A strongly correlated metal from coupled SYK models

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Transport

$$\boldsymbol{j} = \underline{\sigma}\boldsymbol{E} \qquad\qquad \boldsymbol{j}_e = -\underline{\kappa}\boldsymbol{\nabla}T$$

- Arguably most important aspect of quantum materials: electrical and thermal conductivity (and crossed coefficients)
- Sensitive, versatile
- Probes extreme long wavelength, low frequency

Theory

- *Understanding* of transport mainly through electron **quasiparticle** picture
  - Boltzmann equation:

$$\left[\partial_t + \boldsymbol{v}_n(\boldsymbol{k}) \cdot \boldsymbol{\nabla}_r - e\boldsymbol{E} \cdot \boldsymbol{\nabla}_k\right] f_n = \left.\frac{\partial f_n}{\partial t}\right|_{\text{collision}}$$

Linearizing this around equilibrium gives conductivities in terms of band velocities and scattering rates

Ultra-quantum transport

- How does transport work when quasiparticles are not adiabatically connected to electrons?
- Or when quasiparticles scatter very strongly?
- Or if there are no quasiparticles at all?

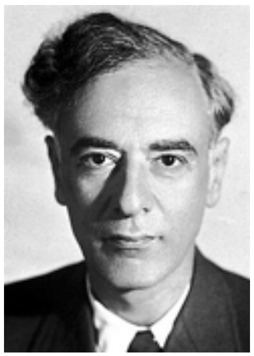
Convergence of ideas and experiments

- Experiments on
  - non-Fermi liquid metals
  - ultra-clean semi-metals
  - thermal conductivity in quantum magnets
  - electron spin resonance of spin liquids
- Theoretical approaches
  - Gauge/gravity duality
  - SYK model and related large N theories
  - Quantum hydrodynamics
  - Field theory

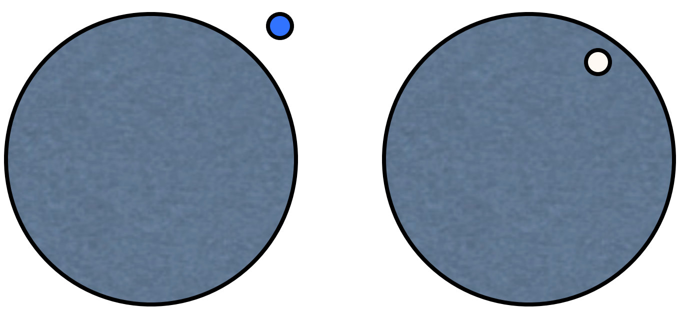
This Talk

- Strongly correlated metals from SYK models - a route to calculable non-quasiparticle transport

Fermi Liquid Theory



Landau provided justification for quasiparticle picture in metals when $T << E_F$



Low energy excitations act like electrons and holes but with wavefunction dressing (Z<1), effective mass, and Landau interactions

$$E = \sum_k \epsilon_k \delta n_k + \frac{1}{2V} \sum_{k,k'} U_{k,k'} \delta n_k \delta n_{k'}$$

scattering is weak because not so many low energy qp states to scatter to

Heavy Fermi Liquids

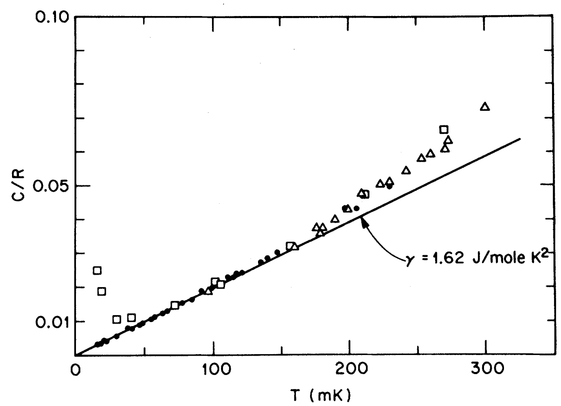


FIG. 1. Specific heat of $CeAl_3$ at very low temperatures in zero field (●, Δ) and in 10 kOe (□).

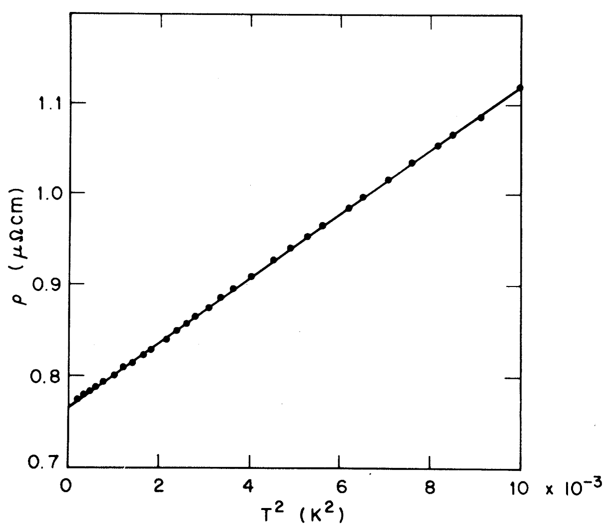


FIG. 3. Electrical resistivity of $CeAl_3$ below 100 mK, plotted against T^2.

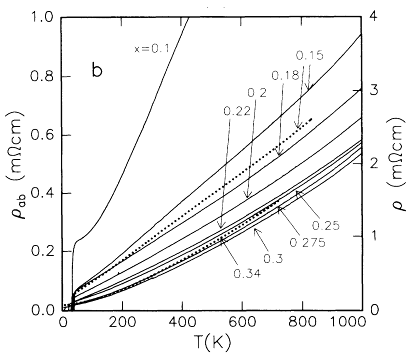
$$C \sim \gamma T$$

$$\rho(T) - \rho(0) \sim AT^2$$

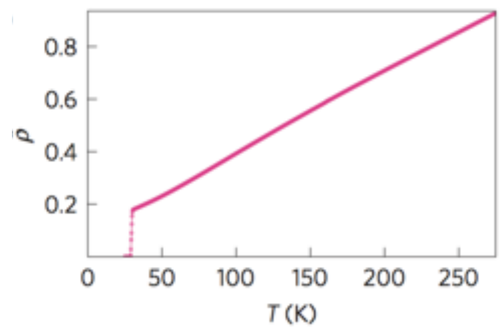
Both γ and A **huge**

Behave like Fermi liquid with tiny E_F and large electron mass, but only for $T \ll E_F$

Non-Fermi Liquids



LSCO Takagi *et al, 1992*



$BaFe_2(As_{1-x}P_x)_2$, Hayes *et al, 2016*

$$\frac{1}{\tau} \sim T\,?$$

T-linear resistivity:

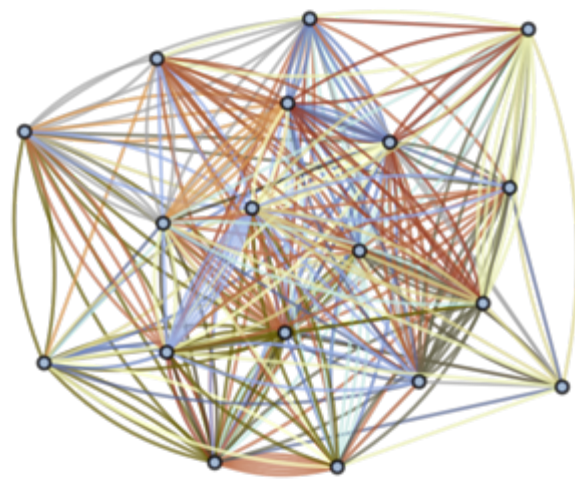
- Many materials
- Often nearby to unconventional superconductivity
- Symptom of a different type of metal?
- If no quasiparticles exist, what is the starting point?

