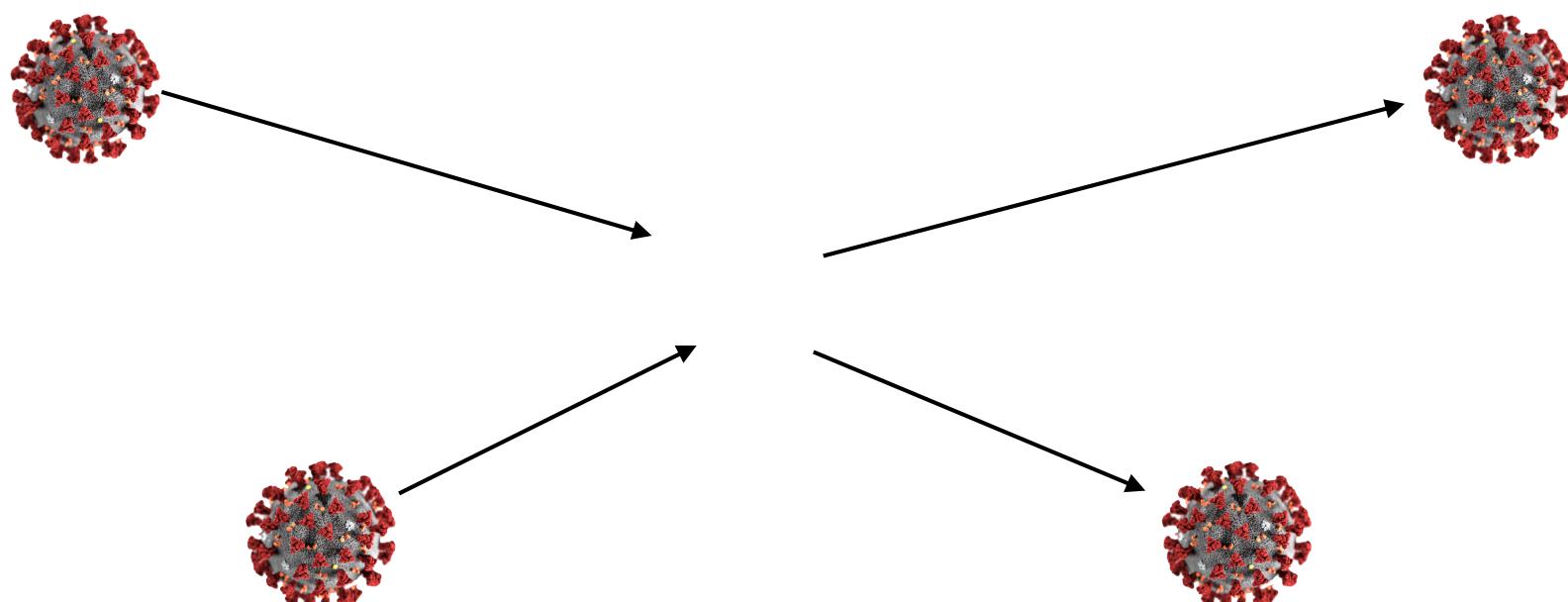


Spectral Signatures of Quasiparticle Interactions in Spin Liquids and Heisenberg Chains



Collaborators



Oleg Starykh, U. Utah



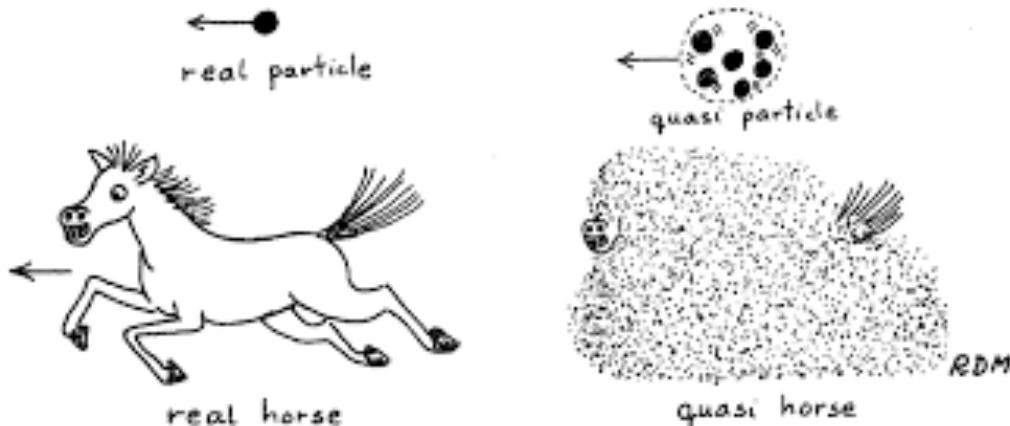
Anna Keselman, KITP

Outline

- A bit about quasiparticles
- Dynamical susceptibility of a spinon Fermi surface in a small Zeeman field
 - Interactions induce a gap between two “optical” modes
- Dynamical susceptibility of 1d spin chains
 - Similar effect at low fields, new effects at high fields

Quasiparticles

- Fundamental excitations of a many body ground state
- Behave like particles: single quasiparticle is long-lived



(c) RD Mattuck

Quasiparticles

- Fundamental excitations of a many body ground state
- Behave like particles: single quasiparticle is long-lived
- Example: semiconductor

$$H_0 = \sum_{n,k} \epsilon_{nk} c_{nk}^\dagger c_{nk} \quad H' = \frac{1}{2V} \sum_{n_1 \dots n_4} \sum_{k_1 \dots k_4} U_{n_1 n_2 n_3 n_4} (k_1 k_2 k_3 k_4) c_{n_1 k_1}^\dagger c_{n_2 k_2}^\dagger c_{n_3 k_3} c_{n_4 k_4} \delta_{k_1+k_2, k_3+k_4}$$

Adiabatic continuity

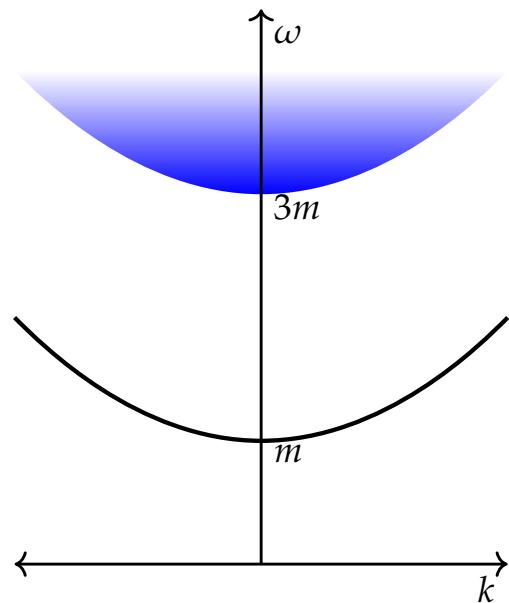
$$|0\rangle = \left| \begin{array}{c} \text{wavy line} \\ \text{dots} \end{array} \right\rangle = \left| \begin{array}{c} \text{wavy line} \\ \text{dots} \end{array} \right\rangle_0 + \left| \begin{array}{c} \bullet \\ \text{wavy line} \\ \text{dots} \end{array} \right\rangle_0 + \left| \begin{array}{c} \bullet \\ \text{wavy line} \\ \text{dots} \end{array} \right\rangle_0 + \dots$$

$$|k\rangle = \left| \begin{array}{c} \text{wavy line} \\ \bullet \end{array} \right\rangle = \left| \begin{array}{c} \text{wavy line} \\ \bullet \end{array} \right\rangle_0 + \left| \begin{array}{c} \bullet \\ \text{wavy line} \\ \bullet \end{array} \right\rangle_0 + \dots$$

Quasiparticles

- Fundamental excitations of a many body ground state
- Behave like particles: single quasiparticle is long-lived
- Example: semiconductor

1-e spectral function



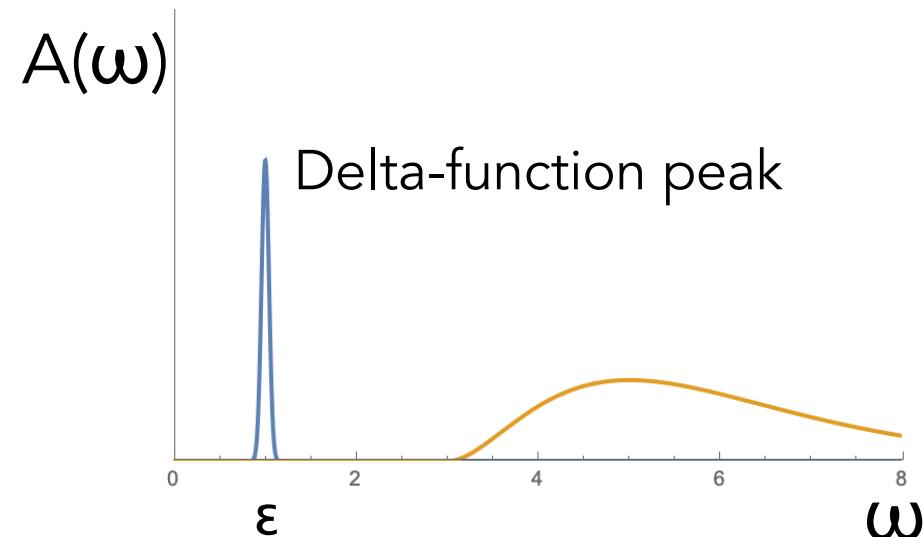
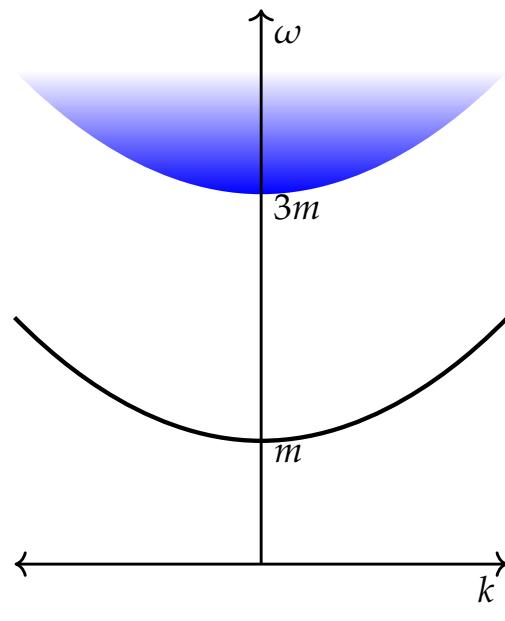
2 quasi-electrons+1 quasi-hole

1 quasi-electron

Quasiparticles

- Fundamental excitations of a many body ground state
- Behave like particles: single quasiparticle is long-lived
- Example: semiconductor

1-e spectral function



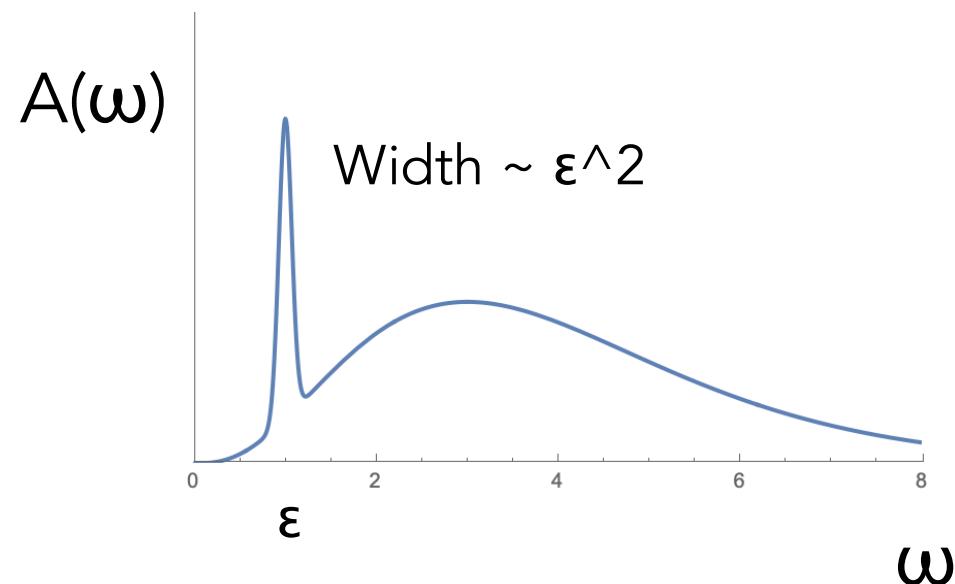
Quasiparticles

- Fundamental excitations of a many body ground state
- Behave like particles: single quasiparticle is long-lived
- Example: metal

1-e spectral function

Fermi Liquid

Quasi electron decay
rate is much smaller
than its energy



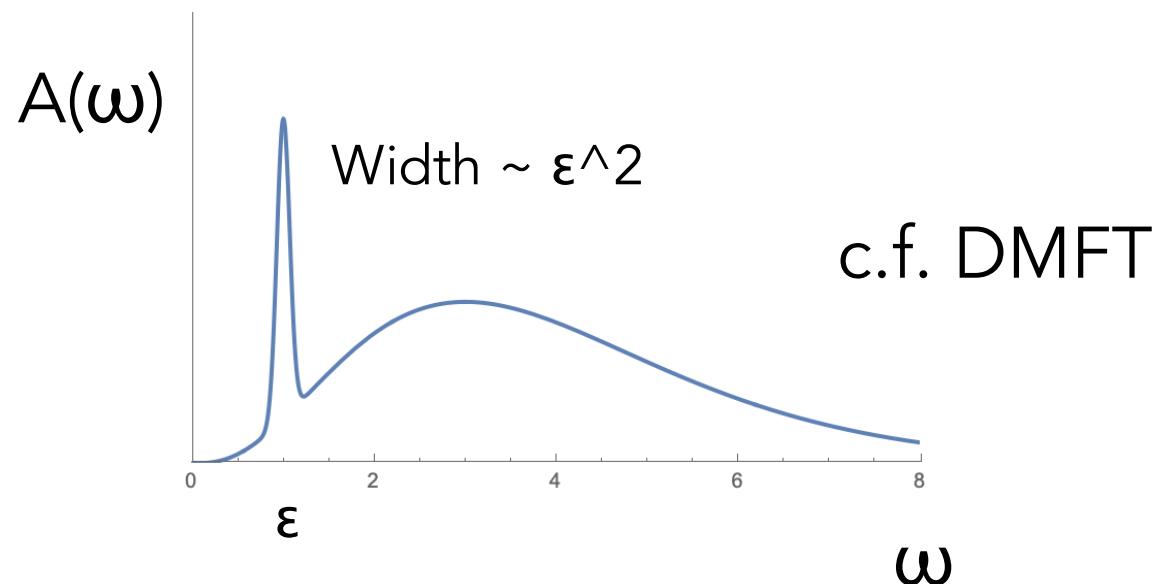
Quasiparticles

- Fundamental excitations of a many body ground state
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1-e spectral function

