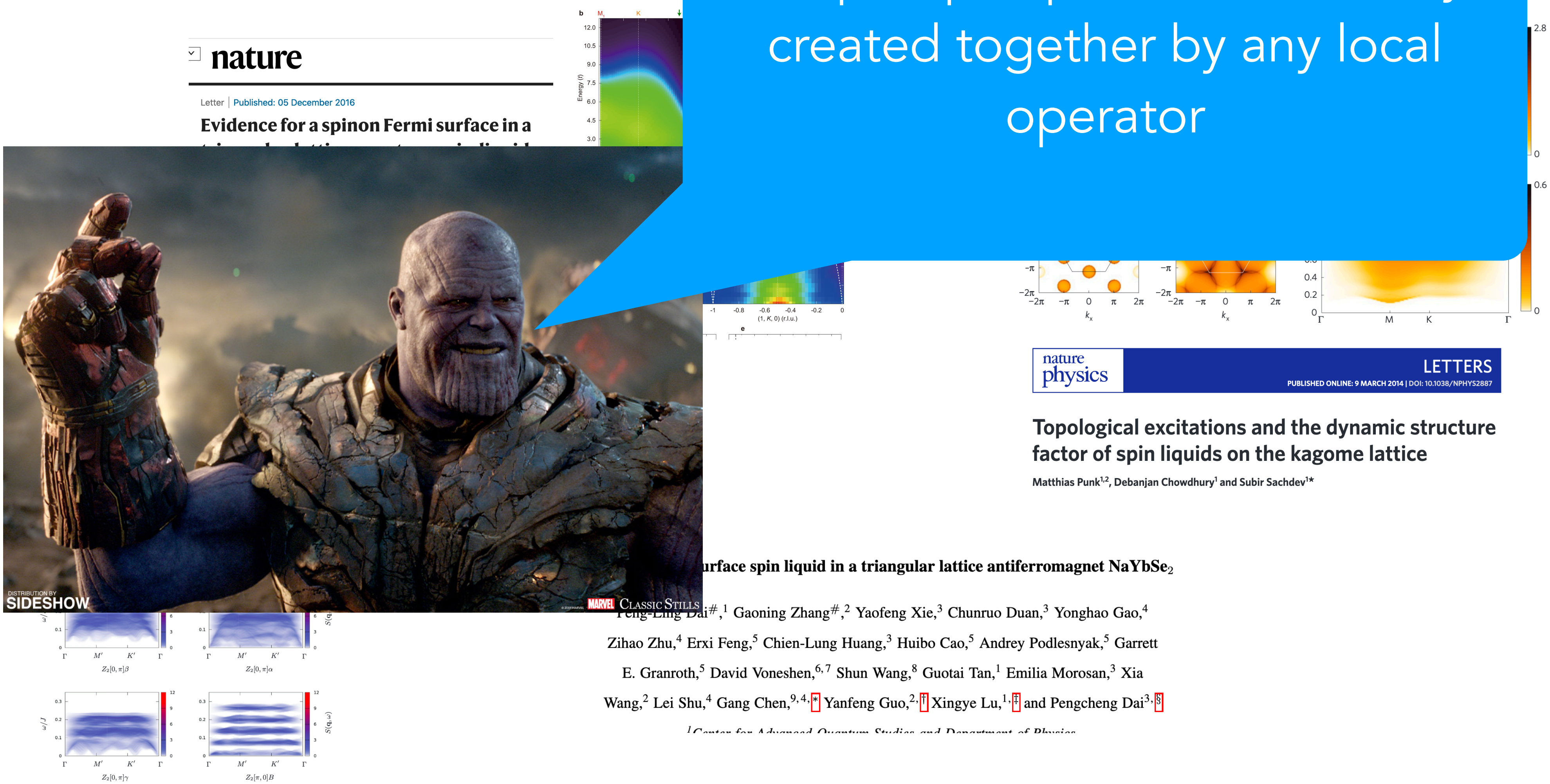
Peng-Ling Dai#,[1] Gaoning Zhang#,[2] Yaofeng Xie,[3] Chunruo Duan,[3] Yonghao Gao,[4] Zihao Zhu,[4] Erxi Feng,[5] Chien-Lung Huang,[3] Huibo Cao,[5] Andrey Podlesnyak,[5] Garrett E. Granroth,[5] David Voneshen,[6,7] Shun Wang,[8] Guotai Tan,[1] Emilia Morosan,[3] Xia Wang,[2] Lei Shu,[4] Gang Chen,[9,4,*] Yanfeng Guo,[2,†] Xingye Lu,[1,‡] and Pengcheng Dai[3,§]

[1]Center for Advanced Quantum Studies and Department of Physics

Parton MFT

- Spins $\vec{S}_r = \frac{1}{2} c_r^\dagger \vec{\sigma} c_r$

Interactions are **inevitable** since multiple quasiparticles are always created together by any local operator



Larmor Theorem

$$H = H_0 - B \cdot S^z_{\mathrm{TOT}} \qquad \left[H_0, \vec{S}_{\mathrm{TOT}}\right] = 0$$

$$\Rightarrow \quad [H, S^\pm_{\mathrm{TOT}}] = \pm B S^\pm_{\mathrm{TOT}}$$

$$\Rightarrow \quad \langle 0 | S^+_{\mathrm{TOT}}\, \delta(\omega - H + E_0)\, S^-_{\mathrm{TOT}} | 0 \rangle$$

$$= \delta(\omega - B) \langle 0 | S^+_{\mathrm{TOT}}\, S^-_{\mathrm{TOT}} | 0 \rangle$$

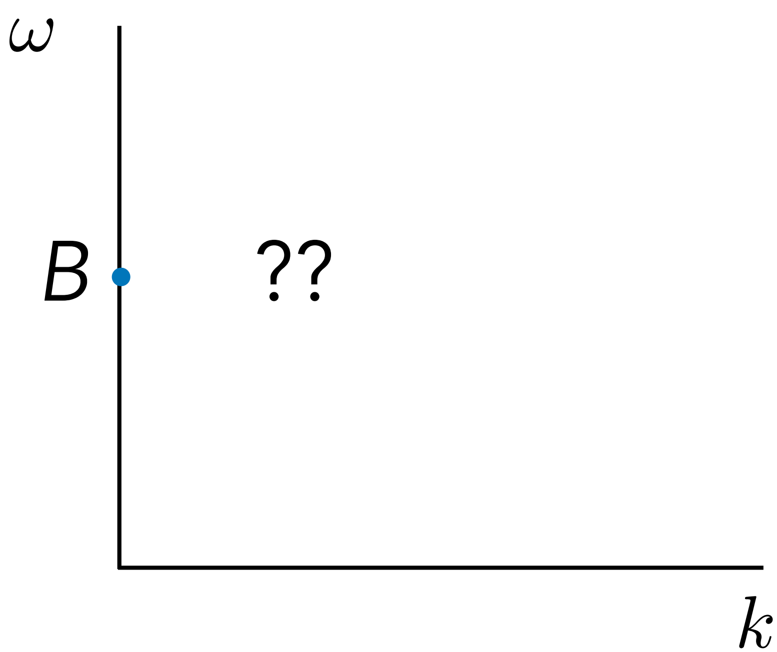
Dynamical spin susceptibility

$$\Rightarrow \quad \chi_\pm(k = 0, \omega) = A\delta(\omega - B)$$

Larmor Theorem

Dynamical spin susceptibility

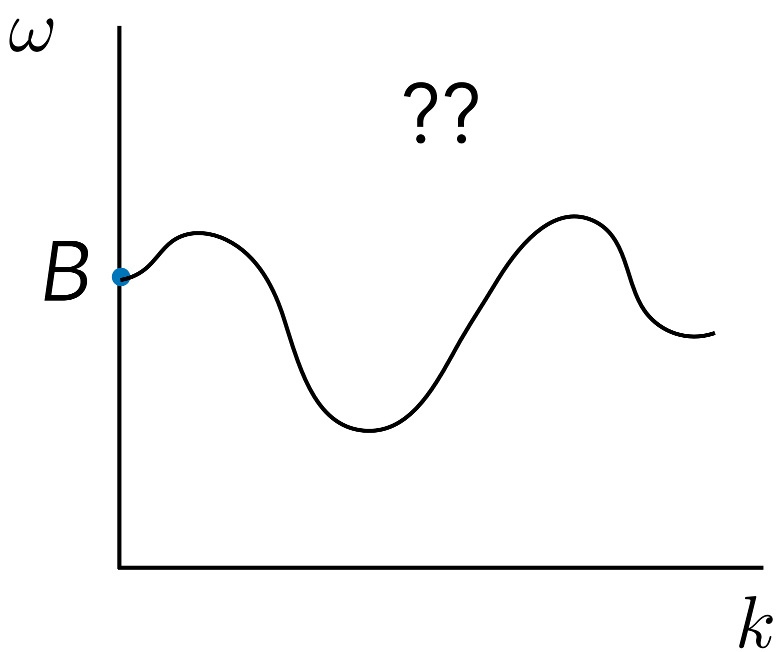
$$\chi_{\pm}(k = 0, \omega) = A\delta(\omega - B)$$



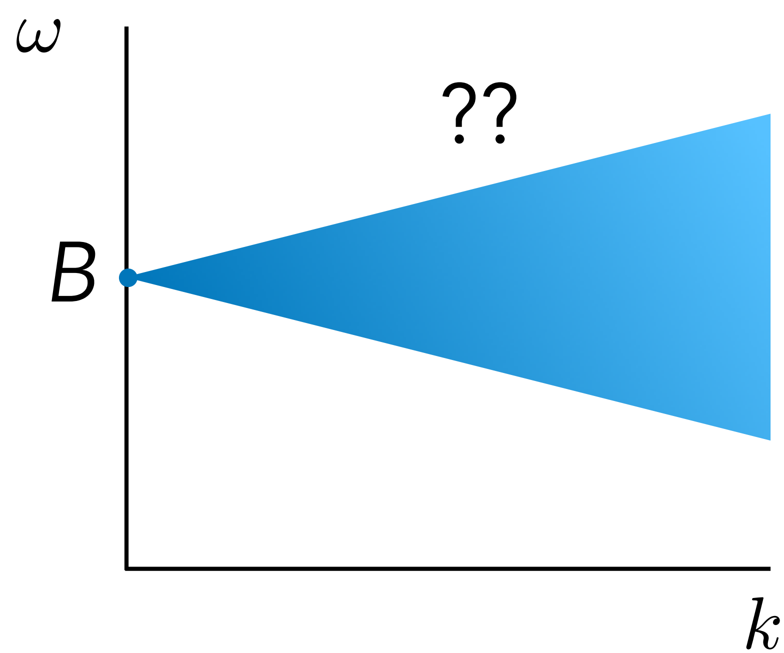
Larmor Theorem

Dynamical spin susceptibility

$$\chi_{\pm}(k=0,\omega) = A\delta(\omega - B)$$



mode

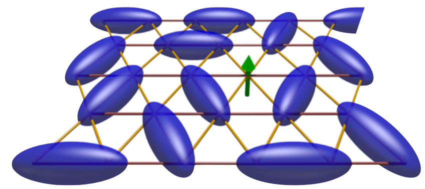


multi-particle continuum

…

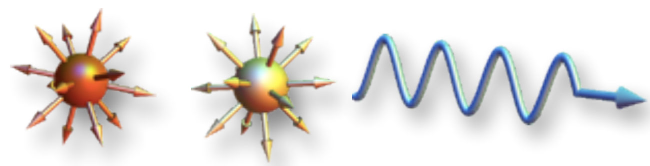
Classes of QSLs

- Topological QSLs



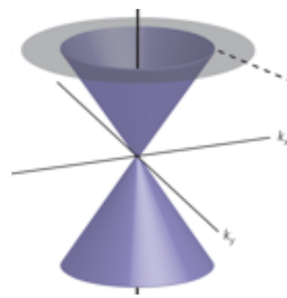
anyons,
spinons

- U(1) QSL



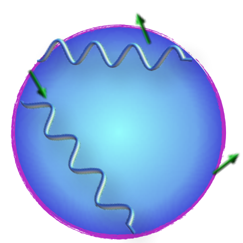
compact U(1)

- Dirac QSLs



QED_3

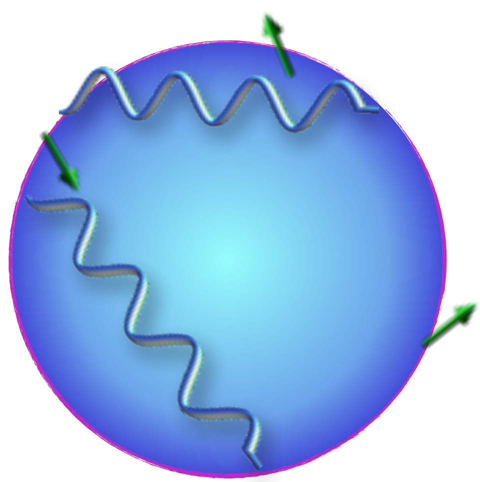
- **Spinon Fermi surface**



non-Fermi
liquid "spin
metal"

