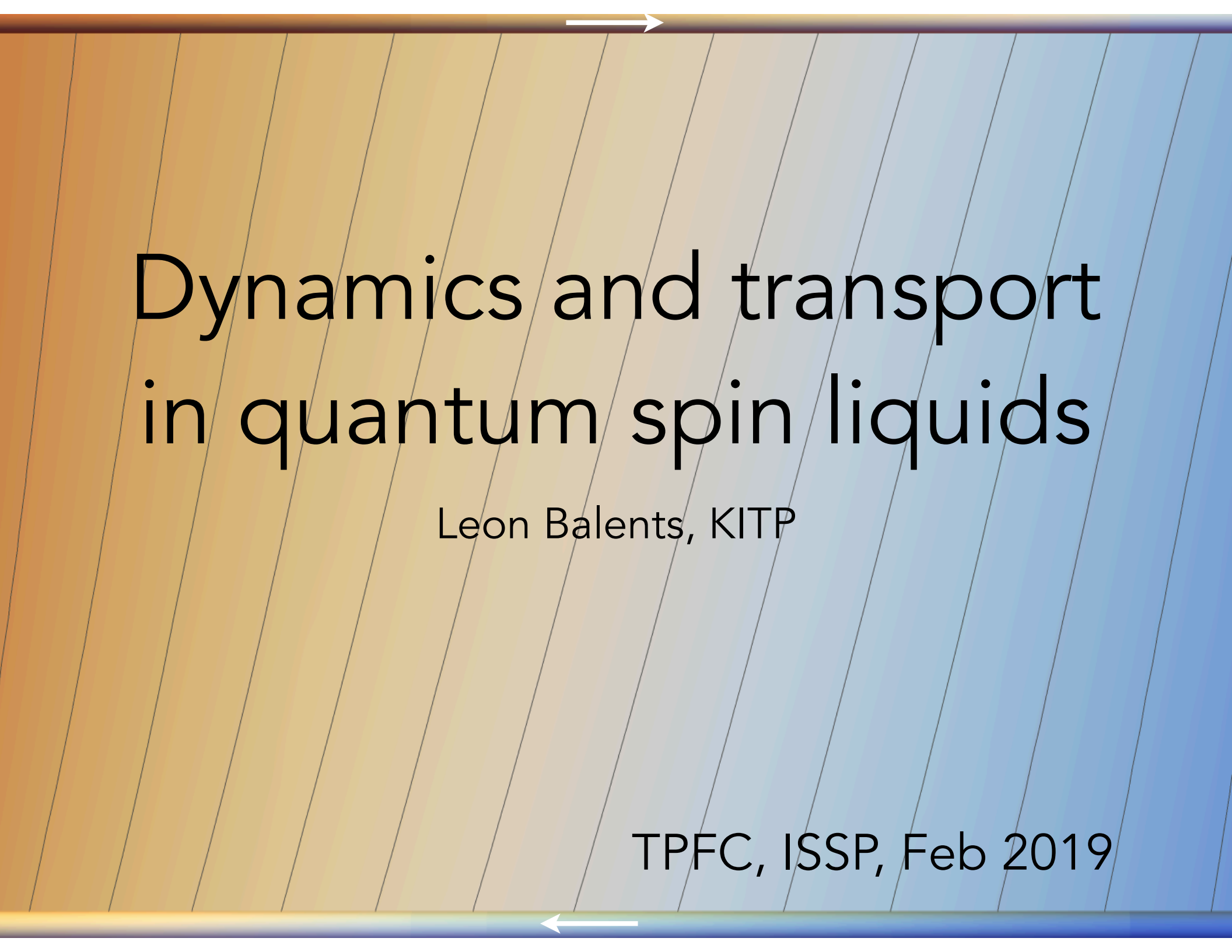


The background features a gradient of diagonal stripes transitioning from orange on the left to blue on the right. At the top, a white arrow points to the right, and at the bottom, a white arrow points to the left.

Dynamics and transport in quantum spin liquids

Leon Balents, KITP

TPFC, ISSP, Feb 2019

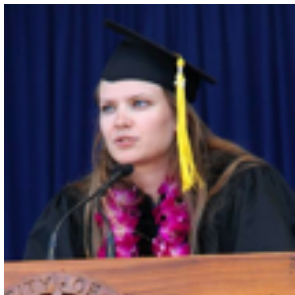
Collaborators



Mengxing Ye, UMn



Gábor Halász, ORNL



Lucile Savary, ENS Lyon



Oleg Starykh, U. Utah

Quantum Spin Liquid



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

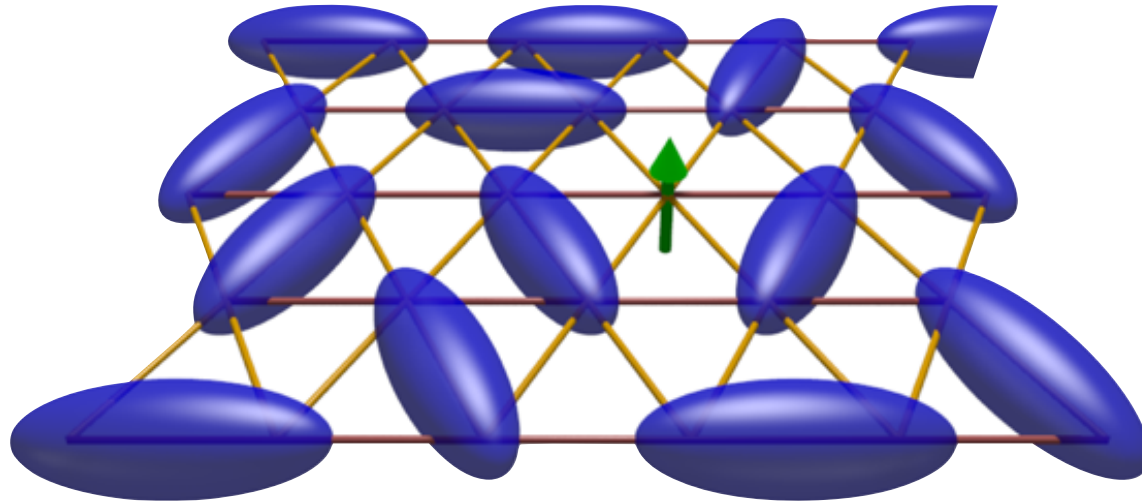
$$\Psi = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$

The diagram shows two 4x4 grids of triangles. In each grid, blue ovals are placed on the bonds between triangles. In the first grid, the ovals are arranged in a regular, repeating pattern. In the second grid, the ovals are arranged in a different, non-repeating pattern. The two grids are separated by a plus sign, and the entire expression is followed by a plus sign and three dots.

Resonating **V**alence **B**ond state

Fractional quantum number

spinon

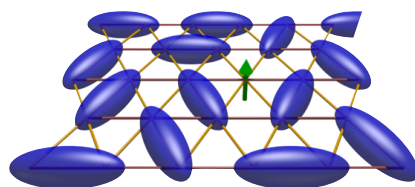


excitation with $\Delta S = 1/2$ not possible for any finite cluster of spins. May be a fermion

always created in pairs by any local operator

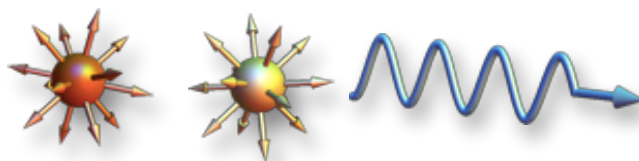
Classes of QSLs

- Topological QSLs



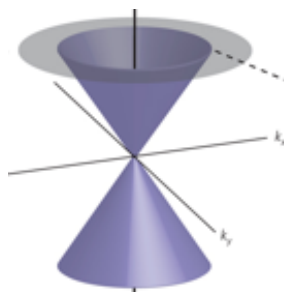
anyons,
spinons

- U(1) QSL



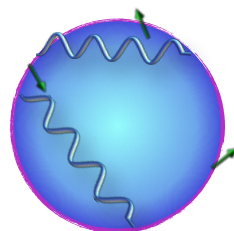
compact U(1)

- Dirac QSLs



QED₃

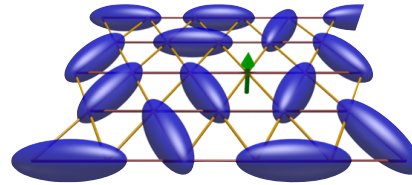
- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

Classes of QSLs

- Topological QSLs



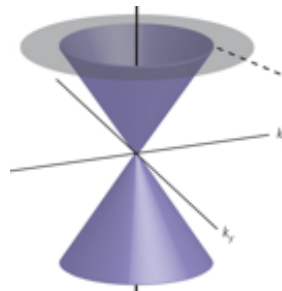
anyons,
spinons

- U(1) QSL



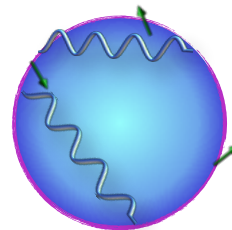
compact U(1)

- Dirac QSLs



QED₃

- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

Smoking Gun?

Difficult to find incontrovertible and sharp indicator of a QSL!

A few possibilities

- T-linear thermal conductivity in spinon Fermi surface QSL **observed!**
- Transverse, linearly dispersing emergent photon mode in 3d U(1) QSL **not yet observed**

Smoking Gun?

Difficult to find incontrovertible and sharp indicator of a QSL!

Some new ones:

- Quantized thermal Hall conductivity in Kitaev's non-abelian spin liquid **Matsuda**
- Silin spin wave and continuum in field-polarized spinon Fermi surface state **not yet observed**

Smoking Gun?

Difficult to find incontrovertible and sharp indicator of a QSL!

Some new ones:

- Quantized thermal Hall conductivity in Kitaev's non-abelian spin liquid
- Silin spin wave and continuum in field-polarized spinon Fermi surface state

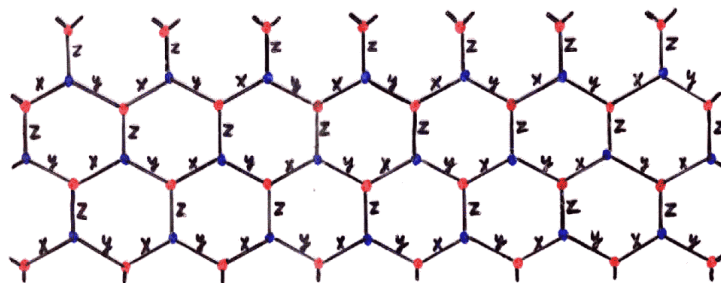
Matsuda

**not yet
observed**

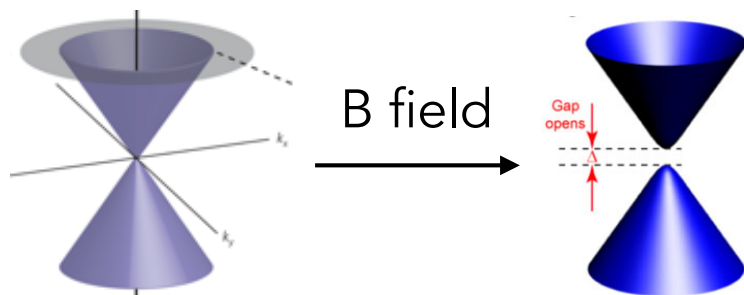
Kitaev Honeycomb



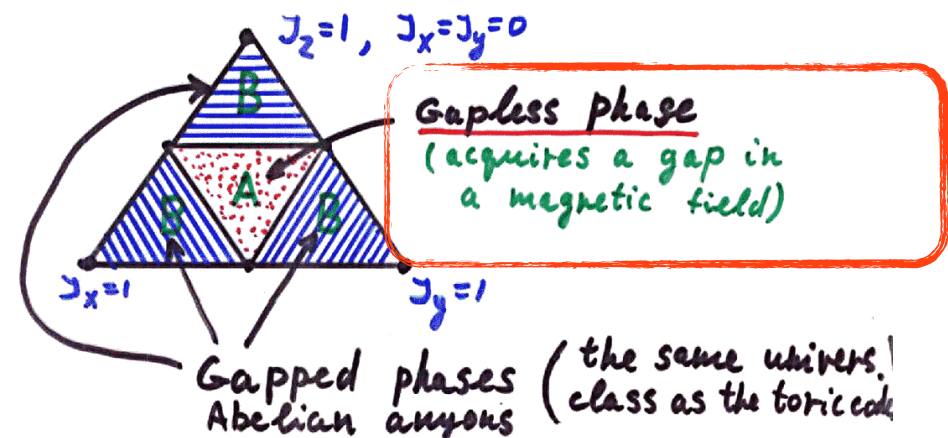
$$H = \sum_{i,\mu} iK_{\mu} c_i c_{i+\mu}$$



~1/4 of graphene

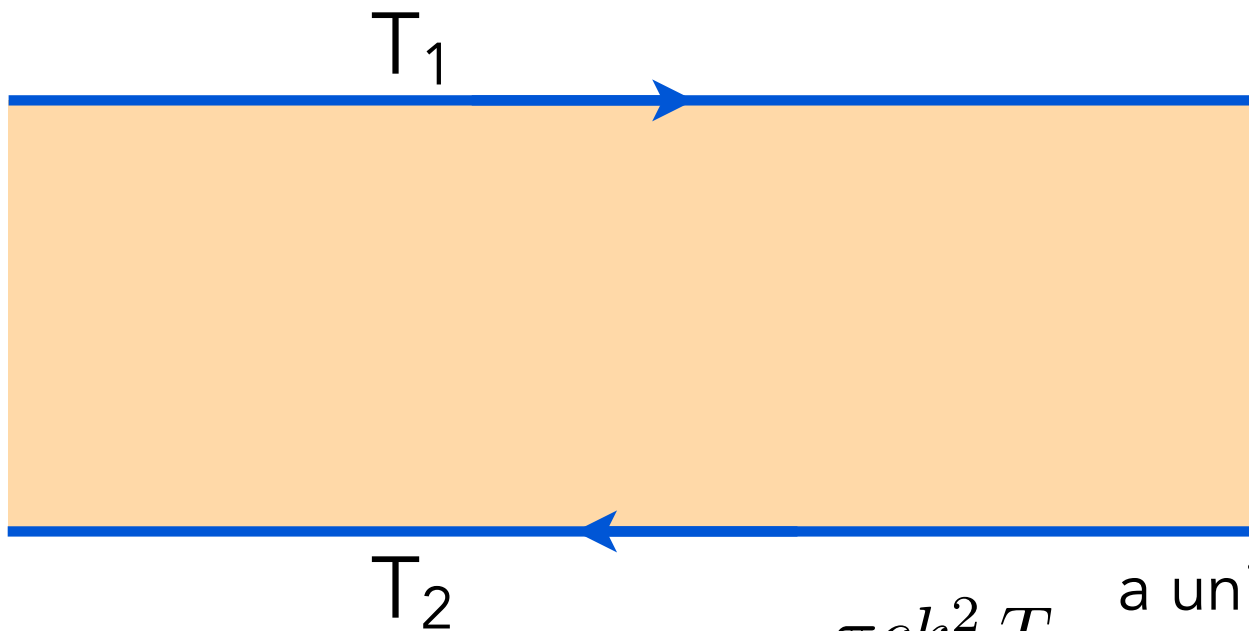


Phase diagram



Thermal Quantum Hall Effect

- No charge. Have to study heat transport!



