GRAPHENE LECTURES

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Graphene lectures.

1. Single layer, Bilayer, Dirac, QBT, topology, Berry curvature, Chern number
   Edge states
2. TBG — Continuum model, magic angle, Experiments.
3. Lattice effects, interaction, topology in TBG.
4. Correlation physics... QAHE, Correlated Insulators, SCivity

Egel

Intro:

TBG - Meeting of two worlds

Graphene - Textbook band theory
- Beautiful example of 2DEG
  - Lots of people recreating physics of 80's-90's from GaAl/AlGaAs But w/ a bit of Dirac
  - Almost everything explainable as either 1e⁻ physics or 2DEG stuff.

∴ What do I mean by "2DEG stuff"
→ all physics of lattice scale subsumed in Dirac dispersion

Correlated e⁻'s - Very large on-site repulsion / localized e⁻'s

Hubbard-Model ~ Giovanni

Spins (fully localized) - Premi, Lucile, Amalia, Rabu, Collin

Quantum Materials - Cuprates, Pnictides, Heavy Fermions, Cobaltates, Ruthenates

TMOs etc.

Band theory is either bad starting pt or strongly renormalized.

TBG → Bringing the two together! Caveats:

∴ Moiré materials > TBG
trilayers, bi-bi, etc.
TMDs – WSe_2 etc.
not sure if I will talk about these.

Some of this stuff is still 1e⁻ physics

∴ Fuchs – 1e⁻ physics is still interesting

∴ Also – 1e⁻ physics is usually the part we understand!

A lot of correlation physics is trying to describe effects of interactions in terms of renormalized 1e⁻ problems.

e.g. BCS, Hartree-Fock ⟺ MFTs

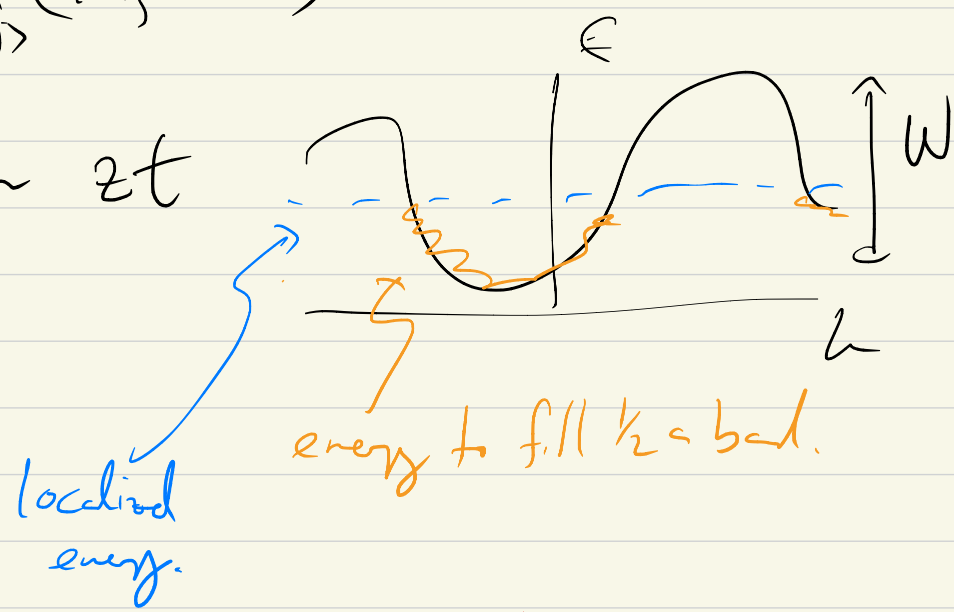
(What's beyond this? QSLs mostly
1d LLs
Spin-charge separation
Incoherent NFLs?)

Anyway, why does TBG make graphene correlated?

Basic Problem: in $s+p$-$\bar{e}$ systems, orbitals are too big & overlap too strongly.

$$H \sim t \sum_{\langle ij \rangle} (c_i^\dagger c_j + h.c.)$$

Bandwidth $W \sim zt$



* Need $U \gtrsim W$ to correlate $\bar{e}$'s, localize them, etc.

W huge for graphene $\sim 3t \sim 8\,eV$
(or 18 eV for both sides)

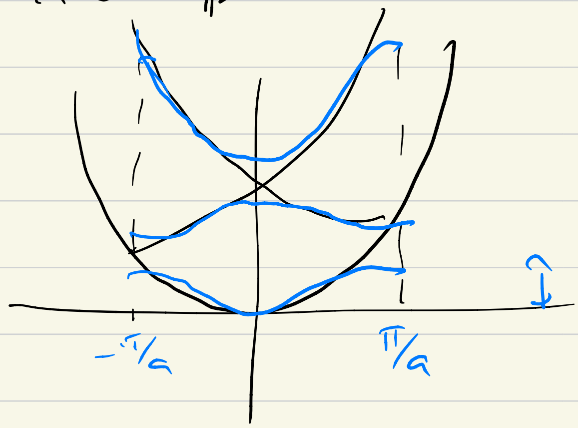
So interactions don't rearrange $\bar{e}$'s over whole band.
(Can still perturb the low-ϵ electrons)

What if we could reduce w?

