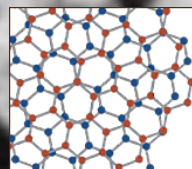


# Towards UQM in experiments

Leon Balents, KITP



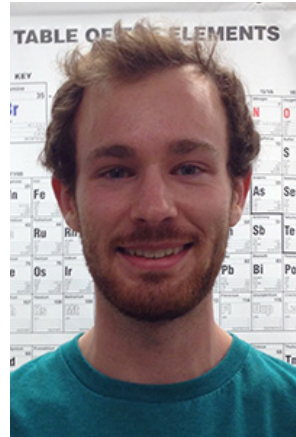
Simons Collaboration on  
Ultra-Quantum Matter



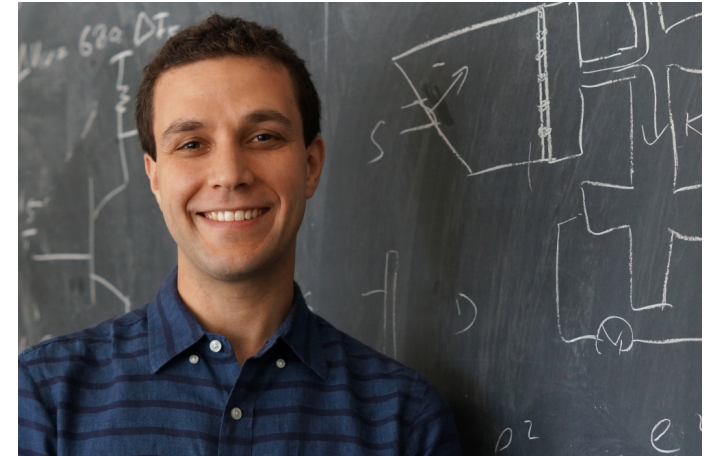
# Collaborators



Stephen Wilson



Mitchell Bordelon



Andrea Young



Chunxiao Liu



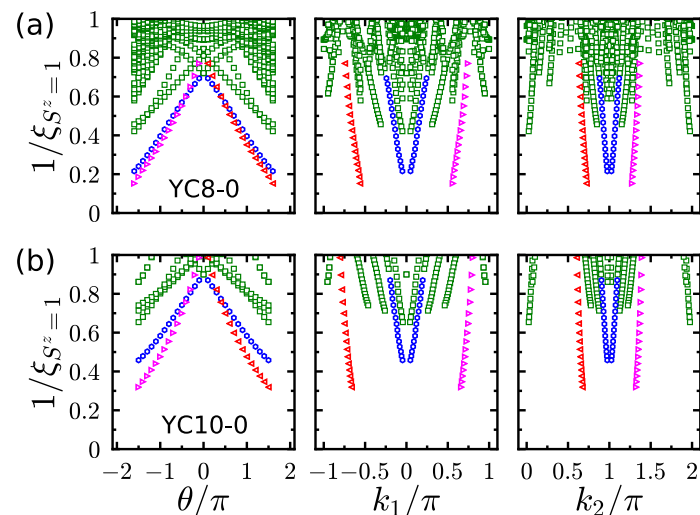
Kasra Hejazi

# Outline

- NaYbO<sub>2</sub>: a possible new quantum spin liquid
- Continuum model for twisted bilayer graphene for dummies
- Quantum anomalous Hall effect in TBG: controlling the order parameter with current

# Spin Liquids

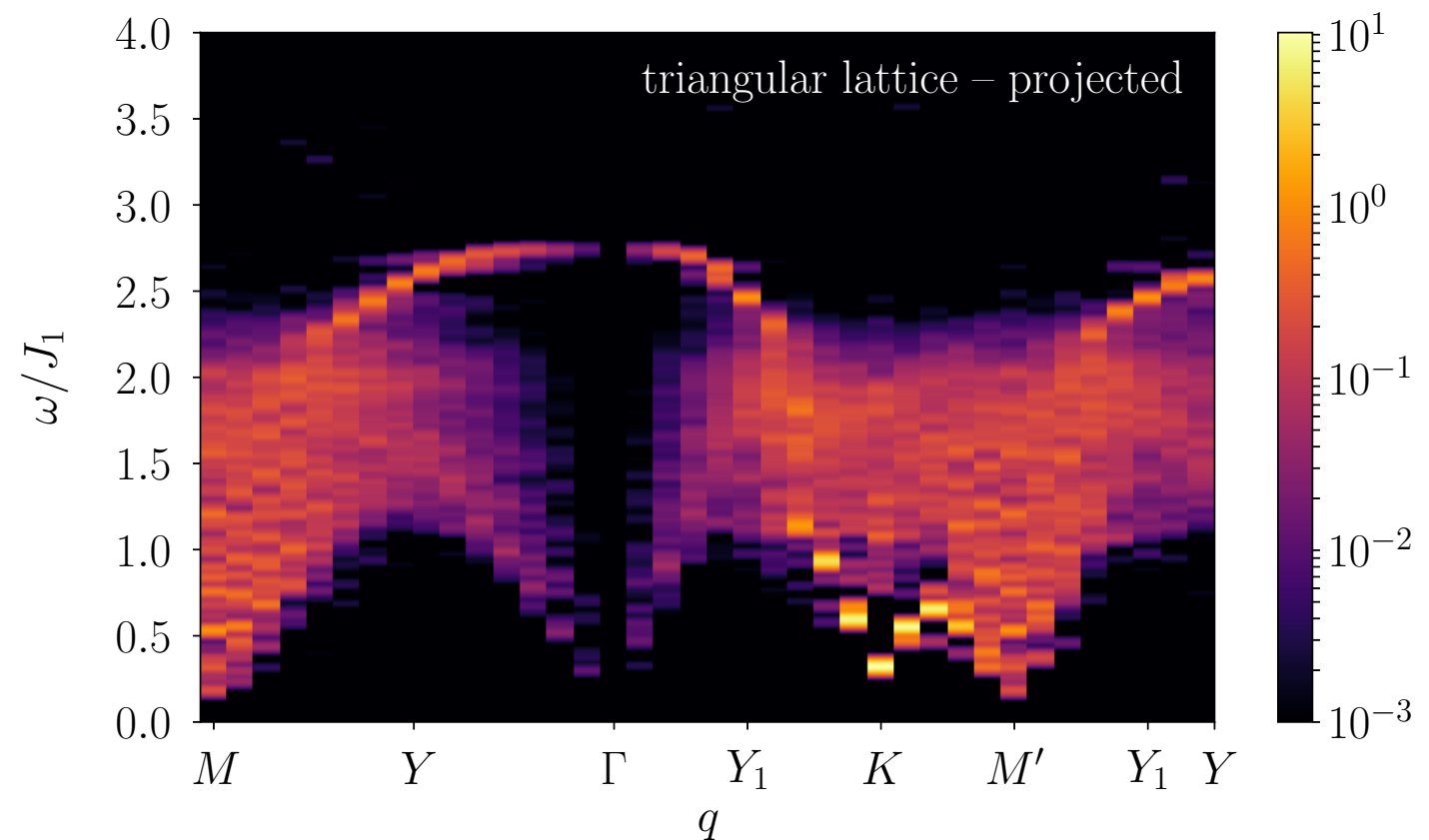
- Possible U(1) Dirac QSL on the triangular lattice ( $J_1$ - $J_2$ )



Shijie Hu *et al*, 2019

	$T_1$	$T_2$	$R$	$C_6$	$\mathcal{T}$
$M_{00}$	+	+	-	+	-
$M_{i0}$	+	+	+	-	+
$M_{01}$	-	-	$-M_{03}$	$-M_{02}$	+
$M_{02}$	+	-	$M_{02}$	$M_{03}$	+
$M_{03}$	-	+	$-M_{01}$	$M_{01}$	+
$M_{i1}$	-	-	$M_{i3}$	$M_{i2}$	-
$M_{i2}$	+	-	$-M_{i2}$	$-M_{i3}$	-
$M_{i3}$	-	+	$M_{i1}$	$-M_{i1}$	-
$\Phi_1^\dagger$	$e^{-i\frac{\pi}{3}}\Phi_1^\dagger$	$e^{i\frac{\pi}{3}}\Phi_1^\dagger$	$-\Phi_3^\dagger$	$\Phi_2$	$\Phi_1$
$\Phi_2^\dagger$	$e^{i\frac{2\pi}{3}}\Phi_2^\dagger$	$e^{i\frac{\pi}{3}}\Phi_2^\dagger$	$\Phi_2^\dagger$	$-\Phi_3$	$\Phi_2$
$\Phi_3^\dagger$	$e^{i\frac{-\pi}{3}}\Phi_3^\dagger$	$e^{i\frac{-2\pi}{3}}\Phi_3^\dagger$	$-\Phi_1^\dagger$	$-\Phi_1$	$\Phi_3$
$\Phi_{4/5/6}^\dagger$	$e^{i\frac{2\pi}{3}}\Phi_{4/5/6}^\dagger$	$e^{-i\frac{2\pi}{3}}\Phi_{4/5/6}^\dagger$	$\Phi_{4/5/6}^\dagger$	$-\Phi_{4/5/6}$	$-\Phi_{4/5/6}$

X-Y Song *et al*, 2018



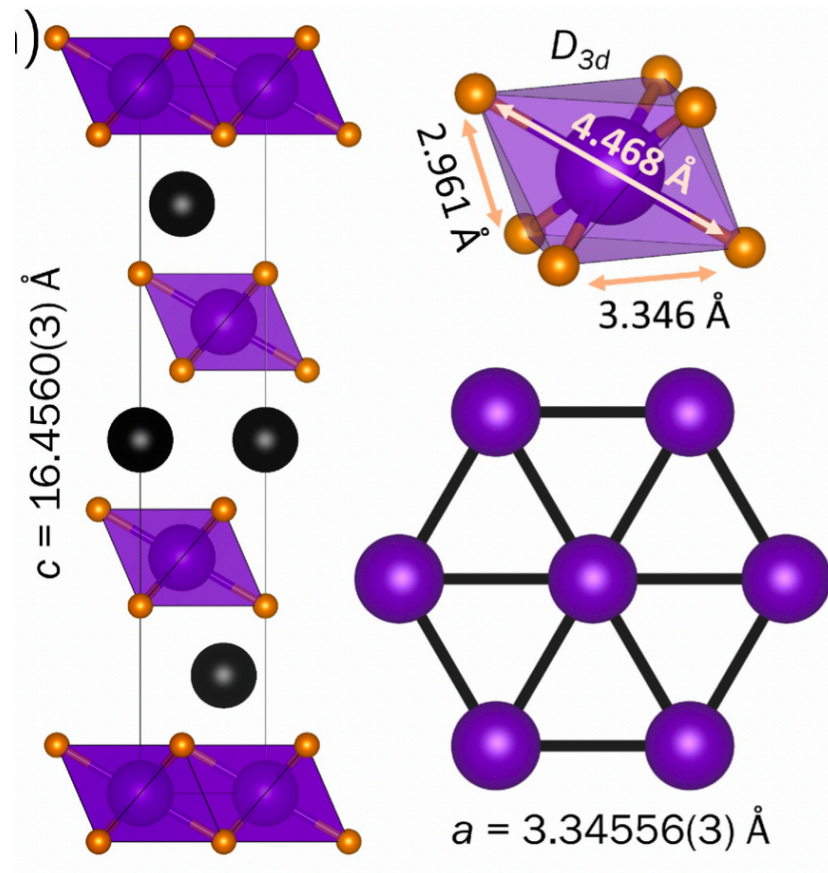
Federico Becca, unpublished



# Spin Liquids?

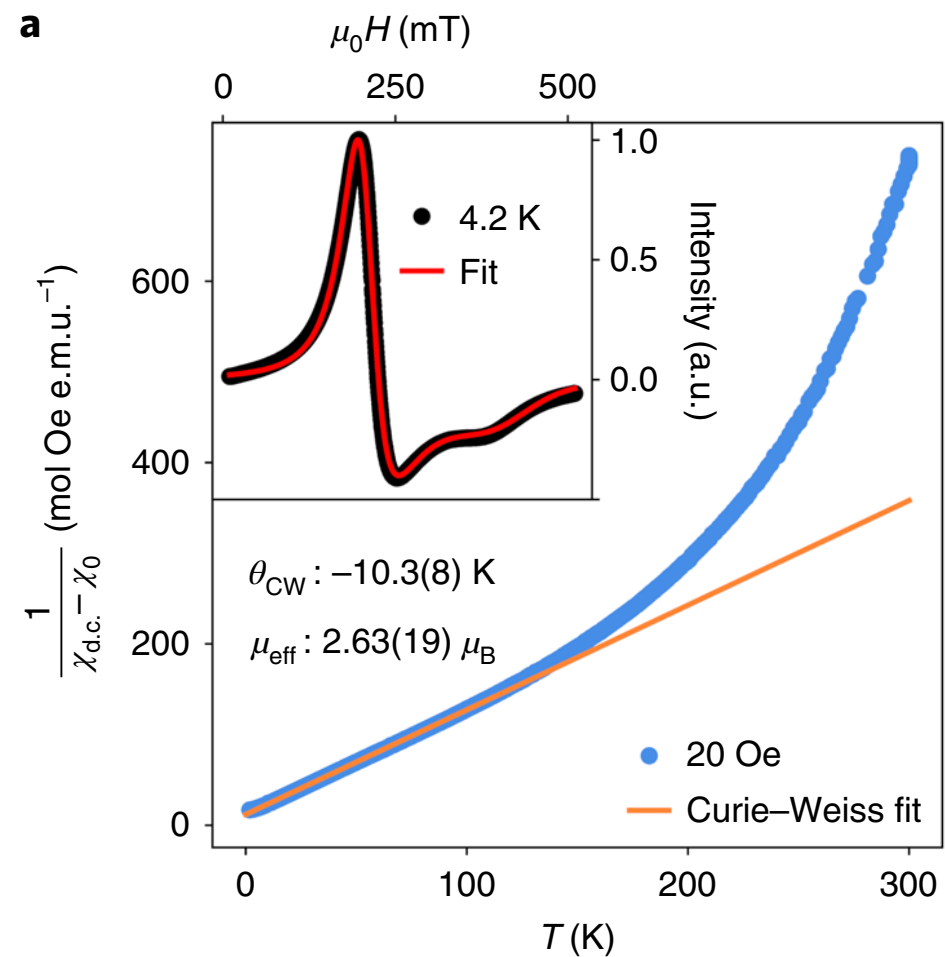
Model	QSL	Material	Issue
Kagome Heisenberg	U(1) Dirac	Herbertsmithite	Interlayer disorder
Kitaev honeycomb	Gapless Z2/Ising	$\alpha$ -RuCl <sub>3</sub>	Un-reproduced key result
Triangular Hubbard	U(1) Fermi surface	k-(ET), dmit organics	Interlayer disorder/ unreproduced result
Triangular J <sub>1</sub> -J <sub>2</sub> Heisenberg	U(1) Dirac?	YbMgGaO <sub>4</sub> ?	Random alloy Mg/Ga disorder

# NaYbO<sub>2</sub>



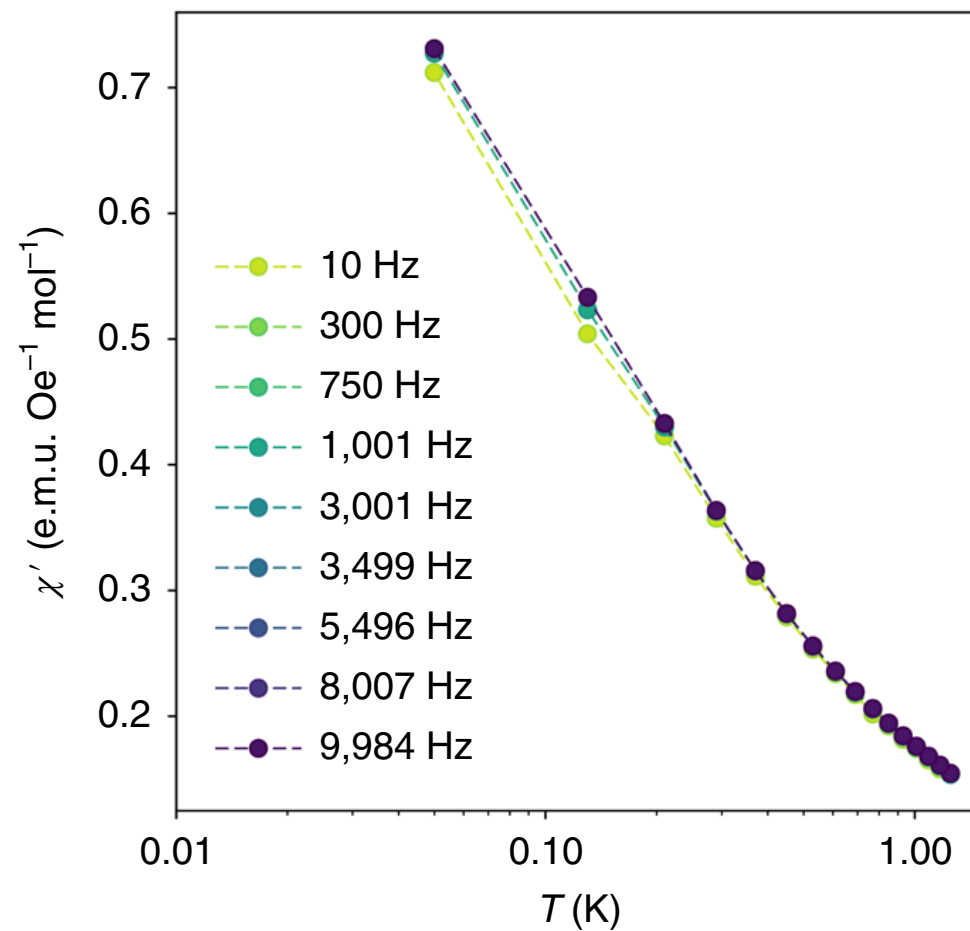
Isotropic triangular lattice of Yb<sup>3+</sup>  
effective  $S=1/2$  moments

