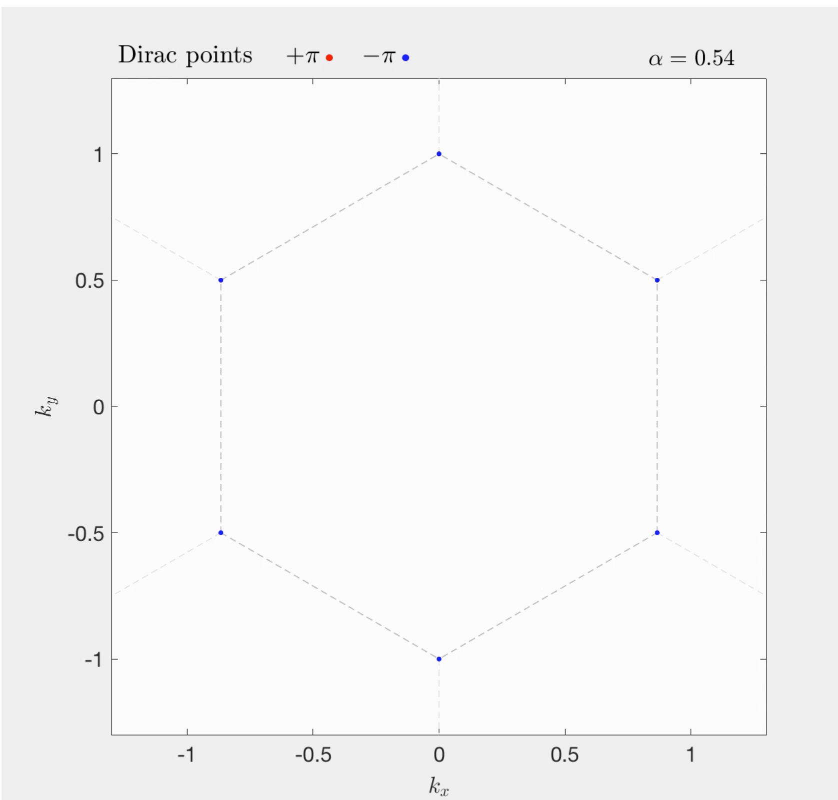
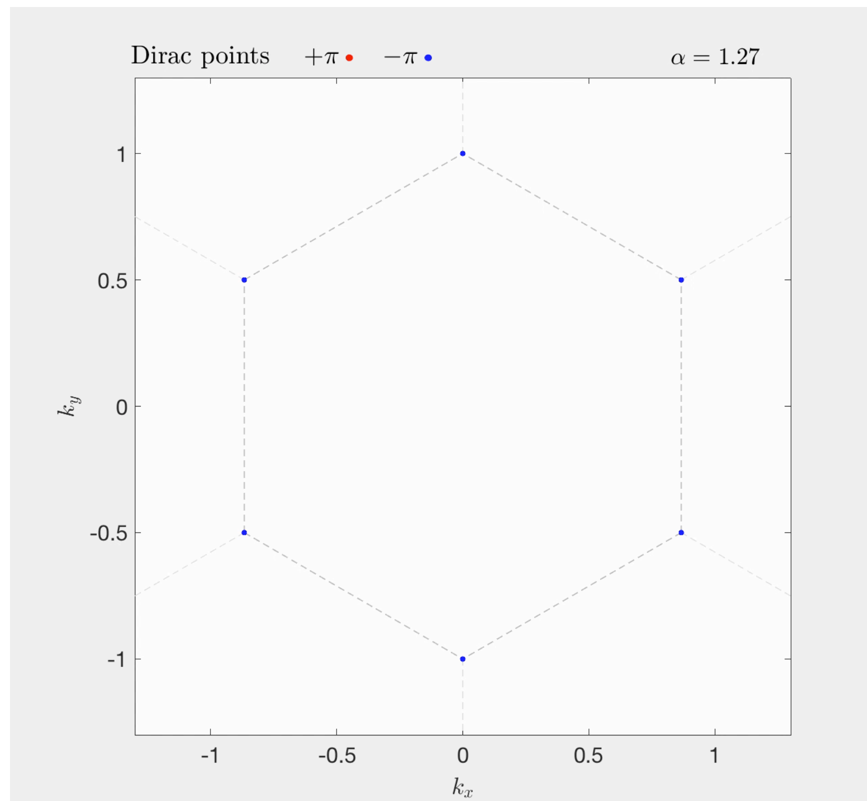
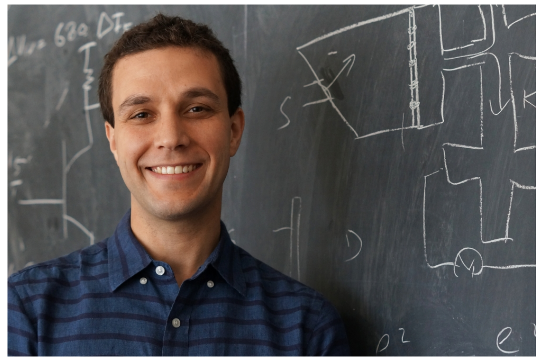
Quantum anomalous Hall effect in twisted bilayer graphene

Leon Balents, KITP





Collaborators



Andrea Young

Chunxiao Liu

Kasra Hejazi

Hassan Shapourian
Harvard/MIT
(for the movies)

Outline

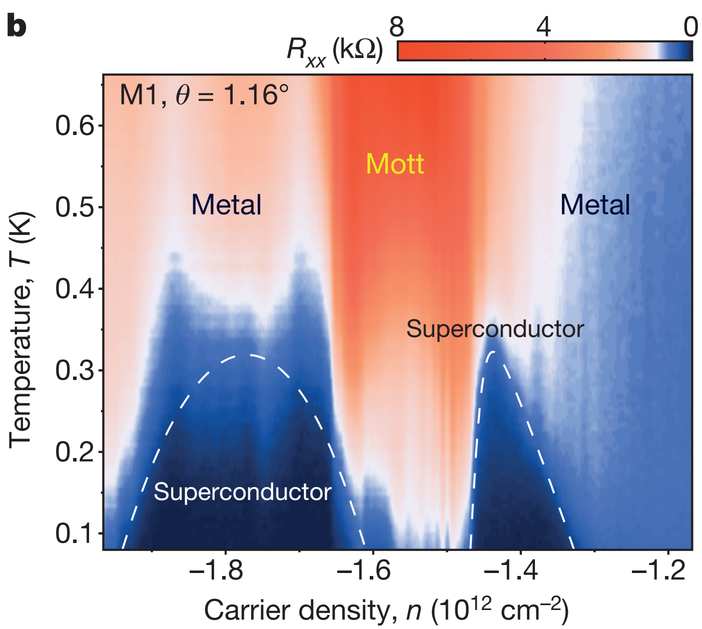
- Summary of twisted bilayer graphene
- Continuum model for TBG for dummies
- Quantum anomalous Hall effect in TBG: controlling the order parameter with current

Magic angle graphene

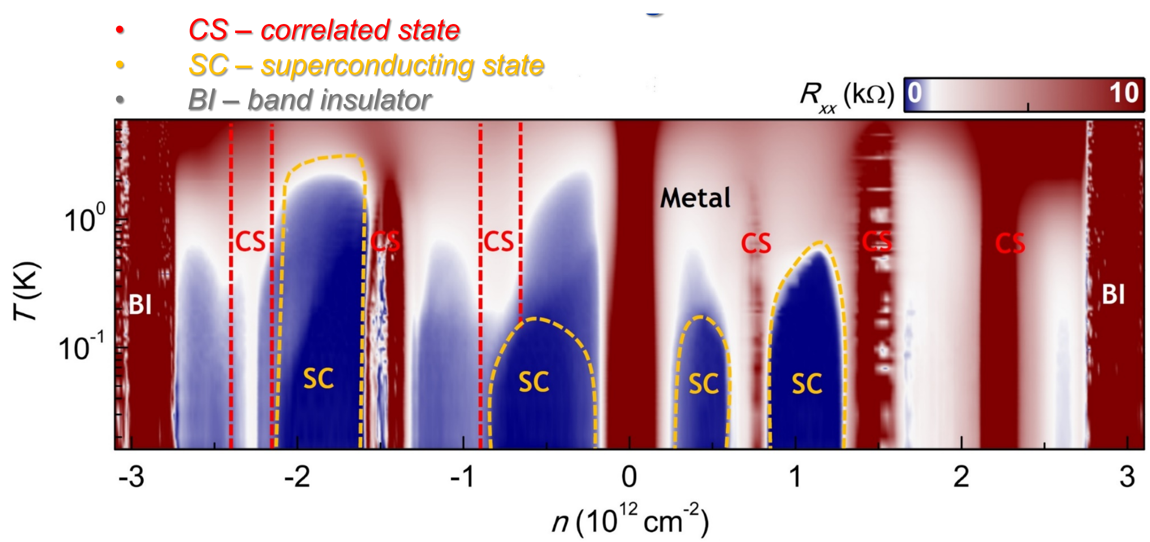
Pablo Jarillo-Herrero
(MIT)

Physics World
Breakthrough of the year,
2018

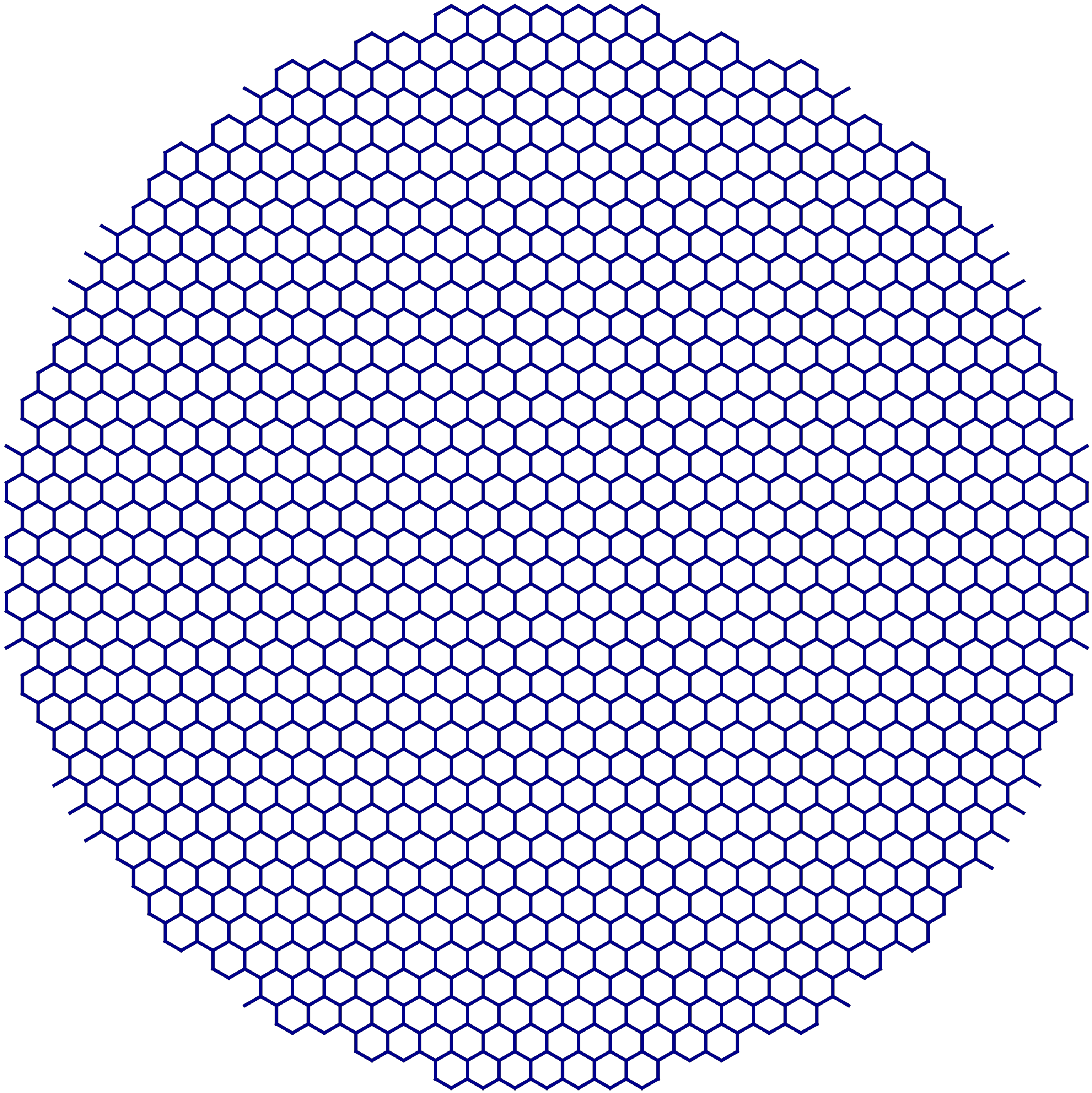
Dimitri Efetov
(Barcelona)