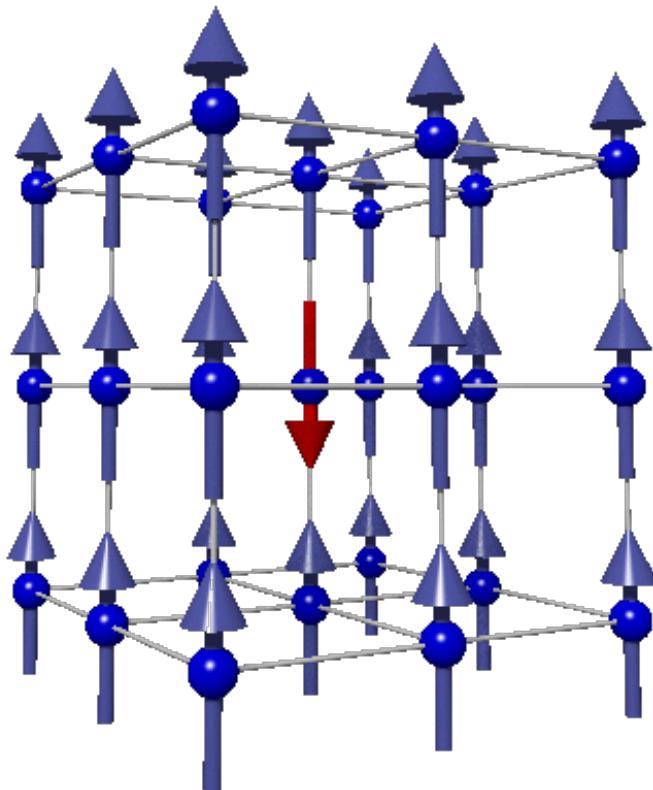
Fermi Liquid

Quasi electron decay
rate is much smaller
than its energy



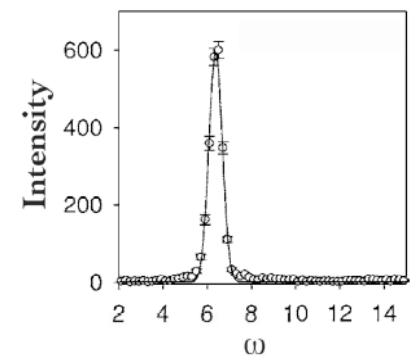
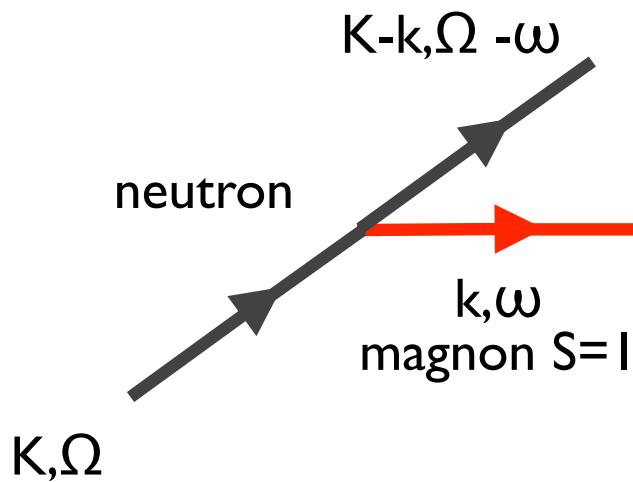
Quasiparticles

- Spin wave: bosonic quasiparticle in a magnet



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

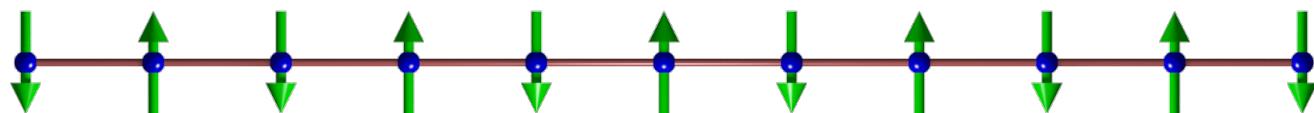
$$|f\rangle = S_k^+ |i\rangle$$



Line shape in Rb_2MnF_4

Exotic quasiparticles

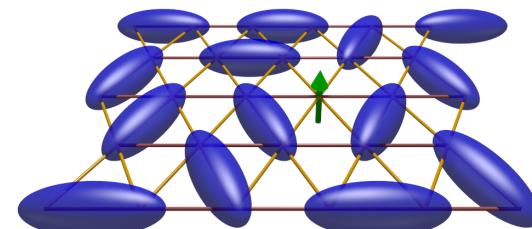
- Fractional/non-local quasiparticles can be emergent
 - 1d domain walls (Ising AF, SSH model)



- Laughlin QPs



- Spinon in 2d spin liquid



Exotic quasiparticles

- Fractional/non-local quasiparticles can be emergent
 - Still long-lived when isolated
 - Not adiabatically connected to any bare particle
 - *Any local operator creates at least 2 of them at a time*

There is no physical “1 particle spectral function” for these quasiparticles

Interactions

- Even though quasiparticles are long-lived, they interact
- e.g. Semiconductor electron gas

$$H = \sum_i \frac{p_i^2}{2m^*} + \frac{1}{2} \sum_{i < j} \frac{e^2}{\varepsilon |r_i - r_j|}$$

- e.g. Fermi liquid

$$H = \sum_k \epsilon_k n_k + \frac{1}{2V} \sum_{k,k'} f_{k,k'} n_k n_{k'}$$

Landau parameters affect 2-particle responses, e.g. compressibility, susceptibility

Interactions

- Exotic quasiparticles:
 - Interactions are more *inevitable* since multiple quasiparticles are always created together (and nearby)
 - Yet surprisingly it is common to ignore quasiparticle interactions in spin liquids

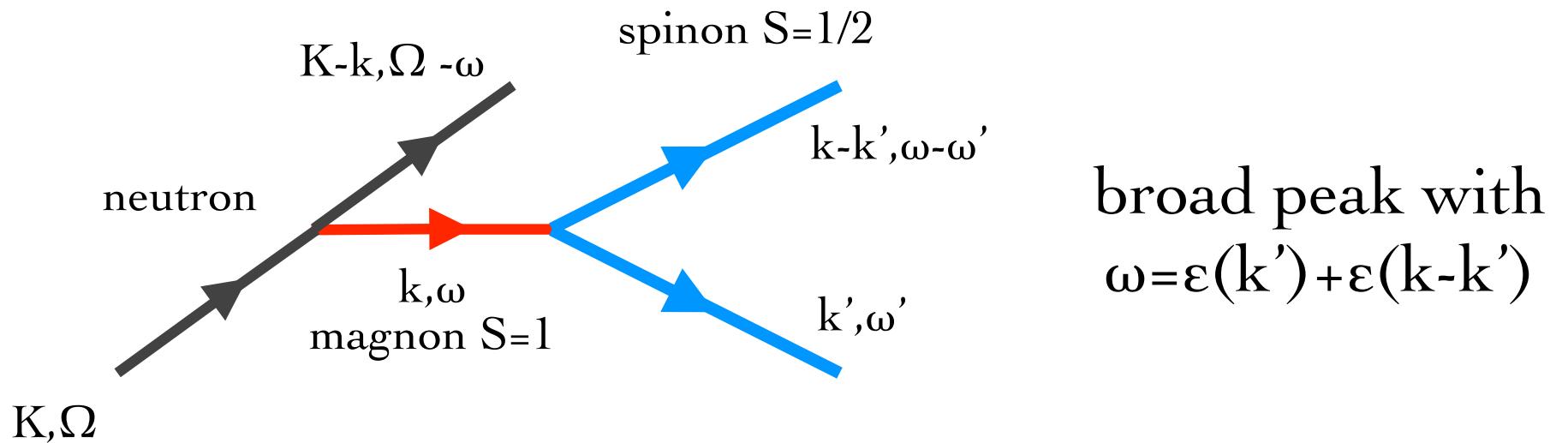
Structure Factor

- Inelastic neutron scattering

$$S(k, \omega) = FT \left[\langle \vec{S}(r, t) \cdot \vec{S}(0, 0) \rangle \right]$$

- Naïve approach $\vec{S}_r = \frac{1}{2} c_r^\dagger \vec{\sigma} c_r$ free spinons

- Structure factor basically measures 2-particle DOS



Structure Factor

- Structure factor basically measures 2-particle DOS?

nature

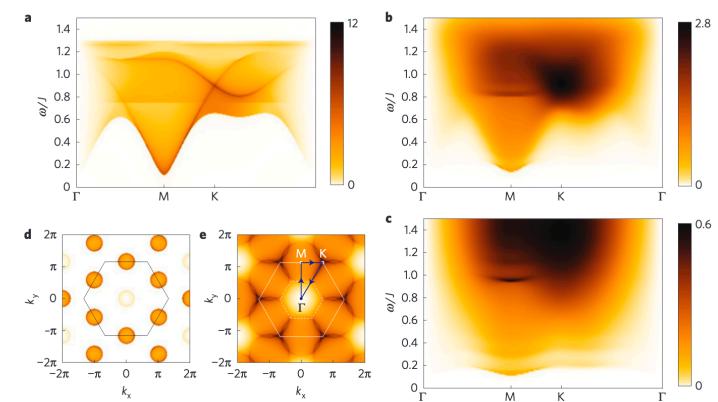
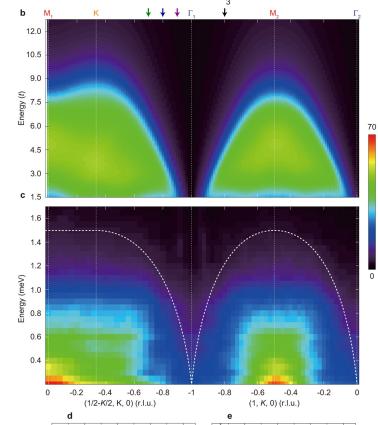
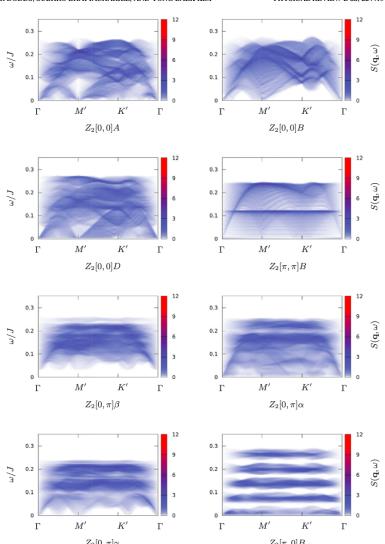
Letter | Published: 05 December 2016

Evidence for a spinon Fermi surface in a triangular-lattice quantum-spin-liquid candidate

Yao Shen, Yao-Dong Li, Hongliang Wo, Yuesheng Li, Shoudong Shen, Bingyin Pan, Qisi Wang, H. C. Walker, P. Steffens, M. Boehm, Yiqing Hao, D. L. Quintero-Castro, L. W. Harriger, M. D. Frontzek, Lijie Hao, Siqin Meng, Qingming Zhang, Gang Chen & Jun Zhao

TYLER DODDS, SUBHRO BHATTACHARJEE, AND YONG BAEK KIM

PHYSICAL REVIEW B **88**, 224413 (2013)



nature
physics

PUBLISHED ONLINE: 9 MARCH 2014 | DOI: 10.1038/NPHYS2887

LETTERS

Topological excitations and the dynamic structure factor of spin liquids on the kagome lattice

Matthias Punk^{1,2}, Debanjan Chowdhury¹ and Subir Sachdev^{1,*}

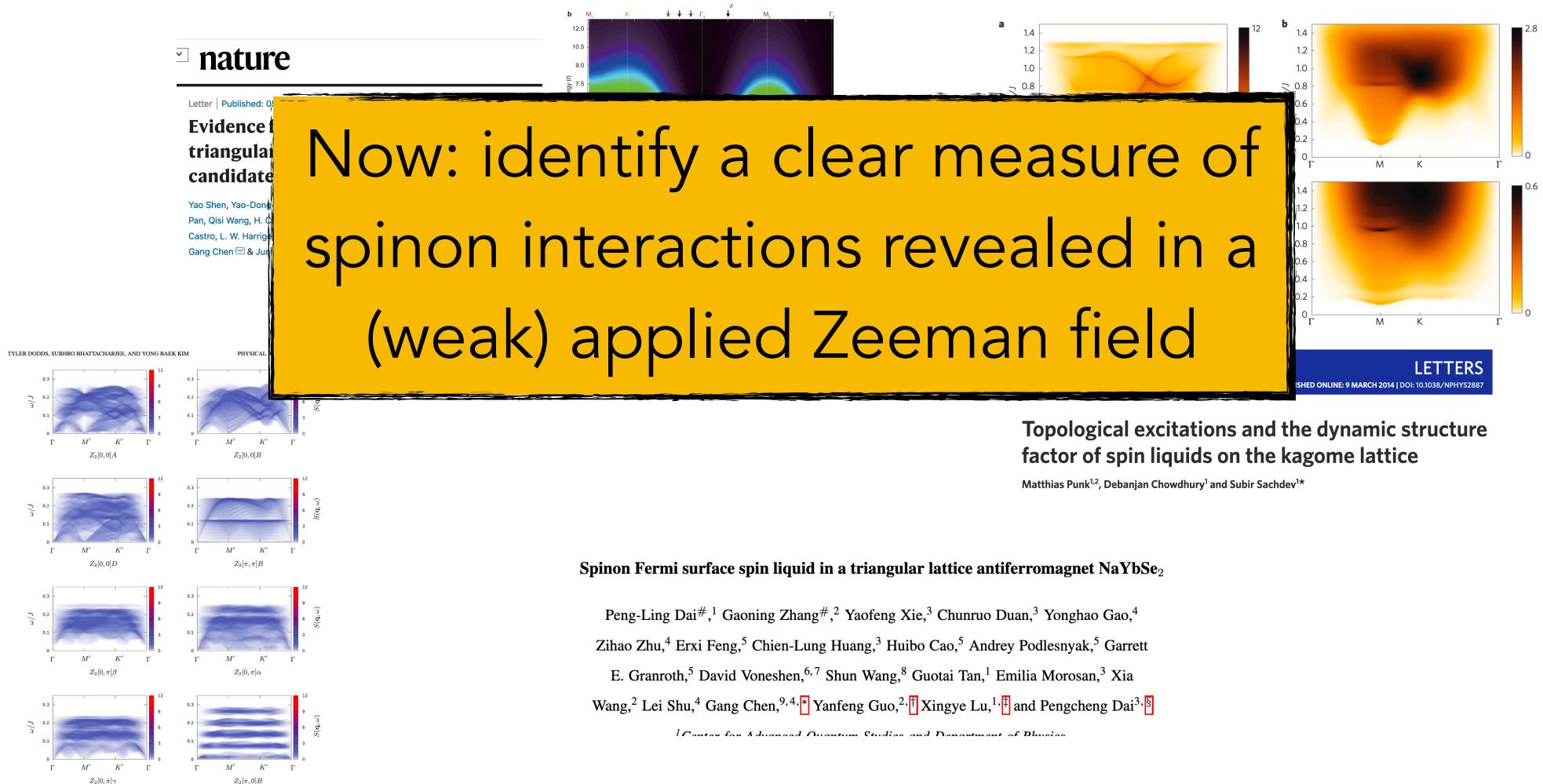
Spinon Fermi surface spin liquid in a triangular lattice antiferromagnet NaYbSe₂

Peng-Ling Dai^{#,1}, Gaoning Zhang^{#,2}, Yaofeng Xie³, Chunruo Duan³, Yonghao Gao⁴, Zihao Zhu⁴, Erxi Feng⁵, Chien-Lung Huang³, Huibo Cao⁵, Andrey Podlesnyak⁵, Garrett E. Granroth⁵, David Voneshen^{6,7}, Shun Wang⁸, Guotai Tan¹, Emilia Morosan³, Xia Wang², Lei Shu⁴, Gang Chen^{9,4,*}, Yanfeng Guo^{2,10}, Xingye Lu^{1,11} and Pengcheng Dai^{3,8}

¹Center for Advanced Quantum Studies and Department of Physics

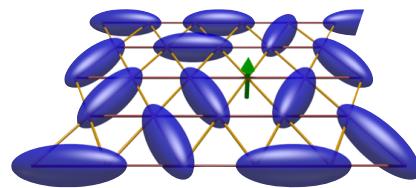
Structure Factor

- Structure factor basically measures 2-particle DOS?