Sachdev-Ye-Kitaev model

A toy *exactly soluble* model
of a non-Fermi liquid

$$H = \sum_{i<j,k<l} U_{ijkl}\, c_i^\dagger c_j^\dagger c_k c_l$$

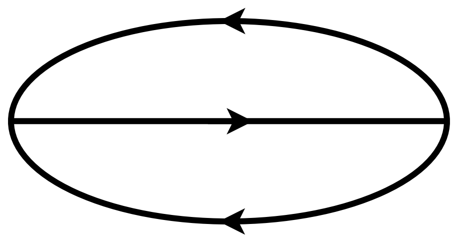
$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3}$$



Like a strongly interacting quantum dot
or atom with complicated Kanamori
interactions between many "orbitals"

SYK Model

Sachdev-Ye, 1993: Model has a soluble large-N limit

$$\Sigma =$$  $$+O(1/N)$$

In equations: very similar to DMFT:

$$G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)} \qquad \Sigma(\tau) = -U^2 G(\tau)^2 G(-\tau)$$

Solution:

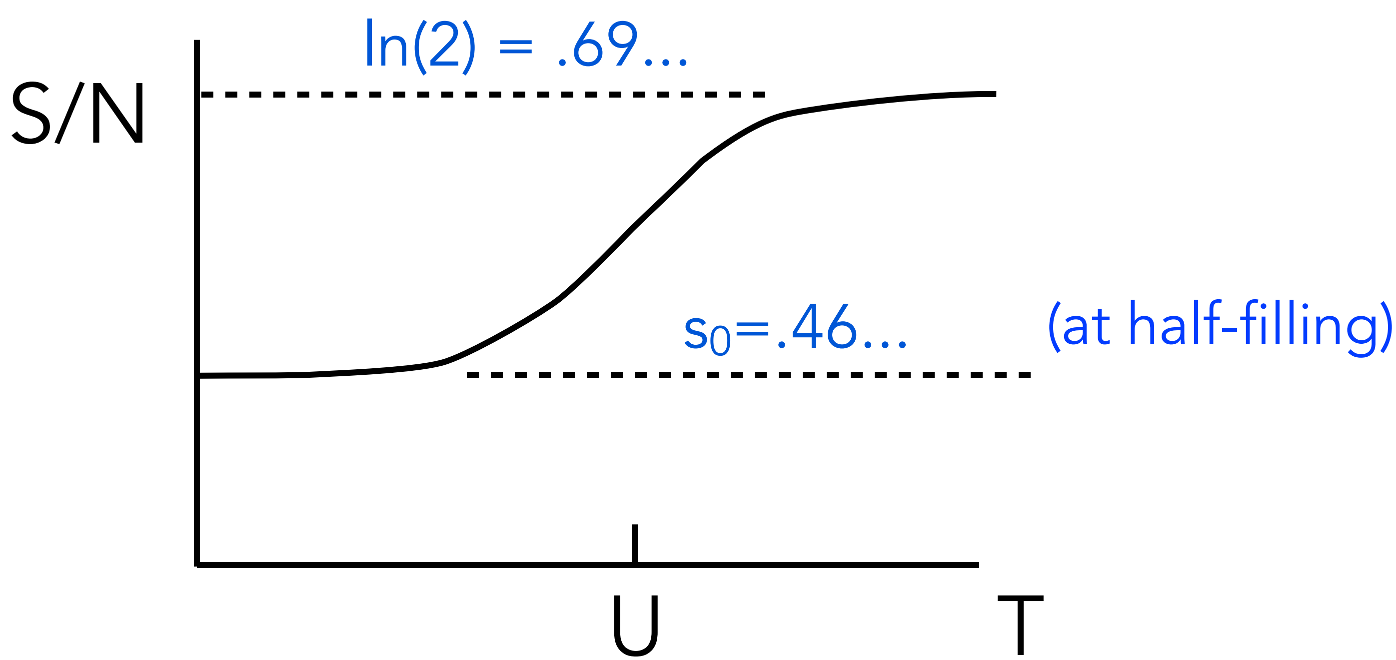
$$G(i\omega) \sim 1/\sqrt{\omega}$$

not a pole: non-Fermi liquid

SYK Model

Why not quasiparticles?

Georges, Parcollet, Sachdev, 2001: **ground state entropy!**



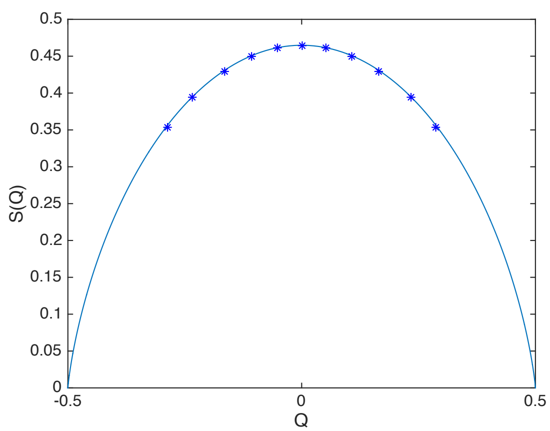
Many states available for scattering

"level spacing" ~ $U \exp(-N s_0)$

Density dependence

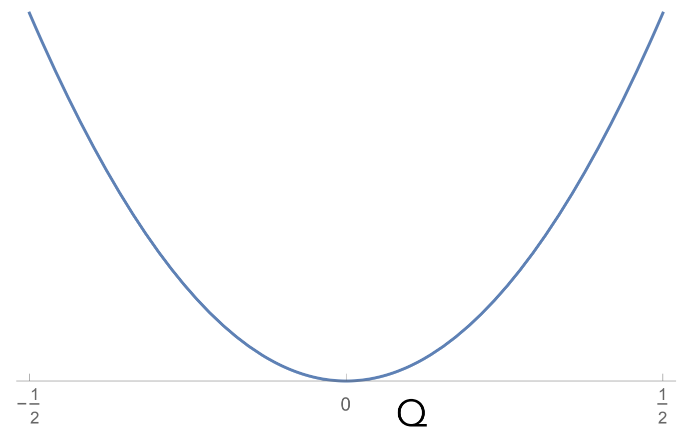
$$H \to H - \mu\mathcal{N} \qquad \mathcal{N} = \sum_i c_i^\dagger c_i \qquad \mathcal{Q} = \frac{\mathcal{N}}{N} - \frac{1}{2}$$

Entropy



Davison *et al*, arXiv:1612.00849

Energy



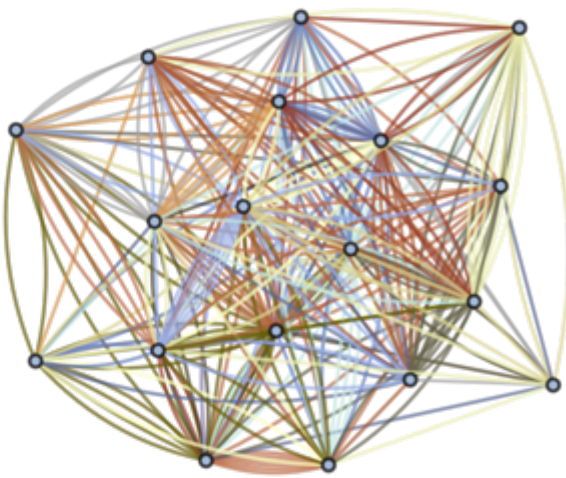
schematic

- Compressibility is constant at T=0

$$K = \left.\frac{\partial \mathcal{Q}}{\partial \mu}\right|_{\mu=0} = \frac{1.04}{U}$$

curvature = 1/K

SYK Summary



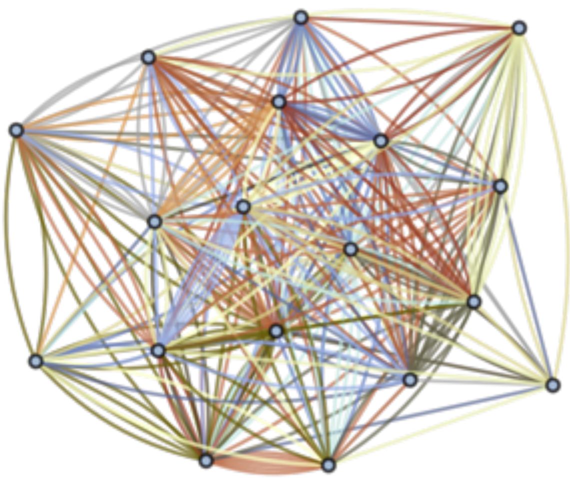
- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T=0) = \frac{1.04}{U}$$

$$S(T=0)/N = .46\ldots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

SYK Summary



- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T=0) = \frac{1.04}{U}$$

$$S(T=0)/N = .46\ldots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

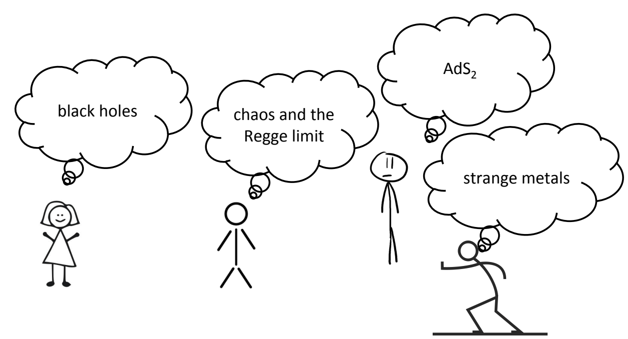
circa 2015

Chaos

$$\left\langle [\mathcal{O}(0), \tilde{\mathcal{O}}^{\dagger}(t)]^2 \right\rangle \sim \frac{1}{N} e^{\lambda_L t}$$