Y. Cao *et al*, 2018

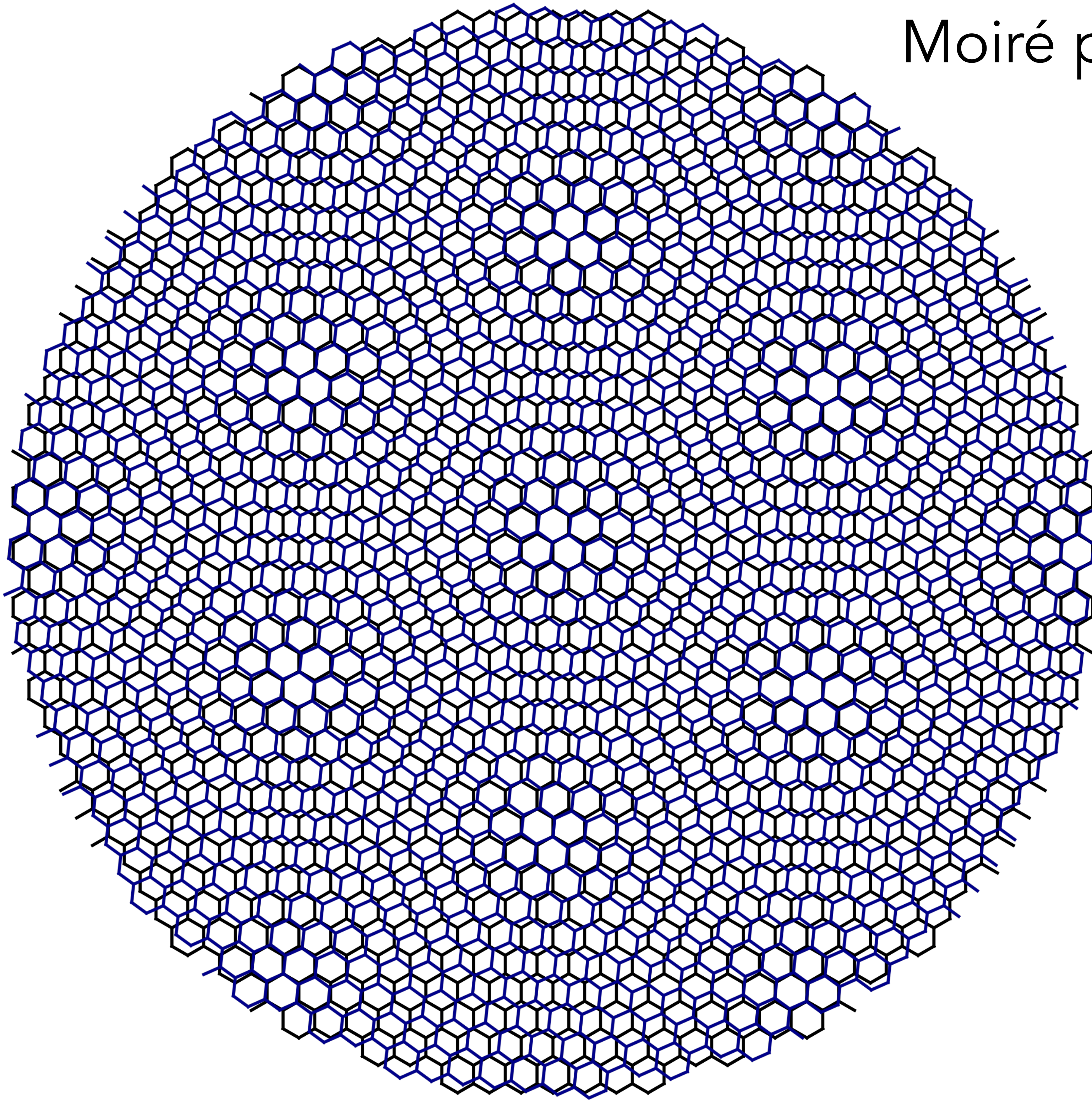


YX. Lu *et al*, 2019



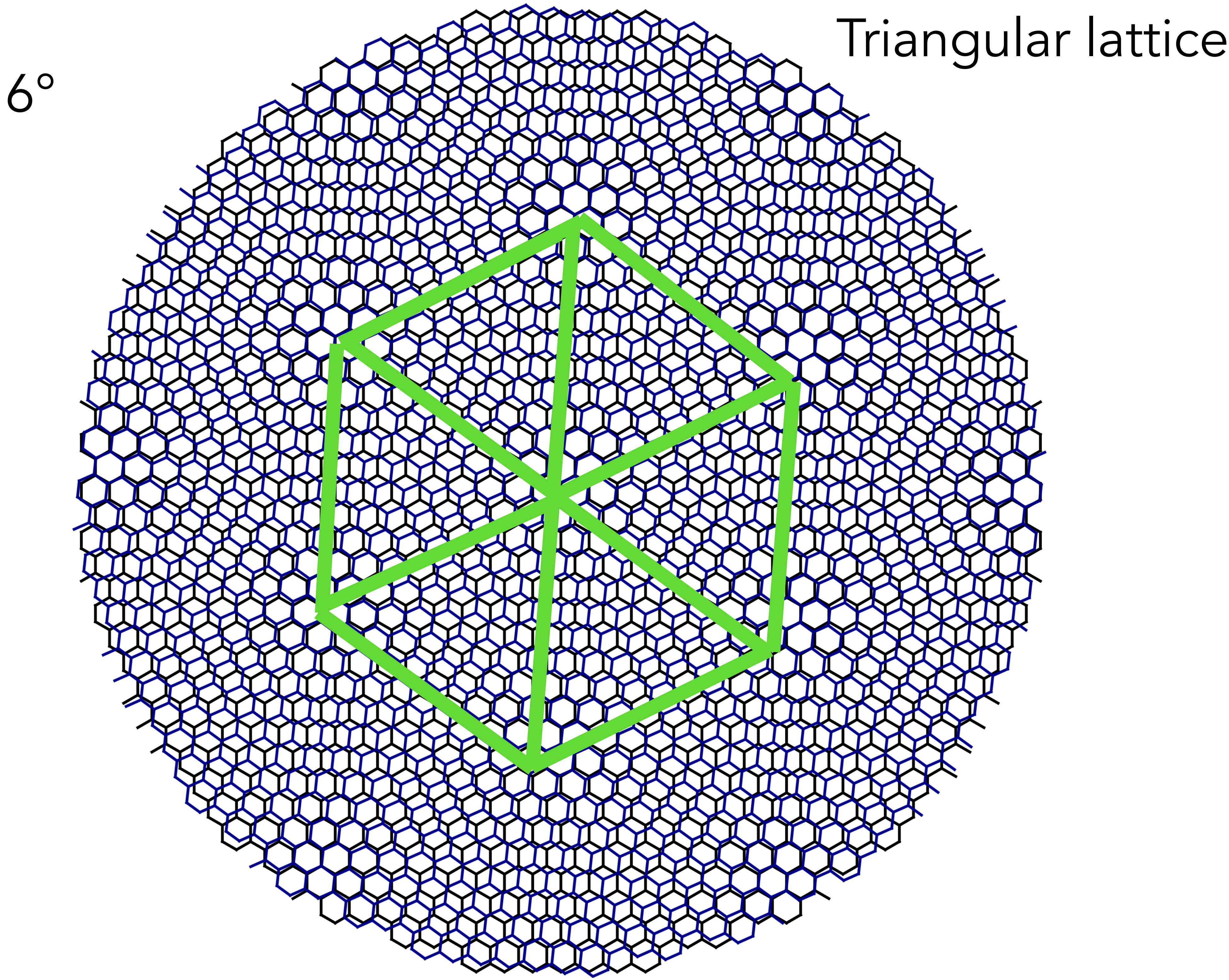
Moiré pattern

6°

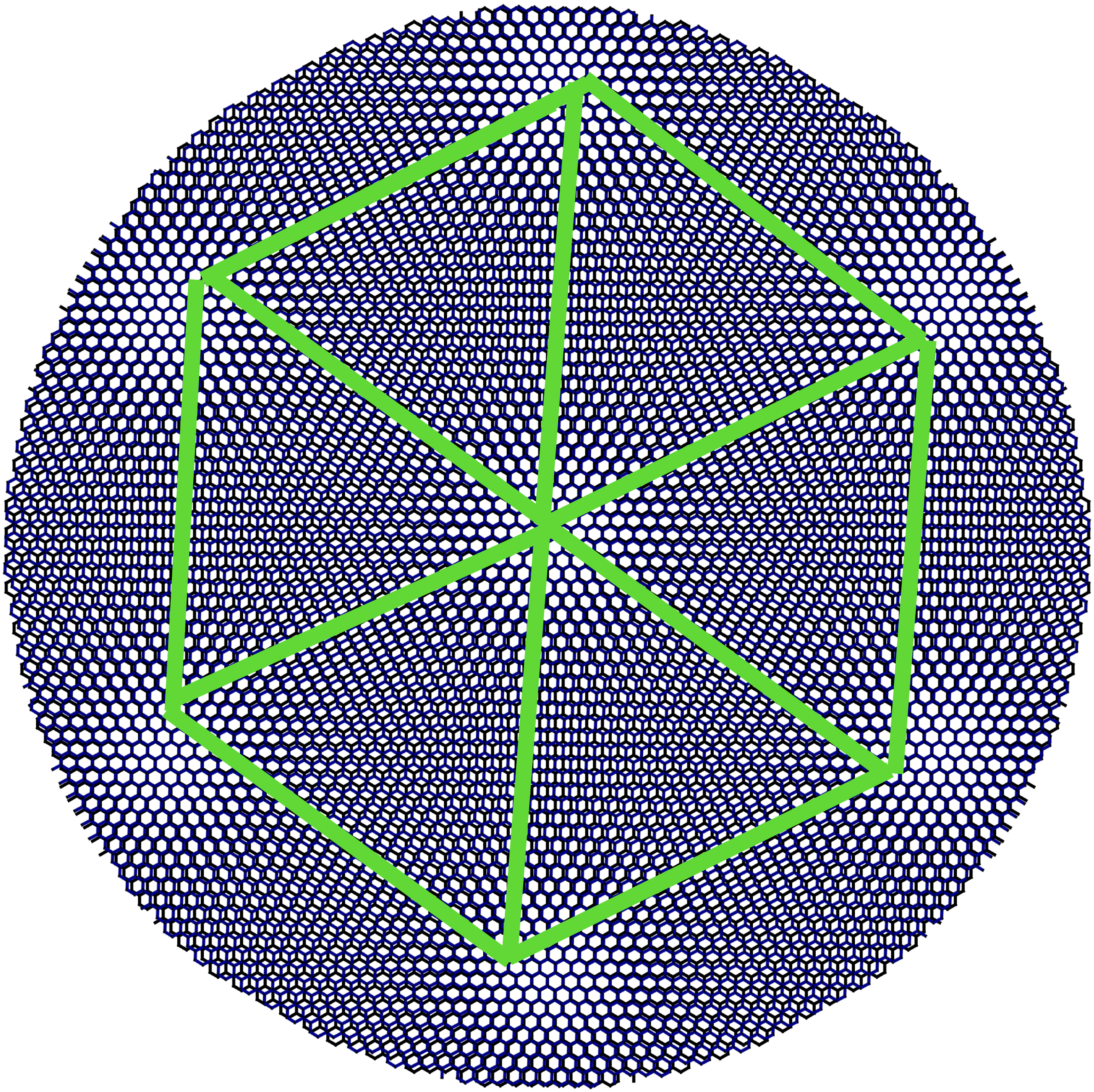


Moiré

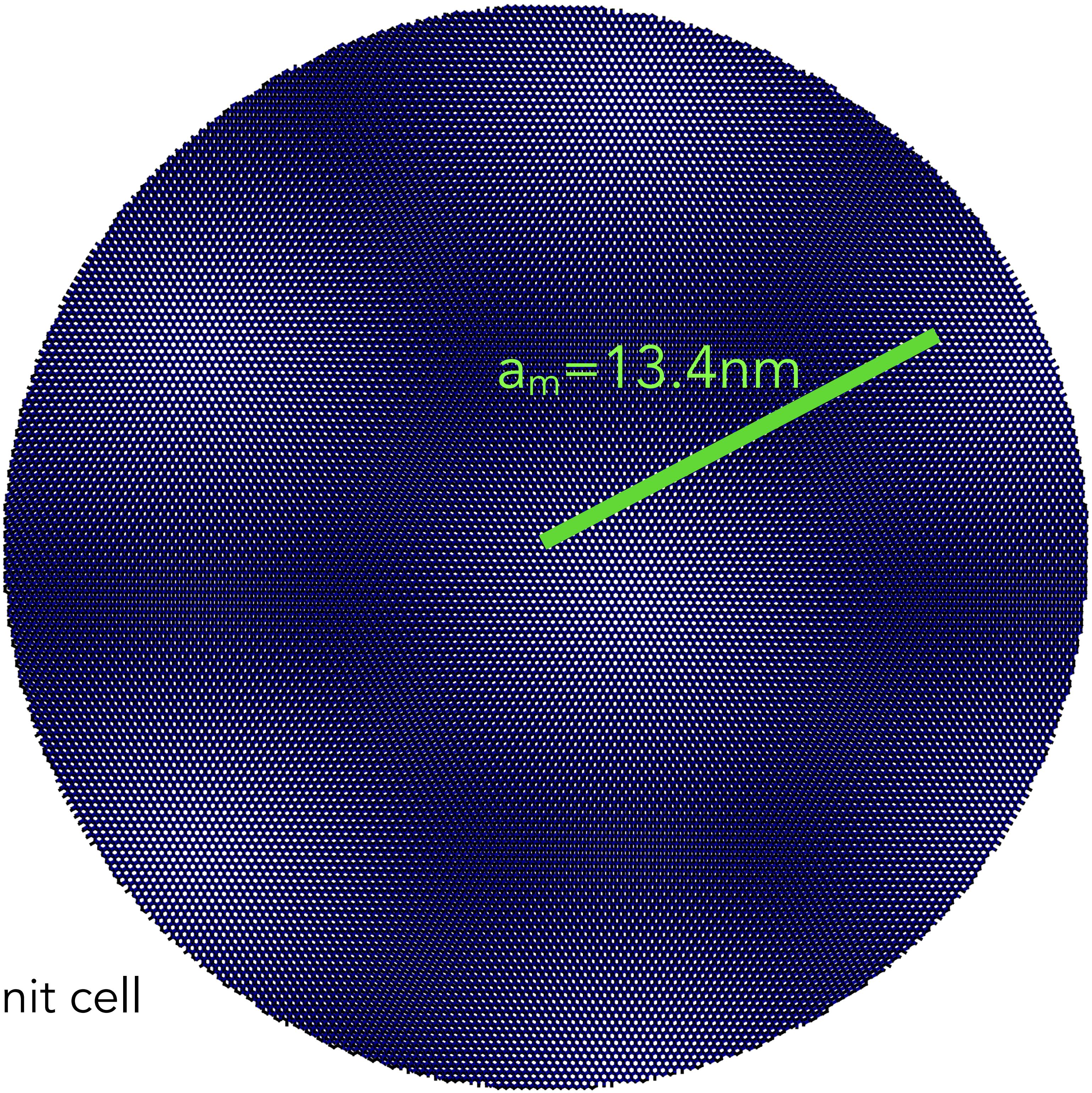
mohair


6°
Triangular lattice

2°

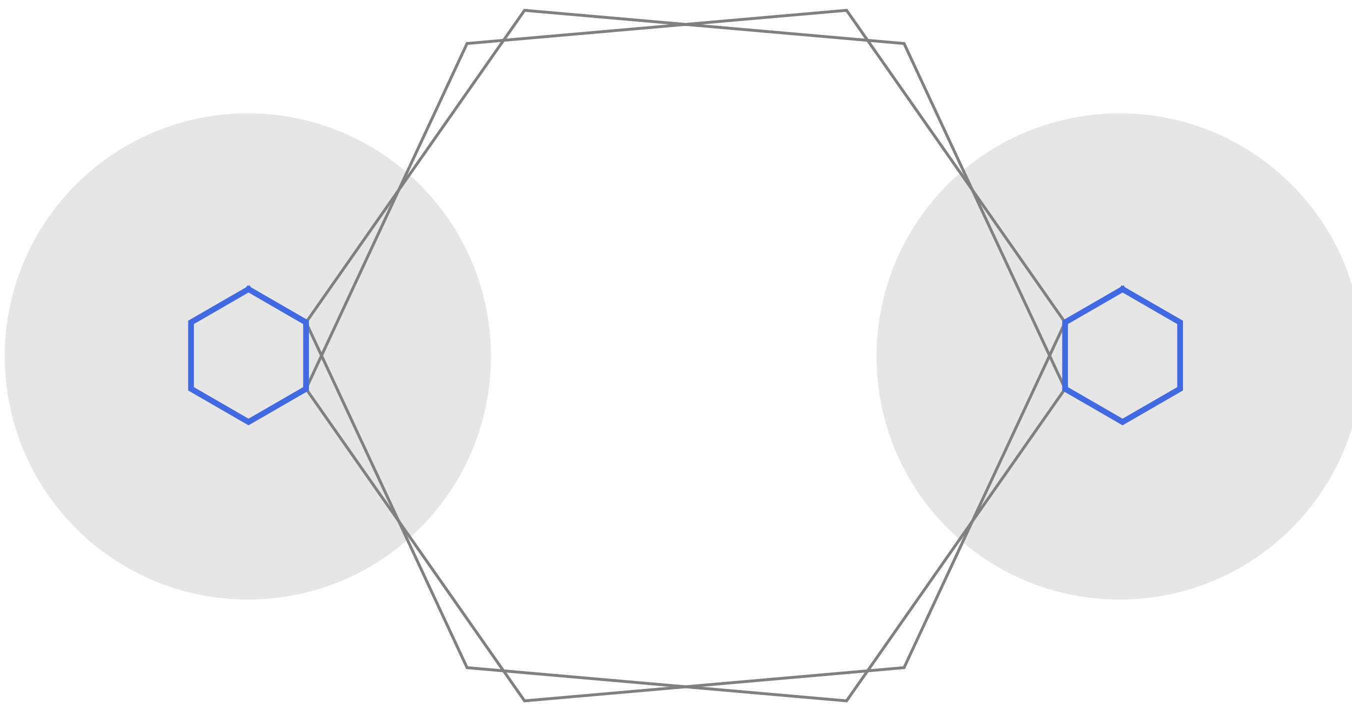


1°



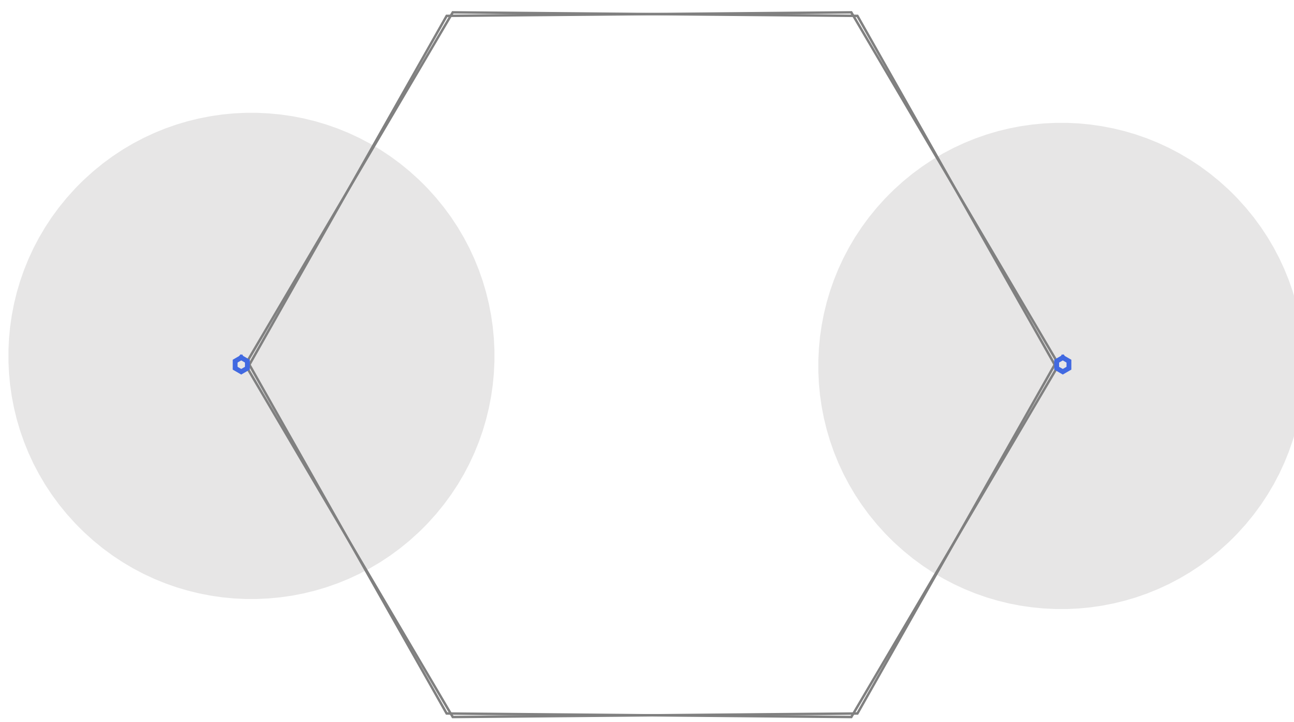
huge unit cell

Continuum model



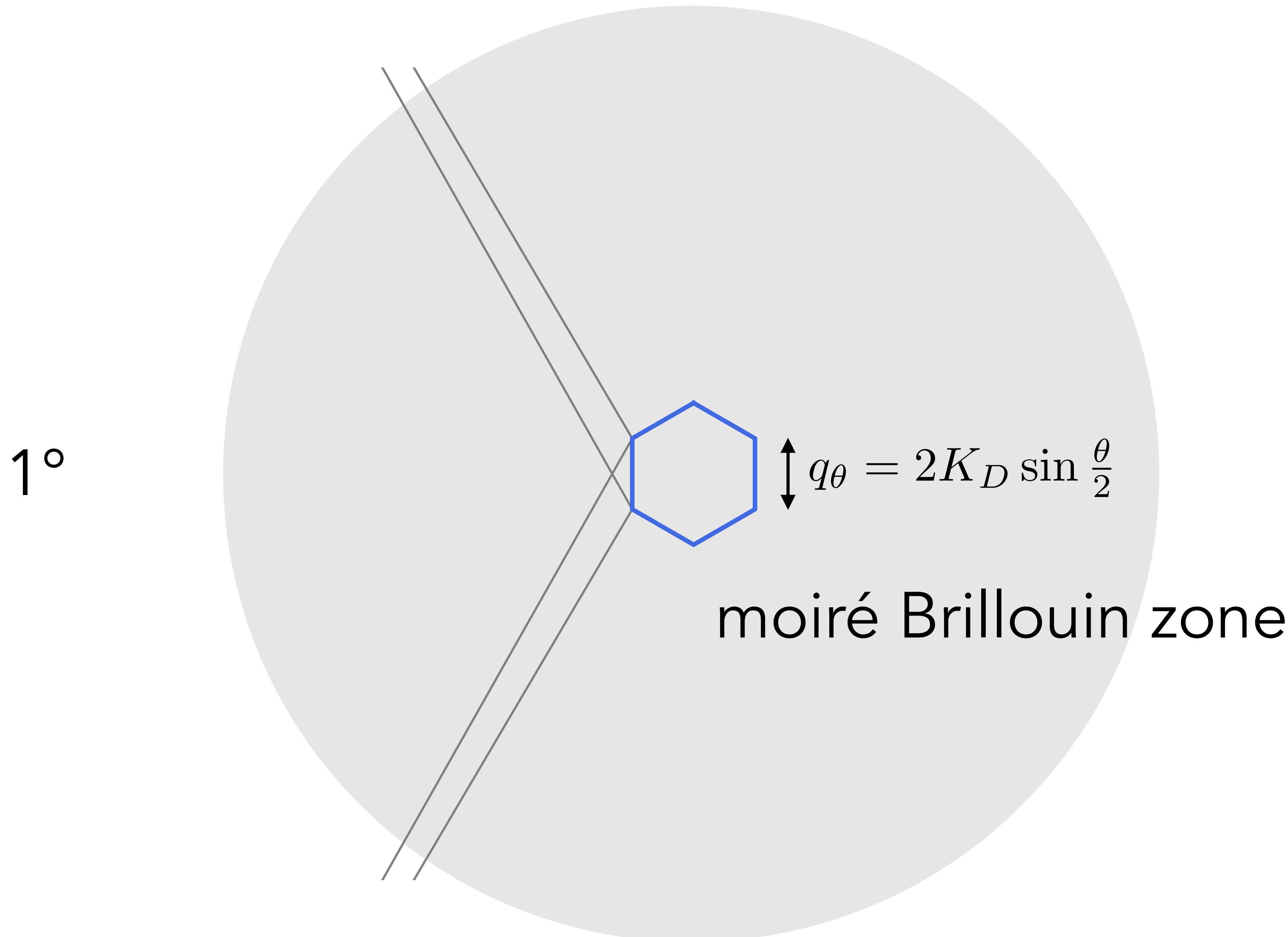
approximate single layer as Dirac cone
no mixing from one valley to the other

Continuum model



1°

One valley



Continuum model

Lopes dos Santos *et al* (2007),
Bistritzer+MacDonald (2011)

$$\mathcal{H} = \psi_1^\dagger \left[-iv\boldsymbol{\tau}(\tfrac{\theta}{2}) \cdot \boldsymbol{\nabla} - \frac{vk_\theta}{2}\tau^y \right] \psi_1 + \psi_2^\dagger \left[-iv\boldsymbol{\tau}(-\tfrac{\theta}{2}) \cdot \boldsymbol{\nabla} + \frac{vk_\theta}{2}\tau^y \right] \psi_2$$

$$+ \sum_j \left[e^{-i\boldsymbol{q}_j \cdot \boldsymbol{x}} \psi_2^\dagger \, \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$

$$\mathsf{T}_j = u\,\mathbb{I} + w\left(\bar{\zeta}^j \tau^+ + \zeta^j \tau^-\right), \qquad j = 0, 1, 2,$$

$$\boldsymbol{q}_j = -\theta \hat{\boldsymbol{z}} \times \boldsymbol{Q}_j$$

- Restores periodicity
- Reveals dimensionless parameter, w/vk_θ
- Predicts flat bands at magic angles

Calculations become possible!