ESR shows  $g_{xy}=3.3$ ,  $g_z=1.7$ : expect  
XY-like spins (common for Yb<sup>3+</sup>)

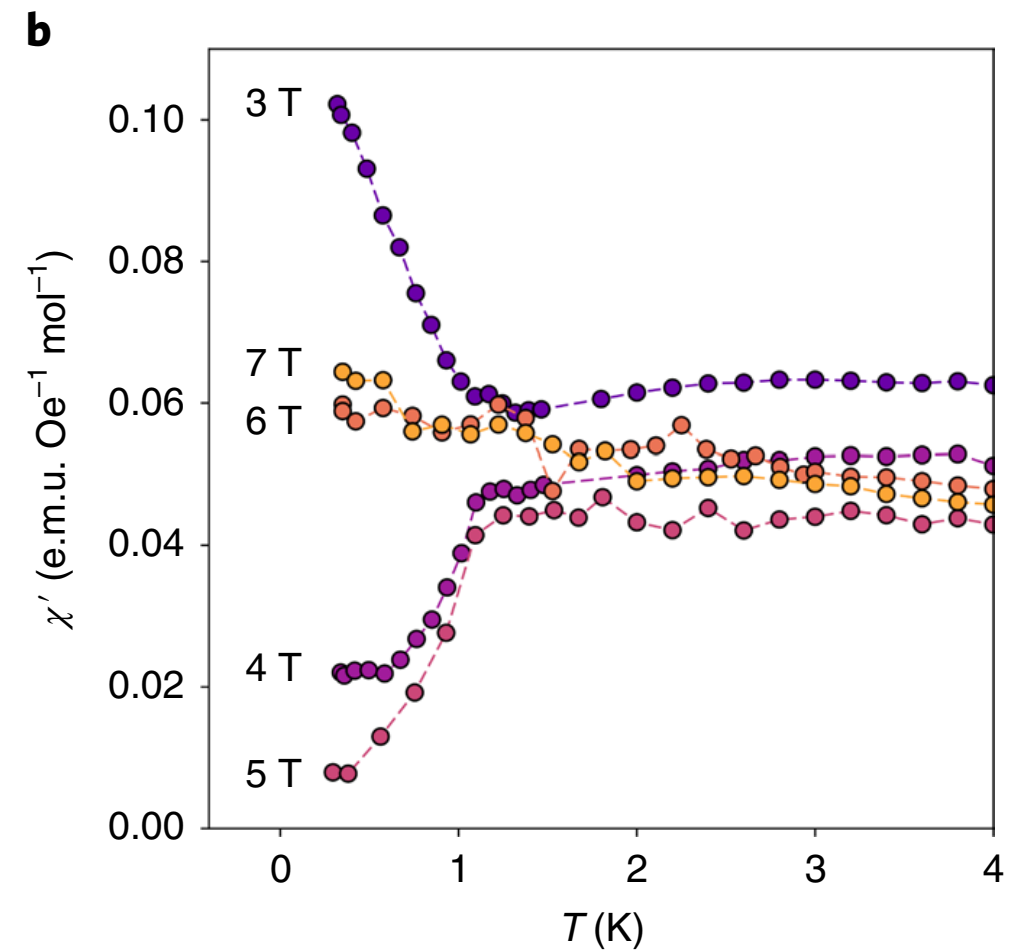


Susceptibility shows antiferromagnetic  
exchange with  $\sim 10\text{K}$  scale

# NaYbO<sub>2</sub>



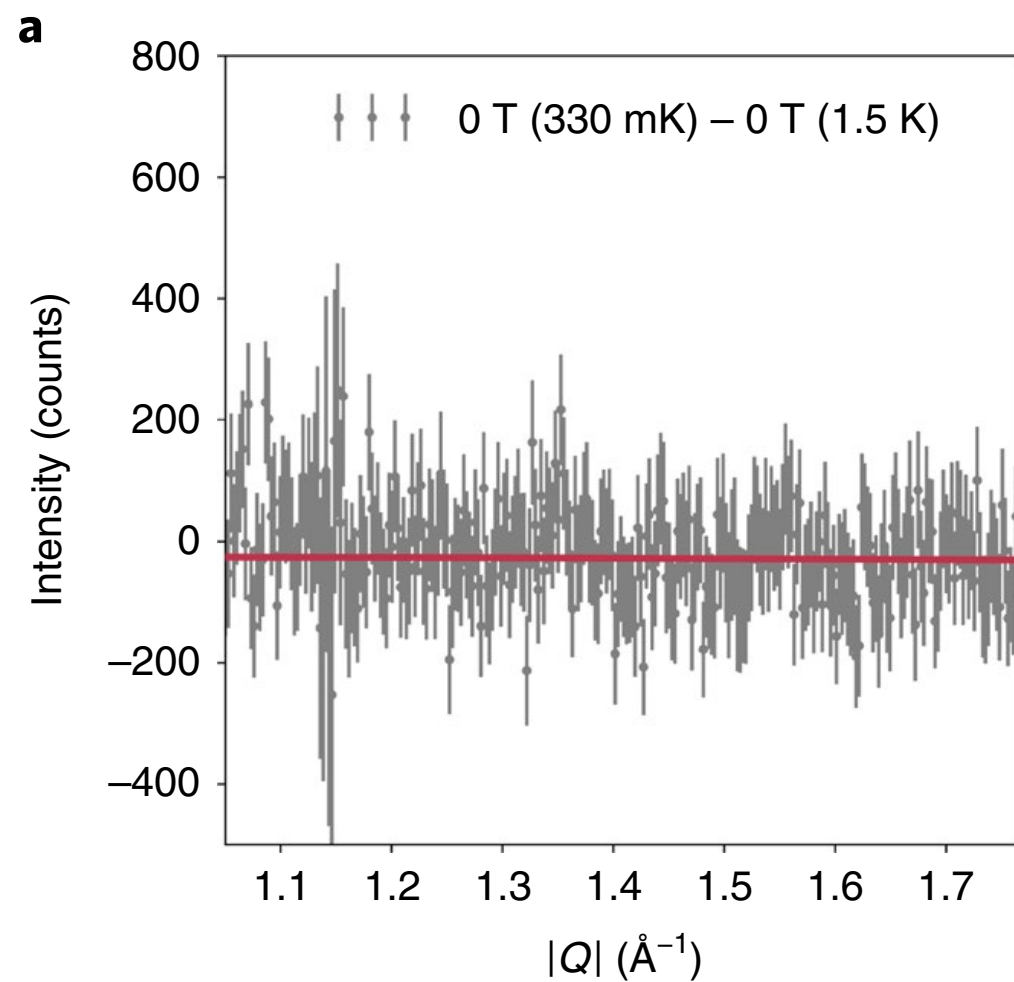
No ordering or freezing at  $B=0$



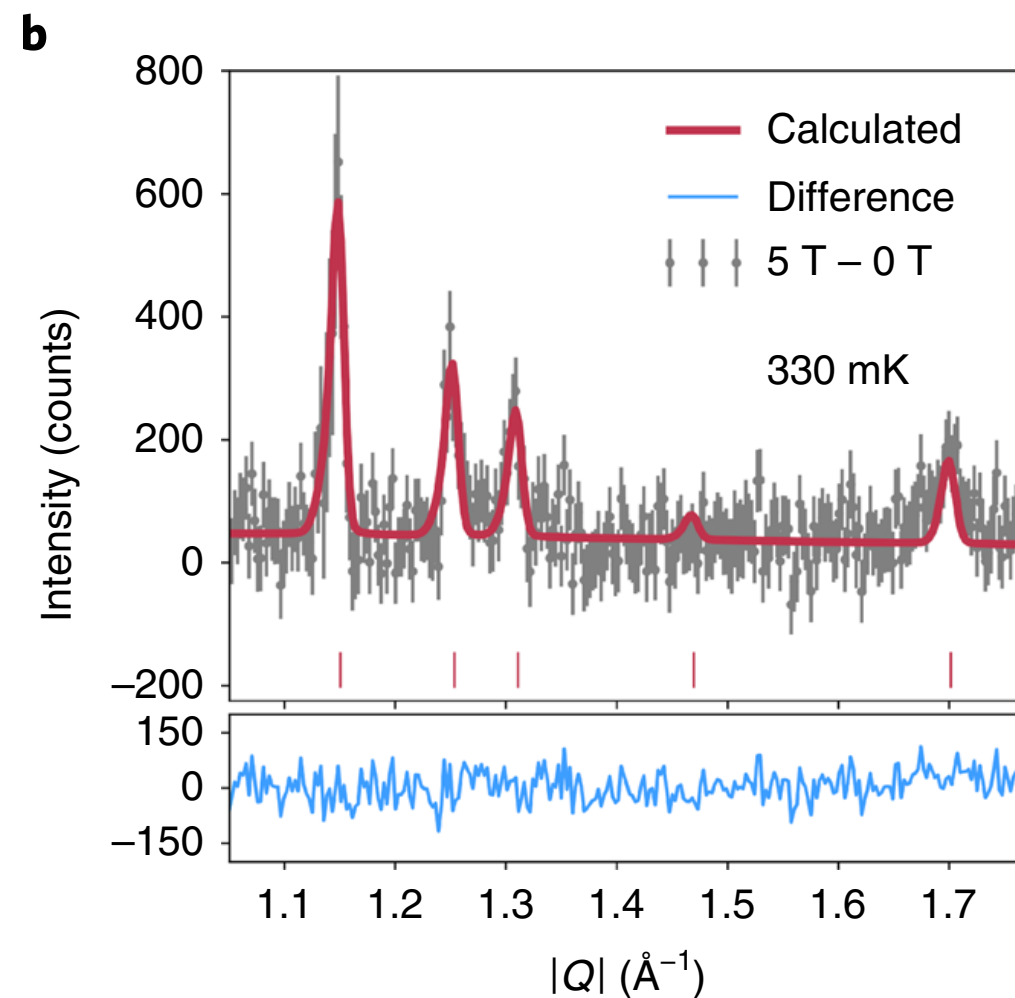
"action" in  $B > 2$  T.

