Measuring thermal Hall effect in a chiral spin liquid

Leon Balents, KITP

Collaborators

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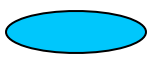
- Gábor Halász (KITP -> ORNL)

- Lucile Savary (ENS Lyon)

Quantum Spin Liquid

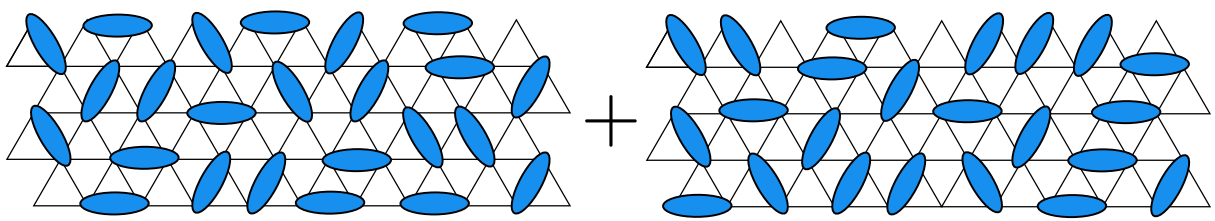
Phil Anderson, 1973

a "quantum liquid" of spins



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Ψ =

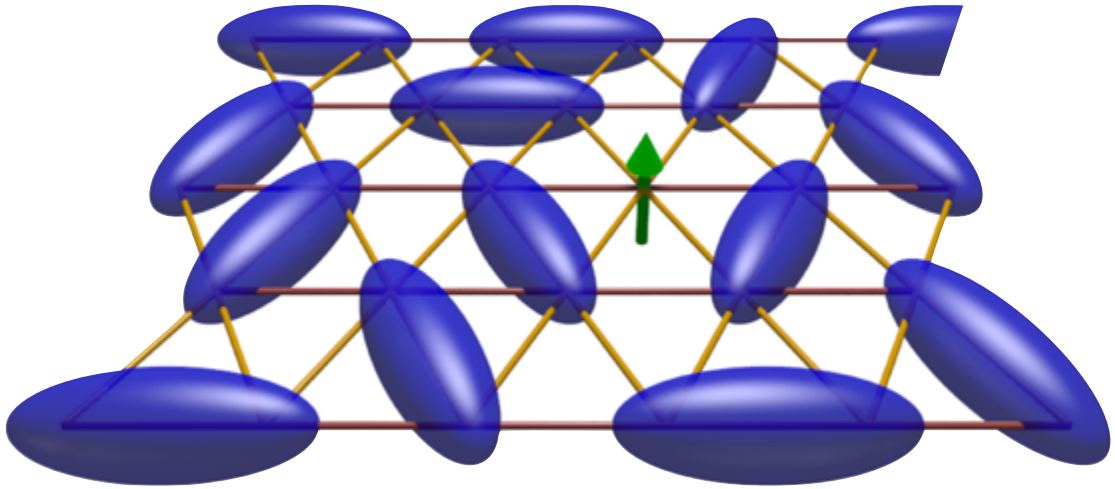


+...

Resonating **V**alence **B**ond state

Fractional quantum number

spinon



excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins

always created in pairs by any
local operator

Smoking Gun?

Difficult to find incontrovertible and sharp indicator of a QSL!

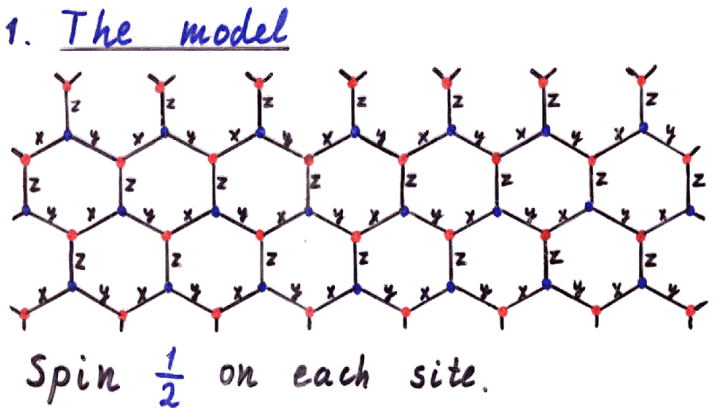
A few possibilities

- T-linear thermal conductivity in spinon Fermi surface QSL **observed!**
- Transverse, linearly dispersing emergent photon mode in 3d U(1) QSL **not yet observed**

Kitaev model

Kitaev's honeycomb model $$H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$$

KITP, 2003



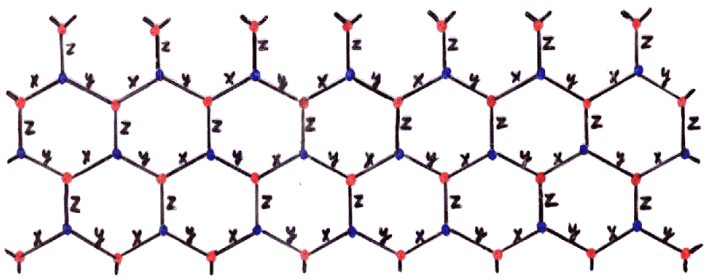
exact parton construction $\sigma_i^\mu = i c_i c_i^\mu \quad c_i c_i^x c_i^y c_i^z = 1$

effective quadratic Hamiltonian

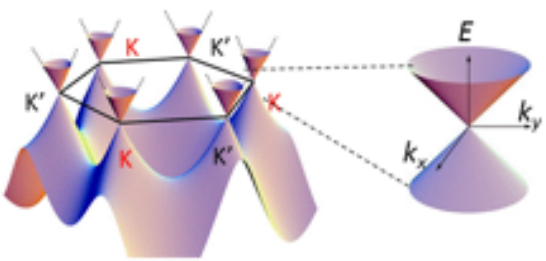
$$H = \sum_{i,\mu} i K_\mu c_i c_{i+\mu}$$

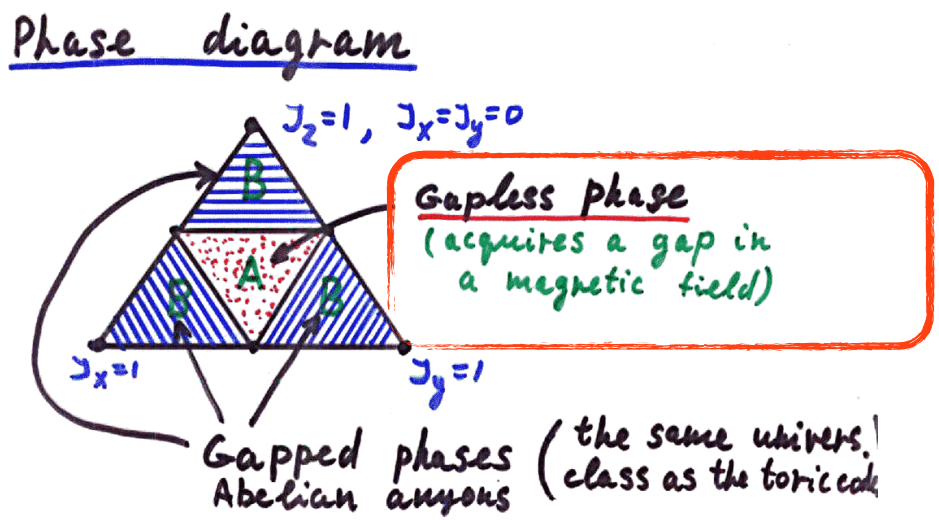
Gapless QSL

$$H = \sum_{i,\mu} iK_\mu c_i c_{i+\mu}$$

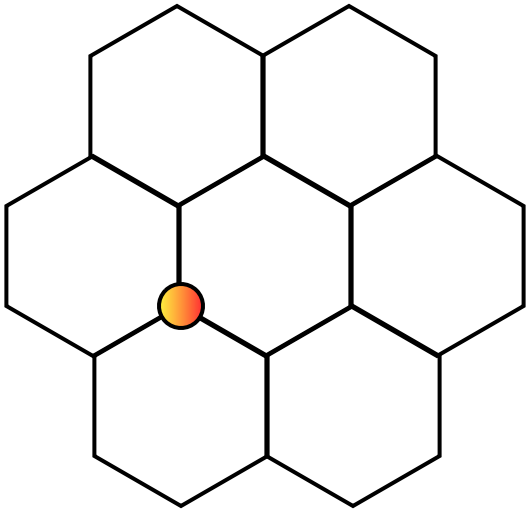


~1/4 of graphene

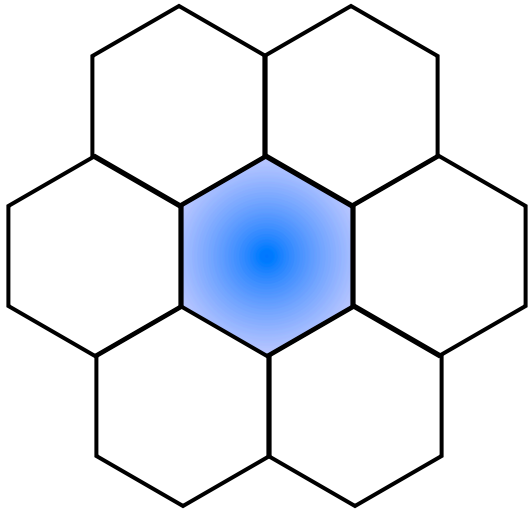




Non-local excitations



Majorana ε



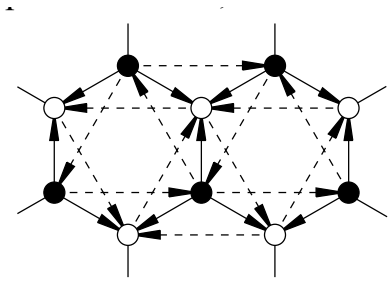
Flux e, m

In Kitaev's model:

- Majorana's dispersion ~ K and Dirac-like
- Fluxes are localized with small gap

Non-Abelian Phase

- In an applied magnetic field, the Majoranas acquire a gap

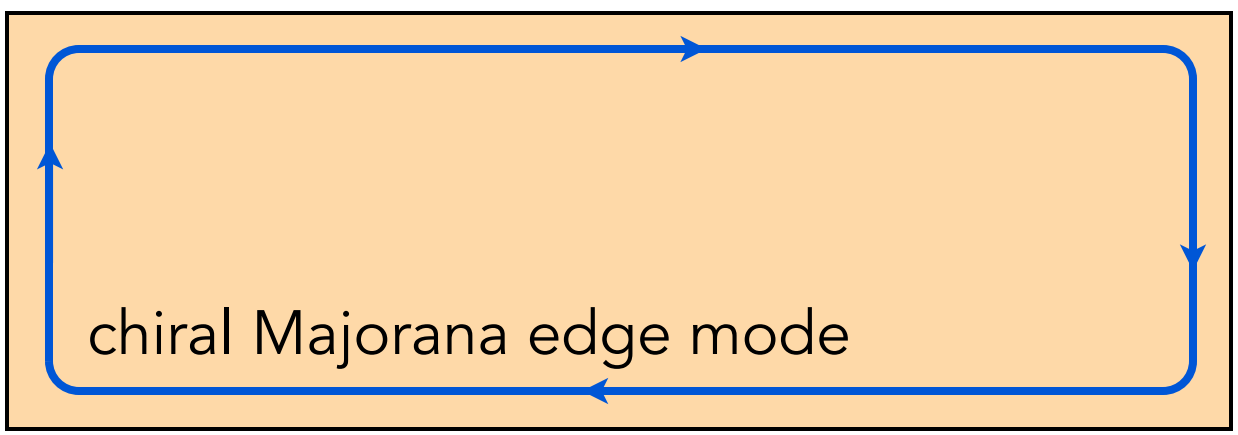


$$H_{\text{eff}} = \frac{\mathrm{i}}{4} \sum_{j,k} A_{jk} c_j c_k,$$

$$A = 2J\,(\longleftarrow) + 2\kappa\,(\leftarrow\cdots),$$

$$\kappa \sim \frac{h_x h_y h_z}{J^2}.$$

field induces a fermion mass, very similar to the Haldane model (except Majorana)



$$H_e = -\frac{iv}{4} \int dx\, \eta \partial_x \eta$$

Quantum Hall Effect?