Basic idea: Add periodic modulation → minibands

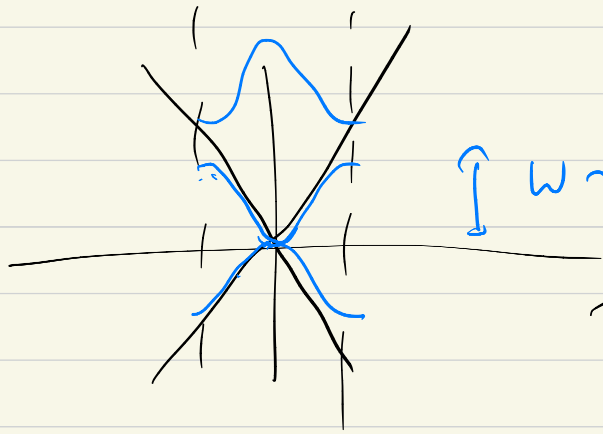
e.g. 1d

free e^- gas



$w \sim \frac{1}{ma^2}$

Dirac

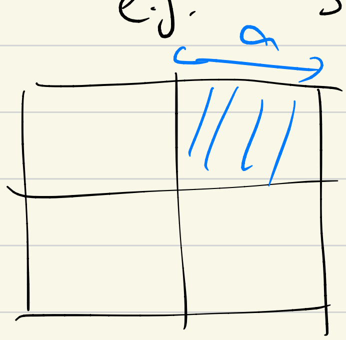


$w \sim v/a$

This is expected for any superlattice

Commonly studied in semiconductor heterostructures

Does it work? e.g. Square superlattice



What is U? $U \sim \frac{e^2}{a}$

$$U/W \sim \frac{e^2/a}{v/a} \sim \frac{e^2}{v}$$

Independent of a! So not so easy.

N.B. For quadratic $W \sim \frac{1}{ma^2}$ $\quad U/W \sim \frac{e^2/a}{1/ma^2} \sim a m e^2 = a/a_{Bohr} \equiv$ "r_s"

Large for large a!

This is rather naive, no screening, no details of potential etc.

But this is basic reason Dirac is rather stable even at low density.

[Always works for strong potential]

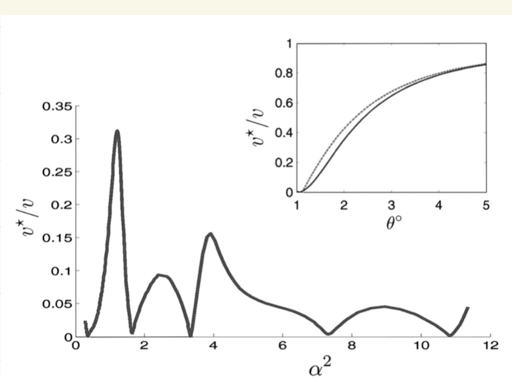
TBG is a bit more special

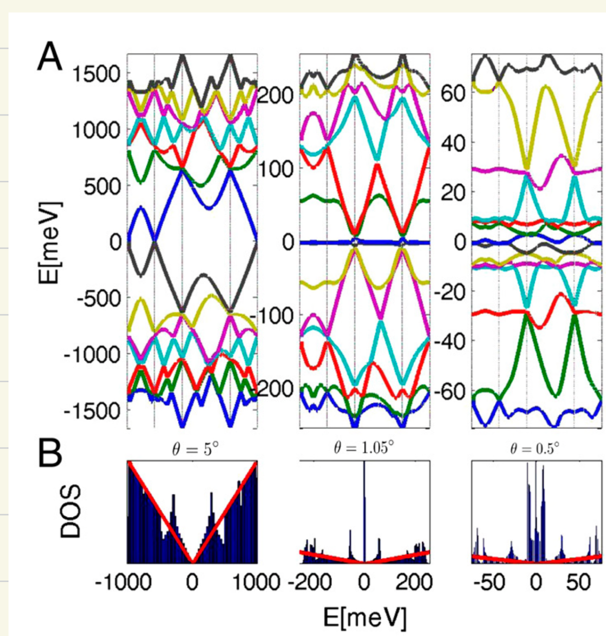
Basically $W \sim v/a \times f\left(\frac{v/a}{w}\right)$

this is some non-trivial dependence on angle → fixing prefactor "magic" $f \ll 1$!