# NaYbO<sub>2</sub>

## Neutron scattering



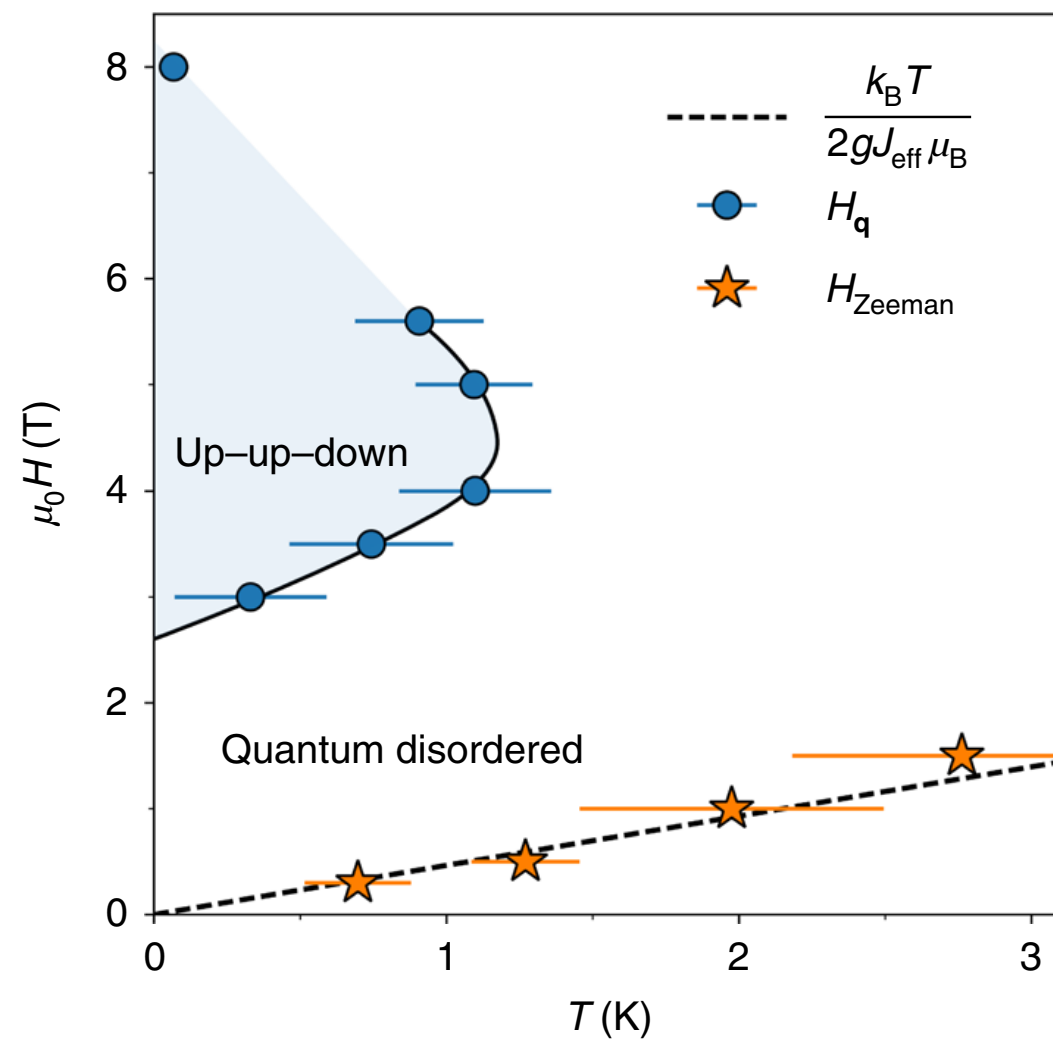
Featureless at B=0



Field-induced order



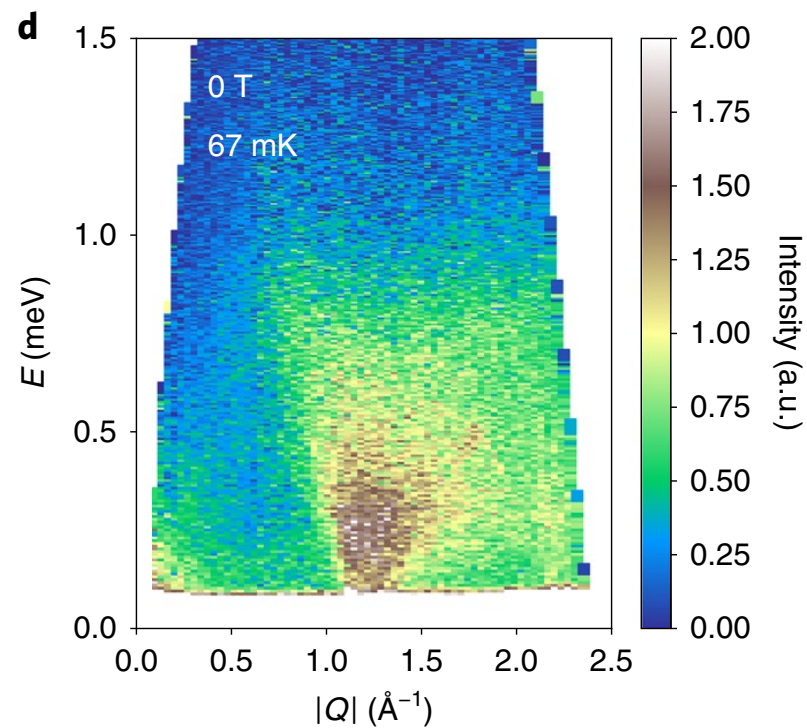
# NaYbO<sub>2</sub>



Field-induced  
3-sublattice  
order

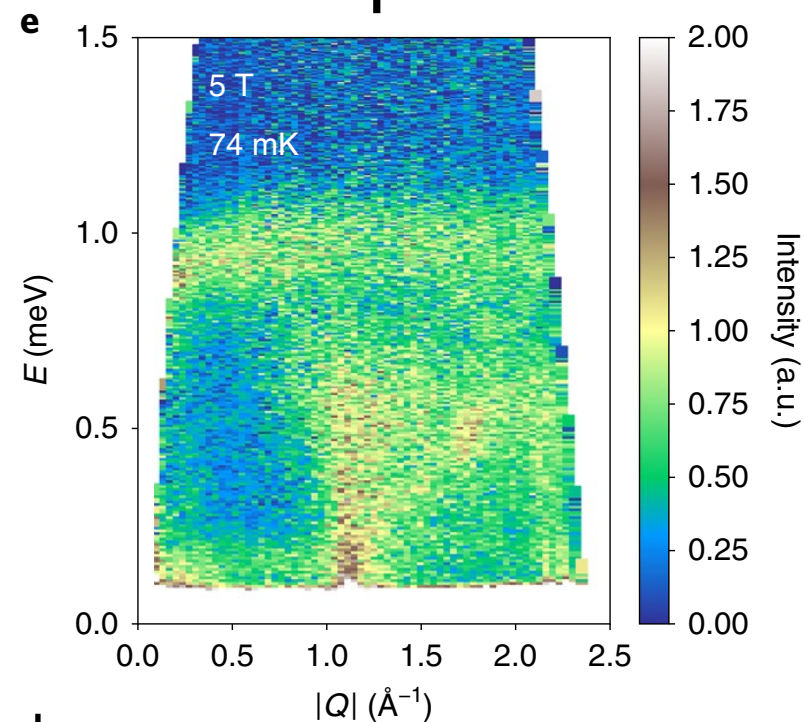
# NaYbO<sub>2</sub>

## Powder INS

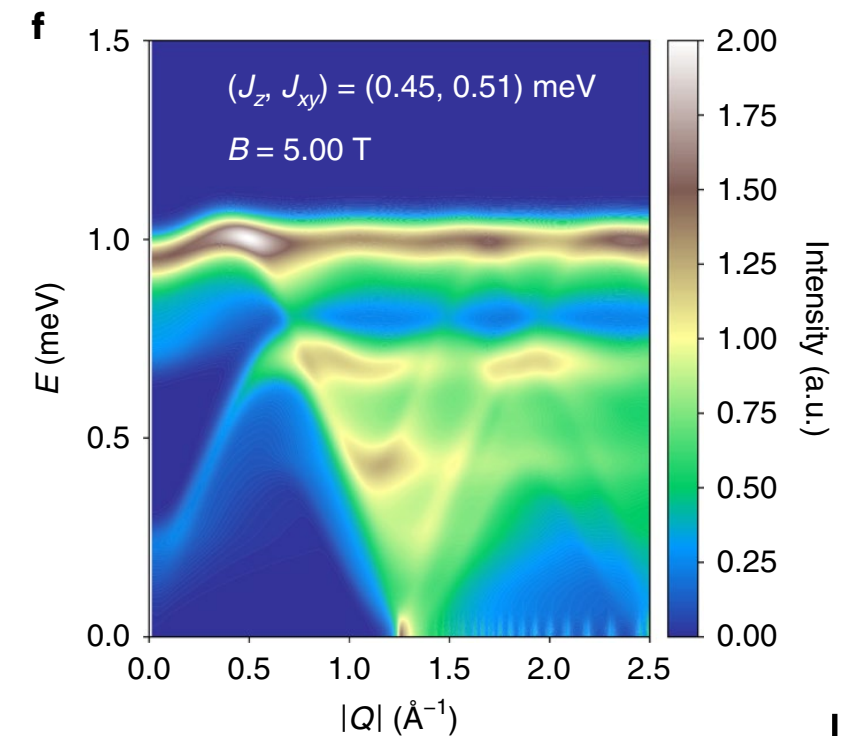


QSL?

expt



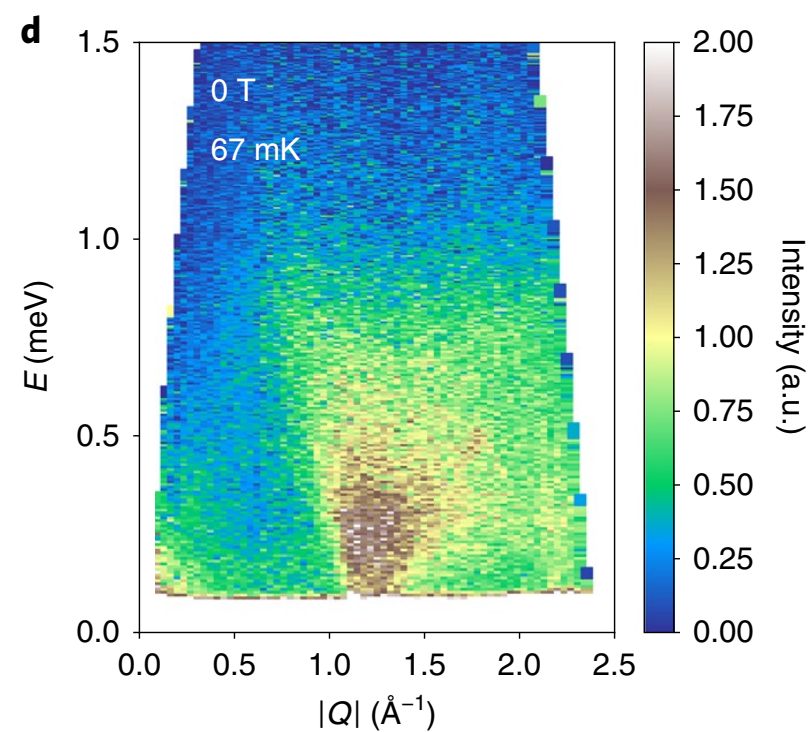
SWT



In the ordered state

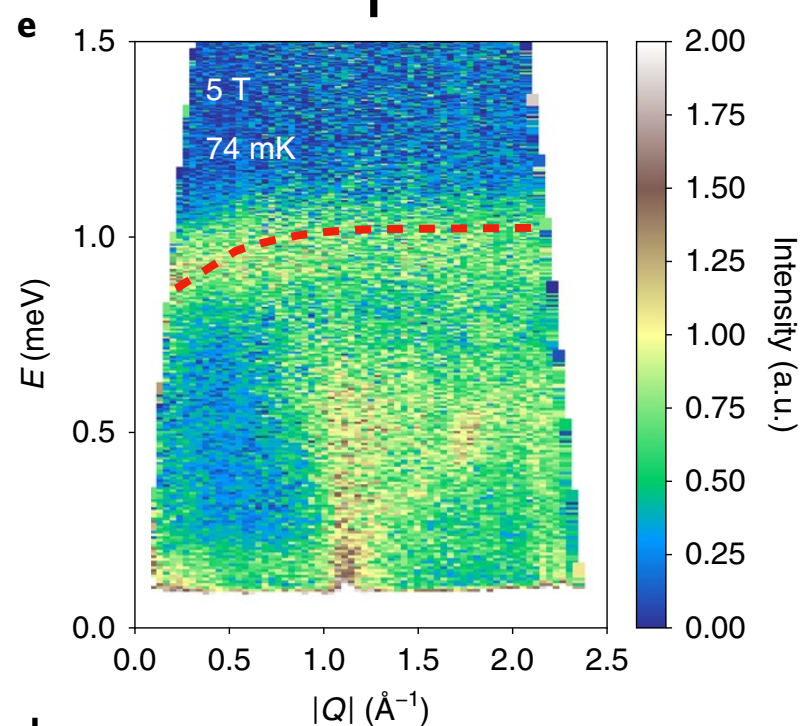
# NaYbO<sub>2</sub>

## Powder INS

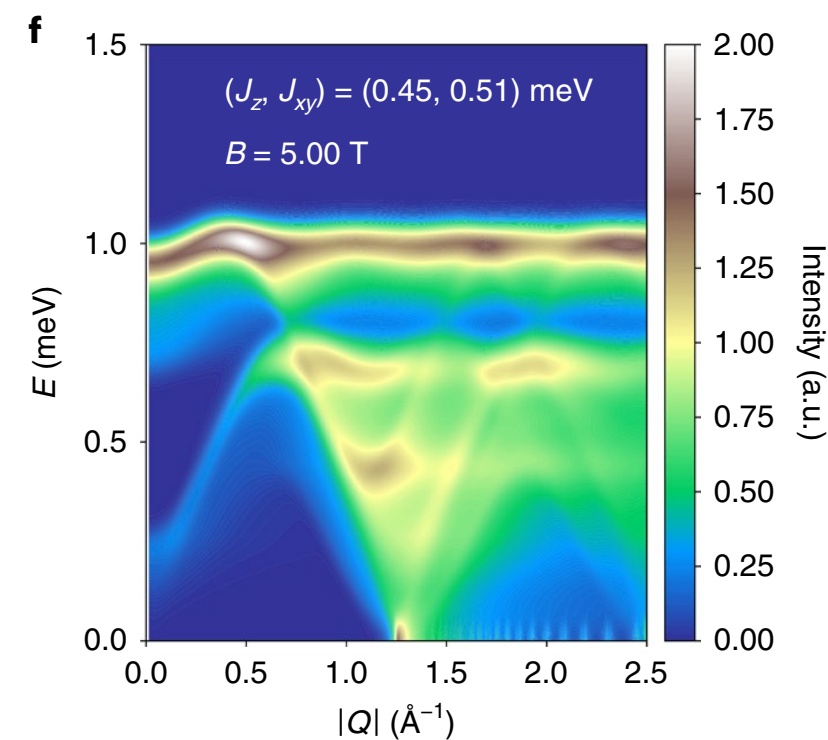


QSL?

expt



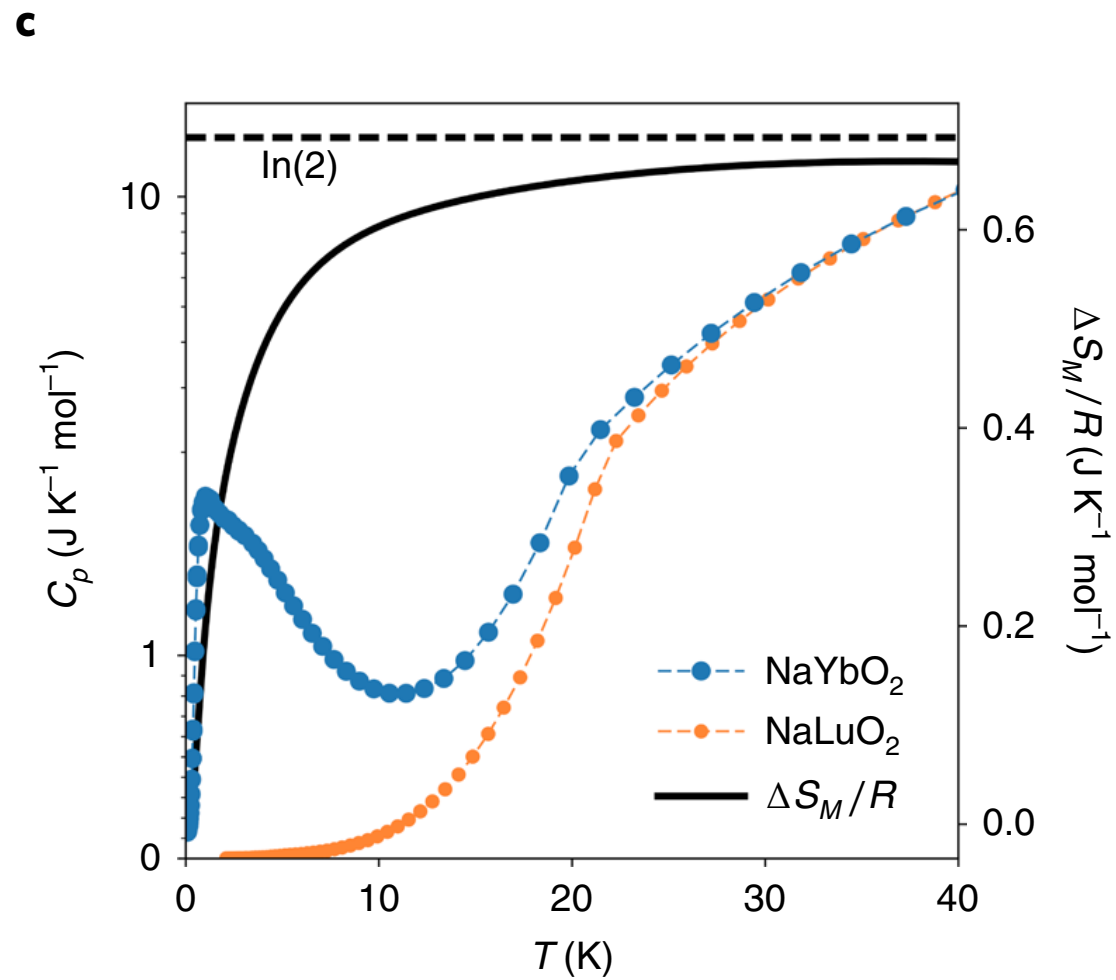
SWT



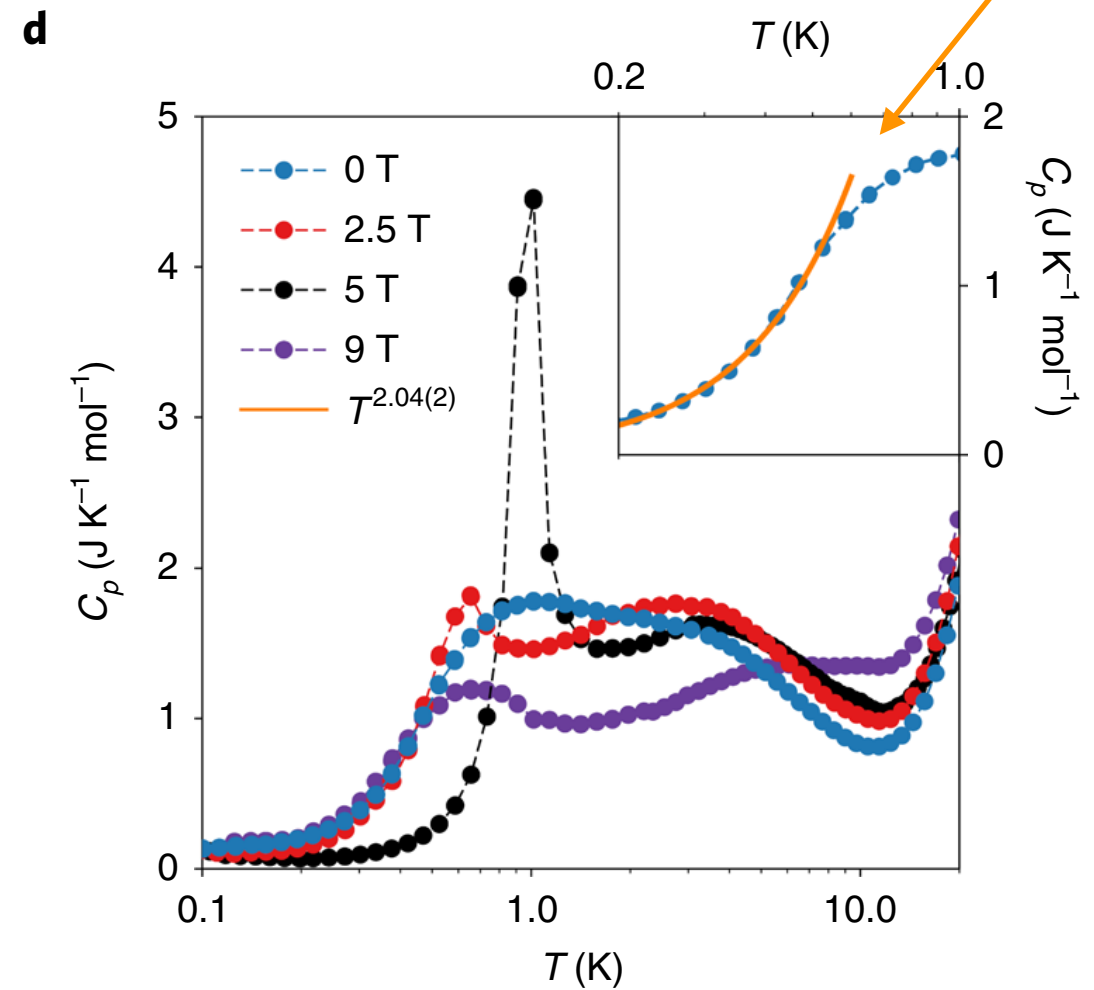
In the ordered state

# NaYbO<sub>2</sub>

$C \sim T^2$ ?