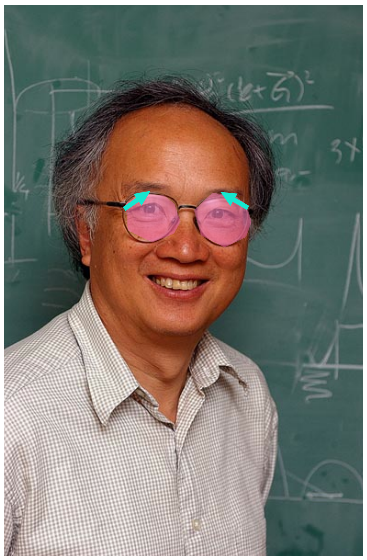
Spinon Fermi surface

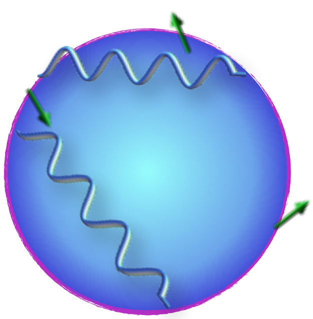
$$|\Psi\rangle = \prod_i \hat{n}_i(2-\hat{n}_i) \prod_{k<k_F} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow} |0\rangle$$

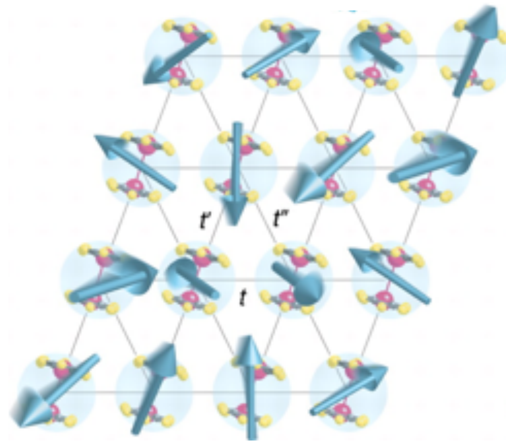


- The most gapless/highly entangled QSL state
- Like a "metal" of neutral fermions w/ a U(1) gauge field
- Prototype "non-Fermi liquid" state of great theoretical interest

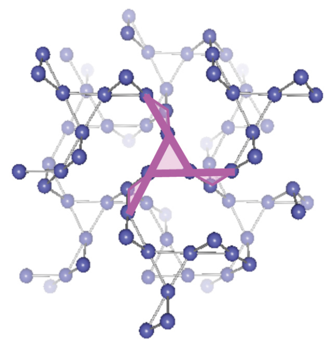
Spinon Fermi surface



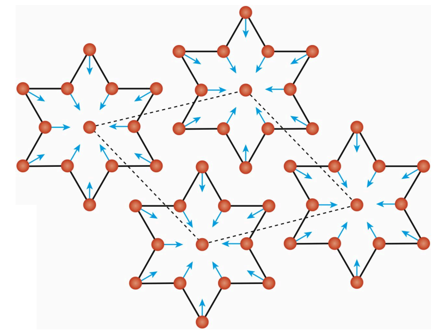




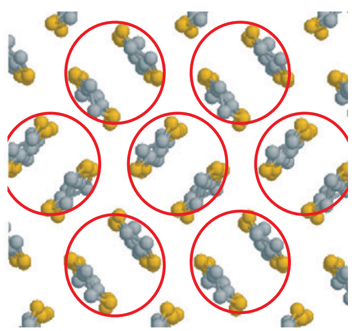
k-ET



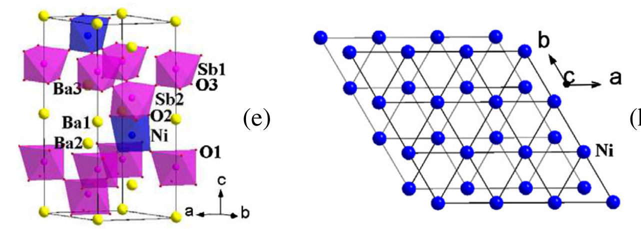
$Na_4Ir_3O_8$



1T-TaS_2



dmit



$Ba_3NiSb_2O_9$

Effective field theory

Basis for diagrammatics

- Spinon Fermi surface: “uniform RVB”

Zeeman term

$$S_\psi = \int d^3x\, \psi^\dagger \Big(\partial_\tau - \mu - \frac{1}{2m} (\boldsymbol{\nabla}_{\mathbf{r}} - i\mathbf{A})^2 - \omega_B \sigma^3 \Big) \psi,$$

Kinetic energy   Emergent gauge field

$$S_A = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2} \left(\gamma |\omega_n|/q + \chi q^2 \right) |A(q)|^2,$$

Landau damping

$$S_u = \int d^3x\, u\, \psi^\dagger_\uparrow \psi_\uparrow \psi^\dagger_\downarrow \psi_\downarrow.$$

Short-range repulsion (from a_0)

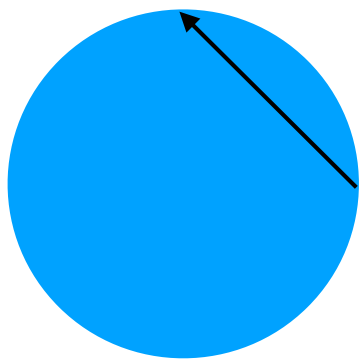
Ioffe,Larkin 1989   Nagaosa 1999

Kim, Furusaki, Lee, Wen 1994

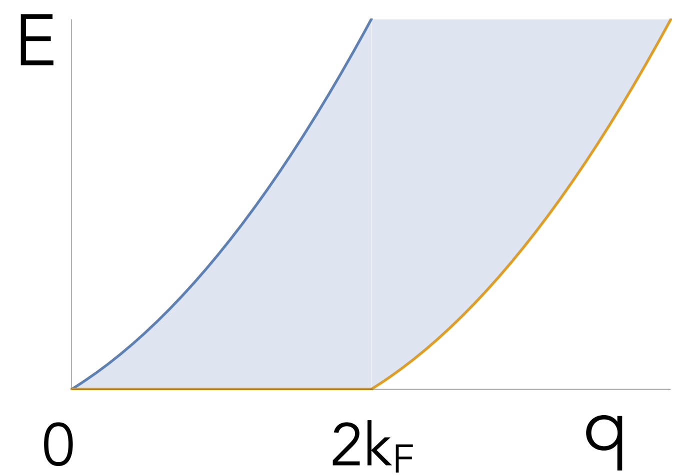
Sachdev, Metlitski, Senthil, McGreevy…very possibly >1/2 the UQM panel??

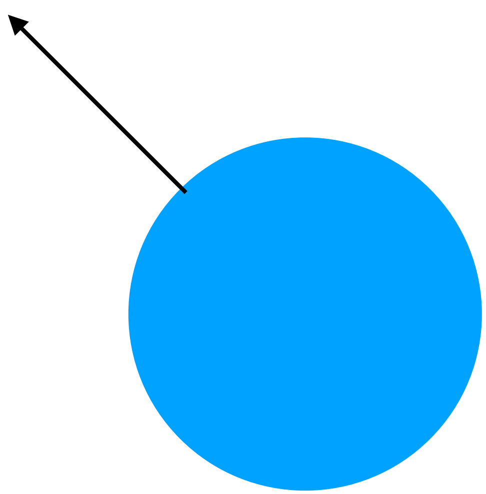
Free particles: p/h continuum

Fermi surface



lowest energy for $k<2k_F$



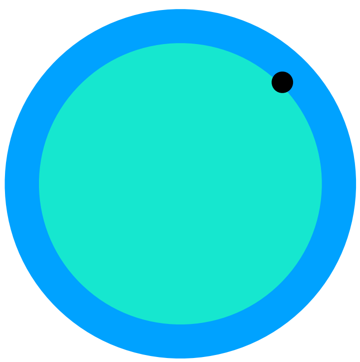


maximum energy

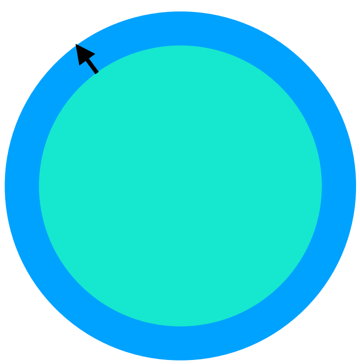
particle-hole continuum

With Zeeman field

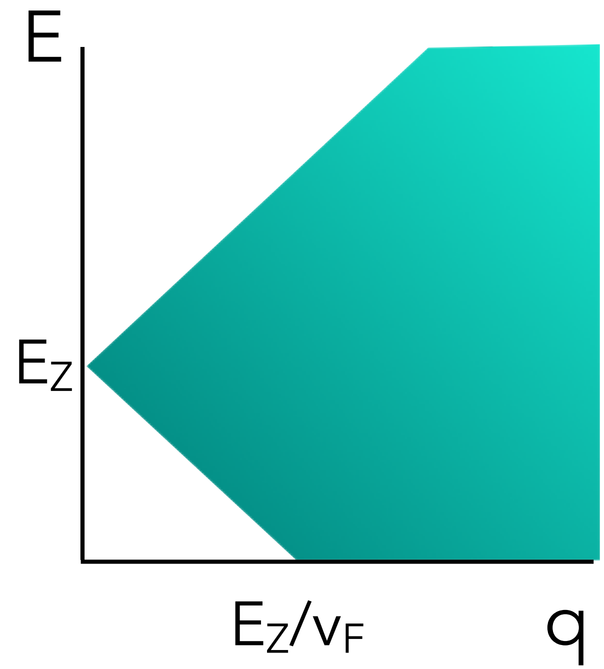
$$\chi_{\pm}(\boldsymbol{q},\omega) = i\int_0^\infty dt\langle[S^\dagger_{\mathbf{q}}(t), S^-_{-\mathbf{q}}(0)]\rangle e^{i\omega t}$$



q=0 costs Zeeman energy



zero energy when $v_F q$ = Zeeman



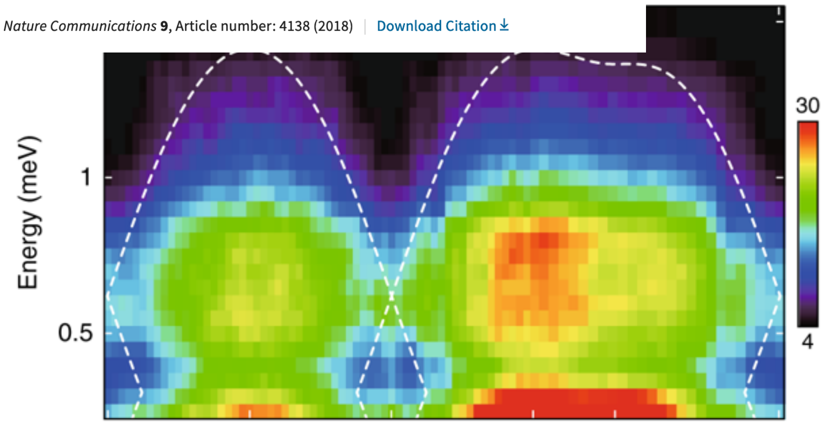
$YbMgGaO_4$

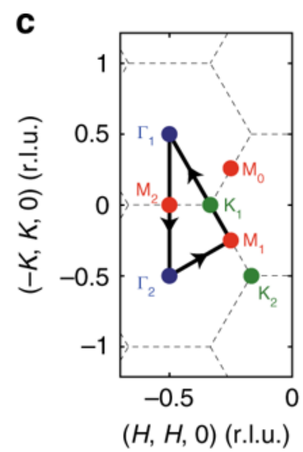
Article | OPEN | Published: 08 October 2018

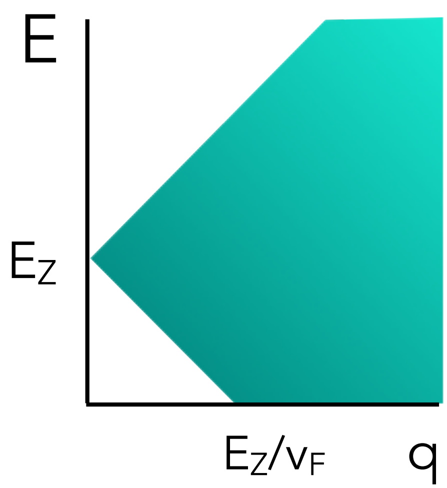
Fractionalized excitations in the partially magnetized spin liquid candidate $YbMgGaO_4$

Yao Shen, Yao-Dong Li, H. C. Walker, P. Steffens, M. Boehm, Xiaowen Zhang, Shoudong Shen, Hongliang Wo, Gang Chen & Jun Zhao

Nature Communications **9**, Article number: 4138 (2018) | Download Citation







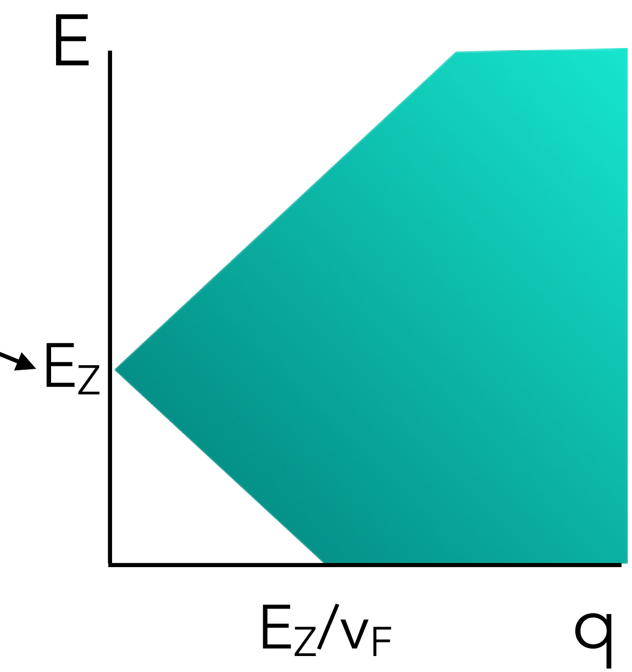
???