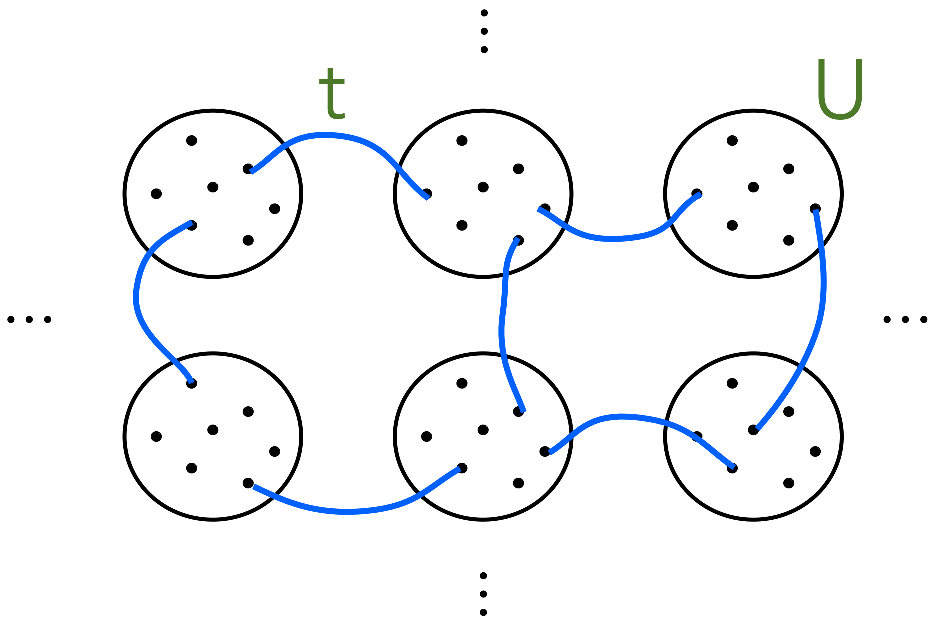
$$\lambda_L = \frac{2\pi k_B T}{\hbar}$$

Holography



slide from D. Stanford, IAS, 2017

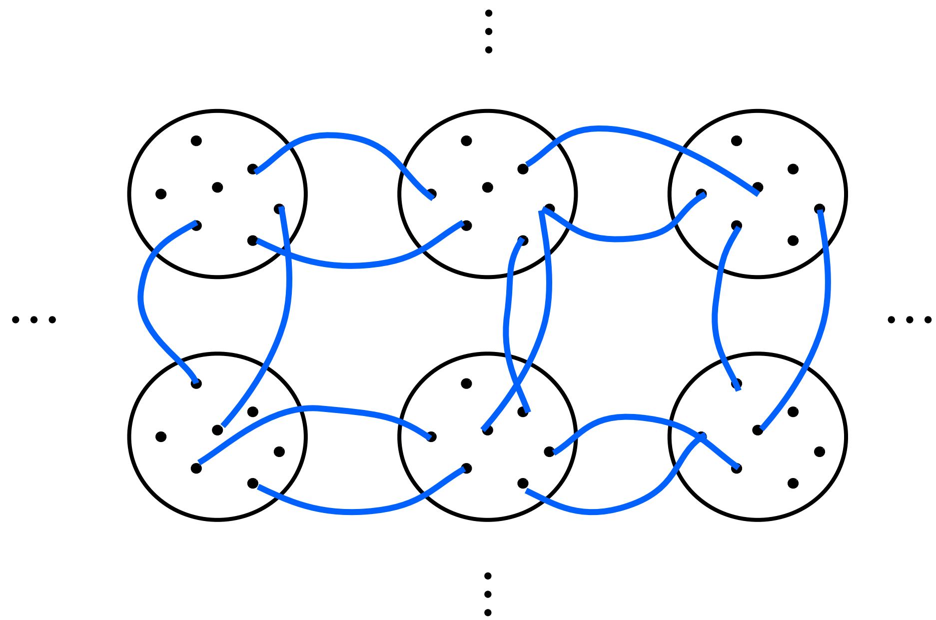
Building a metal



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Building a metal



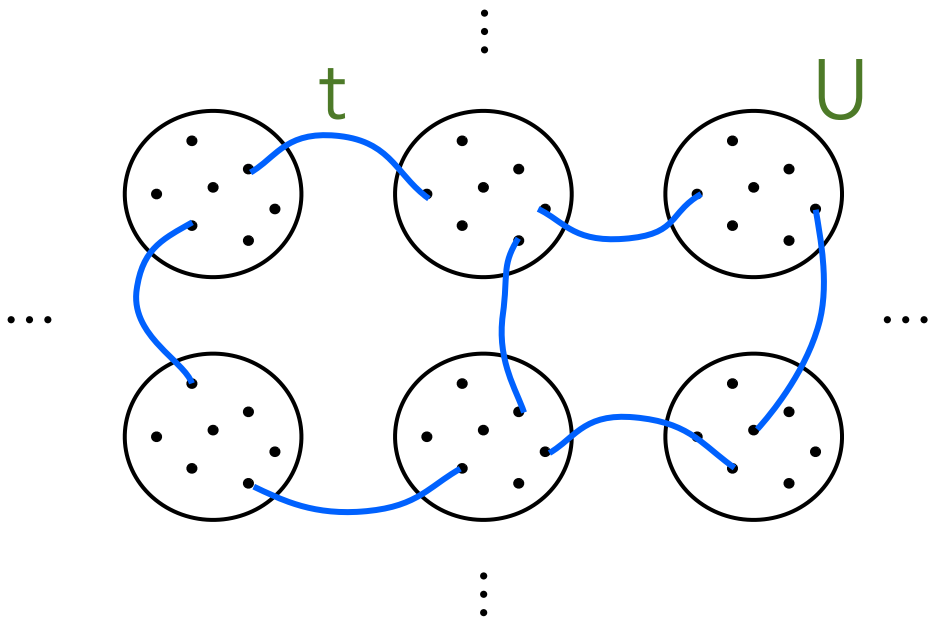
Other work: 2-electron hopping

Y.Gu *et al*, arXiv:1609.07832
R. Davison *et al*, arXiv:1612.00849

$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ijkl,xx'} c^\dagger_{i,x} c^\dagger_{j,x} c_{k,x'} c_{l,x'} + \text{h.c.}$$