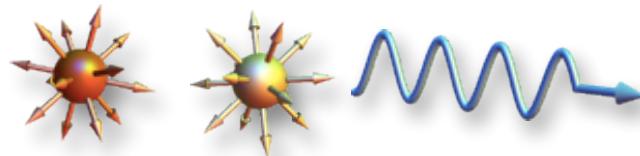
Classes of QSLs

- Topological QSLs



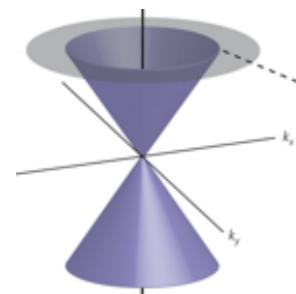
anyons,
spinons

- $U(1)$ QSL



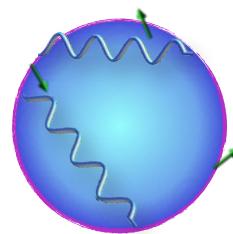
compact $U(1)$

- Dirac QSLs



QED_3

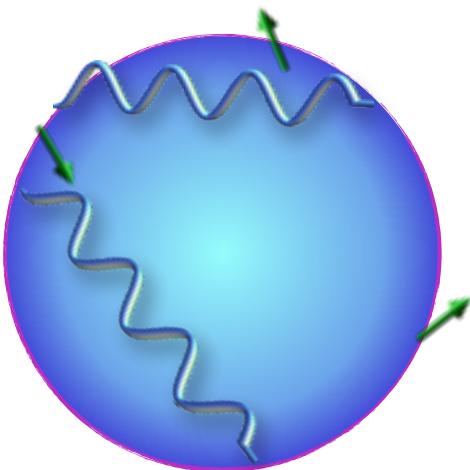
- Spinon Fermi surface



non-Fermi
liquid “spin
metal”

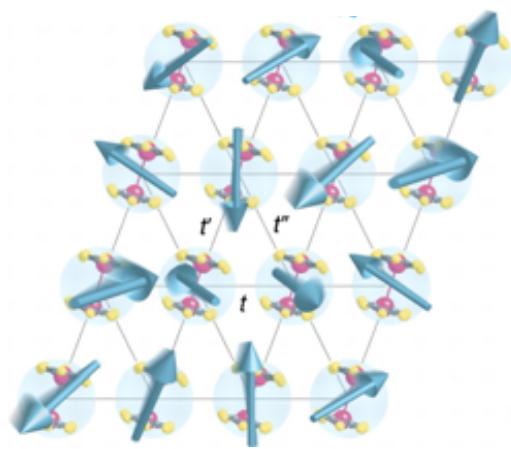
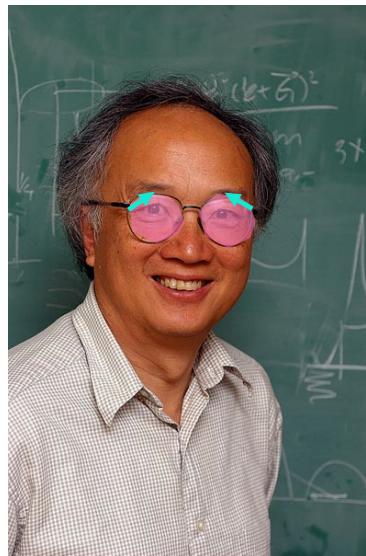
Spinon Fermi surface

$$|\Psi\rangle = \prod_i \hat{n}_i (2 - \hat{n}_i) \prod_{k < k_F} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$

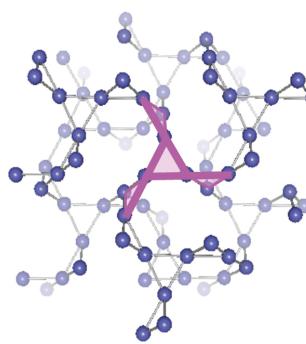


- The most gapless/highly entangled QSL state
- Like a “metal” of neutral fermions w/ a U(1) gauge field
- Prototype “non-Fermi liquid” state of great theoretical interest

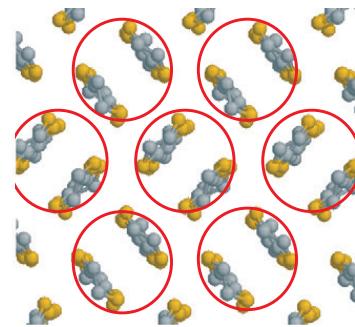
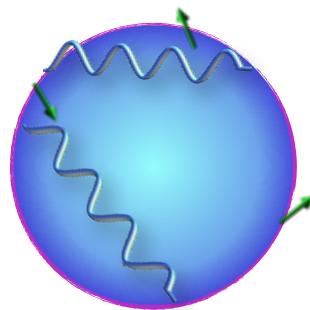
Spinon Fermi surface



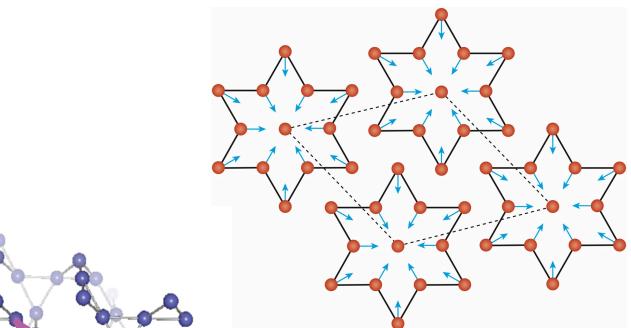
k -ET



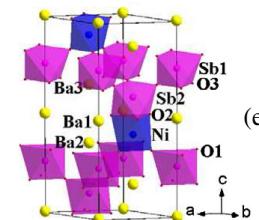
$\text{Na}_4\text{Ir}_3\text{O}_8$



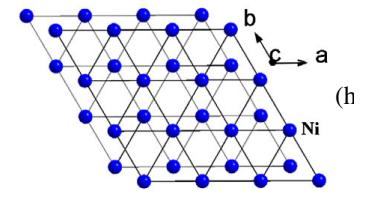
dmit



1T-TaS₂



$\text{Ba}_3\text{Ni}\text{Sb}_2\text{O}_9$



(e) (h)

Effective field theory

- Spinon Fermi surface: “uniform RVB”

Zeeman term

$$S_\psi = \int d^3x \psi^\dagger \left(\partial_\tau - \mu - \frac{1}{2m} (\nabla_{\mathbf{r}} - i\mathbf{A})^2 - \omega_B \sigma^3 \right) \psi,$$

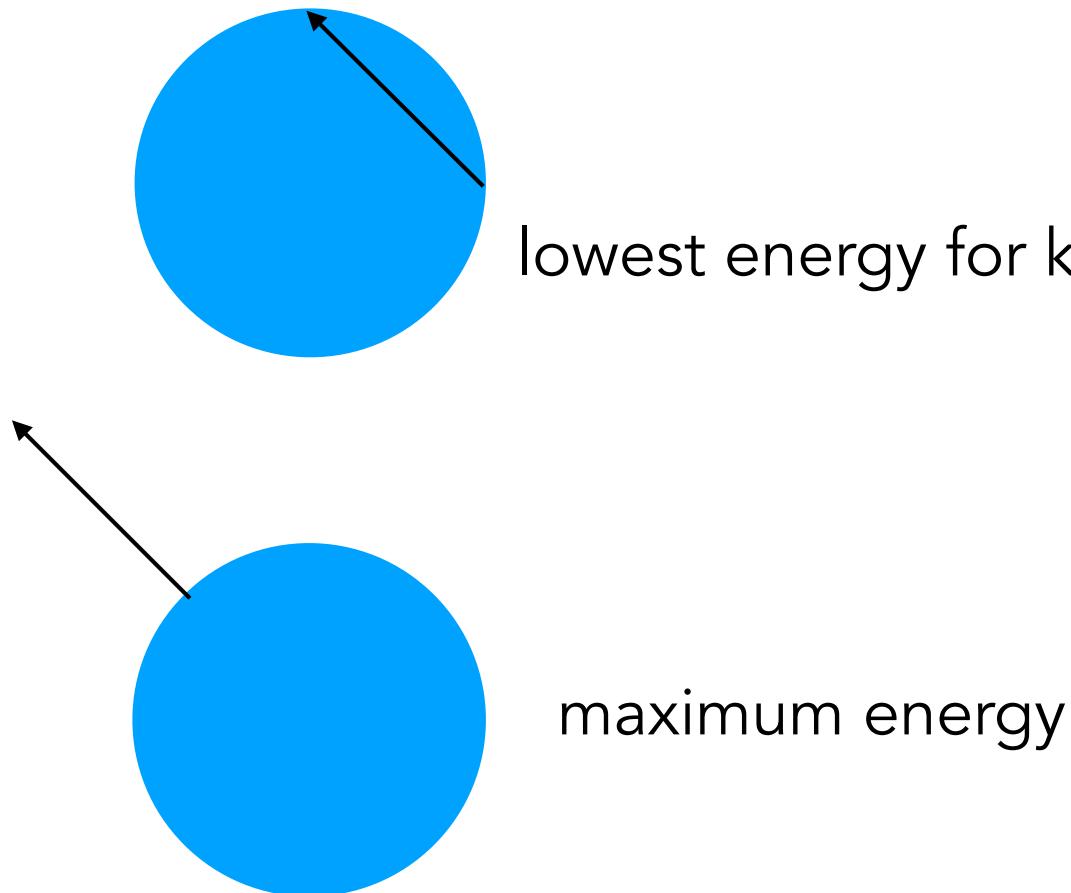
Kinetic energy Emergent gauge field

$$S_A = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2} (\gamma |\omega_n|/q + \chi q^2) |A(q)|^2, \quad \text{Landau damping}$$

$$S_u = \int d^3x u \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow. \quad \text{Short-range repulsion (from } a_0 \text{)}$$

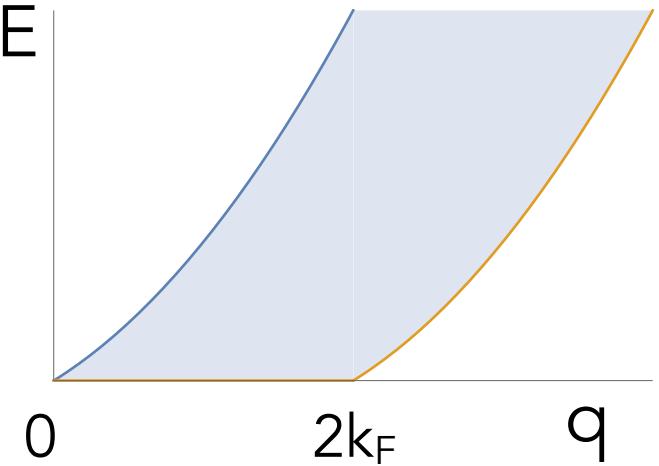
Free particles: p/h continuum

Fermi surface



lowest energy for $k < 2k_F$ E

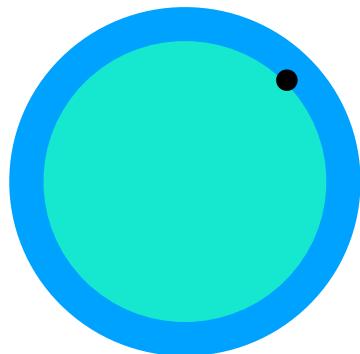
maximum energy



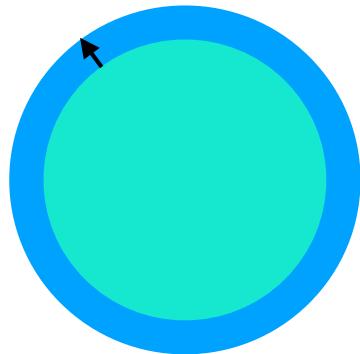
particle-hole continuum

With Zeeman field

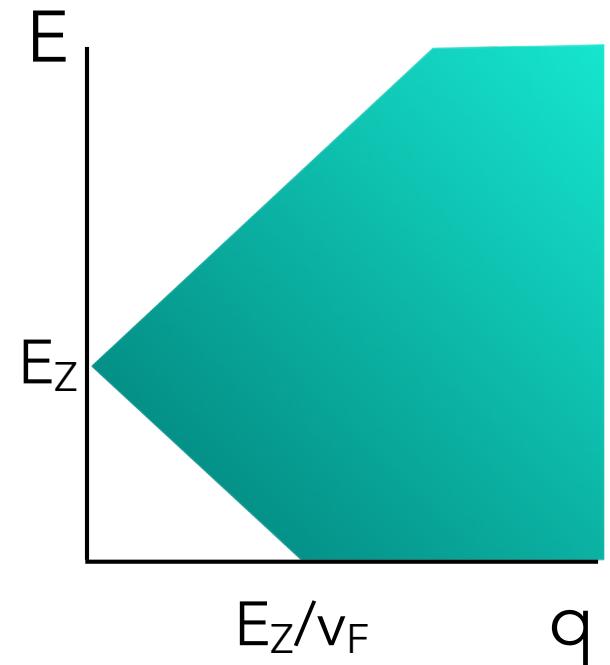
$$\chi_{\pm}(\mathbf{q}, \omega) = i \int_0^{\infty} dt \langle [S_{\mathbf{q}}^{\dagger}(t), S_{-\mathbf{q}}^-(0)] \rangle e^{i\omega t}$$