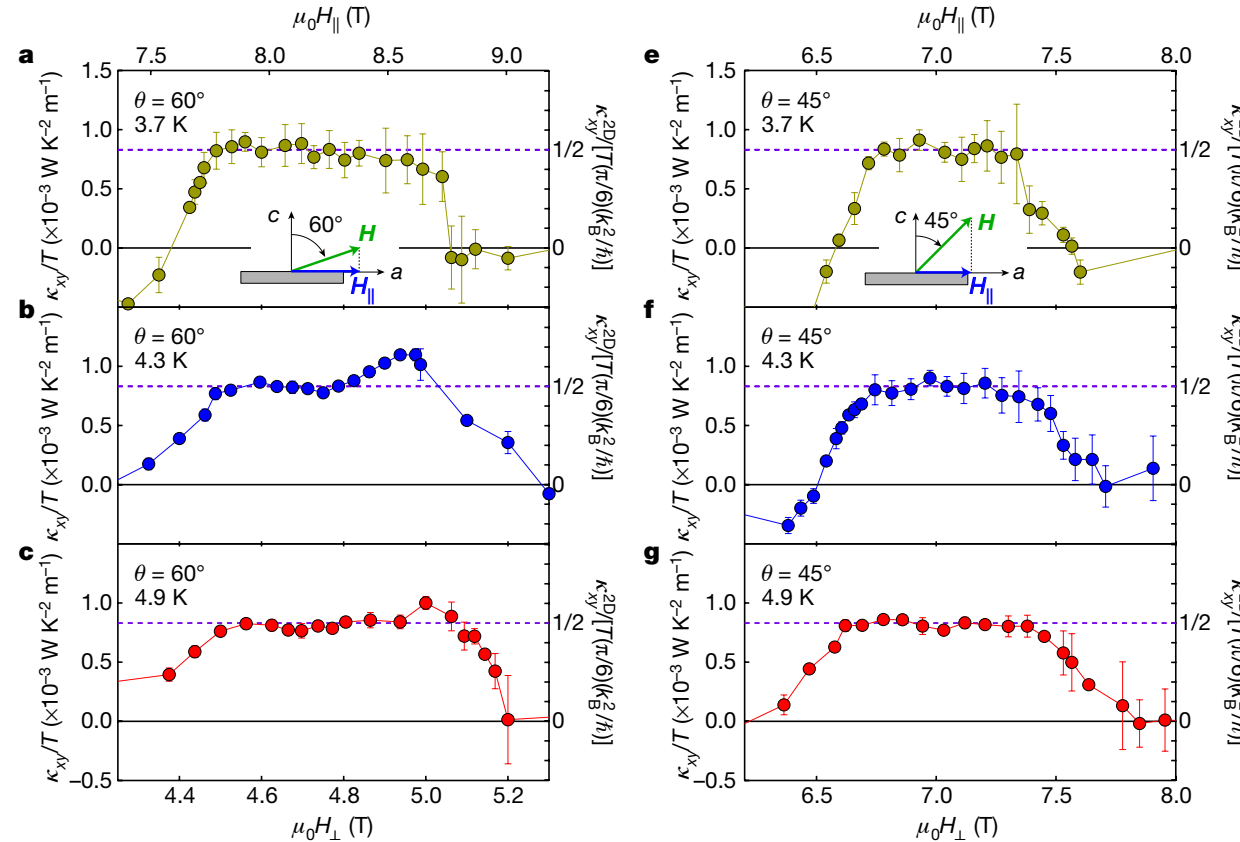
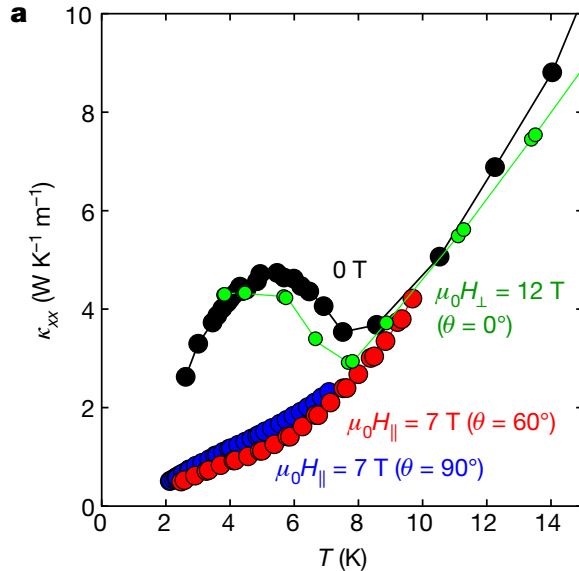
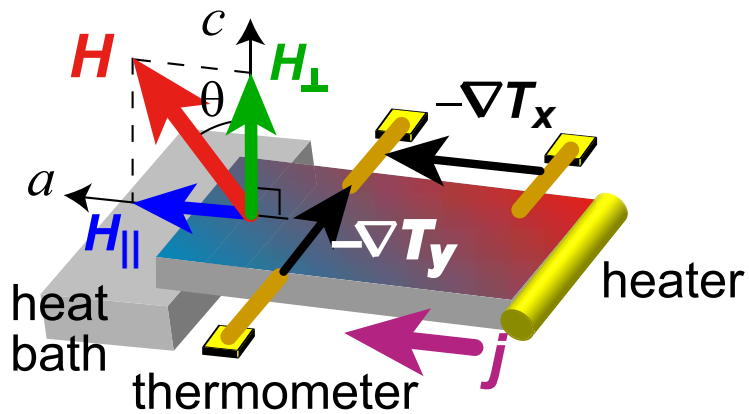
$$I_x = \kappa_H \Delta T_y$$

$$\kappa_H = \frac{\pi c k_B^2 T}{6\hbar}$$

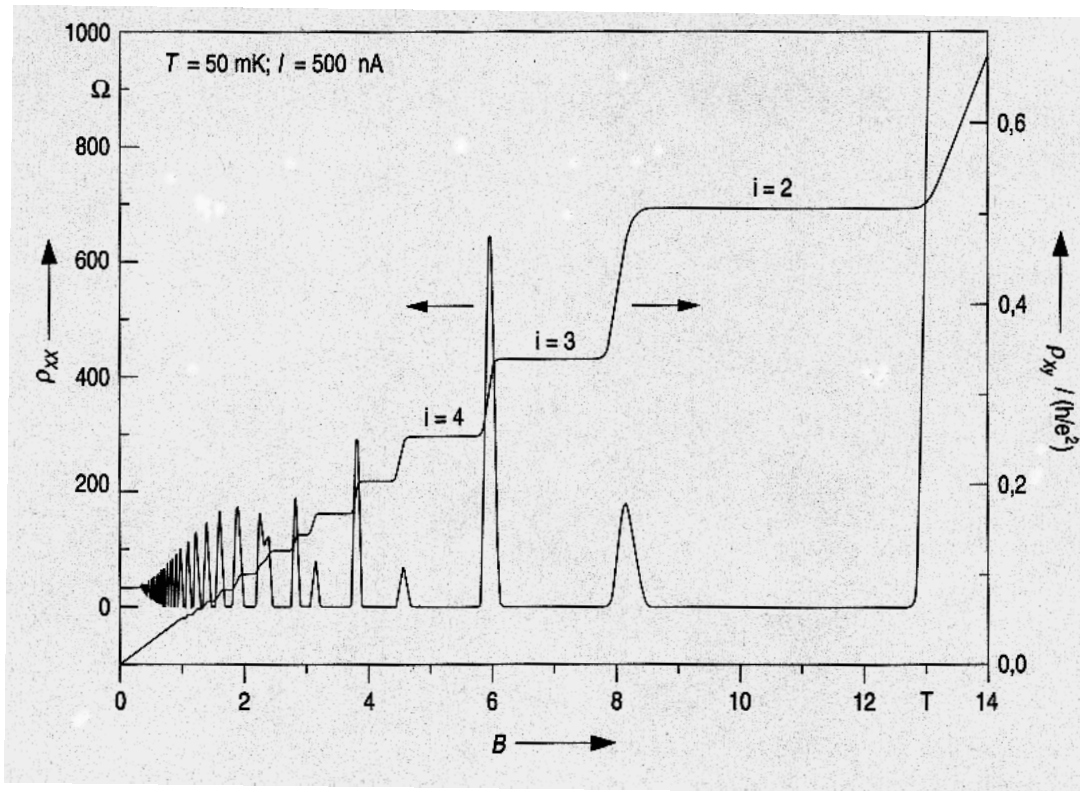
a universal prediction for chiral
"Ising anyon" phase: *agnostic to
microscopic spin interactions*

Thermal Hall Effect

Y. Kasahara *et al*, Nature 2018



Why worry?



c.f. IQHE

Quantization is *only* visible when diagonal conductivity is small, i.e. Hall angle $\sim 90^\circ$

semicircle "law"

$$\sigma_{xx}^2 + \left(\sigma_{xy} - \frac{\sigma_1 + \sigma_2}{2} \right)^2 = \left(\frac{\sigma_1 - \sigma_2}{2} \right)^2$$

But for RuCl_3 , thermal Hall angle $\sim 10^{-3} !!$

Theoretical Problem

From Y. Kasahara *et al*

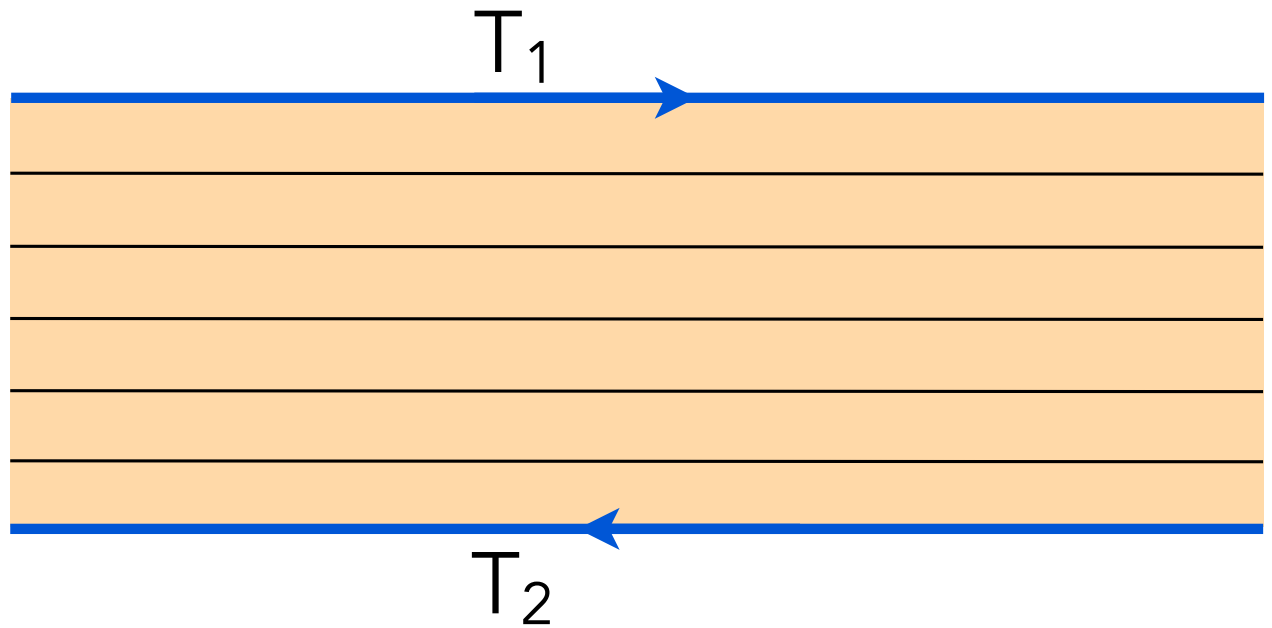
In the plateau regime of κ_{xy} , no anomaly is observed in κ_{xx} , probably because **phonon contributions largely dominate** over fermionic excitations arising from spins in κ_{xx} in the whole temperature range^{28,29}.

Q: Can quantized thermal Hall effect persist when phonons dominate bulk conductivity?

A: **Yes**, and phonons actually *help* to make the effect observable.