Omitting *relevant* 1-electron hopping leaves system NFL even at T=0

Building a metal

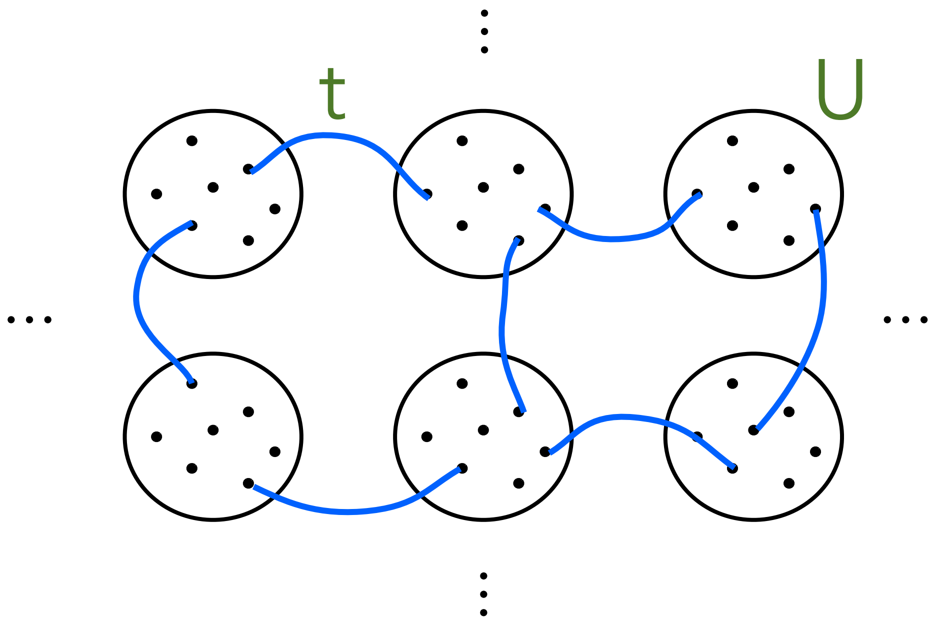


$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x'}$$

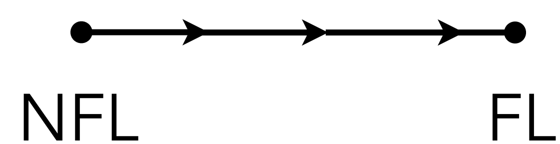
competition!

t/U≪1 interesting

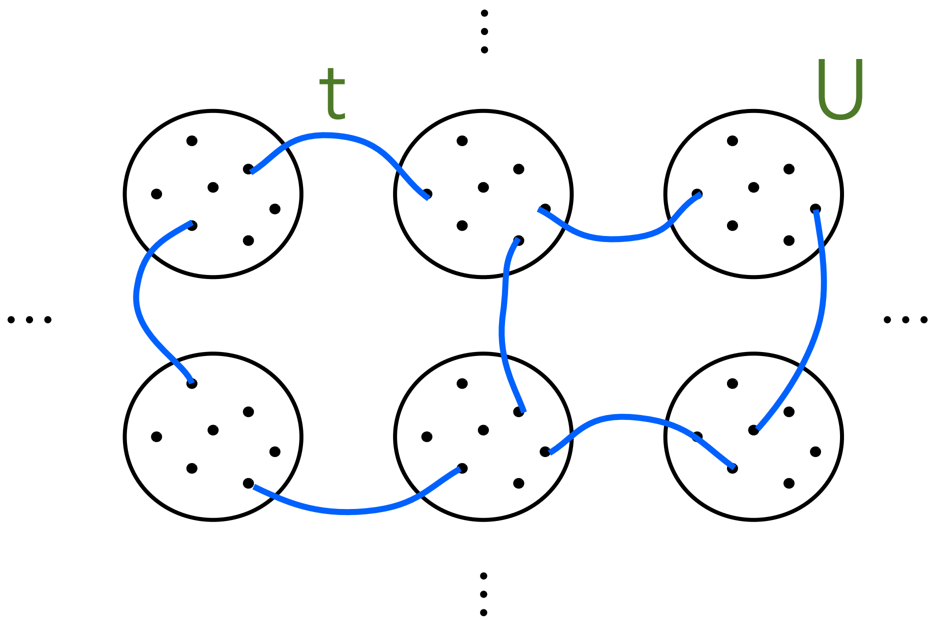
Building a metal



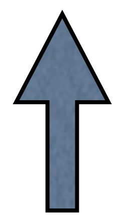
$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$



Self-consistent equations



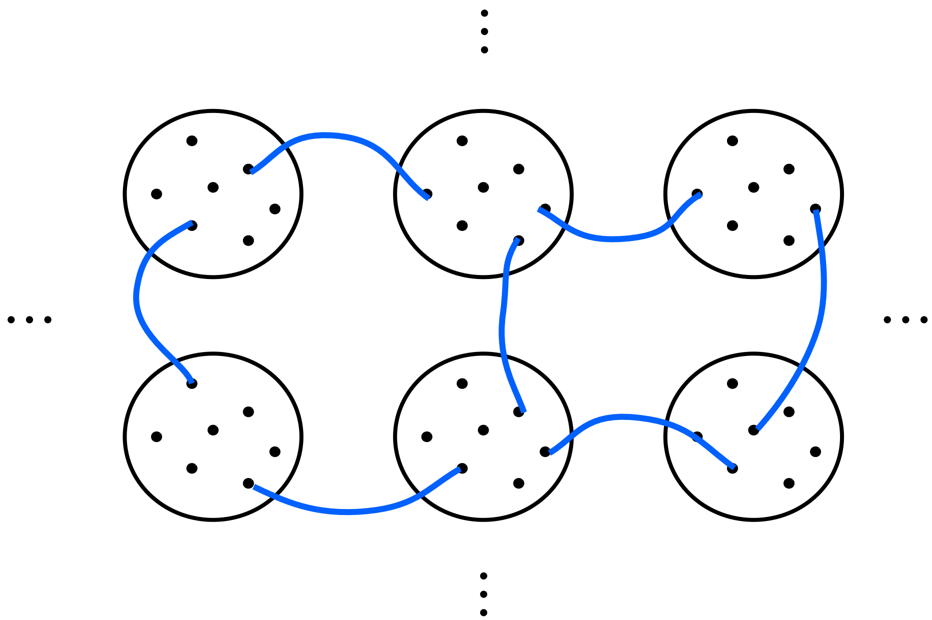
$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$
$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$



strong similarities to DMFT equations

mathematical structure appeared in early study of doped t-J model with *double* large N and infinite dimension limits: O. Parcollet+A. Georges, 1999

Coherence scale



$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$
$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

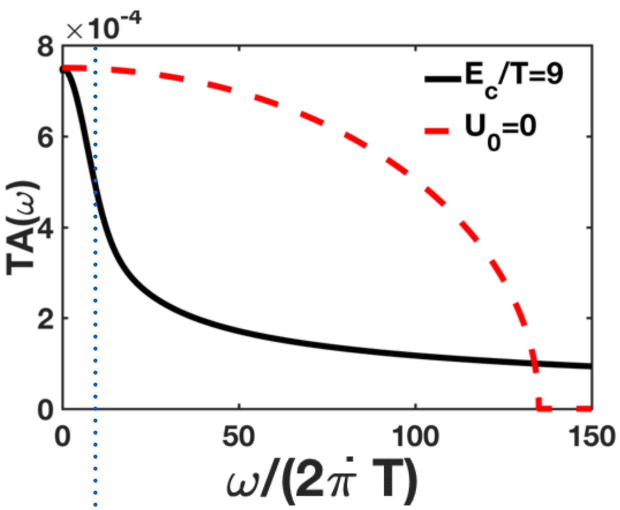
Rescaling $\bar{\omega} = \frac{\omega}{\tilde{E}_c}, \quad \bar{\tau} = \tau \tilde{E}_c, \quad \bar{G}(i\bar{\omega}) = \tilde{t} G(i\omega) \quad \bar{\Sigma}(i\bar{\omega}) = \Sigma(i\omega)/\tilde{t}$ $\quad \tilde{t} = \left(\frac{z}{2}\right)^{\frac{1}{2}} t$

$$\bar{G}(i\bar{\omega}) = \frac{\tilde{t}}{U} i\bar{\omega} - \bar{\Sigma}(i\bar{\omega}) \qquad \bar{\Sigma}(\bar{\tau}) = -\bar{G}(\bar{\tau})^2 \bar{G}(-\bar{\tau}) + 2\bar{G}(\bar{\tau}),$$

For t≪U, a single universal coherence scale appears $\tilde{E}_c = \frac{\tilde{t}^2}{U}$

Coherence scale

We solve these equations in a real time Keldysh formulation numerically and determine asymptotics analytically.