$q=0$ costs Zeeman energy



zero energy when $v_F q$
= Zeeman



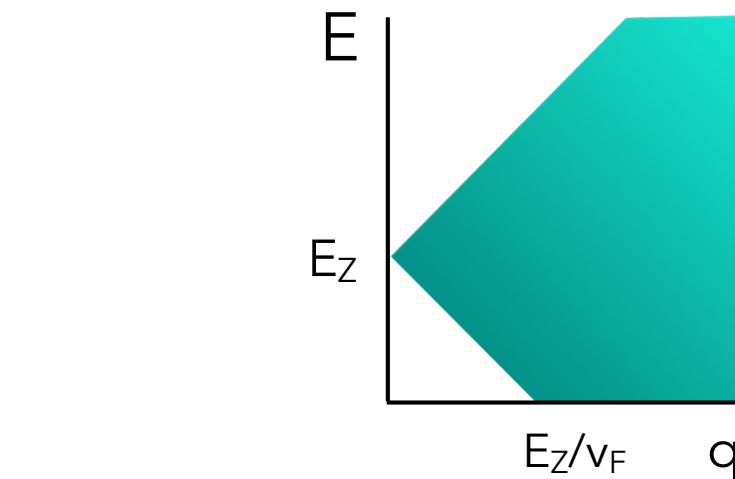
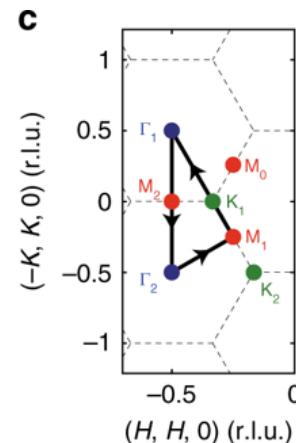
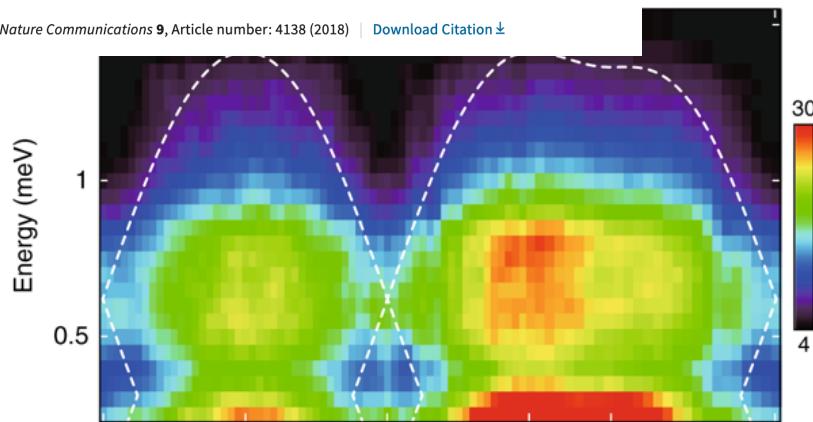
YbMgGaO₄

Article | OPEN | Published: 08 October 2018

Fractionalized excitations in the partially magnetized spin liquid candidate YbMgGaO₄

Yao Shen, Yao-Dong Li, H. C. Walker, P. Steffens, M. Boehm, Xiaowen Zhang, Shoudong Shen,
Hongliang Wo, Gang Chen & Jun Zhao

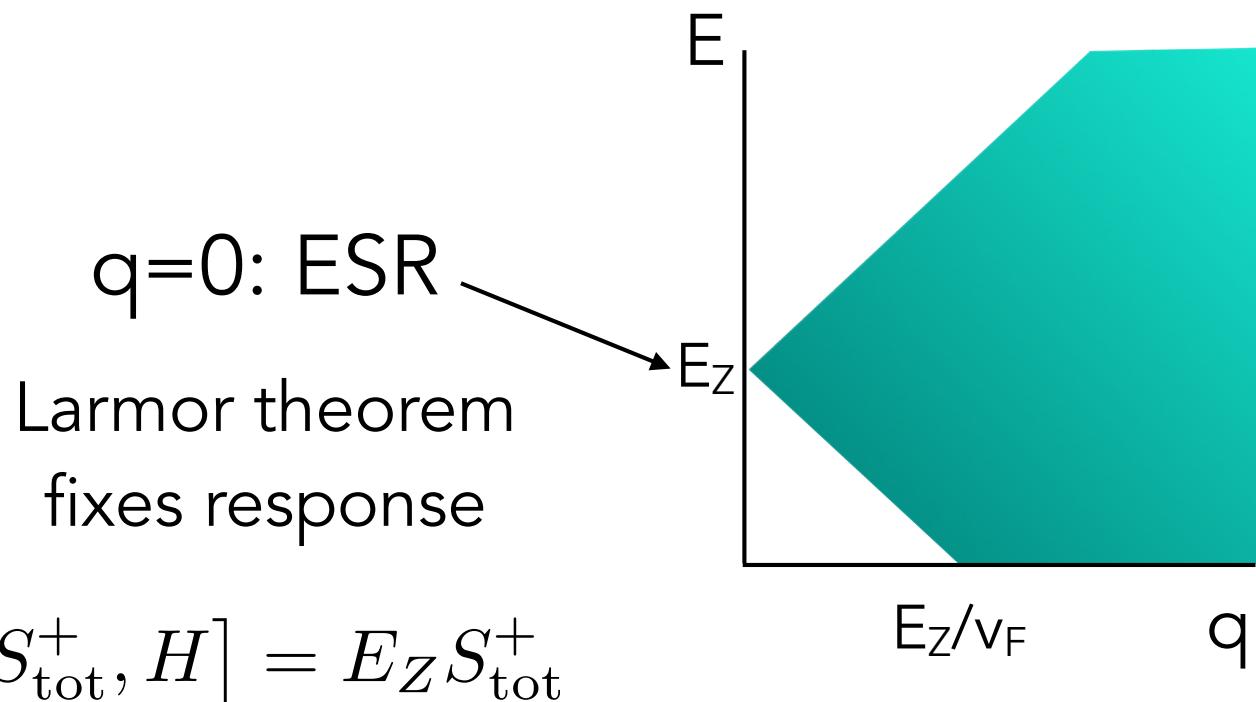
Nature Communications 9, Article number: 4138 (2018) | Download Citation ↴



???

Effects of interactions?

Free spinons



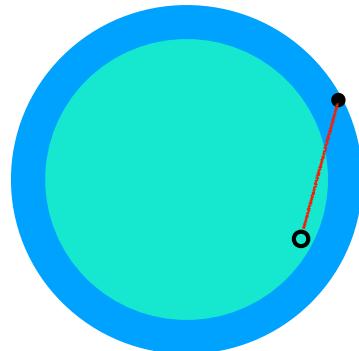
Naïvely Larmor theorem suggests free results good

Interactions

- Longitudinal

$$a_0 \psi^\dagger \psi$$

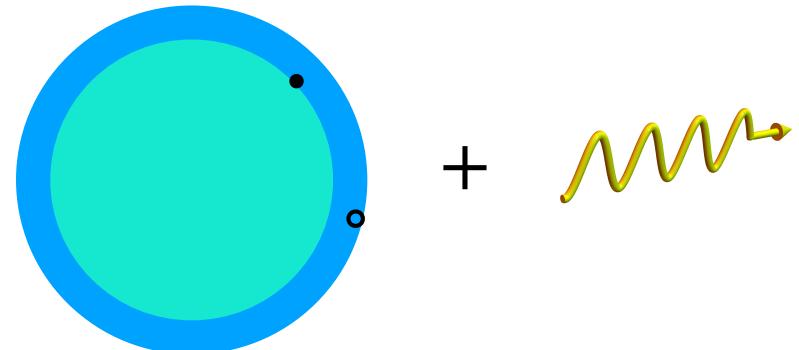
screened Coulomb
interaction



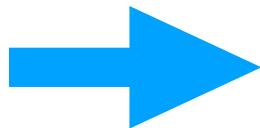
- Transverse

$$i \mathbf{A} \cdot (\psi^\dagger \nabla \psi - \nabla \psi^\dagger \psi)$$

coupling to dynamical
photons



Interactions

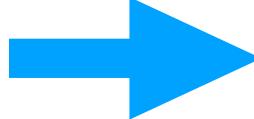
- Longitudinal $a_0 \psi^\dagger \psi$  $u \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow$

$$= -um \left(\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow \right) + u : \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow :$$

self-energy

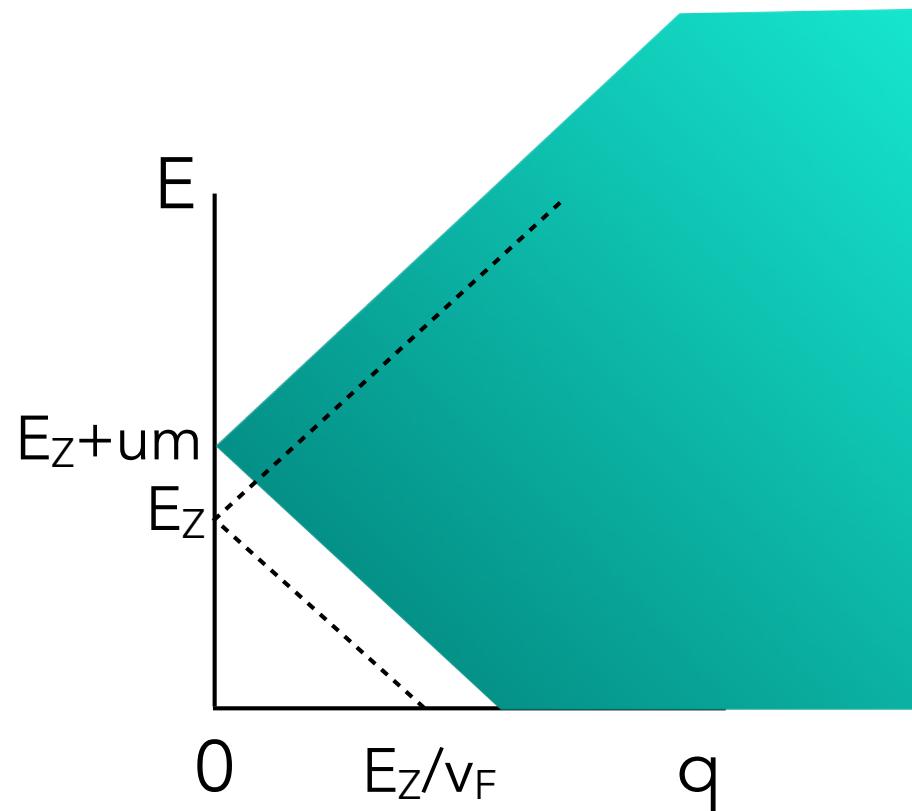
interaction

Self energy

- Longitudinal $a_0 \psi^\dagger \psi$  $u \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow$

$$= -um (\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow)$$

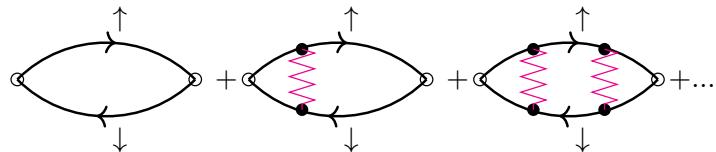
mean field shift



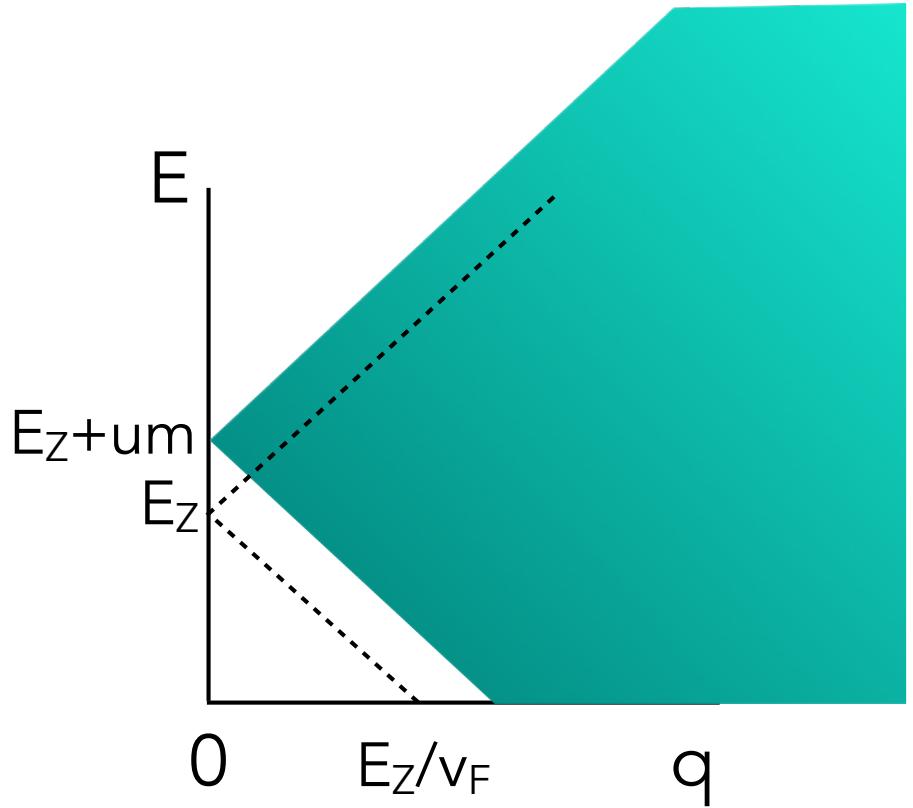
Interaction

Larmor theorem: $q=0$
excitation *must* be at E_Z

RPA



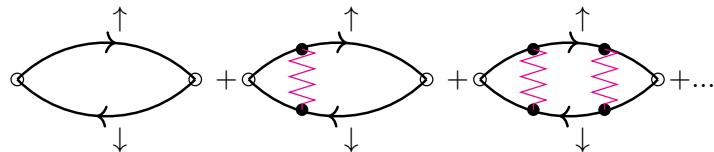
$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi^0(\mathbf{q}, i\omega_n)}{1 + u\chi^0(\mathbf{q}, i\omega_n)}$$



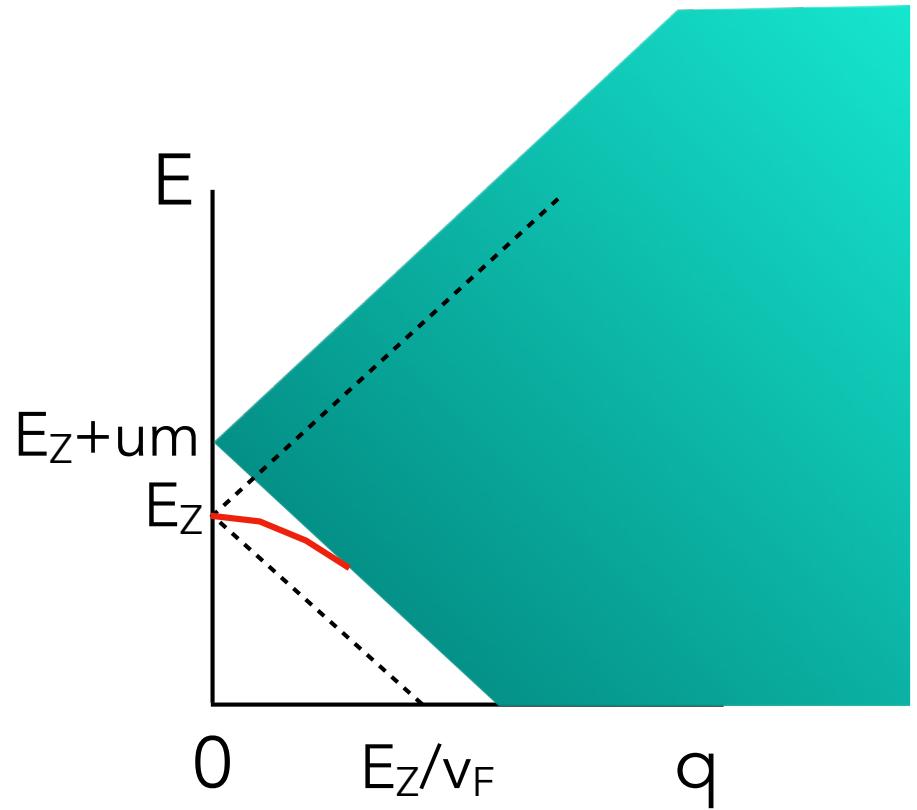
Silin spin wave

Larmor theorem: $q=0$
excitation *must* be at E_Z

RPA



$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi^0(\mathbf{q}, i\omega_n)}{1 + u\chi^0(\mathbf{q}, i\omega_n)}$$