- No charge. Have to study heat transport!

$$I = \int_0^\infty \frac{dq}{2\pi} v^2 q f(vq) = \frac{c\pi k_B^2}{12\hbar} T^2$$

central charge c=1/2

<u>c.f. c=1 for both IQHE *and* FQHE abelian states</u>

implies the existence of bulk non-abelian excitations (the fluxes, bound to MZMs)

Quantum Hall Effect?

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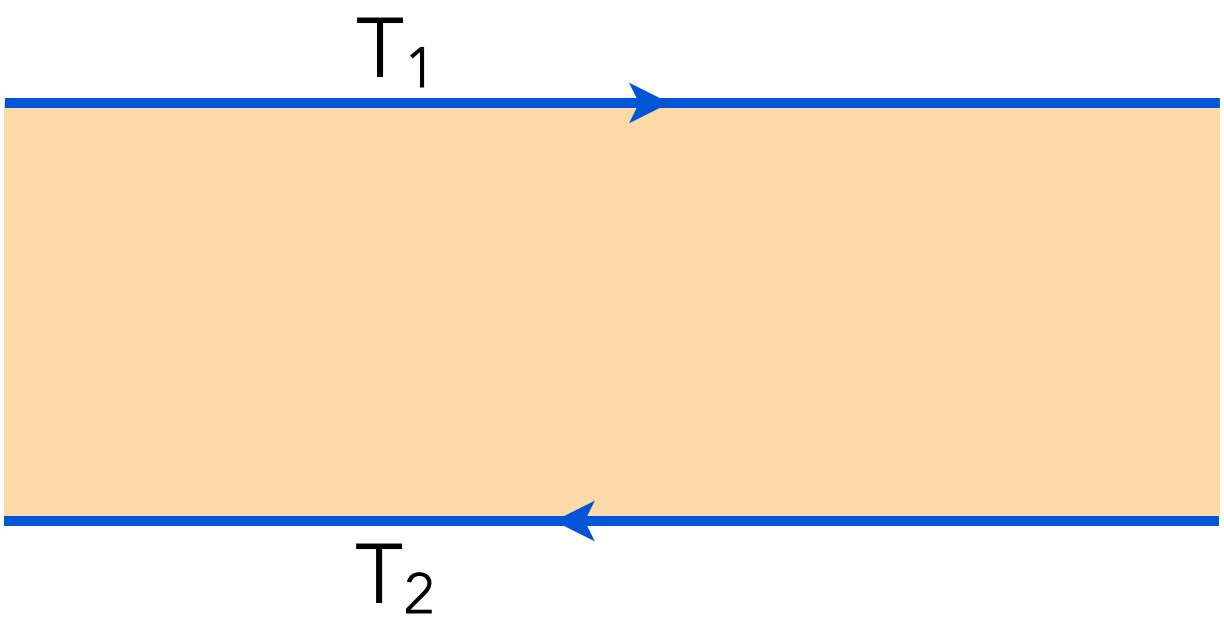

$$I = \frac{c\pi k_B^2}{12\hbar}\left(T_1^2 - T_2^2\right)$$

$$\approx \frac{c\pi k_B^2 T}{6\hbar}\left(T_1 - T_2\right)$$

Quantum Hall Effect?

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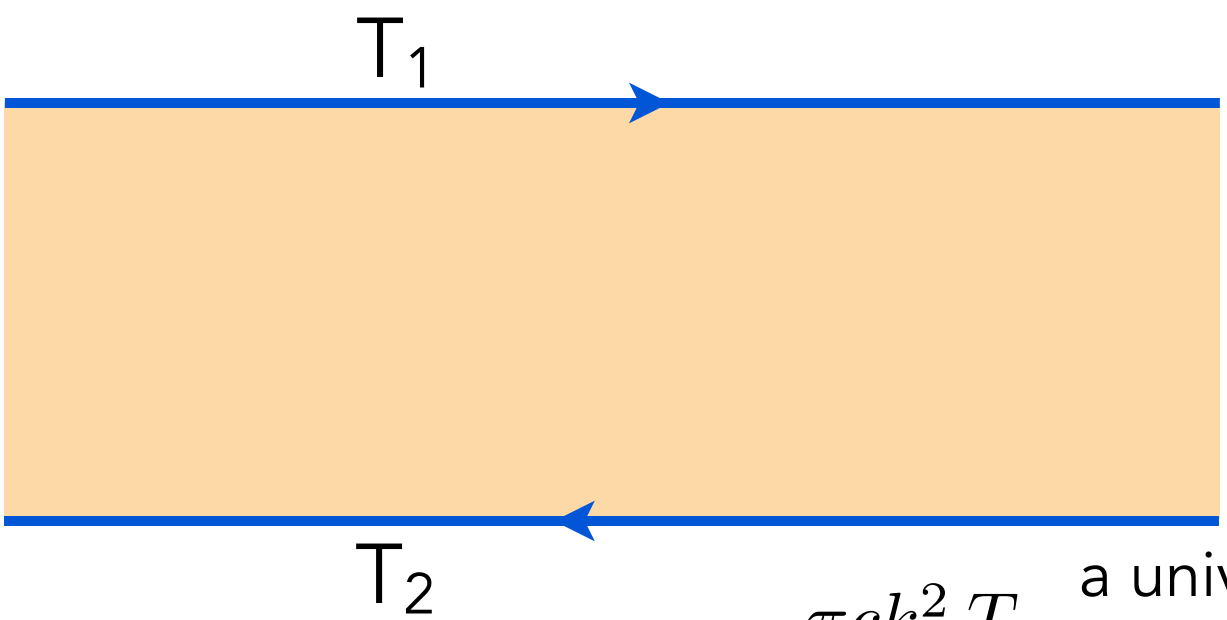


$$I_x = \kappa_H \Delta T_y$$

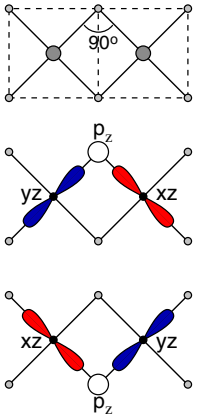
$$\kappa_H = \frac{\pi c k_B^2 T}{6\hbar}$$

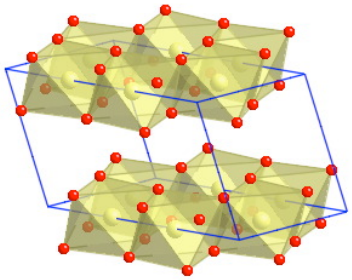
a universal prediction for chiral "Ising anyon" phase: *agnostic to microscopic spin interactions*

Kitaev Materials

Jackeli, Khaliullin
2009

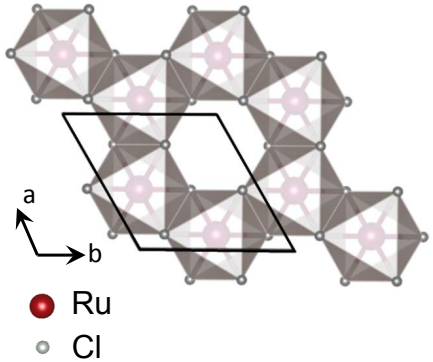
Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling





P.Gegenwart
H. Takagi
...

Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3



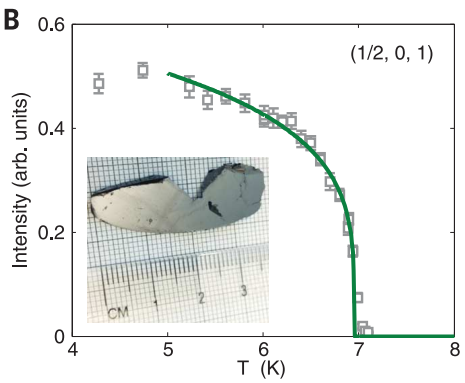
α-$RuCl_3$

Y.-J. Kim...

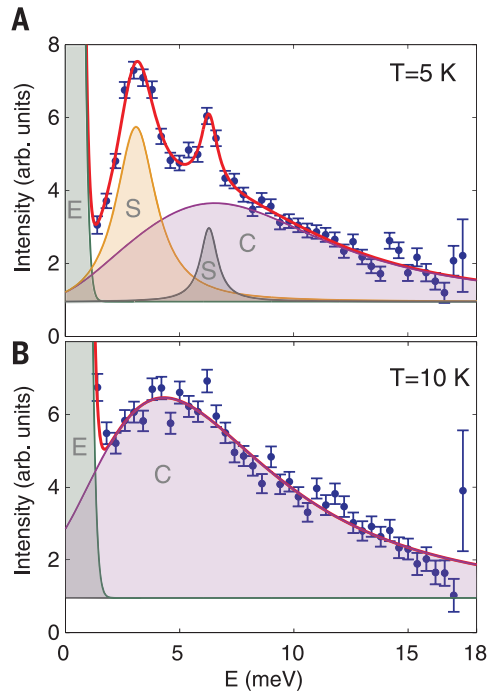
Honeycomb and hyper-honeycomb structures

