UQM from theory to experiment

Leon Balents, KITP

UQMII, August 22-24, 2018

ICTP Dirac Medal 2018

Congratulations to:

Subir Sachdev
HARVARD UNIVERSITY, USA

Dam Thanh Son
UNIVERSITY OF CHICAGO, USA

Xiao-Gang Wen
MIT, USA

The Abdus Salam International Centre for Theoretical Physics

IAEA

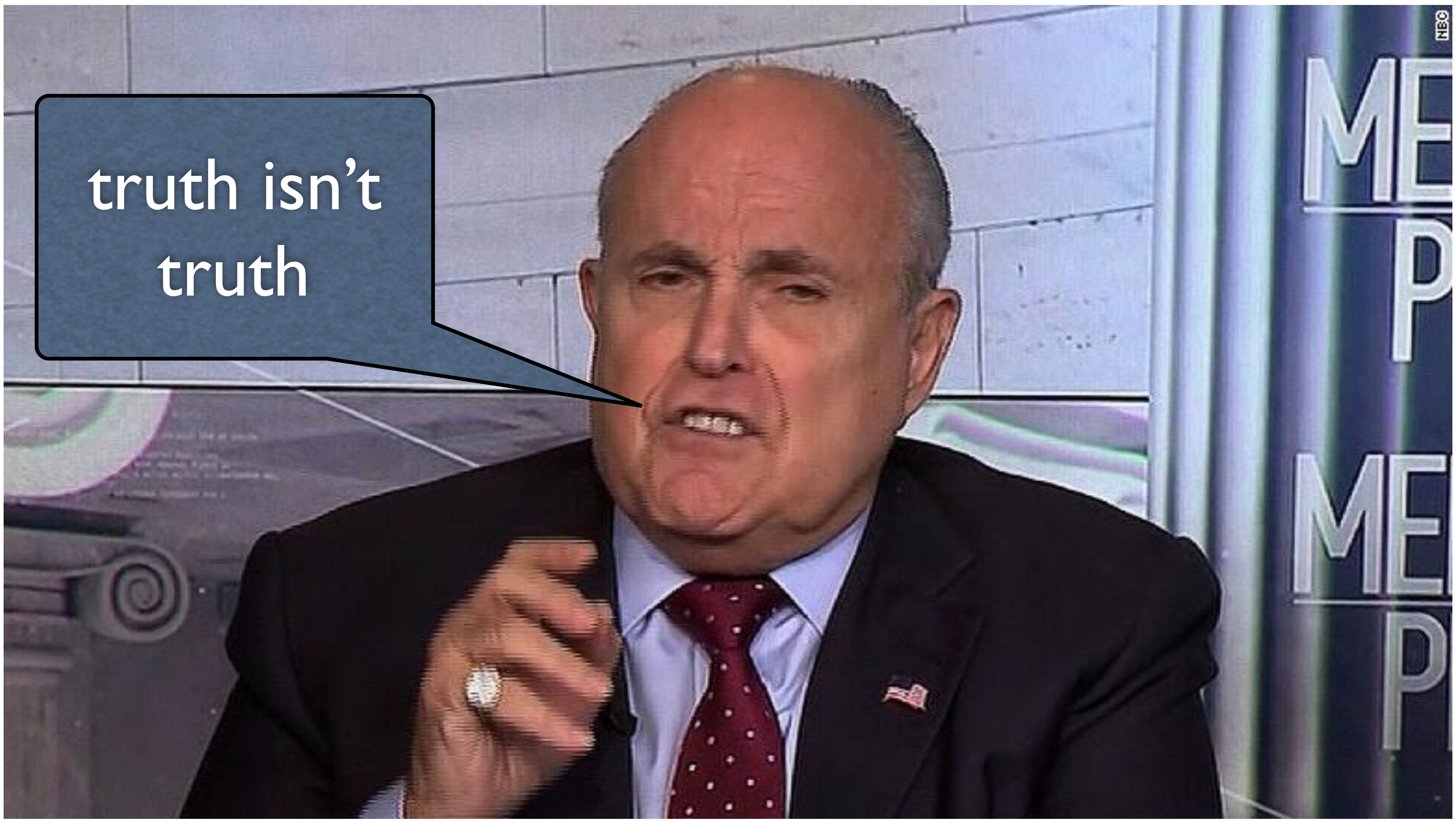
What is UQM?

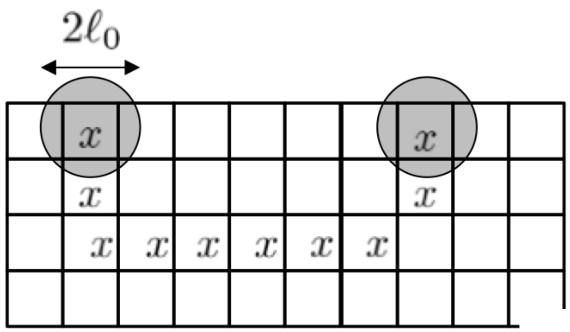
What is UQM?

- Matter that is “more quantum” than usual
- Matter which hosts persistent and non-trivial long-range entanglement or quantum non-locality
- Host for exotic excitations
- The physical realization of exotic or strongly coupled field theories

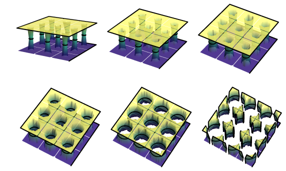
It might seem evasive…

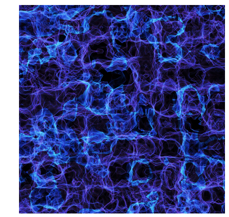
but it could be worse

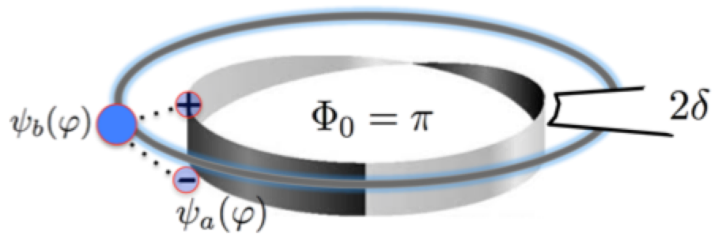




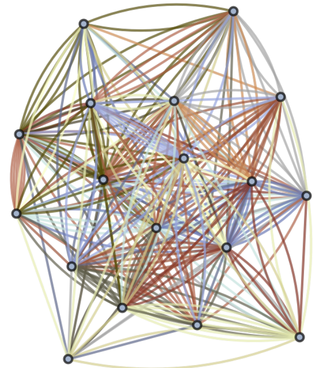
$2\ell_0$

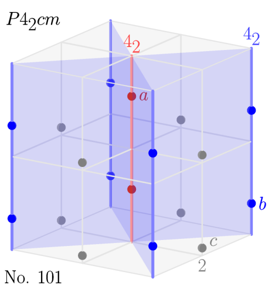




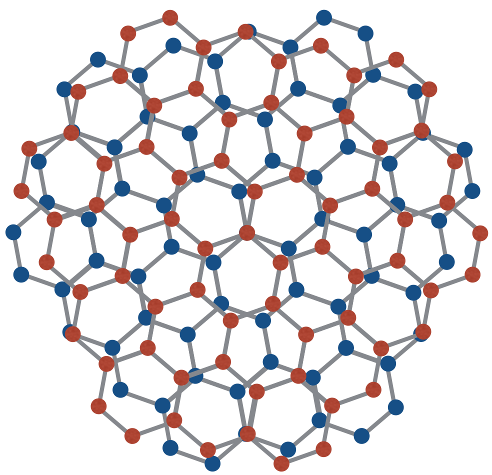


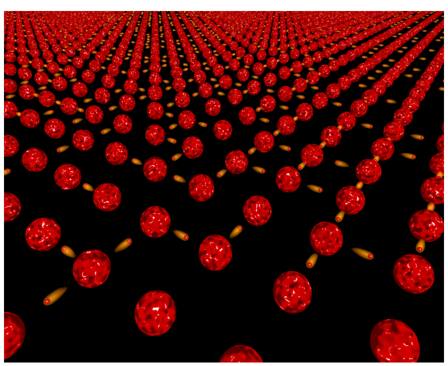
$\psi_b(\varphi)$
$\Phi_0 = \pi$
2δ
$\psi_a(\varphi)$

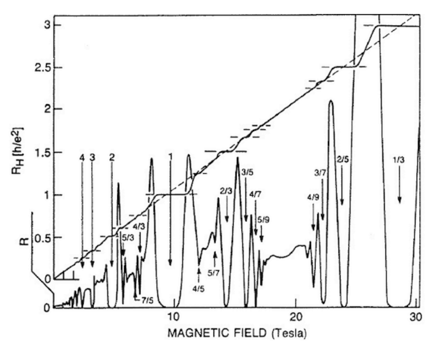




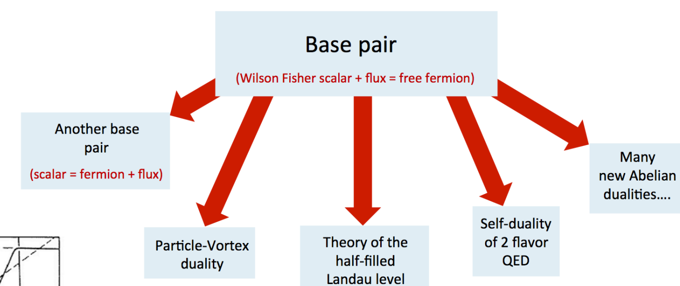
$P4_2cm$
No. 101



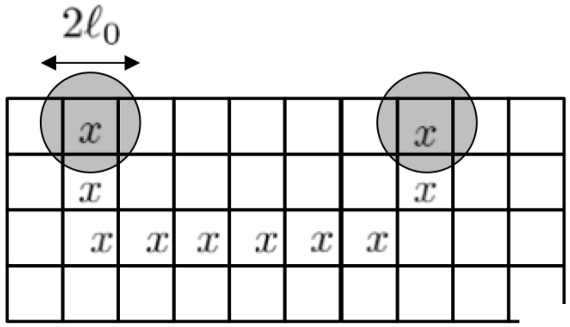


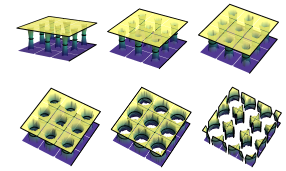


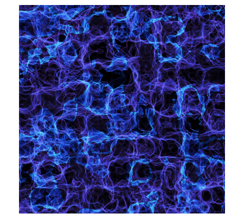
R_H [h/e^2]
MAGNETIC FIELD (Tesla)

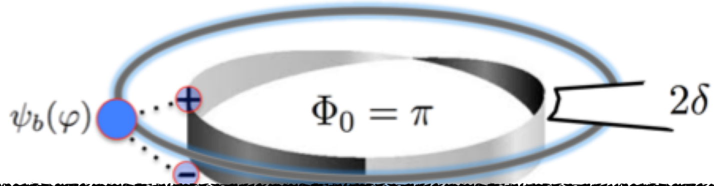


Base pair
(Wilson Fisher scalar + flux = free fermion)
Another base pair
(scalar = fermion + flux)
Particle-Vortex duality
Theory of the half-filled Landau level
Self-duality of 2 flavor QED
Many new Abelian dualities....

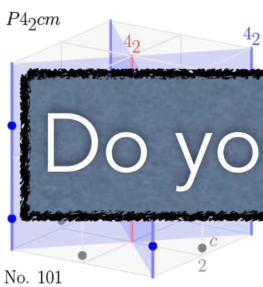


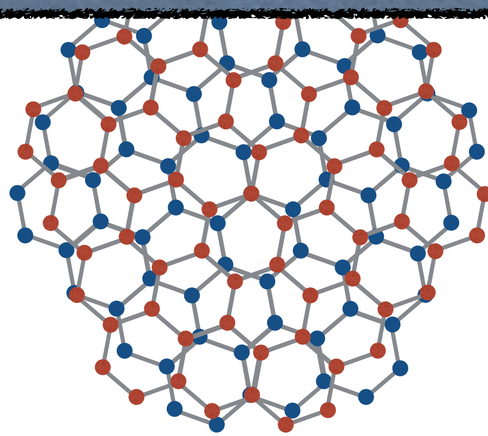


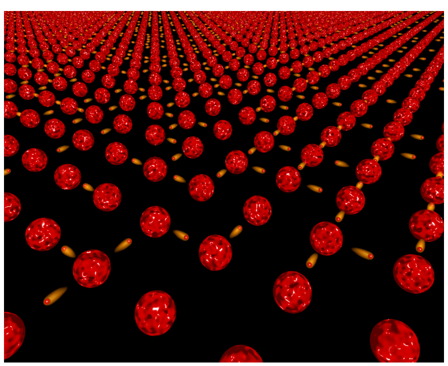


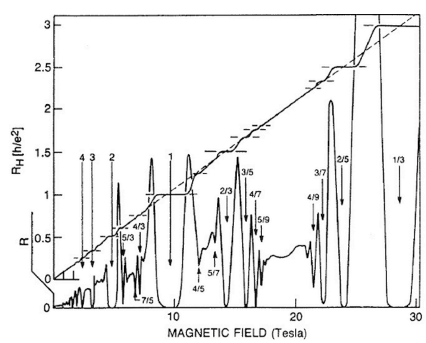


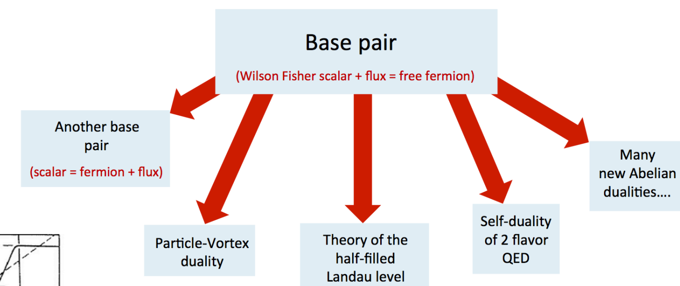
Do you know it when you see it…in experiment?











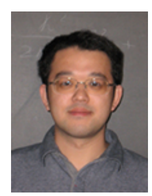
Smoking Gun?

- Inherently, non-locality is not so directly probed
- We need to look for consequences of this structure… not always obvious
- One of the exciting things about UQM now is that it is becoming increasingly accessible to experiment: we *need* to understand UQM better to interpret these
- I'll discuss two examples where experiment surprised me

Two examples

- Singular angular magnetoresistance in a magnetic Weyl semimetal
- Quantized thermal Hall effect in a nonabelian chiral spin liquid

Collaborators

- Mengxing Ye (Minnesota) 
- Gábor Halász (KITP -> ORNL) 
- Lucile Savary (ENS Lyon) 
- Jianpeng Liu (KITP -> HKUST) 
- Joe Checkelsky (MIT) 
- Takehito Suzuki (MIT) 

Two examples

- **Singular angular magnetoresistance in a magnetic Weyl semimetal**
- Quantized thermal Hall effect in a nonabelian chiral spin liquid

Weyl semimetal

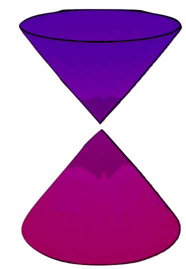
937 PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING
Princeton University, Princeton, New Jersey
(Received June 16, 1937)

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neigborhood of a point $\mathbf{k}$ where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa\rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

$$H = v\vec{\sigma}\cdot\vec{k}$$



A two-component spinor in three dimensions: "half" of a Dirac fermion. Weyl fermions have a chirality and *must* be massless

Weyl semimetal

937 PHYSICAL REVIEW

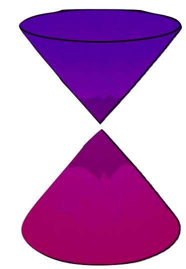
Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING
Princeton University, Princeton, New Jersey

(Received June 16, 1937)

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neigborhood of a point $\mathbf{k}$ where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa\to 0$, for all directions of $\boldsymbol{\kappa}$.

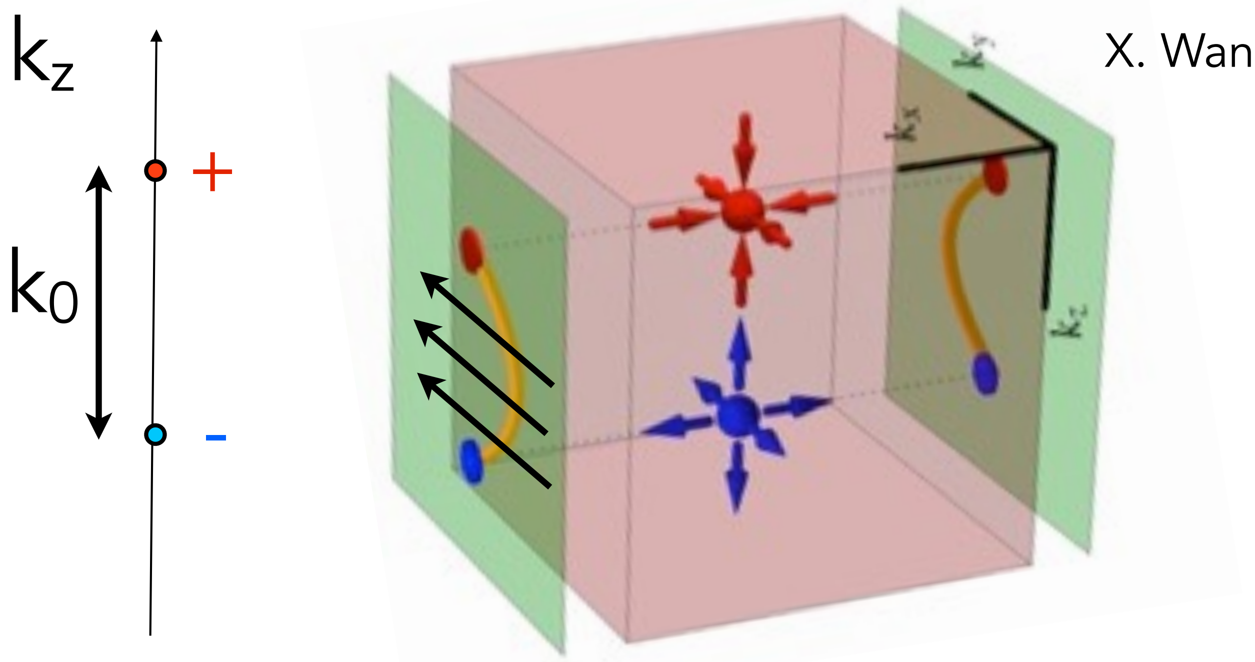
$$H = v\vec{\sigma}\cdot\vec{k}$$



A Weyl point is a “topological defect” in momentum space: a monopole for the Berry curvature

$$\boldsymbol{\nabla}_{\boldsymbol{k}}\cdot\mathcal{B} = \pm 2\pi q$$

Weyl semimetal



Fermi arc = chiral edge state

X. Wan, Vishwanath, Savrasov, 2011
A.A. Burkov+LB, 2011

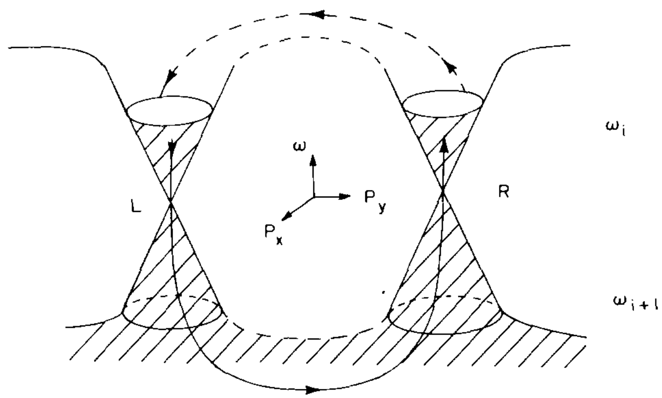
$$\sigma_{\mu\nu} = \frac{e^2}{2\pi h}\epsilon_{\mu\nu\lambda}Q_\lambda$$