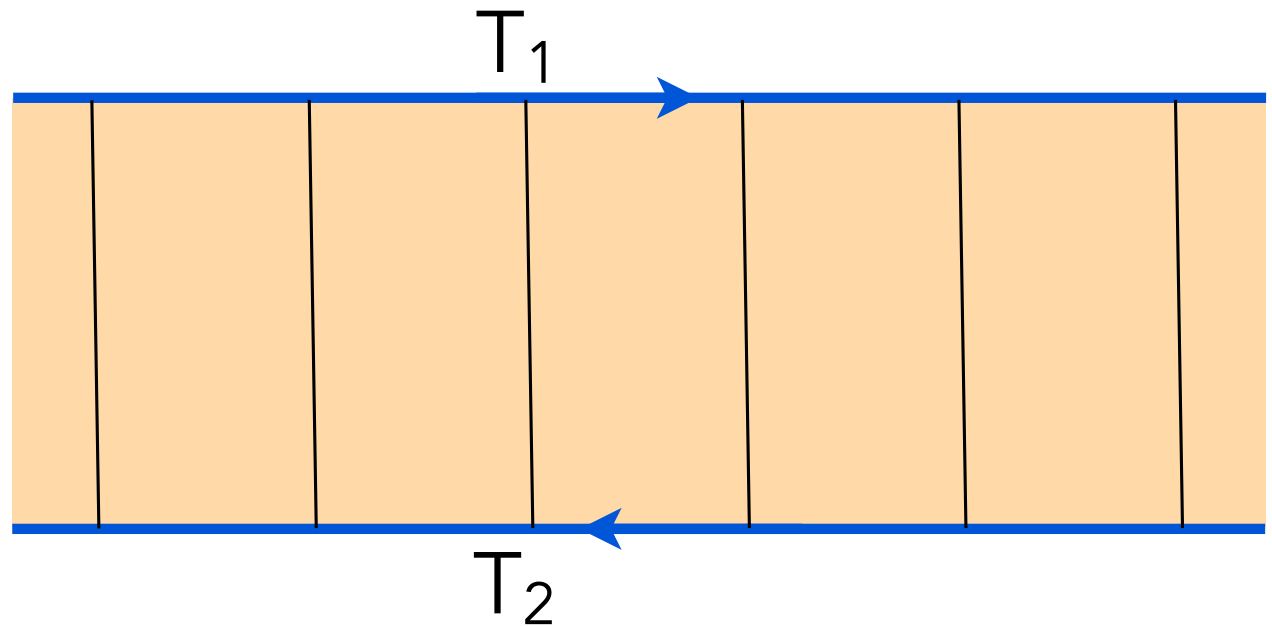
Theoretical Problem

Recall
derivation of
QTHE



Theoretical Problem

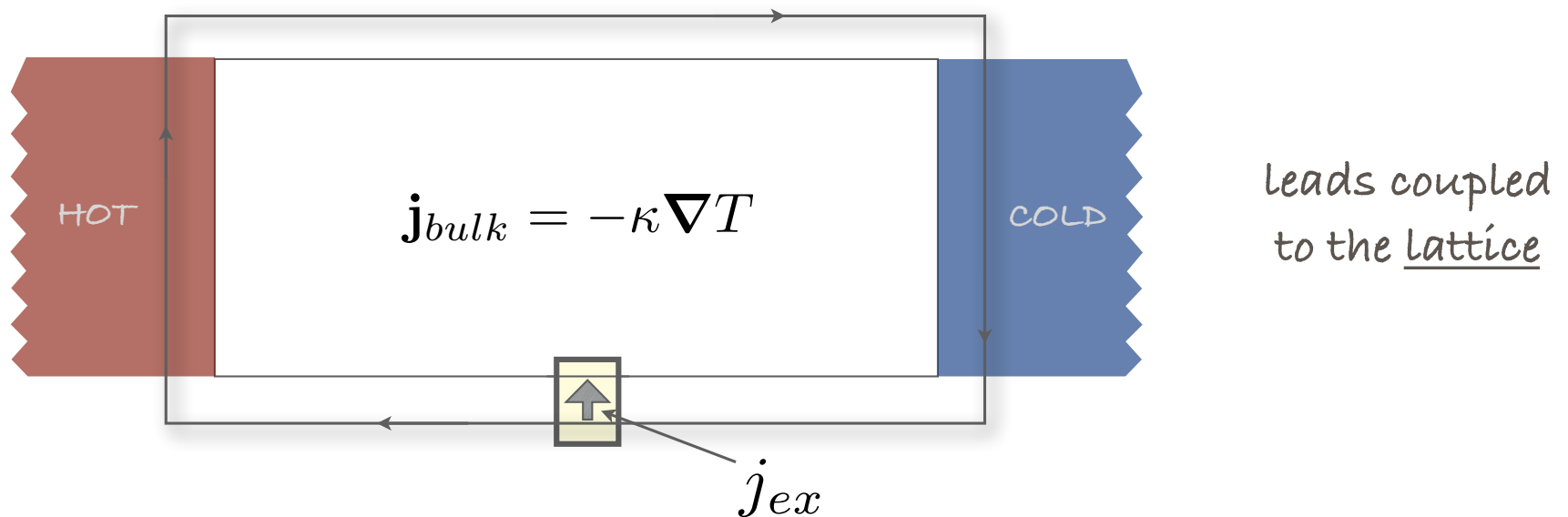
Recall
derivation of
QTHE



$\kappa_{xx} \gg \kappa_{xy}$: Temperature of phonons is *not* constant at the edge
?? Could edge be out of equilibrium with the phonons and have constant T ??

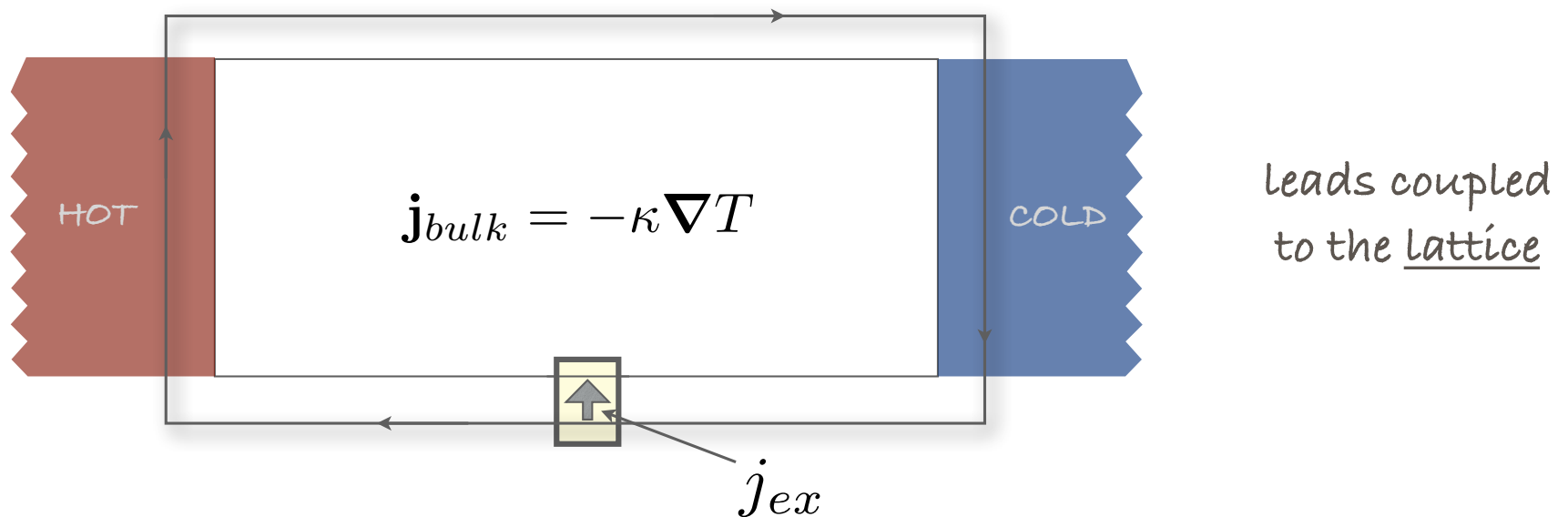
- but: in that case, which T is measured?

Formulation



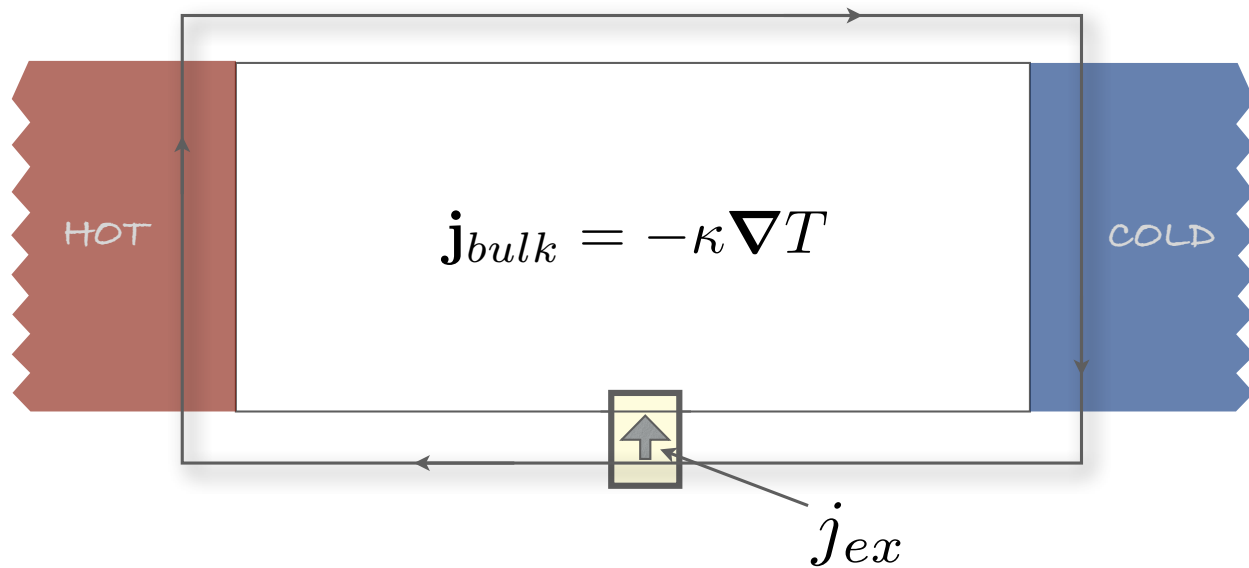
- Assume spins go into nonabelian QSL phase: bulk spin gap, chiral edge state
- Include bulk phonons with *no* Hall conductivity
- Majoranas and phonons can exchange energy at the edge of the sample

Formulation



- Variables:
 - $T_{ph}(x,y)$ = phonon temperature in bulk
 - $T_f(x,L_y)$ etc = fermion temperature at edge

Formulation



leads coupled
to the lattice

- Currents:

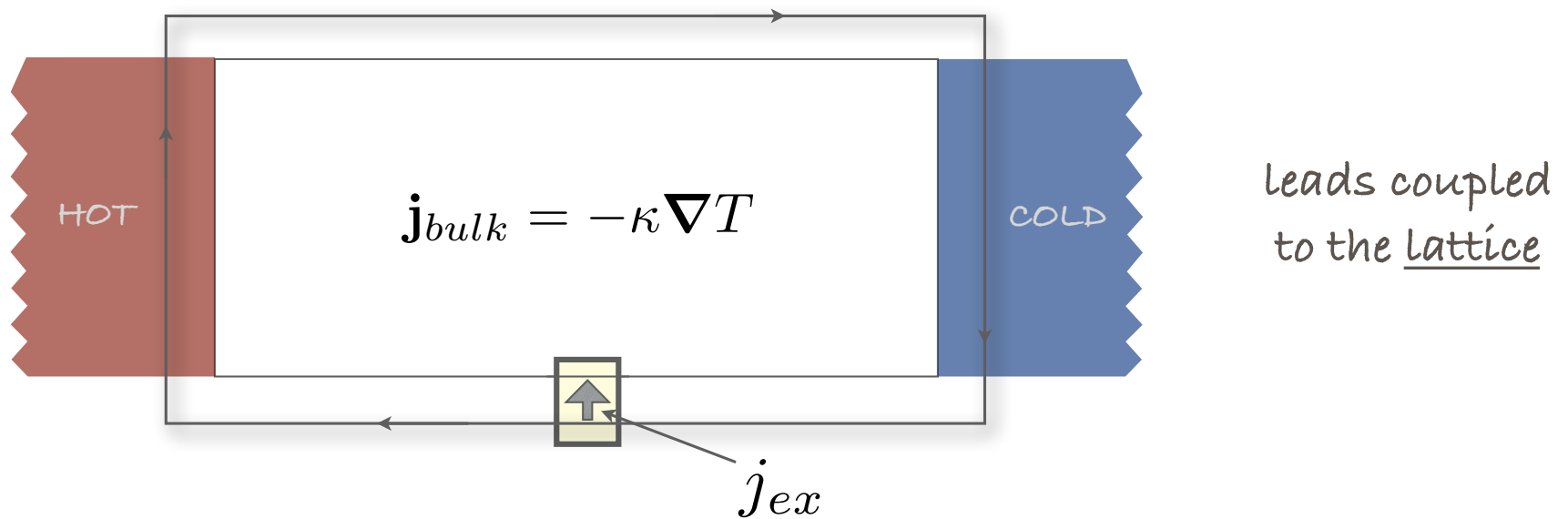
- Bulk

$$\mathbf{j}_{ph} = -\kappa \nabla T_{ph}$$

- Edge

$$I_f = \frac{\pi}{24} T_f^2$$

Formulation

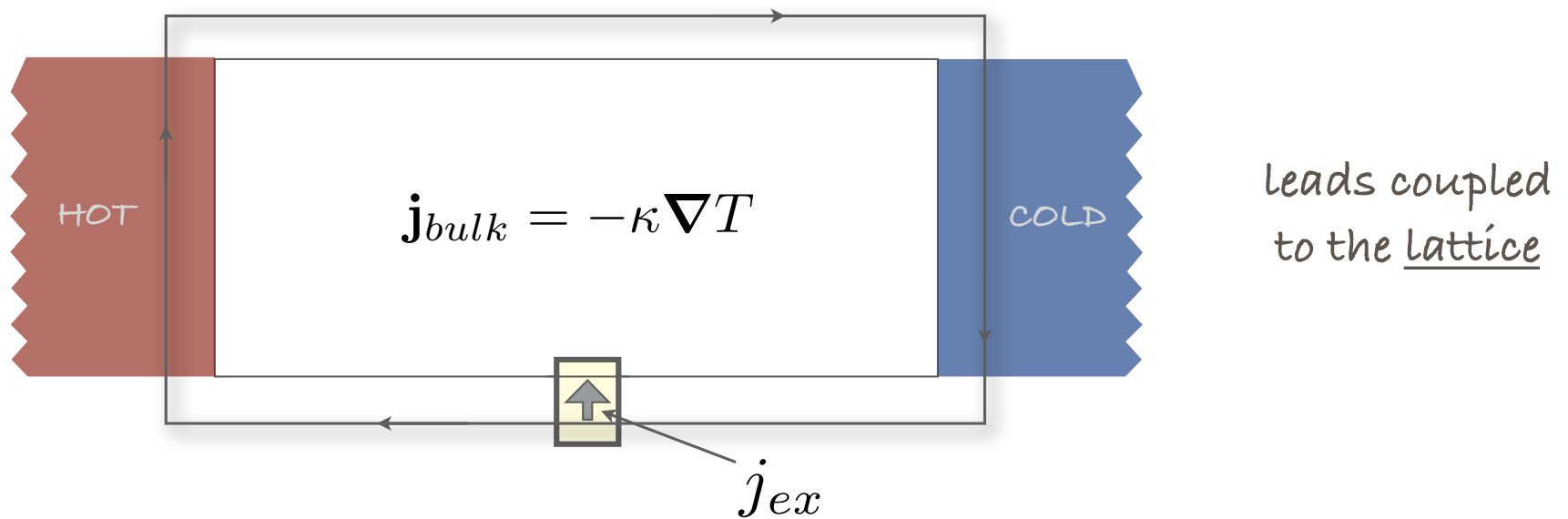


- Continuity:

- Bulk $\nabla \cdot \mathbf{j}_{ph} = 0$

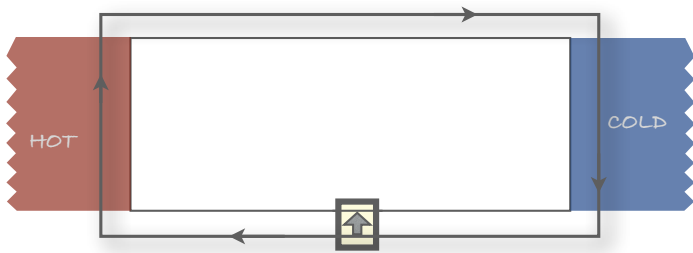
- Edge $\partial_x I_f = \pm j_{ex} , \quad \pm j_{ph}^y(x, \pm y_0) = j_{ex}(x, \pm y_0).$

Formulation



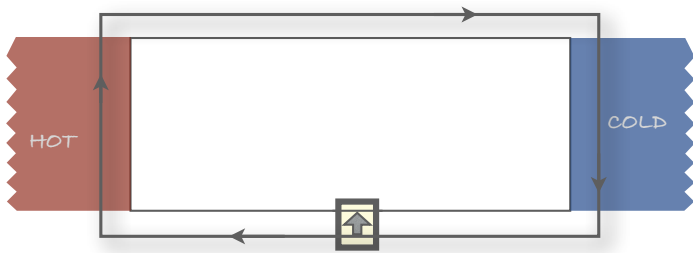
- Coupling:
 - linear response $j_{ex} = \lambda(T)(T_{ph} - T_f)$

Formulation



- Full equations (for *infinite* Hall bar)
 - bulk $\nabla^2 T_{ph} = 0$
 - edge $\kappa_{xy}^q \partial_x T_f(x, \pm y_0) = -\kappa \partial_y T_{ph}(x, \pm y_0)$
 $= \pm \lambda(T) (T_{ph} - T_f)|_{x, \pm y_0}$

Solution



- Solution

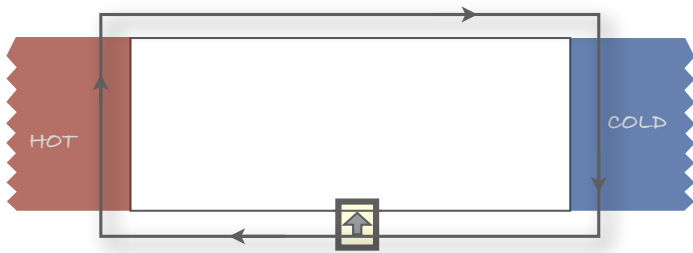
- Bulk $T_{ph}(x, y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$

- edge $T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$

$$\frac{\partial T_{ph}}{\partial y} = -\frac{\kappa_{xy}^q}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

Solution



- Solution

- Bulk $T_{ph}(x, y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$

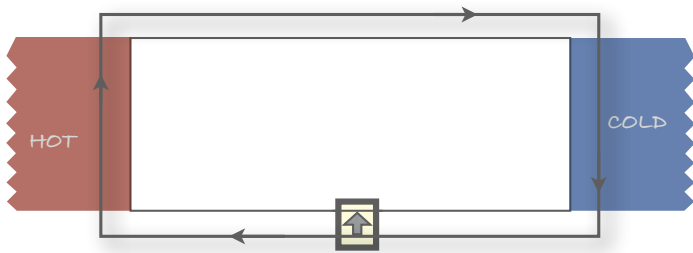
- edge $T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$

$$\frac{\partial T_{ph}}{\partial y} = - \frac{\kappa_{xy}^q}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

phonon Hall angle corresponds to quantization! **Implies**
quantization in 3 probe experiment measuring T_{ph}

Solution



- Solution

- Bulk $T_{ph}(x, y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$

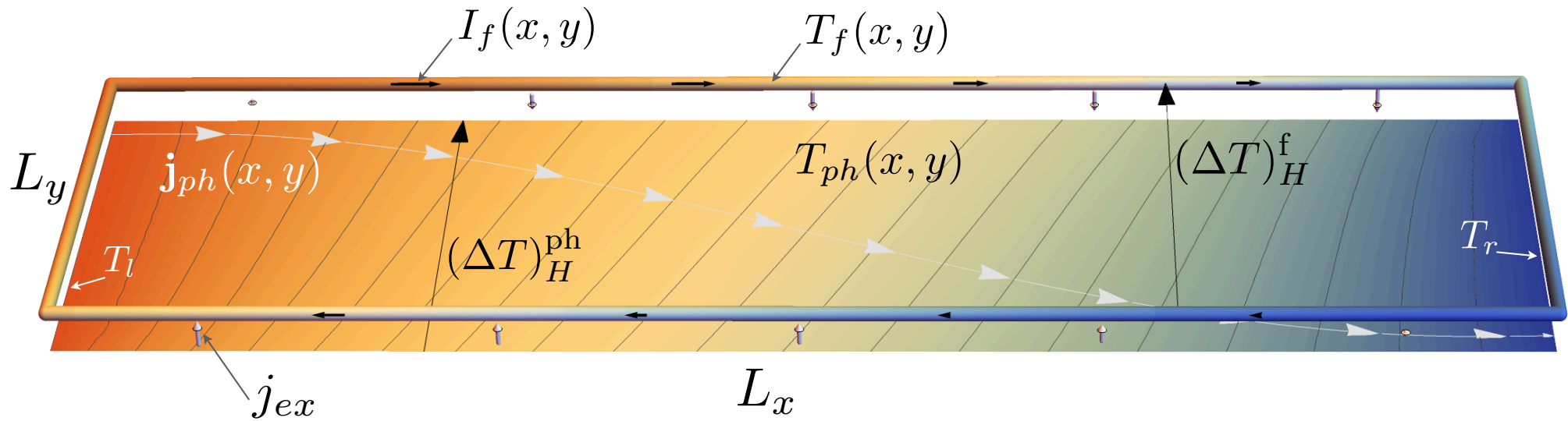
- edge $T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$

$$\frac{\partial T_{ph}}{\partial y} = -\frac{\kappa_{xy}^q}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

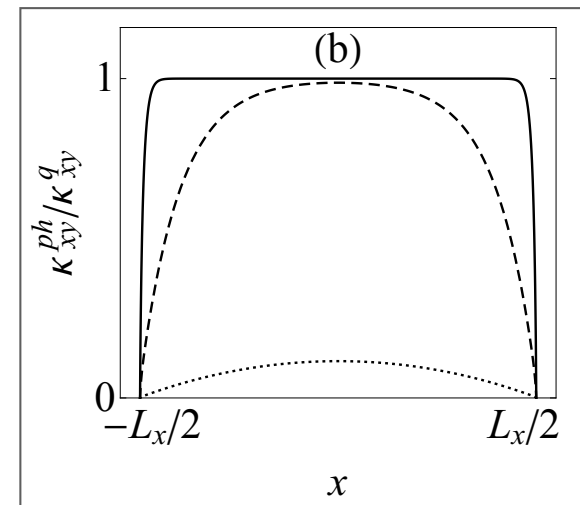
Majorana temperature *not* equal to phonon one. It is actually *better* to measure phonon temperature!

Finite Hall bar



- Solve coupled Laplace and boundary equations, with constant temperature leads.
- Reveals dependence on "thermalization length"

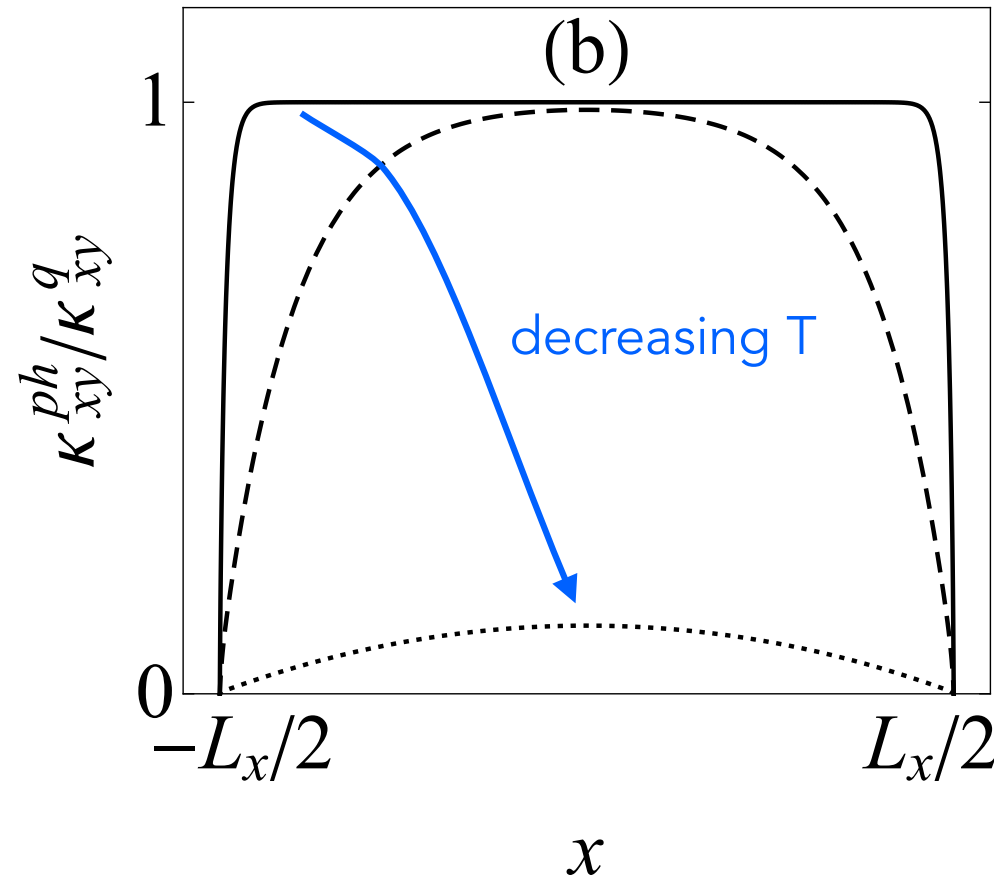
$$\ell_x = \frac{\kappa_{xy}^q}{\lambda}$$



$$L_y/L_x = 0.01$$

$$\ell_x/L_x = \underline{0.01}, \quad \text{---} 0.1, \quad \cdots 1$$

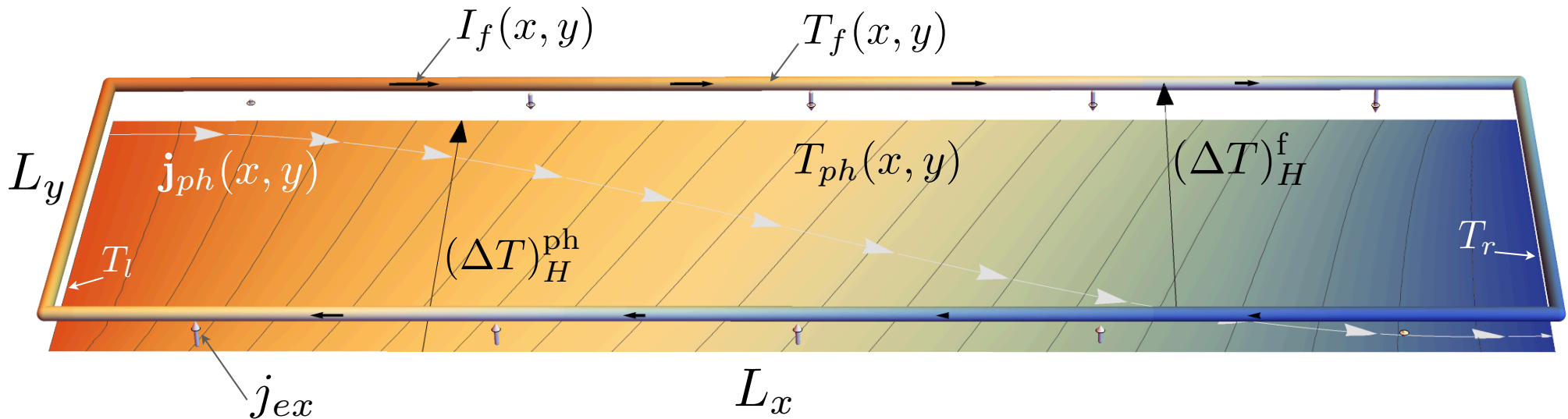
Bulk-edge decoupling



$$\lambda \sim T^6$$

Expect *failure* of quantization at low enough temperature!