α-$RuCl_3$

Clearly not described by ideal Kitaev model, but still interesting



zig-zag order

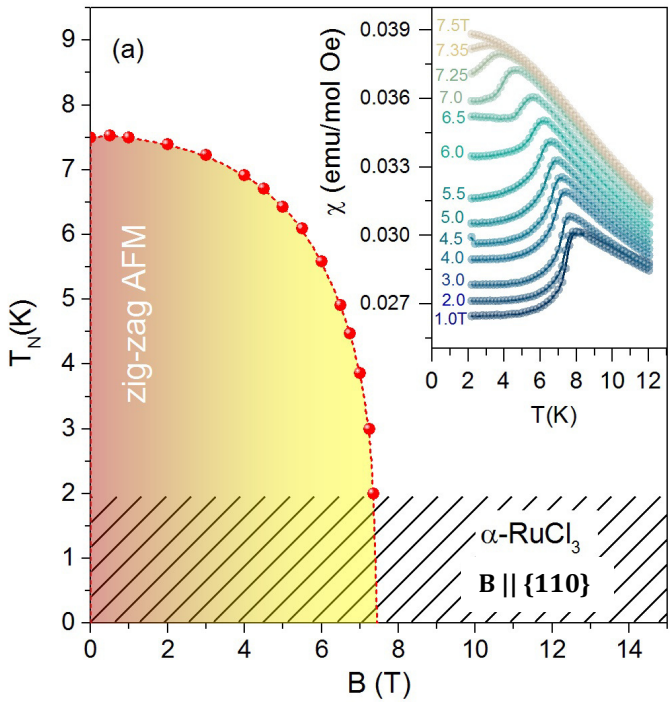


BUT: excitations to ~15meV = 170K

Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017

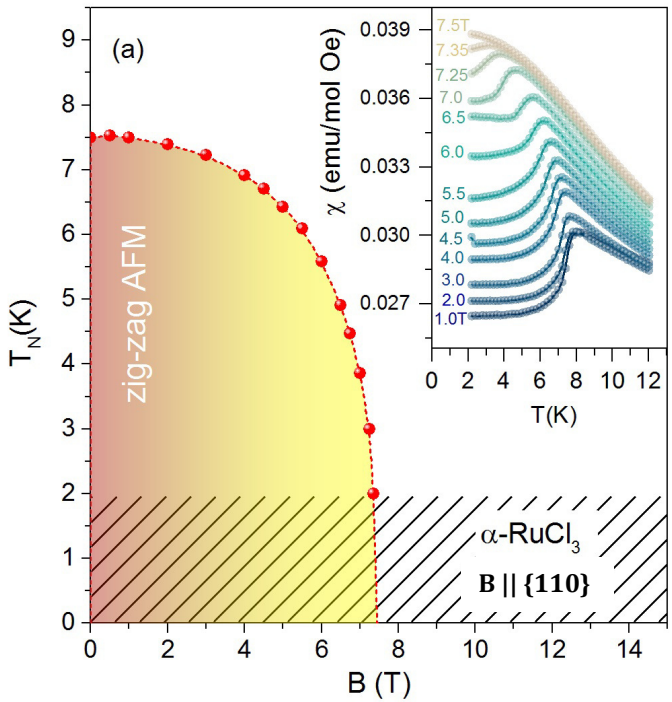


spin liquid or boring paramagnet?

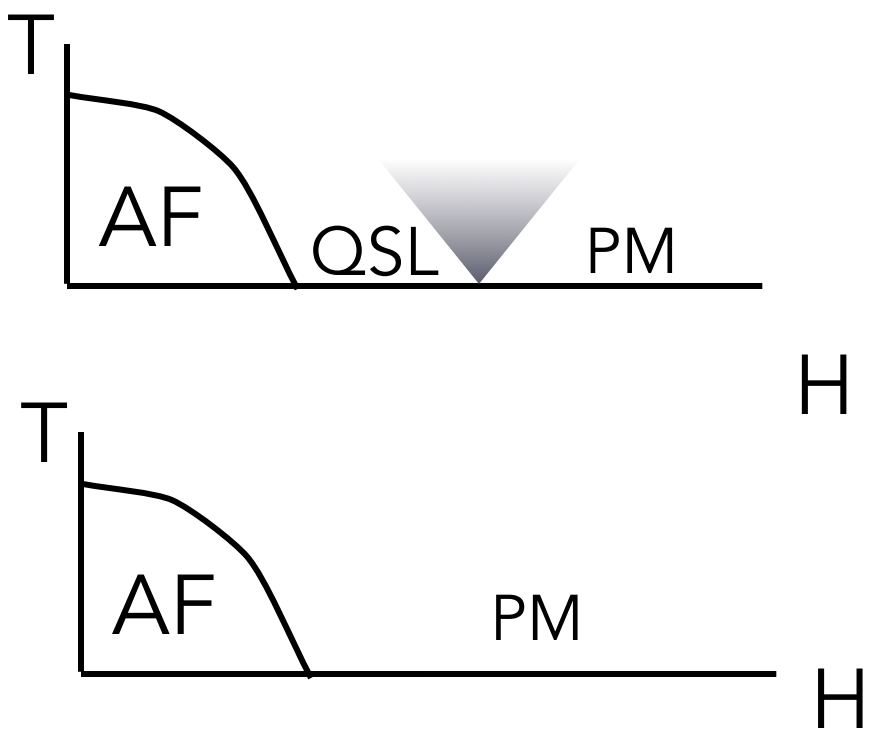
Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017

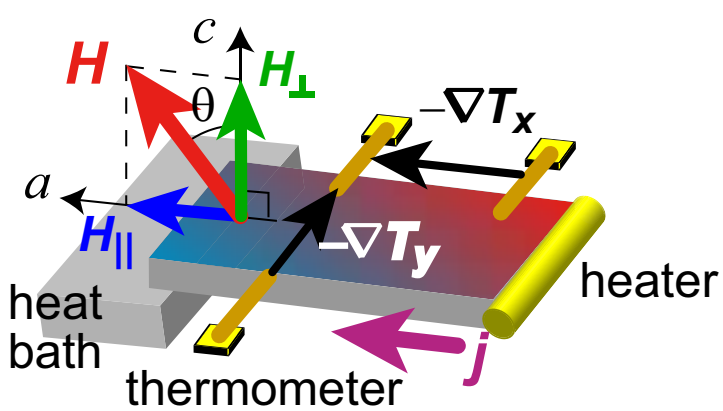


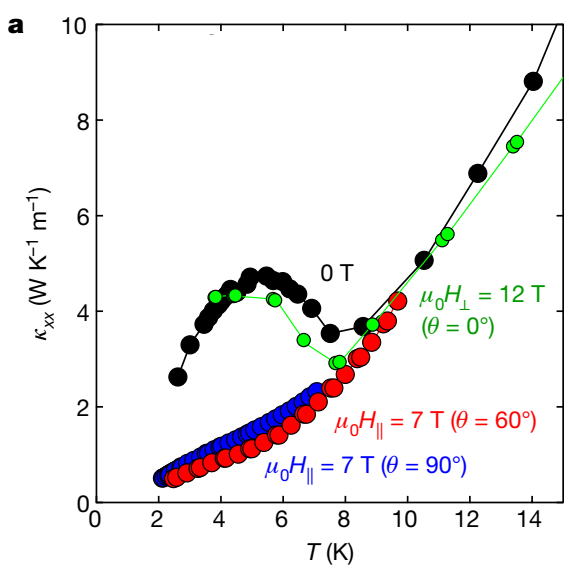
theoretical possibilities



Thermal Hall Effect

Y. Kasahara *et al*, Nature 2018





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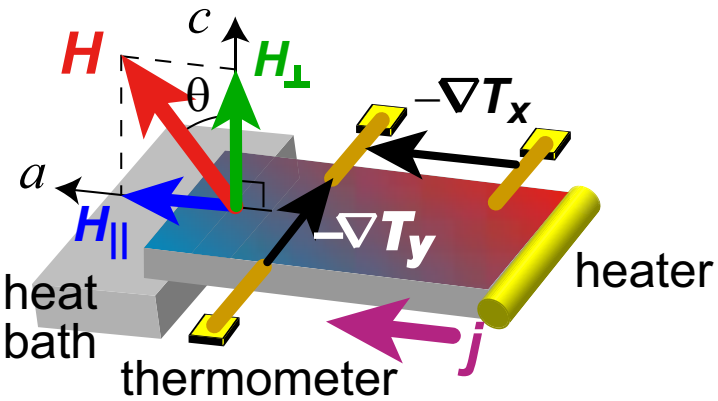