Effects of interactions?

Free spinons

q=0: ESR

Larmor theorem fixes response



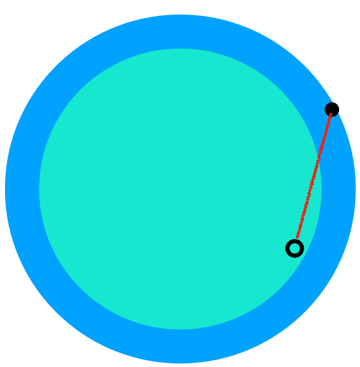
Naïvely Larmor theorem suggests free results good

Interactions

- Longitudinal

$$a_0 \psi^\dagger \psi$$

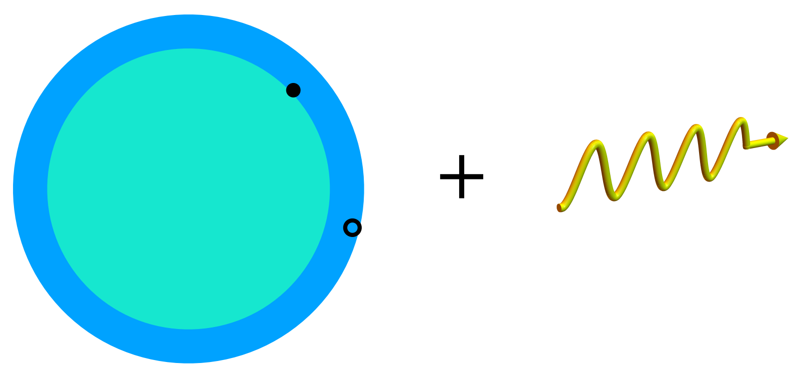
screened Coulomb interaction



- Transverse

$$i\boldsymbol{A} \cdot \left(\psi^\dagger \boldsymbol{\nabla} \psi - \boldsymbol{\nabla} \psi^\dagger \psi\right)$$

coupling to dynamical photons



Interactions

- Longitudinal $a_0\psi^\dagger\psi$ → $u\psi^\dagger_\uparrow\psi_\uparrow\psi^\dagger_\downarrow\psi_\downarrow$

$$= -um\left(\psi^\dagger_\uparrow\psi_\uparrow - \psi^\dagger_\downarrow\psi_\downarrow\right) + \; u :\psi^\dagger_\uparrow\psi_\uparrow\psi^\dagger_\downarrow\psi_\downarrow:$$

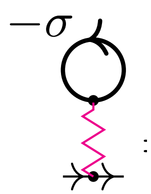
self-energy interaction

Self energy

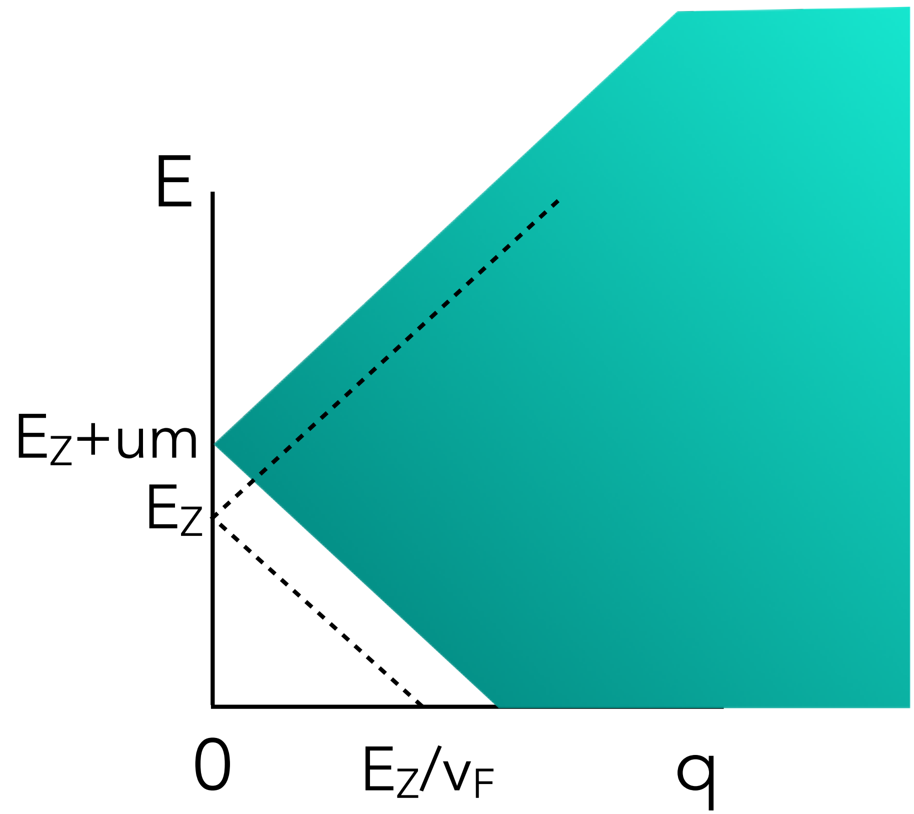
- Longitudinal $a_0 \psi^\dagger \psi \rightarrow u \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow$

$$= -um \left(\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow \right)$$

mean field shift

$\Sigma_\sigma =$ 

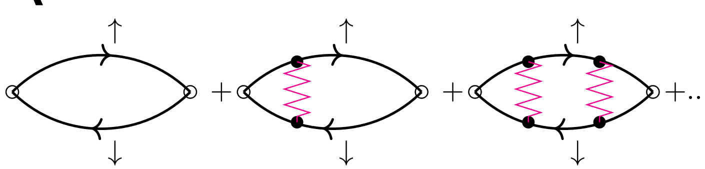
(= Hartree self-energy)



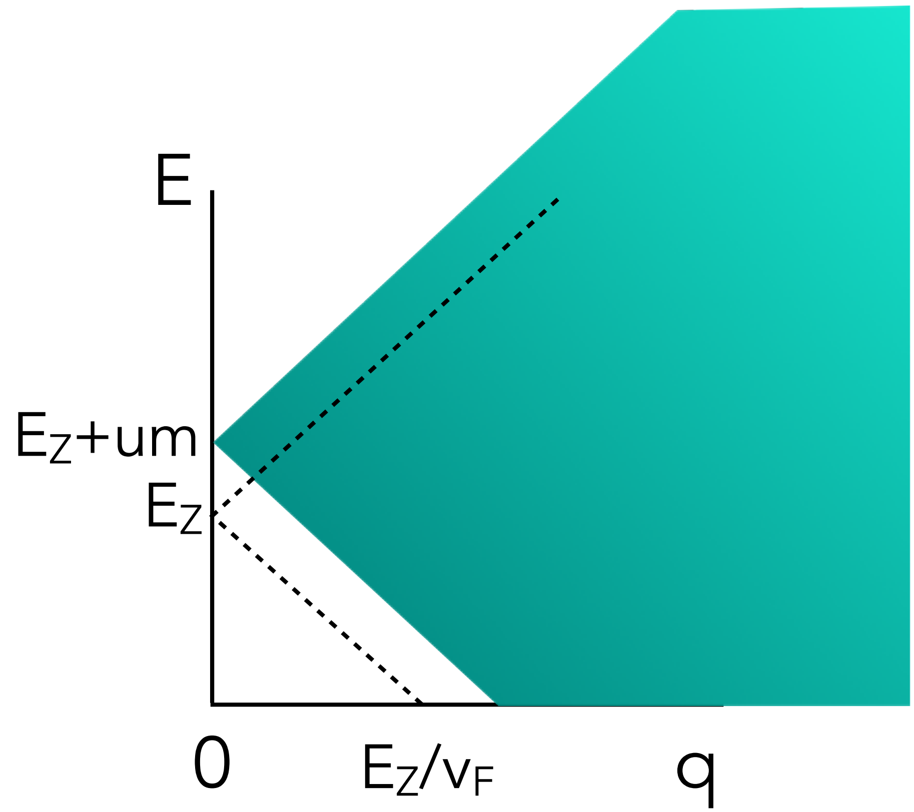
Interaction

Larmor theorem: q=0 excitation *must* be at E_Z

RPA



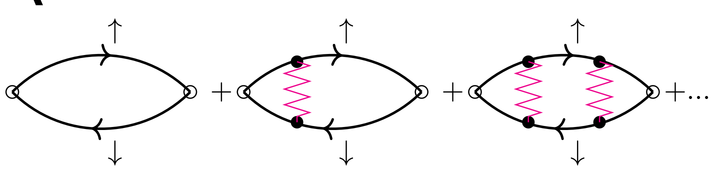
$$\chi(\boldsymbol{q}, i\omega_n) = \frac{\chi^0(\boldsymbol{q}, i\omega_n)}{1 + u\chi^0(\boldsymbol{q}, i\omega_n)}$$



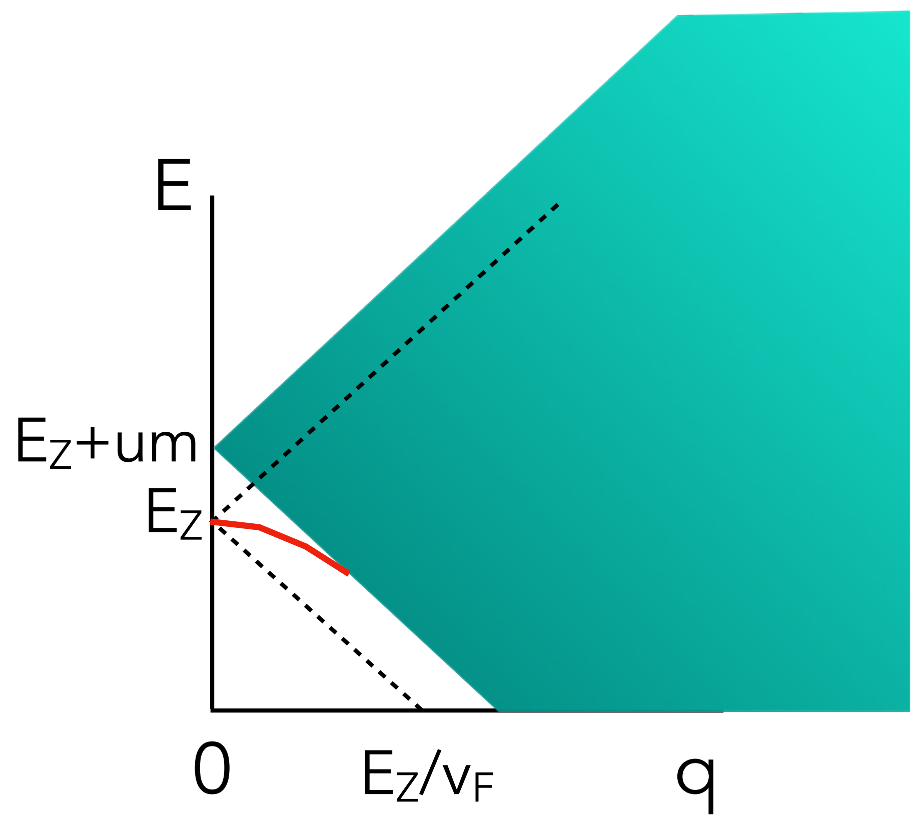
Silin spin wave

<u>Larmor theorem</u>: q=0 excitation *must* be at E_Z

RPA



$$\chi(\boldsymbol{q}, i\omega_n) = \frac{\chi^0(\boldsymbol{q}, i\omega_n)}{1 + u\chi^0(\boldsymbol{q}, i\omega_n)}$$