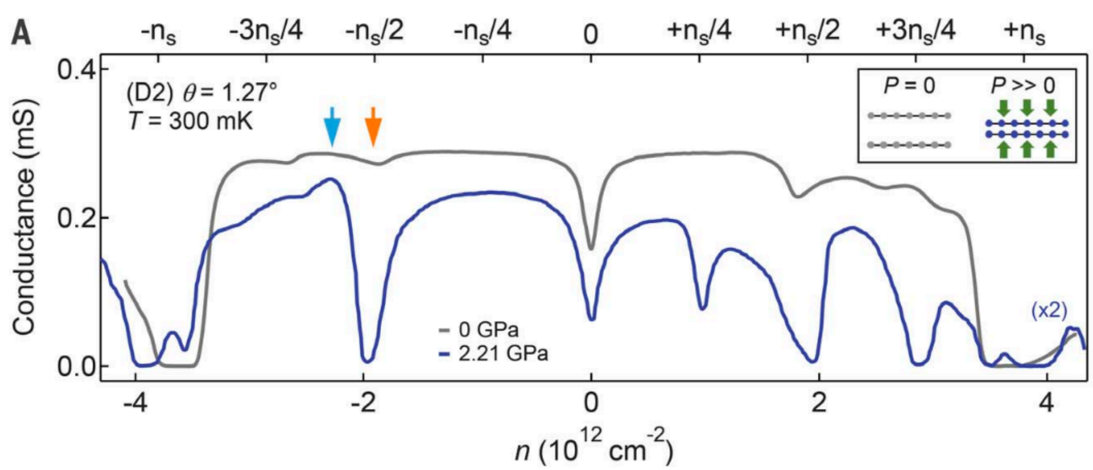
Continuum model

Lopes dos Santos *et al* (2007),
Bistritzer+MacDonald (2011)

One length scale: $a_m = 2\pi / q_\theta$

Dimensionless parameter: $\alpha \equiv \frac{w}{v q_\theta} = \frac{q}{2 v K_D \sin \frac{\theta}{2}}$

Decreasing angle is the same as increasing hopping



Expt proof: M. Yankowitz *et al*, Science 2019

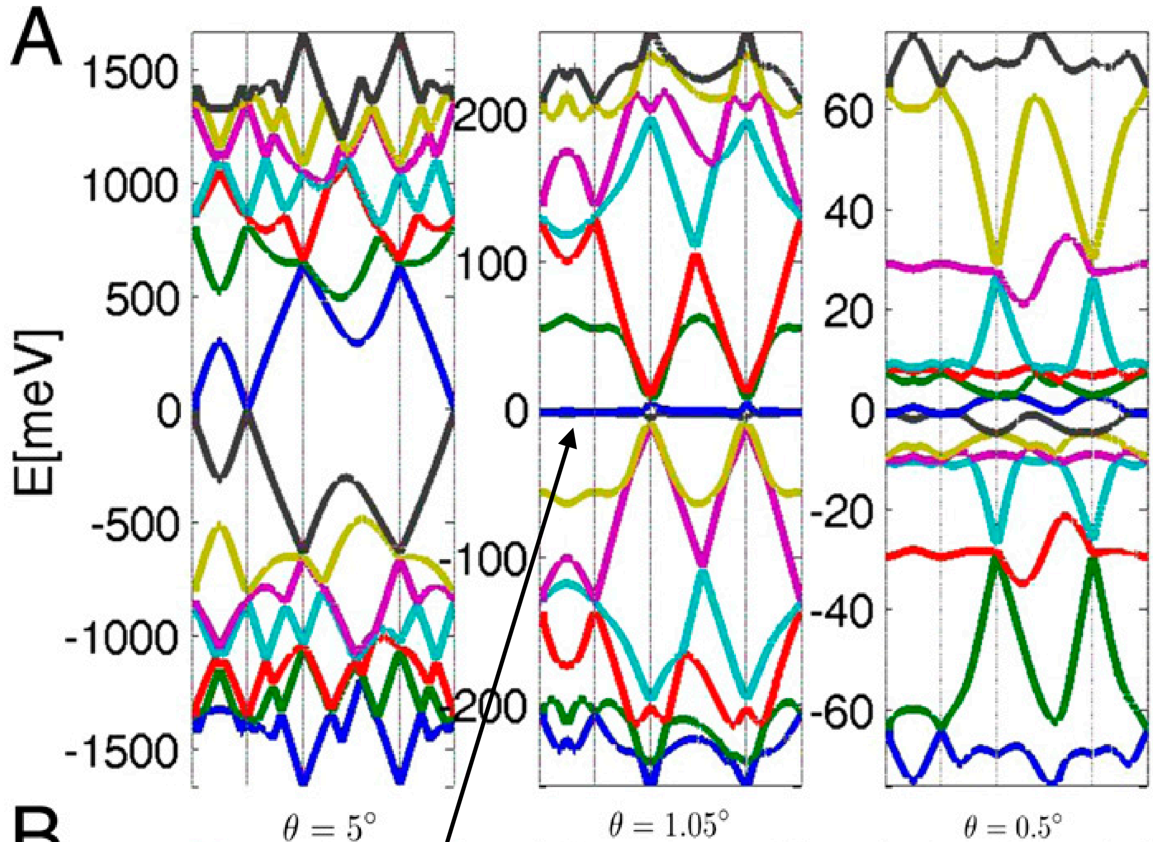
Continuum model

longer moiré period

Just solve it!

(via plane wave expansion)

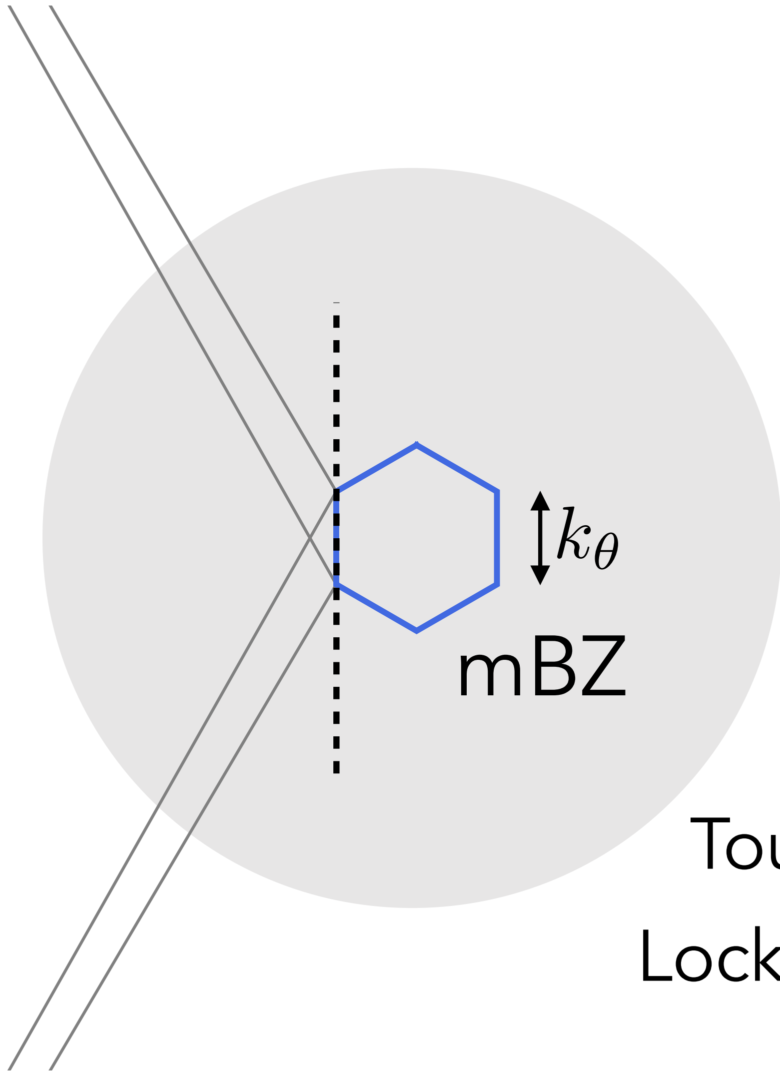
From BM

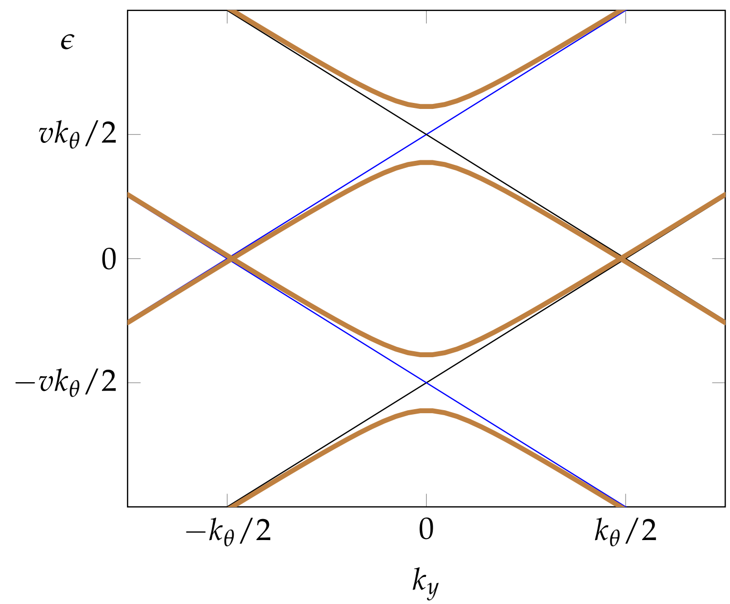


extremely flat bands. W ~ 10meV

Dirac Points

There is always a band touching at K_m,K_m' for *any* angle



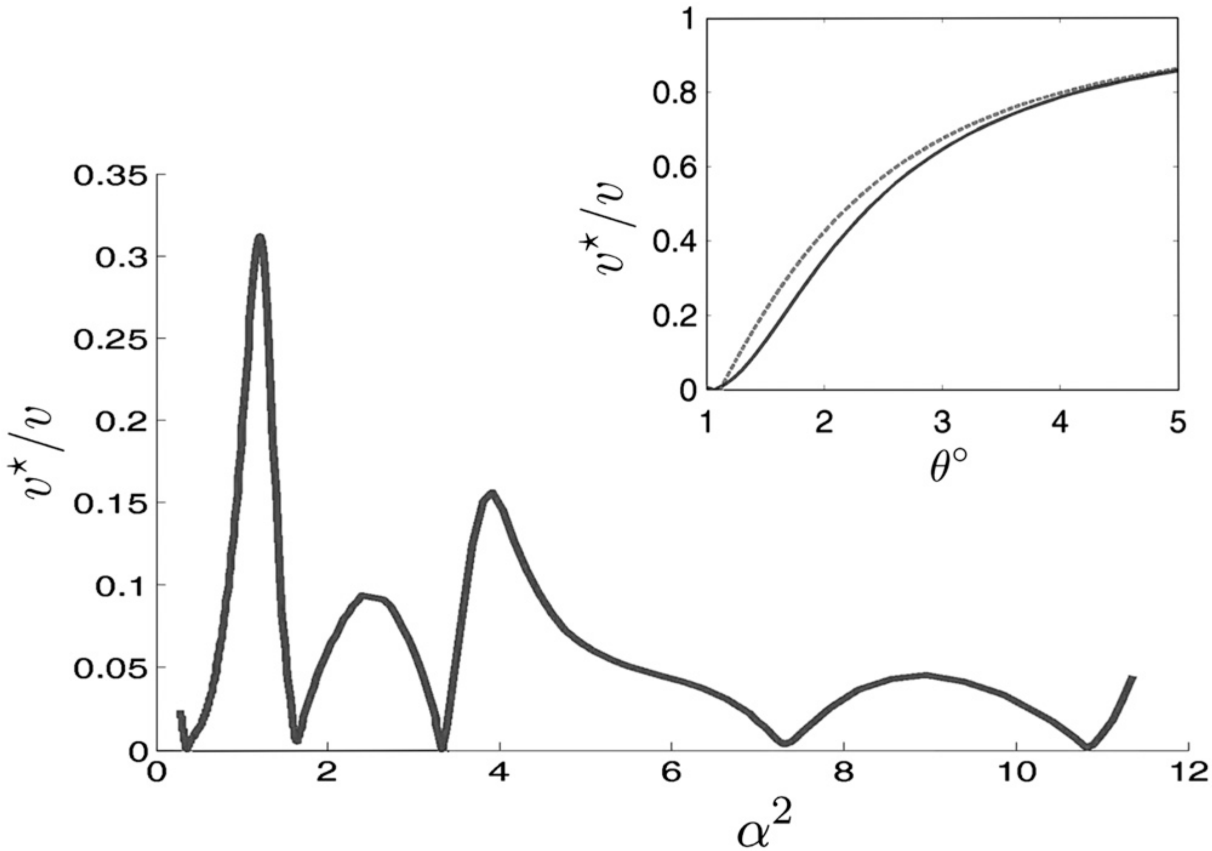


Touching protected by C_2T symmetry

Locked to mBZ corner by C_3 symmetry

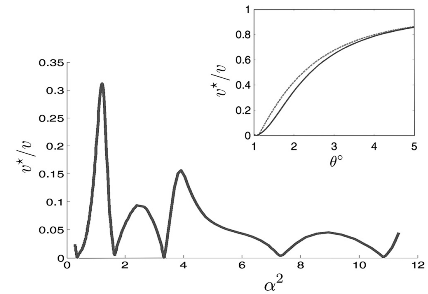
“Magic angles”

From BM



Dirac velocity (at mBZ corners) vanishes at a whole series of angles

Why magic angles?