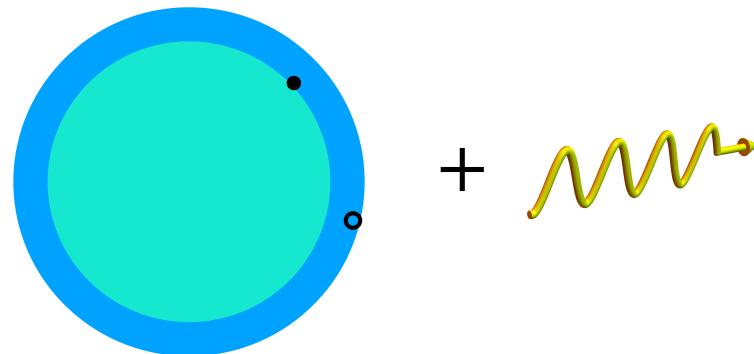


“Silin spin wave”

pole: collective mode

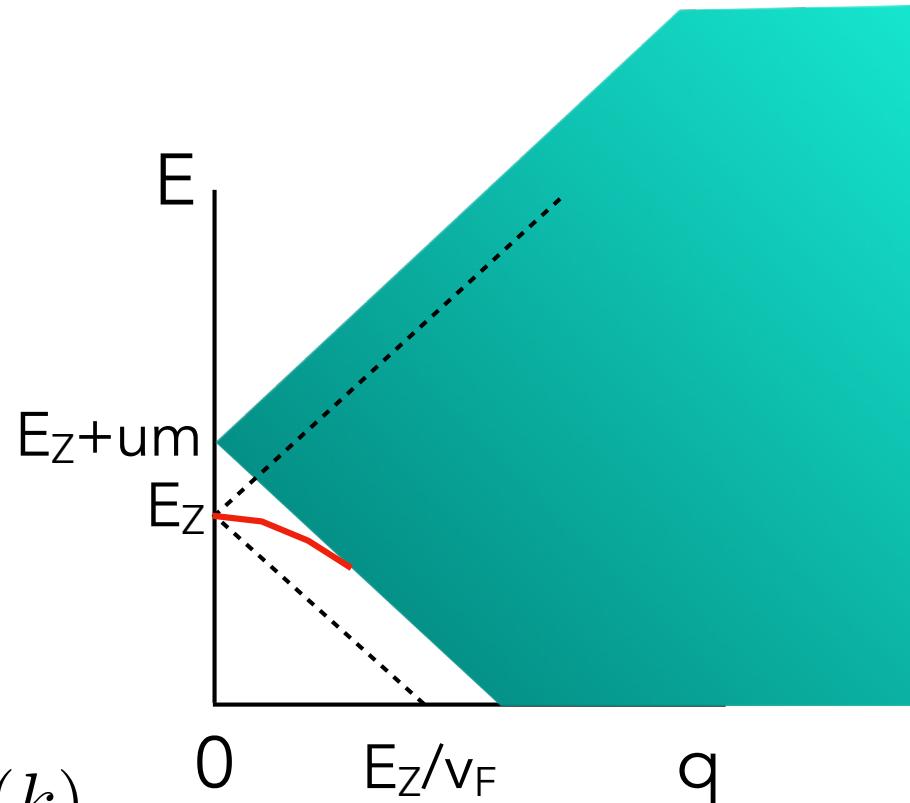
$$\omega = E_Z + um - \sqrt{u^2 m^2 + v_F^2 q^2}$$

Transverse gauge coupling



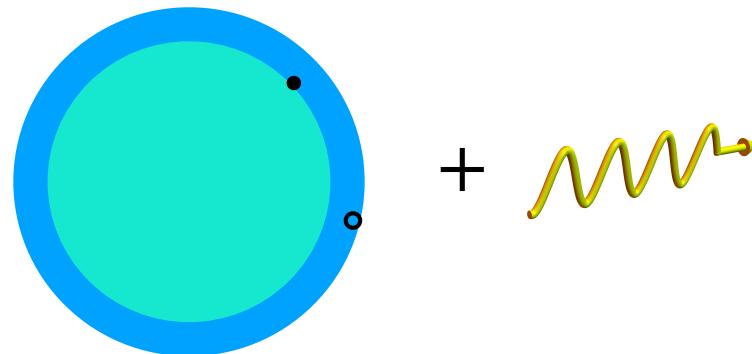
Simple picture:
3-particle process:

$$E = E_{p/h}(q - k) + E_{\text{photon}}(k) \sim c k^3$$

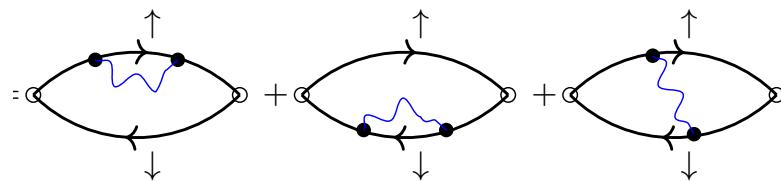


Does this smear out all the Fermi liquid structure?

Transverse gauge coupling

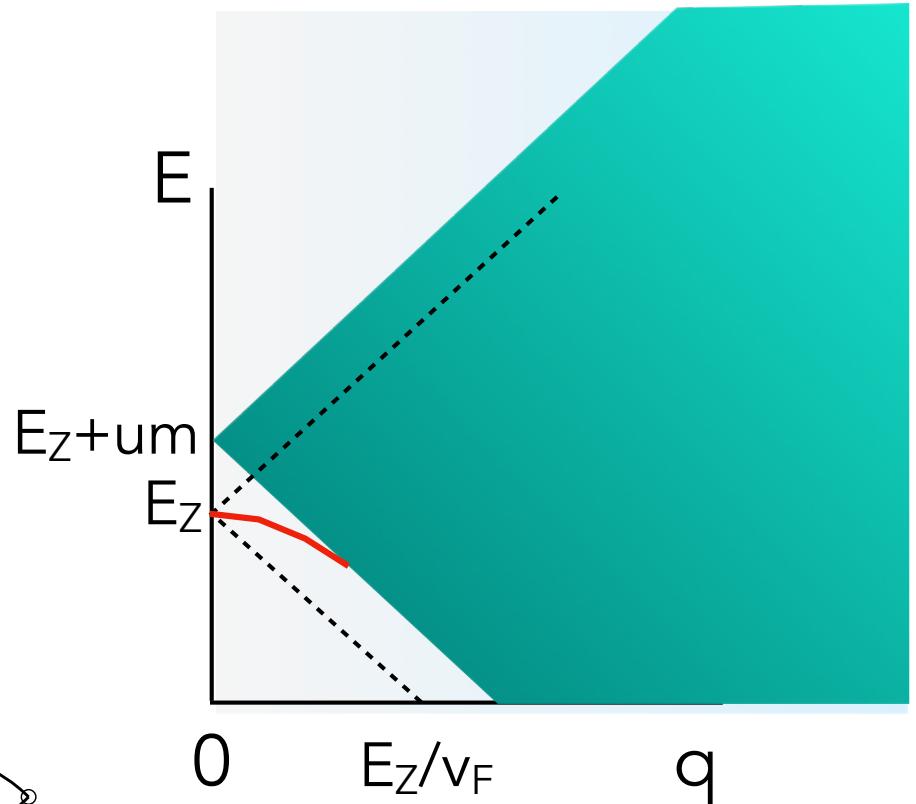


Actual calculation:



c.f. Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, [Phys Rev. B 50, 17917 \(1994\)](#).

$$\text{Im}\chi_{\pm} \sim q^2 \omega^{7/3}$$

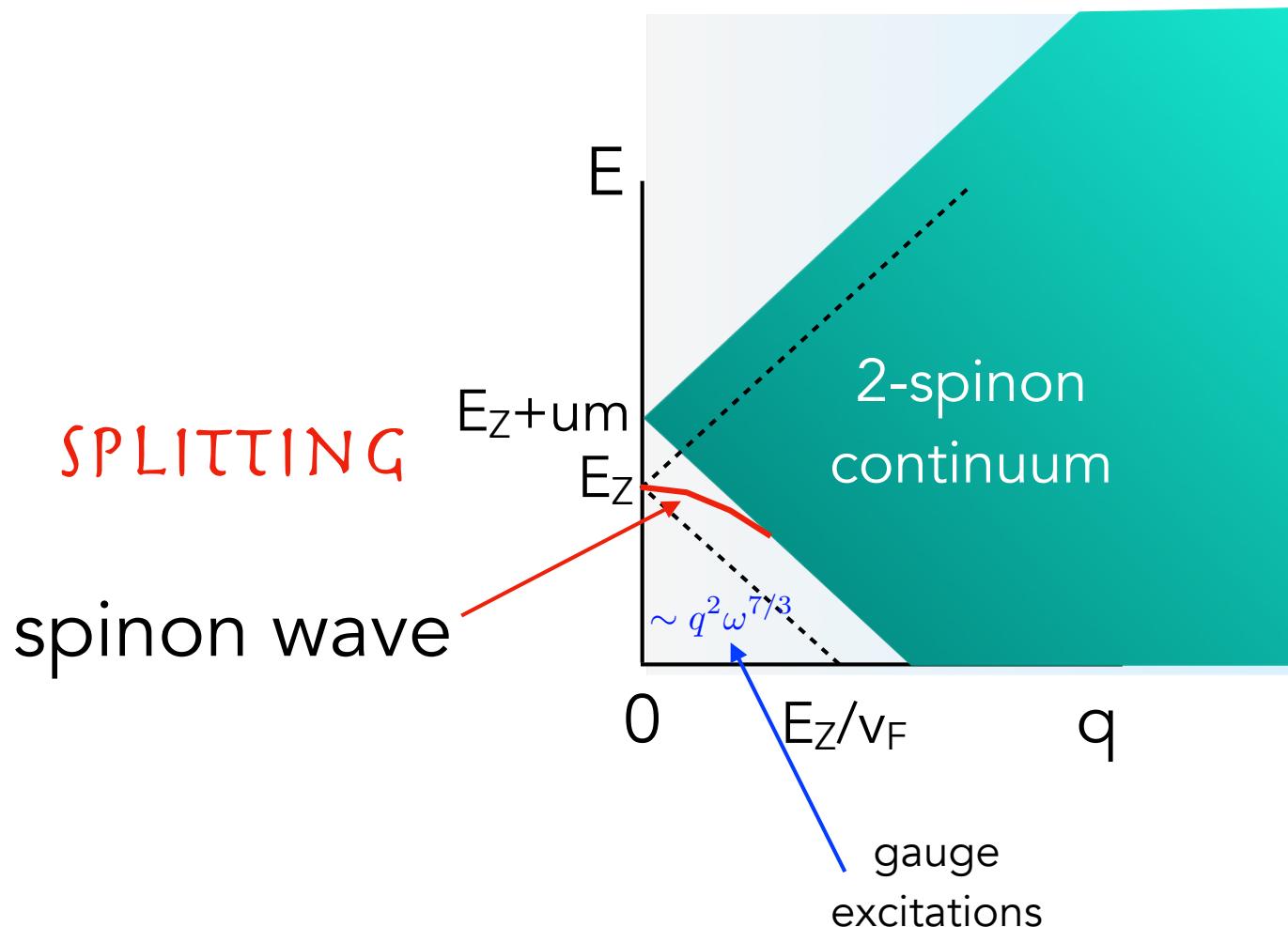


weight at all $q \neq 0$

but weak enough to
preserve structure

Summary

Distinct signatures of spinons,
interactions, and gauge fields



O.Starykh + LB,
arXiv:1904.02117
PRB **101**, 020401 (2020)

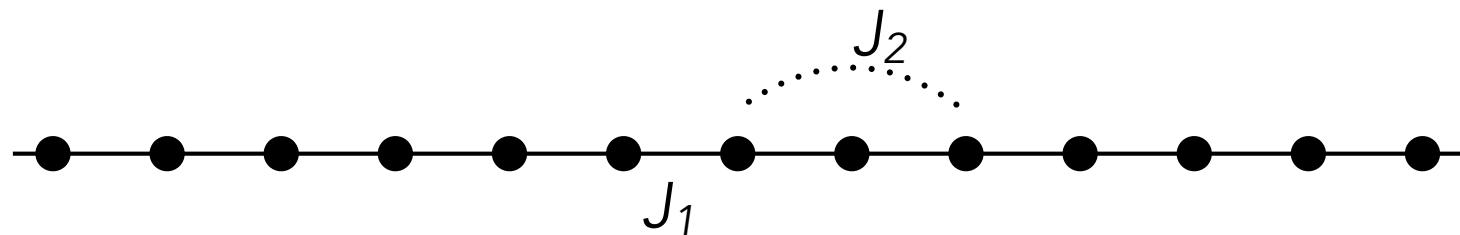
One dimension

- New results: these ideas apply to *one dimensional spin chains* in low magnetic fields and can be tested there!
- Bonus: we also will find signatures of interacting magnons in the high field regime

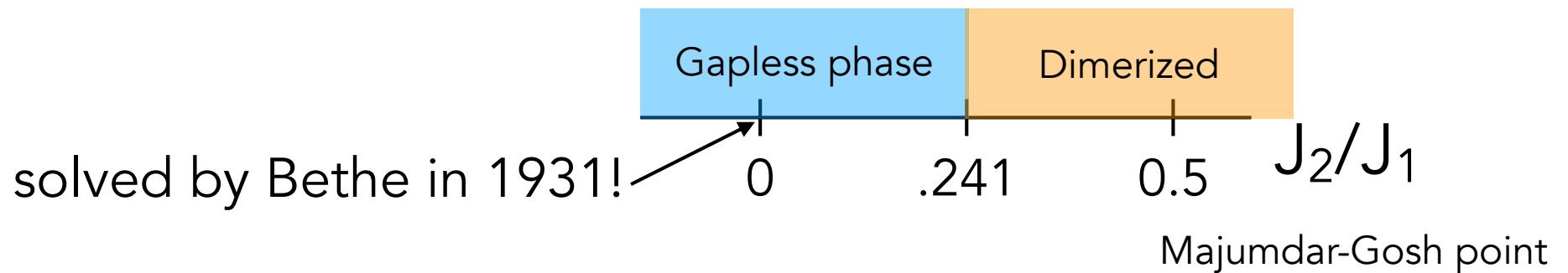
One dimension

- J1-J2 Chain

$$H = \sum_i \left[J_1 \vec{S}_i \cdot \vec{S}_{i+1} + J_2 \vec{S}_i \cdot \vec{S}_{i+2} - B S_i^z \right]$$



- Phase diagram for B=0



Gapless phase

- Wess-Zumino-Witten $SU(2)_1$ CFT
- Many representations:
 - matrix non-linear sigma model
 - free masses scalar field theory (abelian bosonization)
 - Sugawara (current algebra) form
 - Free fermions (most useful today)

Fermion representation

- Spins $\vec{S}_i \sim \vec{J}_R(x_i) + \vec{J}_L(x_i) + (-1)^i \vec{N}(x_i)$

$$\vec{J}_{R/L} = \frac{1}{2} \psi_{R/L}^\dagger \vec{\sigma} \psi_{R/L}$$

- Hamiltonian $H = H_0 + V$

$$H_0 = v \int dx \left(\psi_R^\dagger (-i\partial_x) \psi_R + \psi_L^\dagger (i\partial_x) \psi_L \right) \quad \psi_R = (\psi_{R\uparrow}, \psi_{R\downarrow})^T$$

$$V = -g \int dx \vec{J}_R \cdot \vec{J}_L = -g \int dx \left[J_R^z J_L^z + \frac{1}{2} J_R^+ J_L^- + \frac{1}{2} J_R^- J_L^+ \right]$$