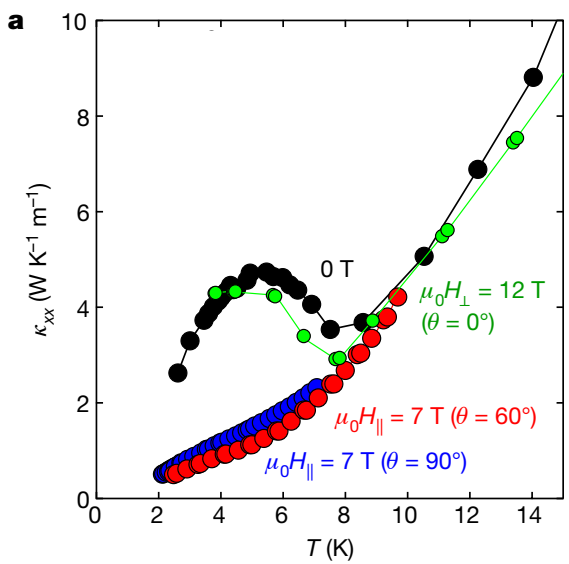

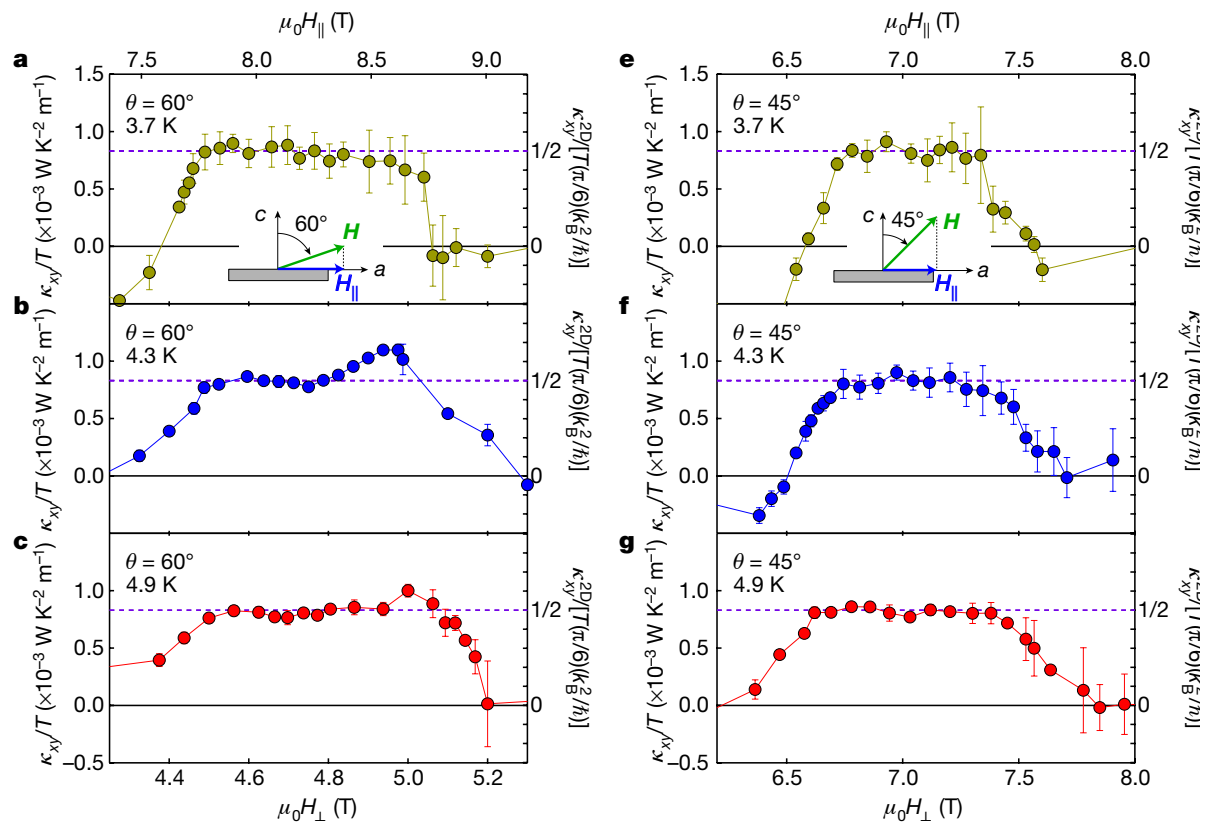

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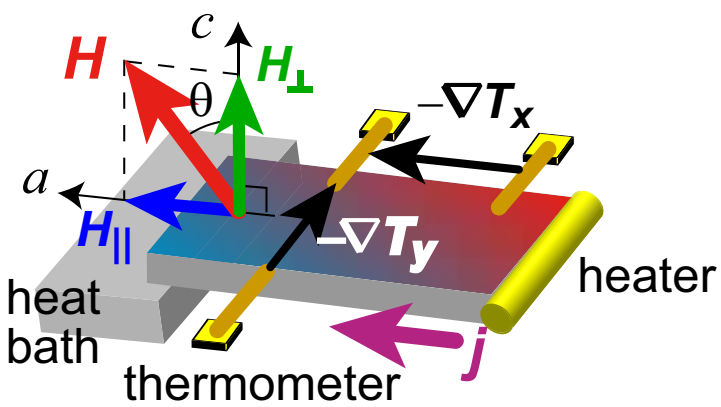

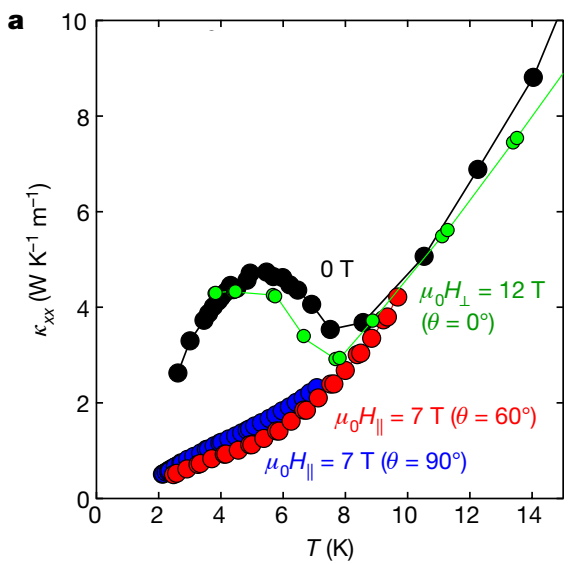

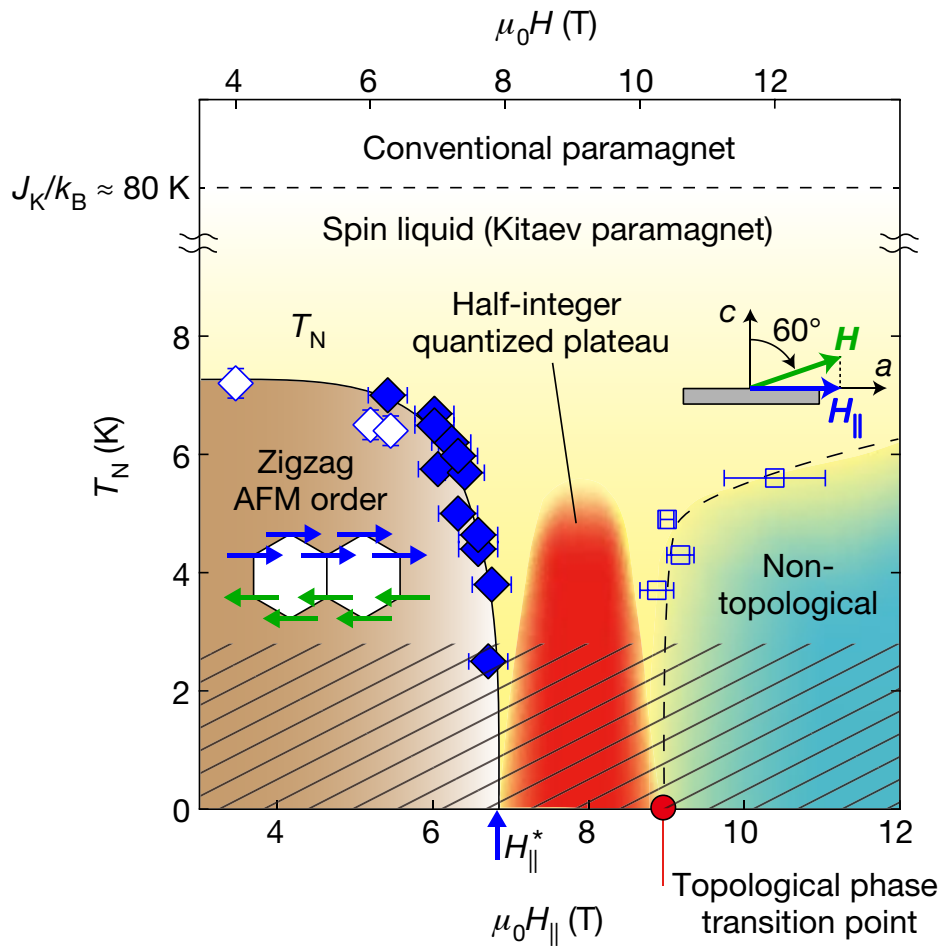

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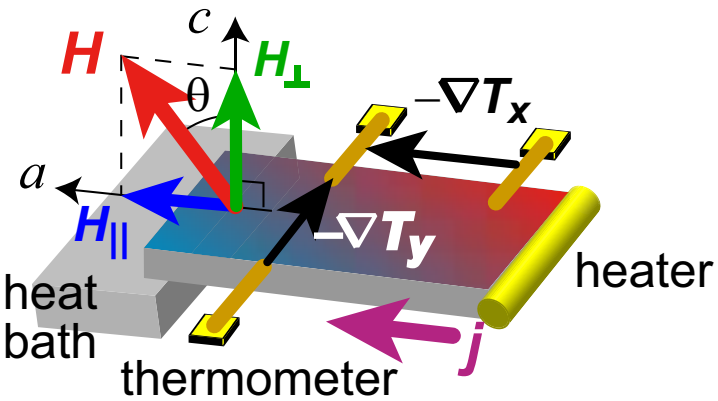
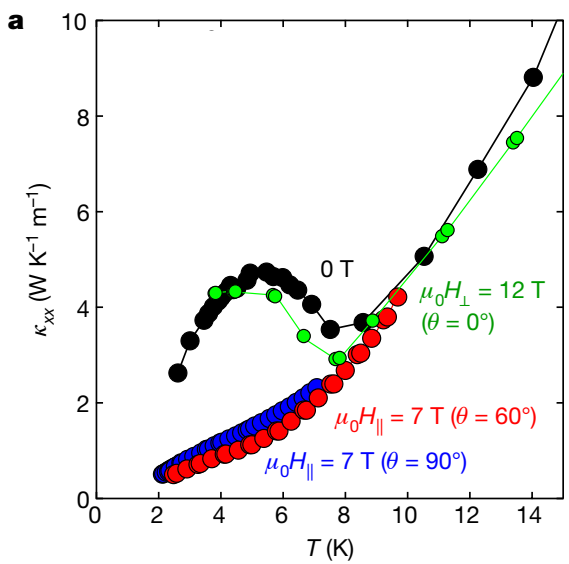
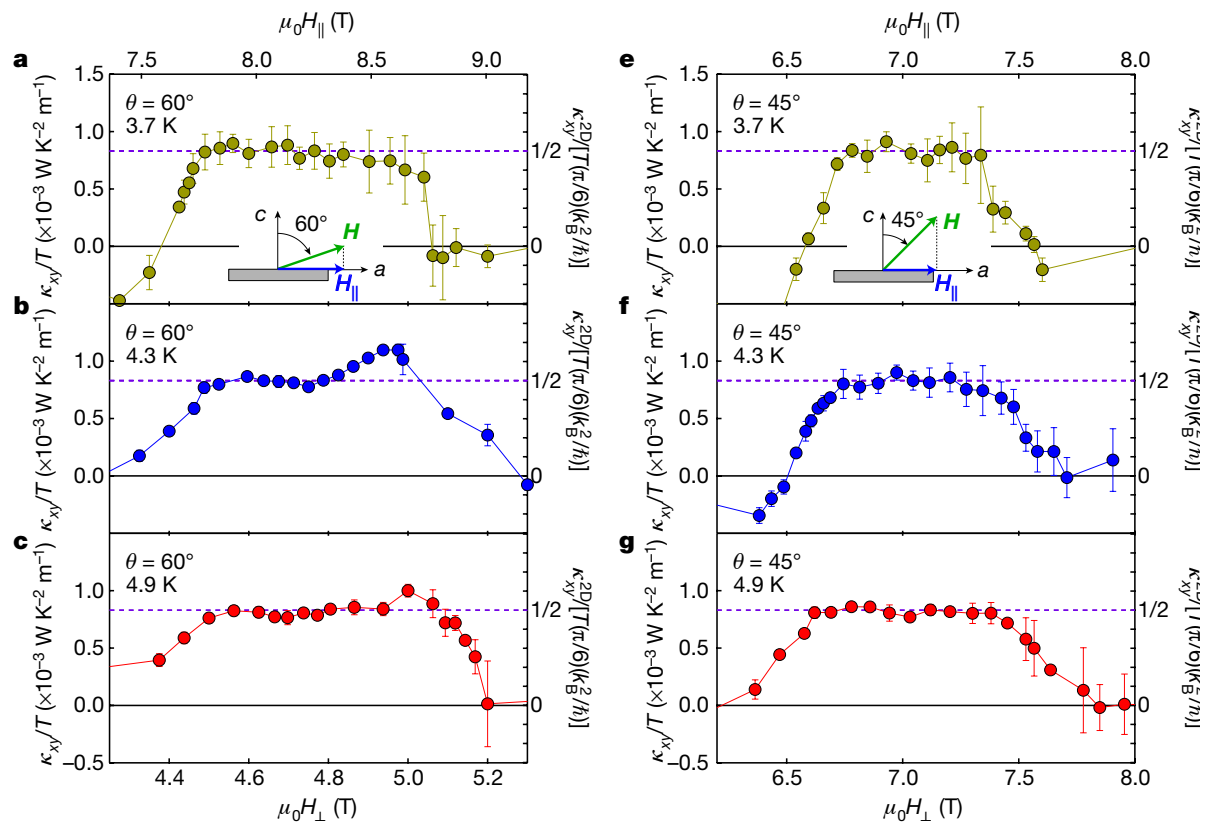
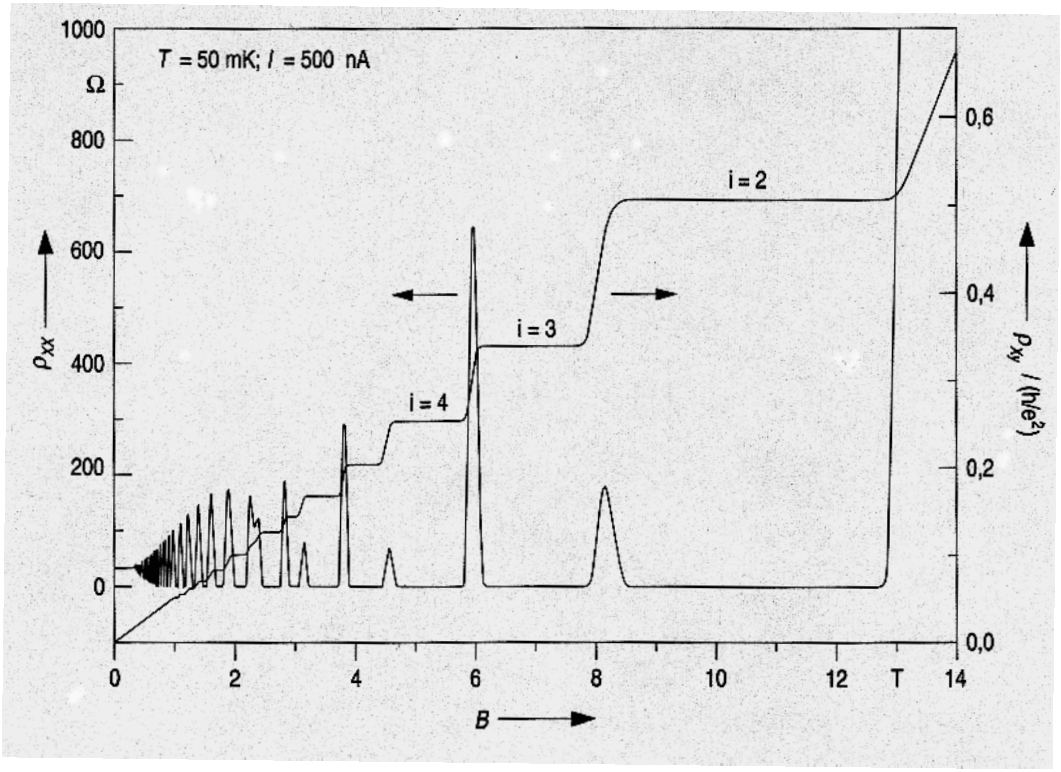