- A1: Just solve it!
- A2: Tufted cushion effect

K. Hejazi *et al*, 2019

- A3: Close to a chiral model with elliptic function solutions

G. Tarnopolsky *et al*, 2019

Where does continuum model come from?

From BM

We derive a continuum model for the tunneling term by assuming that the interlayer tunneling amplitude between π-orbitals is a smooth function $t(r)$ of spatial separation projected onto the graphene planes. The matrix element

$$T^{\alpha\beta}_{\mathbf{k}\mathbf{p}'} = \langle \Psi^{(1)}_{\mathbf{k}\alpha} | H_{\mathrm{T}} | \Psi^{(2)}_{\mathbf{p}'\beta} \rangle \quad [1]$$

of the tunneling Hamiltonian H_{T} describes a process in which an electron with momentum $\mathbf{p}' = M\mathbf{p}$ residing on sublattice β in one layer hops to a momentum state $\mathbf{k}$ and sublattice α in the other layer. In a π-band tight-binding model the projection of the wave functions of the two layers to a given sublattice are

$$|\psi^{(1)}_{\mathbf{k}\alpha}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} e^{i\mathbf{k}(R+\tau_\alpha)} |\mathbf{R} + \tau_\alpha\rangle \quad [2]$$

and

$$|\psi^{(2)}_{\mathbf{p}\beta}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}'} e^{i\mathbf{p}(R'+\tau'_\beta)} |\mathbf{R}' + \tau'_\beta\rangle. \quad [3]$$

Here $\tau_{\mathrm{A}} = 0$, $\tau_{\mathrm{B}} = \tau$, and $\mathbf{R}$ is summed over the triangular Bravais lattice. Substituting Eqs. **2** and **3** in Eq. **1** and invoking the two-center approximation,

$$\langle \mathbf{R} + \tau_\alpha | H_{\mathrm{T}} | \mathbf{R}' + \tau'_\beta \rangle = t(\mathbf{R} + \tau_\alpha - \mathbf{R}' - \tau'_\beta), \quad [4]$$

for the interlayer hopping amplitude in which t depends on the difference between the positions of the two carbon atoms we find that

$$T^{\alpha\beta}_{\mathbf{k}\mathbf{p}'} = \sum_{\mathbf{G}_1\mathbf{G}_2} \frac{t_{\bar{\mathbf{k}}+\mathbf{G}_1}}{\Omega} e^{i[\mathbf{G}_1\tau_\alpha - \mathbf{G}_2(\tau_\beta - \tau) - \mathbf{G}'_2\mathbf{d}]} \delta_{\bar{\mathbf{k}}+\mathbf{G}_1, \bar{\mathbf{p}}'+\mathbf{G}'_2}. \quad [5]$$

Here Ω is the unit cell area, $t_{\mathbf{q}}$ is the Fourier transform of the tunneling amplitude $t(\mathbf{r})$, the vectors $\mathbf{G_1}$ and $\mathbf{G_2}$ are summed over reciprocal lattice vectors, and $\mathbf{G}'_2 = M\mathbf{G_2}$. The bar notation over momenta in Eq. **5** indicates that momentum is measured relative to the center of the Brillouin zone and not relative to the Dirac point. Note that crystal momentum is conserved by the tunneling process because t depends only on the difference between lattice positions.*

Directly calculate overlap of every C orbital in layer 1 with every C orbital in layer 2

Assume rigid rotation of layers

Obtain hopping matrix in momentum space by Poisson resummation formula

At the end of the calculation Fourier transform back to obtain simple real space formula

Effective field theory

Describes low energy, long wavelength physics, can include effects of any perturbations that are small and slowly varying

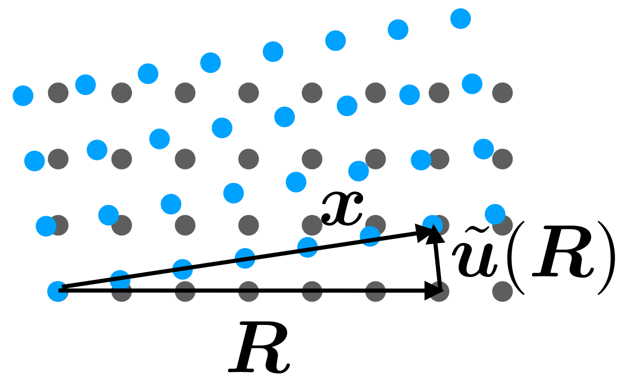
Here:

- Unperturbed system: isolated graphene layers
- Perturbations:
  - Interlayer tunneling
  - Slowly varying displacements of the layers

Rotation ⊂ Displacement Gradient

Ashcroft-Mermin: phonons

$$\boldsymbol{x} = \boldsymbol{R} + \tilde{\boldsymbol{u}}(\boldsymbol{R})$$



Rotation

$$\theta = \frac{1}{2}\left(\partial_x u_y - \partial_y u_x\right)$$

$$\tilde{\boldsymbol{u}} = \theta \hat{\boldsymbol{z}} \times \boldsymbol{R}$$

Twisting is just a subset of elastic deformations of two layers

Effective field theory

Locality:

$$H = \int d^2x\, \mathcal{H}[\psi, \boldsymbol{u}, w]$$

Hamiltonian density is a *local* functional of the fields, *analytic*, and expandable in powers of small parameters — here field gradients and hopping strength

Effective field theory

Locality:

$$H = \int d^2x \, \mathcal{H}[\psi, \boldsymbol{u}, w]$$

Hamiltonian density is a ***local*** functional of the fields, *analytic*, and expandable in powers of small parameters — here field gradients and hopping strength

Small problem:

$$\boldsymbol{x} = \boldsymbol{R} + \tilde{\boldsymbol{u}}(\boldsymbol{R})$$

R is not the actual real space location - physics is local in **x** not **R**

Effective field theory

Locality:

$$H = \int d^2x \, \mathcal{H}[\psi, \boldsymbol{u}, w]$$

Hamiltonian density is a ***local*** functional of the fields, *analytic*, and expandable in powers of small parameters — here field gradients and hopping strength

Solution: Eulerian coordinates

$$\boldsymbol{x} = \boldsymbol{R} + \boldsymbol{u}(\boldsymbol{x})$$

Effective field theory

- Three effects:

  1. Coordinate change: transformation of local frames to global one

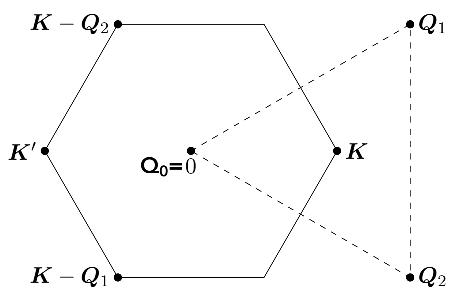
  2. Strains: modification of energetics of each layer due to changes in electron hopping

  3. Tunneling: strong dependence of relative *local* alignment

Result

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[-iv\left(\tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu}\tau^\nu\right)\frac{\partial}{\partial x_\mu} + v\left(\boldsymbol{K}\cdot\partial_\mu \boldsymbol{u}_l + \boldsymbol{\mathcal{A}}_l\right)\tau^\mu\right]\psi_l$$

$$+\sum_j \left[e^{-i\boldsymbol{Q}_j\cdot(\boldsymbol{u}_1-\boldsymbol{u}_2)}\psi_2^\dagger \mathsf{T}_j\psi_1 + \text{h.c.}\right]$$



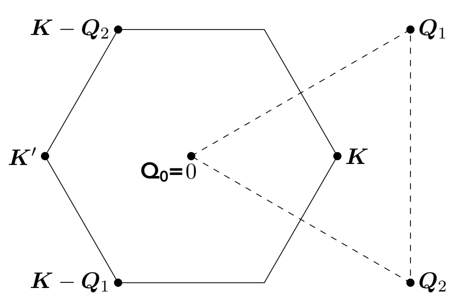
Correct to first order in strain gradients and hopping

Result

coordinate change

$$\mathcal{H}=\sum_l \psi_l^\dagger \left[-iv\left(\tau^\mu+\frac{\partial u_{l,\mu}}{\partial x_\nu}\tau^\nu\right)\frac{\partial}{\partial x_\mu}+v\left(\boldsymbol{K}\cdot\partial_\mu\boldsymbol{u}_l+\boldsymbol{\mathcal{A}}_l\right)\tau^\mu\right]\psi_l$$

$$+\sum_j\left[e^{-i\boldsymbol{Q}_j\cdot(\boldsymbol{u}_1-\boldsymbol{u}_2)}\psi_2^\dagger \mathsf{T}_j\psi_1+\text{h.c.}\right]$$



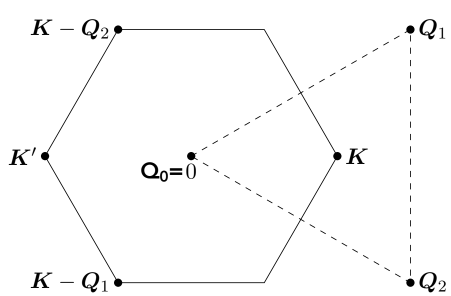
Correct to first order in strain gradients and hopping

Result

Strain gauge field

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[-iv\left(\tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu}\tau^\nu\right)\frac{\partial}{\partial x_\mu} + v\left(\boldsymbol{K}\cdot\partial_\mu \boldsymbol{u}_l + \boldsymbol{\mathcal{A}}_l\right)\tau^\mu \right]\psi_l$$

$$+ \sum_j \left[e^{-i\boldsymbol{Q}_j\cdot(\boldsymbol{u}_1 - \boldsymbol{u}_2)}\psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.}\right]$$



Correct to first order in strain gradients and hopping

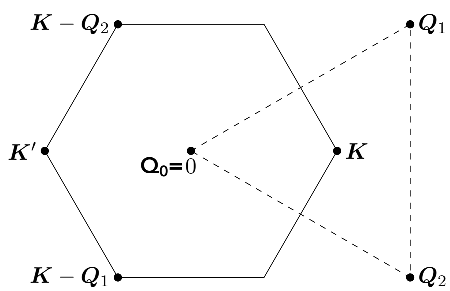
Result

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[-iv\left(\tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu}\tau^\nu\right)\frac{\partial}{\partial x_\mu} + v\left(\boldsymbol{K}\cdot\partial_\mu \boldsymbol{u}_l + \boldsymbol{\mathcal{A}}_l\right)\tau^\mu\right]\psi_l$$

$$+\sum_j \left[e^{-i\boldsymbol{Q}_j\cdot(\boldsymbol{u}_1-\boldsymbol{u}_2)}\psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.}\right]$$

← tunneling. Form fixed by space group symmetries

$$\mathsf{T}_j = u\,\mathbb{I} + w\left(\bar{\zeta}^j\tau^+ + \zeta^j\tau^-\right), \qquad j = 0,1,2,$$



Correct to first order in strain gradients and hopping

Apply to rigid twist

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[-iv \left(\tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v \left(\boldsymbol{K} \cdot \partial_\mu \boldsymbol{u}_l + \boldsymbol{\mathcal{A}}_l \right) \tau^\mu \right] \psi_l$$
$$+ \sum_j \left[e^{-i\boldsymbol{Q}_j \cdot (\boldsymbol{u}_1 - \boldsymbol{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$

Evaluate for $\boldsymbol{u}_1 = -\boldsymbol{u}_2 = \frac{\theta}{2} \hat{\boldsymbol{z}} \times \boldsymbol{x}.$ →

$$\mathcal{H} = \psi_1^\dagger \left[-iv\boldsymbol{\tau}(\tfrac{\theta}{2}) \cdot \boldsymbol{\nabla} - \frac{vk_\theta}{2} \tau^y \right] \psi_1 + \psi_2^\dagger \left[-iv\boldsymbol{\tau}(-\tfrac{\theta}{2}) \cdot \boldsymbol{\nabla} + \frac{vk_\theta}{2} \tau^y \right] \psi_2$$

$$+ \sum_j \left[e^{-i\boldsymbol{q}_j \cdot \boldsymbol{x}} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right] \qquad \boldsymbol{q}_j = -\theta \hat{\boldsymbol{z}} \times \boldsymbol{Q}_j$$

Exactly the BM model.