Summary



- Quantized value of thermal Hall conductivity can be measured even when large phonon conductivity is present
- Quantization is only power-law good, and deviations are controlled by edge-bulk equilibration

M. YE ET AL, PRL 2018

see also: Y. WINKLER-AVIV, PRX 2018

Smoking Gun?

Difficult to find incontrovertible and sharp indicator of a QSL!

Some new ones:

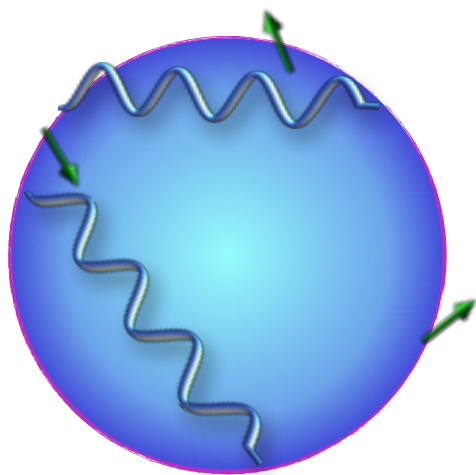
- Quantized thermal Hall conductivity in Kitaev's non-abelian spin liquid
- Silin spin wave and continuum in field-polarized spinon Fermi surface state

Matsuda

**not yet
observed**

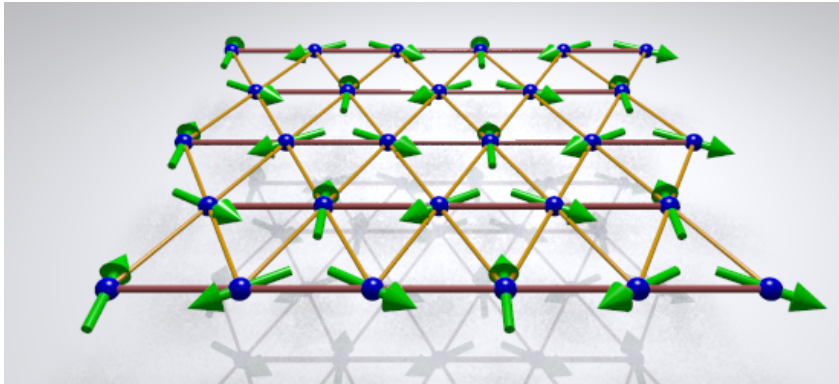
Spinon Fermi surface

$$|\Psi\rangle = \prod_i \hat{n}_i (2 - \hat{n}_i) \prod_{k < k_F} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$

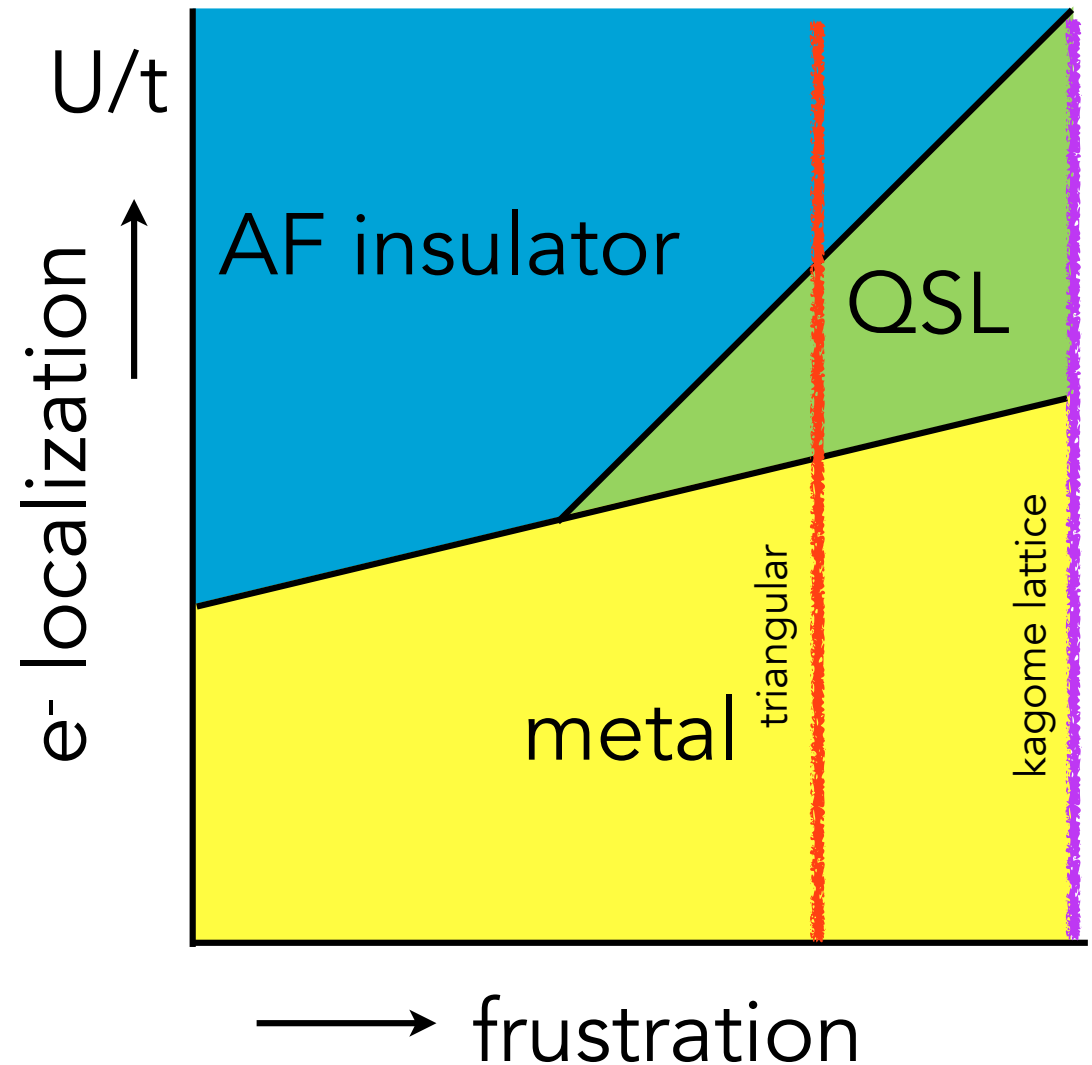


- The most gapless/highly entangled QSL state
- Like a “metal” of neutral fermions w/ a U(1) gauge field
- Prototype “non-Fermi liquid” state of great theoretical interest

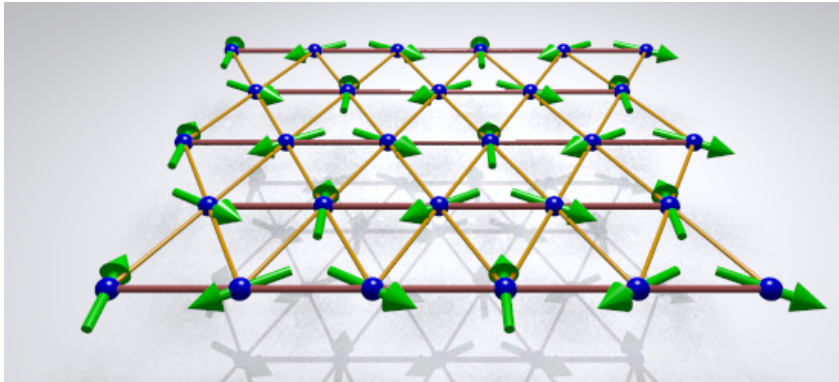
Triangular lattice w/ ring exchange



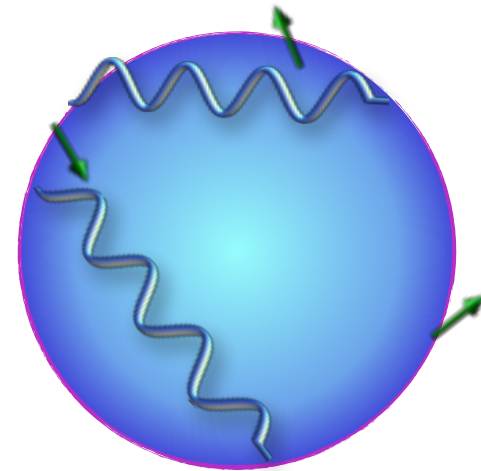
- Motrunich (2005): ring exchange stabilizes a spin liquid



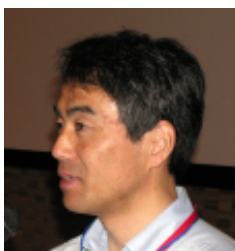
Triangular lattice w/ ring exchange



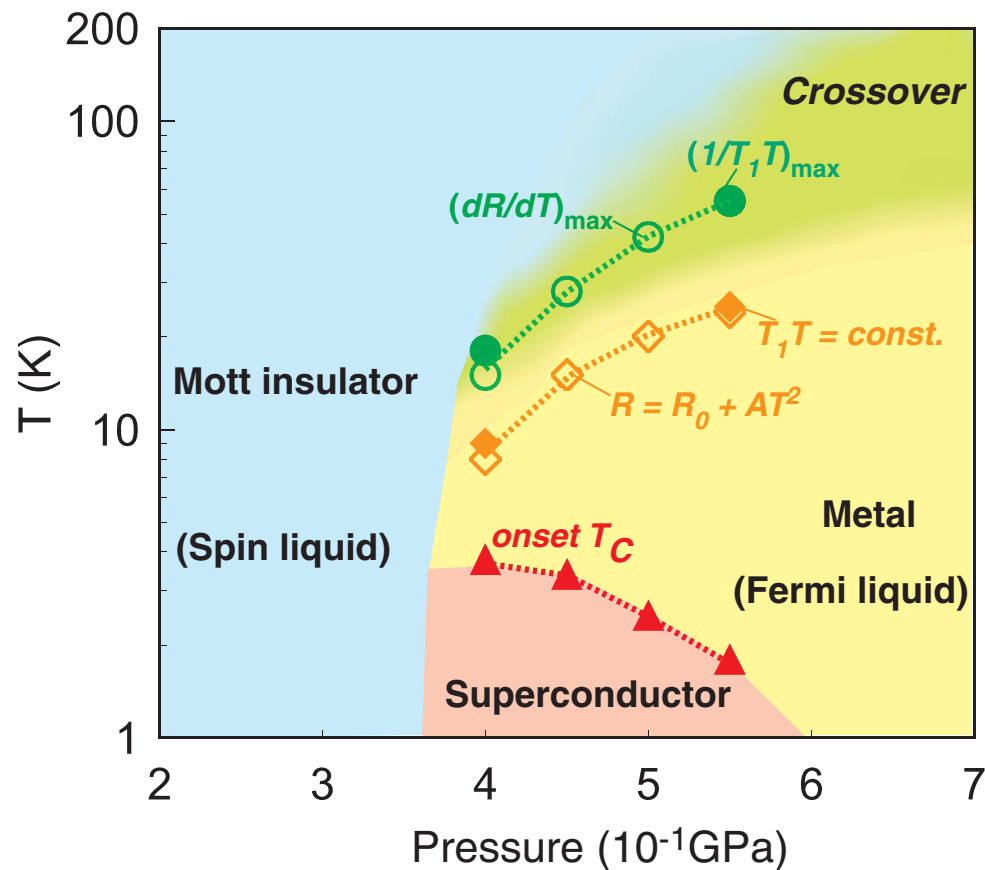
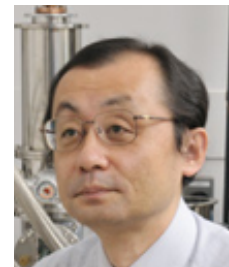
- Motrunich (2005): ring exchange stabilizes a spin liquid



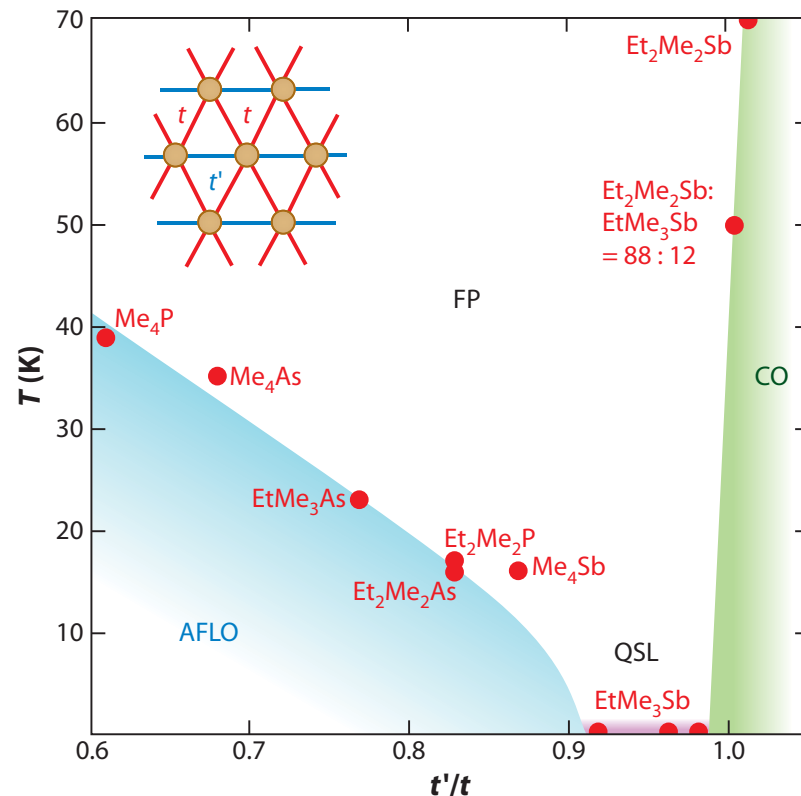
- Motrunich, Lee/Lee: spin liquid state favored by ring exchange is the "spinon Fermi sea" state



triangular organics



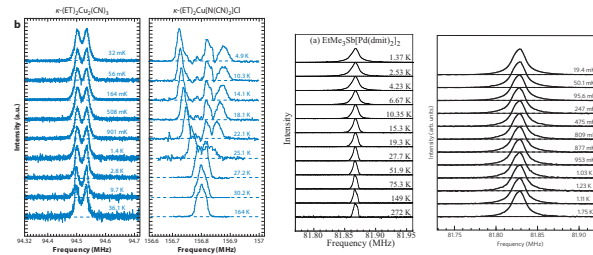
K. Kanoda group (2003-)