We'll see why.

But preview:





So : ◦ Reminder of graphene fundamentals (Today)

◦ TBG + flat bands

   * New derivation of continuum model of Bistritzer + Macdonald. Simpler, less technical.

   * Magic angles.

   * Expts → is this basically correct?

   * Special features of TBG flat bands?

◦ Interactions. What is going on?

   [CA: Nobody knows, or at least we don't agree on it, &]

   - What are the ingredients?

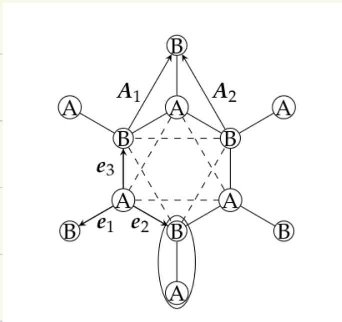
   - What can we deduce from observations?

   - Finally, hope to tell you about some recent results on QAHE & what I find interesting

* Graphene Fundamentals - What you need to know

c.f. Castro-Neto et al, RMP 81, 109 (2009).

1. Single layer graphene



$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j + h.c.$$

$$c_{Rs} = \frac{1}{\sqrt{N}} \sum_k c_{ks}\, e^{ik\cdot R}$$

$$H = \sum_k c_{ks}^\dagger\, h_{ss'}(k)\, c_{ks'}$$

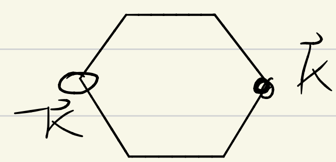
$$h_k(s) = \begin{pmatrix} 0 & f(k) \\ f^*(k) & 0 \end{pmatrix}$$

$$f(\boldsymbol{k}) = -t\left(1 + e^{-i\boldsymbol{k}\cdot \boldsymbol{A}_2} + e^{-i\boldsymbol{k}\cdot \boldsymbol{A}_1}\right).$$

$$\Rightarrow \quad \varepsilon = \pm |f(k)|$$

$$f(\pm\vec{K} + \vec{k}) \simeq v(\pm k_x - i k_y)$$

$$v = \frac{3}{2} t$$

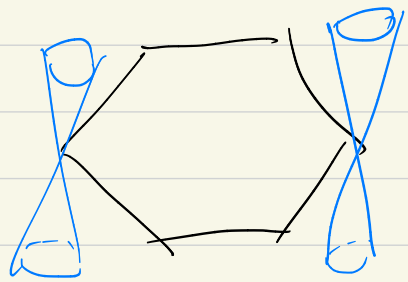


$$c_{Rs} \simeq e^{i\vec{K}\cdot\vec{r}}\, \psi_{+Rs} + e^{-i\vec{K}\cdot\vec{r}}\, \psi_{-Rs}$$

$$H \simeq \int d^2x \; -iv\psi^\dagger (\mu^z \tau^x \partial_x + \tau^y \partial_y)\psi$$

$$= \sum_k v \psi_k^\dagger (\mu^z \tau^x k_x + \tau^y k_y) \psi_k \longrightarrow \varepsilon = \pm v|k|$$

$$v \simeq 10^6 \ m/s$$



Dirac Point. Governs low energy physics.

Many confirmations of Dirac points in graphene.
Maybe clearest is LL structure / IQHE.

Real nature of LLs in graphene?

→ Back to QMs

Consider 1 Dirac pt (K_+) & 1 spin

$$H = v\vec{q}\cdot\vec{\tau} = -iv\vec{\tau}\cdot\vec{\nabla}$$

In Field

$$H = -iv\vec{\tau}\cdot\left(\vec{\nabla} - \frac{ie\vec{A}}{c}\right)$$

$$= -iv\begin{pmatrix} 0 & \partial_x - i\partial_y + \frac{1}{2\ell^2}(x - iy) \\ \partial_x + i\partial_y - \frac{1}{2\ell^2}(x+iy) & 0 \end{pmatrix}$$

$$-iv\left[\partial_x - i\partial_y + \frac{1}{2\ell^2}(x-iy)\right]u_B = \varepsilon u_A$$

$$-iv\left[\partial_x + i\partial_y - \frac{1}{2\ell^2}(x+iy)\right]u_A = \varepsilon u_B$$

$$u_A = \phi_A e^{-r^2/4\ell^2} \qquad u_B = \phi_B e^{-r^2/4\ell^2}$$

$$-iv(\partial_x - i\partial_y)\phi_B = \varepsilon\phi_A$$

$$-iv\left[\partial_x + i\partial_y - \frac{1}{\ell^2}(x+iy)\right]\phi_A = \varepsilon\phi_B$$

$$-i\sqrt{2}\, v/\ell \; a^- \phi_B = \varepsilon \phi_A$$

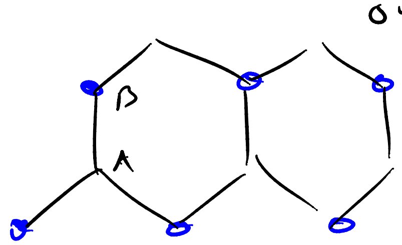
$$+i\sqrt{2}\, v/\ell \; a^+ \phi_A = \varepsilon \phi_B$$