Why worry?



Quantization is *only* visible when diagonal conductivity is small, i.e. Hall angle ~ 90degrees

semicircle "law"

$$\sigma_{xx}^2 + \left(\sigma_{xy} - \frac{\sigma_1 + \sigma_2}{2}\right)^2 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2$$

c.f. IQHE

But for $RuCl_3$, thermal Hall angle ~ 10^{-3} !!

Theoretical Problem

From Y. Kasahara *et al*

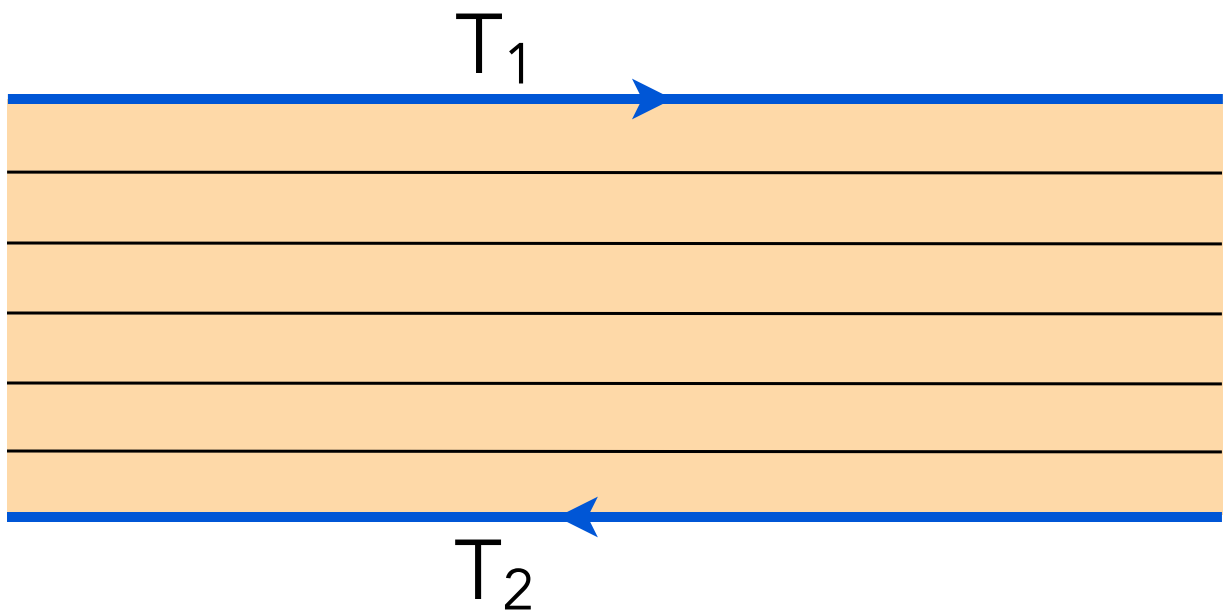
In the plateau regime of κ_{xy}, no anomaly is observed in κ_{xx}, probably because phonon contributions largely dominate over fermionic excitations arising from spins in κ_{xx} in the whole temperature range[28,29].

Q: Can quantized thermal Hall effect persist when phonons dominate bulk conductivity?

A: **Yes**, and phonons actually *help* to make the effect observable.

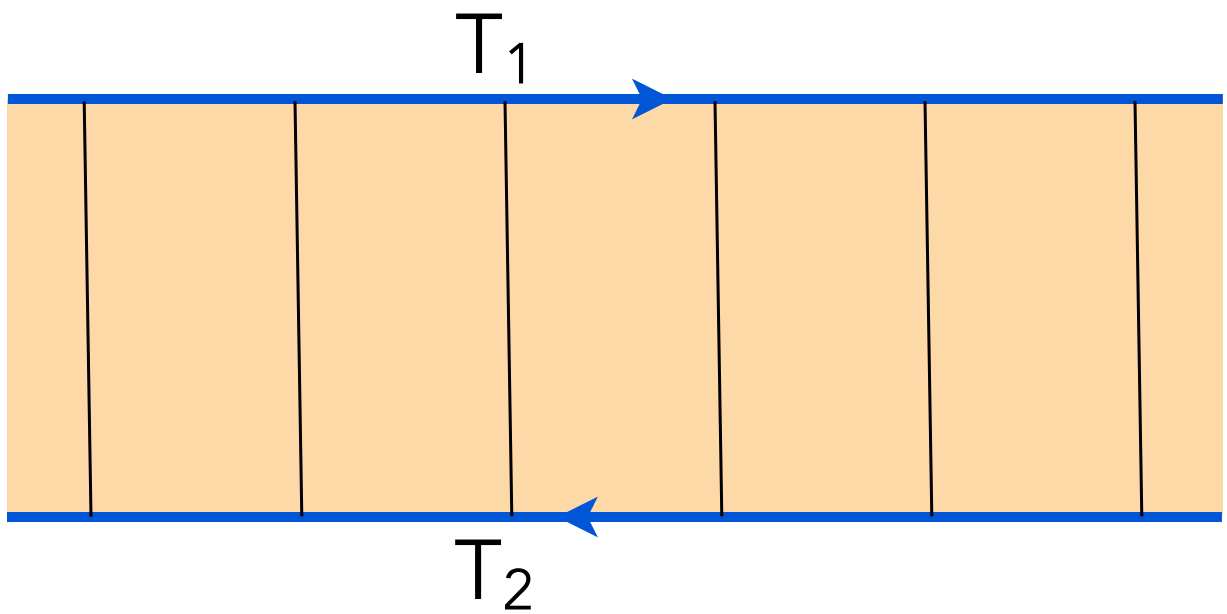
Theoretical Problem

Recall *derivation* of QTHE



Theoretical Problem

Recall *derivation* of QTHE

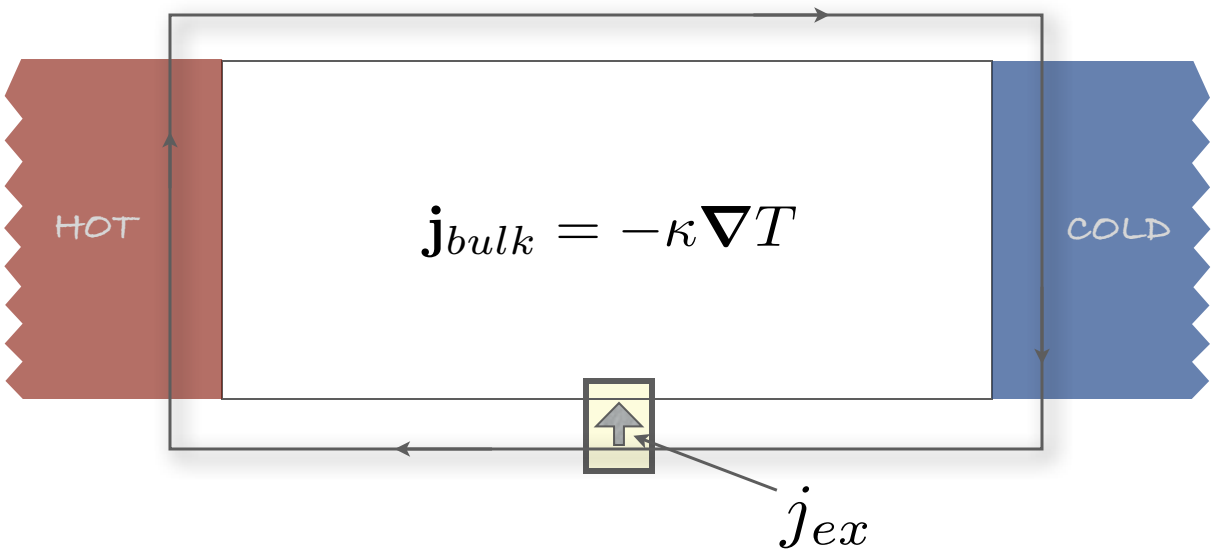


$\kappa_{xx} \gg \kappa_{xy}$: Temperature of phonons in *not* constant at the edge

?? Could edge be out of equilibrium with the phonons and have constant T??

- but: in that case, which T is measured?