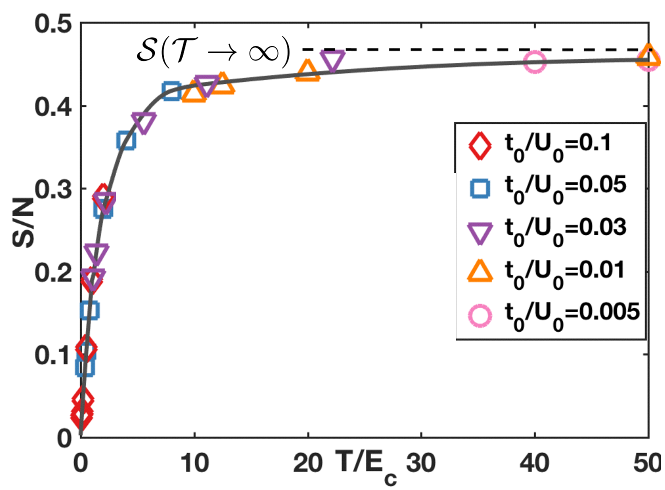
$\omega = E_c$

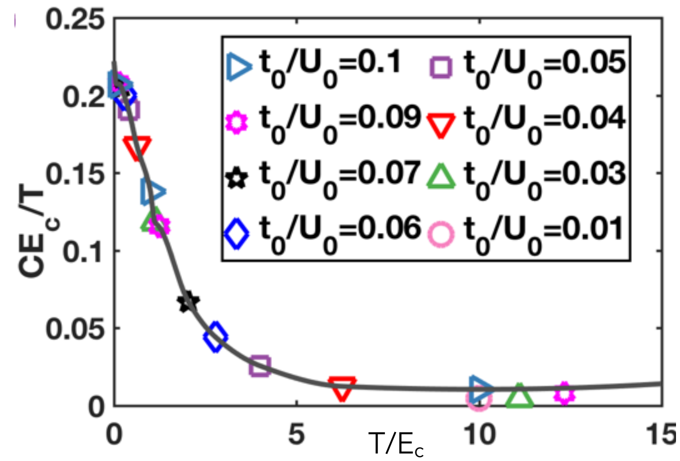
Narrow "coherence peak" appears in spectral function: heavy quasiparticles form for $T \ll E_c$

Quasiparticle weight $Z \sim t/U$

Entropy

Level repulsion: entropy is released for T<E_c!





Universal scaling forms

$$\mathsf{S}/N = \mathcal{S}(T/E_c)$$

$$C/N = T/E_c \mathcal{S}'(T/E_c)$$

$$\gamma \equiv \lim_{T\to 0} \frac{C}{T} = \frac{\mathcal{S}'(0)}{E_c}$$

$$m^*/m \sim U/t$$

Sommerfeld enhancement

Compressibility

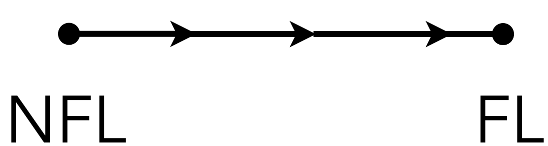
For t≪U, compressibility is almost unaffected by hopping

$$K = \frac{\partial \mathcal{Q}}{\partial \mu}\bigg|_{\mu=0} = \frac{1.04}{U} \ll \gamma \sim \frac{U}{t^2}$$

??How to reconcile with Sommerfeld enhancement??

$$K = g(\epsilon_F)$$

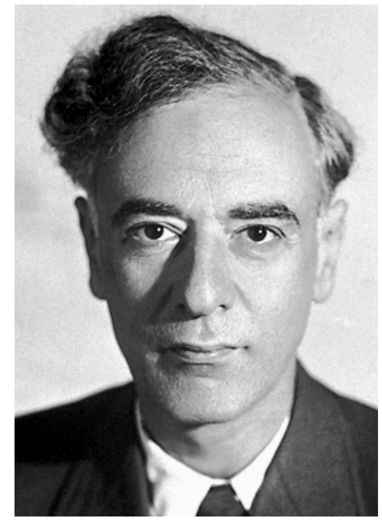
Free fermions



??

$$\gamma = \frac{\pi^2}{3} g(\epsilon_F)$$

Landau Fermi Liquid



- Landau Fermi Liquid Theory

$$\delta E = \sum_a \epsilon_a \delta n_a + \frac{1}{2}\sum_{a,b} f_{ab}\delta n_a \delta n_b$$

- Compressibility is renormalized by Fermi liquid parameter F = g(E_F) f

$$\gamma/K = \frac{\pi^2}{3}(1 + F)$$

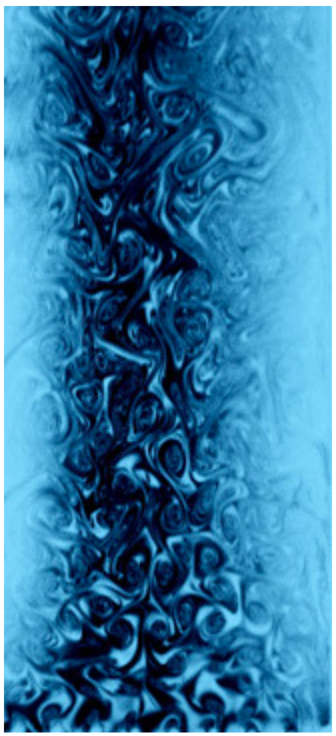
$$\gamma/K \sim (U/t)^2$$

$$F \sim \left(\frac{U}{t}\right)^2 \gg 1$$

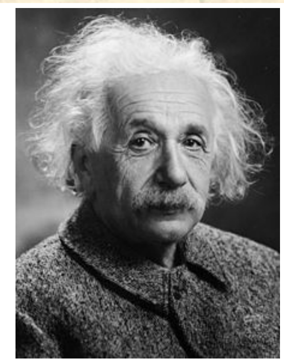
Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics



Einstein-Smoluchowski relation



$$\sigma = e^2 \frac{\partial n}{\partial \mu} D$$

Transport

Quasiparticle picture applies only for $T \ll E_c$

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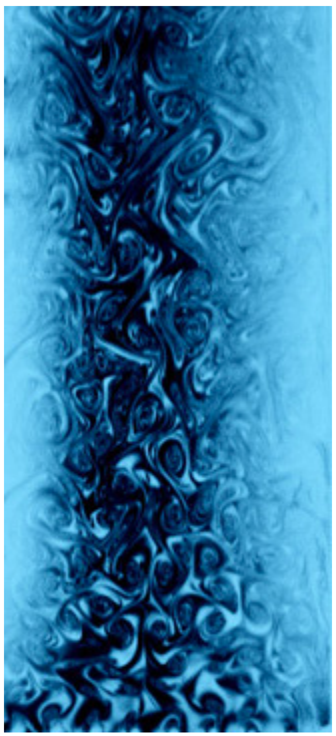


$$\sigma = \lim_{\omega \to 0} \lim_{p \to 0} \frac{-i\omega}{p^2} D_{Rn}(p, \omega)$$

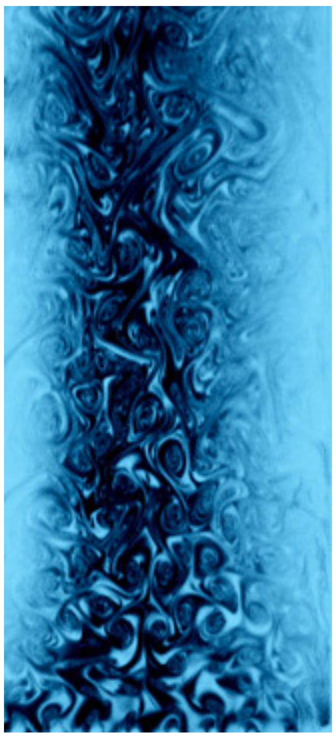
- ✦ Calculate density response using Keldysh method.
- ✦ Do analogously for thermal conductivity
- ✦ Very convenient collective field formulation - fully non-perturbative calculations possible