Entropy confirms  $S=1/2$



So far consistent with U(1) Dirac



# Magic angle graphene

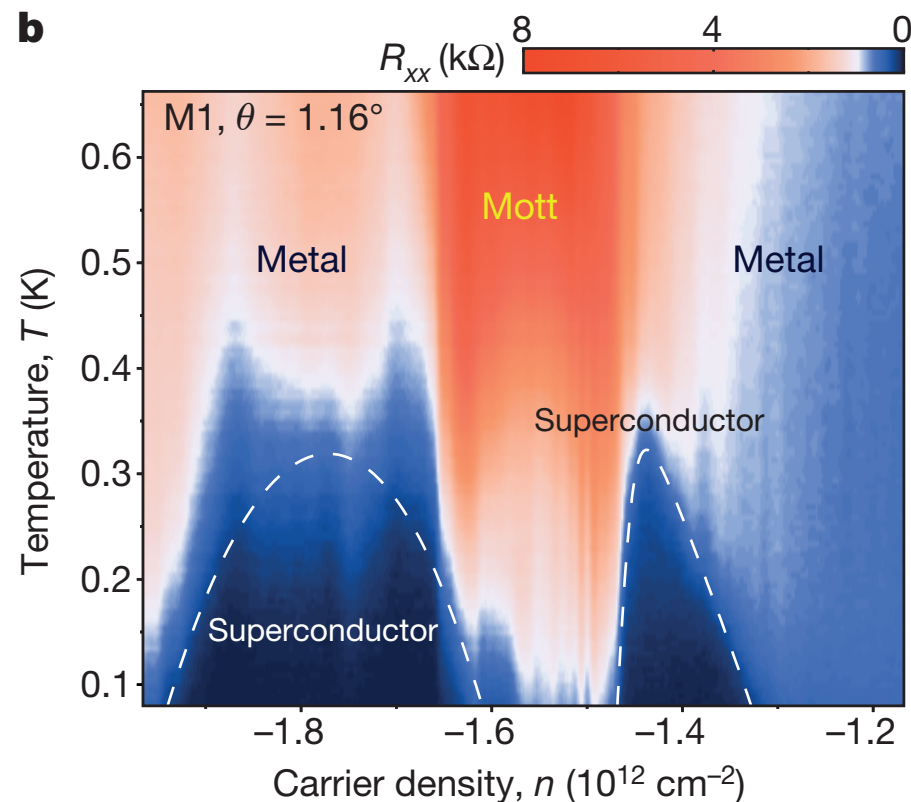


Pablo Jarillo-Herrero  
(MIT)

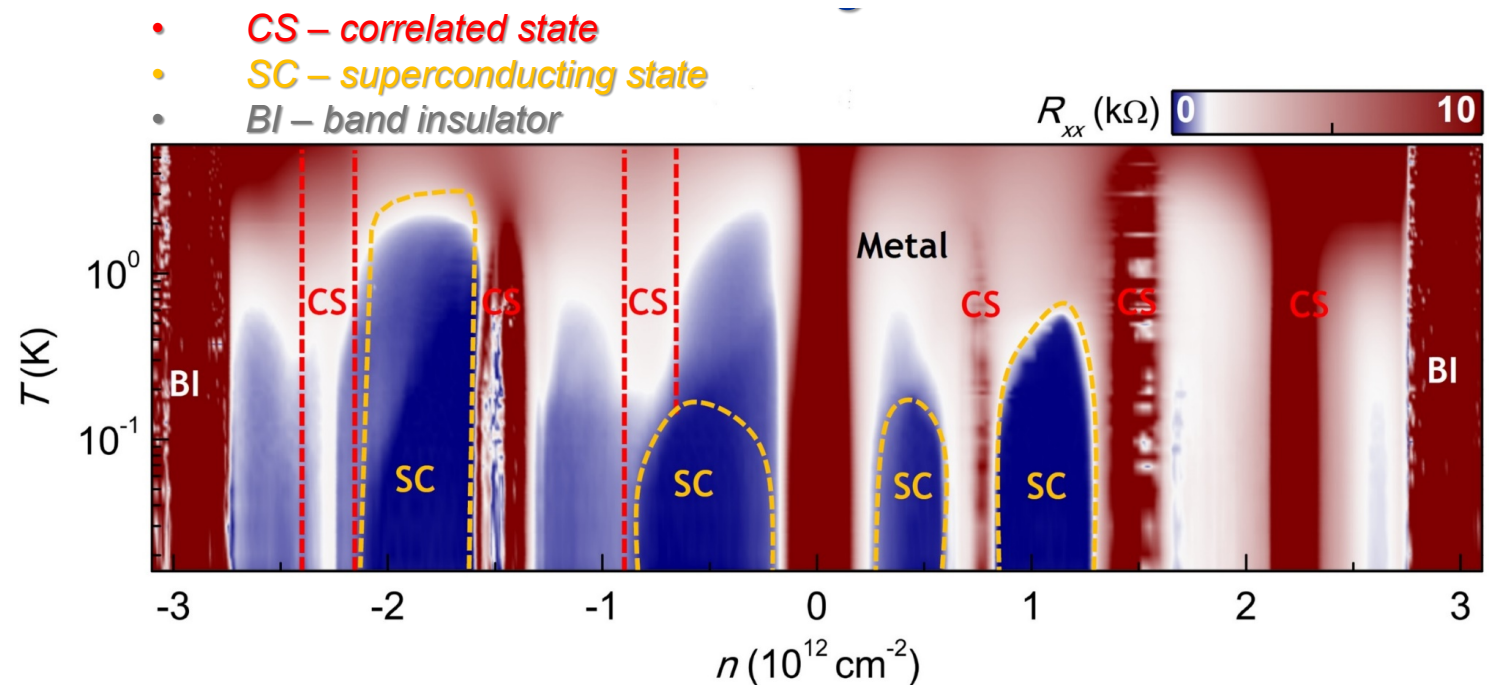
Physics World  
Breakthrough of the year,  
2018



Dimitri Efetov  
(Barcelona)



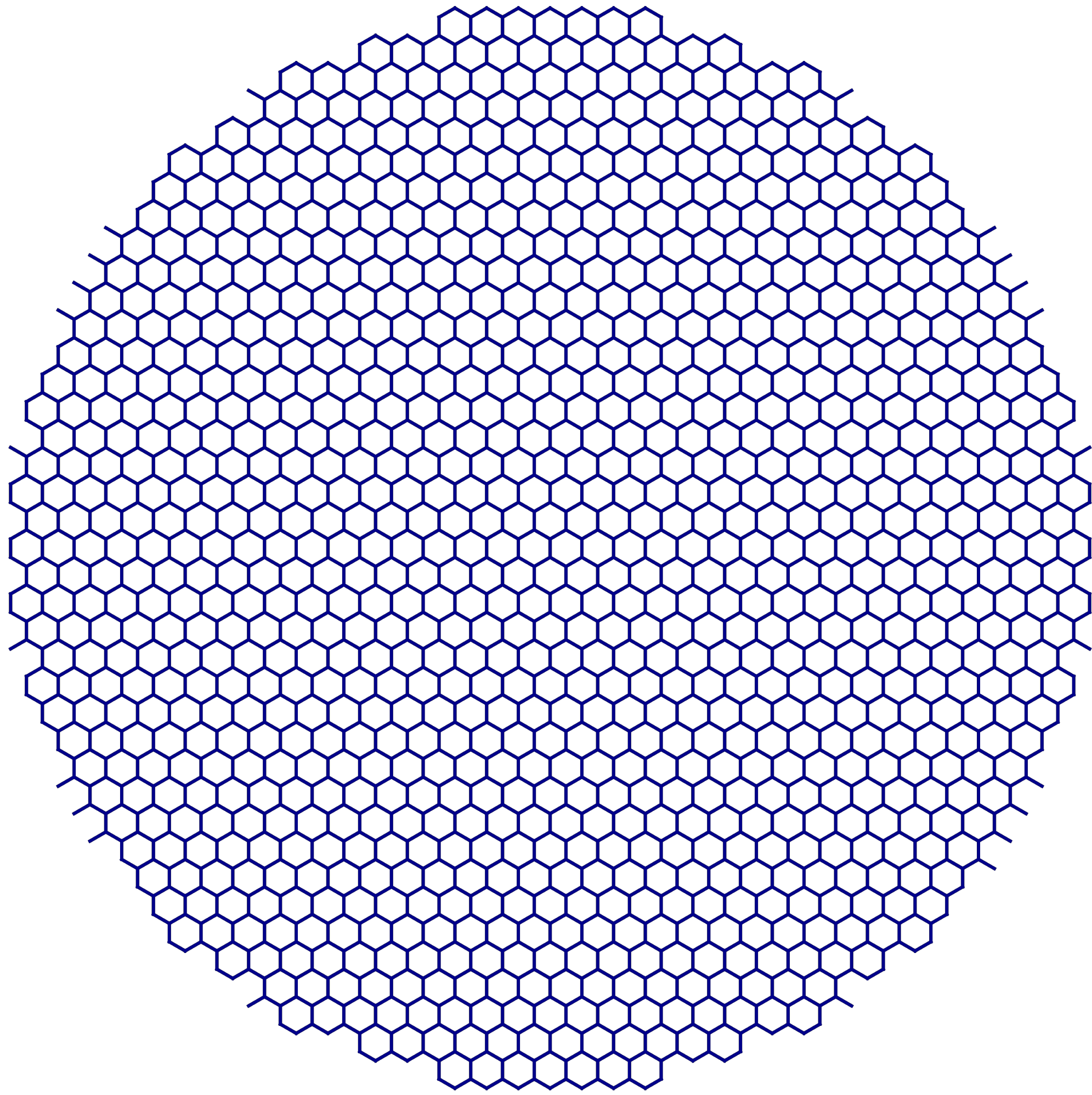
Y. Cao et al, 2018



YX. Lu et al, 2019

# Two topics in TBG

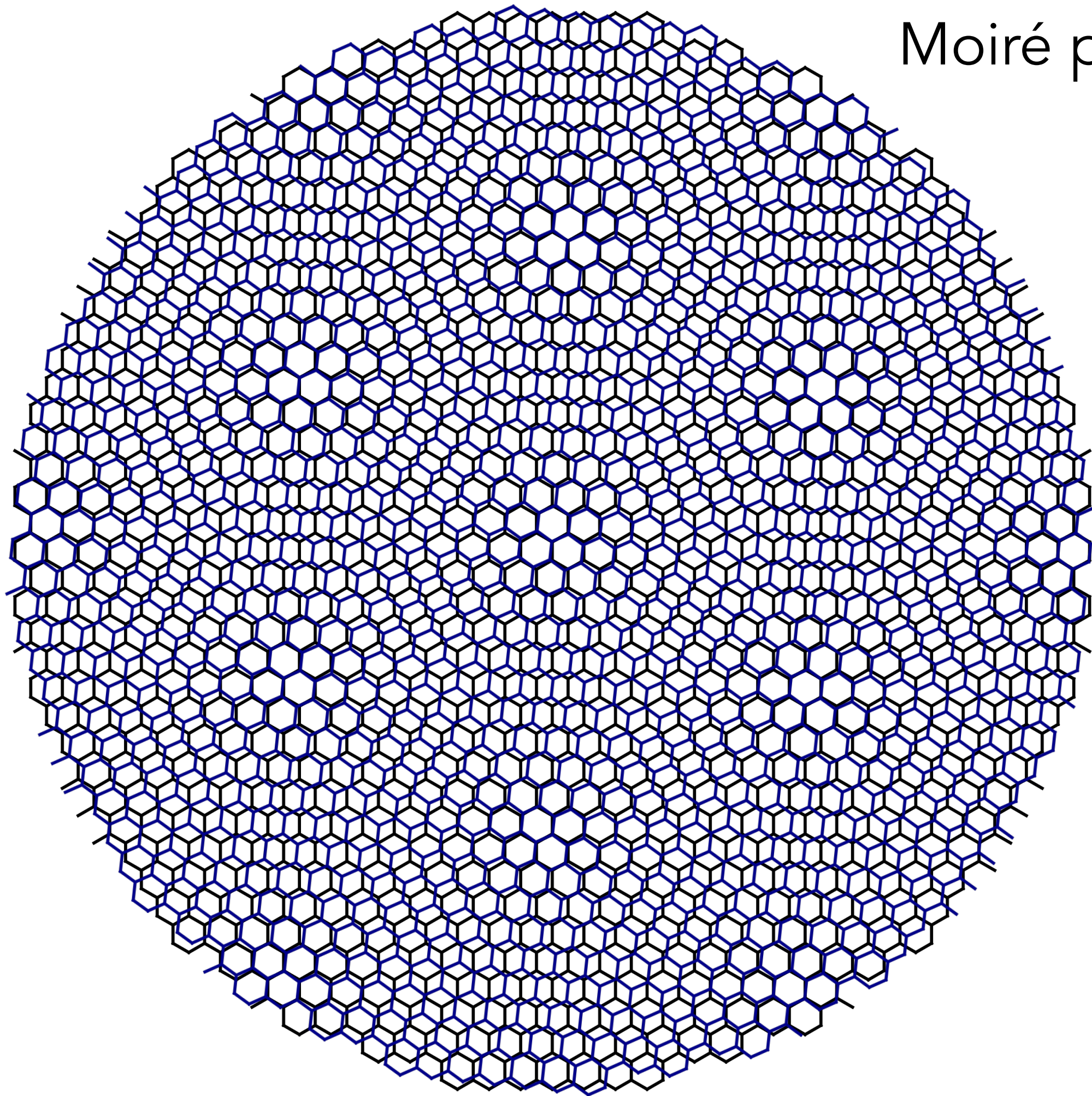
1. Continuum model for TBG as effective field theory
2. Non-equilibrium driving a QAHE state