R. Kato group (2008-)

Evidence

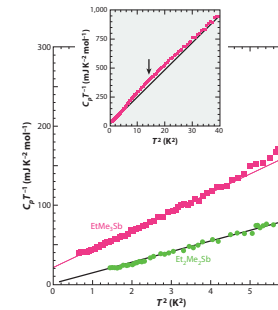
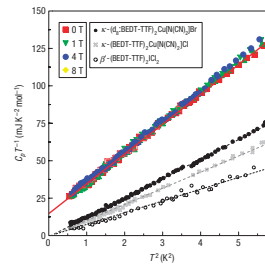
- NMR



no magnetic order

Y. Shimizu *et al*, 2003 T. Itou *et al*, 2008,2010

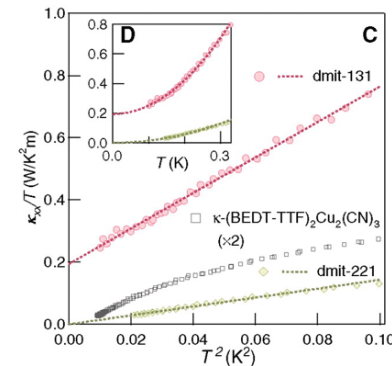
- Specific heat



Sommerfeld law

S. Yamashita *et al*, 2008

- Thermal conductivity

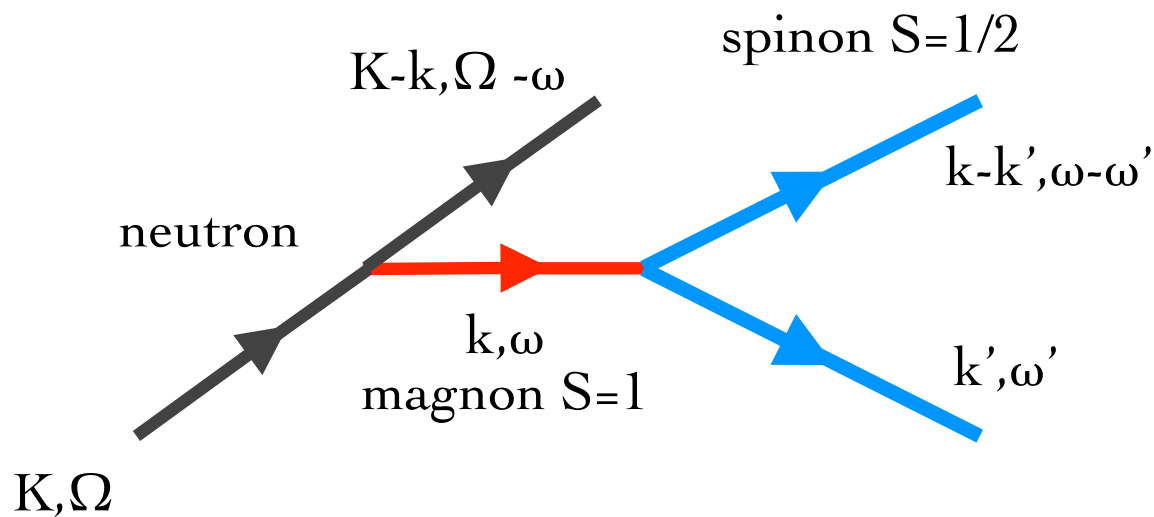


*itinerant
fermions?*

M. Yamashita *et al*, 2010

Spectra

Common expectation in a QSL:
broad continuum scattering

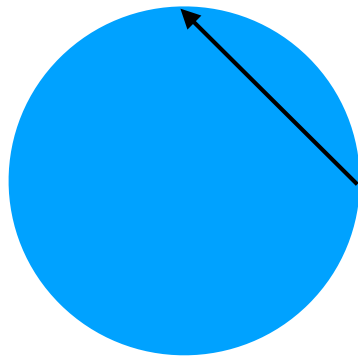


broad peak with
 $\omega = \epsilon(k') + \epsilon(k-k')$

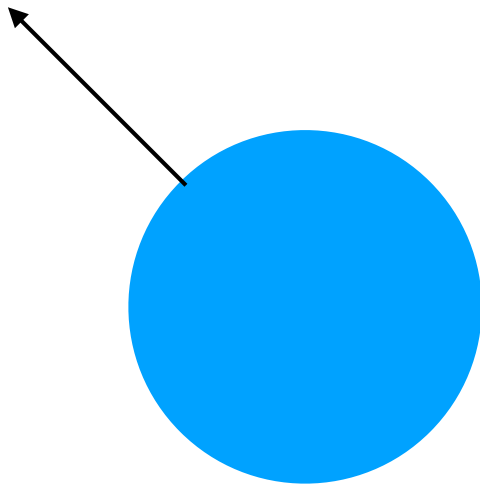
Not very inspiring?

particle-hole continuum

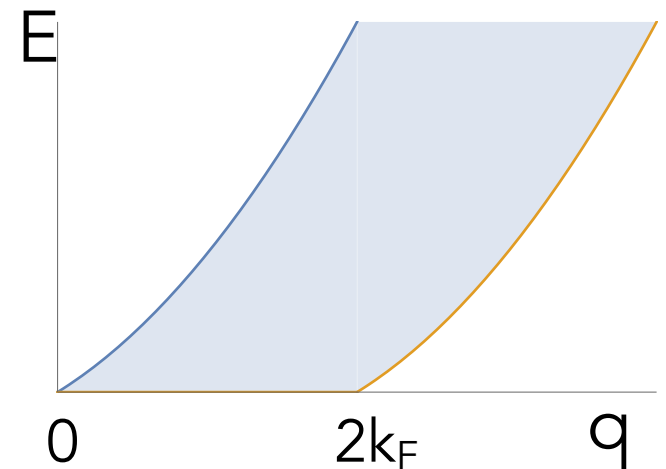
Free fermions



lowest energy for $k < k_F$



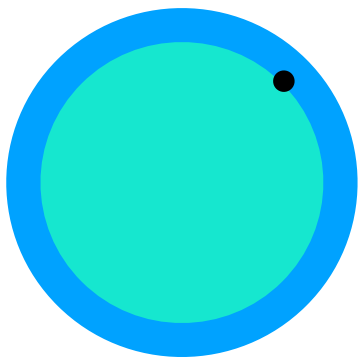
maximum energy



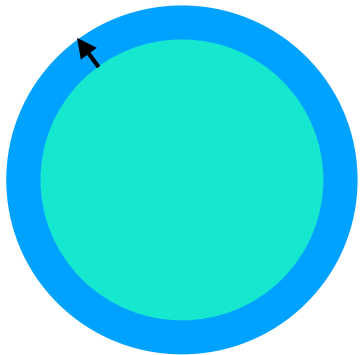
particle-hole continuum

With Zeeman field

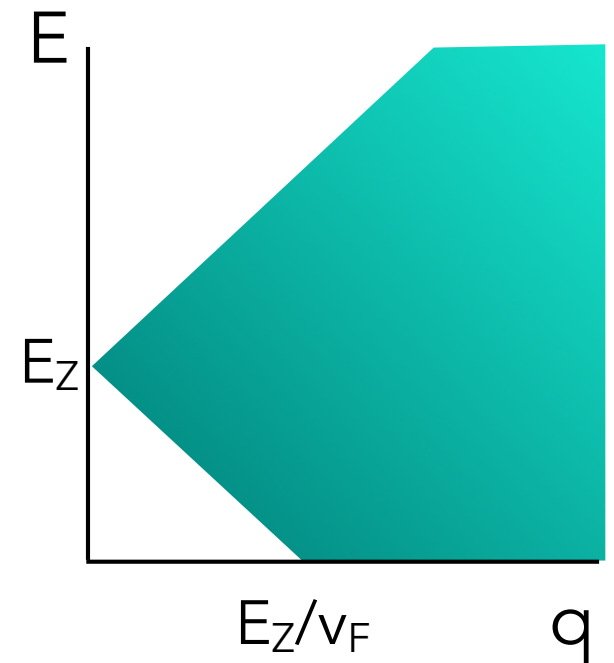
$$\chi_{\pm}(\mathbf{q}, \omega) = i \int_0^{\infty} dt \langle [S_{\mathbf{q}}^{\dagger}(t), S_{-\mathbf{q}}^{-}(0)] \rangle e^{i\omega t}$$



$q=0$ costs Zeeman energy



zero energy when $v_F q$
= Zeeman



Inorganic analogs?

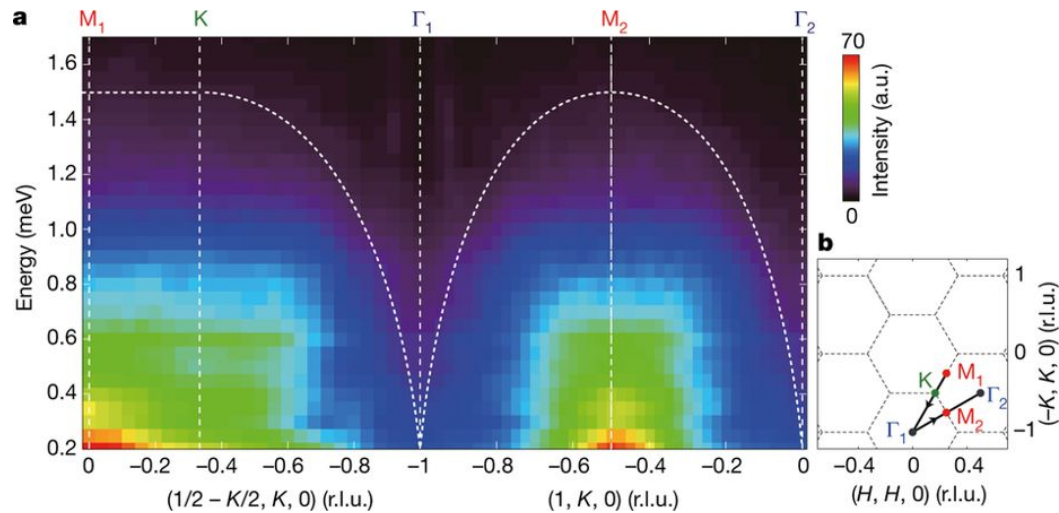
YbMgGaO₄

Letter | Published: 05 December 2016

Evidence for a spinon Fermi surface in a triangular-lattice quantum-spin-liquid candidate

Yao Shen, Yao-Dong Li, Hongliang Wo, Yuesheng Li, Shoudong Shen, Bingying Pan, Qisi Wang, H. C. Walker, P. Steffens, M. Boehm, Yiqing Hao, D. L. Quintero-Castro, L. W. Harriger, M. D. Frontzek, Lijie Hao, Siqin Meng, Qingming Zhang, Gang Chen & Jun Zhao

Nature **540**, 559–562 (22 December 2016) | [Download Citation](#)

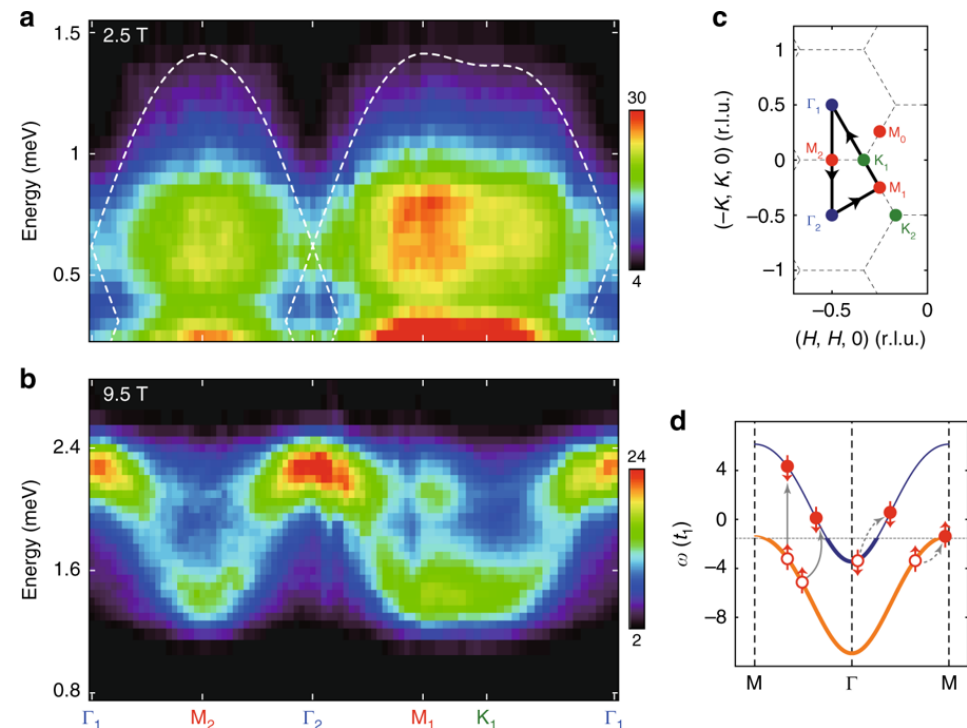


Article | OPEN | Published: 08 October 2018

Fractionalized excitations in the partially magnetized spin liquid candidate YbMgGaO₄