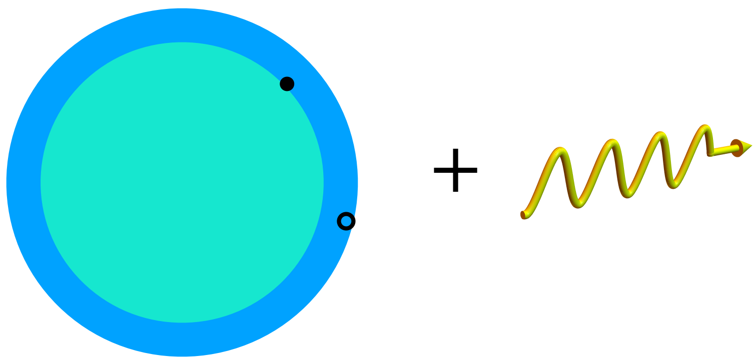


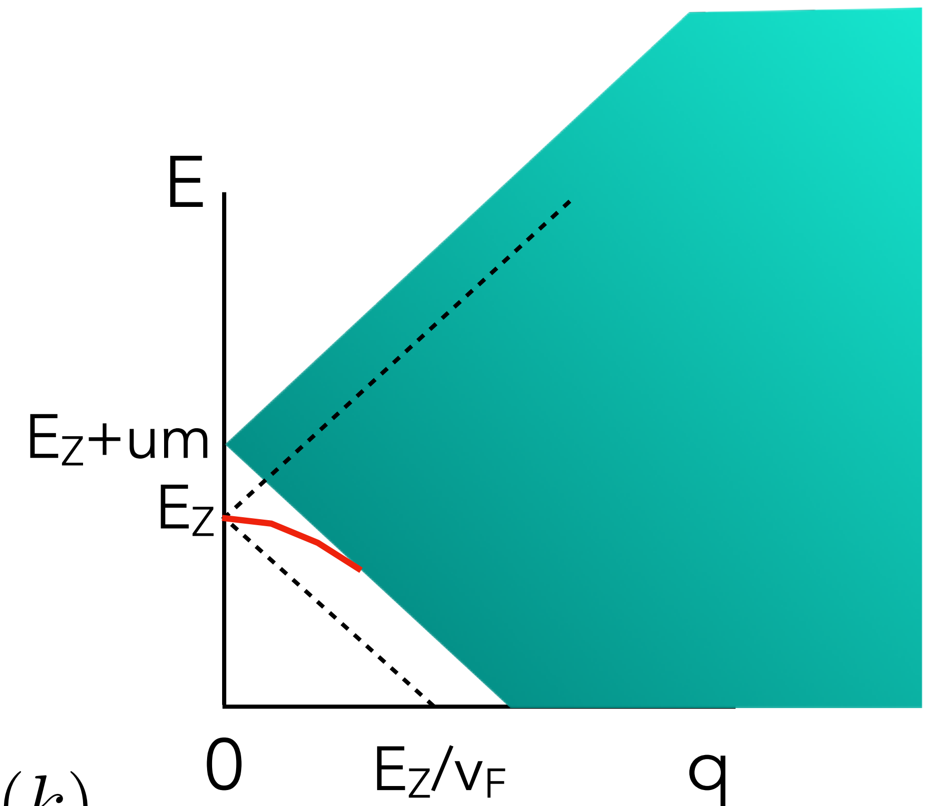
pole: collective mode

$$\omega = E_Z + um - \sqrt{u^2m^2 + v_F^2q^2}$$

"Silin spin wave"

Transverse gauge coupling





Simple picture:

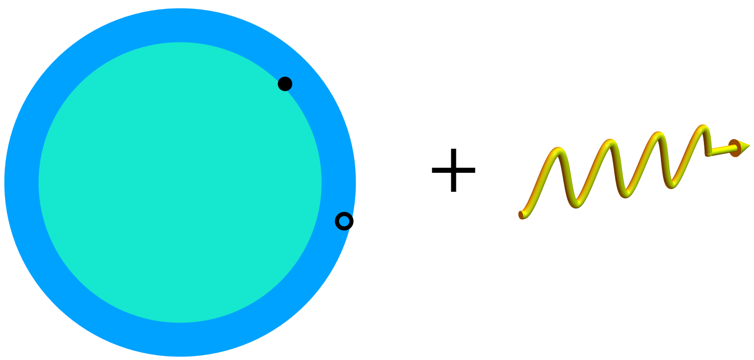
3-particle process:

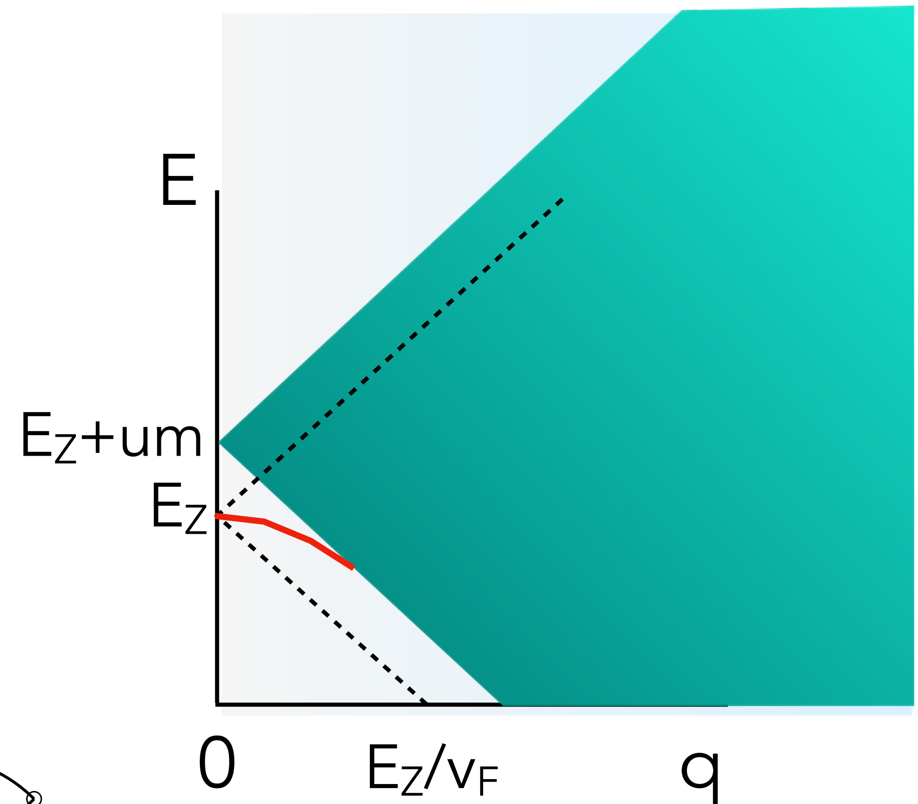
$$E = E_{\mathrm{p/h}}(q-k) + E_{\mathrm{photon}}(k)$$

$$\sim c\,k^3$$

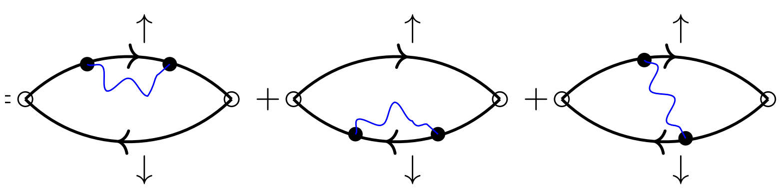
Does this smear out all the Fermi liquid structure?

Transverse gauge coupling



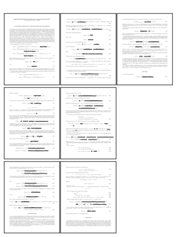


Actual calculation:



weight at all q != 0

Extends



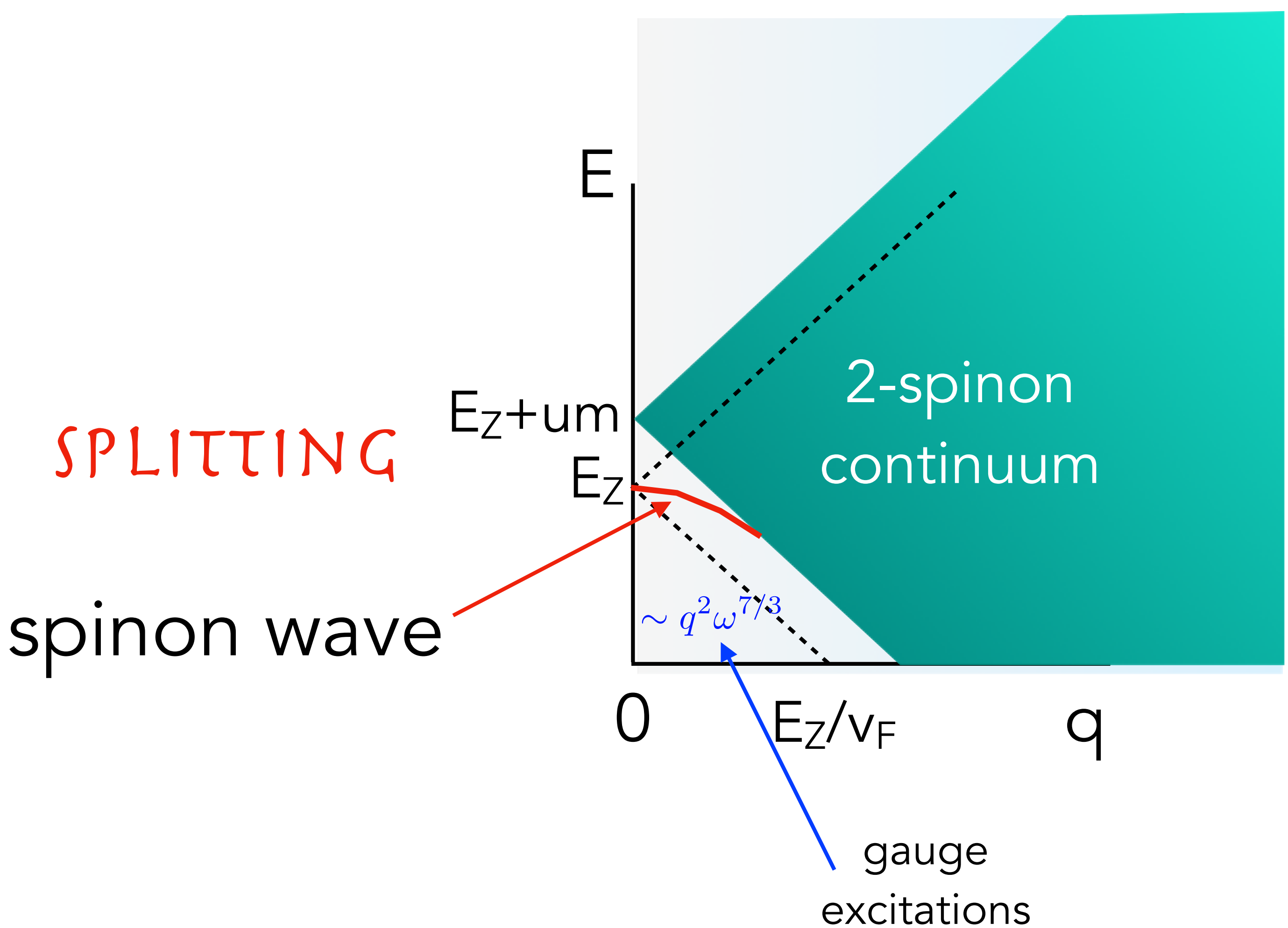
Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, Phys Rev. B **50**, 17917 (1994).

$$\mathrm{Im}\chi_{\pm} \sim q^2 \omega^{7/3}$$

but weak enough to preserve structure

Summary

Distinct signatures of spinons, interactions, and gauge fields



O.Starykh + LB,
arXiv:1904.02117
PRB **101**, 020401 (2020)

One dimension

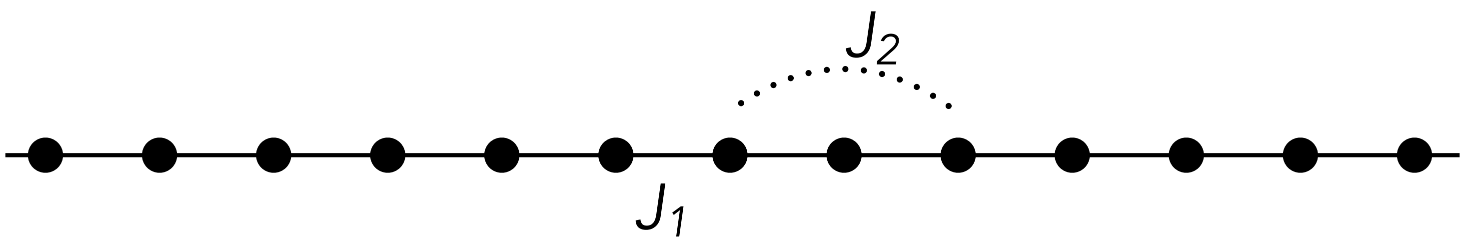
- New results: these ideas apply to *one dimensional spin chains* in low magnetic fields and can be tested there!
    - Bonus: we also will find signatures of interacting *magnons* in the high field regime

1. arXiv:2005.12399 [pdf, other] cond-mat.str-el
   Dynamical signatures of quasiparticle interactions in quantum spin chains
   Authors: Anna Keselman, Leon Balents, Oleg A. Starykh

One dimension

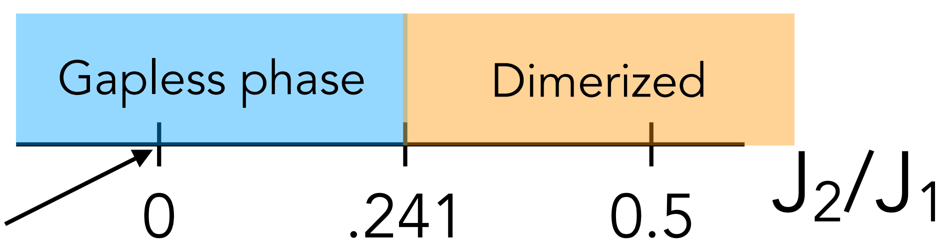
- J1-J2 Chain

$$H = \sum_i \left[J_1 \vec{S}_i \cdot \vec{S}_{i+1} + J_2 \vec{S}_i \cdot \vec{S}_{i+2} - B S_i^z \right]$$



- Phase diagram for B=0

solved by Bethe in 1931!