Moiré pattern

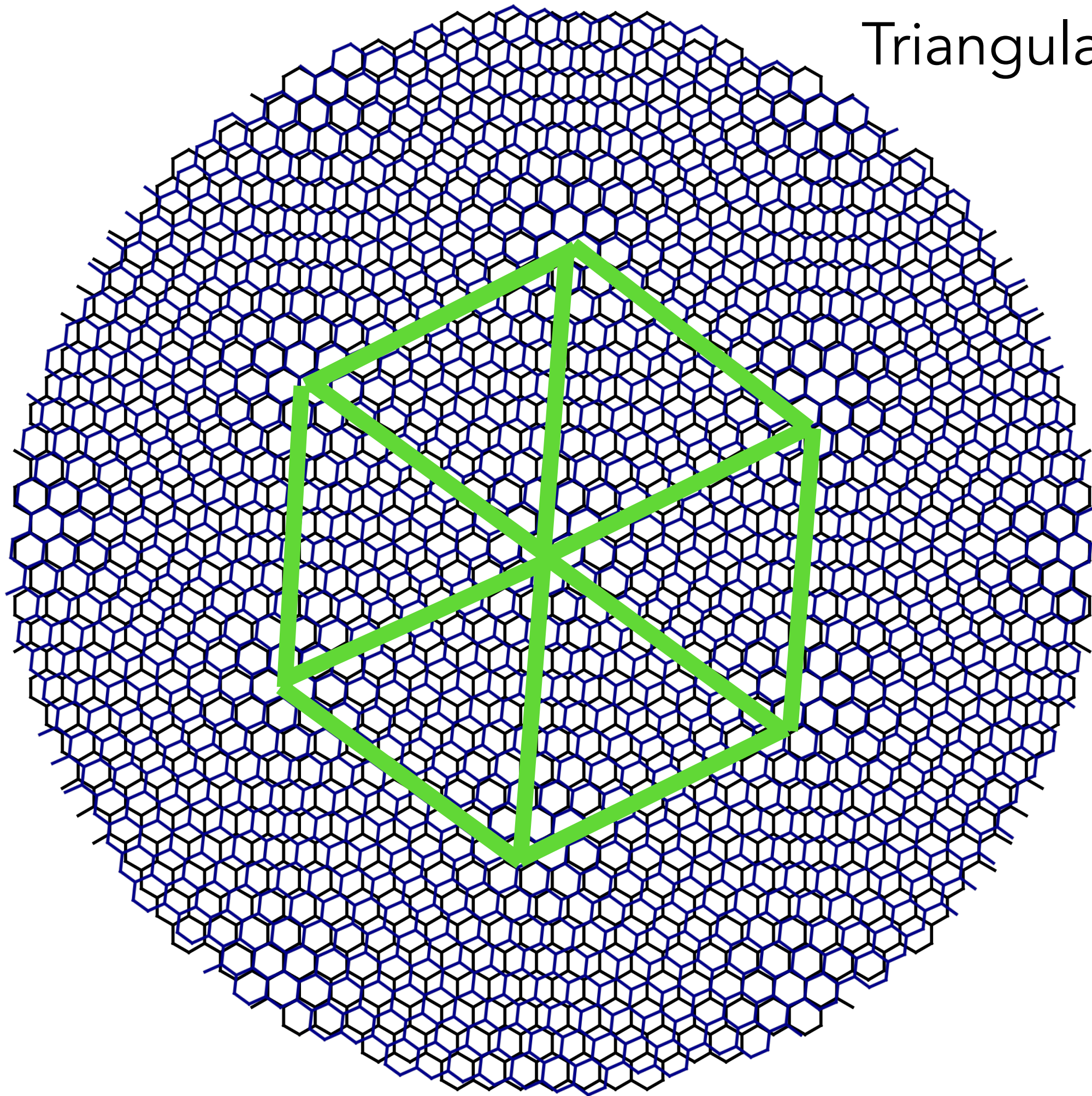
$6^\circ$





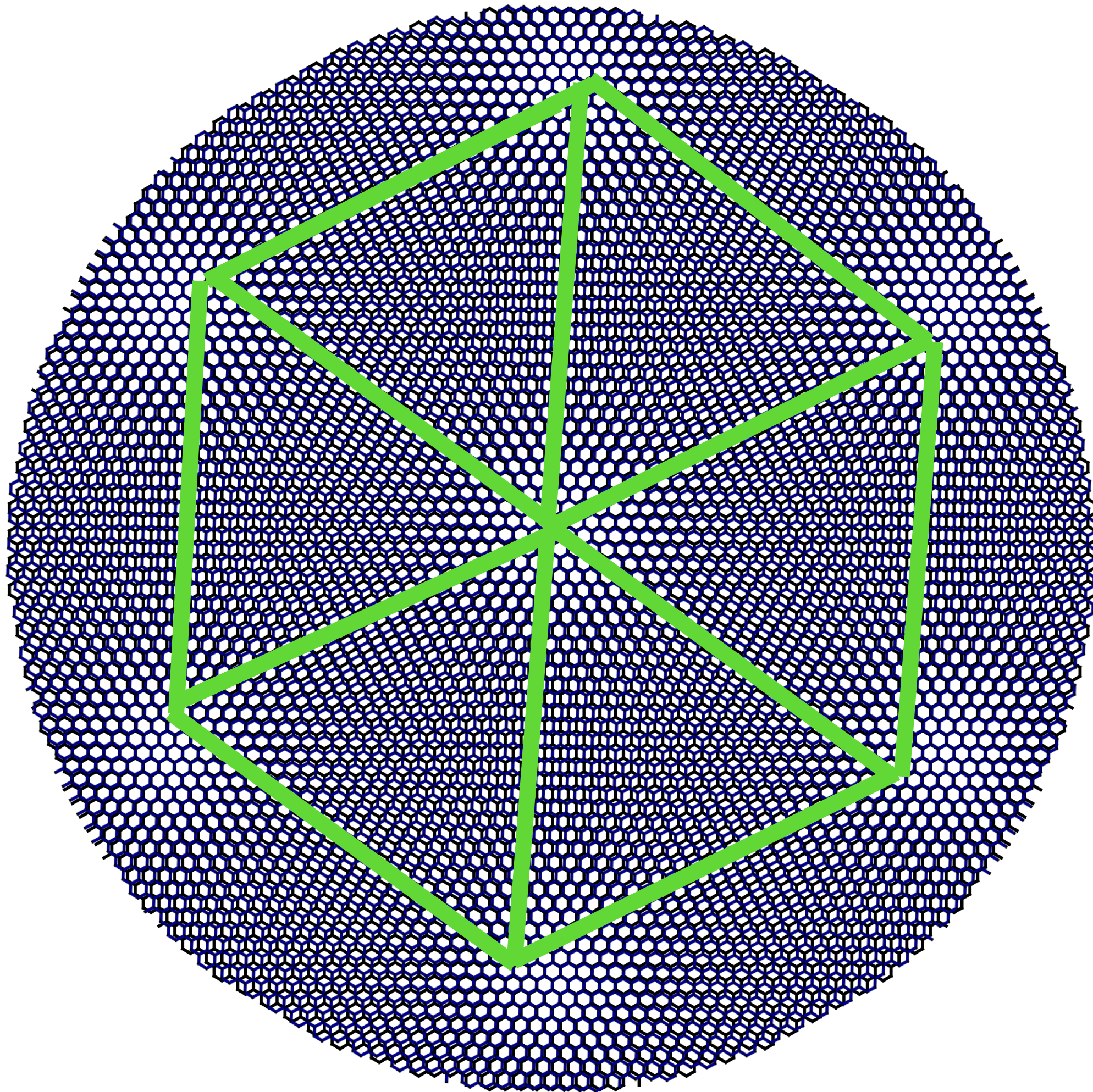
$6^\circ$

Triangular lattice



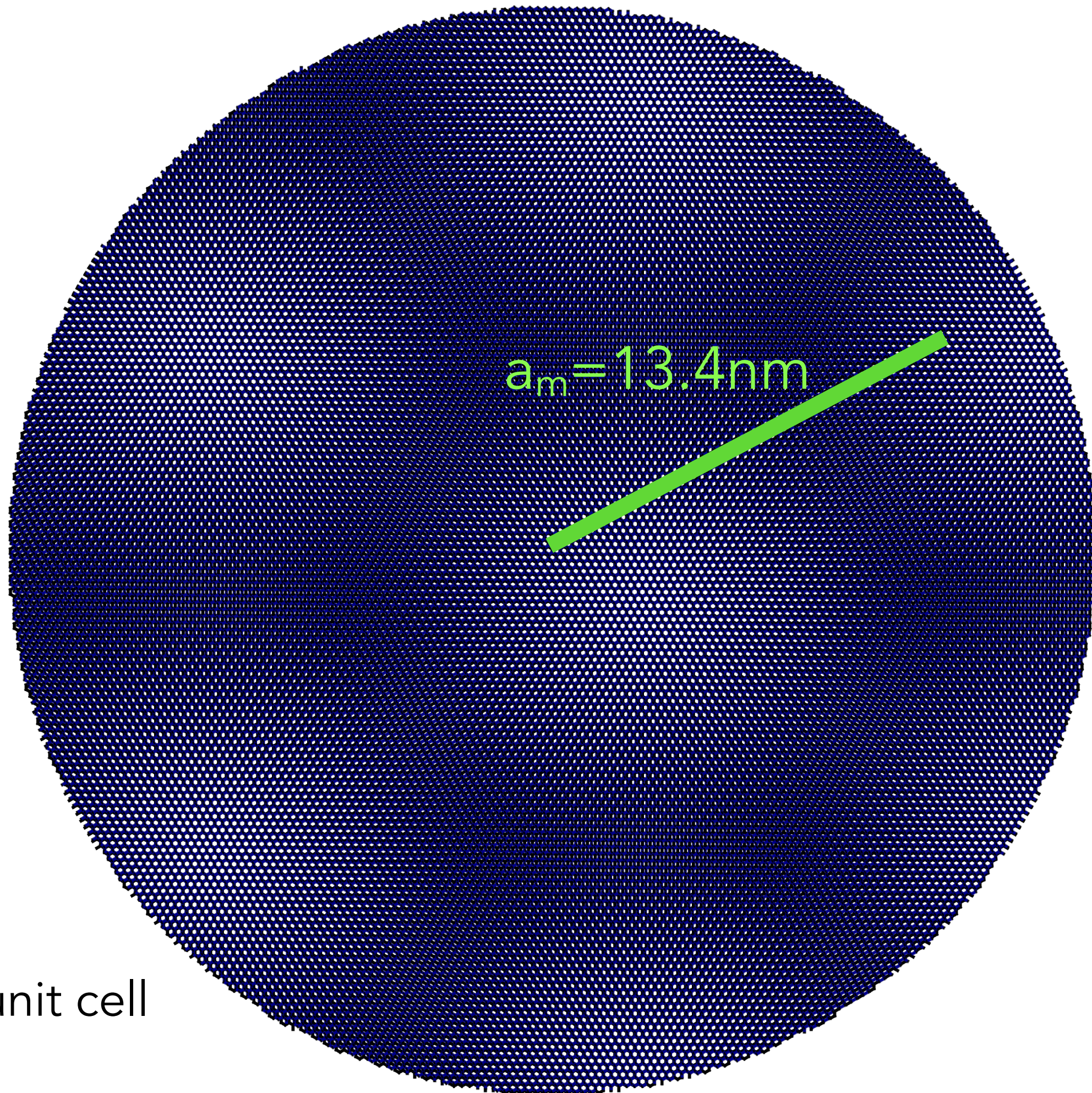


2°





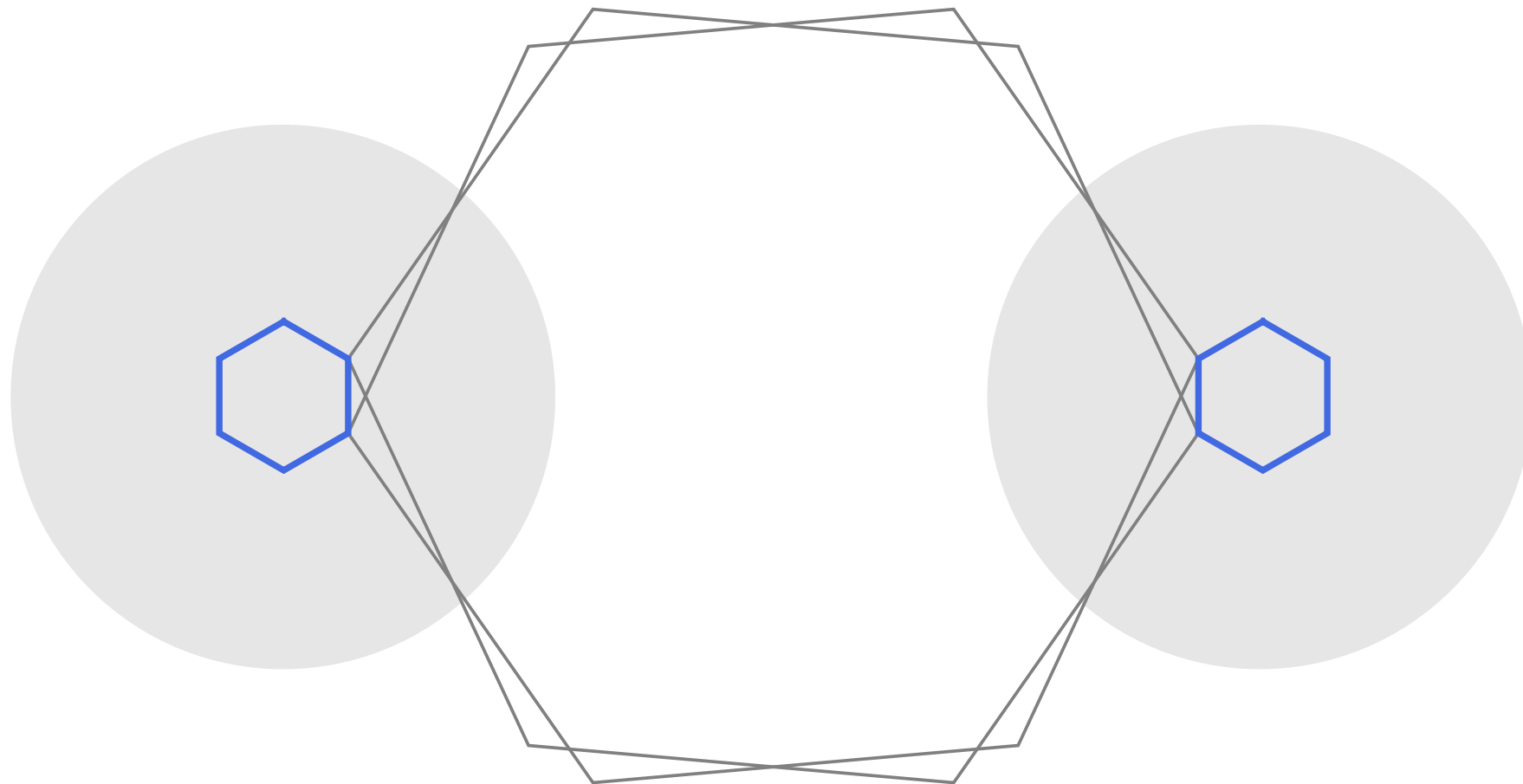
1°



huge unit cell



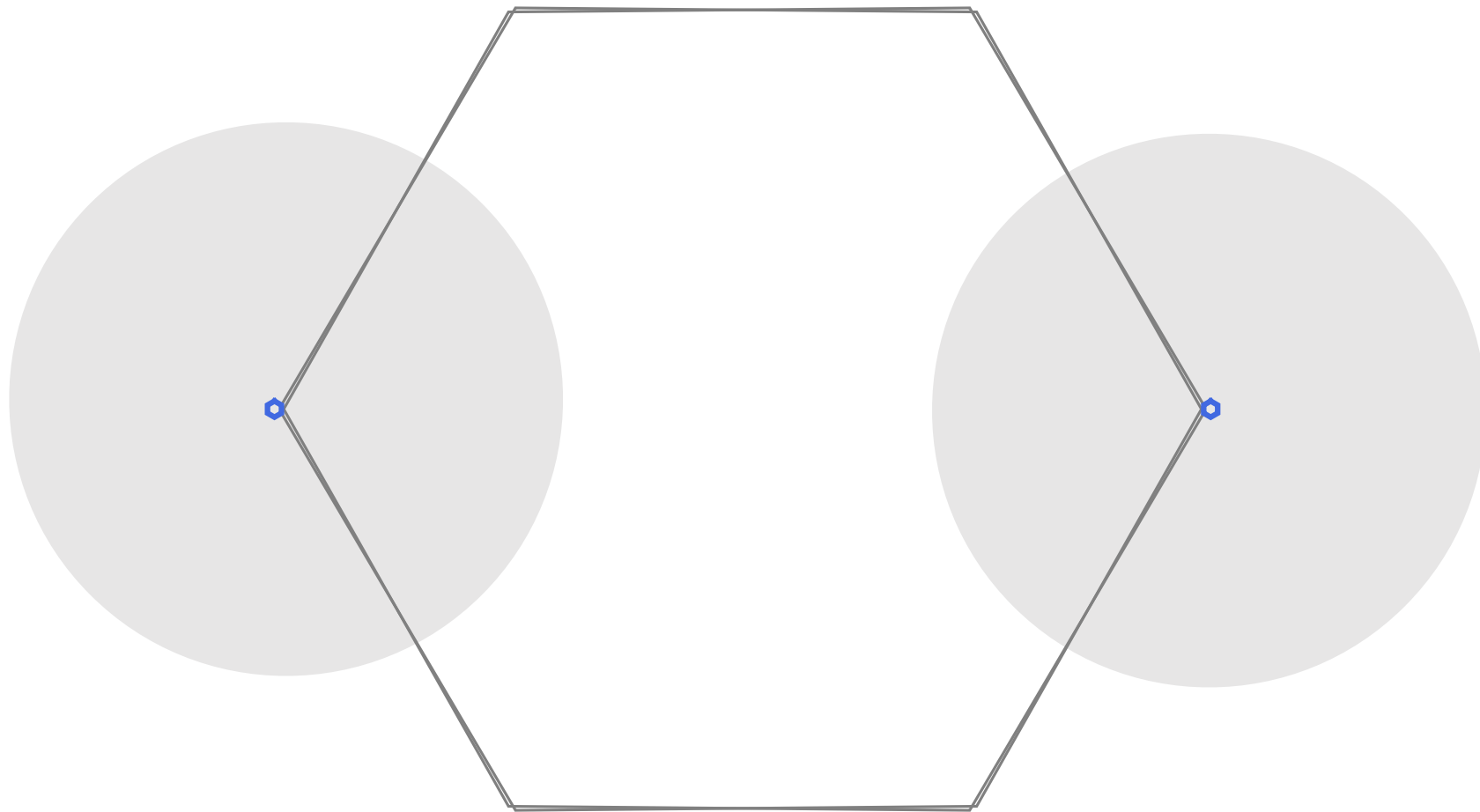
# Continuum model



approximate single layer as Dirac cone  
no mixing from one valley to the other



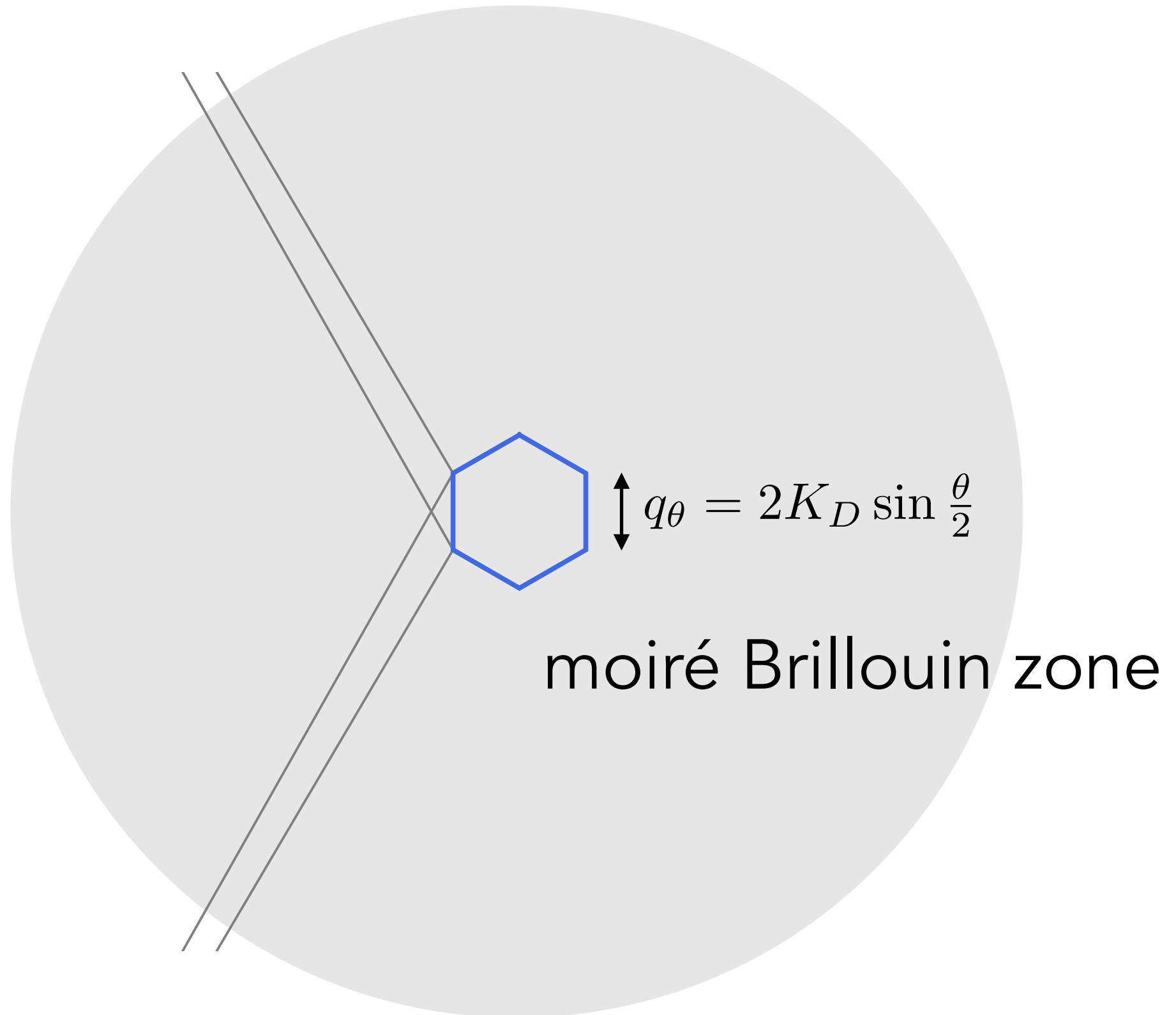
# Continuum model



1°

# One valley

1°



# Continuum model

Bistritzer+MacDonald (2011)