N.B. Because of randomness, momentum is not a hydrodynamic variable

Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics



Effective action:

$$G_{x,ss'}(t,t') \to G_{x,ss'}(t-t')e^{-i(\varphi_s(x,t)-\varphi_{s'}(x,t'))}$$
$$\Sigma_{x,ss'}(t,t') \to \Sigma_{x,ss'}(t-t')e^{-i(\varphi_s(x,t)-\varphi_{s'}(x,t'))},$$

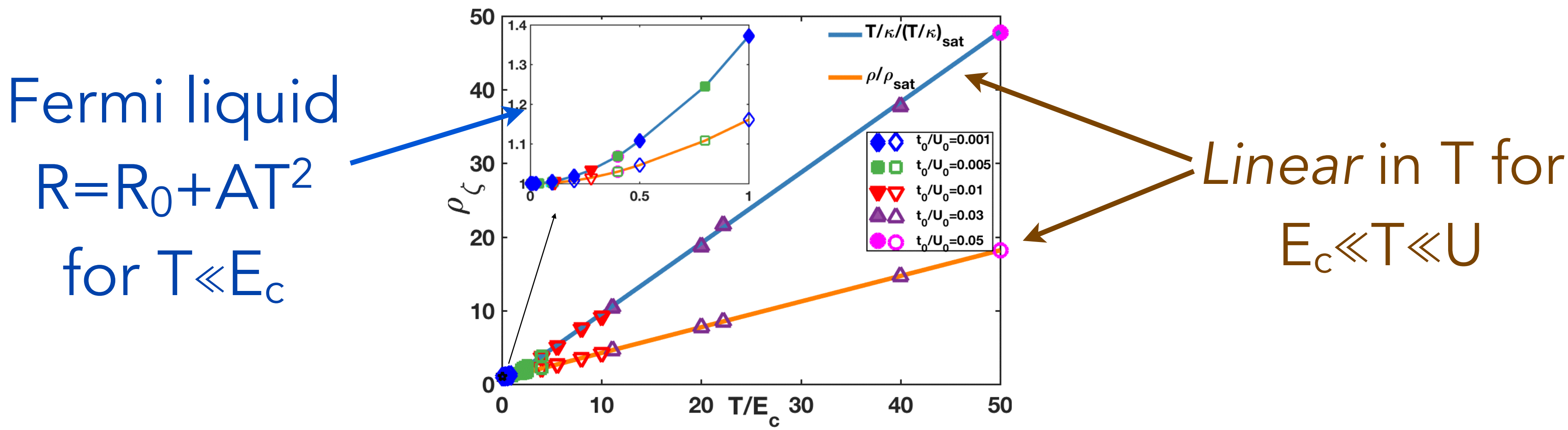
known from SYK solution

$$iS_\varphi = -2K\sum_p \int_{-\infty}^{+\infty} d\omega\, \varphi_{c,\omega}(i\omega^2 - D_\varphi p^2\omega)\varphi_{q,-\omega}.$$

Transport

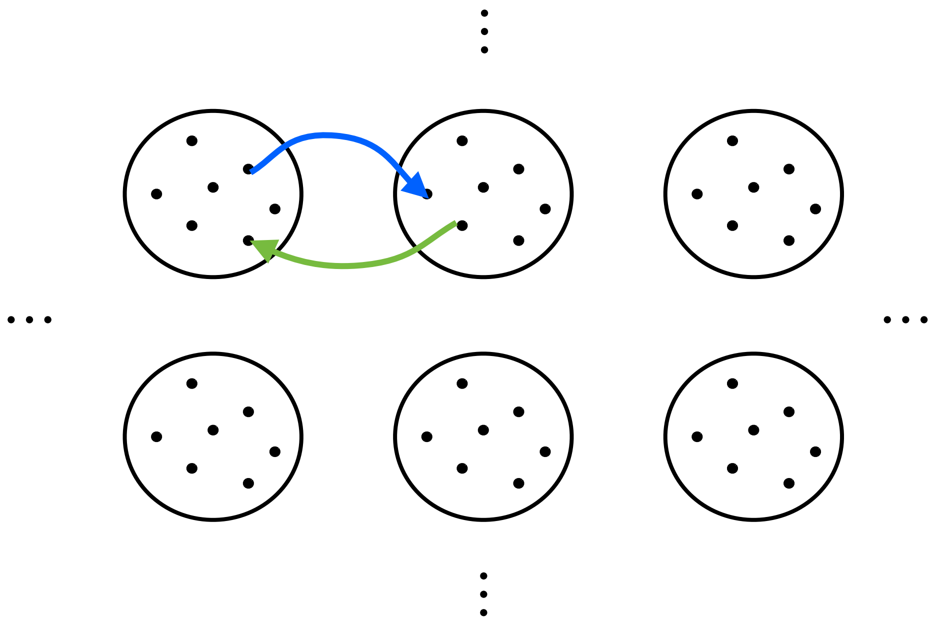
Generalized resistivity $\rho_c = 1/\sigma$ $\rho_e = T/\kappa$

scaling $\rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta(\frac{T}{E_c})$



Crossover from heavy FL to strange metal

Incoherent tunneling



$$J_{x,x'} \sim it\left(c_x^\dagger c_{x'} - \text{h.c.}\right)$$

Kubo $\sigma = \dfrac{\langle JJ \rangle}{\omega}$

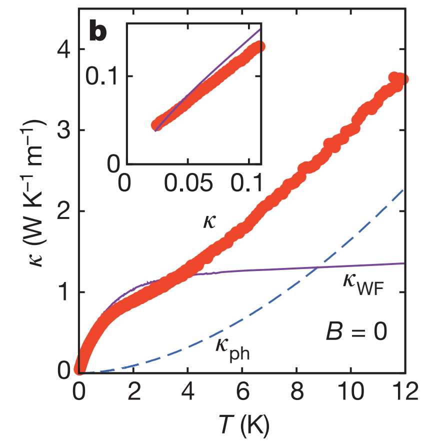
$$\sigma \sim \frac{1}{\omega}\int d\tau\, t^2 \left\langle c_x^\dagger c_{x'} c_{x'}^\dagger c_x \right\rangle \sim \frac{t^2}{E^2} G(\tau)^2 \sim \frac{t^2}{E^2}\left(\frac{1}{\sqrt{U\tau}}\right)^2$$

$$\sim \frac{t^2}{U}\frac{1}{k_B T}$$

Wiedemann-Franz ratio

<u>Lorenz</u> $L = \frac{\kappa}{\sigma T}$

$= \pi^2/3$ for a Fermi liquid

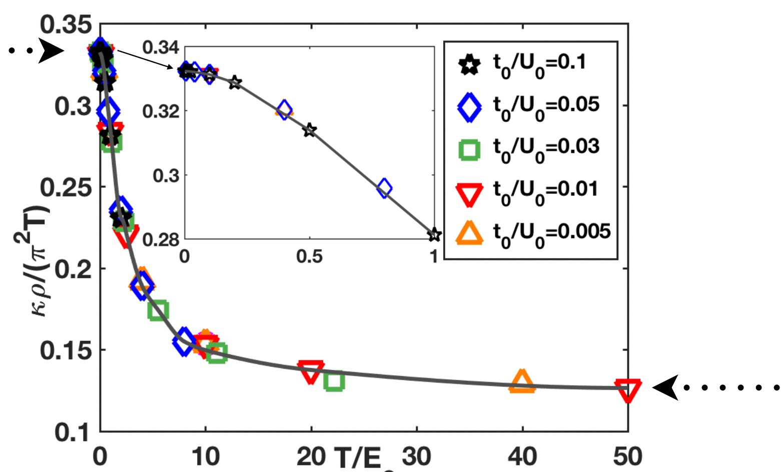


$YbRh_2Si_2$, Pfau *et al* (2012)