Majumdar-Gosh point

Gapless phase

- Wess-Zumino-Witten $SU(2)_1$ CFT
- Many representations:
  - matrix non-linear sigma model
  - free masses scalar field theory (abelian bosonization)
  - Sugarawa (current algebra) form
  - **Free fermions (most useful today)**

Fermion representation

- Spins $$\vec{S}_i \sim \vec{J}_R(x_i) + \vec{J}_L(x_i) + (-1)^i \vec{N}(x_i)$$

$$\vec{J}_{R/L} = \tfrac{1}{2}\psi^\dagger_{R/L}\vec{\sigma}\,\psi_{R/L}$$

- Hamiltonian H = H_0 + V

$$H_0 = v\int dx\Big(\psi^\dagger_R(-i\partial_x)\psi_R + \psi^\dagger_L(i\partial_x)\psi_L\Big) \qquad \psi_R = (\psi_{R\uparrow}, \psi_{R\downarrow})^T$$

$$V = -g\int dx\,\vec{J}_R\cdot\vec{J}_L = -g\int dx\,\left[J^z_R J^z_L + \tfrac{1}{2}J^+_R J^-_L + \tfrac{1}{2}J^-_R J^+_L\right]$$

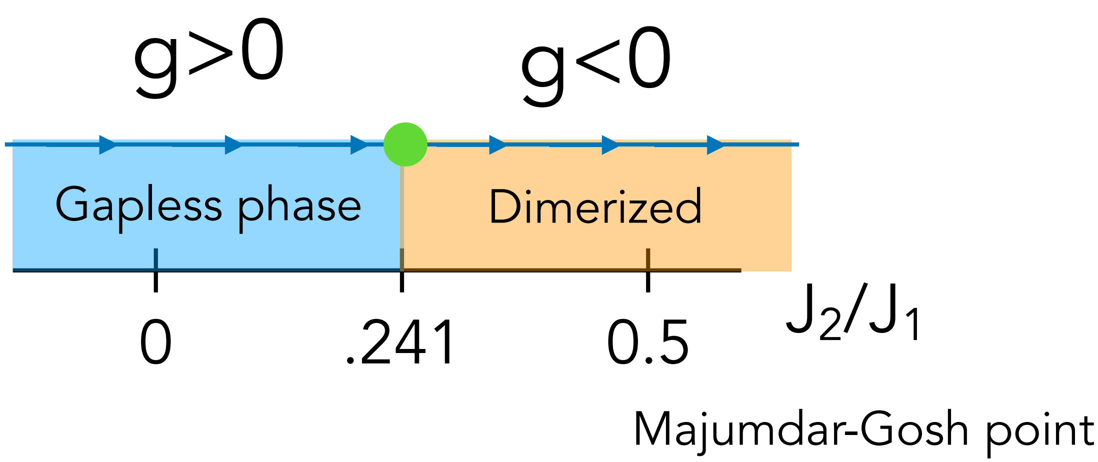
- Fermions contain decoupled charge mode which does not affect spin operators or correlations (spin-charge separation)

Backscattering

- Understanding the phase diagram

$$H_0 = v \int dx \Big(\psi_R^\dagger (-i\partial_x)\psi_R + \psi_L^\dagger (i\partial_x)\psi_L \Big)$$

$$V = -g \int dx\, \vec{J}_R \cdot \vec{J}_L = -g \int dx\, \left[J_R^z J_L^z + \tfrac{1}{2} J_R^+ J_L^- + \tfrac{1}{2} J_R^- J_L^+ \right]$$



- Renormalization group $\dfrac{dg}{d\ell} = -g^2$

"marginally irrelevant"
in critical phase

Free fermions??

PHYSICAL REVIEW B, VOLUME 65, 134410

Electron spin resonance in $S=\frac{1}{2}$ antiferromagnetic chains

Masaki Oshikawa[1] and Ian Affleck[2,*]
[1]Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan
[2]Department of Physics, Boston University, Boston, Massachusetts 02215
(Received 13 August 2001; published 19 March 2002)

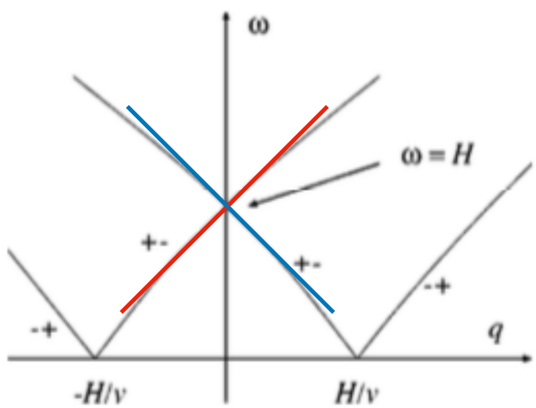


FIG. 2. The zero temperature transverse spin structure factor $S_{xx}(\omega,q)=S_{yy}(\omega,q)$ of the $S=1/2$ Heisenberg antiferromagnetic chain under an applied field H, near $q=0$. It is approximately proportional to $\omega[\delta(\omega-|q-H|)+\delta(\omega-|q+H|)]$, giving the resonance at $q=0$, $\omega=H$. This consists of two branches coming from S_{+-} and S_{-+}, which are marked by $+-$ and $-+$ in the graph. In fact, there is a small spreading of the spectrum and the structure factor is generally not a perfect delta function. However, it is exactly the delta function $\delta(\omega-H)$ at $q=0$, as explained in the text.

$$S_{xx}(\omega,q)=S_{yy}(\omega,q)\propto\omega[\delta(\omega-|q+H|)+\delta(\omega-|q-H|)].$$

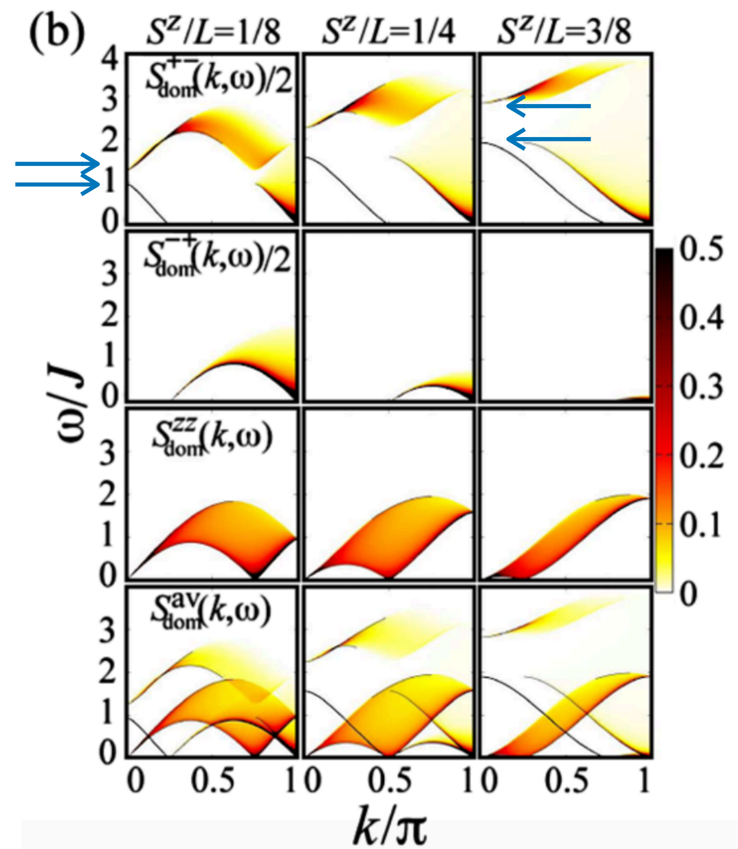
Free fermion S(q,ω) in 1d

BUT

???

Dynamically Dominant Excitations of String Solutions in the Spin-1/2 Antiferromagnetic Heisenberg Chain in a Magnetic Field

Masanori Kohno
Phys. Rev. Lett. **102**, 037203 – Published 22 January 2009



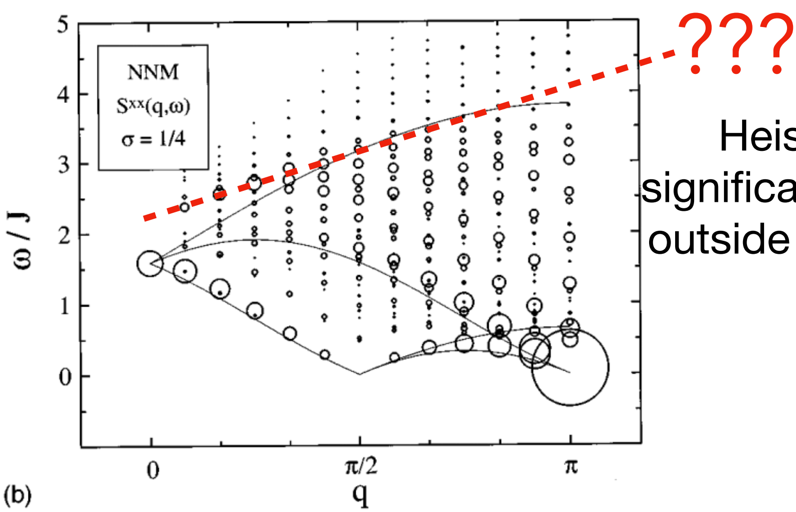
PHYSICAL REVIEW B VOLUME 54, NUMBER 9 1 SEPTEMBER 1996-I

Dynamical correlation functions of the $S=1/2$ nearest-neighbor and Haldane-Shastry Heisenberg antiferromagnetic chains in zero and applied fields

Kim Lefmann*
Department of Solid State Physics, Risø National Laboratory, DK-4000 Roskilde, Denmark

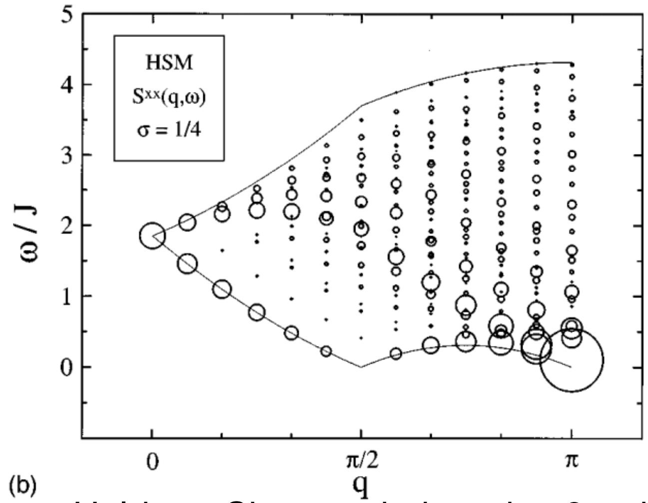
Christian Rischel[†]
Ørsted Laboratory, Niels Bohr Institute, University of Copenhagen, DK-2100 København Ø, Denmark
(Received 12 February 1996)

We present a numerical diagonalization study of two one-dimensional $S=1/2$ antiferromagnetic Heisenberg chains, having nearest-neighbor and Haldane-Shastry $(1/r^2)$ interactions, respectively. We have obtained the $T=0$ dynamical correlation function, $S^{\alpha\alpha}(q,\omega)$, for chains of length $N=8-28$. We have studied $S^{zz}(q,\omega)$ for the Heisenberg chain in zero field, and from finite-size scaling we have obtained a limiting behavior that for large ω deviates from the conjecture proposed earlier by Müller *et al.* For both chains we describe the behavior of $S^{zz}(q,\omega)$ and $S^{xx}(q,\omega)$ for selected values of the applied field, and compare with previous work by Müller *et al.* and Talstra and Haldane. Suggestions for future finite-field neutron scattering experiments are made. [S0163-1829(96)00733-3]



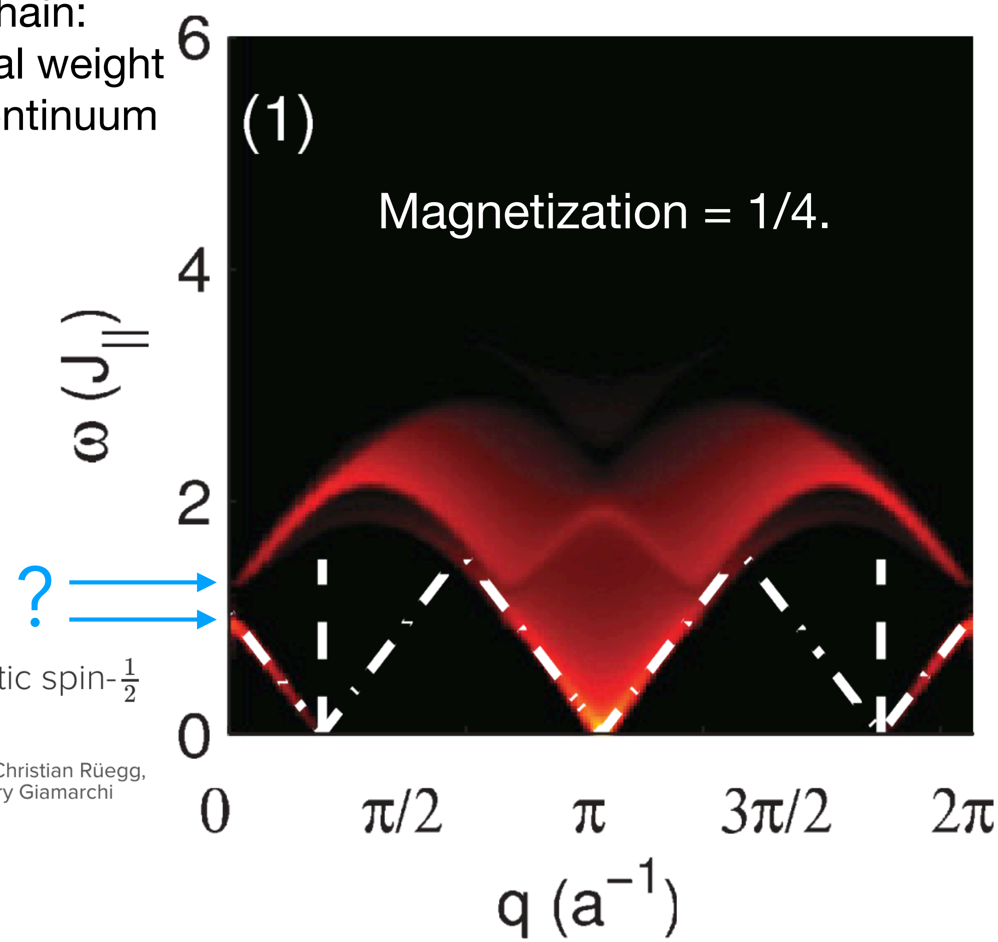
(b)