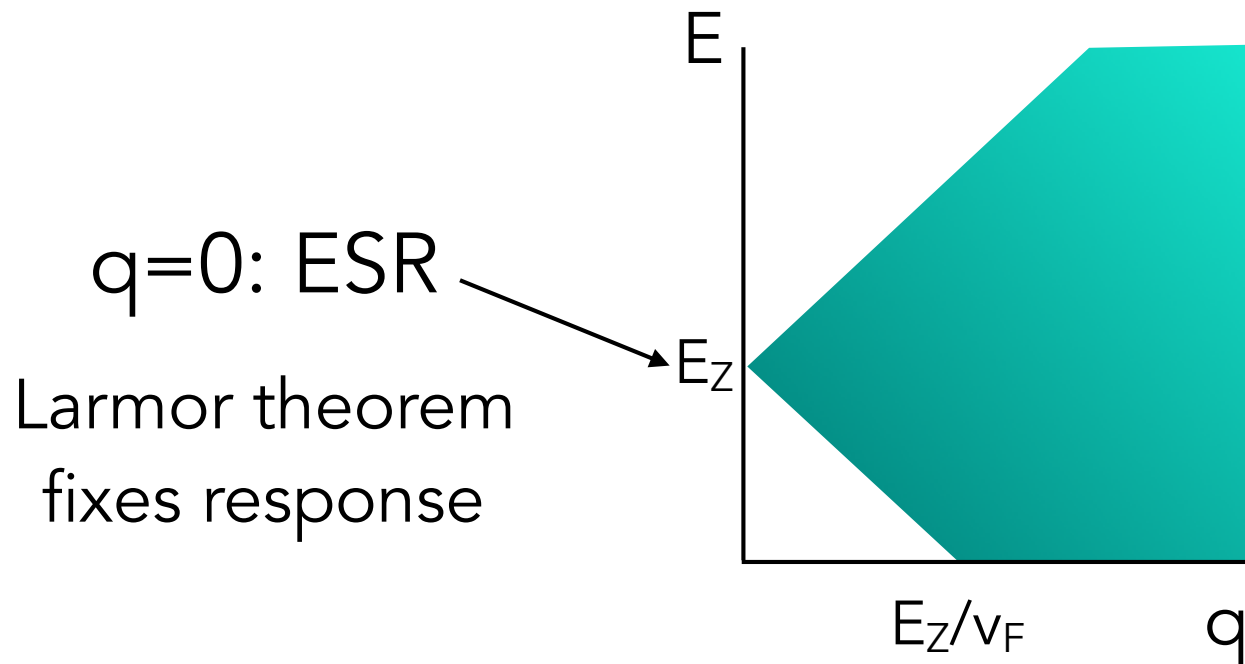
Yao Shen, Yao-Dong Li, H. C. Walker, P. Steffens, M. Boehm, Xiaowen Zhang, Shoudong Shen, Hongliang Wo, Gang Chen & Jun Zhao

Nature Communications **9**, Article number: 4138 (2018) | [Download Citation](#)



particle-hole continuum

With Zeeman field



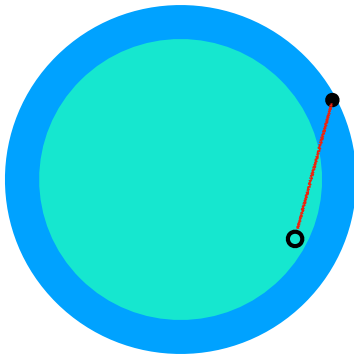
Effects of interactions?

Interactions

- Longitudinal

$$a_0 \psi^\dagger \psi$$

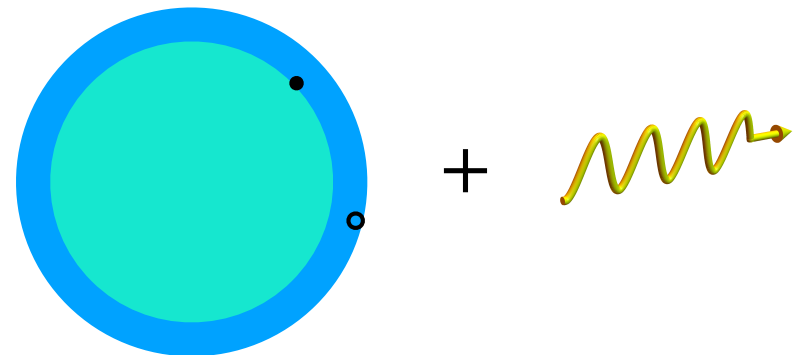
screened Coulomb
interaction



- Transverse

$$i\mathbf{A} \cdot (\psi^\dagger \nabla \psi - \nabla \psi^\dagger \psi)$$

coupling to dynamical
photons



Interactions

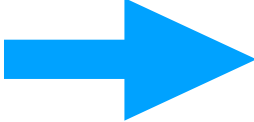
• Longitudinal $a_0 \psi^\dagger \psi \quad \longrightarrow \quad u \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow$

$$= -um \left(\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow \right) + u : \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow :$$

self-energy

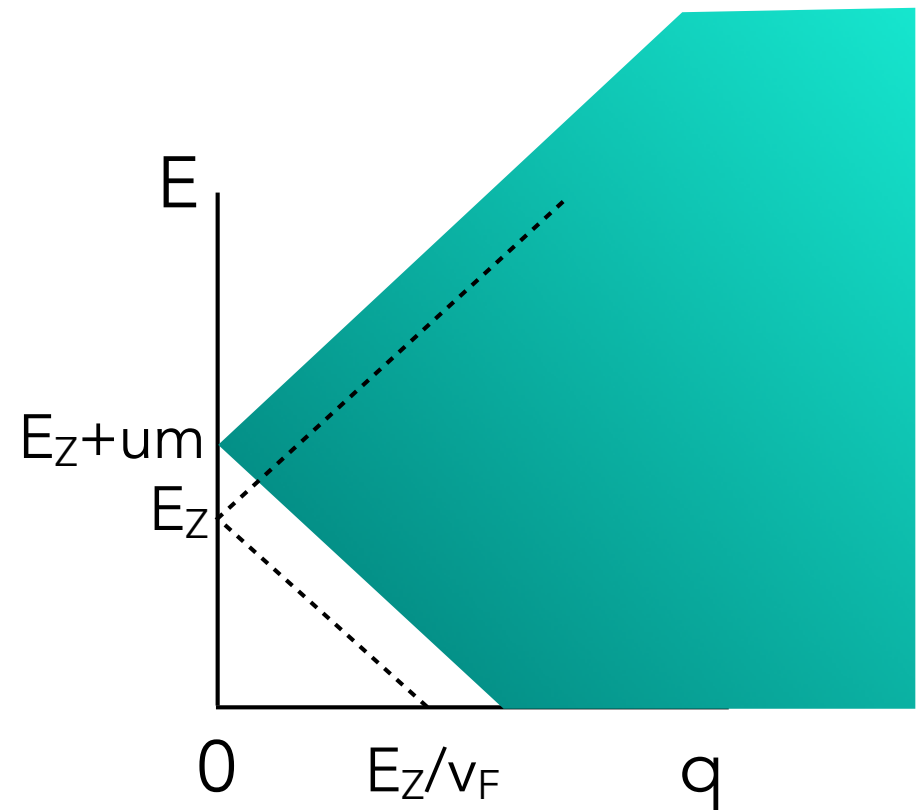
interaction

Self energy

- Longitudinal $a_0 \psi^\dagger \psi$  $u \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow$

$$= -um \left(\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow \right)$$

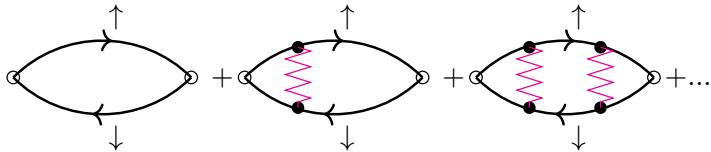
mean field shift



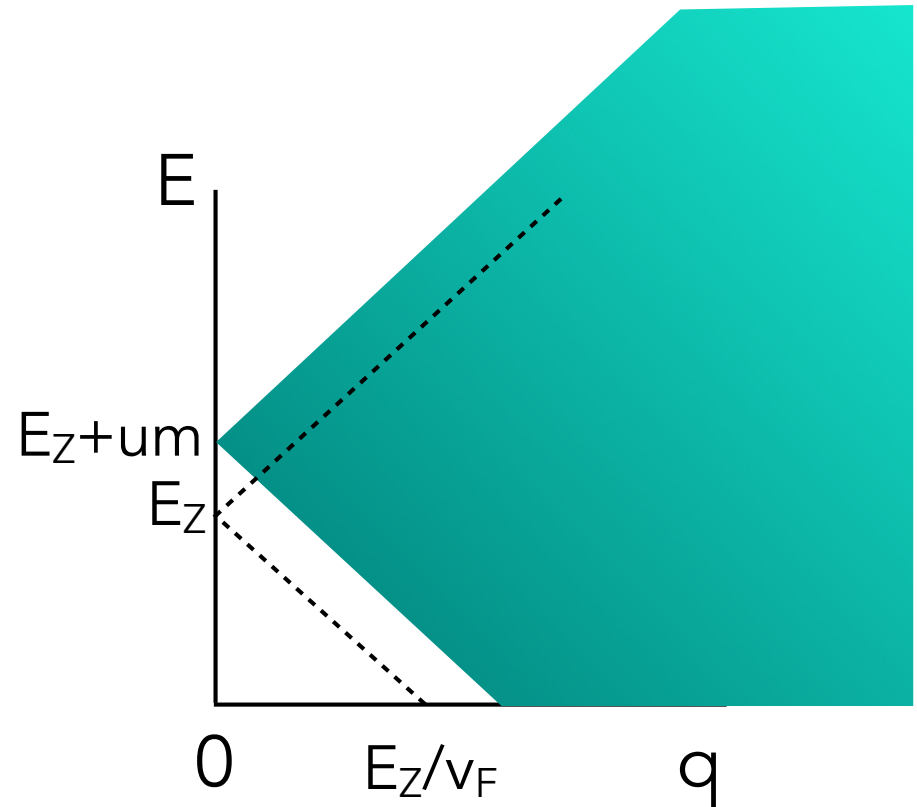
Interaction

Larmor theorem: $q=0$
excitation *must* be at E_Z

RPA



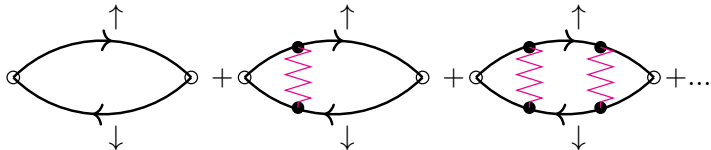
$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi^0(\mathbf{q}, i\omega_n)}{1 + u\chi^0(\mathbf{q}, i\omega_n)}$$



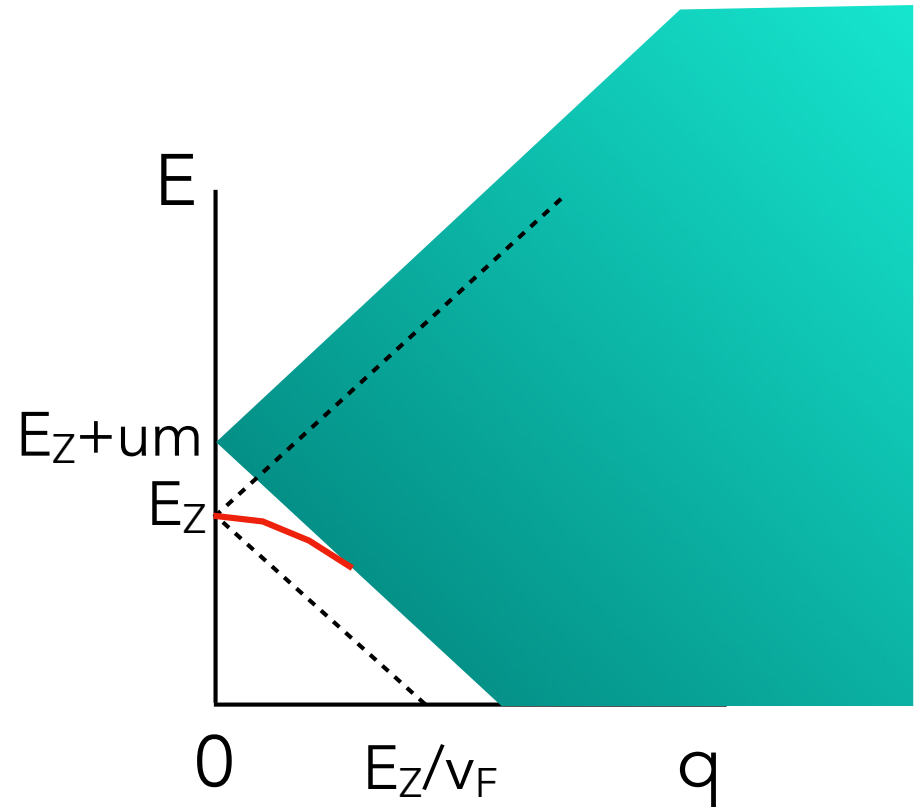
Silin spin wave

Larmor theorem: $q=0$
excitation *must* be at E_Z

RPA



$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi^0(\mathbf{q}, i\omega_n)}{1 + u\chi^0(\mathbf{q}, i\omega_n)}$$