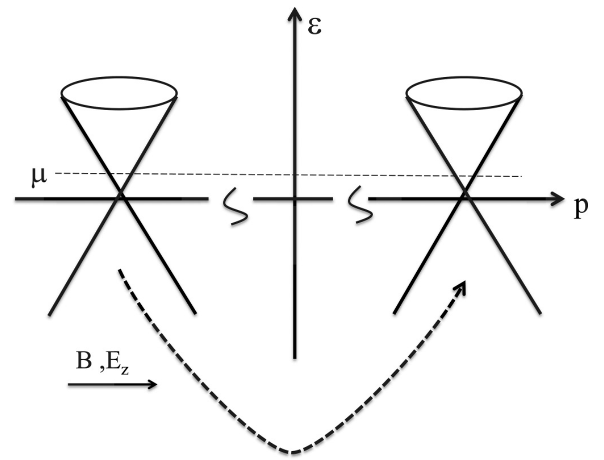
$$\vec{Q} = \sum_i \vec{k}_i q_i + \vec{Q}_{RLV}$$

Hall vector **Q** ~ "dipole moment" of Weyl points

(when E_F away from Weyl points add FS contributions)

Chiral anomaly





Nielsen-Ninomiya

Son-Spivak

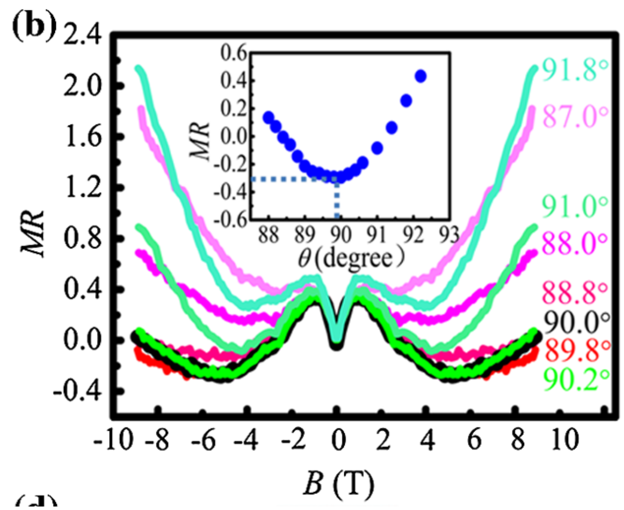
$$\sigma_{zz} = \frac{e^2}{4\pi^2\hbar c}\frac{v}{c}\frac{(eB)^2v^2}{\mu^2}\tau.$$

negative magnetoresistance

Chiral anomaly

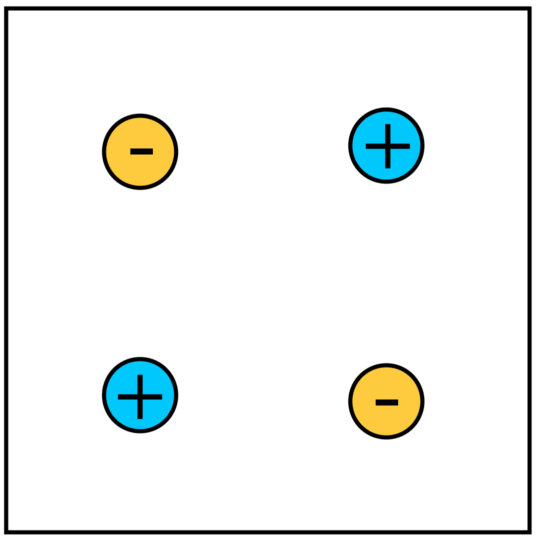
X. Huang *et al*, 2015

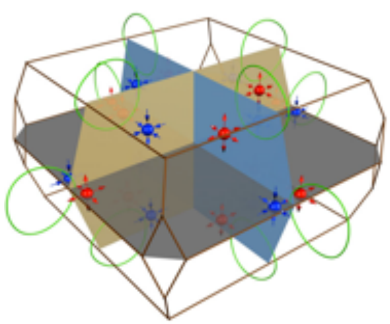
TaAs



Negative MR observed in some Weyl materials but somewhat ambiguous

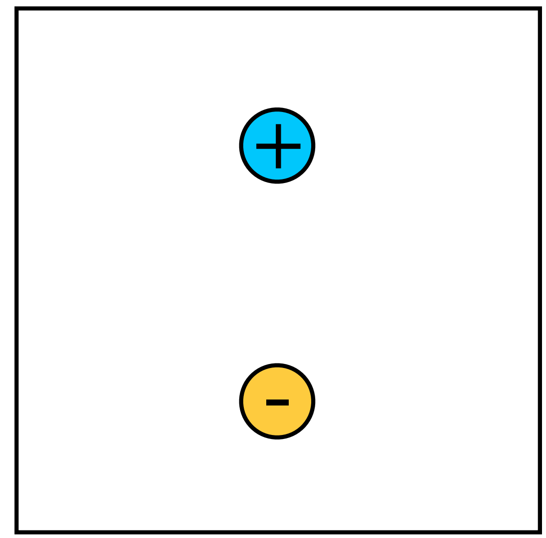
I-breaking
Weyls

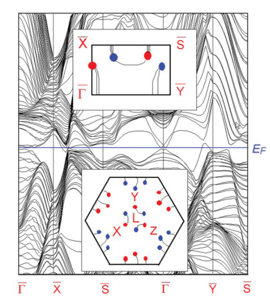




TaAs, Na_3Bi, TaP, WTe_2,...

TR-breaking
Weyls

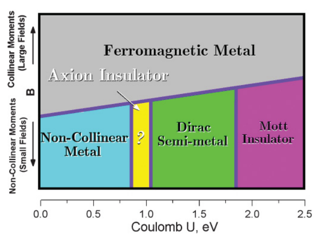




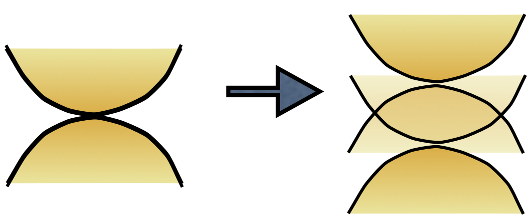
$R_2Ir_2O_7$?, Mn_3(Sn/Ge), RAlGe

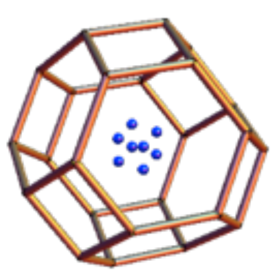
Antiferromagnetic Weyls

Pyrochlore iridates

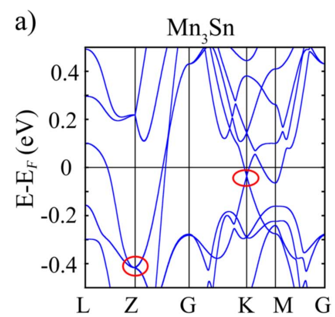


X. Wan *et al*, 2011

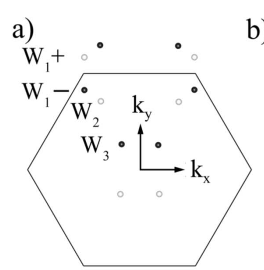




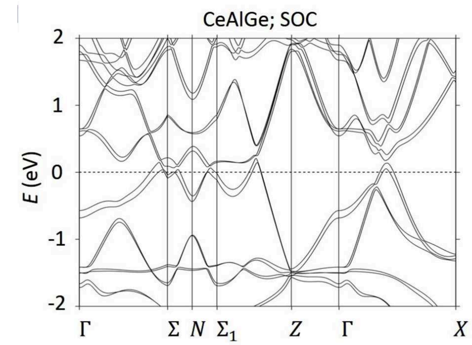
Mn_3Sn, Mn_3Ge



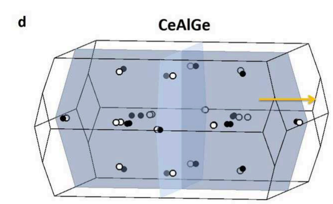
H. Yang *et al*, 2017



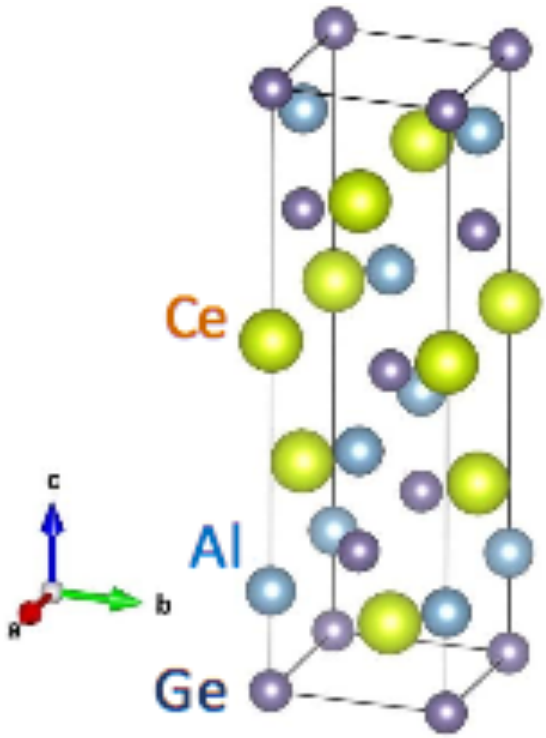
RAlGe



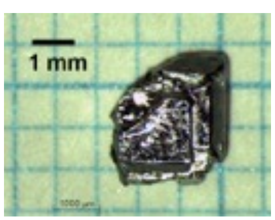
G. Chang *et al*, 2016



CeAlGe

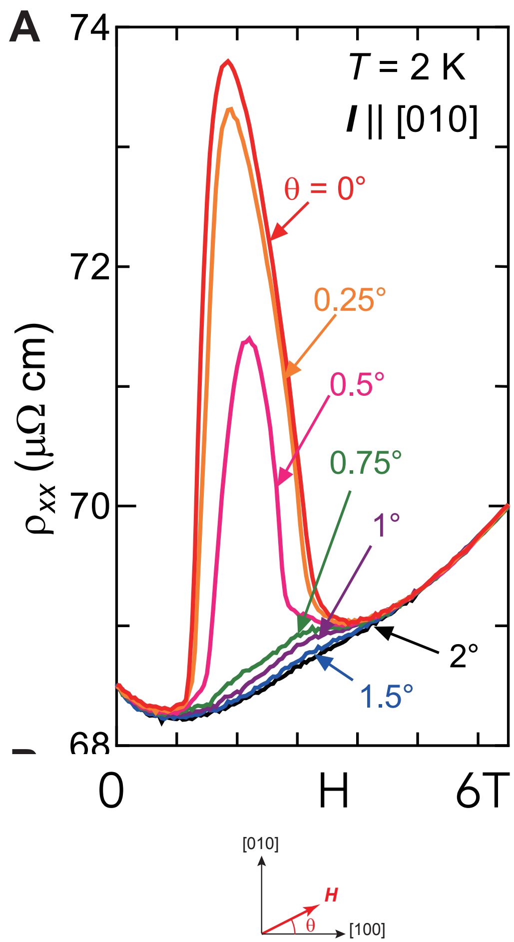


Space group: $I4_1md$



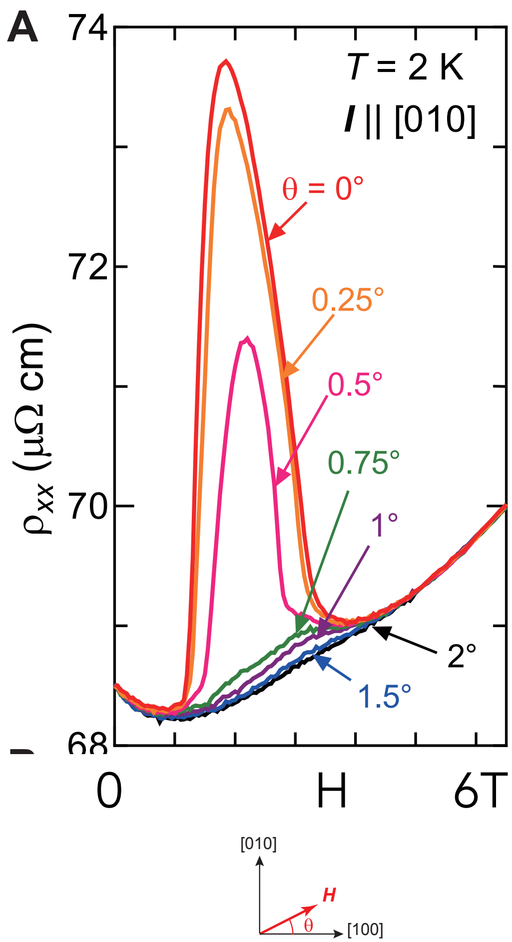
- tetragonal
- Ce $4f^1$ moments
- Semi-metallic band structure

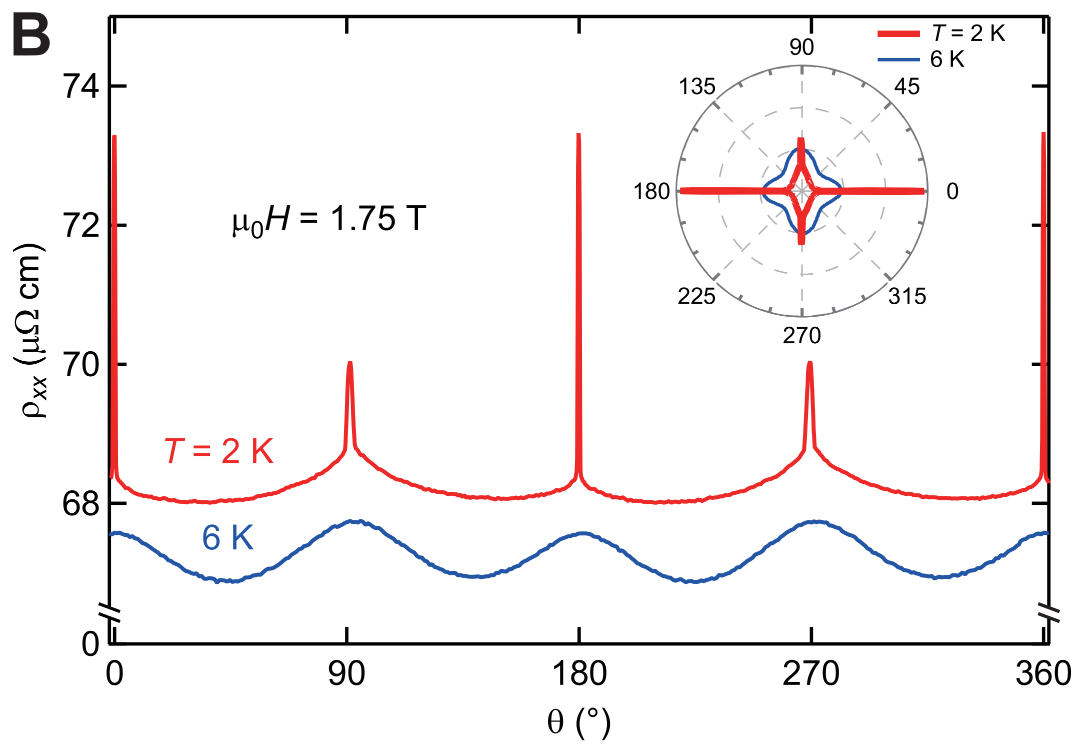
Resistivity



resistivity enhancement at intermediate fields and low T

Resistivity





very narrow angular dependence!
smoking gun of something!

Suzuki Angular Magneto-Resistance

~~Suzuki~~ Angular Magneto-Resistance

Savary Angular Magneto-Resistance

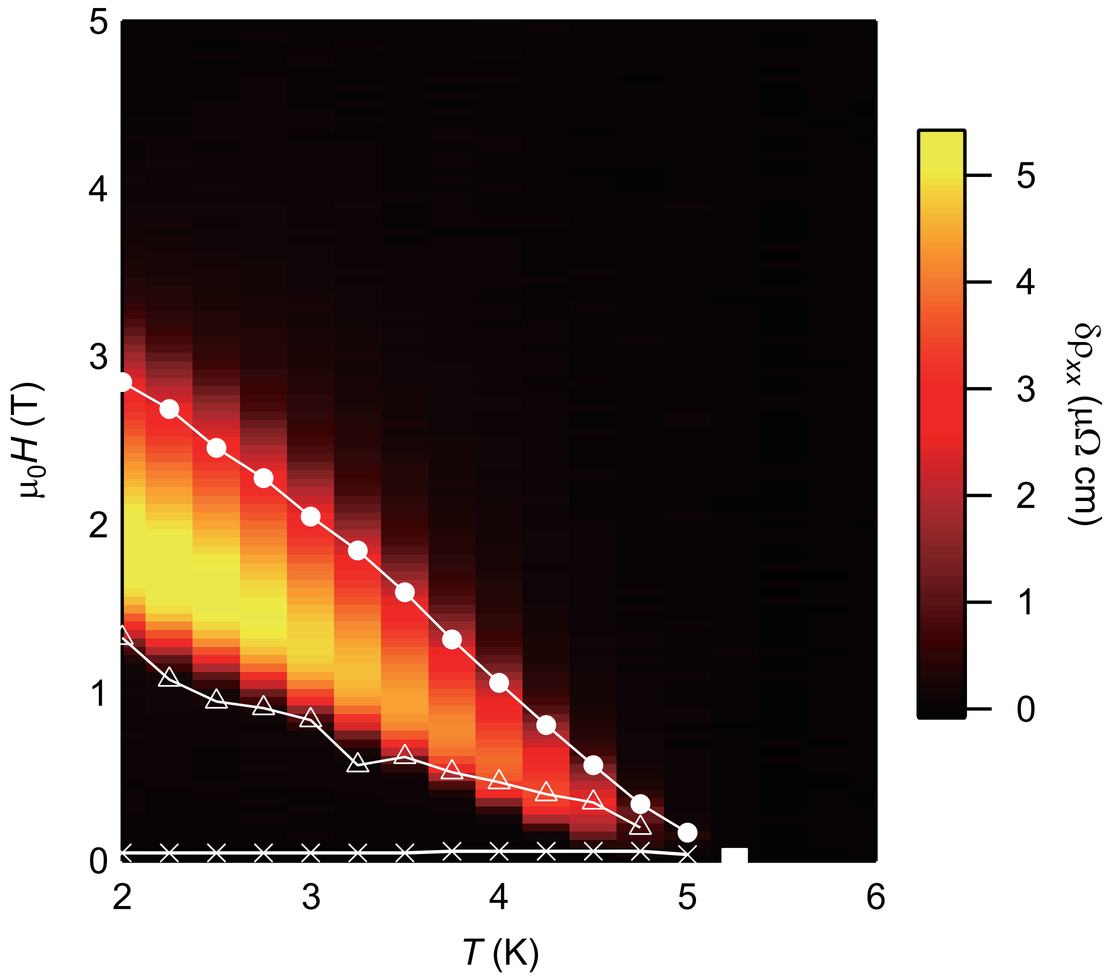
~~Suzuki~~ Angular Magneto-Resistance

~~Savary~~ Angular Magneto-Resistance

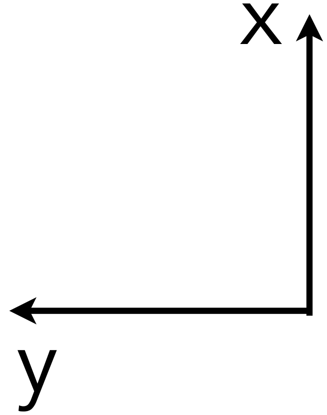
Singular Angular Magneto-Resistance

SAMR

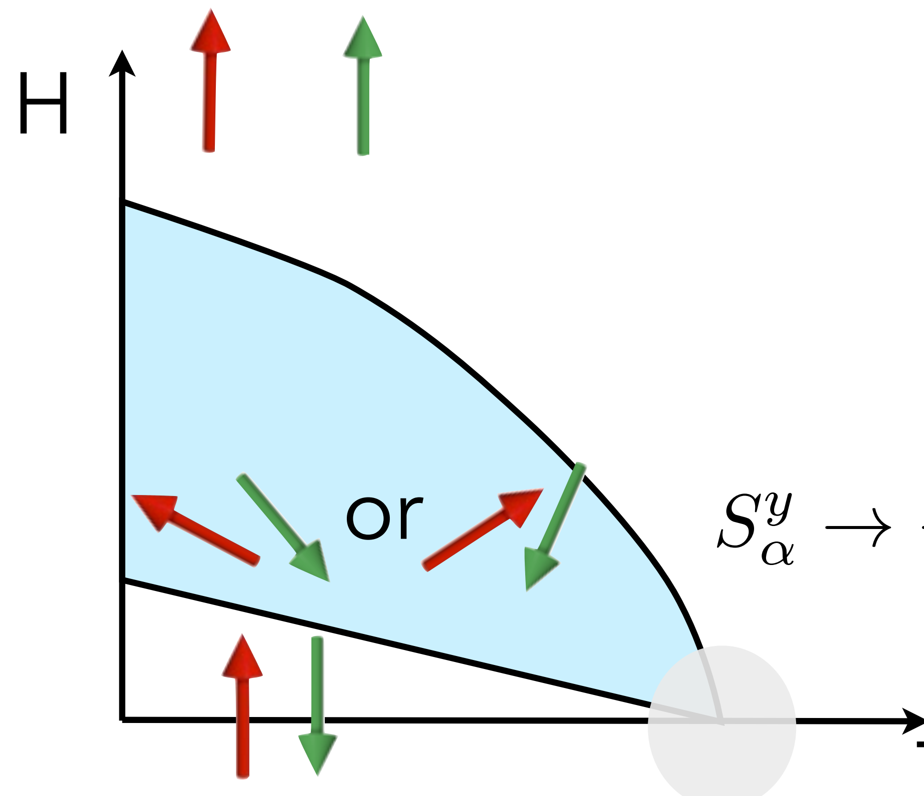
A Phase?