Heisenberg chain: significant spectral weight outside Muller continuum



(b)

Haldane-Shastry chain - nice 2-spinon continuum



Statics and dynamics of weakly coupled antiferromagnetic spin-$\frac{1}{2}$ ladders in a magnetic field

Pierre Bouillot, Corinna Kollath, Andreas M. Läuchli, Mikhail Zvonarev, Benedikt Thielemann, Christian Rüegg, Edmond Orignac, Roberta Citro, Martin Klanjšek, Claude Berthier, Mladen Horvatić, and Thierry Giamarchi
Phys. Rev. B **83**, 054407 – Published 9 February 2011

Backscattering

- RG $$\frac{dg}{d\ell} = -g^2$$

Flow should be cut off by the Zeeman energy

- Interaction:

$$V = -g \int dx\, \vec{J}_R \cdot \vec{J}_L = -g \int dx\, \left[J_R^z J_L^z + \tfrac{1}{2} J_R^+ J_L^- + \tfrac{1}{2} J_R^- J_L^+ \right]$$

Renormalizes *spinon* Zeeman splitting

$$B \to B + gM$$

"vertex corrections": collective modes

RPA-like formula

$$G = \frac{G^0_{RR} + G^0_{LL} - g G^0_{RR} G^0_{LL}}{1 - (g/2)^2 G^0_{RR} G^0_{LL}}$$

Result

- Structure factor

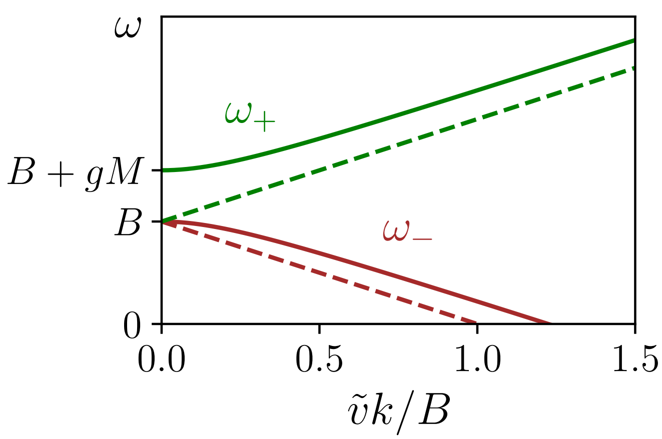
$$\chi^{\pm}(k,\omega) = M\left(\frac{A_+(k)}{\omega-\omega_+(k)}+\frac{A_-(k)}{\omega-\omega_-(k)}\right)$$

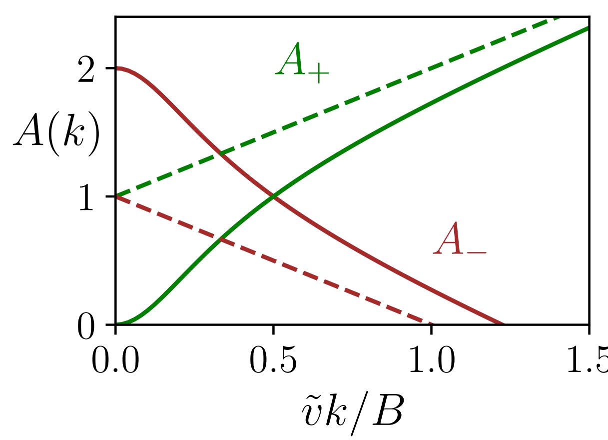
$$A_{\pm}(k) = 1\pm\frac{\tilde{v}^2k^2-B\Delta}{B\sqrt{\Delta^2+\tilde{v}^2k^2}},$$

$$\omega_{\pm}(k) = B+\Delta\pm\sqrt{\Delta^2+\tilde{v}^2k^2}.$$

Mode splitting

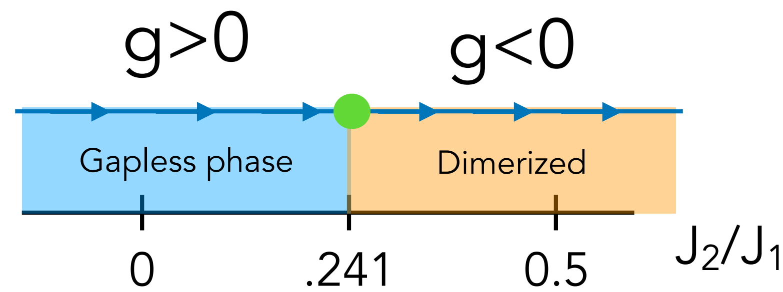
Direct measure of spinon interactions





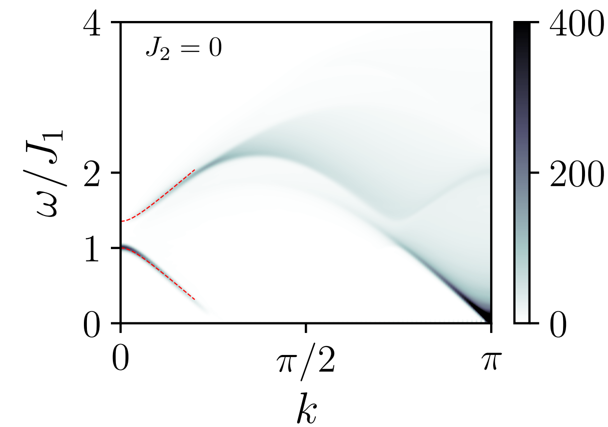
Spectral weight: lower branch dominant (c.f. exciton)