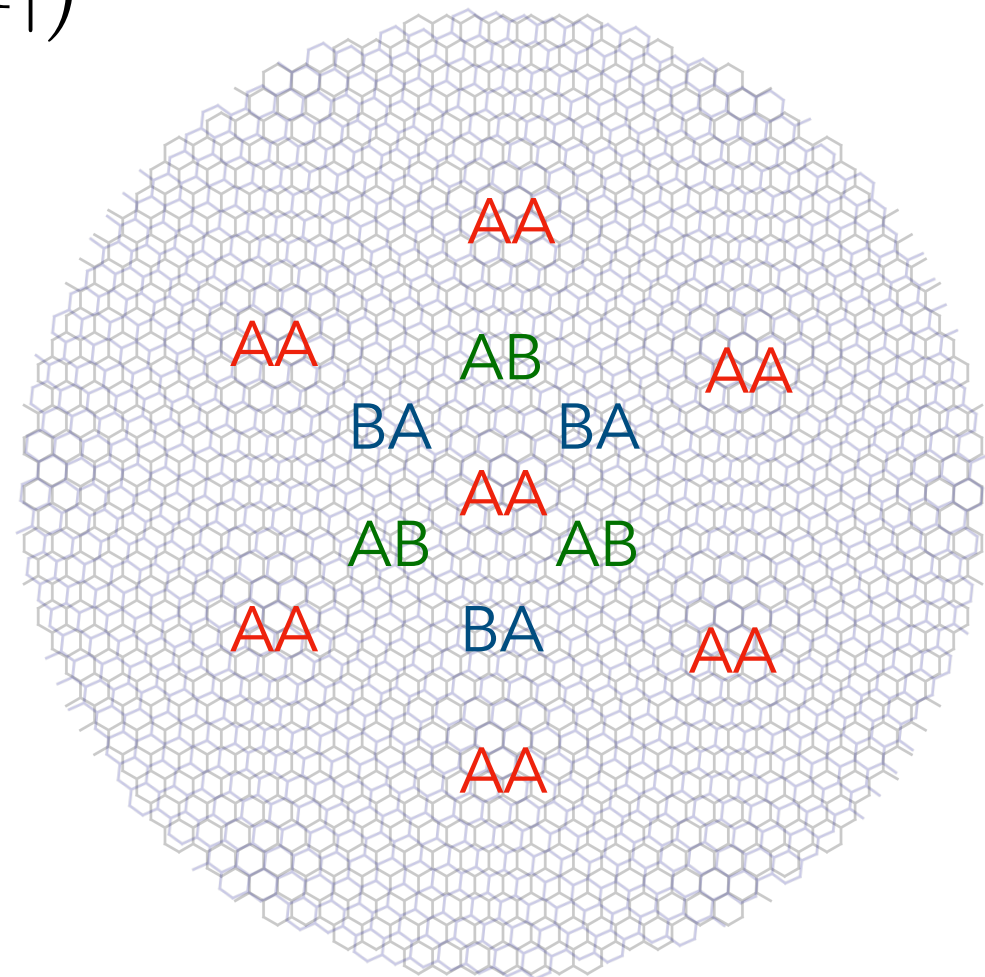
$$H = H_{\text{kin}} + H_{\text{tun}}$$

$$H_{\text{kin}} = v(\mathbf{k} - \mathbf{K}_1) \cdot \boldsymbol{\sigma}_{\theta/2} |1\rangle\langle 1| + v(\mathbf{k} - \mathbf{K}_2) \cdot \boldsymbol{\sigma}_{-\theta/2} |2\rangle\langle 2|$$

$$H_{\text{tun}} = w \left( \mathbf{T}(\mathbf{x}) |1\rangle\langle 2| + \mathbf{T}^\dagger(\mathbf{x}) |2\rangle\langle 1| \right)$$

periodic hopping matrix: smoothly interpolates  
hopping of uniform AA/AB/BA bilayers

- Restores periodicity
- Reveals dimensionless parameter,  $w/vk_\theta$
- Predicts flat bands at magic angles



# BM Derivation

We derive a continuum model for the tunneling term by assuming that the interlayer tunneling amplitude between  $\pi$ -orbitals is a smooth function  $t(r)$  of spatial separation projected onto the graphene planes. The matrix element

$$T_{\mathbf{k}\mathbf{p}'}^{\alpha\beta} = \langle \Psi_{\mathbf{k}\alpha}^{(1)} | H_T | \Psi_{\mathbf{p}'\beta}^{(2)} \rangle \quad [1]$$

of the tunneling Hamiltonian  $H_T$  describes a process in which an electron with momentum  $\mathbf{p}' = M\mathbf{p}$  residing on sublattice  $\beta$  in one layer hops to a momentum state  $\mathbf{k}$  and sublattice  $\alpha$  in the other layer. In a  $\pi$ -band tight-binding model the projection of the wave functions of the two layers to a given sublattice are

$$|\psi_{\mathbf{k}\alpha}^{(1)}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} e^{i\mathbf{k}(\mathbf{R}+\tau_\alpha)} |\mathbf{R} + \tau_\alpha\rangle \quad [2]$$

and

$$|\psi_{\mathbf{p}'\beta}^{(2)}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}'} e^{i\mathbf{p}'(\mathbf{R}'+\tau'_\beta)} |\mathbf{R}' + \tau'_\beta\rangle. \quad [3]$$

Here  $\tau_A = 0$ ,  $\tau_B = \tau$ , and  $\mathbf{R}$  is summed over the triangular Bravais lattice. Substituting Eqs. 2 and 3 in Eq. 1 and invoking the two-center approximation,

$$\langle \mathbf{R} + \tau_\alpha | H_T | \mathbf{R}' + \tau'_\beta \rangle = t(\mathbf{R} + \tau_\alpha - \mathbf{R}' - \tau'_\beta), \quad [4]$$

for the interlayer hopping amplitude in which  $t$  depends on the difference between the positions of the two carbon atoms we find that

$$T_{\mathbf{k}\mathbf{p}'}^{\alpha\beta} = \sum_{\mathbf{G}_1 \mathbf{G}_2} \frac{t_{\mathbf{k}+\mathbf{G}_1}}{\Omega} e^{i[\mathbf{G}_1\tau_\alpha - \mathbf{G}_2(\tau_\beta - \tau) - \mathbf{G}_2'\mathbf{d}]} \delta_{\mathbf{k}+\mathbf{G}_1, \mathbf{p}'+\mathbf{G}_2'}. \quad [5]$$

Here  $\Omega$  is the unit cell area,  $t_{\mathbf{q}}$  is the Fourier transform of the tunneling amplitude  $t(\mathbf{r})$ , the vectors  $\mathbf{G}_1$  and  $\mathbf{G}_2$  are summed over reciprocal lattice vectors, and  $\mathbf{G}_2' = M\mathbf{G}_2$ . The bar notation over momenta in Eq. 5 indicates that momentum is measured relative to the center of the Brillouin zone and not relative to the Dirac point. Note that crystal momentum is conserved by the tunneling process because  $t$  depends only on the difference between lattice positions.\*

Directly calculate overlap of every C orbital in layer 1 with every C orbital in layer 2

Assume rigid rotation of layers

Obtain hopping matrix in momentum space by Poisson resummation formula

At the end of the calculation Fourier transform back to obtain simple real space formula

# Effective field theory

Describes low energy, long wavelength physics,  
can include effects of any perturbations that are  
small and slowly varying

Here:

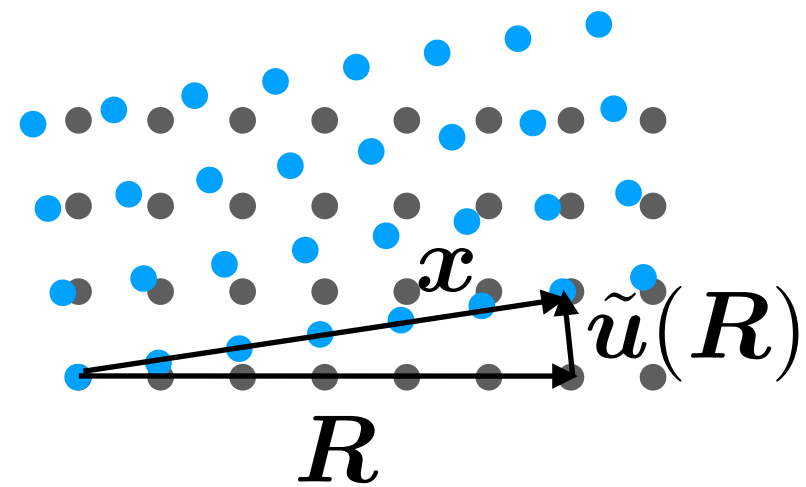
- Unperturbed system: isolated graphene layers
- Perturbations:
  - Interlayer tunneling
  - Slowly varying displacements of the layers



# Rotation $\subset$ Displacement Gradient

Ashcroft-Mermin: phonons

$$\mathbf{x} = \mathbf{R} + \tilde{\mathbf{u}}(\mathbf{R})$$



Rotation

$$\theta = \frac{1}{2} (\partial_x u_y - \partial_y u_x)$$

$$\tilde{\mathbf{u}} = \theta \hat{\mathbf{z}} \times \mathbf{R}$$

Twisting is just a subset of elastic  
deformations of two layers

# Effective field theory

Locality:

$$H = \int d^2x \mathcal{H}[\psi, \boldsymbol{u}, w]$$

Hamiltonian density is a *local* functional of the fields, *analytic*, and expandable in powers of small parameters — here field gradients and hopping strength

# Effective field theory

Locality:

$$H = \int d^2x \mathcal{H}[\psi, \mathbf{u}, w]$$

Hamiltonian density is a **local** functional of the fields, *analytic*, and expandable in powers of small parameters — here field gradients and hopping strength

Small problem:

$\mathbf{x} = \mathbf{R} + \tilde{\mathbf{u}}(\mathbf{R})$        $\mathbf{R}$  is not the actual real space location -  
physics is local in  $\mathbf{x}$  not  $\mathbf{R}$

# Effective field theory

Locality:

$$H = \int d^2x \mathcal{H}[\psi, \boldsymbol{u}, w]$$

Hamiltonian density is a **local** functional of the fields, *analytic*, and expandable in powers of small parameters — here field gradients and hopping strength

Solution: Eulerian coordinates

$$\boldsymbol{x} = \boldsymbol{R} + \boldsymbol{u}(\boldsymbol{x})$$

# Effective field theory

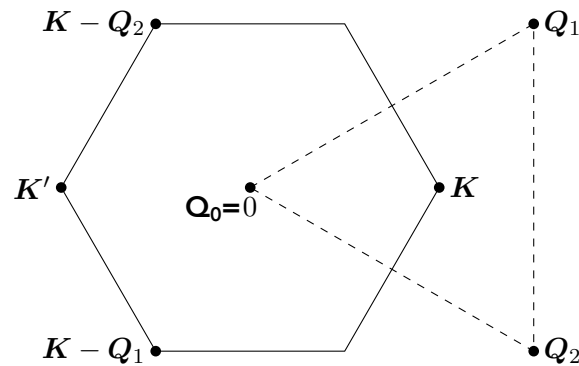
- Three effects:
  1. Coordinate change: transformation of local frames to global one
  2. Strains: modification of energetics of each layer due to changes in electron hopping
  3. Tunneling: strong dependence of relative *local* alignment



# Result

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$



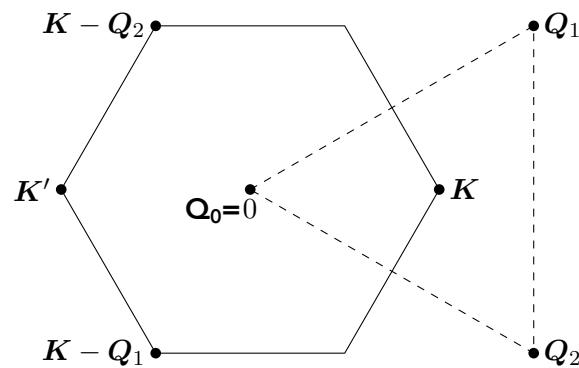
Correct to first order in strain gradients and hopping

# Result

coordinate change

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$



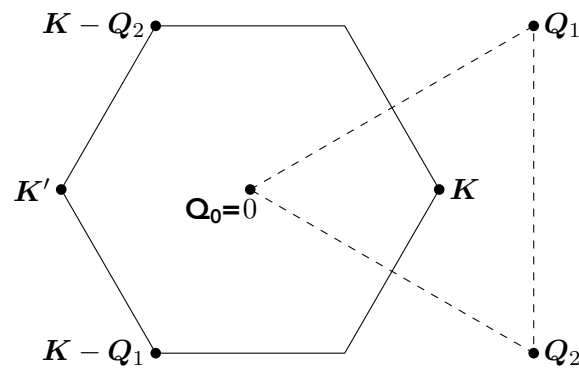
Correct to first order in strain gradients and hopping

# Result

Strain gauge field

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathbf{T}_j \psi_1 + \text{h.c.} \right]$$

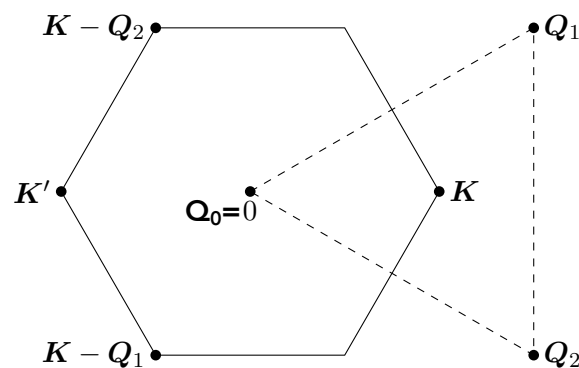


Correct to first order in strain gradients and hopping

# Result

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right] \leftarrow \text{tunneling. Form fixed by space group symmetries}$$



$$\mathsf{T}_j = u \mathbb{I} + w (\bar{\zeta}^j \tau^+ + \zeta^j \tau^-), \quad j = 0, 1, 2,$$

Correct to first order in strain gradients and hopping

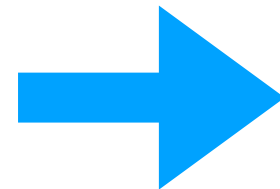


# Apply to rigid twist

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right]$$

Evaluate for  $\mathbf{u}_1 = -\mathbf{u}_2 = \frac{\theta}{2} \hat{\mathbf{z}} \times \mathbf{x}$ .



$$\mathcal{H} = \psi_1^\dagger \left[ -iv \boldsymbol{\tau} \left( \frac{\theta}{2} \right) \cdot \boldsymbol{\nabla} - \frac{vk_\theta}{2} \tau^y \right] \psi_1 + \psi_2^\dagger \left[ -iv \boldsymbol{\tau} \left( -\frac{\theta}{2} \right) \cdot \boldsymbol{\nabla} + \frac{vk_\theta}{2} \tau^y \right] \psi_2$$

$$+ \sum_j \left[ e^{-i\mathbf{q}_j \cdot \mathbf{x}} \psi_2^\dagger \mathsf{T}_j \psi_1 + \text{h.c.} \right] \quad \mathbf{q}_j = -\theta \hat{\mathbf{z}} \times \mathbf{Q}_j$$

Exactly the BM model.