Spin Flop

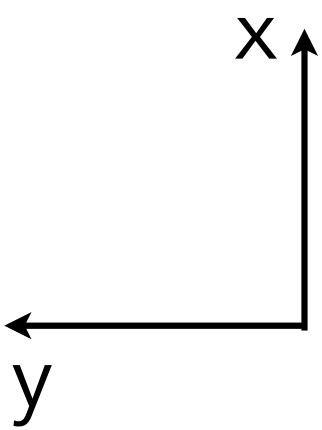


$m_{010} \times \mathrm{TR}$
broken

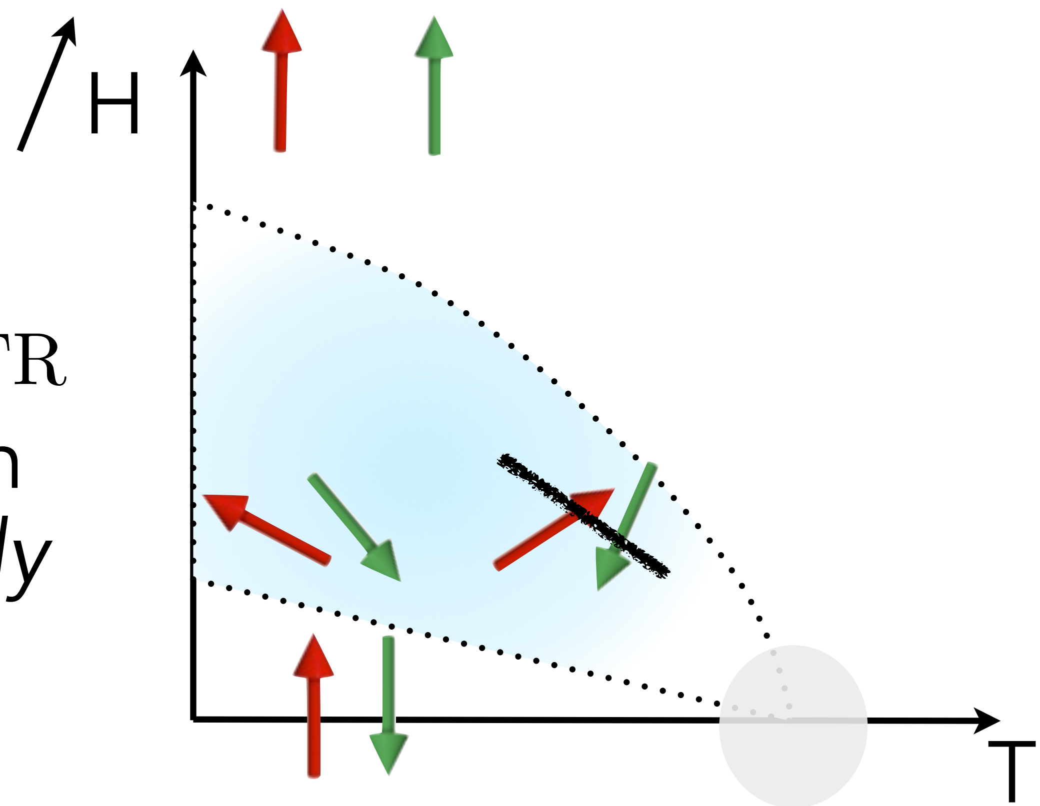


$S^y_\alpha \to -S^y_\alpha$

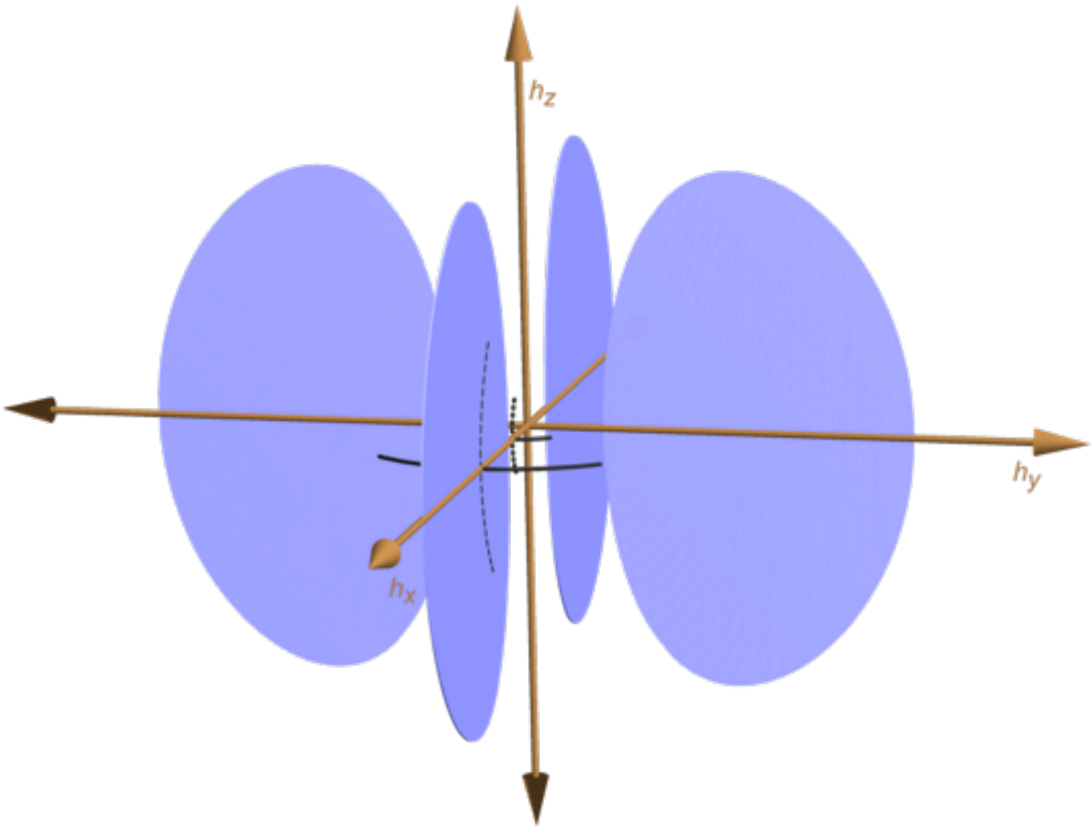
Tilted Field

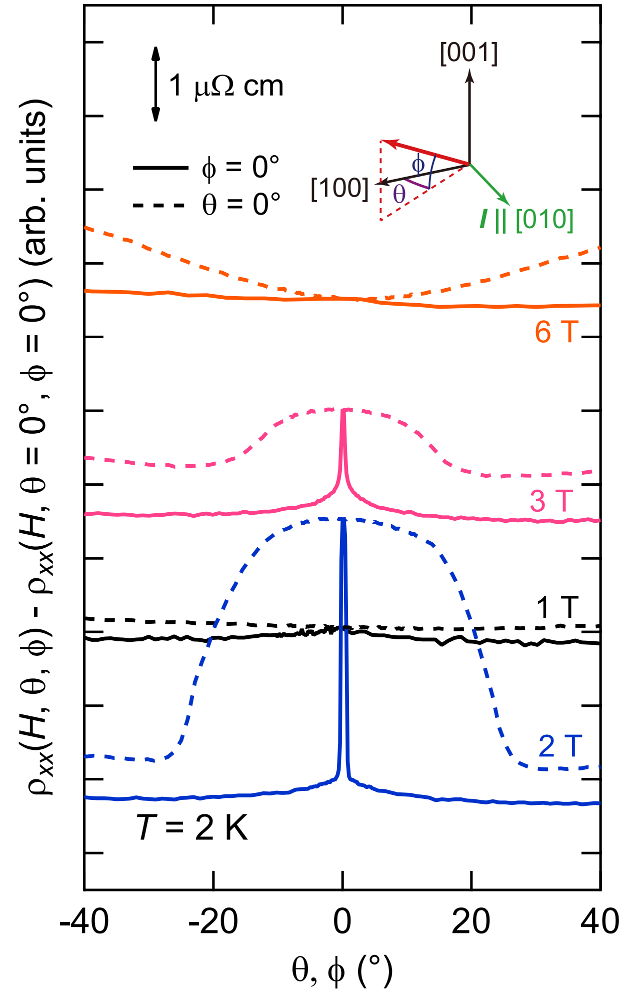


$m_{010} \times \mathrm{TR}$
broken
explicitly



Phase diagram

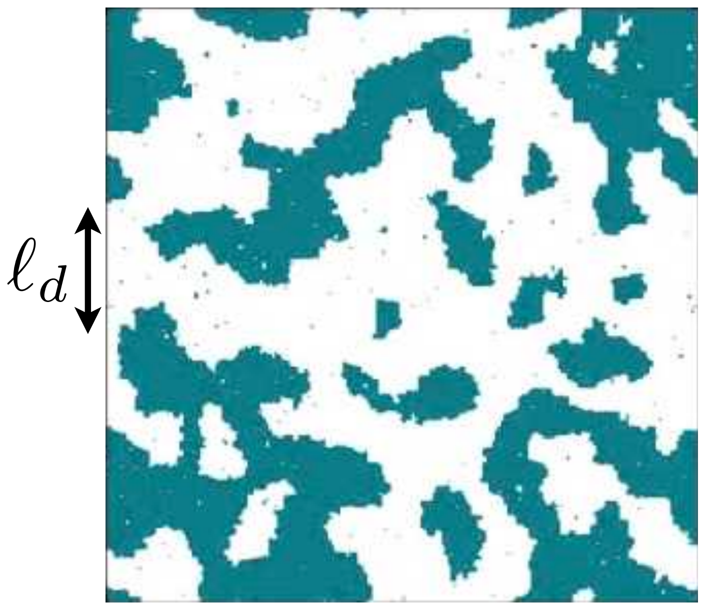




SAMR is an indicator of SSB. Why?

Domains

Extra resistance comes from *domain walls*

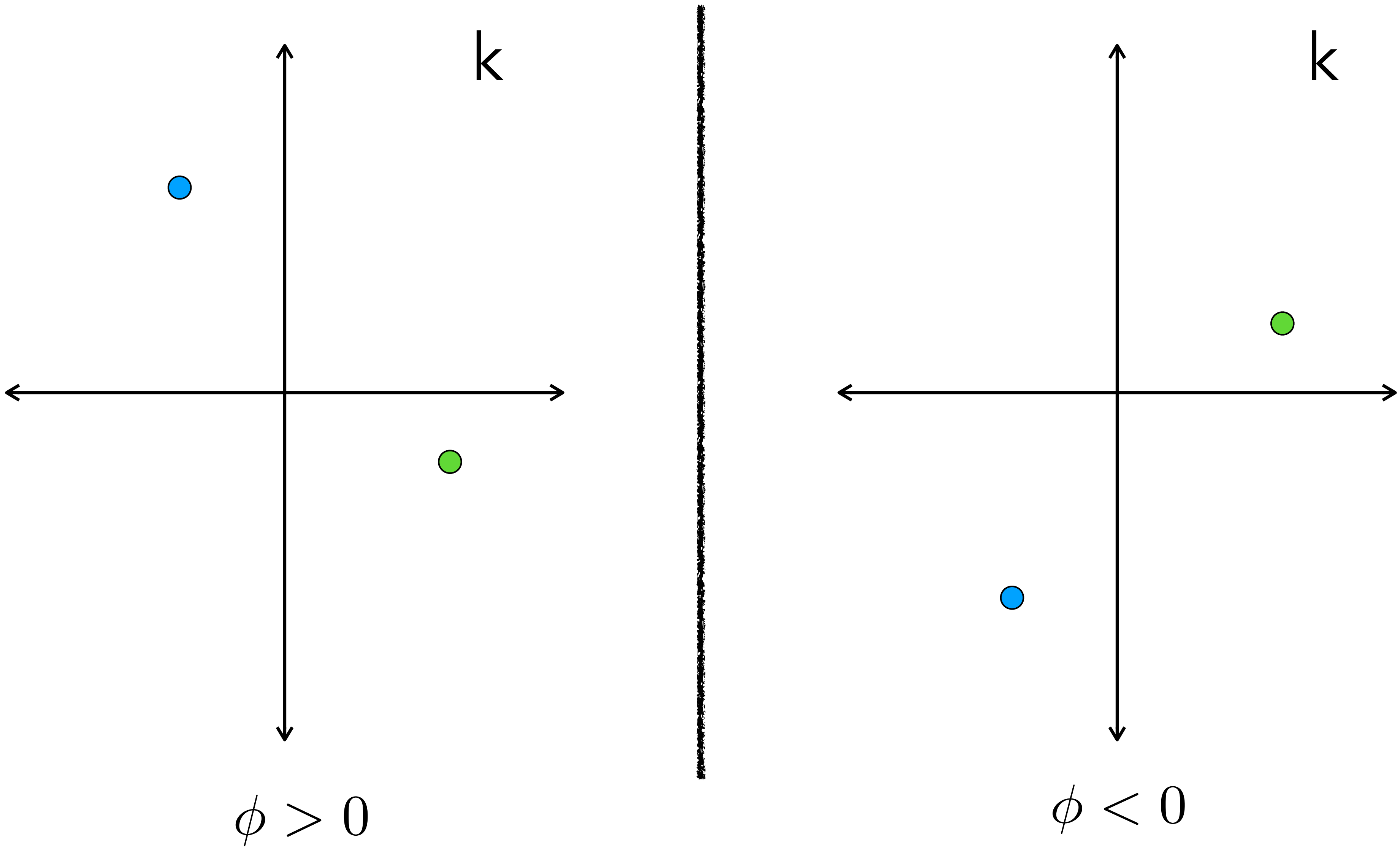


$$\rho_{\text{eff}} = \rho + \frac{\tilde{\rho}_{\text{dw}}}{\ell_d}$$

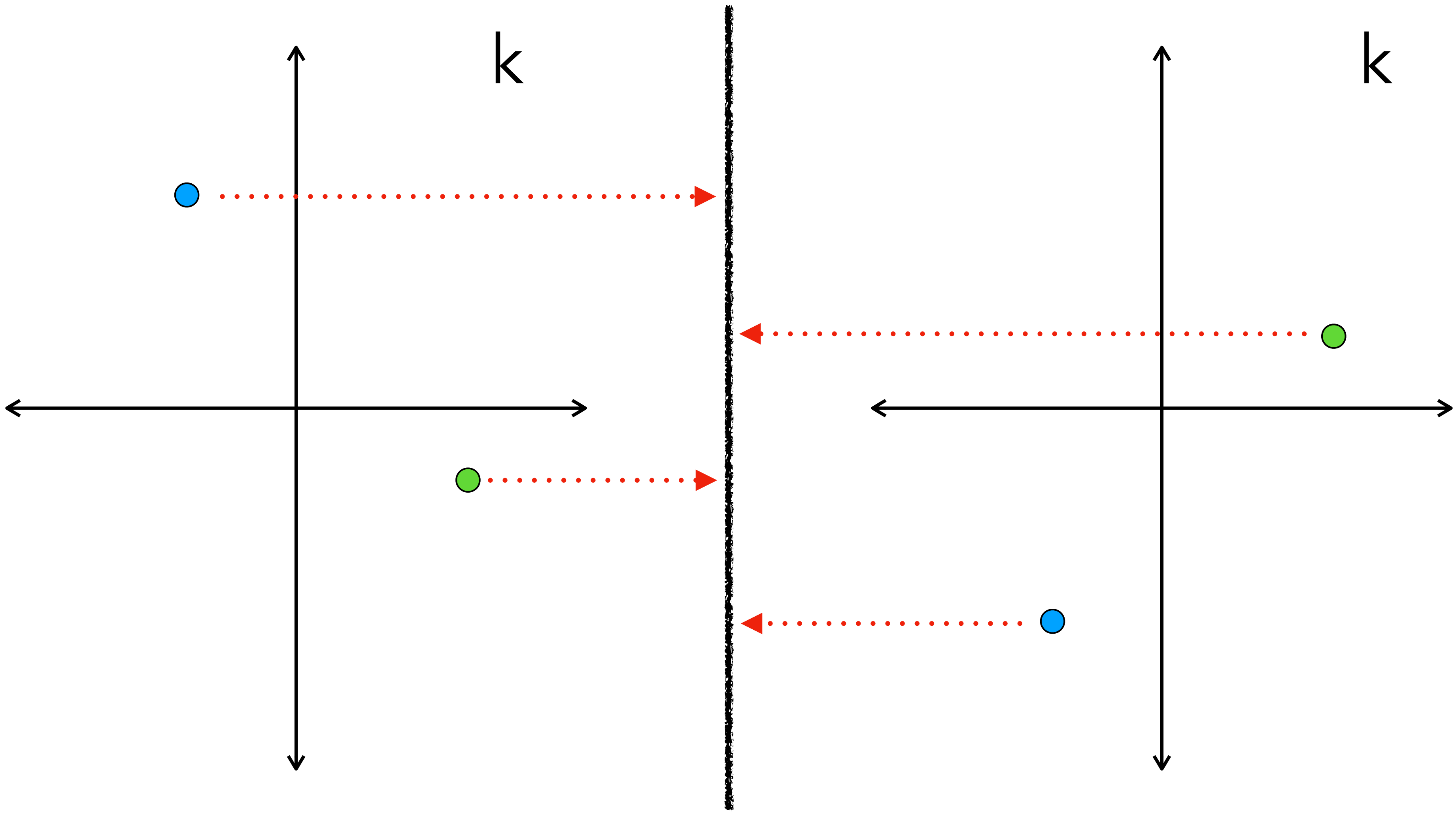
$$V_{\text{dw}} = \tilde{\rho}_{\text{dw}} j$$

Size of the effect depends on size of $\tilde{\rho}_{dw}$

Domain resistance



Domain resistance



$\phi > 0$ mirror is a mirror! $\phi < 0$

ideal Weyl system has zero DW conductance!