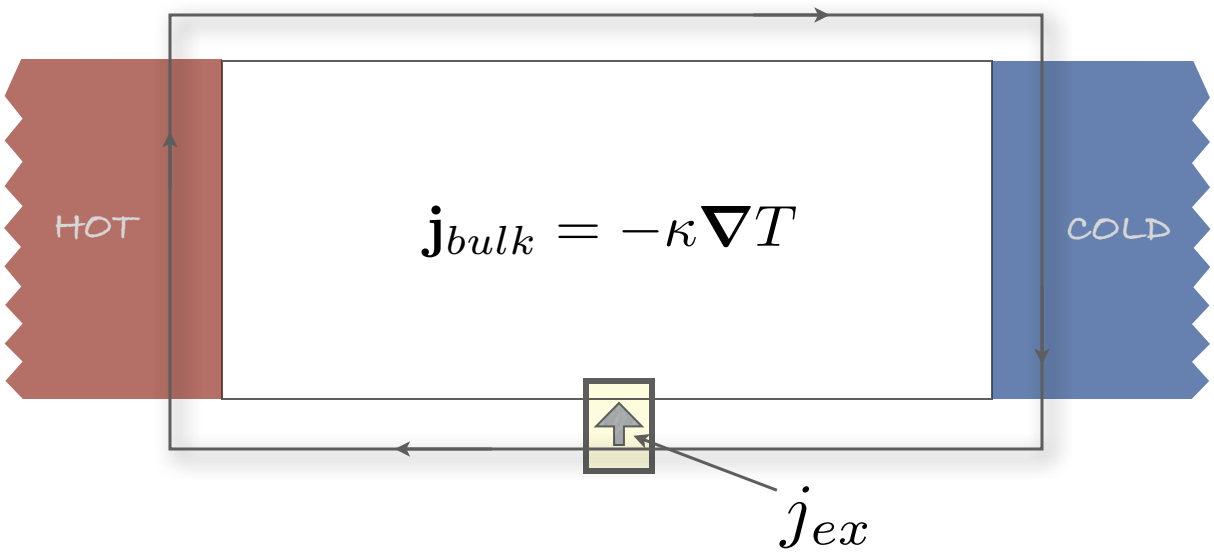
Formulation



leads coupled to the lattice

- Assume spins go into nonabelian QSL phase: bulk spin gap, chiral edge state
- Include bulk phonons with *no* Hall conductivity
- Majoranas and phonons can exchange energy at the edge of the sample

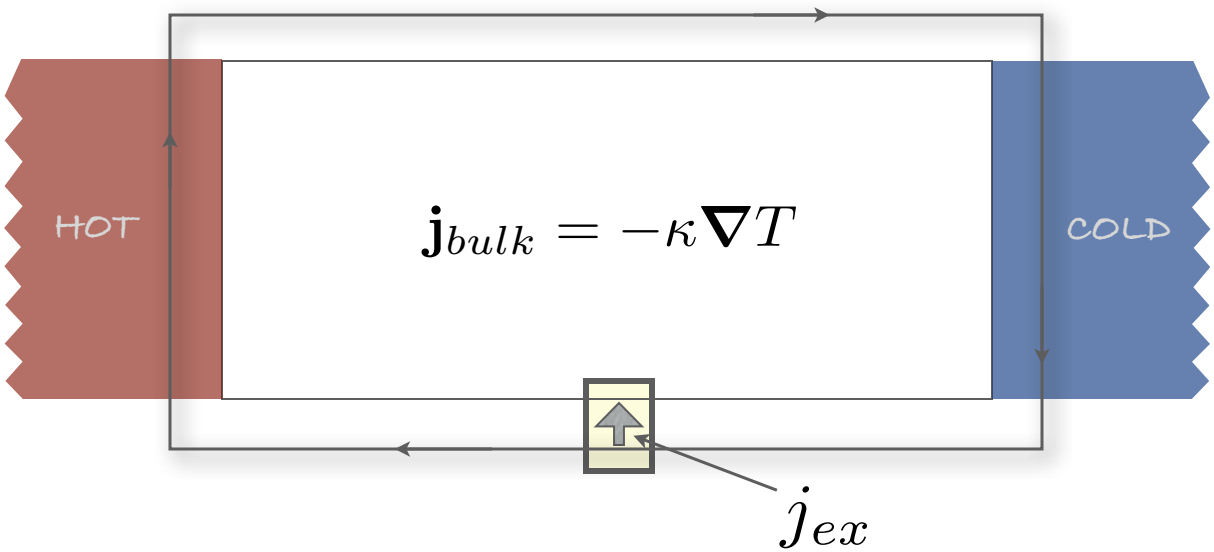
Formulation



leads coupled to the lattice

- Variables:
  - $T_{ph}(x,y)$ = phonon temperature in bulk
  - $T_f(x,L_y)$ etc = fermion temperature at edge

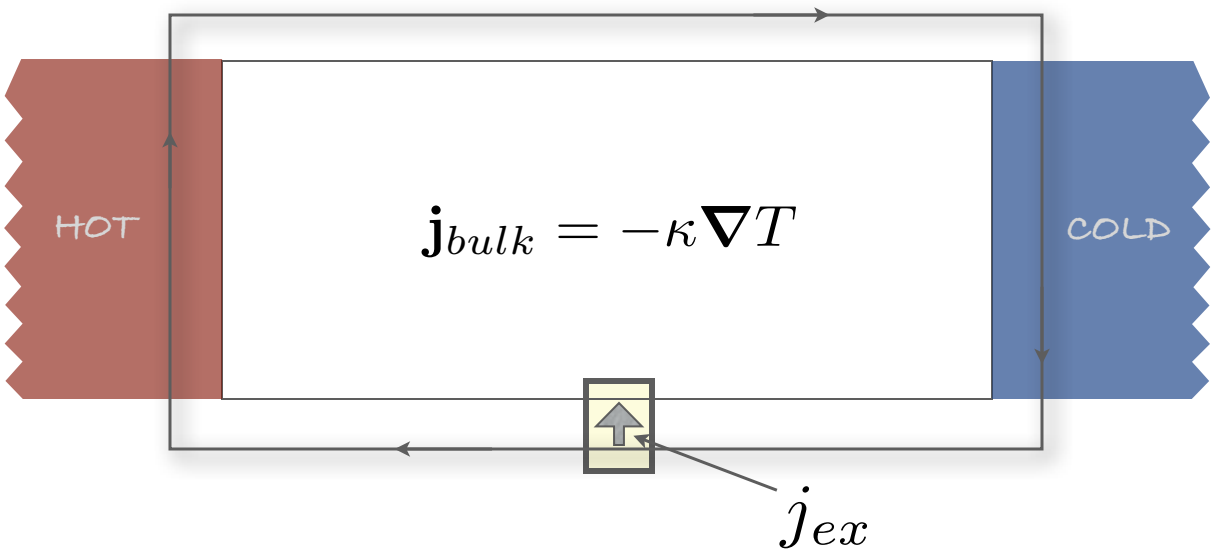
Formulation



leads coupled to the lattice

- Currents:
  - Bulk $\boldsymbol{j}_{ph} = -\kappa \boldsymbol{\nabla} T_{ph}$
  - Edge $I_f = \frac{\pi}{24} T_f^2$

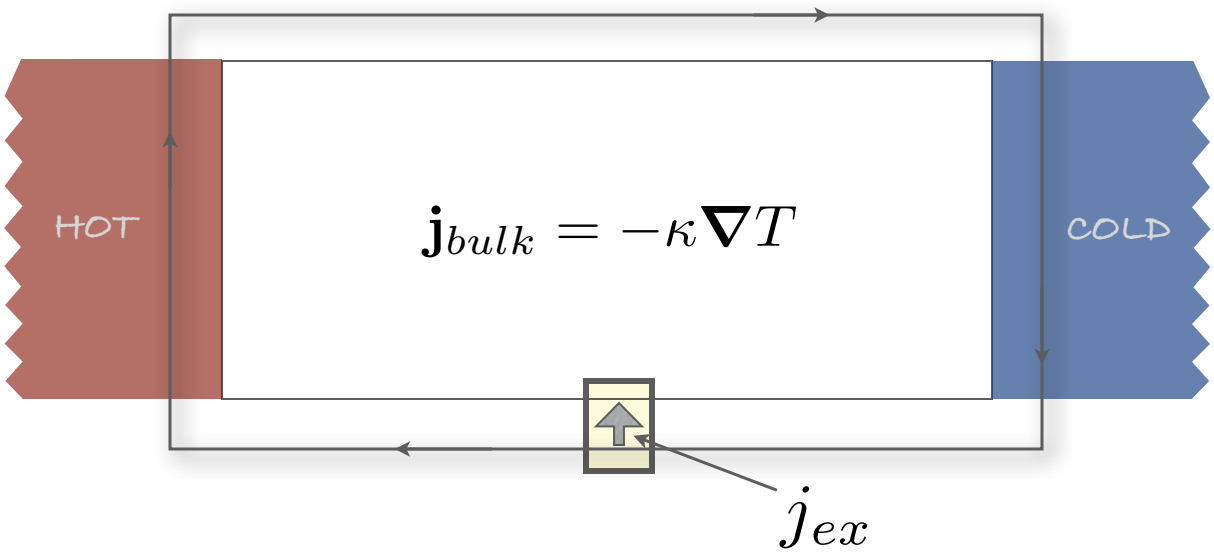
Formulation



leads coupled to the lattice

- Continuity:
  - Bulk $\boldsymbol{\nabla} \cdot \boldsymbol{j}_{ph} = 0$
  - Edge $\partial_x I_f = \pm j_{ex}\,, \quad \pm j^y_{ph}(x, \pm y_0) = j_{ex}(x, \pm y_0).$

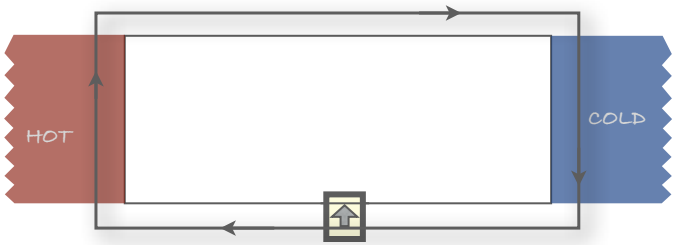
Formulation



leads coupled to the lattice

- Coupling:
  - linear response $j_{ex} = \lambda(T)(T_{ph} - T_f)$

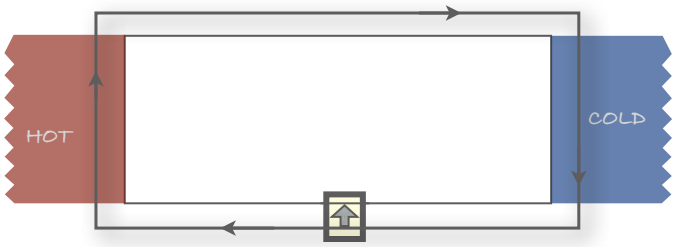
Formulation



- Full equations (for *infinite* Hall bar)
  - bulk $\nabla^2 T_{ph} = 0$
  - edge $\kappa^q_{xy} \partial_x T_f(x, \pm y_0) = -\kappa \partial_y T_{ph}(x, \pm y_0)$

$$= \pm \lambda(T) \left(T_{ph} - T_f\right)\big|_{x, \pm y_0}$$

Solution

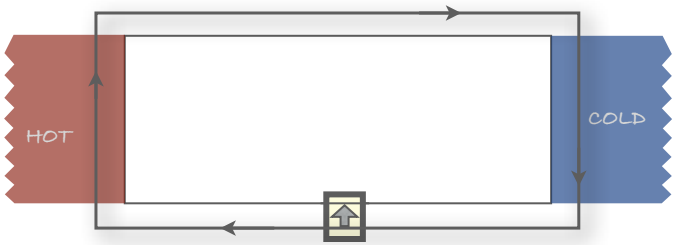


- Solution
  - Bulk $$T_{ph}(x,y) = T_0 + \frac{\partial T_{ph}}{\partial x} x + \frac{\partial T_{ph}}{\partial y} y$$
  - edge $$T_f(x, \pm y_0) = T_{1,\pm} + \frac{\partial T_{ph}}{\partial x} x$$

$$\frac{\partial T_{ph}}{\partial y} = -\frac{\kappa^q_{xy}}{\kappa} \frac{\partial T_{ph}}{\partial x}$$

$$T_{ph} - T_f = \mp \frac{\kappa}{\lambda} \frac{\Delta T_y}{L_y}$$

Solution



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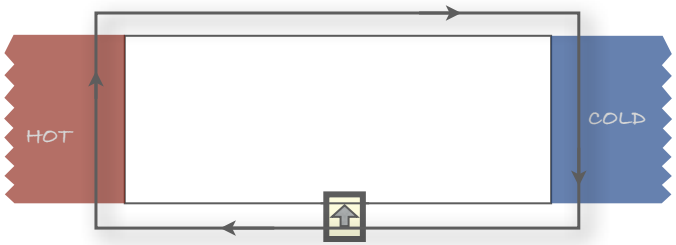
$$T_{ph} - T_f = \mp\frac{\kappa}{\lambda}\frac{\Delta T_y}{L_y}$$

phonon Hall angle corresponds to quantization! **Implies quantization in 3 probe experiment measuring T_{ph}**