<u>SYK lattice:</u>

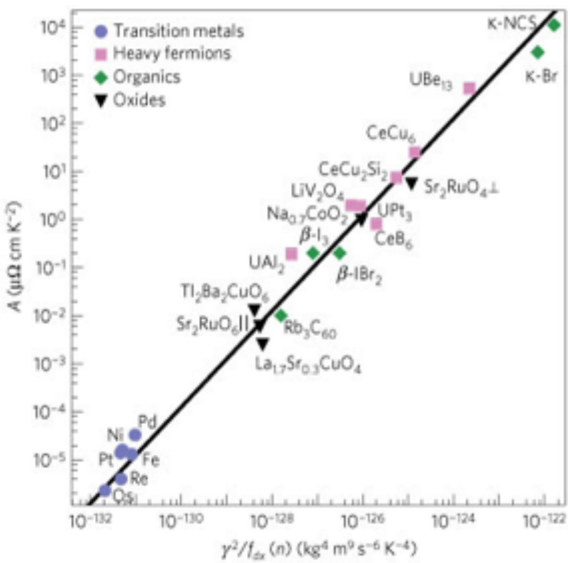
$L = \mathcal{L}(T/E_c)$

FL $\frac{\pi^2}{3}$



$\frac{\pi^2}{8}$ NFL

Kadowaki Woods ratio



$$\rho_\zeta(T \ll E_c) \approx \rho_\zeta(0) + A_\zeta T^2,$$

$$KW = A/\gamma^2$$

approximately constant for many metals

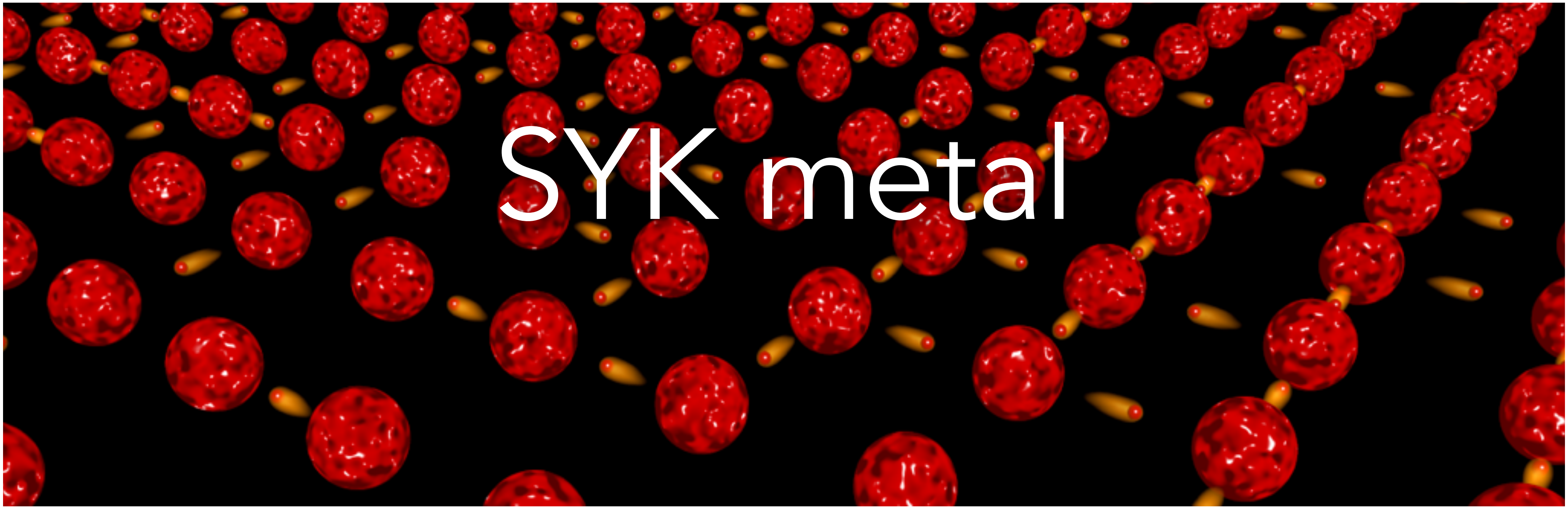
Scaling implies:

$$A \sim 1/(NE_c^2)$$

recall $\gamma \sim 1/E_c$

$$KW = A/(N\gamma)^2 \sim 1/N^3$$

independent of t,U!

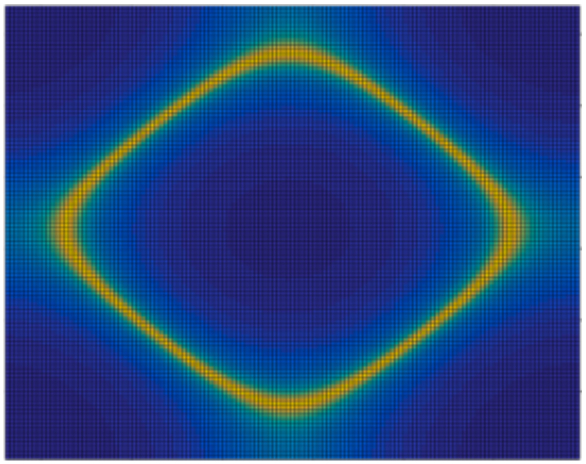


SYK metal

- Small coherence scale $E_c = t^2/U$
- Heavy mass $\gamma \sim m^*/m \sim U/t$
- Small QP weight $Z \sim t/U$
- Kadowaki-Woods A/γ^2 = constant
- Linear in T resistivity and T/κ
- Lorenz ratio crosses over from FL to NFL value

SYK Fermi Surfaces?

- Extension to translationally invariant systems?



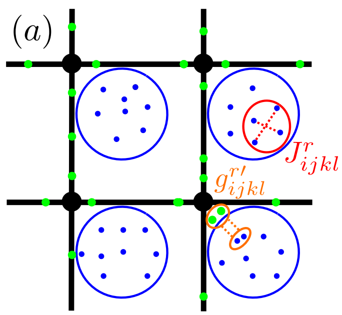
Fermi surface emerging in translationally invariant SYK model by Aavishkar Patel

$$\begin{aligned} G(\boldsymbol{k}, i\omega_n)^{-1} &= i\omega_n - \epsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k}, i\omega_n), \\ \Sigma(\boldsymbol{x}, \tau) &= -U_0^2 G(\boldsymbol{x}, \tau)^2 G(-\boldsymbol{x}, -\tau), \end{aligned}$$

- SYK lattice, tensor models,...
- Momentum space differentiation and realistic applications?
- Relation to methods like DCA, cluster DMFT?

SYK Fermi Surfaces?

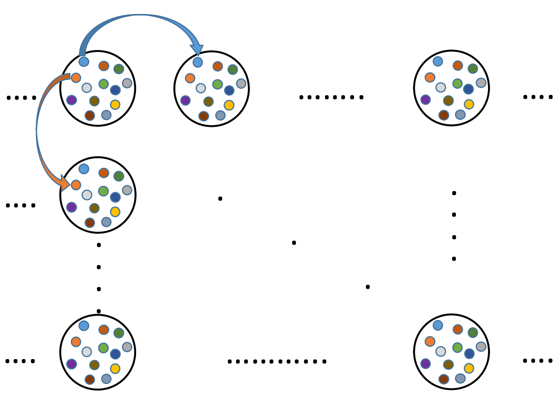
Patel, McGreevy, Arovas, Sachdev, arXiv:1712.05026



SYK dots as strong local scatterers

$$H = -t \sum_{\langle rr' \rangle;\ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r;\ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r;\ i=1}^{N} f_{ri}^\dagger f_{ri}$$
$$+ \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^{r} f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J_{ijkl}^{r} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$