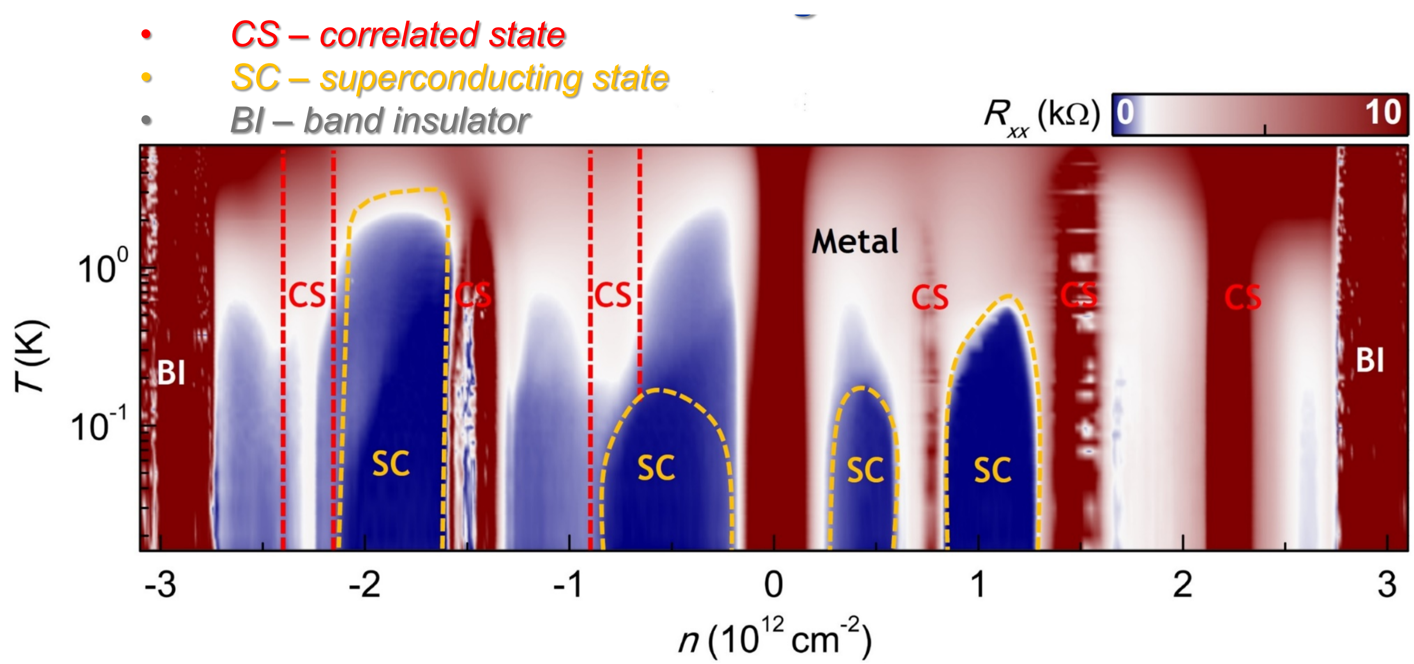
Result

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[-iv \left(\tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v \left(\boldsymbol{K} \cdot \partial_\mu \boldsymbol{u}_l + \boldsymbol{\mathcal{A}}_l \right) \tau^\mu \right] \psi_l$$
$$+ \sum_j \left[e^{-i\boldsymbol{Q}_j \cdot (\boldsymbol{u}_1 - \boldsymbol{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$

- Recovers BM result intuitively
- Subsumes other extensions of BM (Nam+Koshino, Bi,Yuan+Fu…)
- Includes coupling of acoustic phonons
- Can handle arbitrary inhomogeneous strains
- All these things together
- Easy to add more layers
- Very nice for teaching

Interaction physics

YX. Lu *et al*, 2019



All this *within* two flat bands

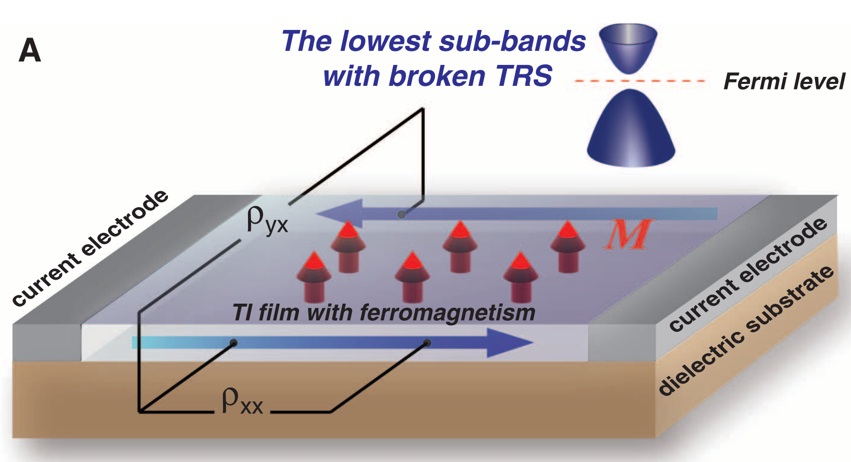
Many correlated insulators

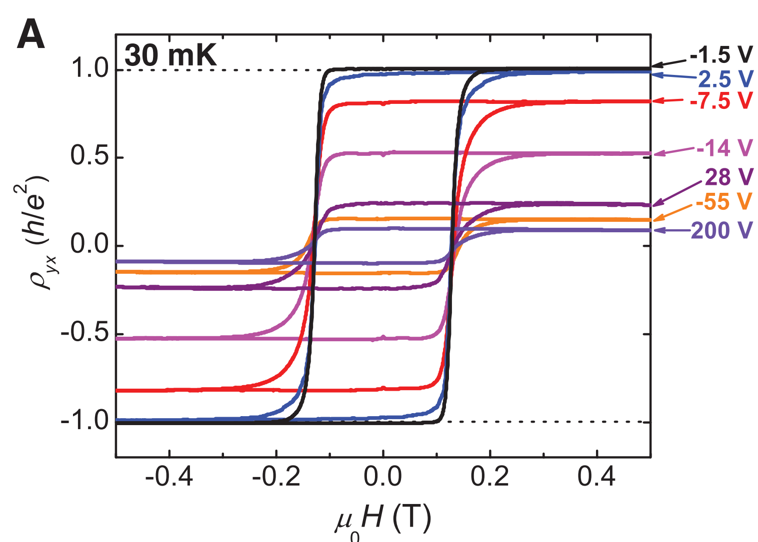
Many superconductors

Nematics?

Quantum Anomalous Hall Effect

This is just the appearance of QHE in zero magnetic field by spontaneous breaking of time-reversal symmetry



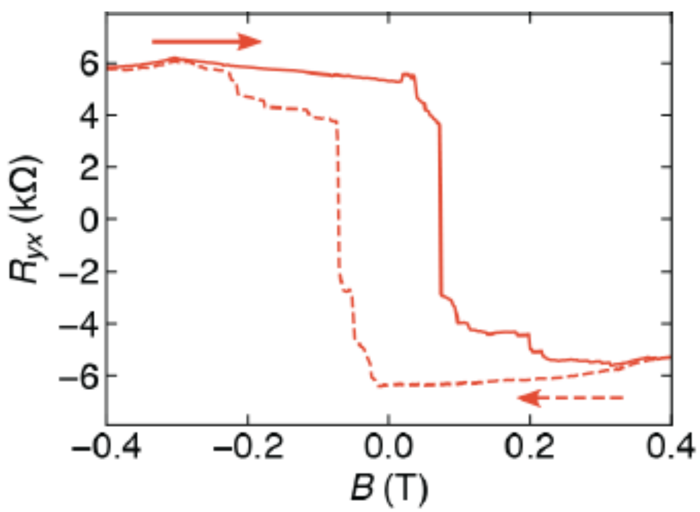


C.-Z. Zhang *et al*, 2013

Cr-doped $(Bi/Sb)_2Te_3$

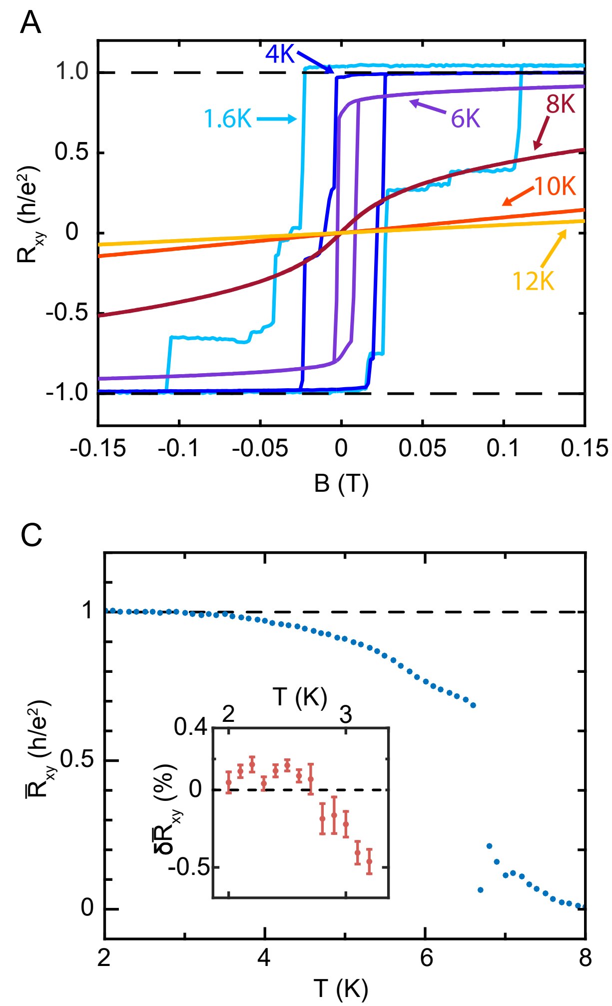
QAHE in TBG

3 e⁻'s per moiré unit cell



A. Sharpe, et. al. ***Science (2019)***;

Spontaneous AHE - not quite quantized

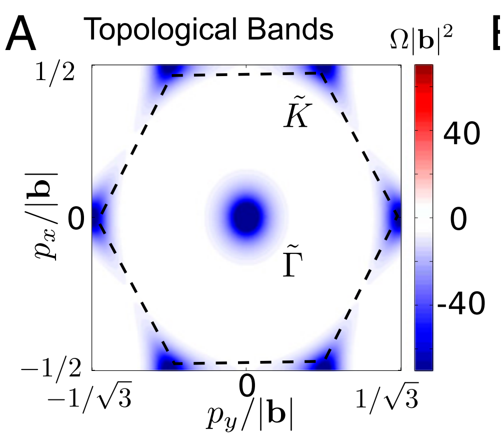


M. Serlin *et al*, unpublished

QAHE to 1/1000 accuracy

Theoretical remarks

- Underlying Dirac fermions of graphene have large incipient Berry curvature
- Curvature is realized by breaking C_2T symmetry



Topological Bloch bands in graphene superlattices

Justin C. W. Song[a,b,c,1], Polnop Samutpraphoot[c], and Leonid S. Levitov[c,1]

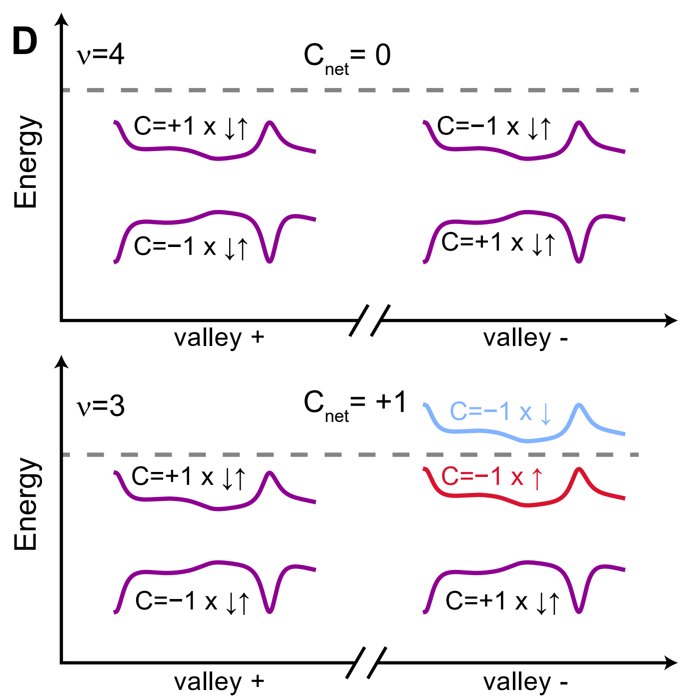
[a]Walter Burke Institute for Theoretical Physics, California Institute of Technology, CA 91125; [b]Institute for Quantum Information and Matter, and Department of Physics, California Institute of Technology, CA 91125; and [c]Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

Edited by Subir Sachdev, Harvard University, Cambridge, MA, and approved June 24, 2015 (received for review December 30, 2014)

No interactions needed - just coupling to hBN - to generate Dirac mass and form valley Chern bands

Theoretical remarks

- Valley polarization gives non-zero AHE.
- Quantization occurs if gap is complete - needs spin polarization



Flat band ferromagnetism

Spin and **valley** exchange splitting

If you have any more questions

arXiv:1901.08110 [pdf, other] cond-mat.str-el cond-mat.mes-hall

Anomalous Hall ferromagnetism in twisted bilayer graphene

Authors: Nick Bultinck, Shubhayu Chatterjee, Michael P. Zaletel

arXiv:1907.13633 [pdf, other] cond-mat.mes-hall cond-mat.str-el

Ferromagnetism and its stability from the one-magnon spectrum in twisted bilayer graphene