Solution



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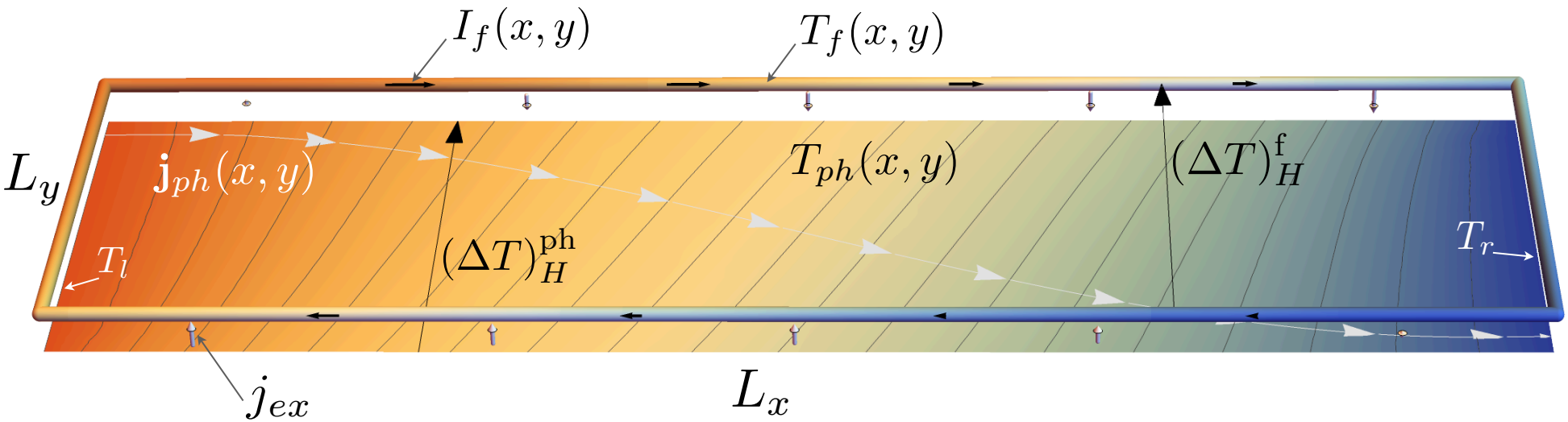
$$T_{ph} - T_f = \mp\frac{\kappa}{\lambda}\frac{\Delta T_y}{L_y}$$

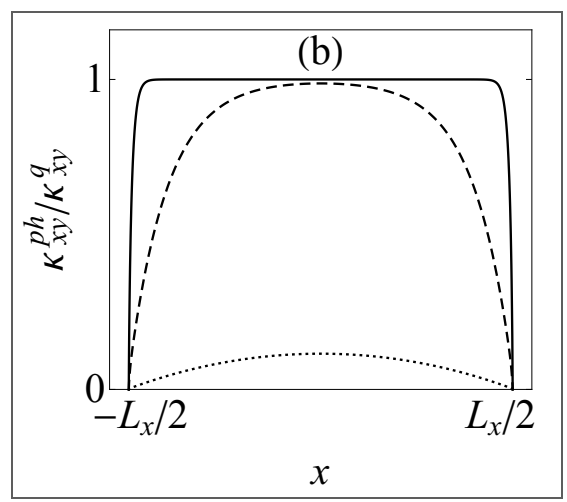
Majorana temperature *not* equal to phonon one. It is actually *better* to measure phonon temperature!

Finite Hall bar



- Solve coupled Laplace and boundary equations, with constant temperature leads.
- Reveals dependence on "thermalization length"

$$\ell_x = \frac{\kappa^q_{xy}}{\lambda}$$



$L_y/L_x = 0.01$

ℓ_x/L_x

$= \underline{0.01},\ \underline{0.1},\ 1$

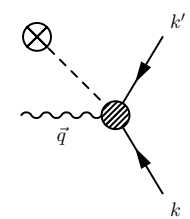
Bulk-edge coupling

local phonon strain couples to Majorana kinetic energy

$$H_{int} = \frac{-igv_f}{4} \int \mathrm{d}x\, \zeta(x) K_{ij} \partial_i u_j(x, y_0) \eta(x) \partial_x \eta(x),$$

We use kinetic equation to calculate

$$\Big(\frac{\partial \mathcal{E}}{\partial t}\Big)_{ph\to f} = -\Big(\frac{\partial \mathcal{E}_{ph}}{\partial t}\Big) = -\sum_{q_x, q_y} \omega_{\vec{q}} \Big(\frac{\partial g(\vec{q})}{\partial t}\Big)_{coll},$$



Rate

$$\Big(\frac{\partial g(\vec{q})}{\partial t}\Big)_{coll} = \frac{2\pi}{\hbar} \times 2 \sum_{k,k'} \{ \langle |\mathcal{M}^+(\vec{q}, k, k')|^2 \rangle_{dis} (1 + g_\omega) f_\epsilon f_{\epsilon'} \delta(-\omega_{\vec{q}} + \epsilon_k + \epsilon_{k'})$$
$$- \langle |\mathcal{M}^-(\vec{q}, k, k')|^2 \rangle_{dis}\, g_\omega f_\epsilon f_{\epsilon'} \delta(\omega_{\vec{q}} + \epsilon_k + \epsilon_{k'}) \}$$

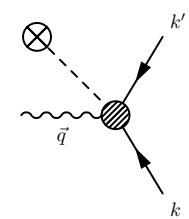
Bulk-edge coupling

local phonon strain couples to Majorana kinetic energy

$$H_{int} = \frac{-igv_f}{4} \int \mathrm{d}x\, \zeta(x) K_{ij} \partial_i u_j(x, y_0) \eta(x) \partial_x \eta(x),$$

We use kinetic equation to calculate

$$j_{ex} \propto \left(\frac{\partial \mathcal{E}}{\partial t}\right)_{ph \to f} = -\left(\frac{\partial \mathcal{E}_{ph}}{\partial t}\right) = -\sum_{q_x, q_y} \omega_{\vec{q}} \left(\frac{\partial g(\vec{q})}{\partial t}\right)_{coll},$$

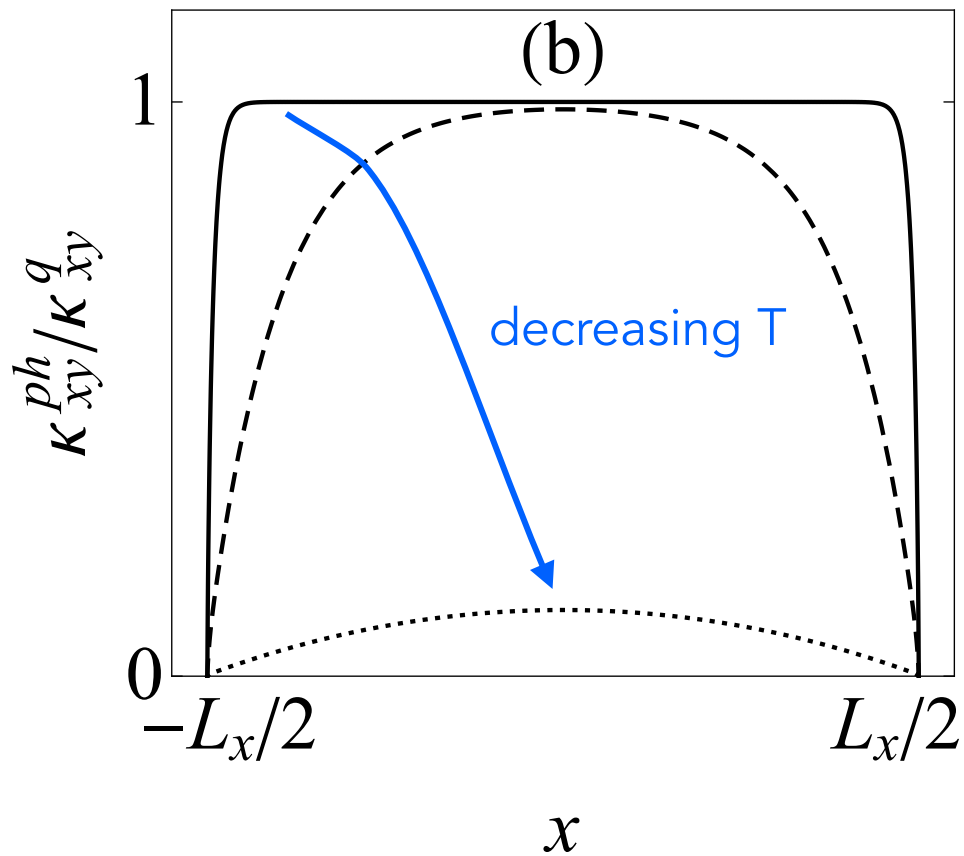


Result

$$\lambda = \frac{g^2 \zeta^2}{32(2\pi)^3 v_{ph}^4 v_f^2 \rho_0} f\, T^6$$

strong T-dependence: thermalization becomes poor at low T.