Simulations



- MPS methods: DMRG+TEBD

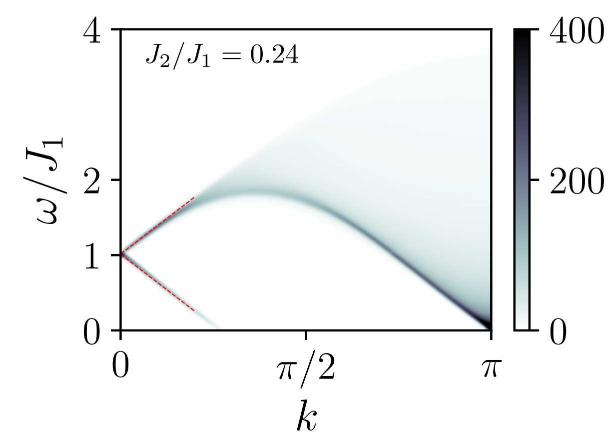
NN chain



$B/J_1=1$

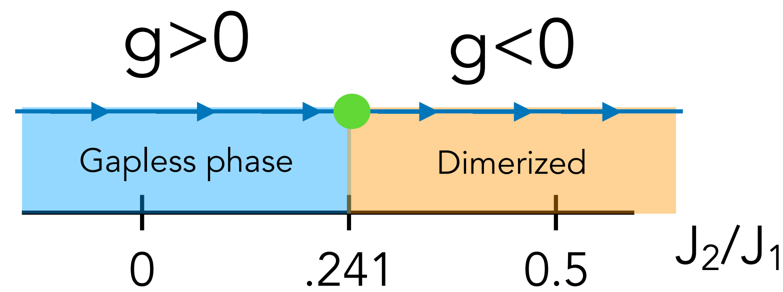
Near QCP

$g \approx 0$



Simulations

- MPS methods: DMRG+TEBD

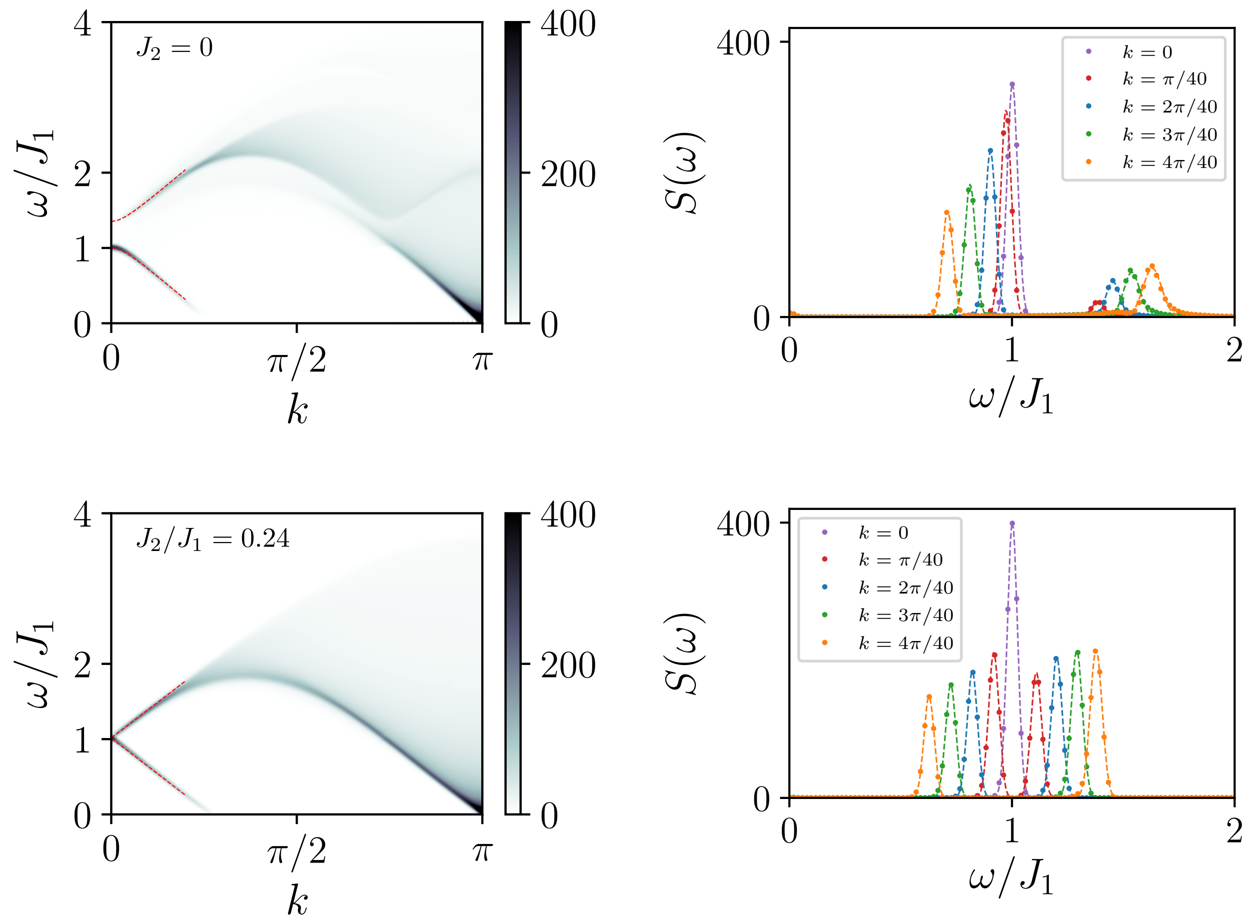


NN chain

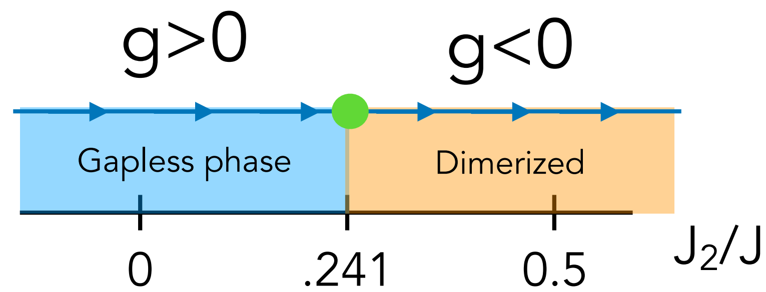
$B/J_1=1$

Near QCP

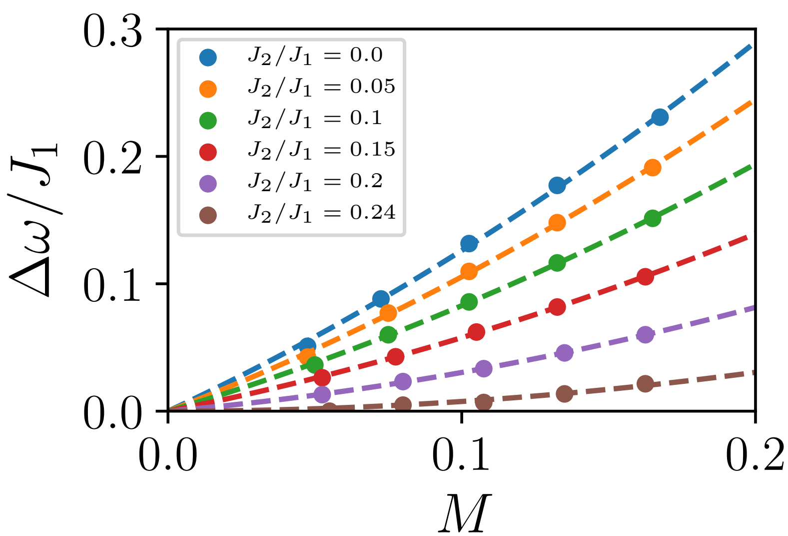
$g \approx 0$



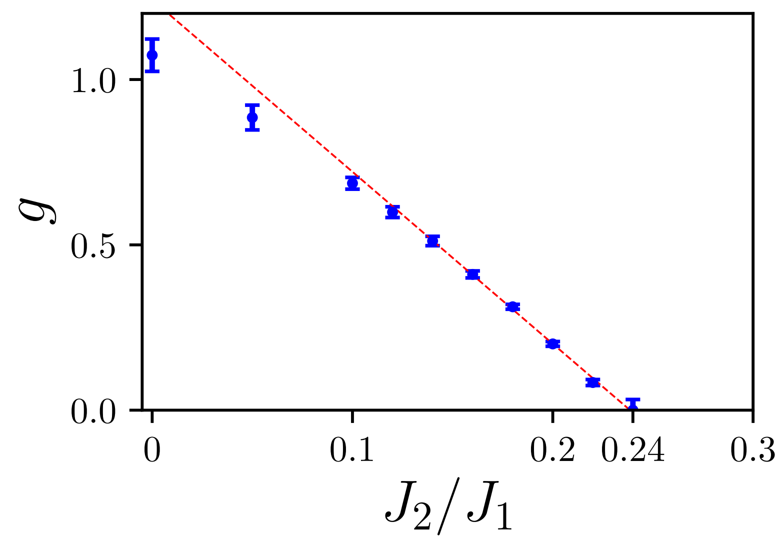
Simulations



- MPS methods: DMRG+TEBD
  - Systematics:

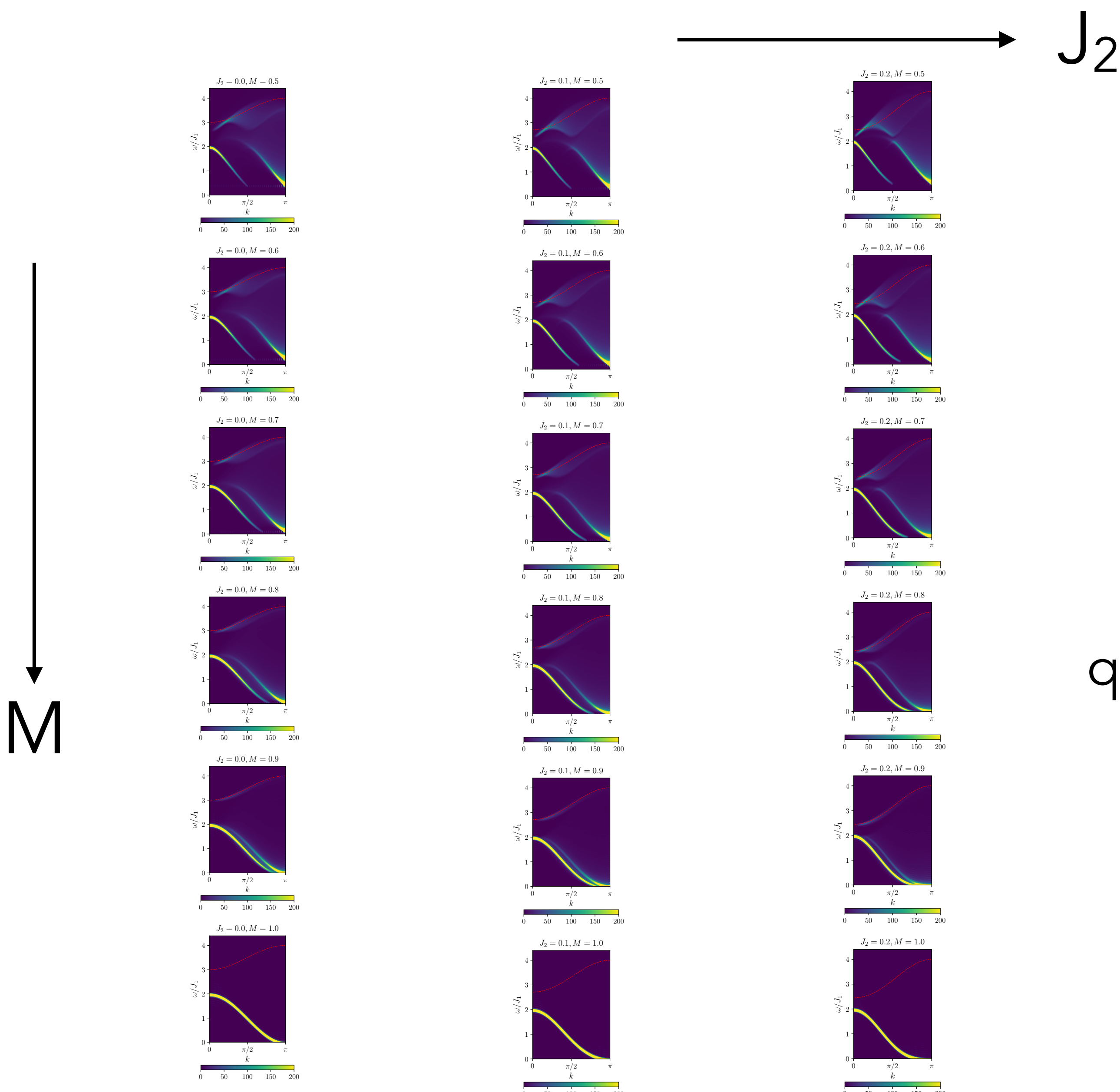


$$\Delta\omega = gM + \alpha M^2$$

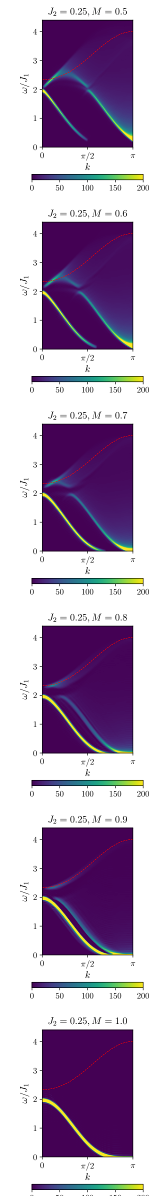


Theory works :)

Higher Magnetization



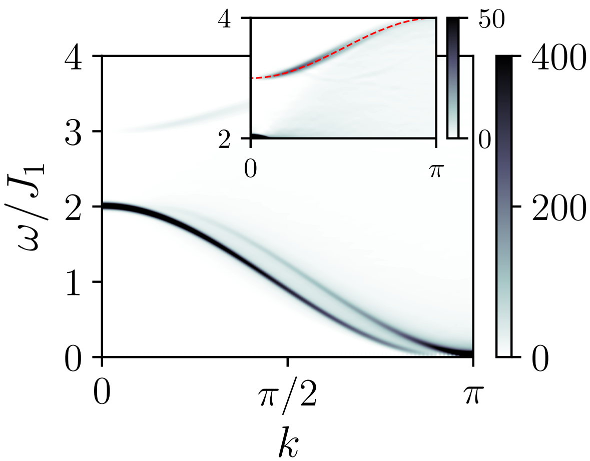
q=0 splitting does not vanish for large M at g=.25



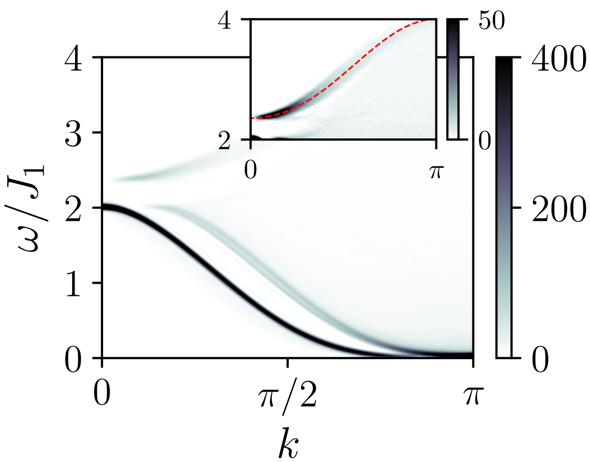
Higher Magnetization

M/M_s=.9

J_2=0



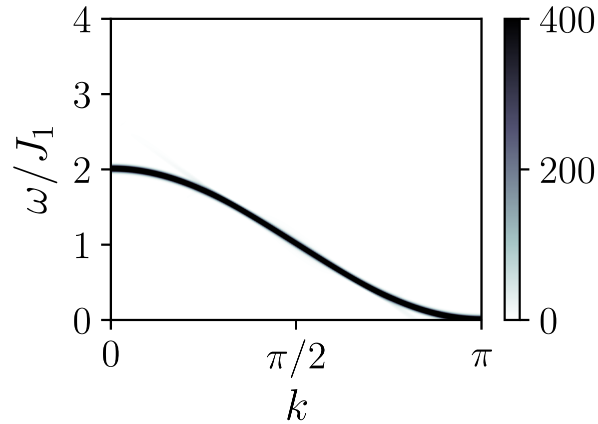
J_2=.24



- k=0 gap persistent
- Lower mode has most of weight and slightly split
- Upper mode with small weight

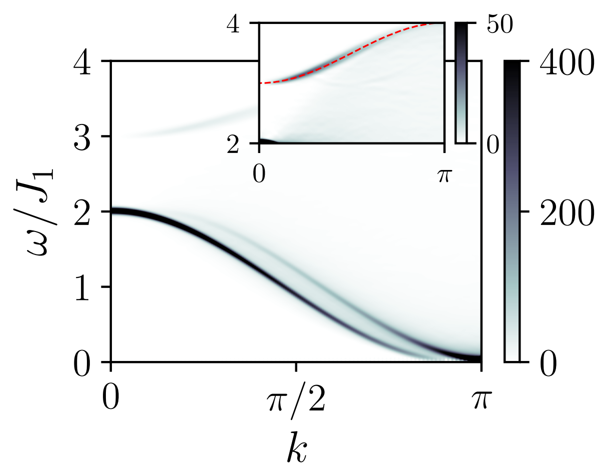
Higher Magnetization

$M/M_s=1$
$J_2=0$



$$|k\rangle = \frac{1}{\sqrt{L}}\sum_x e^{ikx} S_x^- |\uparrow\cdots\uparrow\rangle$$

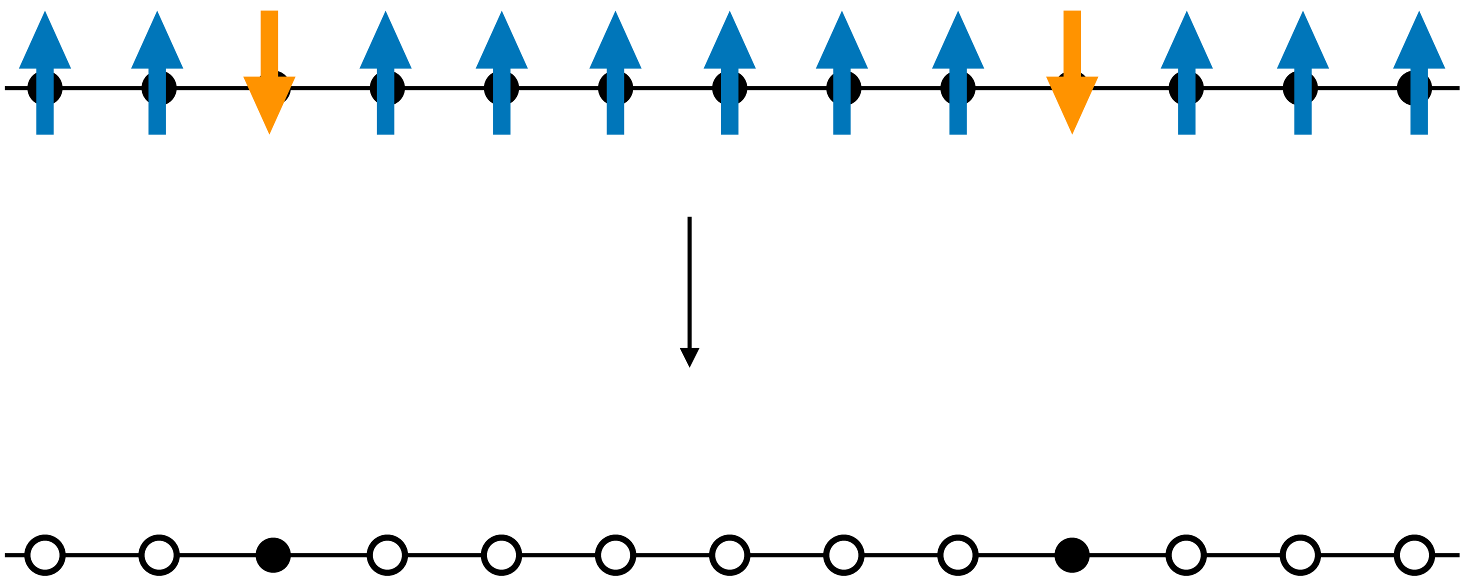
$M/M_s=.9$



- Lower mode(s) clearly descend from single magnon of the ferromagnet
- Upper mode: spectral weight transfer to *large* energy upon small "doping" with spin flips