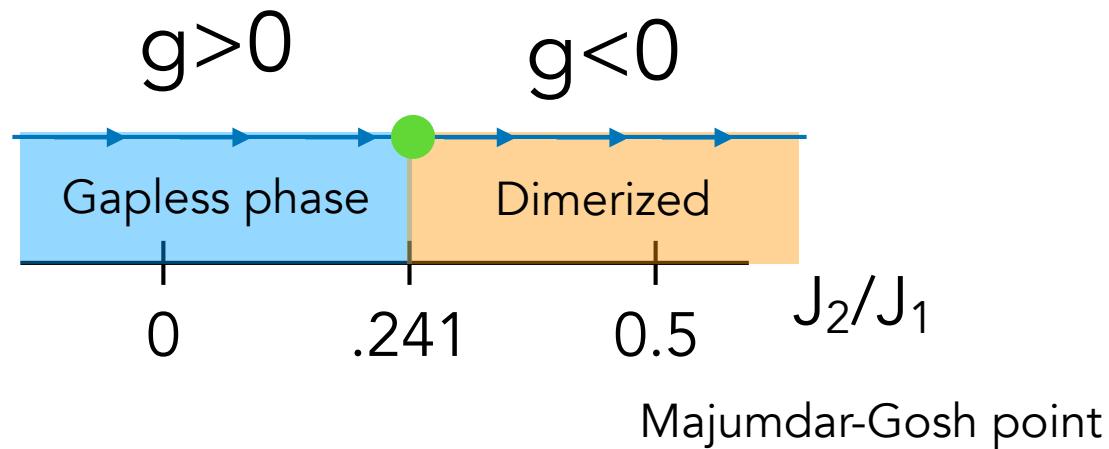
- Fermions contain decoupled charge mode which does not affect spin operators or correlations (spin-charge separation)

Backscattering

- Understanding the phase diagram

$$H_0 = v \int dx \left(\psi_R^\dagger (-i\partial_x) \psi_R + \psi_L^\dagger (i\partial_x) \psi_L \right)$$

$$V = -g \int dx \vec{J}_R \cdot \vec{J}_L = -g \int dx \left[J_R^z J_L^z + \frac{1}{2} J_R^+ J_L^- + \frac{1}{2} J_R^- J_L^+ \right]$$



- Renormalization group $\frac{dg}{d\ell} = -g^2$

“marginally irrelevant”
in critical phase

Free fermions??

PHYSICAL REVIEW B, VOLUME 65, 134410

Electron spin resonance in $S = \frac{1}{2}$ antiferromagnetic chains

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(Received 13 August 2001; published 19 March 2002)

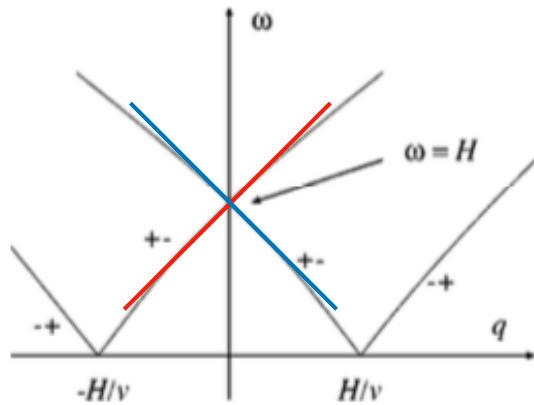
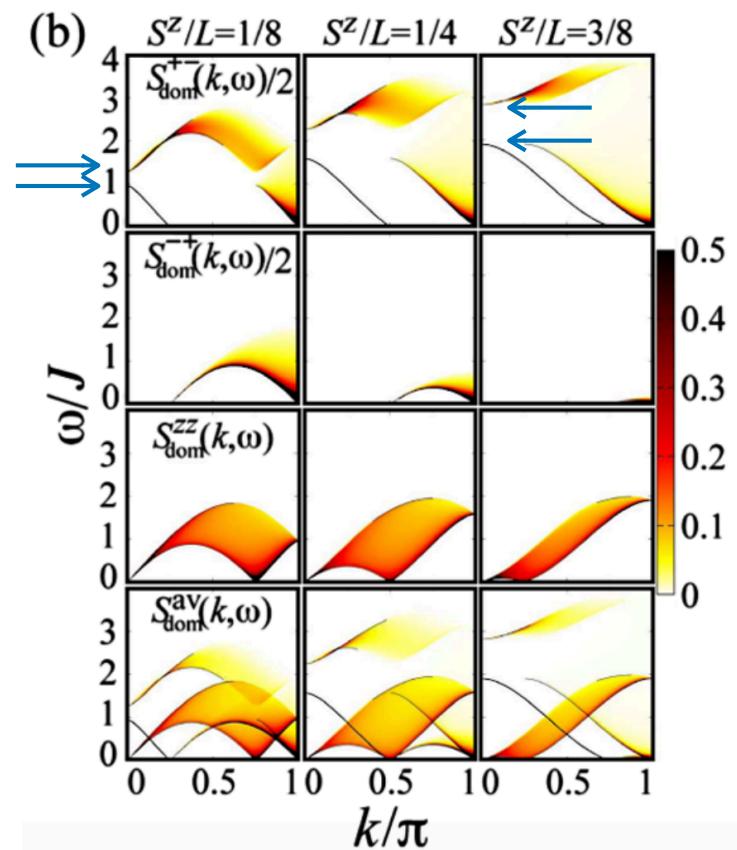


FIG. 2. The zero temperature transverse spin structure factor $S_{xx}(\omega, q) = S_{yy}(\omega, q)$ of the $S = 1/2$ Heisenberg antiferromagnetic chain under an applied field H , near $q = 0$. It is approximately proportional to $\omega[\delta(\omega - |q - H|) + \delta(\omega - |q + H|)]$, giving the resonance at $q = 0, \omega = H$. This consists of two branches coming from S_{+-} and S_{-+} , which are marked by $+-$ and $-+$ in the graph. In fact, there is a small spreading of the spectrum and the structure factor is generally not a perfect delta function. However, it is exactly the delta function $\delta(\omega - H)$ at $q = 0$, as explained in the text.

$$S_{xx}(\omega, q) = S_{yy}(\omega, q) \propto \omega[\delta(\omega - |q + H|) + \delta(\omega - |q - H|)].$$

Free fermion $S(q, \omega)$ in 1d

BUT
???



Dynamically Dominant Excitations of String Solutions in the Spin-1/2 Antiferromagnetic Heisenberg Chain in a Magnetic Field

Masanori Kohno
Phys. Rev. Lett. **102**, 037203 – Published 22 January 2009

**Dynamical correlation functions of the $S=1/2$ nearest-neighbor
and Haldane-Shastry Heisenberg antiferromagnetic chains in zero and applied fields**

Kim Lefmann*

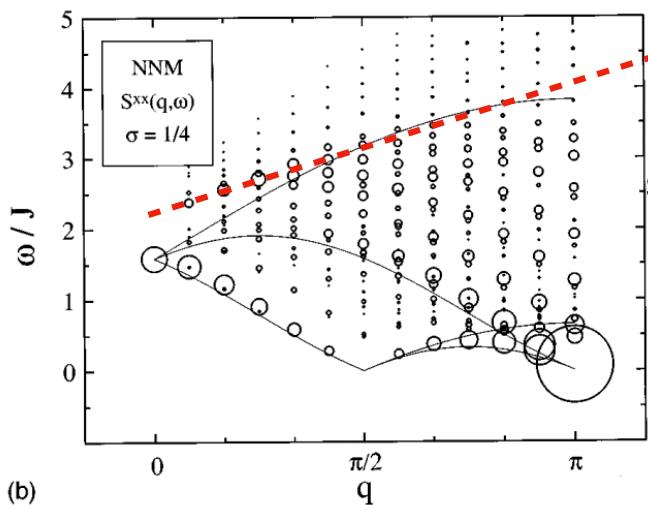
Department of Solid State Physics, Risø National Laboratory, DK-4000 Roskilde, Denmark

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(Received 12 February 1996)

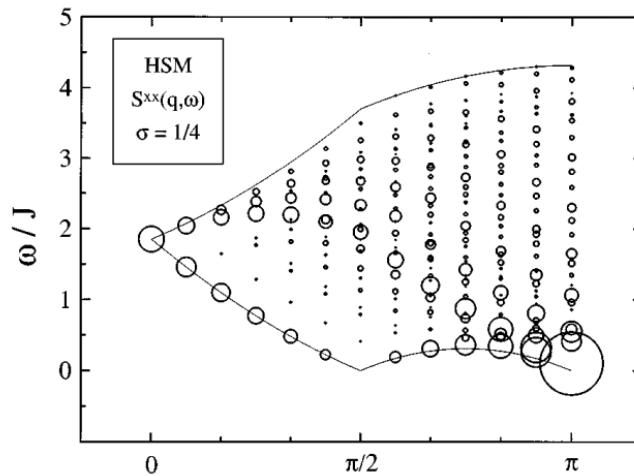
We present a numerical diagonalization study of two one-dimensional $S=1/2$ antiferromagnetic Heisenberg chains, having nearest-neighbor and Haldane-Shastry ($1/r^2$) interactions, respectively. We have obtained the $T=0$ dynamical correlation function, $S^{xx}(q, \omega)$, for chains of length $N=8-28$. We have studied $S^{zz}(q, \omega)$ for the Heisenberg chain in zero field, and from finite-size scaling we have obtained a limiting behavior that for large ω deviates from the conjecture proposed earlier by Müller *et al.* For both chains we describe the behavior of $S^{zz}(q, \omega)$ and $S^{xx}(q, \omega)$ for selected values of the applied field, and compare with previous work by Müller *et al.* and Talstra and Haldane. Suggestions for future finite-field neutron scattering experiments are made. [S0163-1829(96)00733-3]



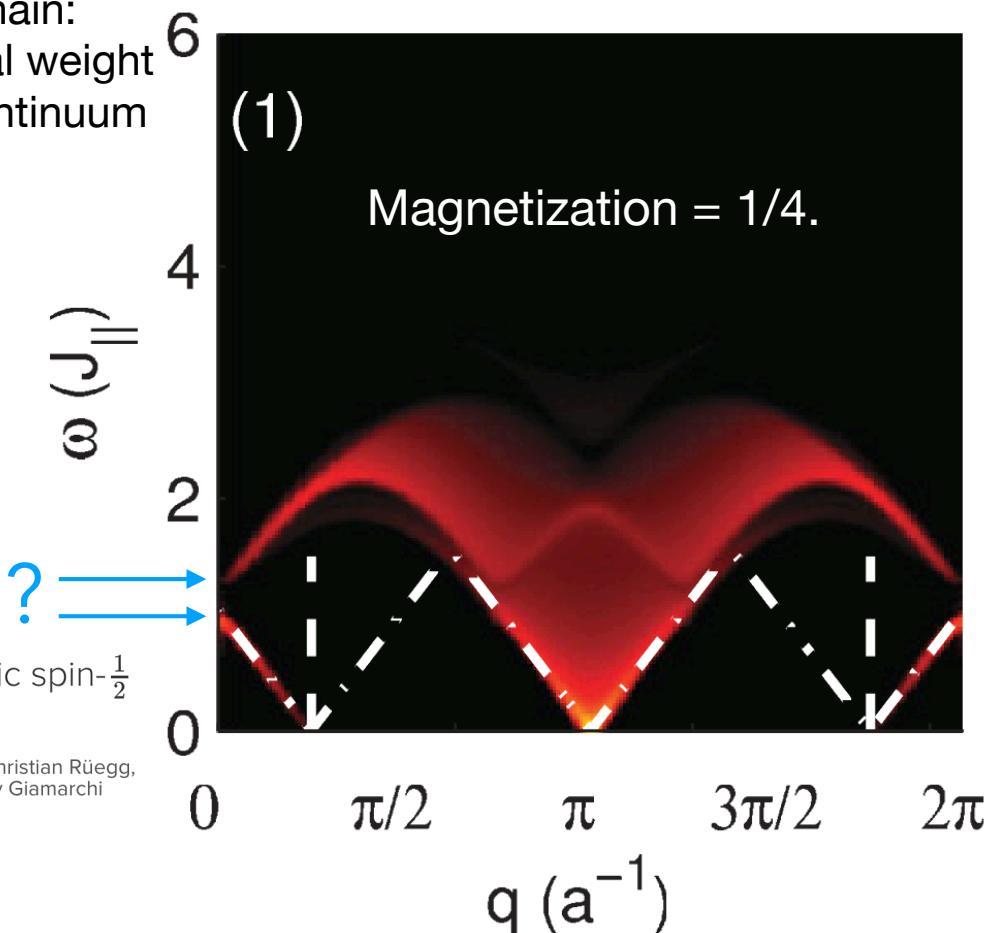
Heisenberg chain:
significant spectral weight
outside Muller continuum

Statics and dynamics of weakly coupled antiferromagnetic spin- $\frac{1}{2}$ ladders in a magnetic field

Pierre Bouillot, Corinna Kollath, Andreas M. Läuchli, Mikhail Zvonarev, Benedikt Thielemann, Christian Rüegg, Edmond Orignac, Roberta Citro, Martin Klanjšek, Claude Berthier, Mladen Horvatić, and Thierry Giamarchi
Phys. Rev. B **83**, 054407 – Published 9 February 2011



(b) Haldane-Shastry chain - nice 2-spinon continuum



Backscattering

- RG $\frac{dg}{d\ell} = -g^2$ Flow should be cut off by the Zeeman energy
- Interaction:

$$V = -g \int dx \vec{J}_R \cdot \vec{J}_L = -g \int dx [J_R^z J_L^z + \frac{1}{2} J_R^+ J_L^- + \frac{1}{2} J_R^- J_L^+]$$

Renormalizes
Zeeman splitting
 $B \rightarrow B + gM$

“vertex corrections”:
collective modes

RPA-like formula

$$G = \frac{G_{RR}^0 + G_{LL}^0 - gG_{RR}^0 G_{LL}^0}{1 - (g/2)^2 G_{RR}^0 G_{LL}^0}$$