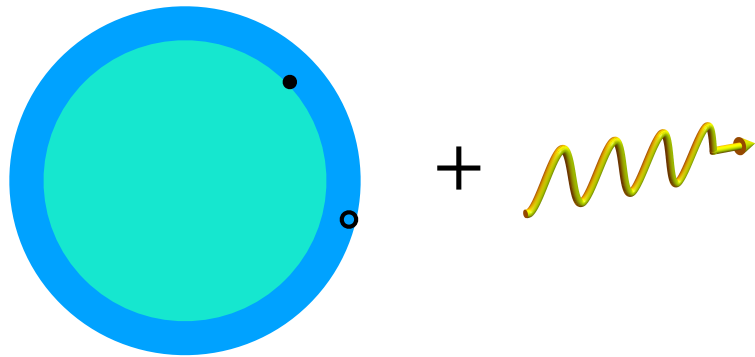


pole: collective mode

“Silin spin wave”

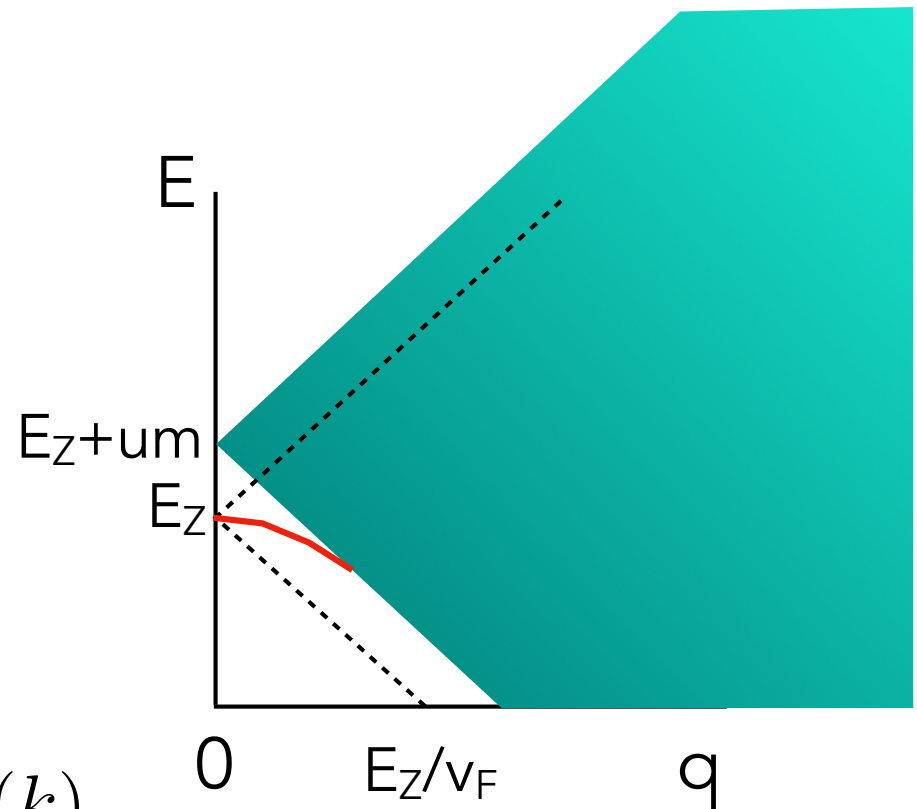
$$\omega = E_Z + um - \sqrt{u^2 m^2 + v_F^2 q^2}$$

Transverse gauge coupling



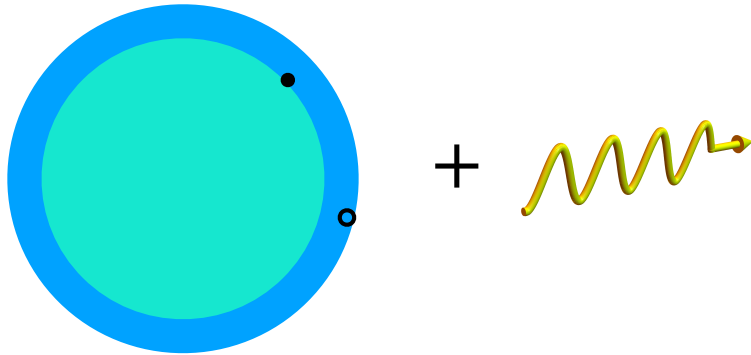
Simple picture:
3-particle process:

$$E = E_{p/h}(q - k) + E_{\text{photon}}(k) \\ \sim ck^3$$

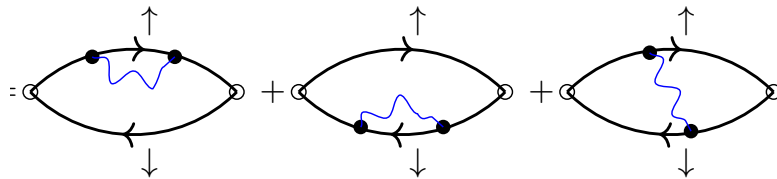


Does this smear out all the Fermi liquid structure?

Transverse gauge coupling

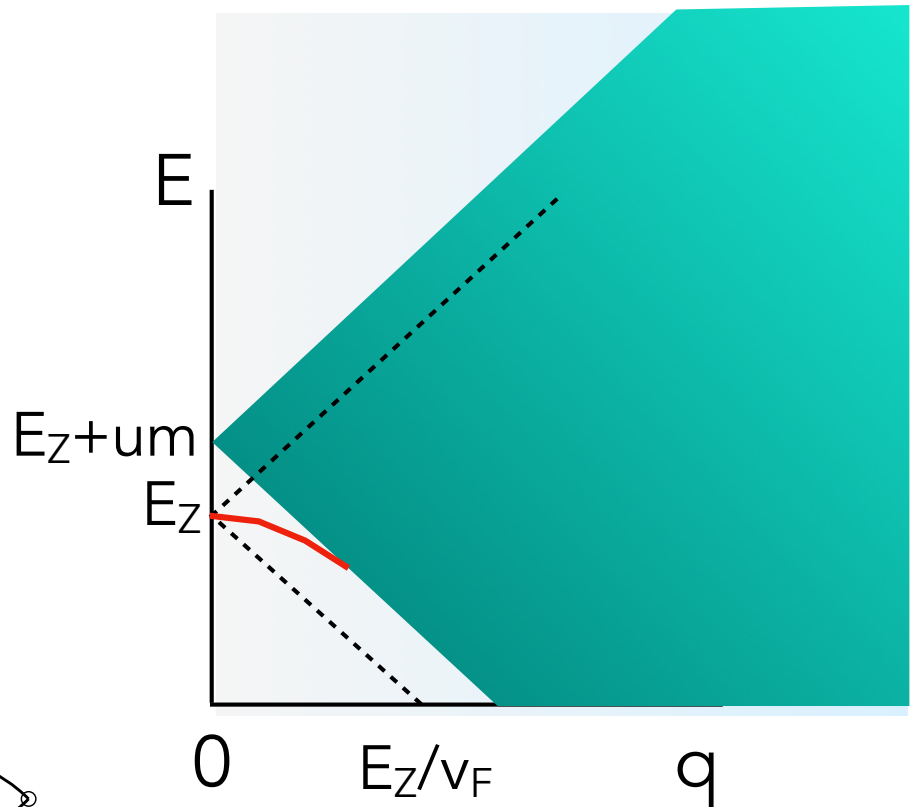


Actual calculation:



c.f. Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, [Phys Rev. B](#) **50**, 17917 (1994).

$$\text{Im}\chi_{\pm} \sim q^2 \omega^{7/3}$$

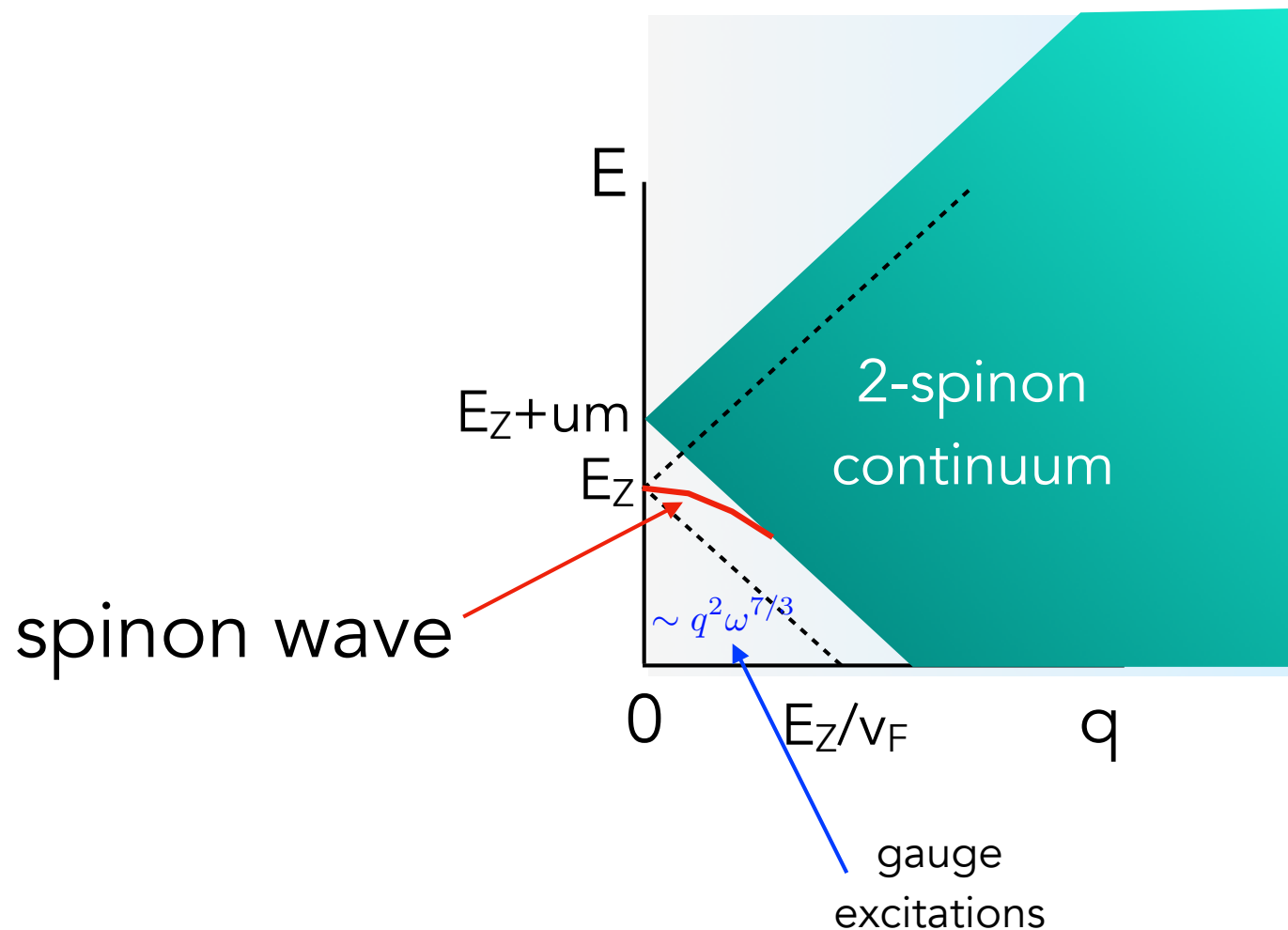


weight at all $q \neq 0$

but weak enough to
preserve structure

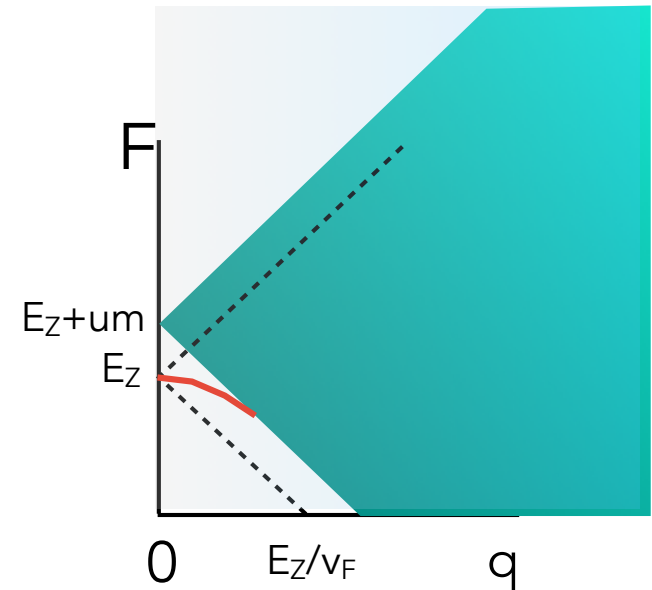
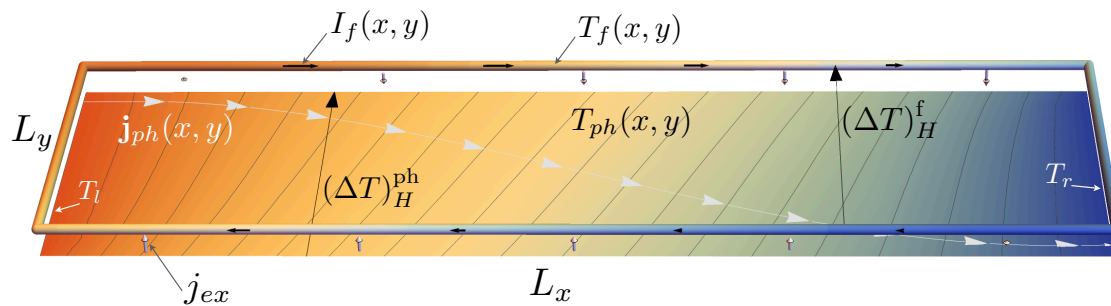
Summary

Distinct signatures of spinons,
interactions, and gauge fields



O.Starykh + LB,
in preparation

Recap



- Quantized value of thermal Hall conductivity *can* be measured even when large phonon conductivity is present
- Quantization is only power-law good, and deviations are controlled by edge-bulk equilibration
- Fermionic two-spinon continuum modified by *Silin spin wave* and *gauge continuum*.
- Further work: modifications near $q=0$ by anisotropies/ SOC - needed to understand ESR, extension to Dirac spin liquids



U.S. DEPARTMENT OF
ENERGY

Office of
Science



GORDON AND BETTY
MOORE
FOUNDATION