Prediction



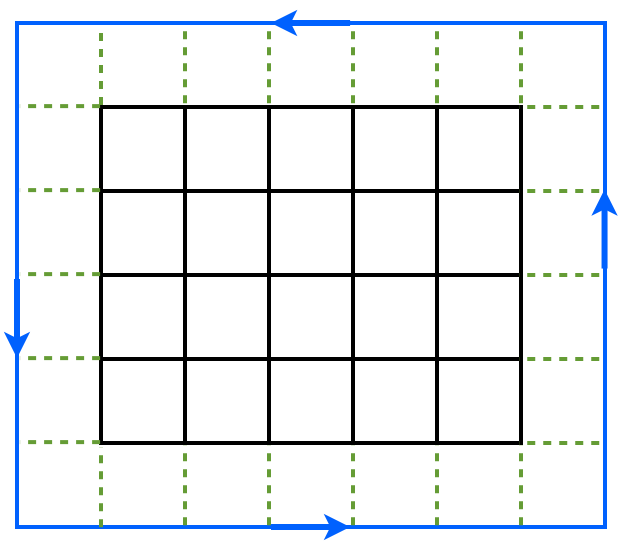
Expect *failure* of quantization at low enough temperature!

Disorder

- Experimental observation:
  - Quantization is observed only in the "best" samples, which have the largest *diagonal* thermal conductivity
- Presumably, sample variations involve lattice disorder that affects the phonon transport
  - What is the effect on thermal Hall effect?

Disorder

- Network model



resistor $I_{i\to j} = \kappa_{ij}(T_i - T_j)$

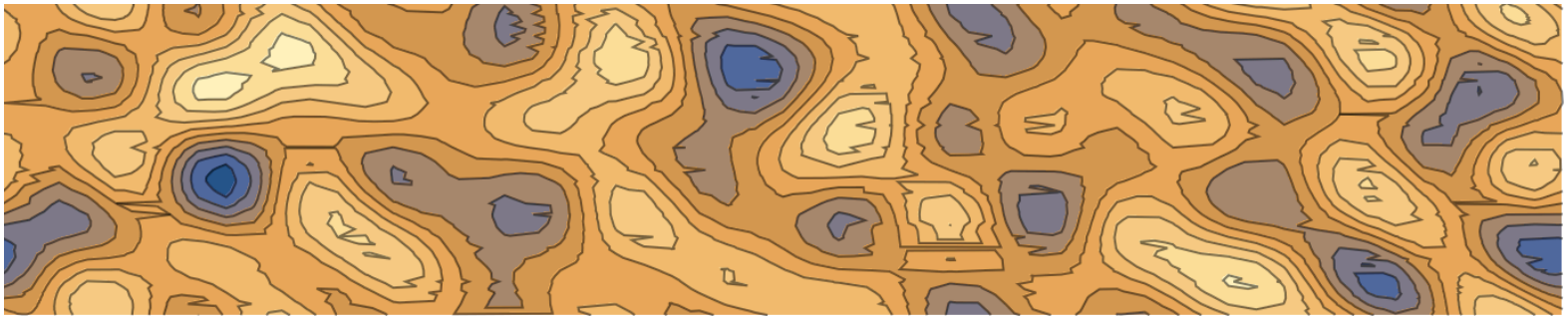
chiral $I_{i\to j} = \pm\kappa_H(T_i + T_j)/2$

resistor $I_{i\to j} = \lambda(T_i - T_j)$

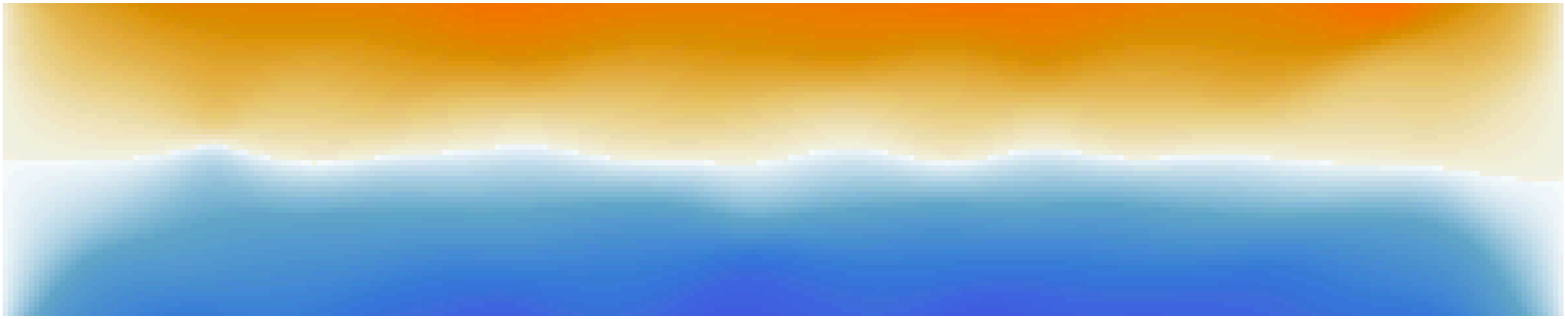
- Disorder included as random resistors

Disorder

Input: smooth random conductivity

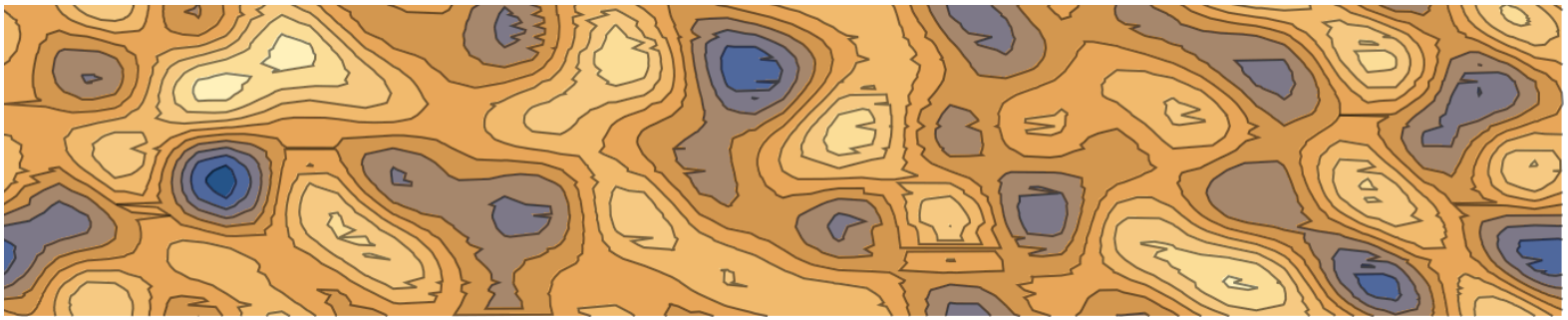


Output: Hall temperature gradient
(anti-symmetrized in field)

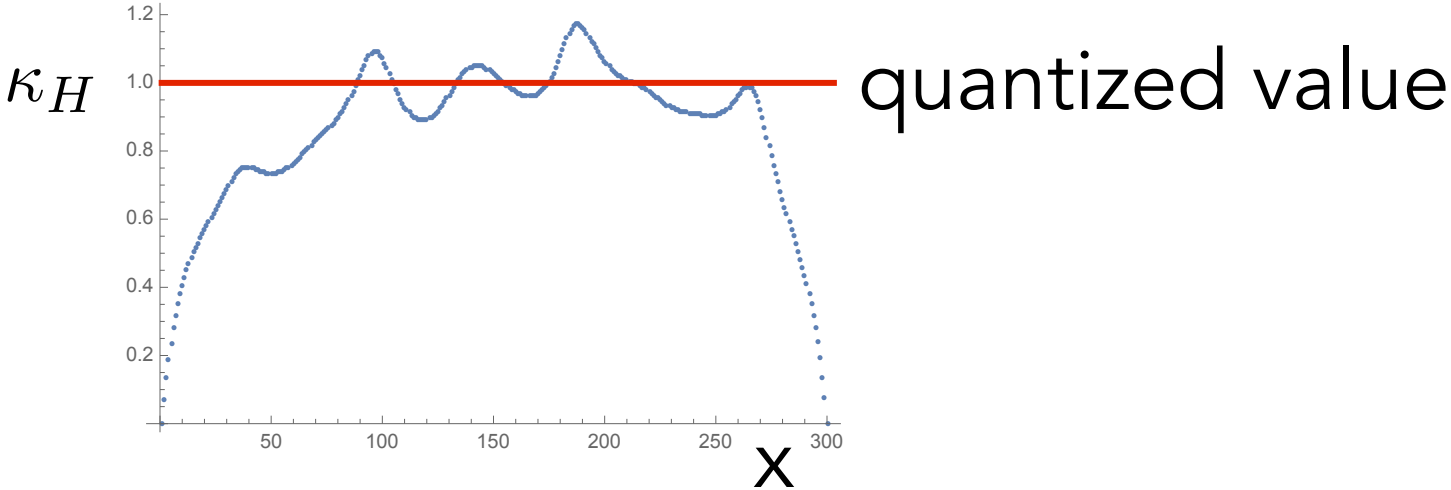


Disorder

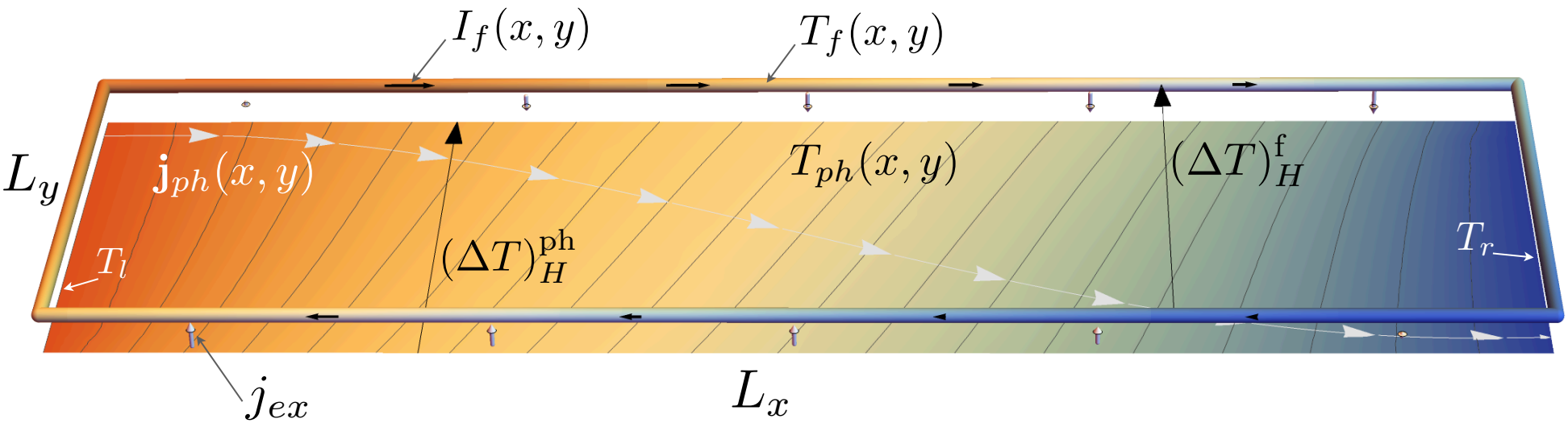
Input: smooth random conductivity



Output: Hall temperature gradient
(anti-symmetrized in field)



Summary



- Quantized value of thermal Hall conductivity *can* be measured even when large phonon conductivity is present
- Quantization is only power-law good, and deviations are controlled by edge-bulk equilibration

ARXIV:1805.10532 see also: ARXIV:1805.11507

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