Result

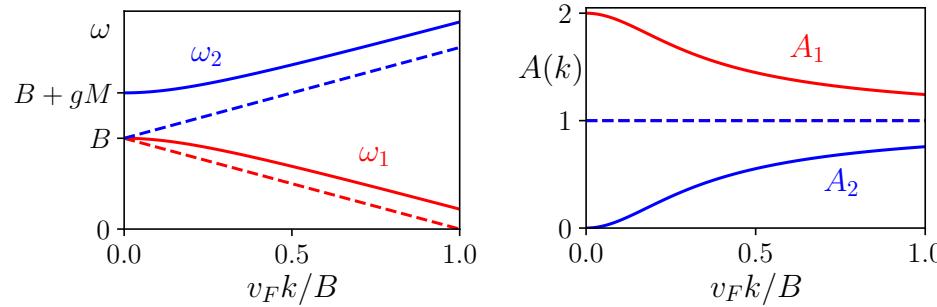
- Structure factor

$$\chi(k, \omega) = M \left(\frac{A_1(k)}{\omega - \omega_1(k)} + \frac{A_2(k)}{\omega - \omega_2(k)} \right)$$

$$\omega_{1(2)}(k) = B + gM/2 \mp \sqrt{g^2M^2/4 + v^2k^2}$$

$$A_{1(2)}(k) = 1 \pm \frac{gM/2}{\sqrt{g^2M^2/4 + v^2k^2}}$$

Mode splitting
Direct measure of
spinon interactions

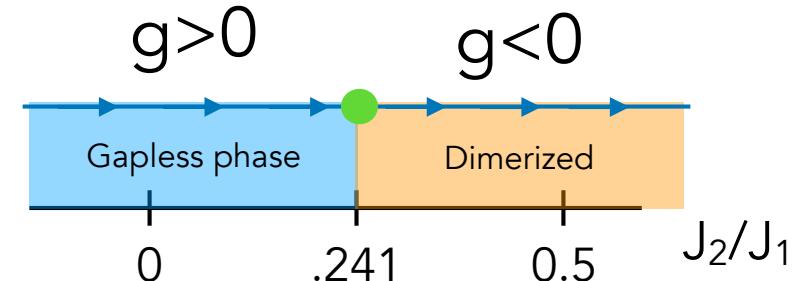


Spectral weight:
lower branch
dominant (c.f.
exciton)



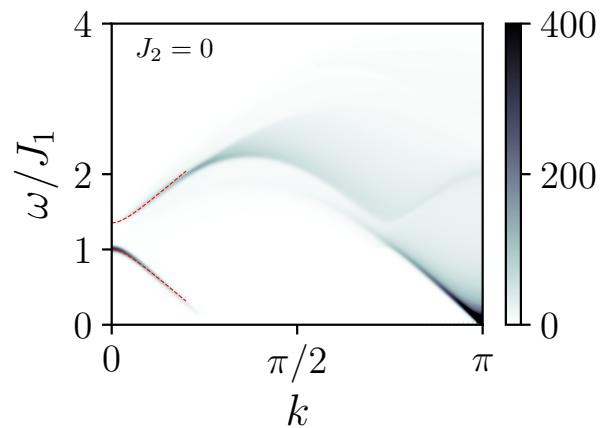
Simulations

- MPS methods: DMRG+TEBD



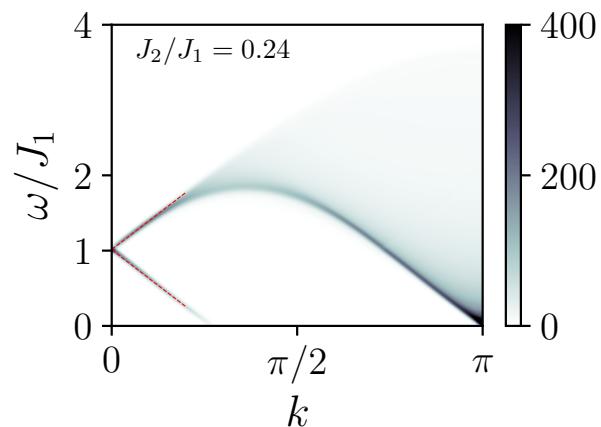
NN chain

$B/J_1 = 1$



Near QCP

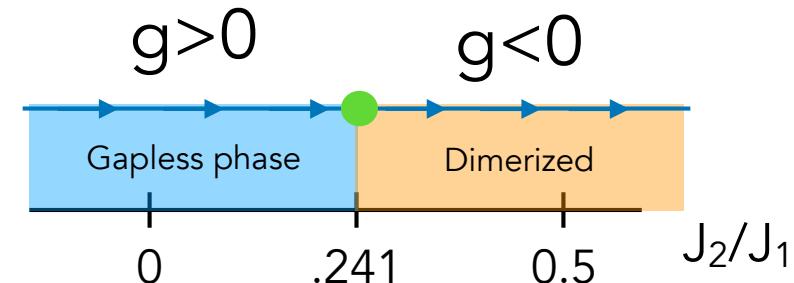
$g \approx 0$





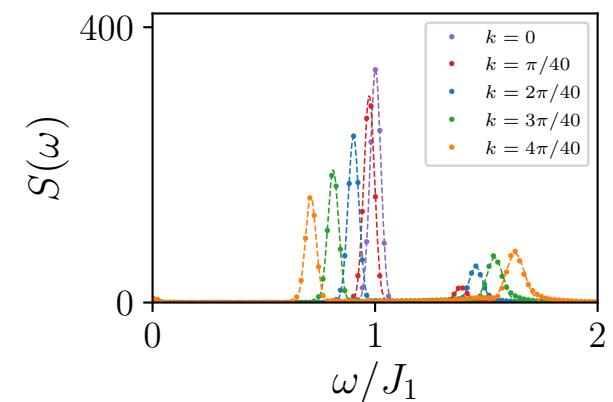
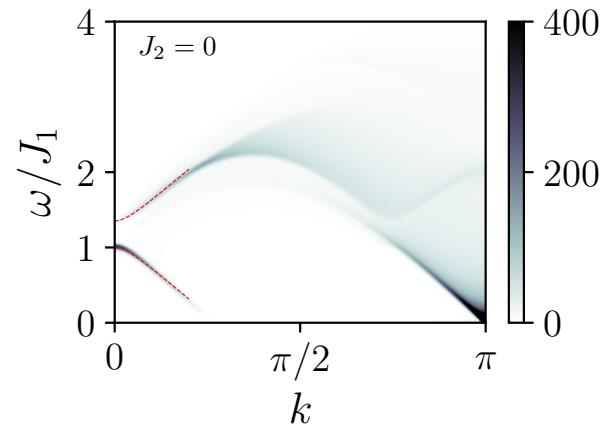
Simulations

- MPS methods: DMRG+TEBD



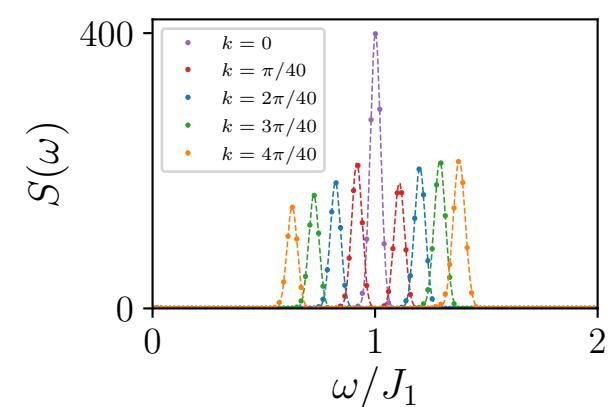
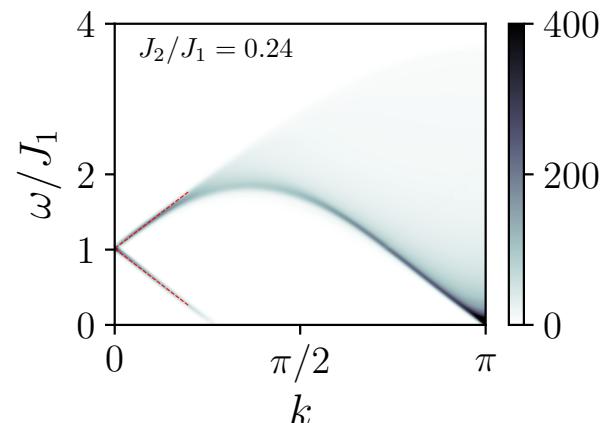
NN chain

$B/J_1 = 1$



Near QCP

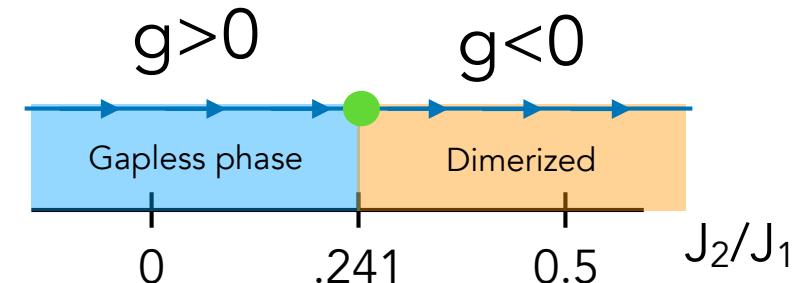
$g \approx 0$



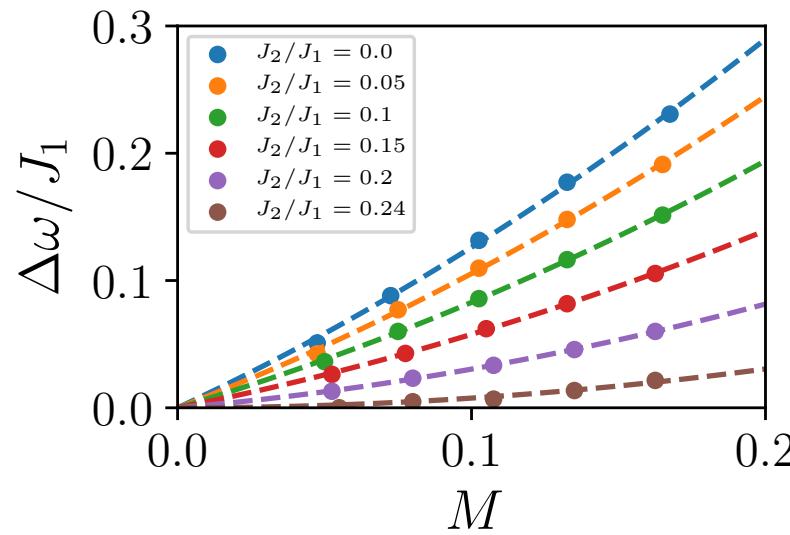


Simulations

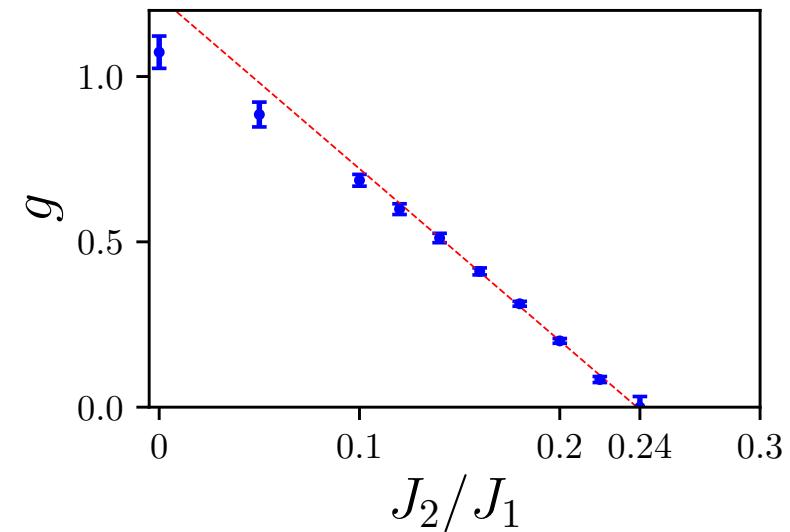
- MPS methods: DMRG+TEBD



- Systematics:

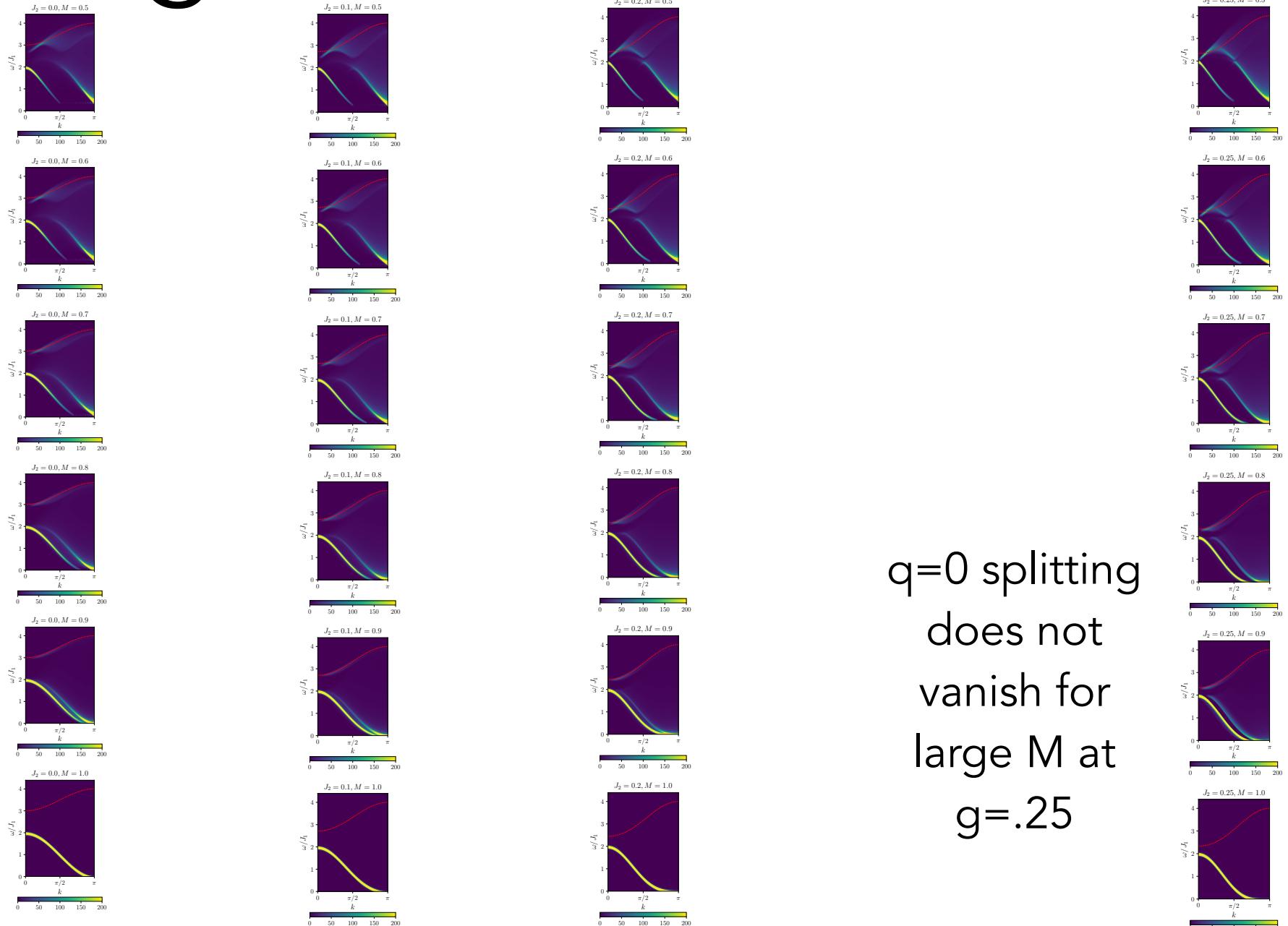


$$\Delta\omega = gM + \alpha M^2$$



Theory works :)