Authors: Yahya Alavirad, Jay D. Sau

arXiv:1907.11723 [pdf, other] cond-mat.str-el

Ferromagnetism in narrow bands of moiré superlattices

Authors: Cécile Repellin, Zhihuan Dong, Ya-Hui Zhang, T. Senthil

arXiv:1812.04213 [pdf, other] cond-mat.str-el

On the nature of the correlated insulator states in twisted bilayer graphene

Authors: Ming Xie, Allan H. MacDonald

arXiv:1810.08642 [pdf, other] cond-mat.str-el doi 10.1103/PhysRevLett.122.246401

Strong coupling phases of partially filled twisted bilayer graphene narrow bands

Authors: Jian Kang, Oskar Vafek

…or one of another 141 papers in the last year

Of course this is the most important one

arXiv:1901.08110 [pdf, other] cond-mat.str-el cond-mat.mes-hall

Anomalous Hall ferromagnetism in twisted bilayer graphene

Authors:

arXiv:190

Ferrom phene

Authors:

arXiv:190

Ferrom

Authors:

arXiv:181

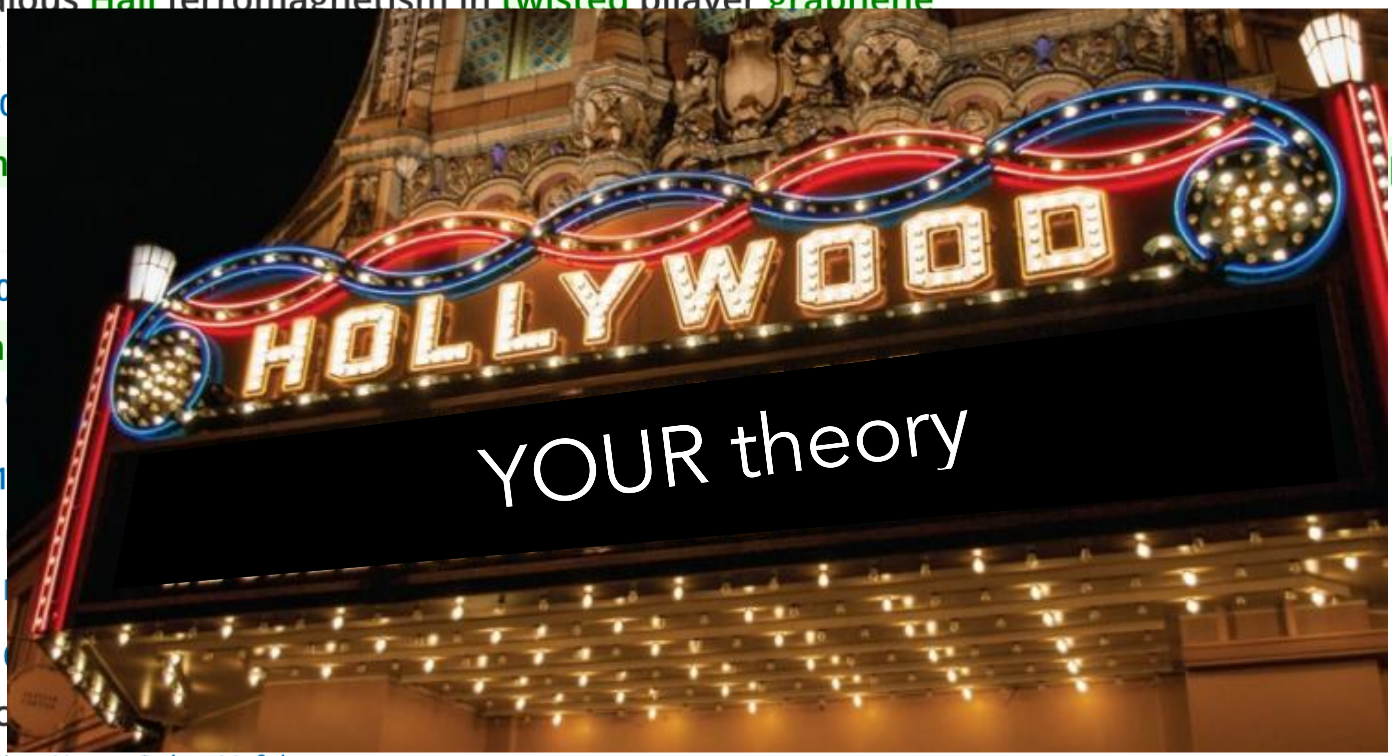
On the

Authors:

arXiv:181

Strong c

Authors: Jian Kang, Oskar Vafek



…

Theoretical Remarks

Effective field theory approach to interactions:

$$H_{\text{int}} = \int d^2x \, d^2x' \, V_{sc}(|x - x'|)\rho(x)\rho(x') + H_{sr}$$

Screened
Coulomb potential

$\xi_s \gtrsim L_m$

Local
interactions

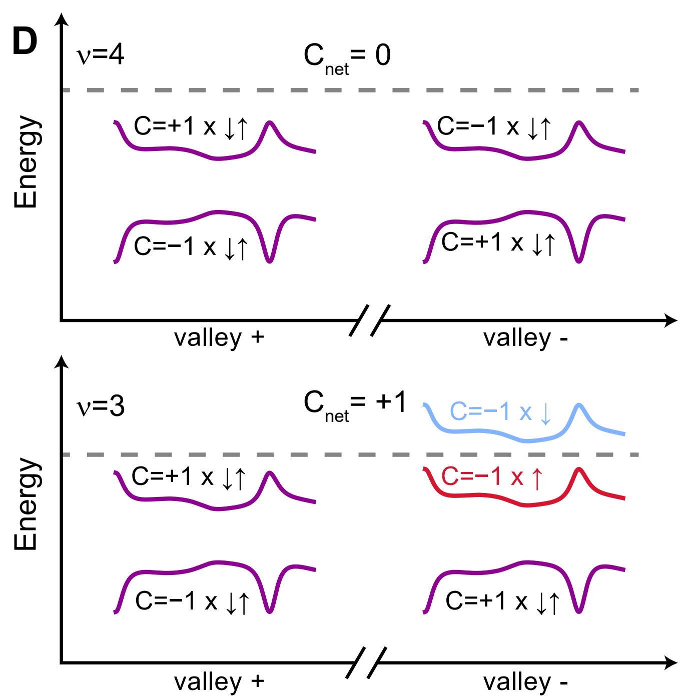
Suppressed by a_0/ξ_s

$$\rho = \sum_{l,v,\sigma} \psi^{\dagger}_{l,v,\sigma}\psi_{l,v,\sigma}$$

Conservation of spin and charge *separately* at each valley *plus* discrete Z_2 valley exchange symmetry (TR,C_2)

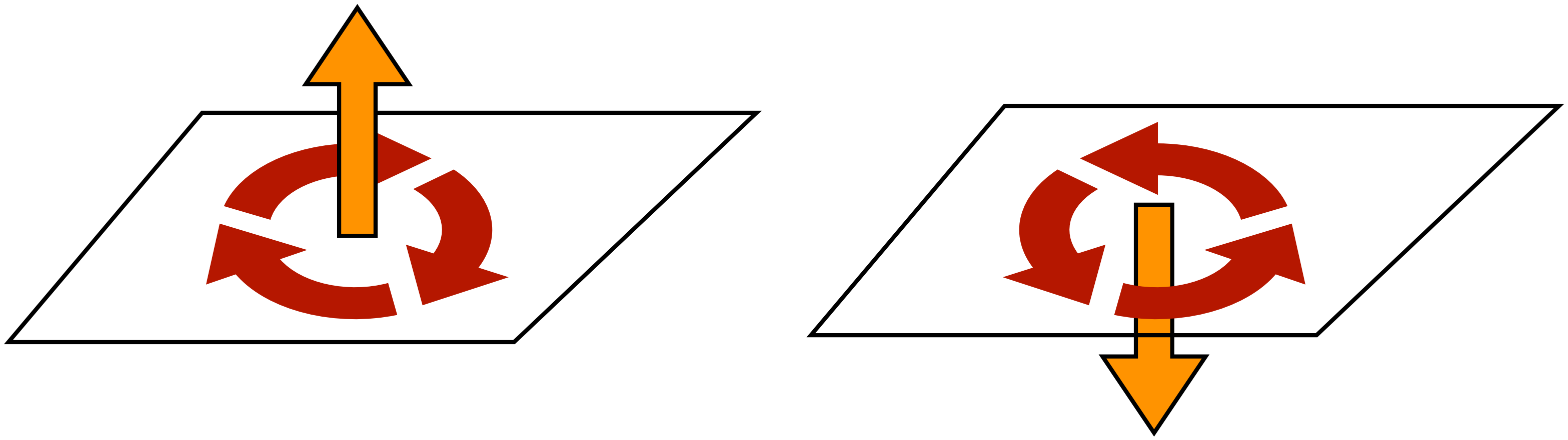
Theoretical remarks

- Valley polarization gives non-zero AHE.
- Quantization occurs if gap is complete - needs spin polarization

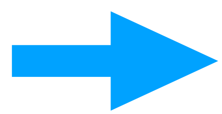


Symmetry breaking =
Z_2 (valley)
x
SU(2) (spin)