Two examples

- Singular angular magnetoresistance in a magnetic Weyl semimetal
- <u>**Quantized thermal Hall effect in a nonabelian chiral spin liquid**</u>

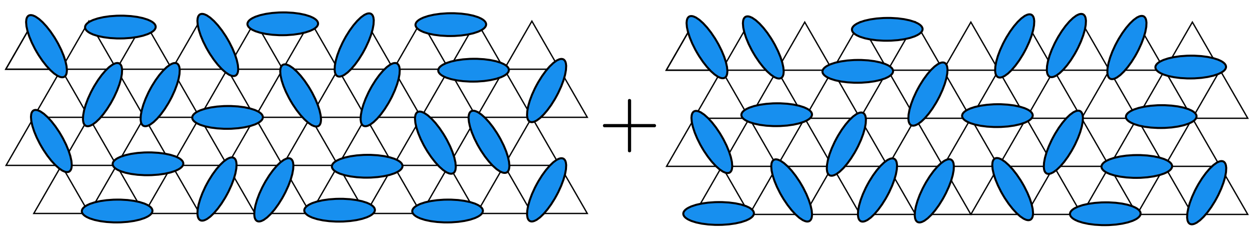
Quantum Spin Liquid

Phil Anderson, 1973

a "quantum liquid" of spins

$$\text{(bond)} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Ψ =



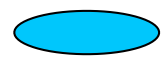
+...

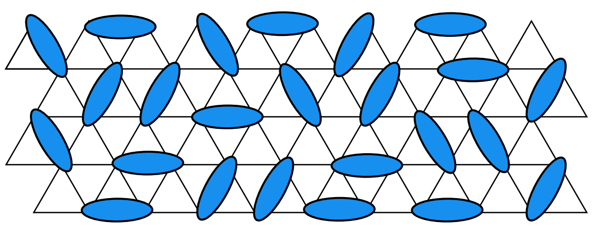
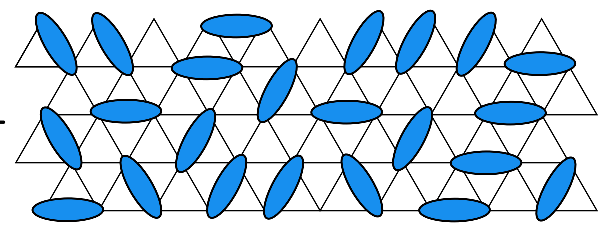
Resonating **V**alence **B**ond state

Quantum Spin Liquid

Phil Anderson, 1973

a "quantum liquid" of spins

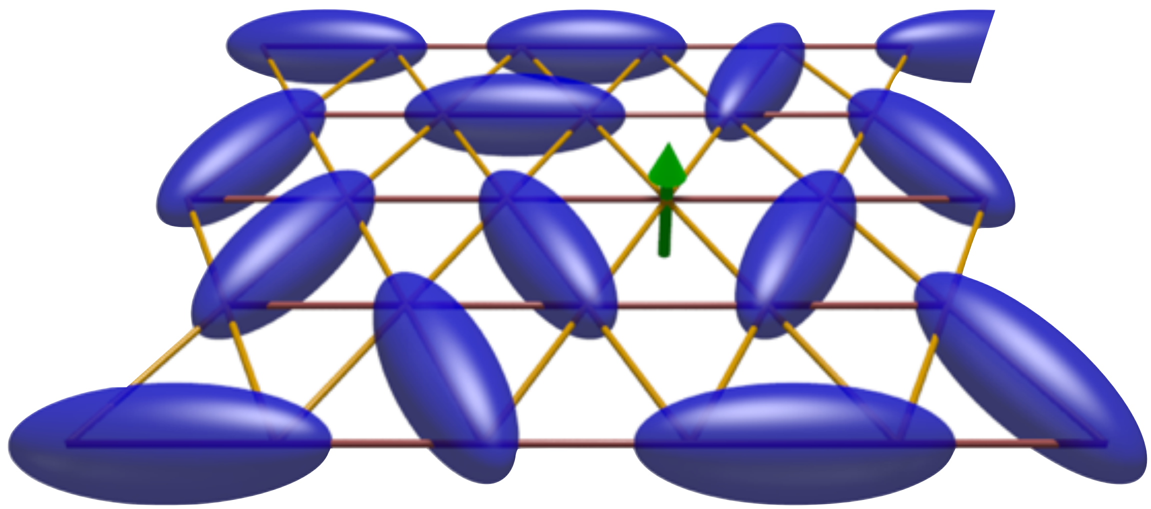
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Ψ =  $+$  $+ \ldots$

"poster child" for UQM

Fractional quantum number

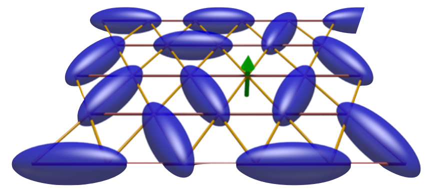
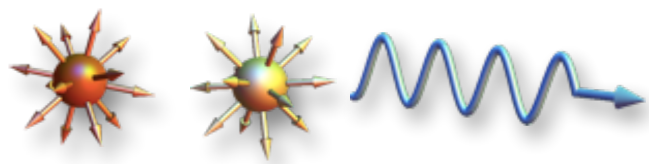
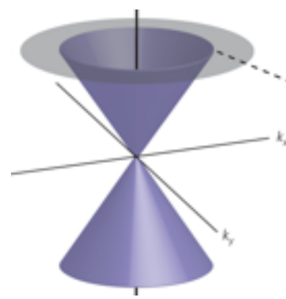
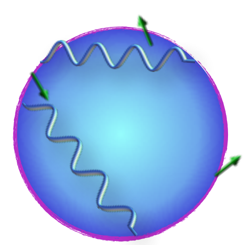
spinon



excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins

always created in pairs by any
local operator

Classes of QSLs

- Topological QSLs  anyons, spinons
- U(1) QSL  compact U(1)
- Dirac QSLs  QED_3
- Spinon Fermi surface  non-Fermi liquid “spin metal”

Smoking Gun?

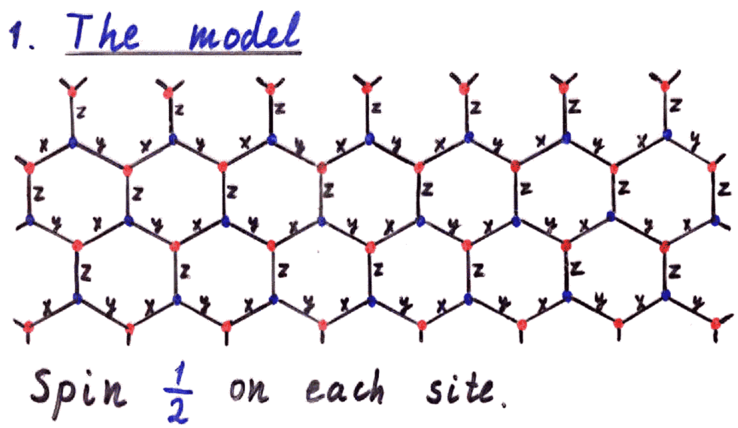
Difficult to find incontrovertible and sharp indicator of a QSL!

A few possibilities

- T-linear thermal conductivity in spinon Fermi surface QSL — **observed!**
- Transverse, linearly dispersing emergent photon mode in 3d U(1) QSL — **not yet observed**

Kitaev model

Kitaev's honeycomb model $$H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$$



KITP, 2003

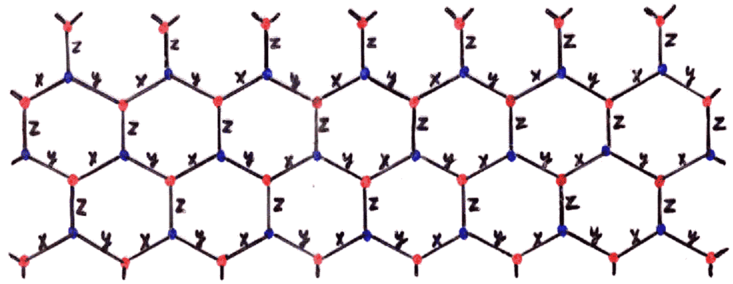
exact parton construction $\sigma_i^\mu = i c_i c_i^\mu \quad c_i c_i^x c_i^y c_i^z = 1$

effective quadratic Hamiltonian

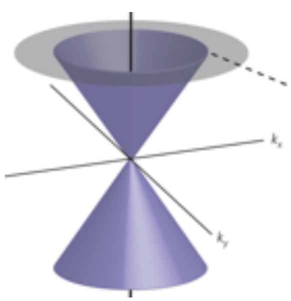
$$H = \sum_{i,\mu} i K_\mu c_i c_{i+\mu}$$

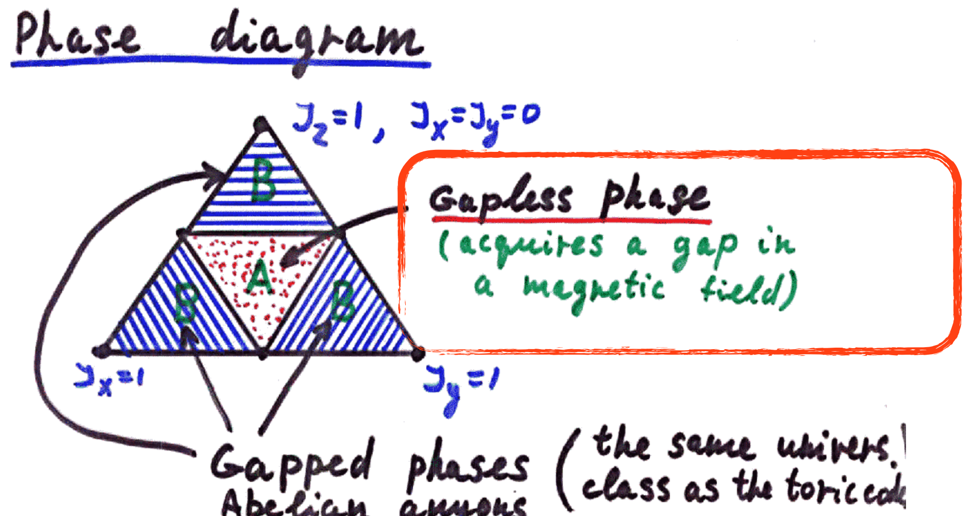
Gapless QSL

$$H = \sum_{i,\mu} i K_\mu c_i c_{i+\mu}$$

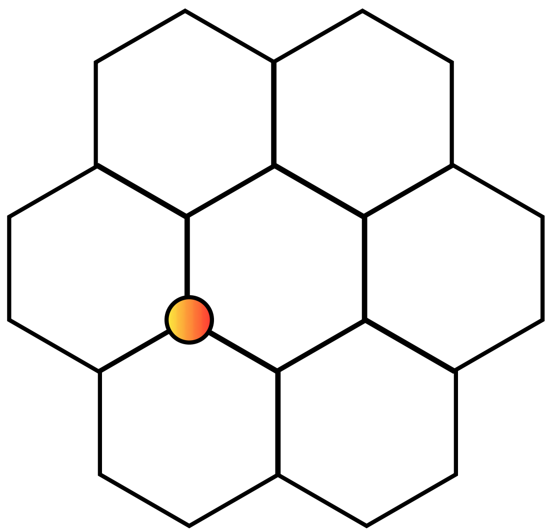


~1/4 of graphene

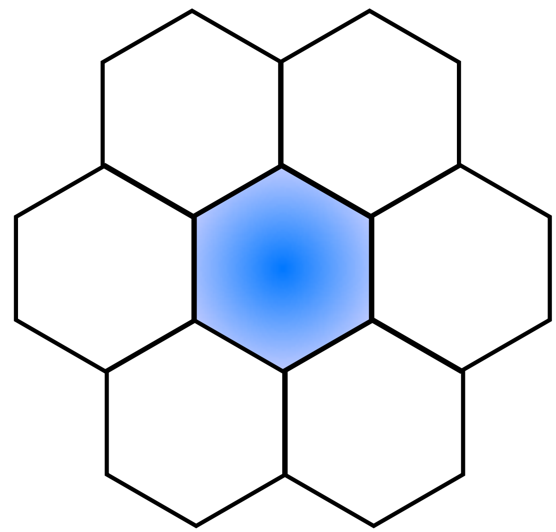




Non-local excitations



Majorana $\mathcal{E}$



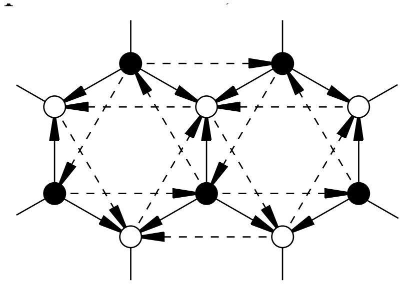
Flux e, m

In Kitaev's model:

- Majorana's dispersion ~ K and Dirac-like
- Fluxes are localized with small gap

Non-Abelian Phase

- In an applied magnetic field, the Majoranas acquire a gap

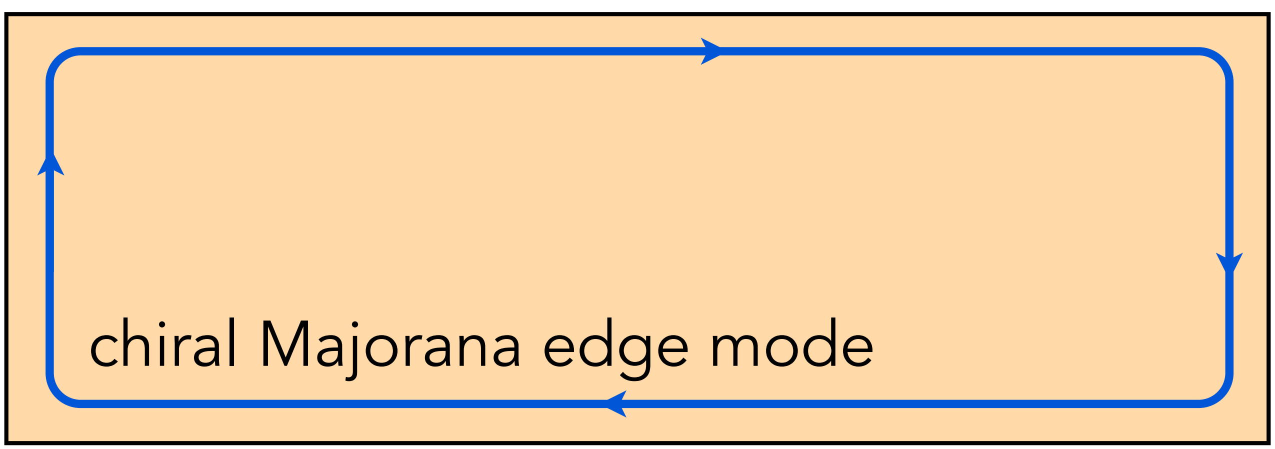


$$H_{\mathrm{eff}} = \frac{\mathrm{i}}{4} \sum_{j,k} A_{jk} c_j c_k,$$

$$A = 2J\,(\longleftarrow) + 2\kappa\,(\dashleftarrow),$$

$$\kappa \sim \frac{h_x h_y h_z}{J^2}.$$

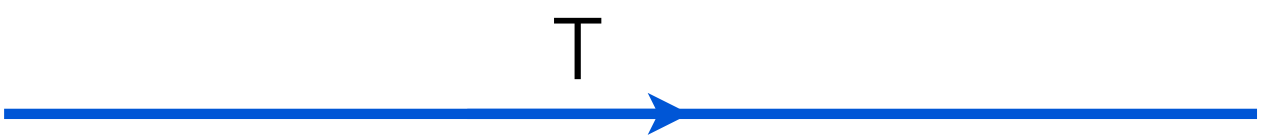
field induces a fermion mass, very similar to the Haldane model (except Majorana)



$$H_e = -\frac{iv}{4} \int dx\, \eta \partial_x \eta$$

Quantum Hall Effect?

- No charge. Have to study heat transport!



$$I = \int_0^\infty \frac{dq}{2\pi} \, v^2 q f(vq) = \frac{c\pi k_B^2}{12\hbar} T^2$$

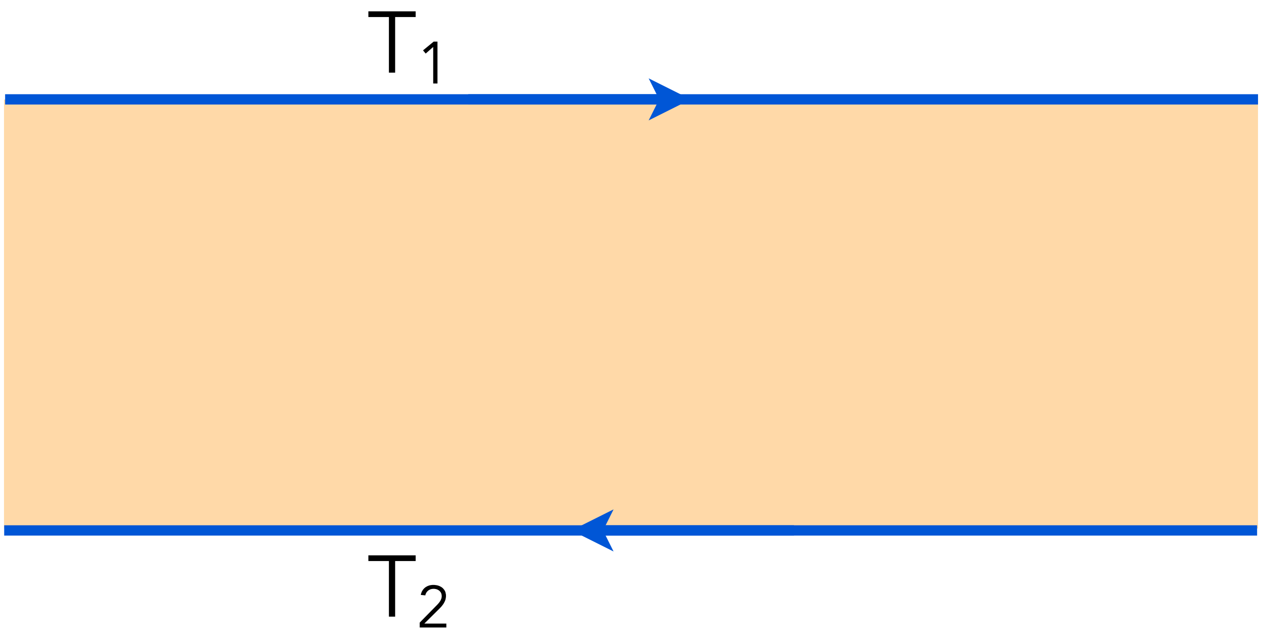
central charge c=1/2

<u>c.f. c=1 for both IQHE *and* FQHE abelian states</u>

implies the existence of bulk non-abelian excitations (the fluxes, bound to MZMs)

Quantum Hall Effect?

- No charge. Have to study heat transport!