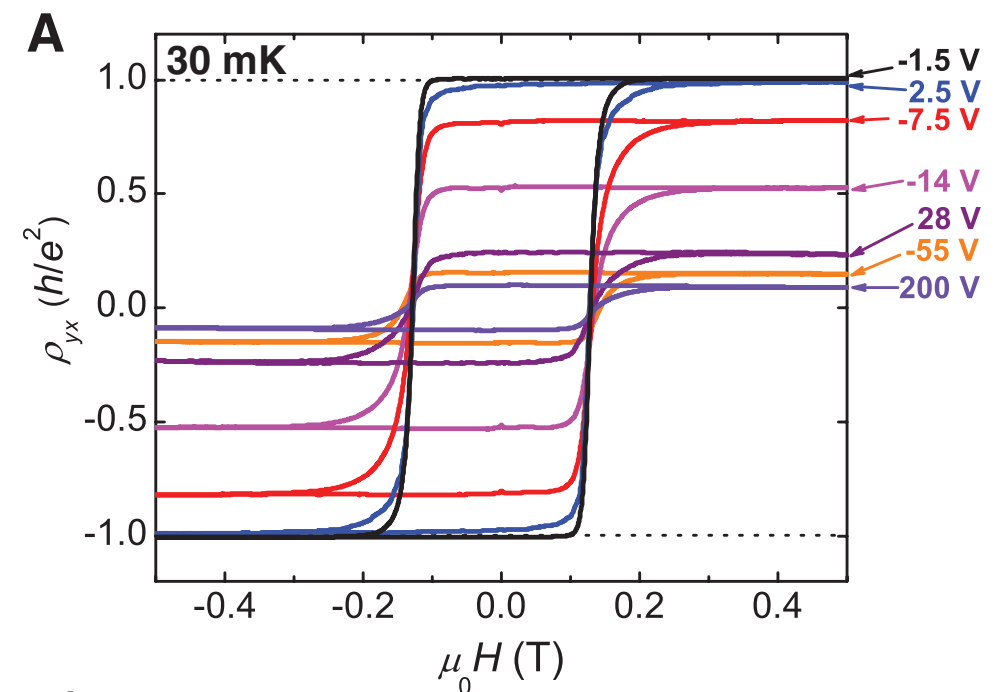
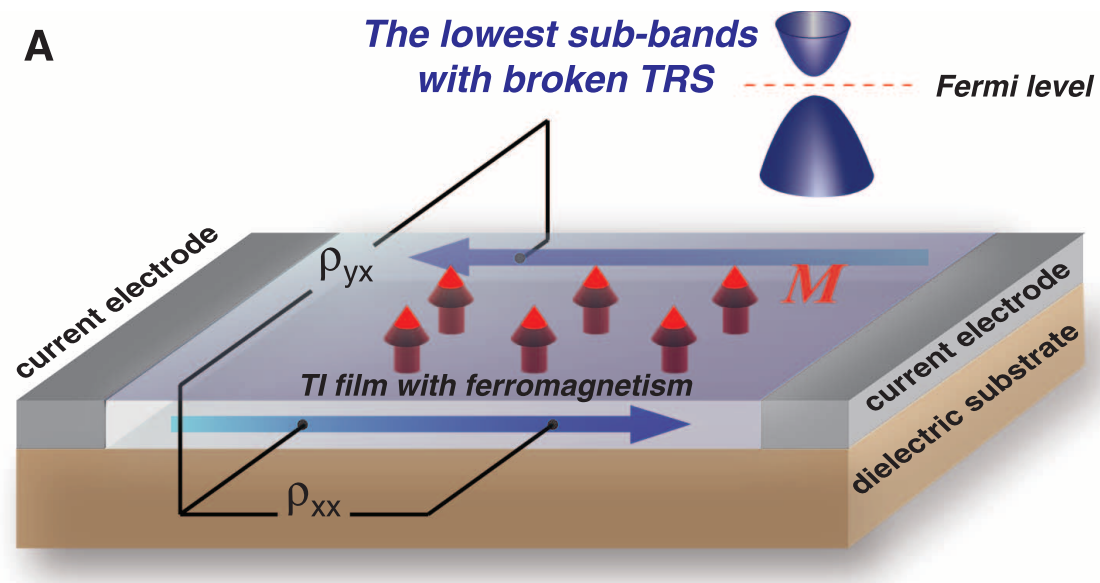
# Result

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l \\ + \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathbf{T}_j \psi_1 + \text{h.c.} \right]$$

- Recovers BM result intuitively
- Subsumes other extensions of BM (Nam+Koshino, Bi,Yuan+Fu...)
- Includes coupling of acoustic phonons
- Can handle arbitrary inhomogeneous strains
- All these things together
- Easy to add more layers
- Very nice for teaching

# Quantum Anomalous Hall Effect

This is just the appearance of QHE in zero magnetic field by spontaneous breaking of time-reversal symmetry

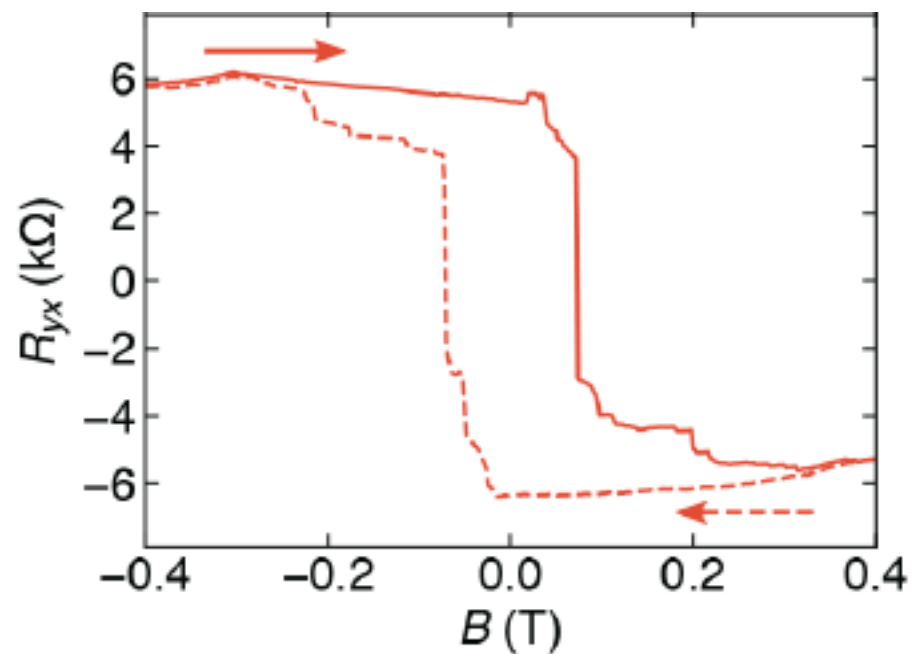


C.-Z. Zhang et al, 2013

Cr-doped  $(\text{Bi/Sb})_2\text{Te}_3$

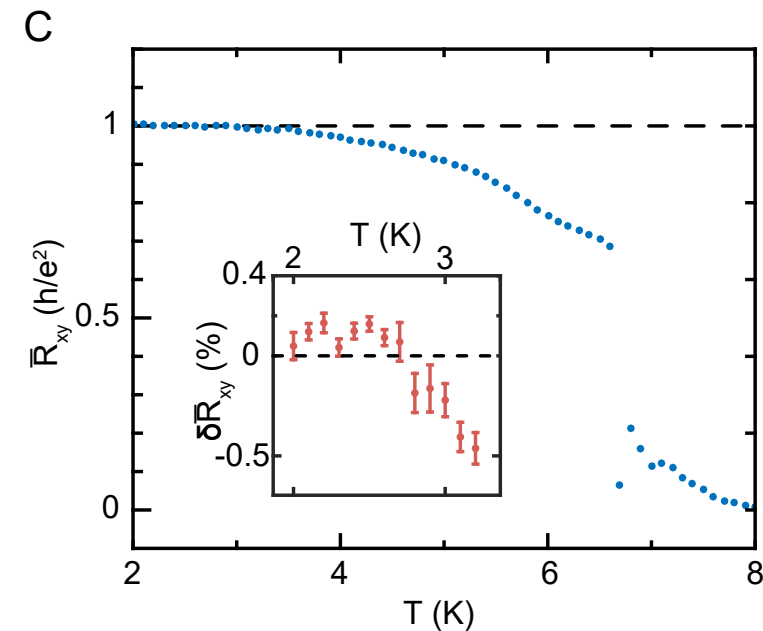
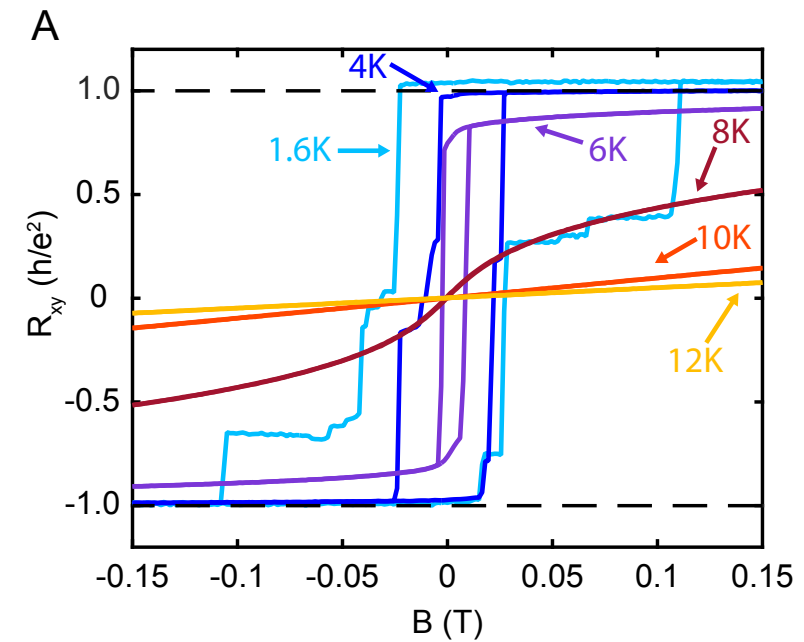
# QAHE in TBG

3 e<sup>-</sup>'s per moiré unit cell



A. Sharpe, et. al. *Science* (2019);

Spontaneous AHE - not quite quantized

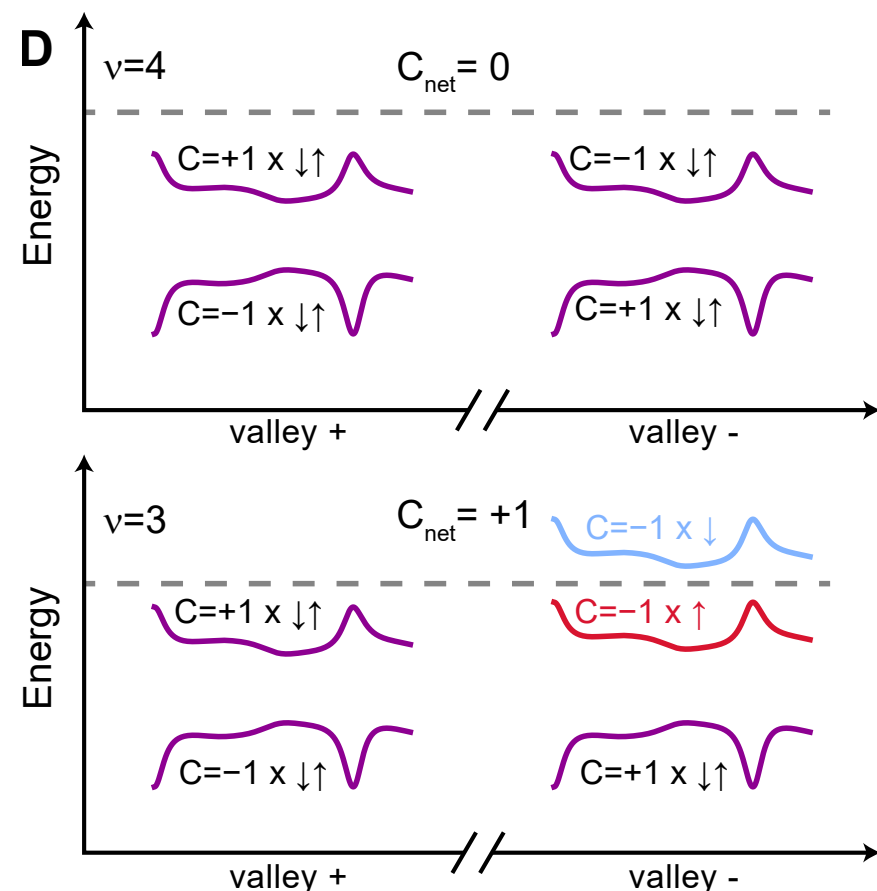


QAHE to 1/1000 accuracy

M. Serlin et al, unpublished

# Theoretical remarks

- Underlying Dirac fermions of graphene have large incipient Berry curvature
- Curvature is realized by breaking  $C_2T$  symmetry
- Valley polarization gives non-zero AHE.
- Quantization occurs if gap is complete - needs spin polarization




Spin and valley split

Symmetry breaking =  
 $Z_2$  (valley)  
 $\times$   
 $SU(2)$  (spin)

A portrait of a man with dark hair, smiling and looking slightly to the right. He is wearing a light blue button-down shirt. The background is a textured, mottled grey.

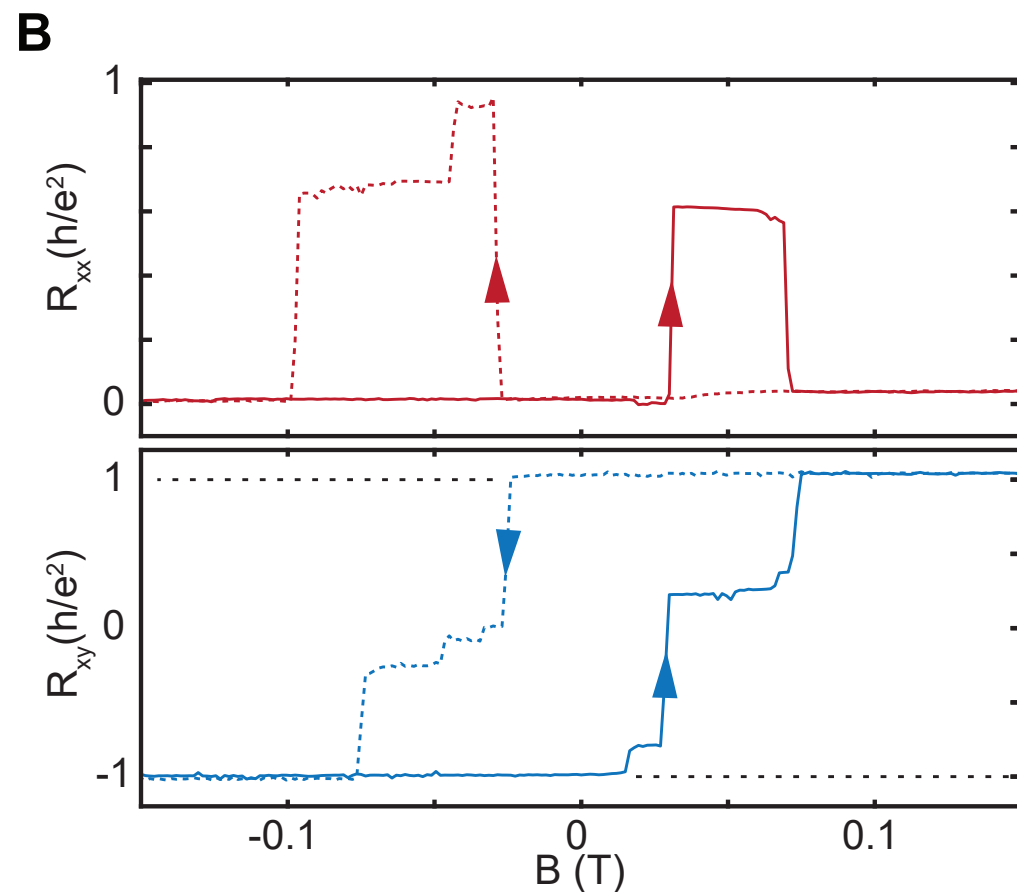
- arXiv:1803.09742 [pdf, other] cond-mat.str-el cond-mat.mtrl-sci cond-mat.supr-con doi 10.1103/PhysRevX.8.031089

- arXiv:1808.02482 [pdf, other] cond-mat.str-el cond-mat.mtrl-sci cond-mat.supr-con doi: 10.1103/PhysRevB.99.195455