Theoretical remarks



Valley polarization

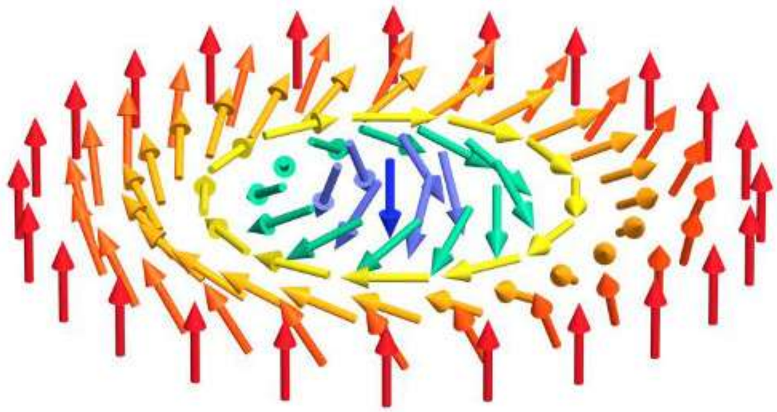


Orbital moment out of plane *Ising*

Associated with T_c robust

Tied to sign of QAHE

Theoretical remarks



Valley polarization → Orbital moment out of plane *Ising*

Associated with T_c robust

Tied to sign of QAHE

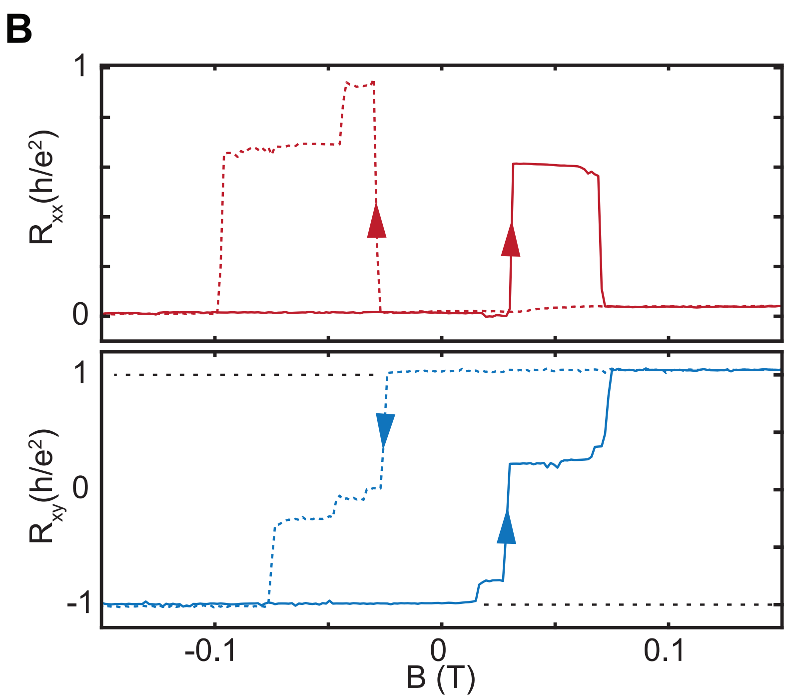
Spin polarization → Spin moment

O(3) Heisenberg-like

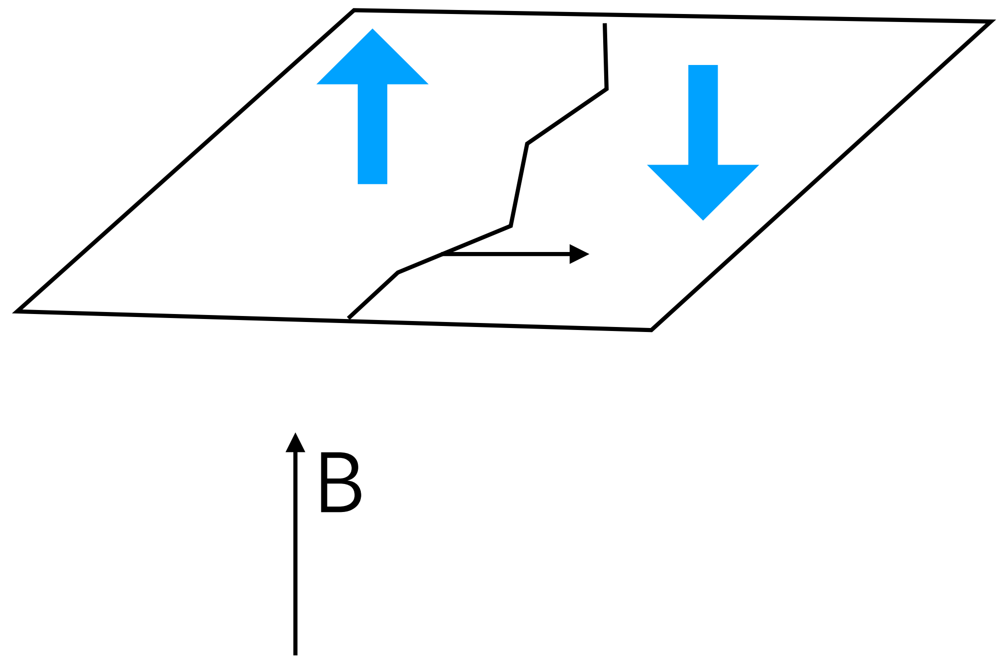
Easily polarized by field at low T

Domain manipulation

Metastability



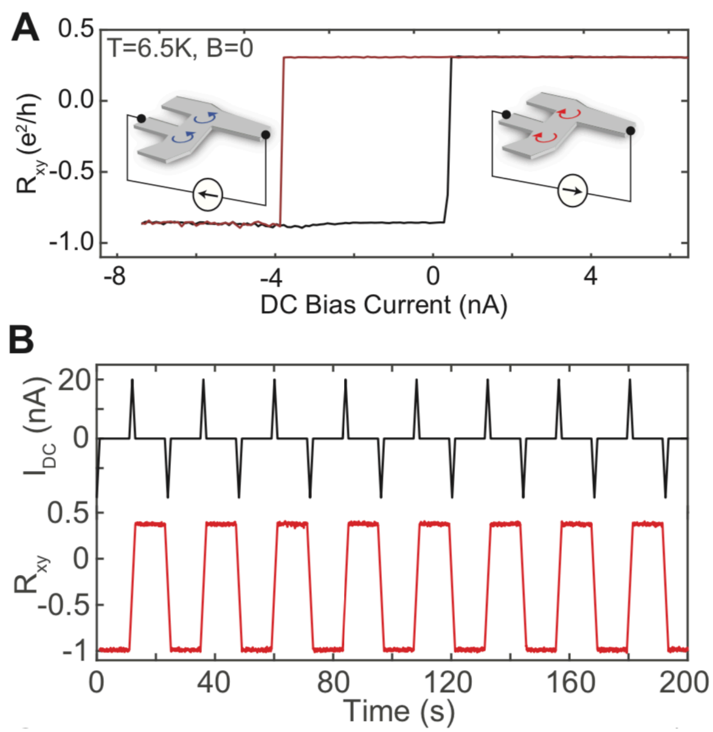
B field biases energy of domains



$$\Delta E = -\boldsymbol{B} \cdot \boldsymbol{M}$$

n.b. domains are *valley* domains

Domain manipulation



(tiny)

Current switches domains. How does this work?

Current

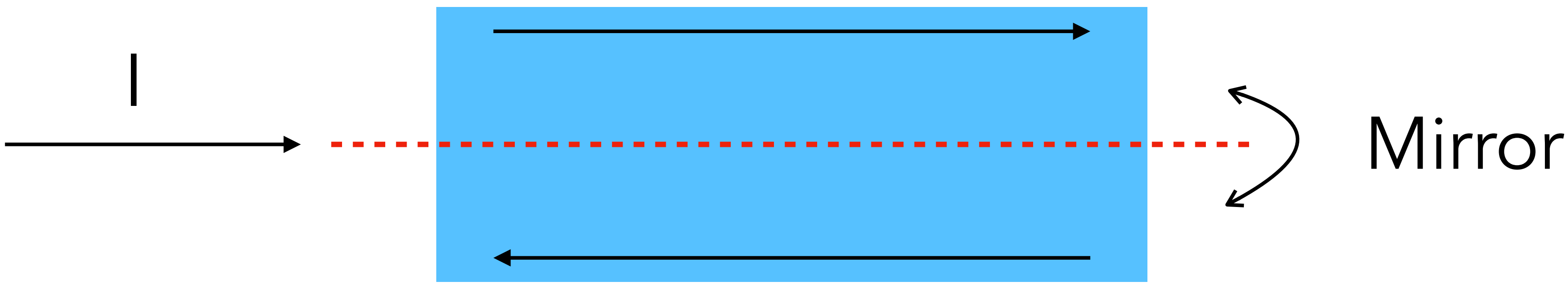
Well-developed IQHE: $\rho_{xx} \ll \rho_{xy}$

- no dissipation, only edge state transport
- Charge of each edge is separately conserved

✤ Can view current-carrying state as quasi-equilibrium ensemble where current determines edge occupation

Can formulate F(I,M)

(A)symmetry



I → I vector

M → -M pseudovector

Energetics

$$n = n_R + n_L$$

$$I_p = v_R n_R - v_L n_L$$

Two chemical potentials

$$H \to H - \mu N - \tilde{\mu} I_p = \sum_k \left(\epsilon(k) - \mu - \tilde{\mu}\epsilon'(k)\right) c_k^\dagger c_k .$$

$$I = e\frac{\partial(F/L)}{\partial\tilde{\mu}}$$

Sommerfeld expansion

$$F/L = \mathcal{F}_0 + \frac{1}{2}\mathcal{F}_2\tilde{\mu}^2 + \frac{1}{6}\mathcal{F}_3\tilde{\mu}^3 + O(\tilde{\mu}^4)$$

$$\mathcal{F}_2 = \int \frac{dk}{2\pi}\,(\varepsilon_k')^2 n_F'(\varepsilon_k) \approx \frac{1}{2\pi}\left(|v_1| + |v_2|\right),$$

$$\mathcal{F}_3 = \int \frac{dk}{2\pi}\,(\varepsilon_k')^3 n_F''(\varepsilon_k) \approx -\frac{1}{\pi}\left(\frac{\mathrm{sign}(v_1)}{m_1} + \frac{\mathrm{sign}(v_2)}{m_2}\right).$$

Result

$$F/L = \mathcal{F}_0 + \frac{1}{2e^2\mathcal{F}_2} I^2 - \frac{\mathcal{F}_3}{3e^3\mathcal{F}_2^3} I^3 + O(I^4).$$

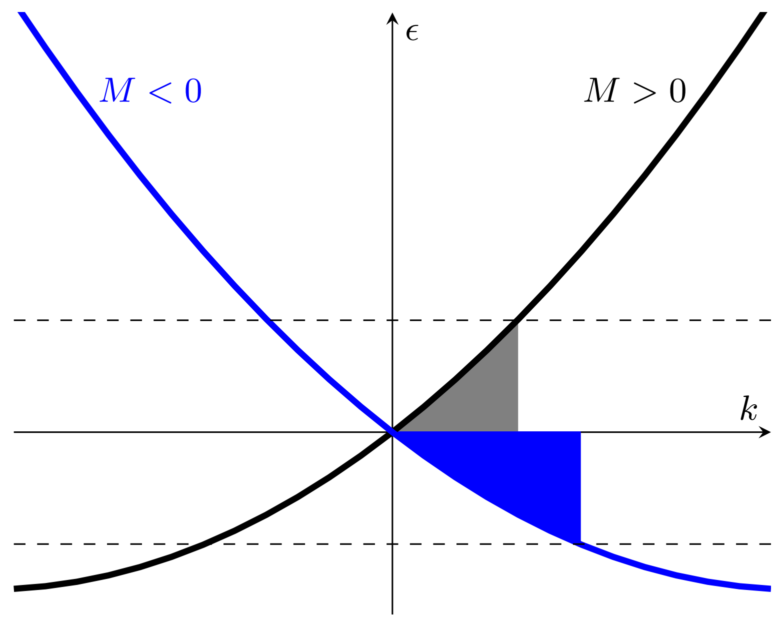
Energetics

Simple limit: one “fast” (costly) edge

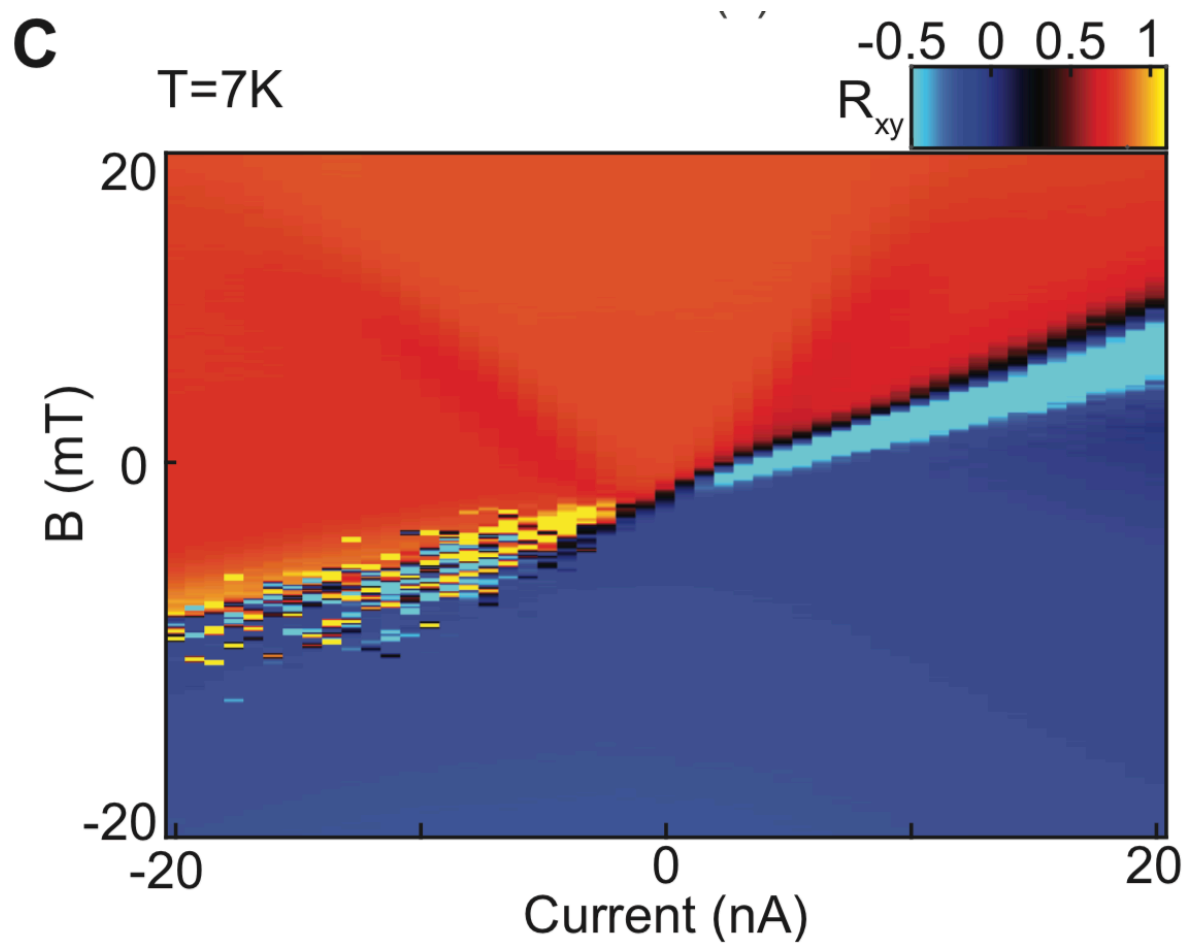
I>0:

- Add right-moving e^{-}s
- Remove left-moving e^{-}s

$$\Delta F \sim \frac{\hbar^2}{me^3v^3} L I^3$$



Dissipative Regime

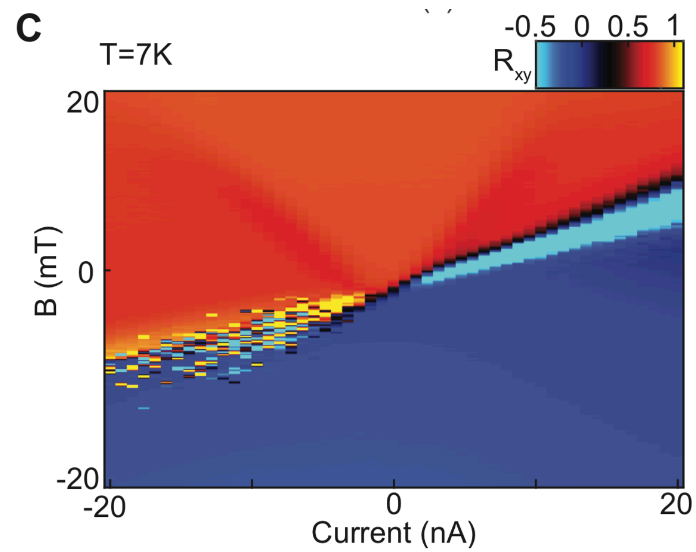


A fully non-equilibrium problem, bulk 2d physics

Thanks

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[-iv \left(\tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v \left(\boldsymbol{K} \cdot \partial_\mu \boldsymbol{u}_l + \boldsymbol{\mathcal{A}}_l \right) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[e^{-i\boldsymbol{Q}_j \cdot (\boldsymbol{u}_1 - \boldsymbol{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$



L.B., arXiv:1909.01545

M. Serlin *et al*, arXiv:1907.00261

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