Picture

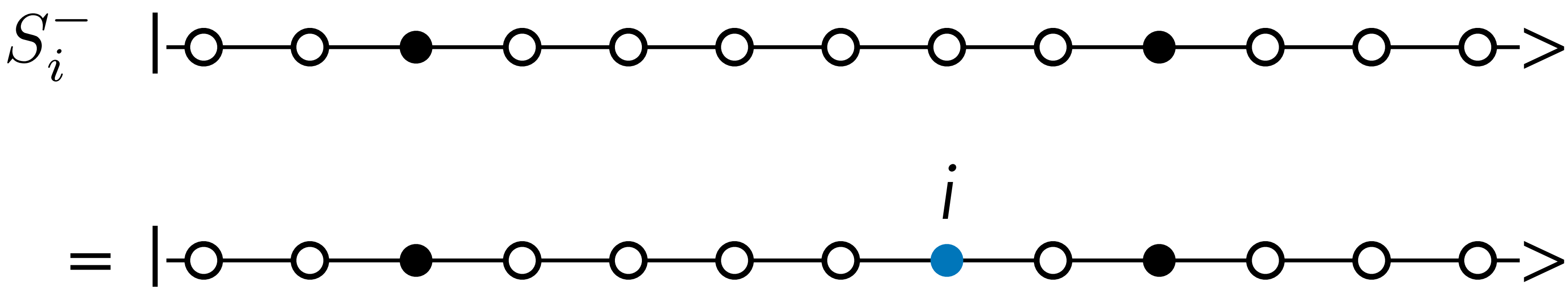
- Spin flip gas



~Tonks gas

Picture

- Spin flip $\quad S_i^- \sim c_i^\dagger$



Extra particle can be ~free *or* bind to one of the existing particles <u>if they interact</u>!

Bound state

- Two magnons $|2_K\rangle = \sum_{m,n} \psi_{m,n} S_m^- S_n^- |0\rangle$ $\psi_{m,n} = e^{iK\left(\frac{m+n}{2}\right)} f(m-n)$
  - Easy to show there is a bound state outside the two-magnon continuum

- Approximation: finite system with one spin flip in box of size 1/(density of spin flips)

$$\langle S_k^+ \delta(\omega - H) S_k^- \rangle_n \sim \langle 1_\pi | S_k^+ \delta(\omega - H) S_k^- | 1_\pi \rangle_{L=1/n}$$

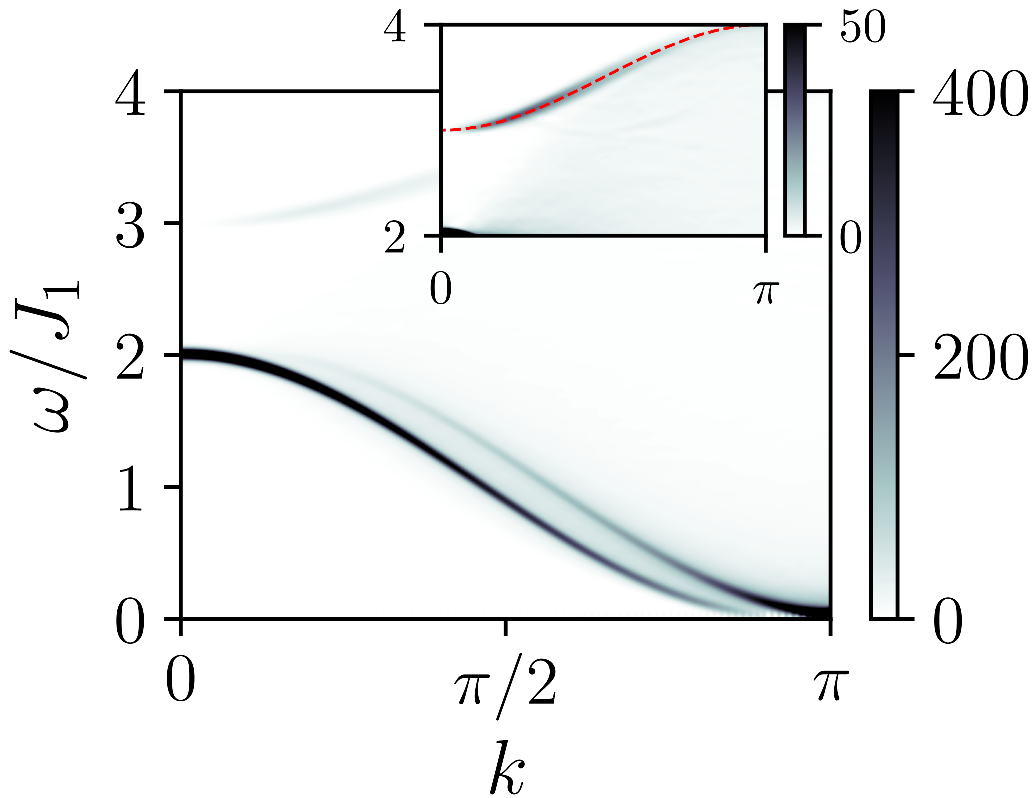
$$\sim \cdots + |\langle 2_{\pi+K} | S_k^- | 1_\pi \rangle|^2_{L=1/n} \delta(\omega - \epsilon_2(k+\pi))$$

2-magnon bound state appears with weight ~ n ~ M_s-M

Bound state

- Check

Red dashed line = calculated bound state energy

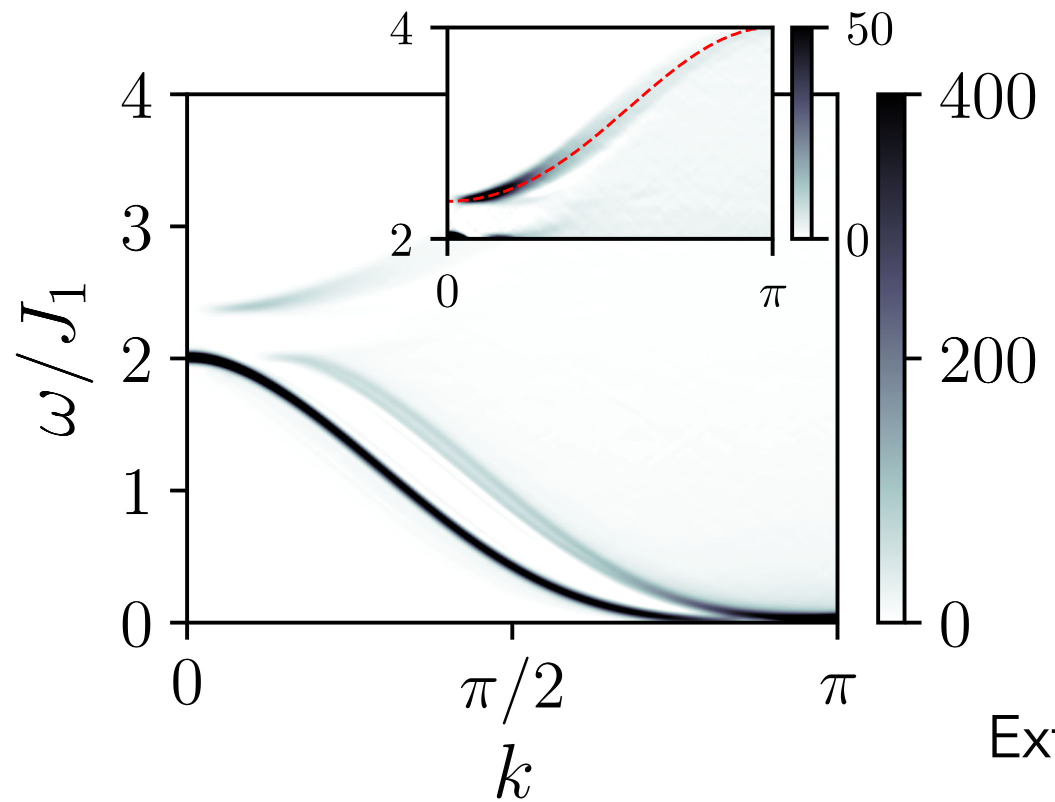


c.f. Bethe ansatz "strings"

Bound state

- Check

Red dashed line = calculated bound state energy



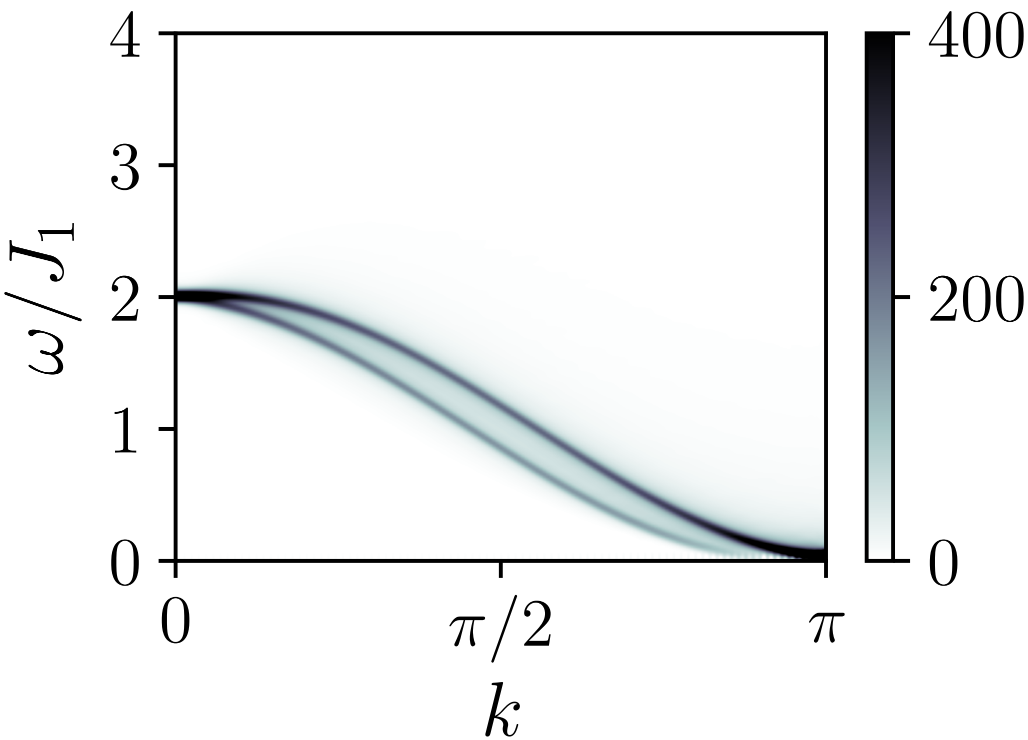
J_2/J_1=.24

But not related to integrability

Extremely general phenomena of spectral weight transfer in low density correlated systems

Bound state

- Check: does it really come from magnon interactions?
  - XX model (equivalent to free fermions)



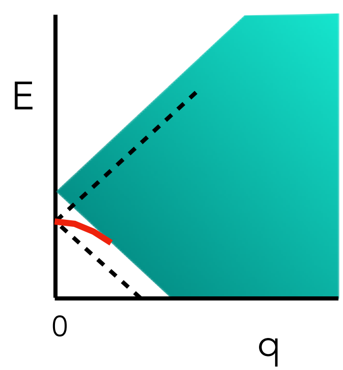
Bound state entirely absent

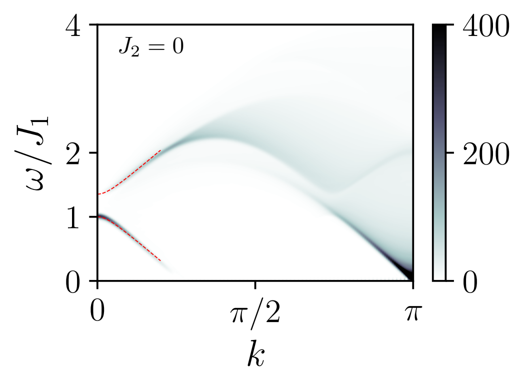
Also see that lower mode splitting is *not* an interaction effect. It arises from Jordan-Wigner string

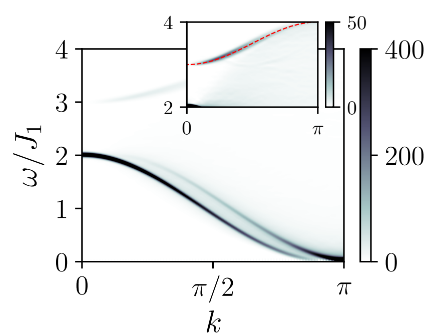
Summary

- General lesson: interactions between emergent quasiparticles are generically strong and can significantly modify spectral response of local operators

- We identified simple spectral signatures of quasiparticle interactions (spinons or magnons) in 1d chains and 2d spin liquids







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