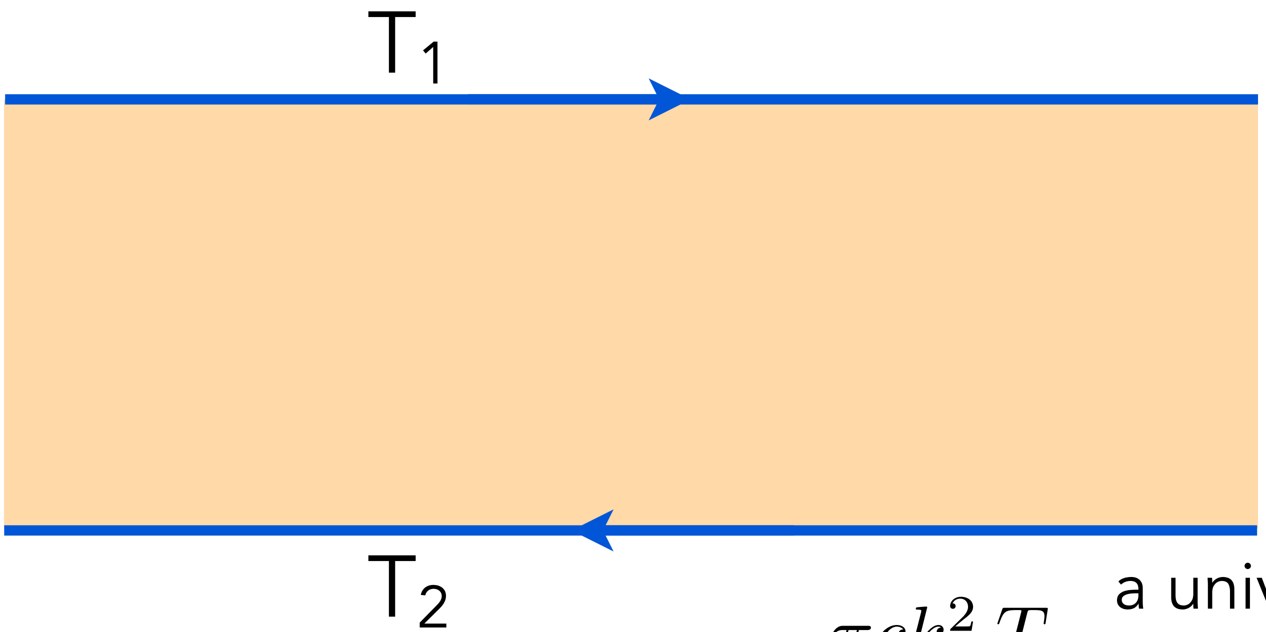


$$I = \frac{c\pi k_B^2}{12\hbar}\left(T_1^2 - T_2^2\right)$$

$$\approx \frac{c\pi k_B^2 T}{6\hbar}\left(T_1 - T_2\right)$$

Quantum Hall Effect?

- No charge. Have to study heat transport!



$$I_x = \kappa_H \Delta T_y$$

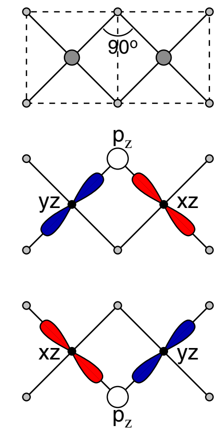
$$\kappa_H = \frac{\pi c k_B^2 T}{6\hbar}$$

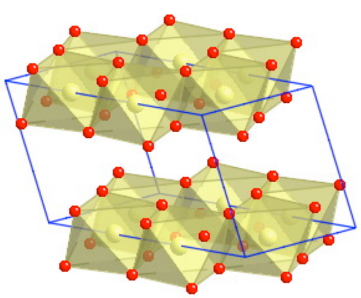
a universal prediction for chiral "Ising anyon" phase: *agnostic to microscopic spin interactions*

Kitaev Materials

Jackeli, Khaliullin
2009

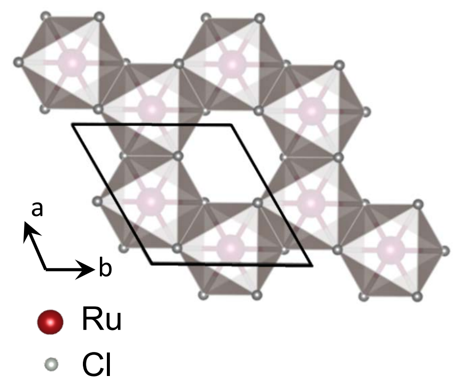
Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling





P.Gegenwart
H. Takagi
...

Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3



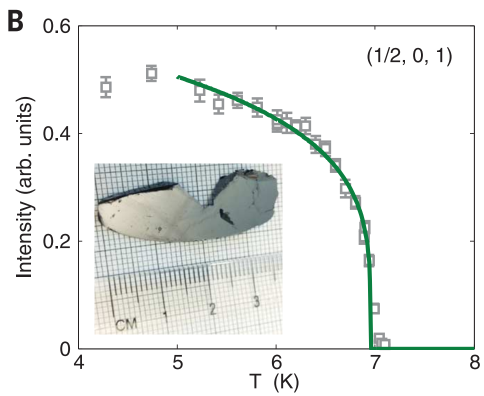
α-$RuCl_3$

Y.-J. Kim...

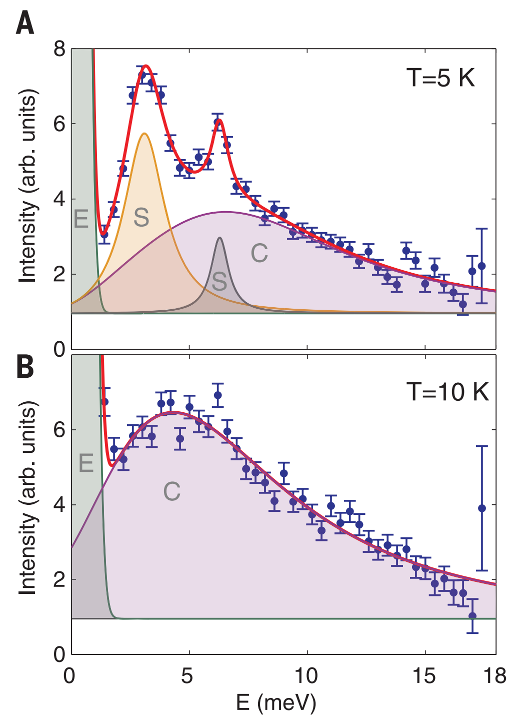
Honeycomb and hyper-honeycomb structures

α-$RuCl_3$

Clearly not described by ideal Kitaev model, but still interesting



zig-zag order

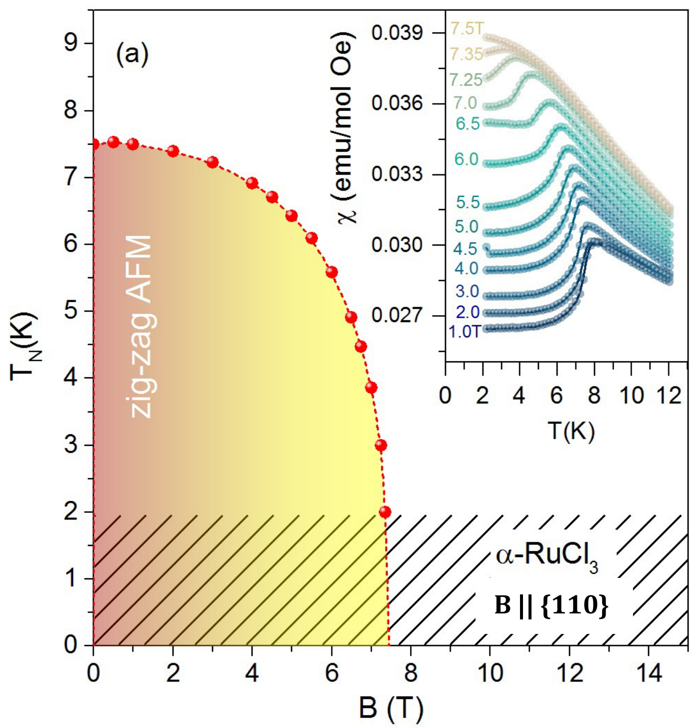


BUT: excitations to ~15meV = 170K

Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017

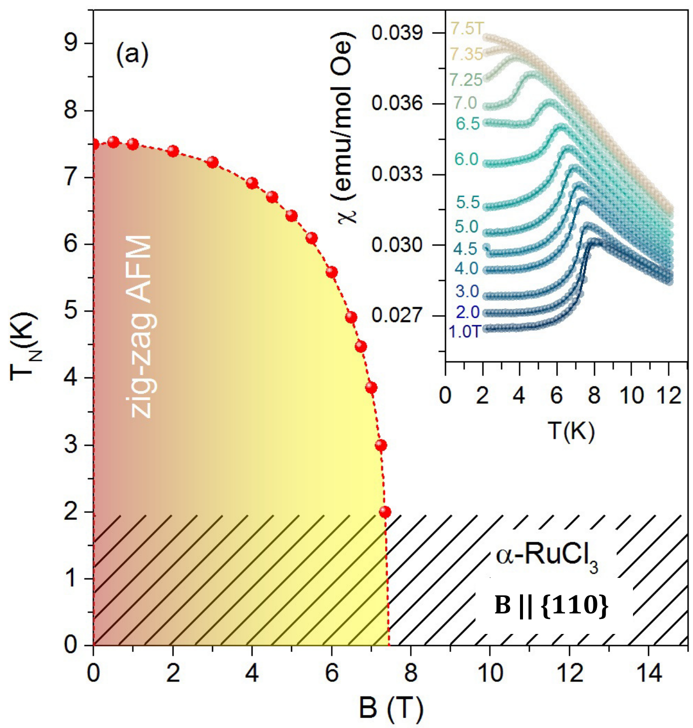


spin liquid or boring paramagnet?

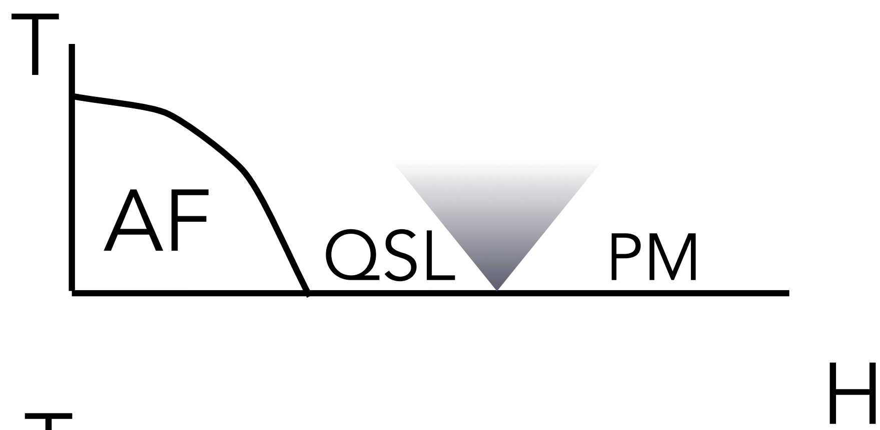
Suppressing order

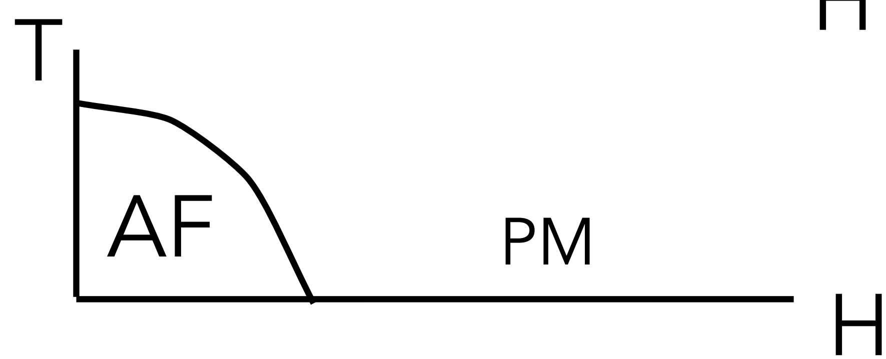
With a magnetic field:

A. Banerjee *et al*, 2017



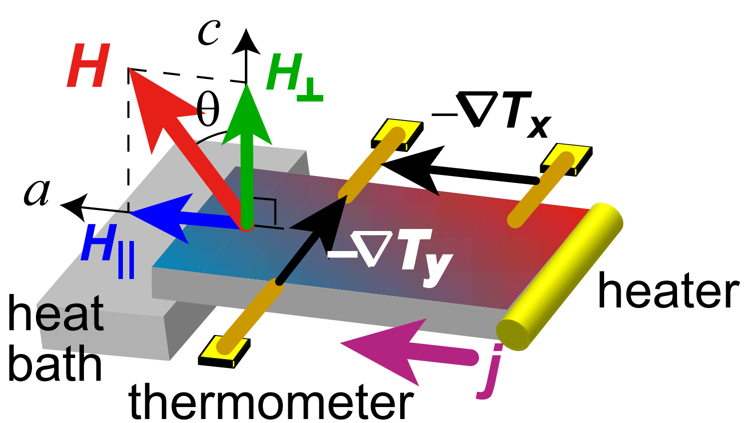
theoretical possibilities

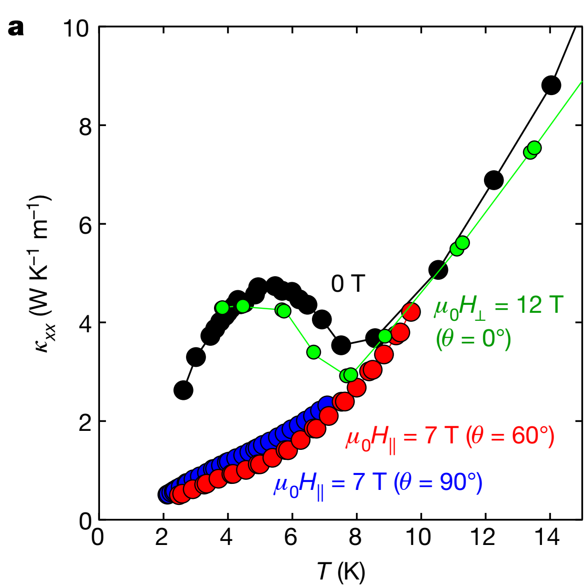




Thermal Hall Effect

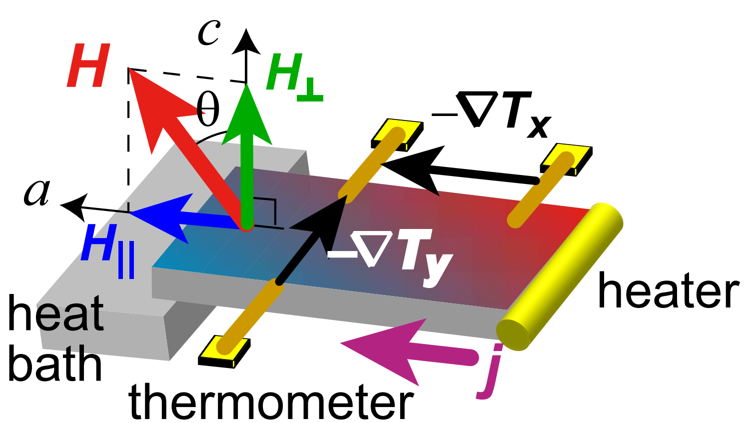
Y. Kasahara *et al*, Nature 2018

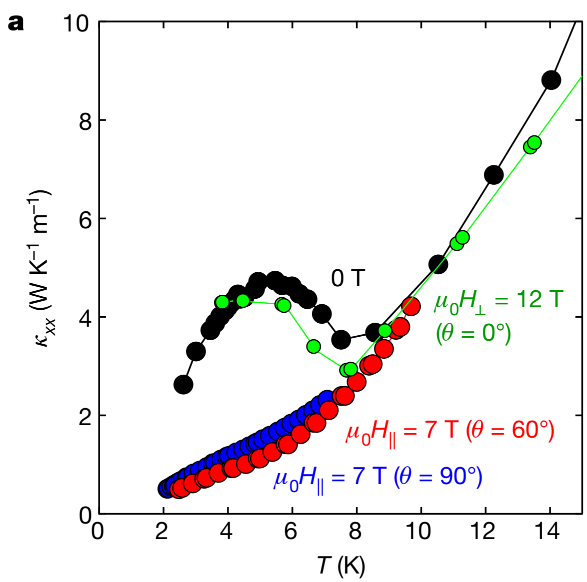


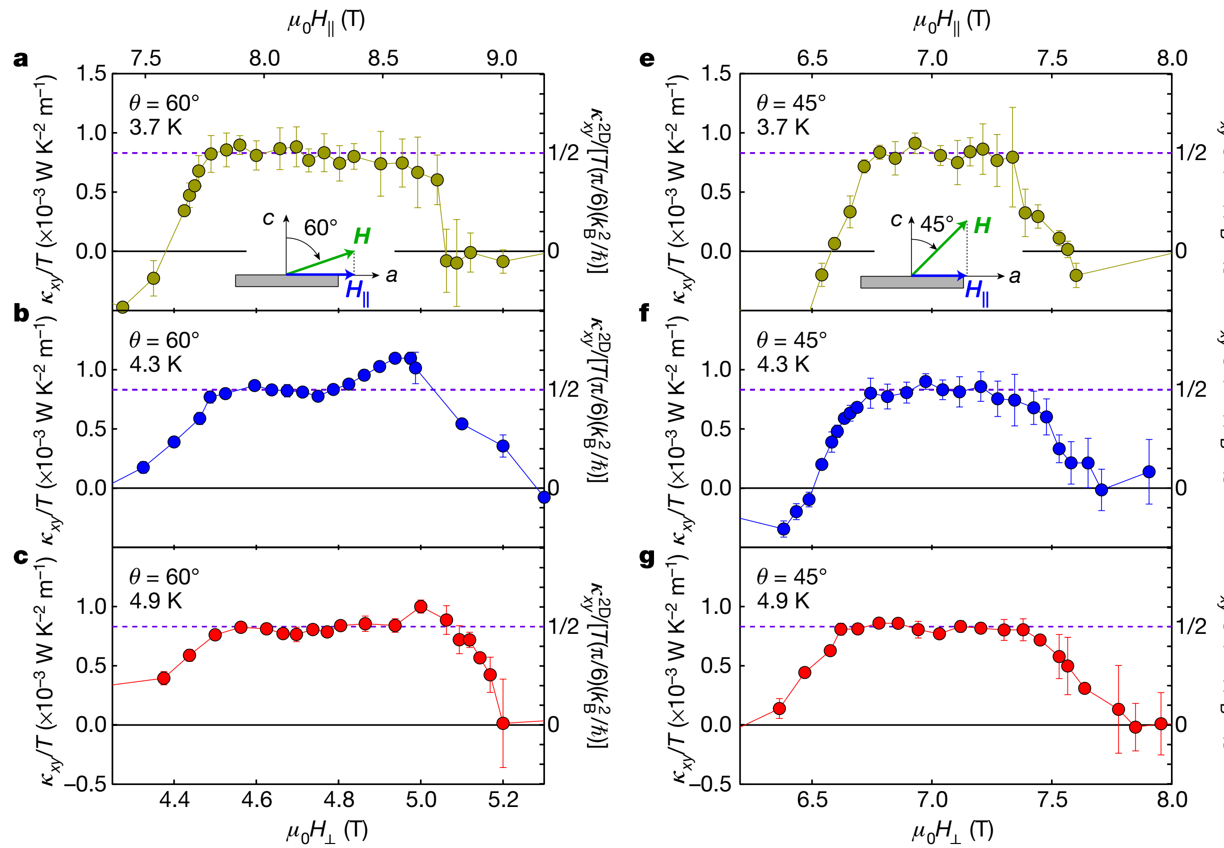


Thermal Hall Effect

Y. Kasahara *et al*, Nature 2018

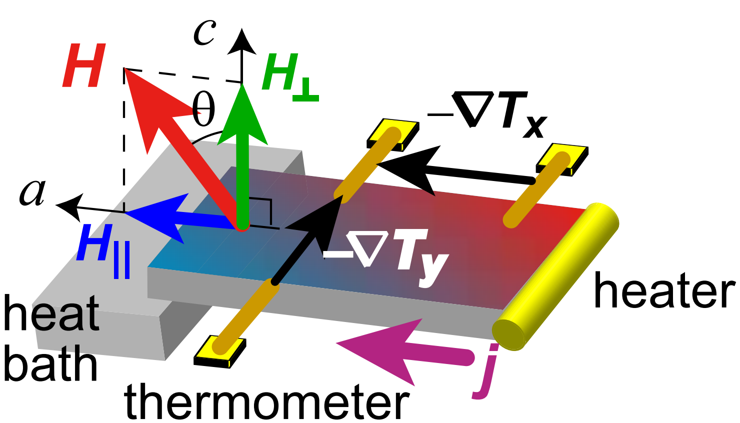


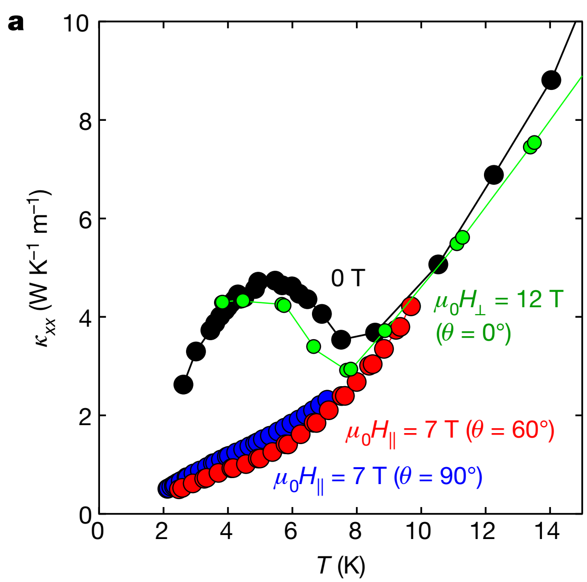


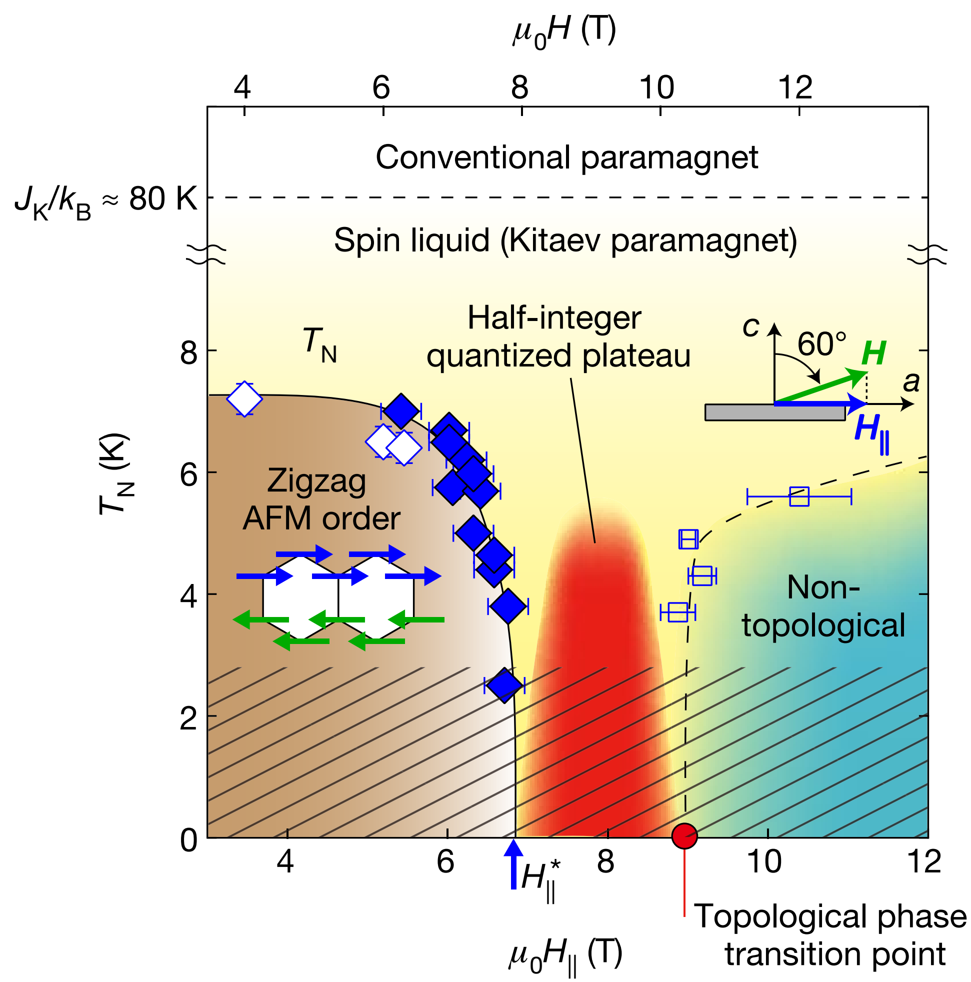


Thermal Hall Effect

Y. Kasahara *et al*, Nature 2018

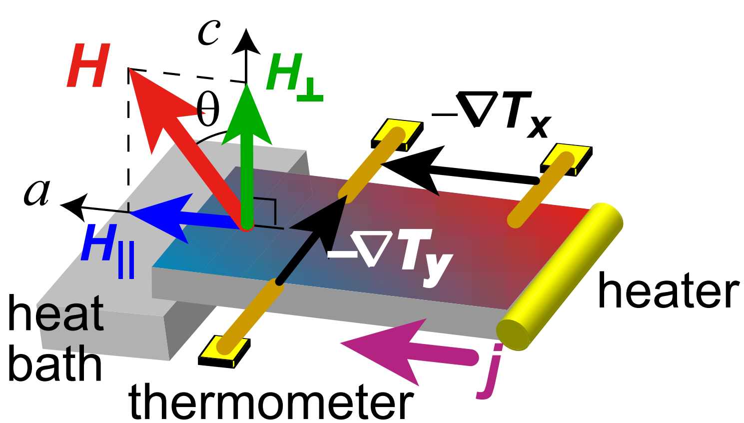


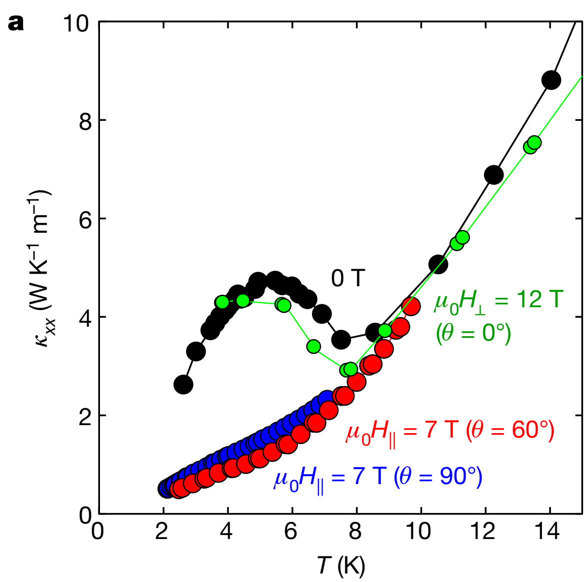


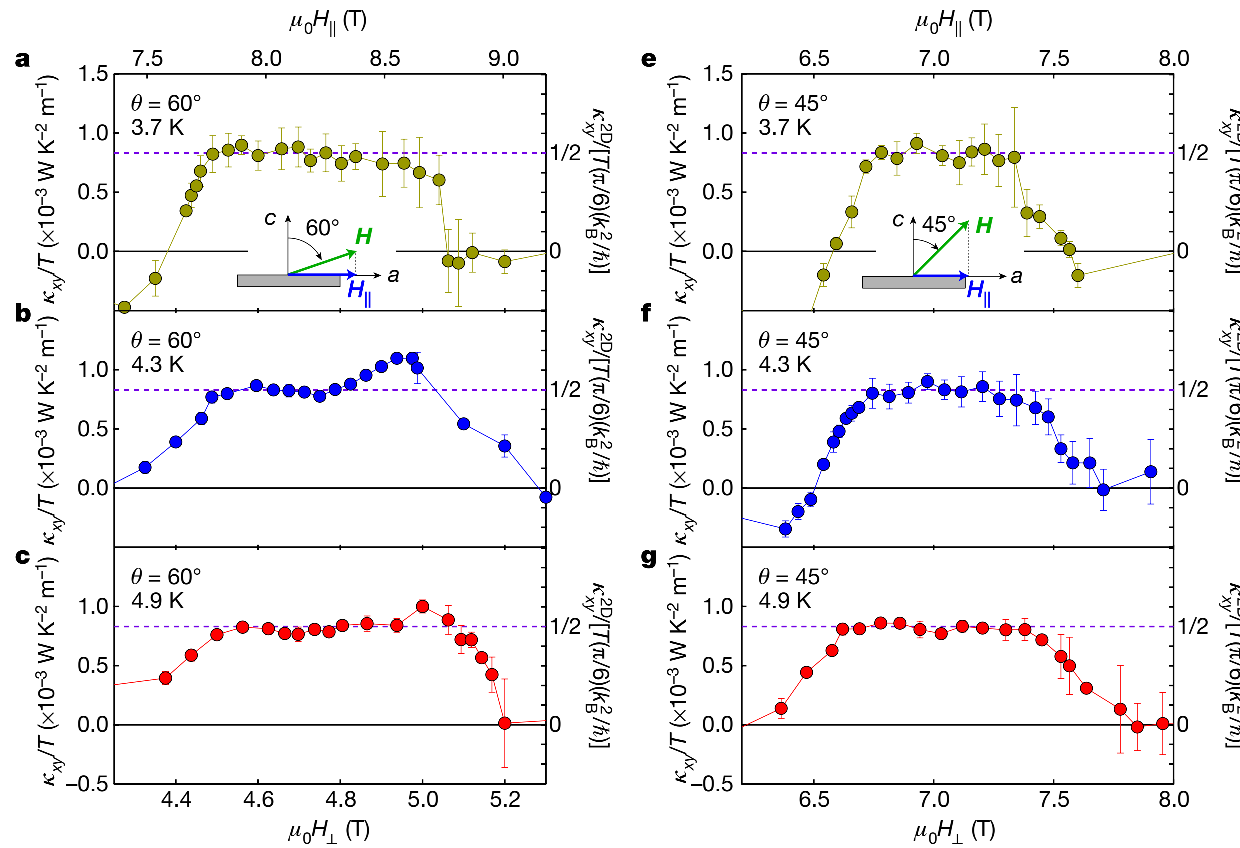


Thermal Hall Effect

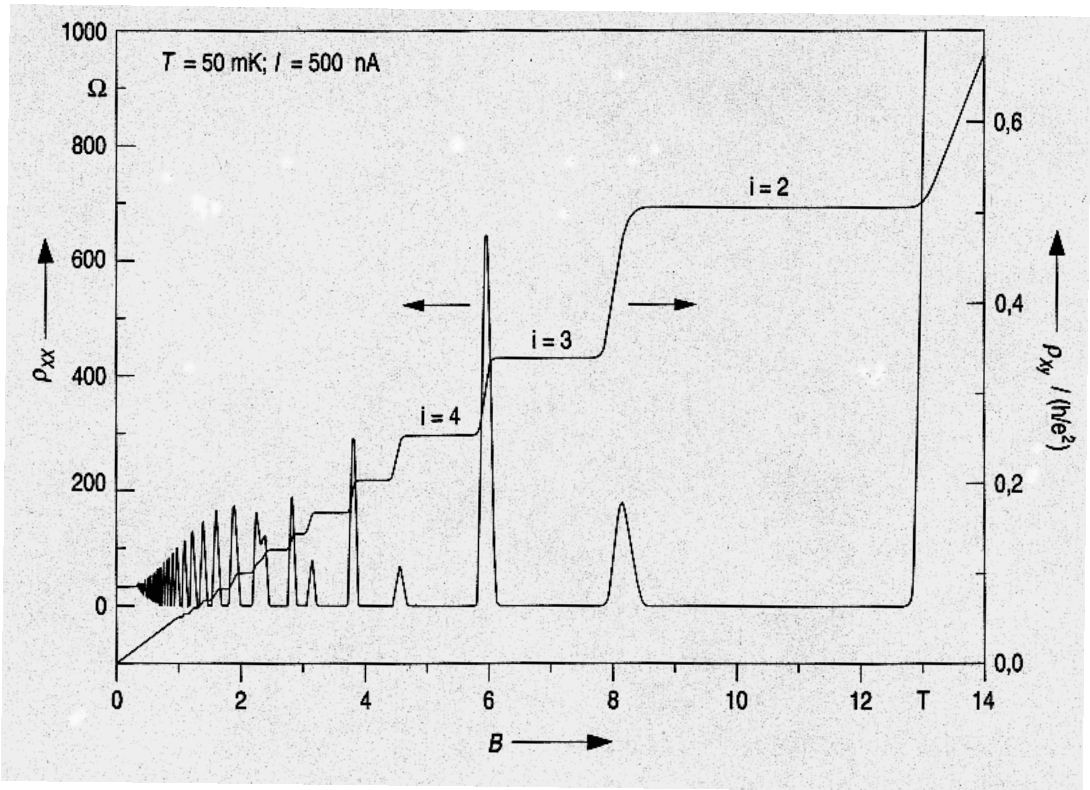
Y. Kasahara *et al*, Nature 2018







Why worry?



Quantization is *only* visible when diagonal conductivity is small, i.e. Hall angle ~ 90degrees

semicircle "law"

$$\sigma_{xx}^2 + \left(\sigma_{xy} - \frac{\sigma_1 + \sigma_2}{2}\right)^2 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2$$

c.f. IQHE

But for $RuCl_3$, thermal Hall angle ~ 10^{-3} !!

Theoretical Problem

From Y. Kasahara *et al*

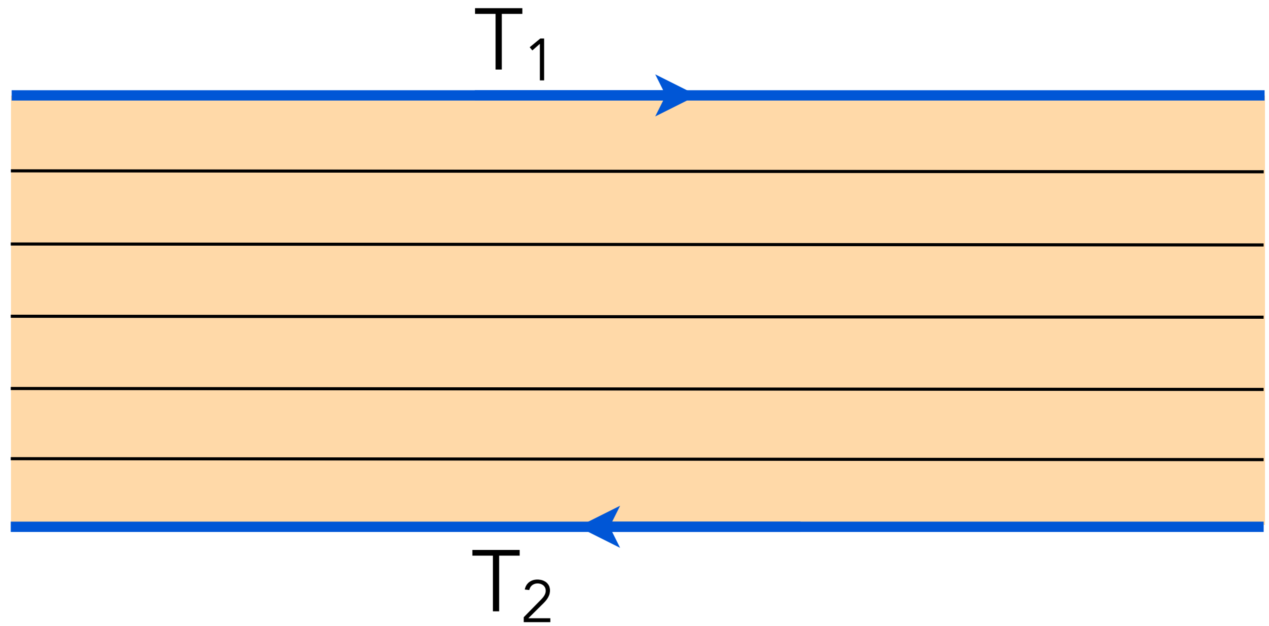
In the plateau regime of κ_{xy}, no anomaly is observed in κ_{xx}, probably because phonon contributions largely dominate over fermionic excitations arising from spins in κ_{xx} in the whole temperature range[28,29].

Q: Can quantized thermal Hall effect persist when phonons dominate bulk conductivity?

A: **Yes**, and phonons actually *help* to make the effect observable.

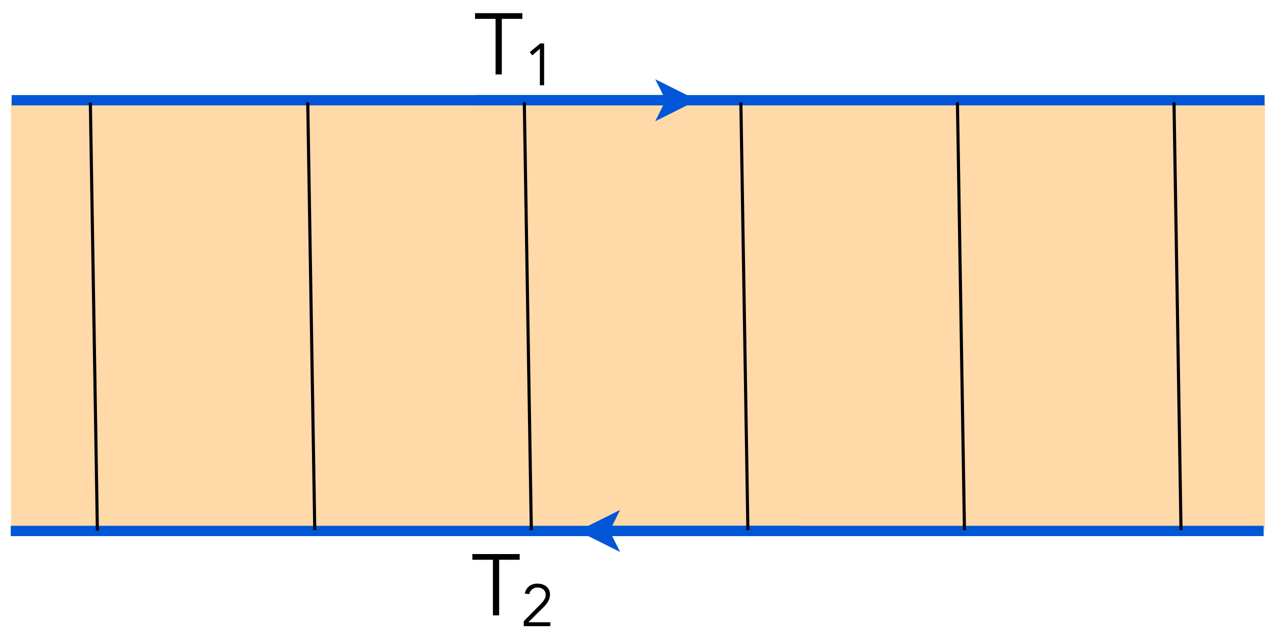
Theoretical Problem

Recall *derivation* of QTHE



Theoretical Problem

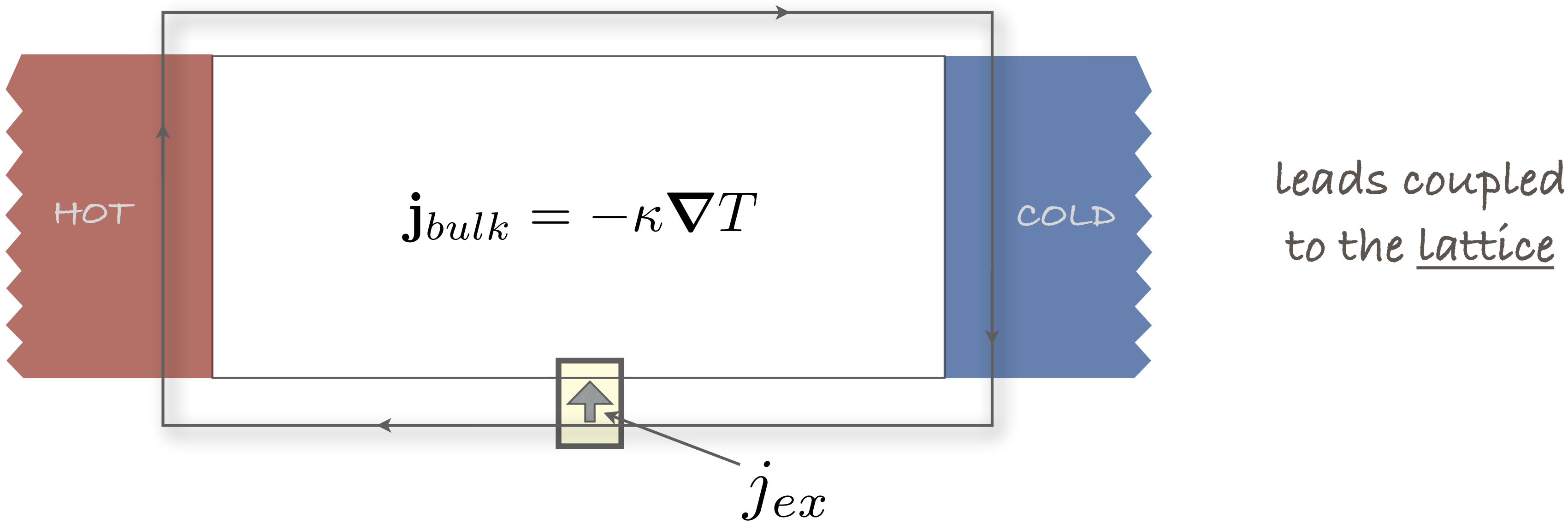
Recall *derivation* of QTHE



$\kappa_{xx} \gg \kappa_{xy}$: Temperature of phonons in *not* constant at the edge ?? Could edge be out of equilibrium with the phonons and have constant T??