Higher Magnetization

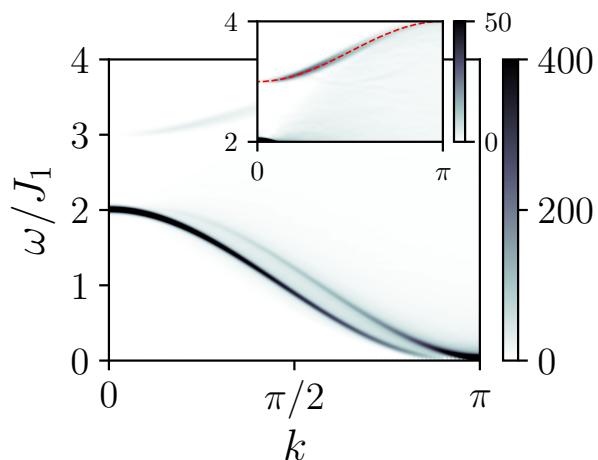


$q=0$ splitting
 does not
 vanish for
 large M at
 $g=.25$

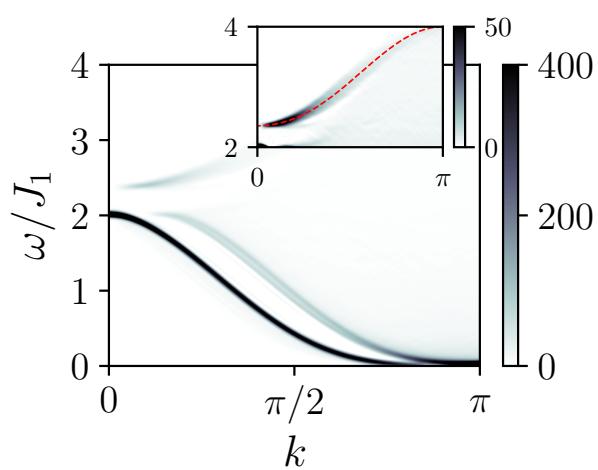
Higher Magnetization

$$M/M_s = .9$$

$$J_2 = 0$$



$$J_2 = .45$$

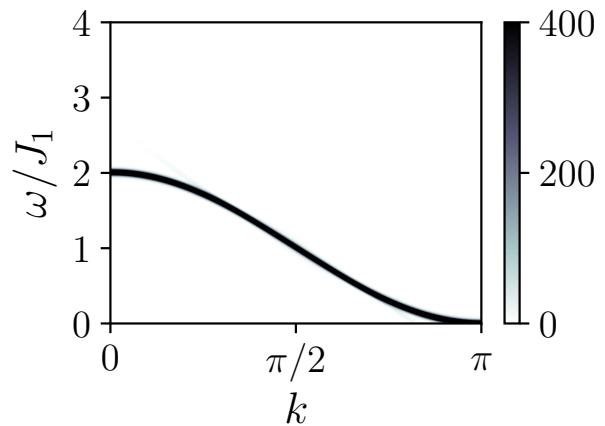


- $k=0$ gap persistent
- Lower mode has most of weight and slightly split
- Upper mode with small weight

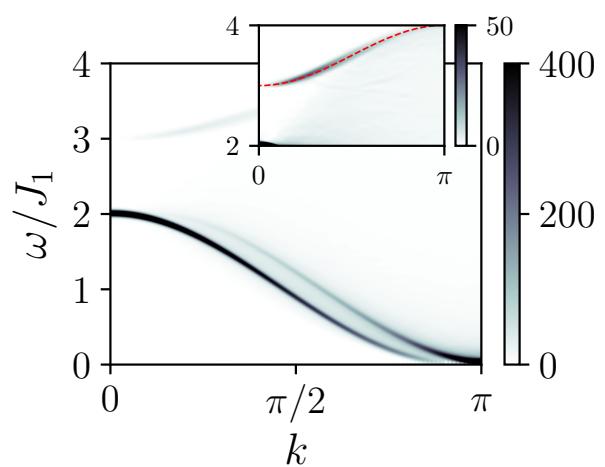
Higher Magnetization

$M/M_s = 1$

$J_2 = 0$



$M/M_s = .9$

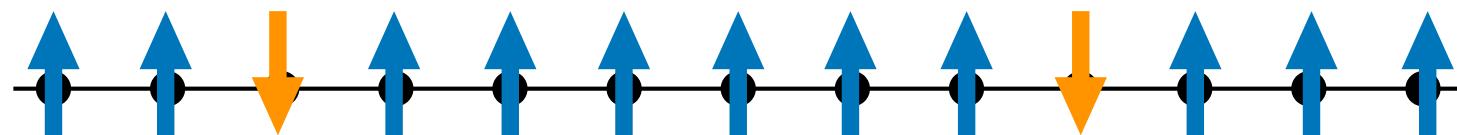


- Lower mode(s) clearly descend from single magnon of the ferromagnet
- Upper mode: spectral weight transfer to *large* energy upon small “doping” with spin flips

$$|k\rangle = \frac{1}{\sqrt{L}} \sum_x e^{ikx} S_x^- |\uparrow \cdots \uparrow \rangle$$

Picture

- Spin flip gas



~Tonks gas

Picture

- Spin flip $S_i^- \sim c_i^\dagger$

$$S_i^- \mid \circ \circ \bullet \circ \circ \circ \circ \circ \bullet \circ \circ \circ \circ \circ \rangle$$

$$= \mid \circ \circ \bullet \circ \circ \circ \circ \circ \color{blue}{\bullet} \circ \bullet \circ \circ \circ \circ \rangle^i$$

Extra particle can be ~free or bind to one of the existing particles if they interact!

Bound state

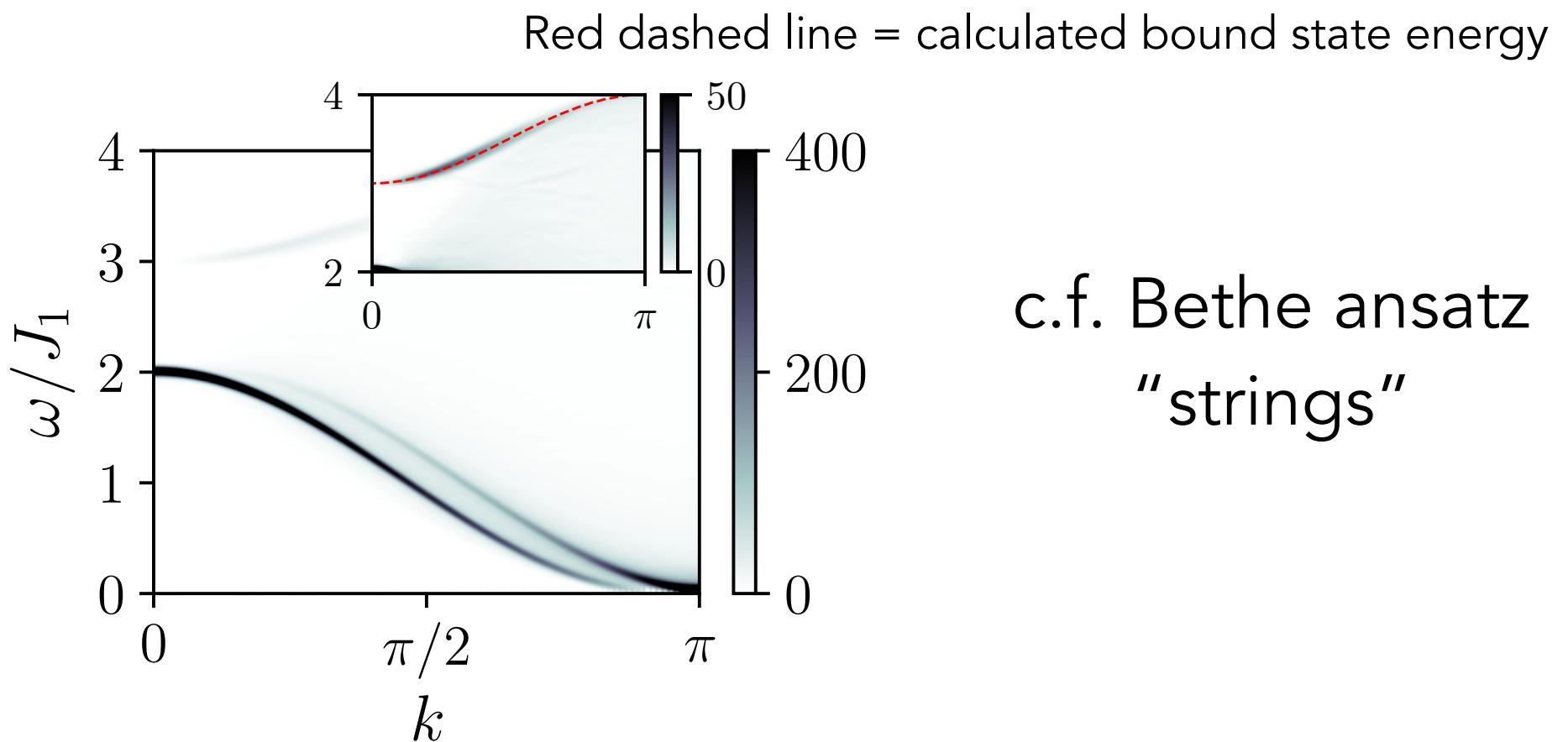
- Two magnons $|2_K\rangle = \sum_{m,n} \psi_{m,n} S_m^- S_n^- |0\rangle$ $\psi_{m,n} = e^{iK(\frac{m+n}{2})} f(m-n)$
- Easy to show there is a bound state outside the two-magnon continuum
- Approximation: finite system with one spin flip in box of size $1/(\text{density of spin flips})$

$$\begin{aligned} \langle S_k^+ \delta(\omega - H) S_k^- \rangle_n &\sim \langle 1_\pi | S_k^+ \delta(\omega - H) S_k^- | 1_\pi \rangle_{L=1/n} \\ &\sim \dots + |\langle 2_{\pi+K} | S_k^- | 1_\pi \rangle|_{L=1/n}^2 \delta(\omega - \epsilon_2(k + \pi)) \end{aligned}$$

2-magnon bound state appears with weight $\sim n \sim M_s - M$

Bound state

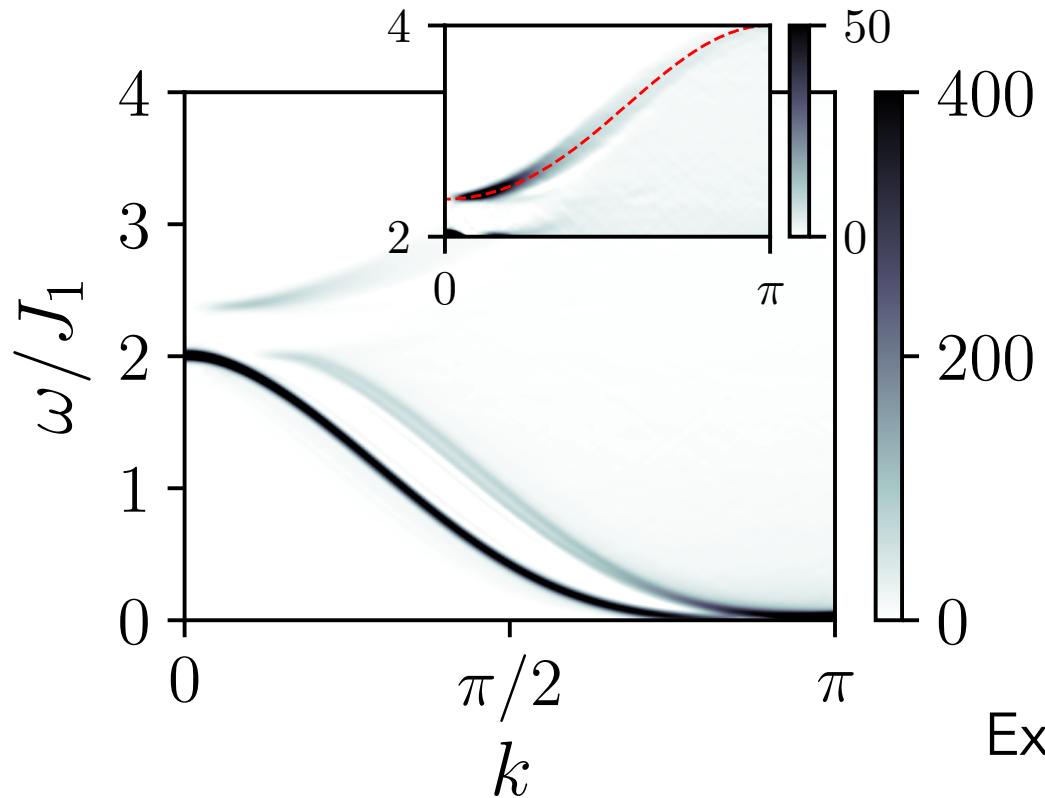
- Check



Bound state

- Check

Red dashed line = calculated bound state energy

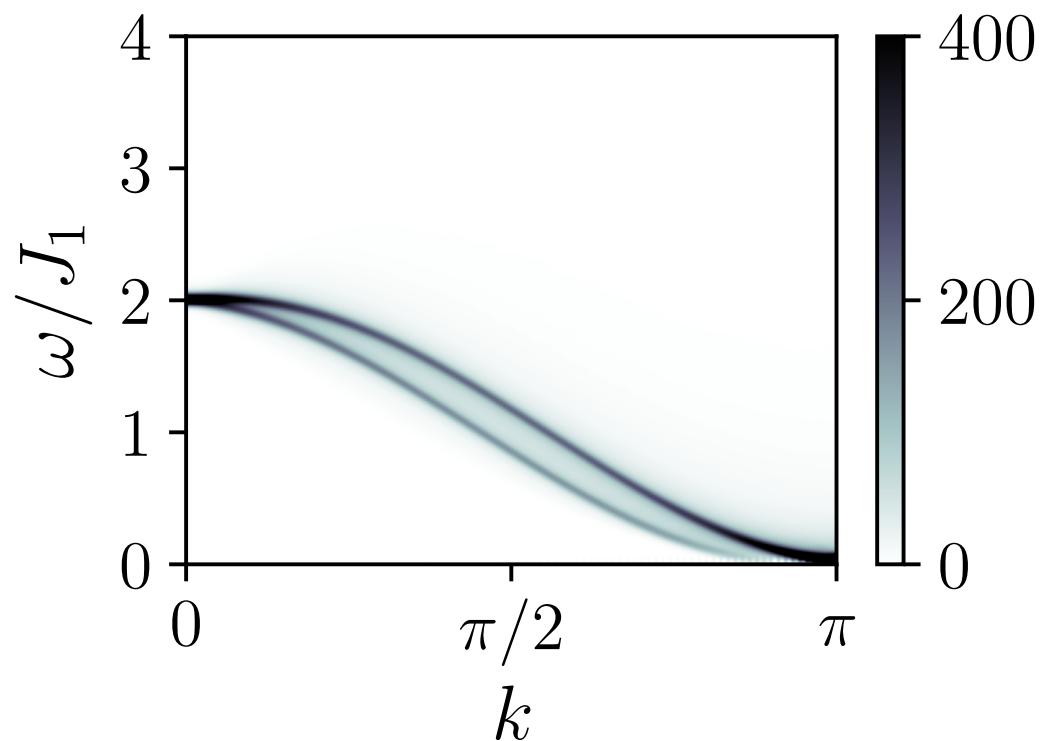


Extremely general phenomena of
spectral weight transfer in low
density correlated systems

But not related
to integrability

Bound state

- Check: does it really come from magnon interactions?
 - XX model (equivalent to free fermions)



Bound state
entirely absent

Also see that lower mode
splitting is *not* an interaction
effect. It arises from Jordan-
Wigner string

Summary

- We identified simple spectral signatures of quasiparticle interactions (spinons or magnons) in 1d chains and 2d spin liquids
- Experiments??