$\underline{\varepsilon = 0}$ $\quad a^- \phi_B = 0 \leftarrow \phi_B = f(z)$ LLL

$\quad a^+ \phi_A = 0 \leftarrow \phi_A = 0$

So $\quad u = \begin{pmatrix} 0 \\ f(z)\, e^{-r^2/4\ell^2} \end{pmatrix}$

$u_A = 0 \longrightarrow$ wf is non-zero only on one sublattice!



If you think of TB model, you see why it has zero energy!

(N.B. if considered K^- D.P., would have found $u_B = 0$)

Other levels?

$\phi_B = \phi_n \qquad$ w.f. in n^{th} LL

$\phi_A = -iv\sqrt{2}/\ell\varepsilon \; a^- \phi_n$

$$\Rightarrow \varepsilon^2 = 2n v^2/\ell^2 \rightarrow \boxed{\varepsilon = \pm\sqrt{2n}\, v/\ell}$$

$$\Rightarrow \phi_A = \mp \frac{i}{\sqrt{n}} a^- \phi_n$$

$$u = \begin{pmatrix} -\frac{\mathrm{sgn}(n)\, i}{\sqrt{|n|}} a^- \phi_{|n|} \\ \phi_{|n|} \end{pmatrix} e^{-r^2/4\ell^2}$$

$$n = 0, \pm 1, \ldots \qquad \varepsilon_n = \mathrm{sgn}(n)\sqrt{2|n|}\,\frac{v}{\ell}$$

Basically all wfs look like mixtures of nth & $n-1$'th LLs.

$\rightarrow$ number of states in each LL is still the same as NR e^-'s indeed.

$\Rightarrow$ Result is $\varepsilon_n = \mathrm{sgn}(n)\sqrt{2|n|}\, v/\ell$

$\ell = \sqrt{\frac{\hbar}{eB}}$ magnetic length.

Recall usual LLs $\varepsilon_n = (n + 1/2)\hbar\omega_c$ $\qquad \omega_c = \frac{eB}{m}$

$eB = \frac{\hbar}{\ell^2}$ $\qquad \hbar\omega_c = \frac{\hbar eB}{m} = \frac{\hbar^2}{m\ell^2}$

Basic facts:

- filling 1 LL changes σ_{xy} by e^2/h
- At $\varepsilon_F = 0$, p/h symmetry requires $\sigma_{xy} = 0$
- Each LL is $2_{\text{spin}} \times 2_{\text{valley}} = 4$-fold degenerate.

We don't need to invoke this but we can.

Let $\sigma_{xy} = G\, e^2/h$

G = 0) p/h Symmetry $\nu = 0$

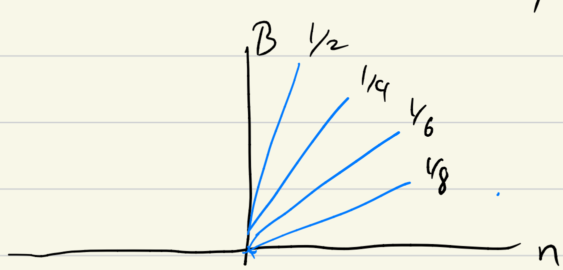
$\nu = 8$

$\nu = 4$

* Sketch Data from old expts.

* The unusual sequence is a sign of Dirac fermions.

Often experimenters plot "fan diagram" — minimum of ρ_{xx} as intensity versus B + n



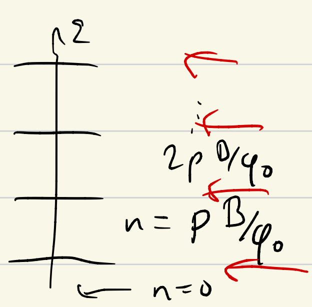
Max or Min?

Max ρ_{xx} → middle of LLs → shape.

$\nu = (n + \frac{1}{2}) \frac{\rho B}{\varphi_0}$

$n = 0, 1, 2, 3, \ldots$

Usual LLs



ρ_{xx} peaks between IQHE states of LLs