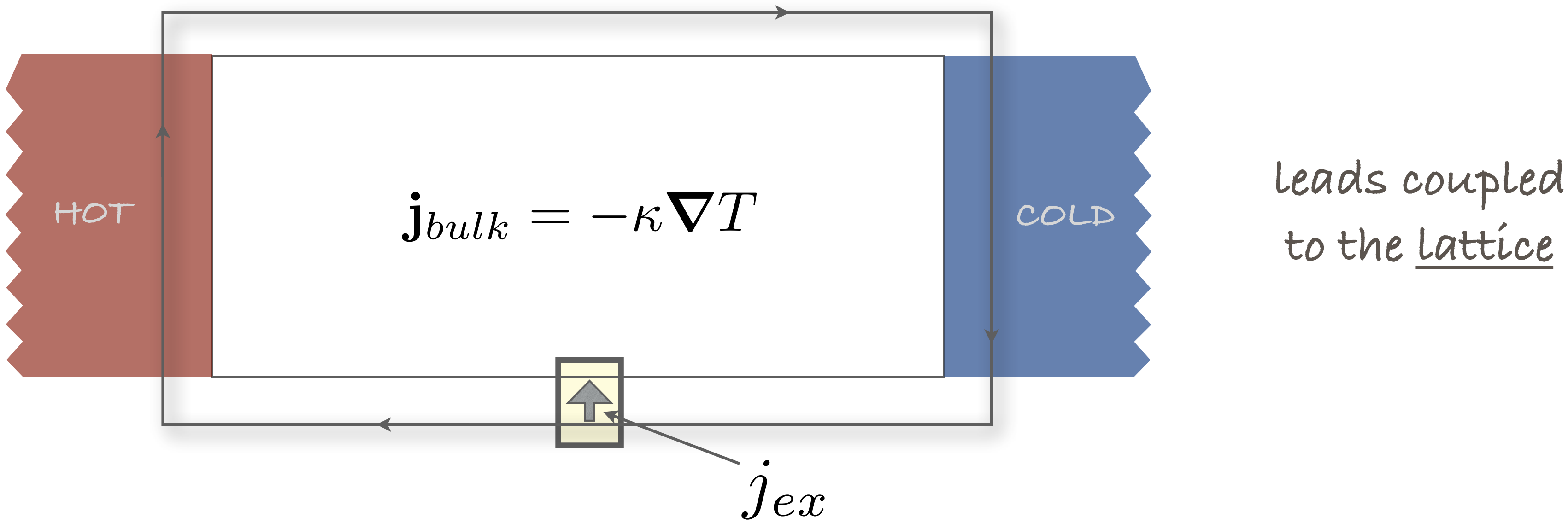
- but: in that case, which T is measured?

Formulation



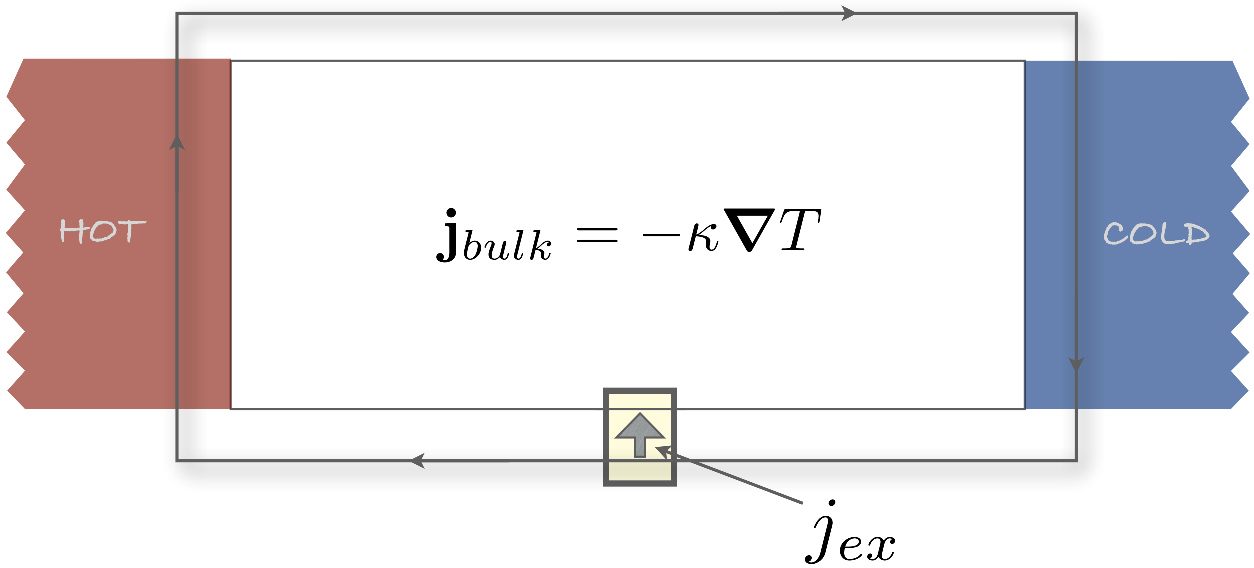
- Assume spins go into nonabelian QSL phase: bulk spin gap, chiral edge state
- Include bulk phonons with *no* Hall conductivity
- Majoranas and phonons can exchange energy at the edge of the sample

Formulation



- Variables:
  - $T_{ph}(x,y)$ = phonon temperature in bulk
  - $T_f(x,L_y)$ etc = fermion temperature at edge

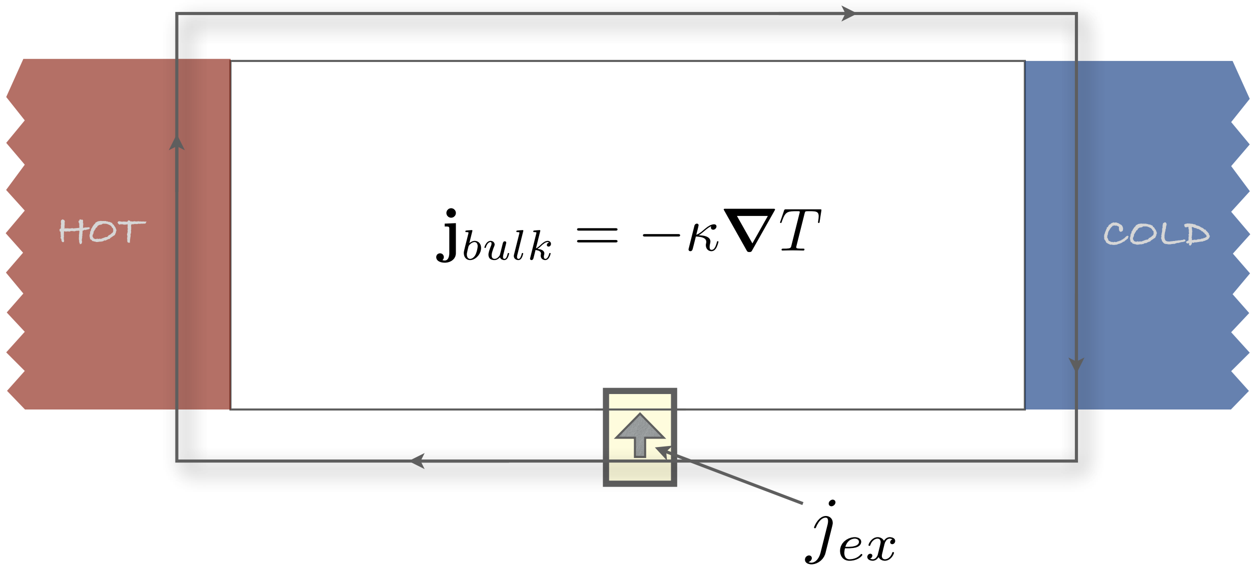
Formulation



leads coupled
to the lattice

- Currents:
  - Bulk $\boldsymbol{j}_{ph} = -\kappa \boldsymbol{\nabla} T_{ph}$
  - Edge $I_f = \frac{\pi}{24} T_f^2$

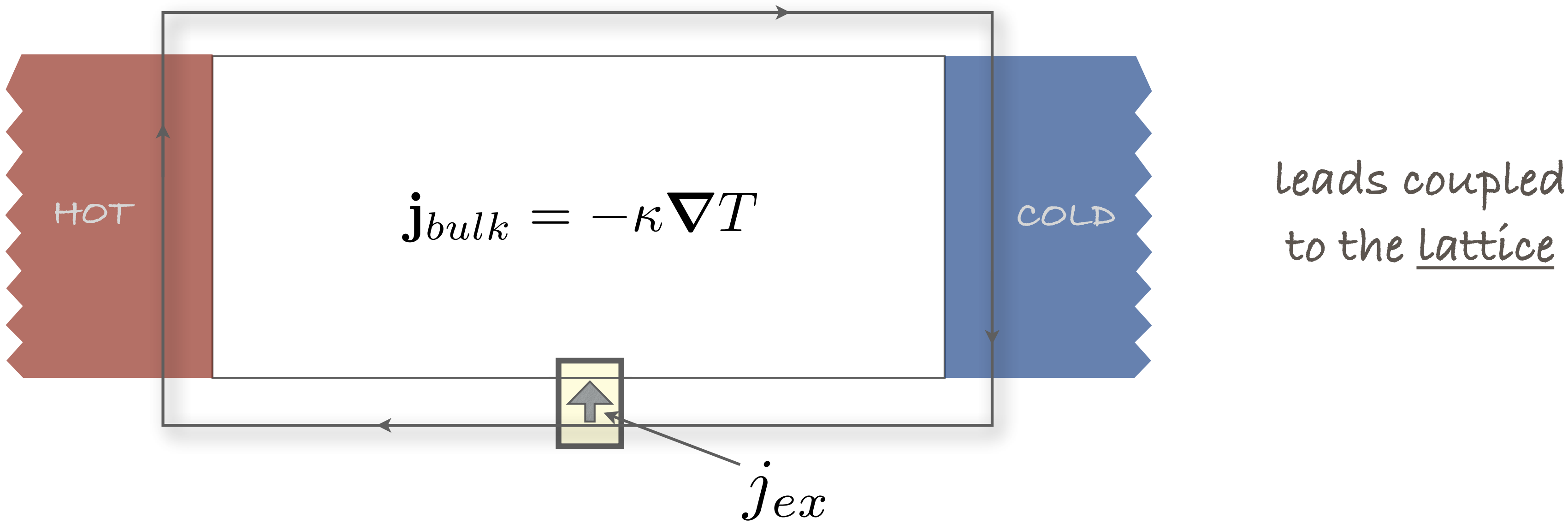
Formulation



leads coupled to the lattice

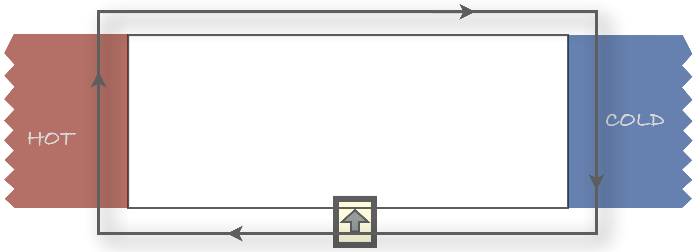
- Continuity:
  - Bulk $\nabla \cdot \boldsymbol{j}_{ph} = 0$
  - Edge $\partial_x I_f = \pm j_{ex}$, $\pm j^y_{ph}(x, \pm y_0) = j_{ex}(x, \pm y_0).$

Formulation



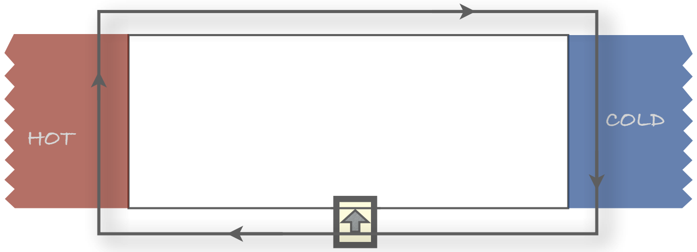
- Coupling:
  - linear response $j_{ex} = \lambda(T)(T_{ph} - T_f)$

Formulation



- Full equations (for *infinite* Hall bar)
  - bulk $\nabla^2 T_{ph} = 0$
  - edge $$\kappa^q_{xy} \partial_x T_f(x, \pm y_0) = -\kappa \partial_y T_{ph}(x, \pm y_0)$$ $$= \pm\lambda(T)\,(T_{ph} - T_f)|_{x, \pm y_0}$$

Solution

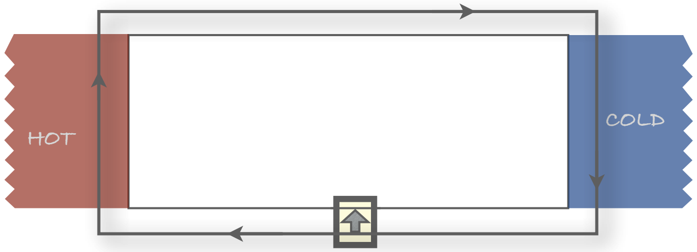


- Solution
  - Bulk $$T_{ph}(x,y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$$
  - edge $$T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$$

$$\frac{\partial T_{ph}}{\partial y} = -\frac{\kappa^q_{xy}}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

Solution



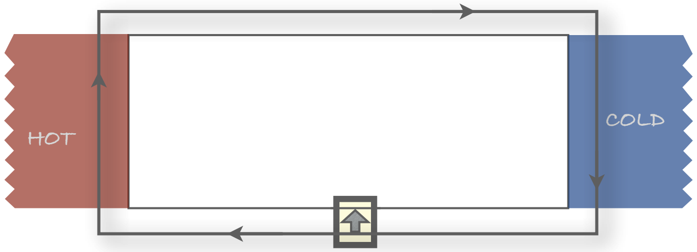
- Solution
  - Bulk $$T_{ph}(x,y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$$
  - edge $$T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$$

$$\frac{\partial T_{ph}}{\partial y} = -\frac{\kappa^q_{xy}}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

phonon Hall angle corresponds to quantization! **Implies quantization in 3 probe experiment measuring T_{ph}**

Solution



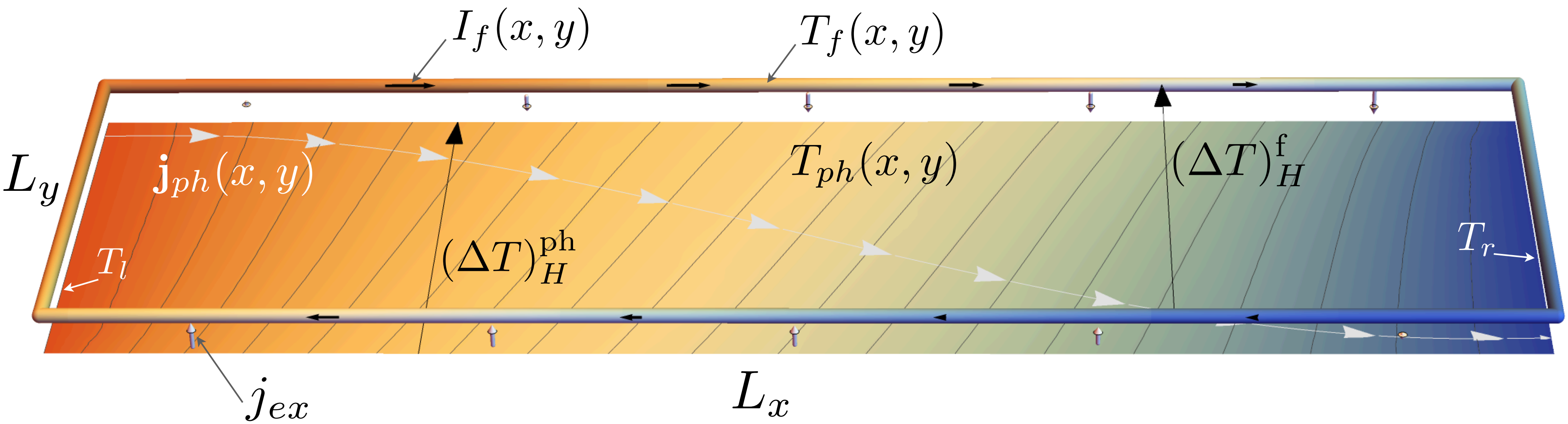
- Solution
  - Bulk $$T_{ph}(x,y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$$
  - edge $$T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$$

$$\frac{\partial T_{ph}}{\partial y} = -\frac{\kappa^q_{xy}}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

Majorana temperature *not* equal to phonon one. It is actually *better* to measure phonon temperature!

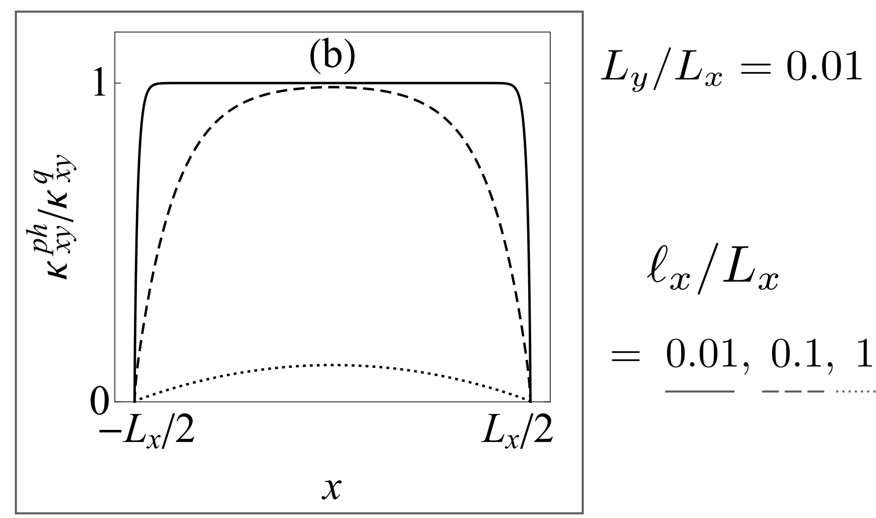
Finite Hall bar



- Solve coupled Laplace and boundary equations, with constant temperature leads.