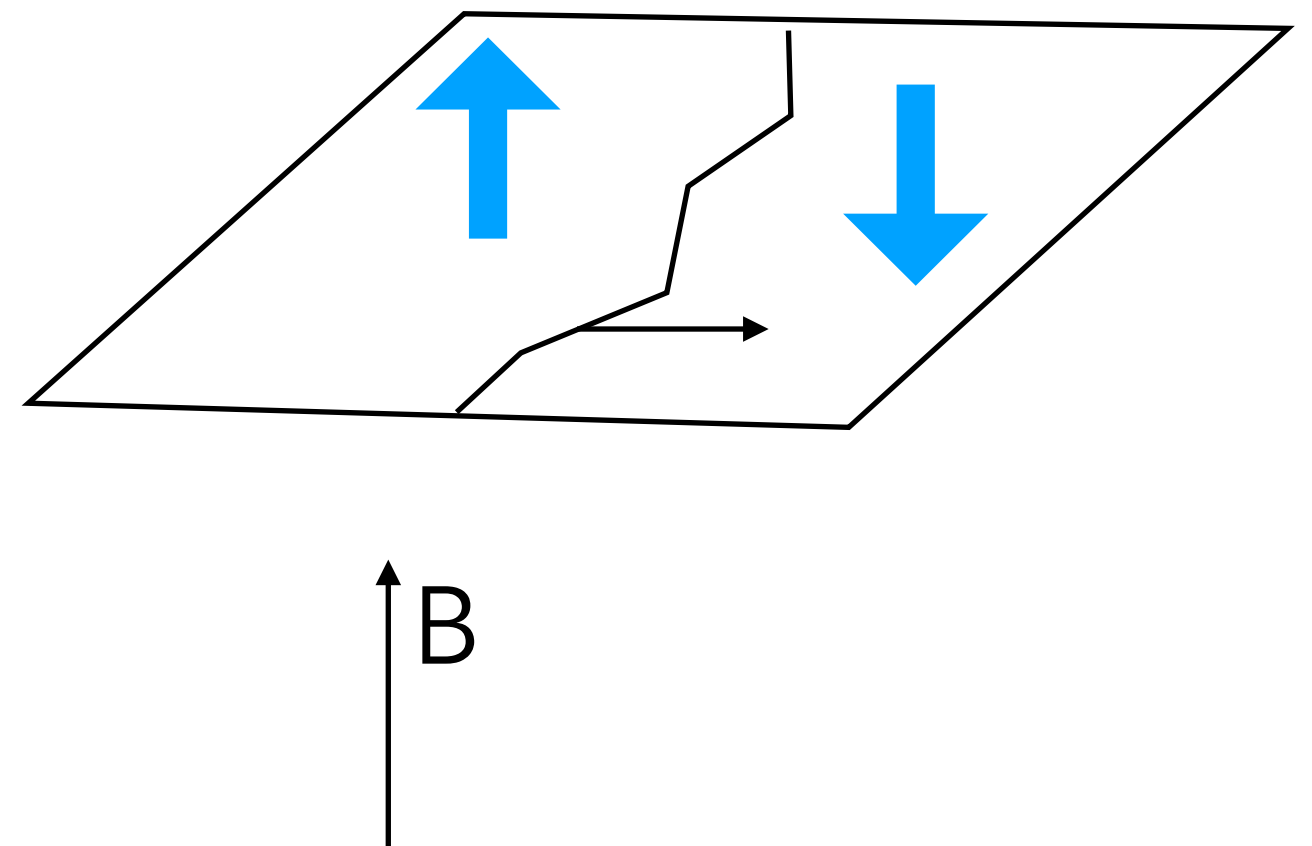
7. [arXiv:1806.07873 \[pdf, other\]](https://arxiv.org/abs/1806.07873), [cond-mat.matrx-hall](https://cond-mat.matrx.org/cond-mat-matrx-hall), [cond-mat-matrx-solid](https://cond-mat.matrx.org/cond-mat-matrx-solid), [10.1103/PhysRevX.8.084543](https://doi.org/10.1103/PhysRevX.8.084543) 
- Band Structure of Twisted Bilayer Graphene: Appearance Symmetries, Commensurate Approximations and Anomalous**
- Abstracts:** [Liqun Zou](#), [Hu Chun Pu](#), [Ashwin Vishwanath](#), [J. Senthil](#)
- Abstract:** A remarkable feature of the band structure of bilayer graphene at small twist angle is the appearance of isolated bands near neutrality, whose bandwidth can be reduced at certain magic angles (e.g.  $\sim 1.05^\circ$ ), in this regime, correlated insulating states and superconductivity have been experimentally observed. A microscopic description of these phenomena requires an understanding of [unref. v1](#).
- Submitted:** 29 August, 2018; **v1 submitted:** 20 June, 2018; **originally announced:** June 2018.
- Comments:** 14 pages + appendices; [e-print published](#) in *Journal of Phys. Rev. B* 98, 084543 (2018).



# Domain manipulation



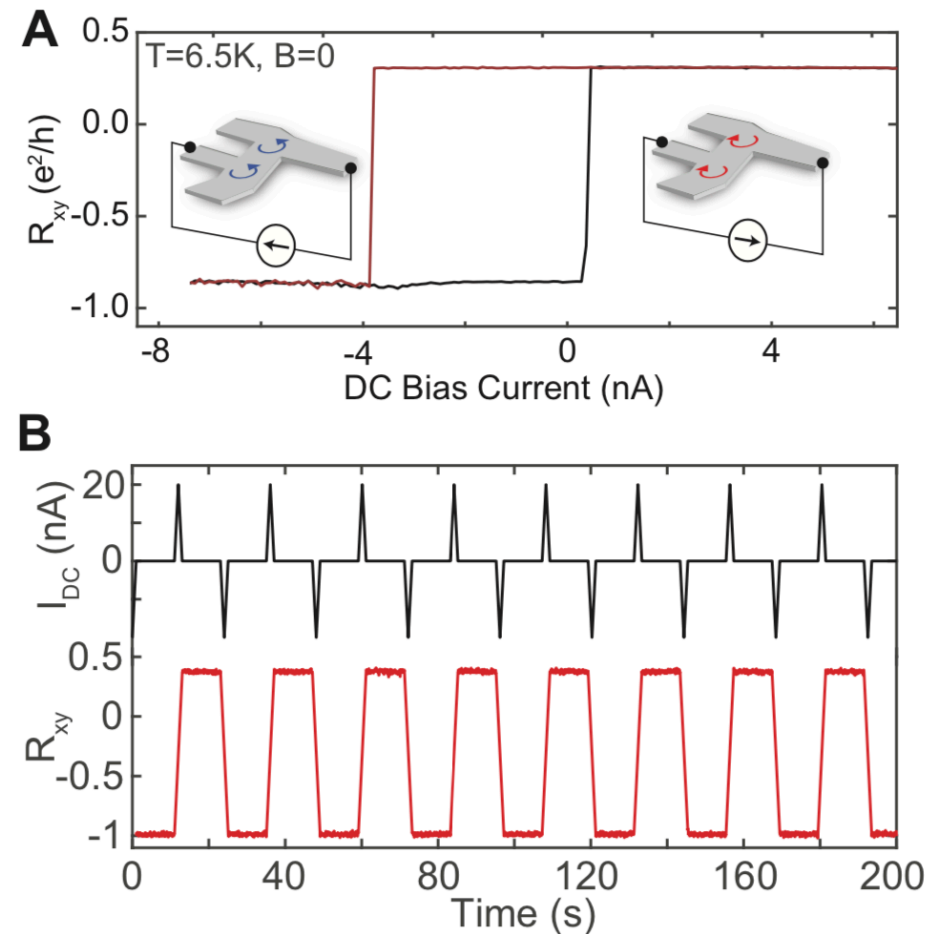
B field biases energy of domains



$$\Delta E = -\mathbf{B} \cdot \mathbf{M}$$

n.b. domains are *valley* domains

# Domain manipulation



(tiny)

Current switches domains. How does this work?

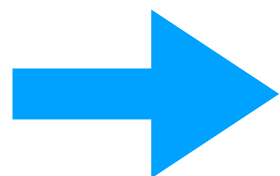
# Current



Well-developed IQHE:

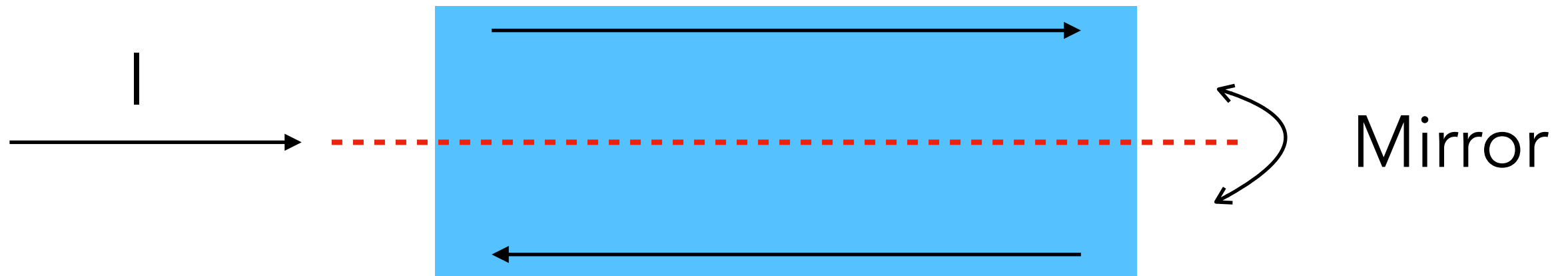
- no dissipation, only edge state transport
- Charge of each edge is separately conserved

✿ Can view current-carrying state as quasi-equilibrium ensemble where current determines edge occupation



Can formulate  $F(I, M)$

# (A)symmetry



$l \rightarrow l$	vector
$M \rightarrow -M$	pseudovector

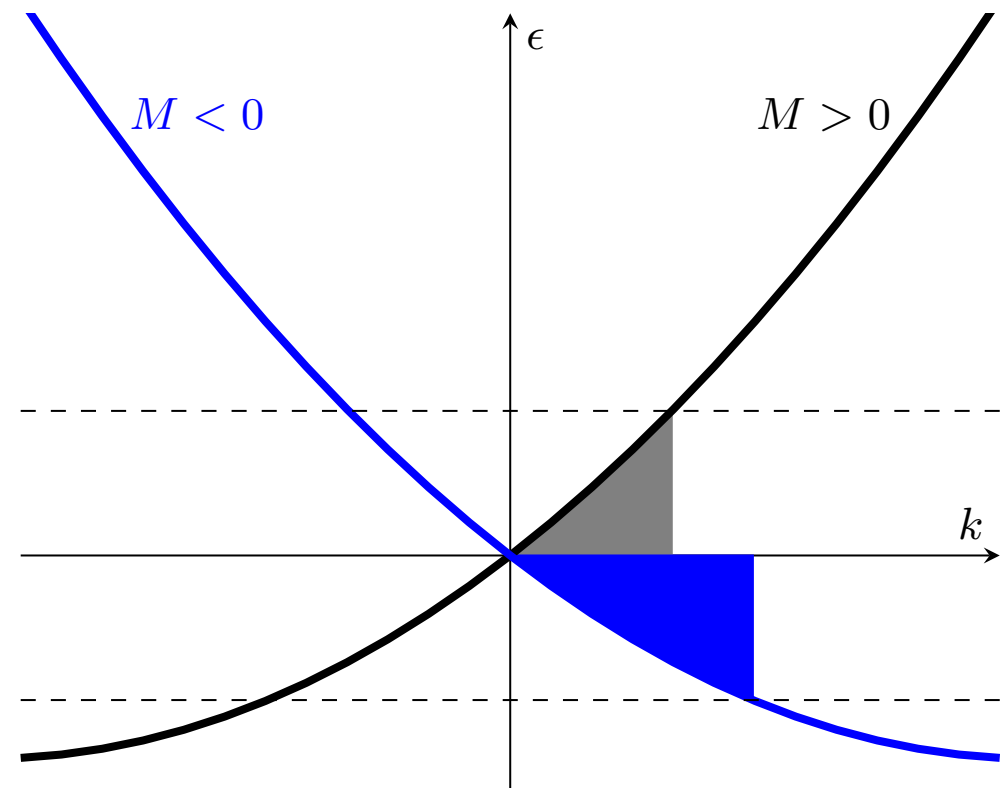
# Energetics

Simple limit: one "fast" (costly) edge

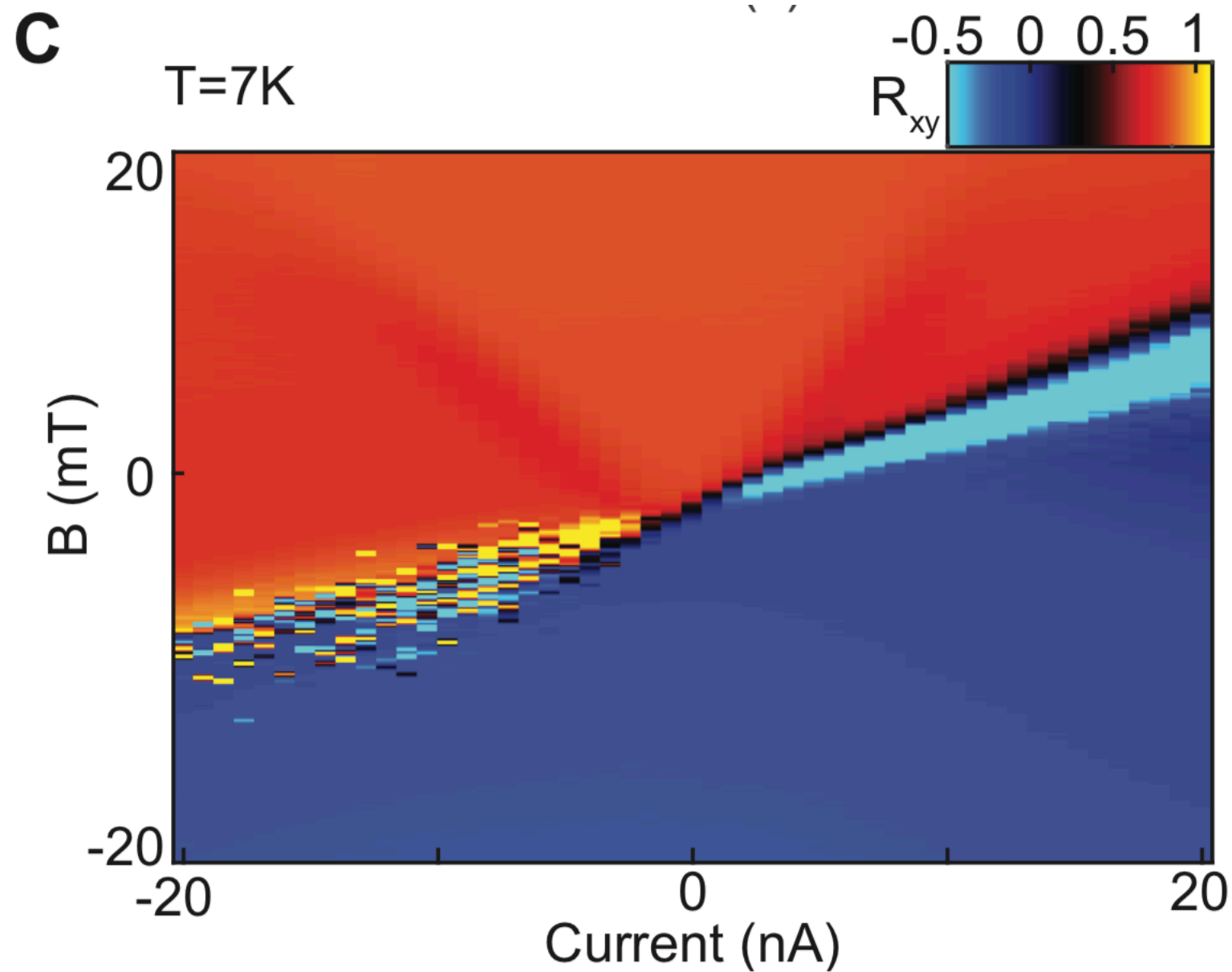
$I > 0$ :

- Add right-moving  $e^-$ s
- Remove left-moving  $e^-$ s

$$\Delta F \sim \frac{\hbar^2}{me^3 v^3} LI^3$$



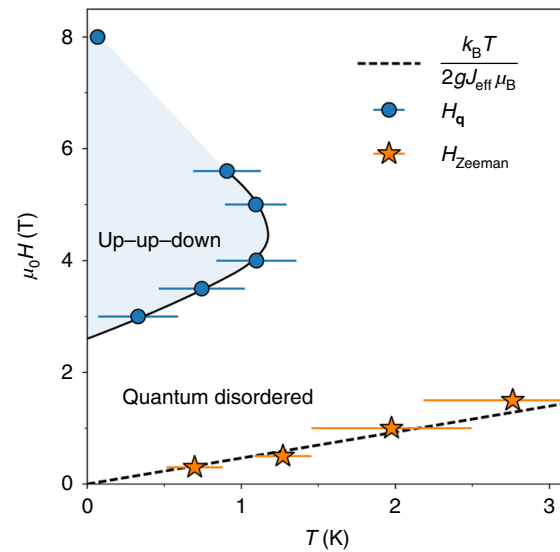
# Dissipative Regime



A fully non-equilibrium problem, bulk 2d physics



# Thanks

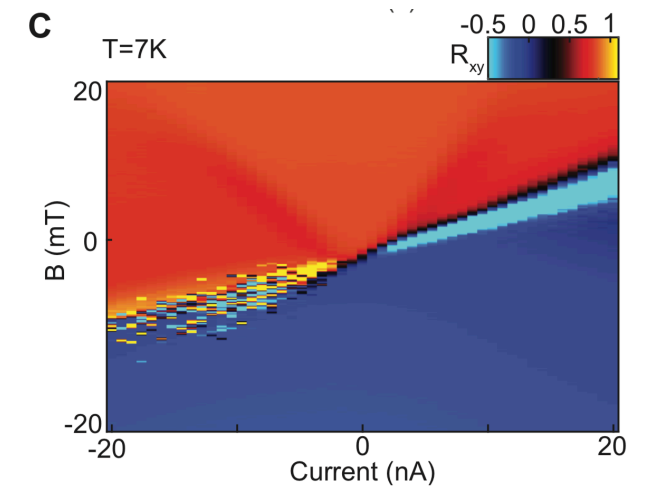


M. Bordelon et al, Nat. Phys. (2019)

$$\mathcal{H} = \sum_l \psi_l^\dagger \left[ -iv \left( \tau^\mu + \frac{\partial u_{l,\mu}}{\partial x_\nu} \tau^\nu \right) \frac{\partial}{\partial x_\mu} + v (\mathbf{K} \cdot \partial_\mu \mathbf{u}_l + \mathcal{A}_l) \tau^\mu \right] \psi_l$$

$$+ \sum_j \left[ e^{-i\mathbf{Q}_j \cdot (\mathbf{u}_1 - \mathbf{u}_2)} \psi_2^\dagger \mathbf{T}_j \psi_1 + \text{h.c.} \right]$$

L.B., arXiv:1909.01545



M. Serlin et al, arXiv:1907.00261