- Reveals dependence on "thermalization length"

$$\ell_x = \frac{\kappa_{xy}^q}{\lambda}$$



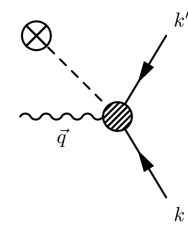
Bulk-edge coupling

local phonon strain couples
to Majorana kinetic energy

$$H_{int} = \frac{-igv_f}{4} \int \mathrm{d}x\, \zeta(x) K_{ij} \partial_i u_j(x, y_0) \eta(x) \partial_x \eta(x),$$

We use kinetic equation to
calculate

$$\Big(\frac{\partial \mathcal{E}}{\partial t}\Big)_{ph \to f} = -\Big(\frac{\partial \mathcal{E}_{ph}}{\partial t}\Big) = -\sum_{q_x, q_y} \omega_{\vec{q}} \Big(\frac{\partial g(\vec{q})}{\partial t}\Big)_{coll},$$



Rate

$$\Big(\frac{\partial g(\vec{q})}{\partial t}\Big)_{coll} = \frac{2\pi}{\hbar} \times 2 \sum_{k,k'} \{ \langle |\mathcal{M}^{+}(\vec{q}, k, k')|^2 \rangle_{dis} (1 + g_\omega) f_\epsilon f_{\epsilon'} \delta(-\omega_{\vec{q}} + \epsilon_k + \epsilon_{k'}) - \langle |\mathcal{M}^{-}(\vec{q}, k, k')|^2 \rangle_{dis}\, g_\omega f_\epsilon f_{\epsilon'} \delta(\omega_{\vec{q}} + \epsilon_k + \epsilon_{k'}) \}$$

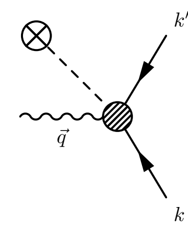
Bulk-edge coupling

local phonon strain couples to Majorana kinetic energy

$$H_{int} = \frac{-igv_f}{4}\int \mathrm{d}x\,\zeta(x)K_{ij}\partial_i u_j(x,y_0)\eta(x)\partial_x\eta(x);$$

We use kinetic equation to calculate

$$j_{ex} \propto \left(\frac{\partial \mathcal{E}}{\partial t}\right)_{ph\to f} = -\left(\frac{\partial \mathcal{E}_{ph}}{\partial t}\right) = -\sum_{q_x,q_y}\omega_{\vec{q}}\left(\frac{\partial g(\vec{q})}{\partial t}\right)_{coll},$$

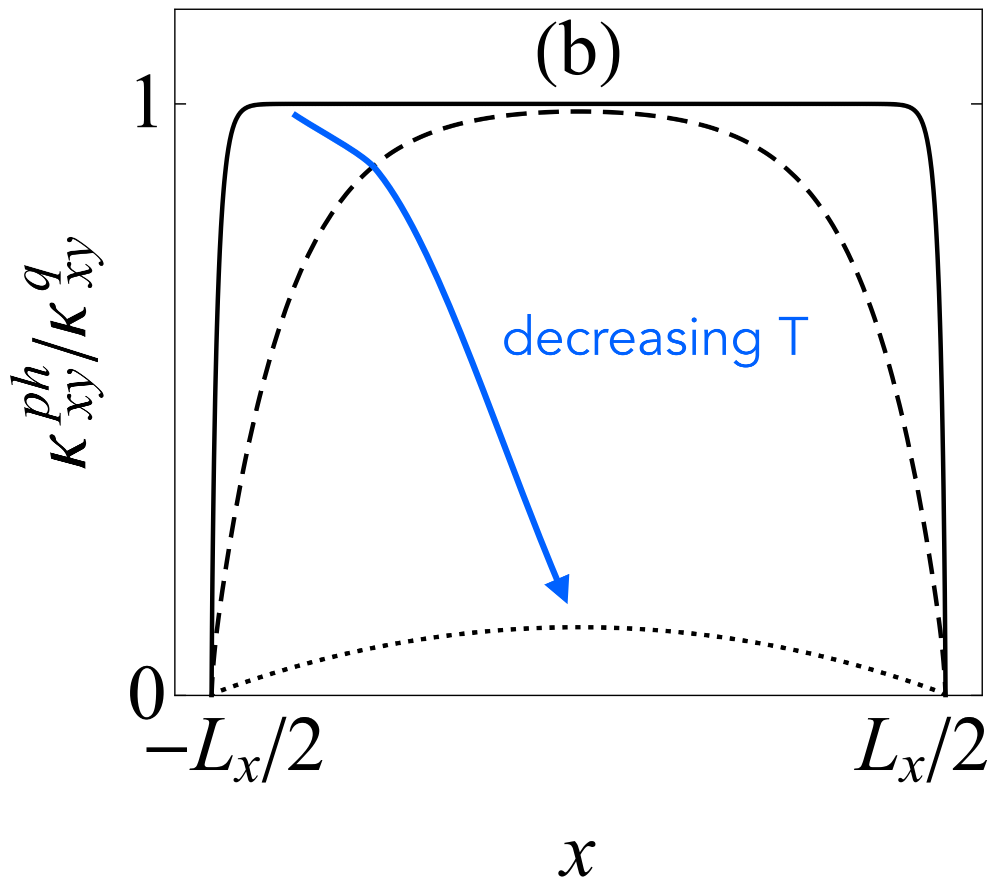


Result

$$\lambda = \frac{g^2\zeta^2}{32(2\pi)^3 v_{ph}^4 v_f^2 \rho_0} f\, T^6$$

strong T-dependence: thermalization becomes poor at low T.

Prediction



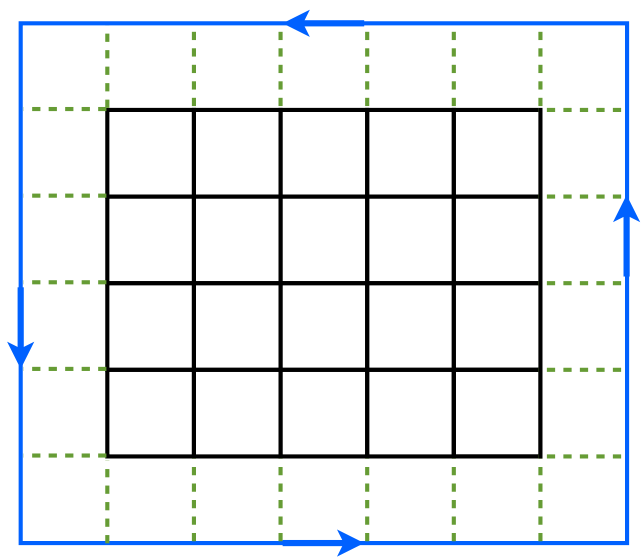
Expect *failure* of quantization at low enough temperature!

Disorder

- Experimental observation:
    - Quantization is observed only in the “best” samples, which have the largest *diagonal* thermal conductivity
- Presumably, sample variations involve lattice disorder that affects the phonon transport
    - What is the effect on thermal Hall measurements?

Disorder

- Network model



— resistor $I_{i\to j} = \kappa_{ij}(T_i - T_j)$

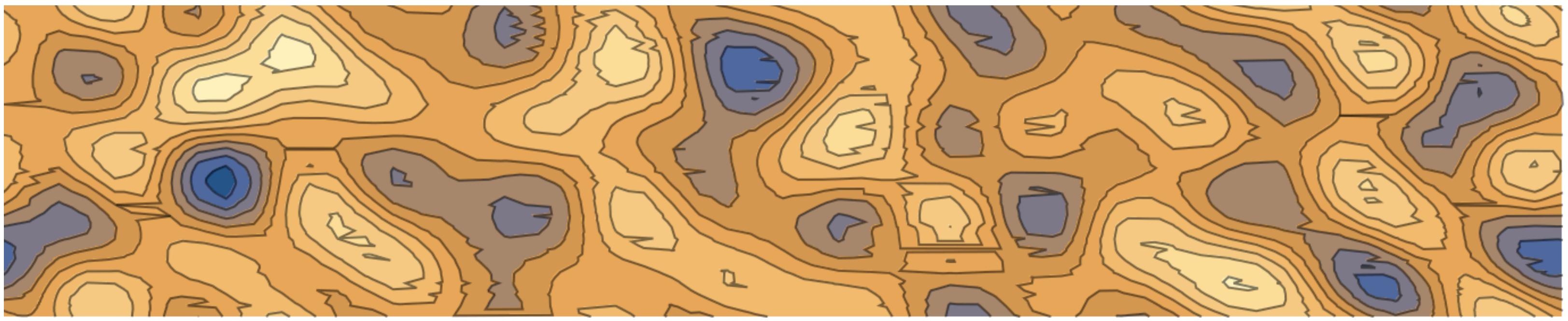
→ chiral $I_{i\to j} = \pm\kappa_H(T_i + T_j)/2$

- - - resistor $I_{i\to j} = \lambda(T_i - T_j)$

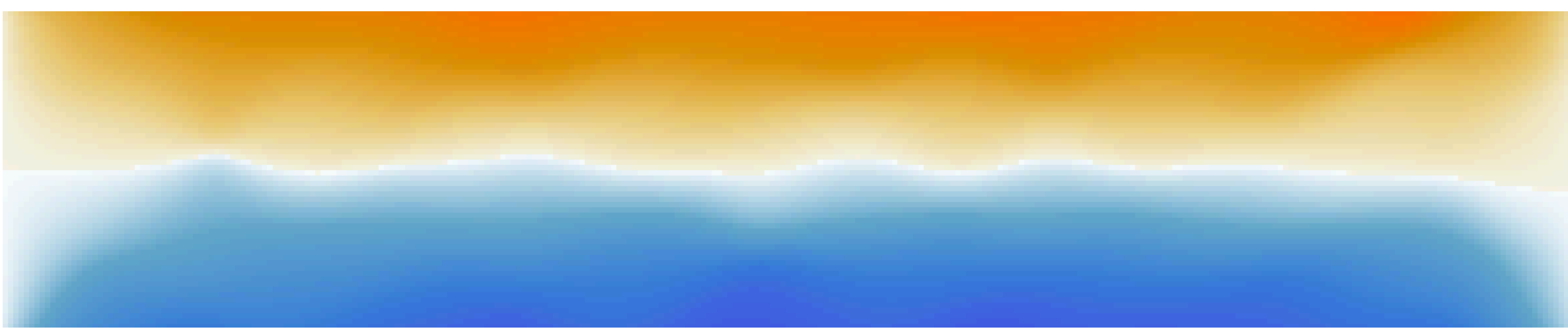
- Disorder included as random resistors

Disorder

Input: smooth random conductivity

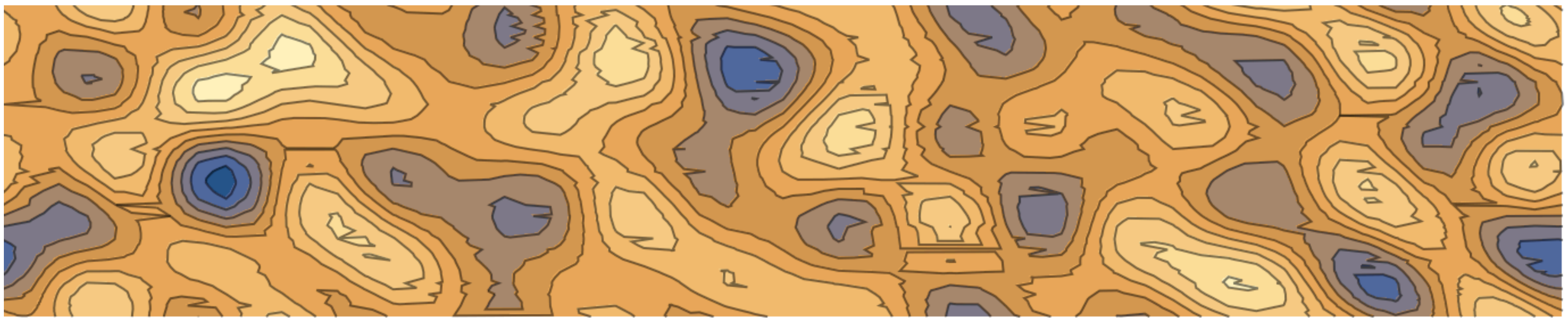


Output: Hall temperature gradient
(anti-symmetrized in field)

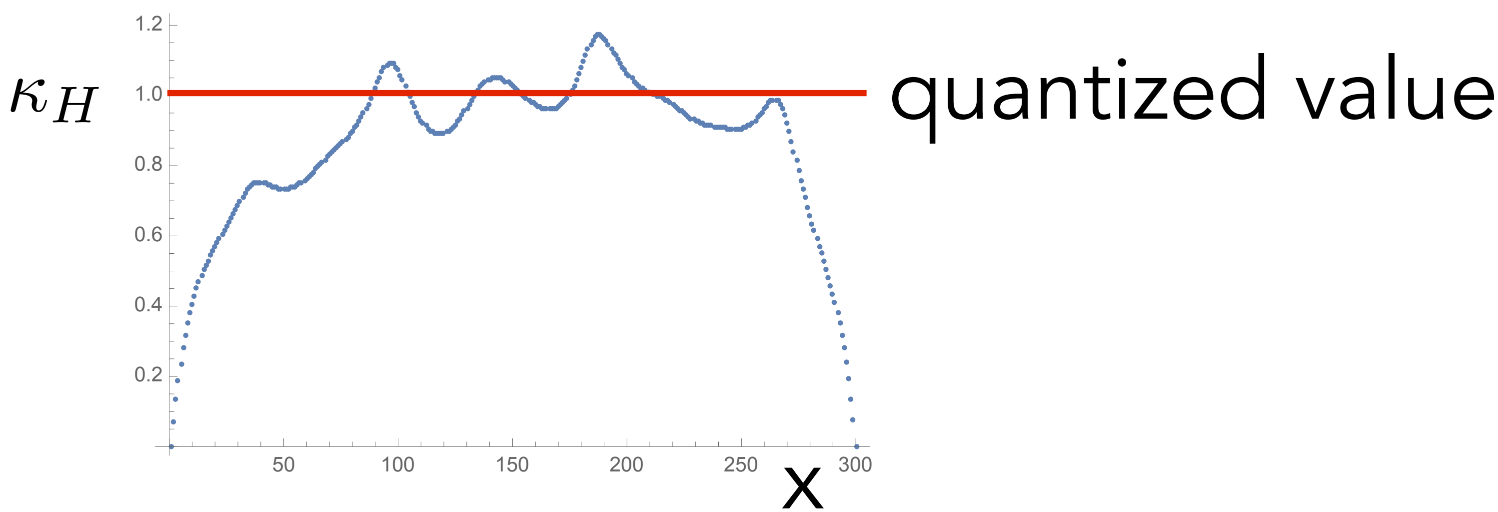


Disorder

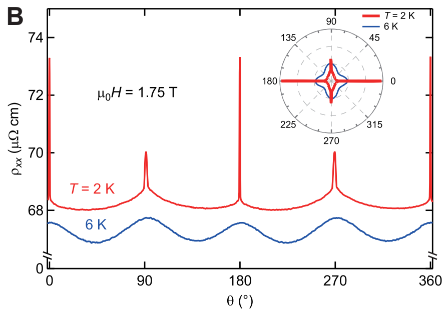
Input: smooth random conductivity



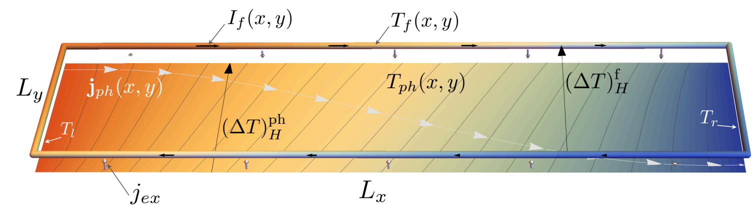
Output: Hall temperature gradient (anti-symmetrized in field)



Summary



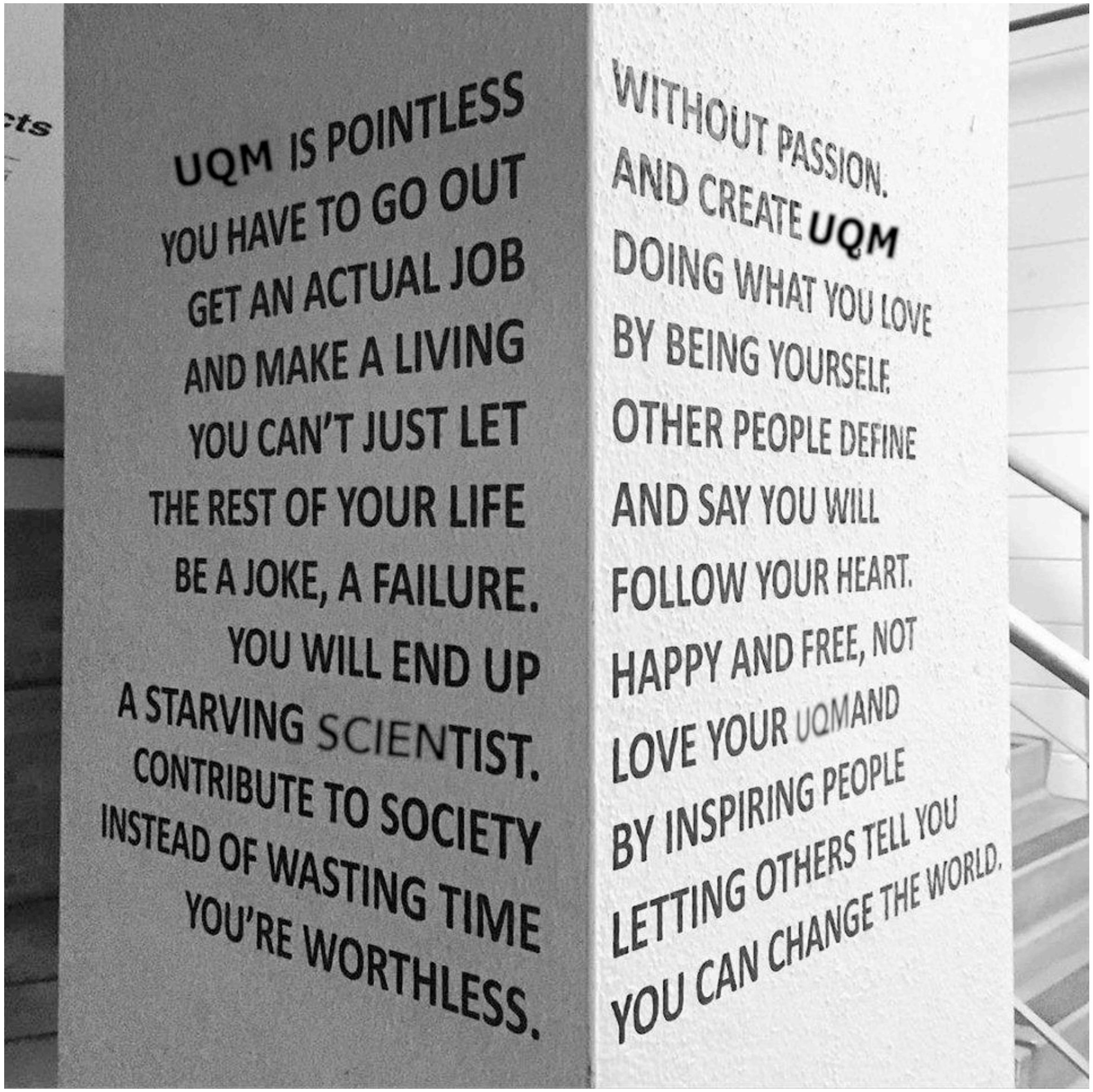
- Singular angular magneto-resistance: the result of SSB combined with the accidental but protected nature of Weyl points



- Quantized value of thermal Hall conductivity *can* be measured even when large phonon conductivity is present
- Quantization is only power-law good, and deviations are controlled by edge-bulk equilibration

ARXIV:1805.10532

U.S. DEPARTMENT OF ENERGY | Office of Science
NSF
GORDON AND BETTY MOORE FOUNDATION



based on the original by Jasmine Kay Uy, 2015