Chowdhury, Werman, Berg, Senthil, arXiv:1801.06178

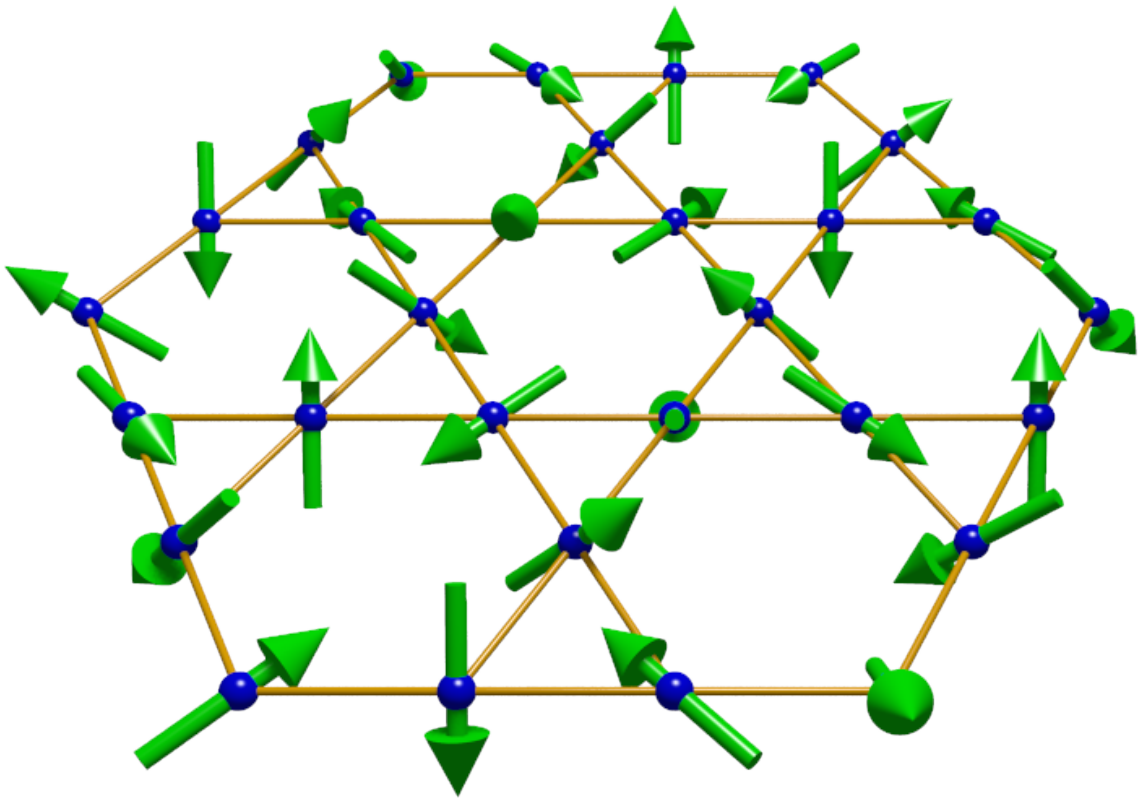


SYK models w/ full translational symmetry

$$H_c = \sum_{\boldsymbol{r},\boldsymbol{r}'} \sum_{\ell} (-t^c_{\boldsymbol{r},\boldsymbol{r}'} - \mu_c \delta_{\boldsymbol{r}\boldsymbol{r}'}) c^\dagger_{\boldsymbol{r}\ell} c_{\boldsymbol{r}'\ell} + \frac{1}{(2N)^{3/2}} \sum_{\boldsymbol{r}} \sum_{ijk\ell} U^c_{ijk\ell} c^\dagger_{\boldsymbol{r}i} c^\dagger_{\boldsymbol{r}j} c_{\boldsymbol{r}k} c_{\boldsymbol{r}\ell},$$

$$H = -t \sum_{\langle rr' \rangle;\ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r;\ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r;\ i=1}^{N} f_{ri}^\dagger f_{ri}$$
$$+ \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl} f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J_{ijkl} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$

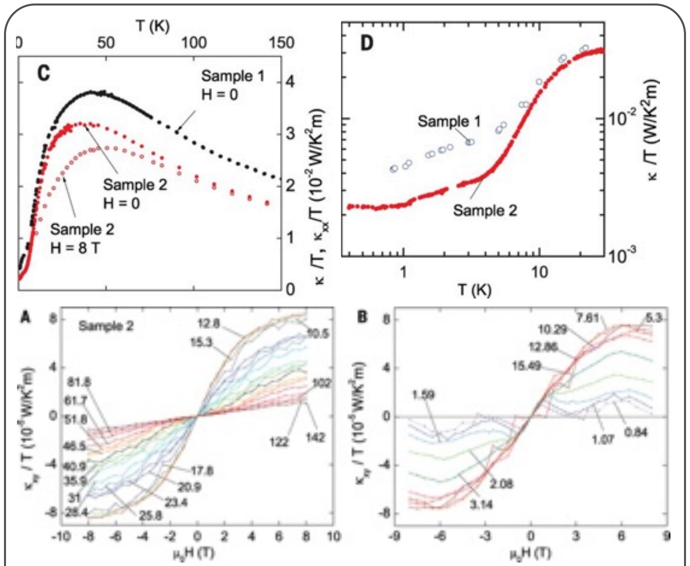
Quantum magnets



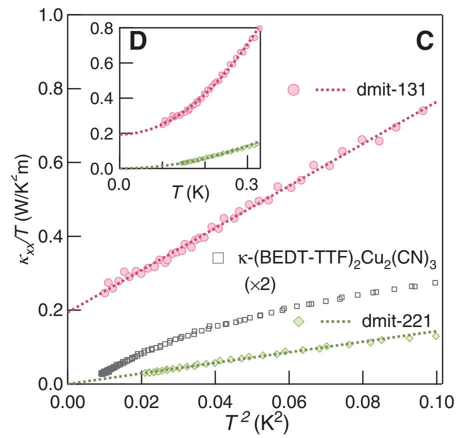
Less obvious ultra-quantum “fluids”

Heat transport in magnets

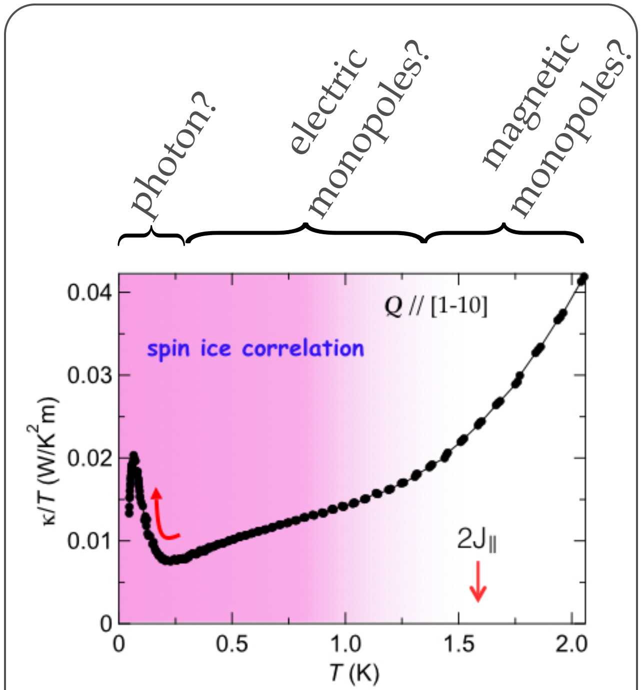
A growing experimental effort valuable for its sensitivity, long-wavelength nature, and automatic removal of localized states



$Tb_2Ti_2O_7$
Ong's group 2015



M. Yamashita *et al*, 2010



$Pr_2Zr_2O_7$
Tokiwa, Matsuda *et al.* (2017?)

SYK magnets?

Original SY paper actually treated spins

VOLUME 70, NUMBER 21 PHYSICAL REVIEW LETTERS 24 MAY 1993

Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

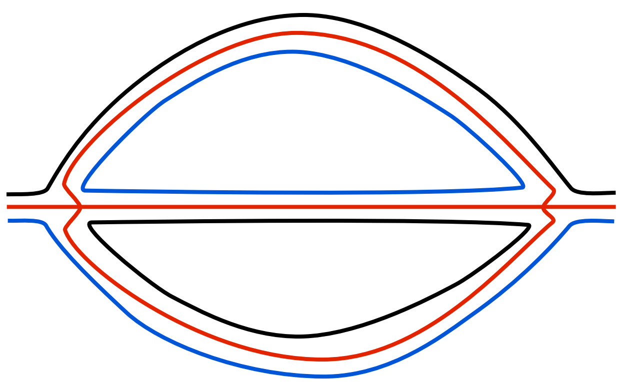
Subir Sachdev and Jinwu Ye

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{\mathcal{S}}_i \cdot \hat{\mathcal{S}}_j$$

Challenge: extend to non-disordered systems

- Similar to problem of SYK Fermi surfaces
- Tensor model for large N control?

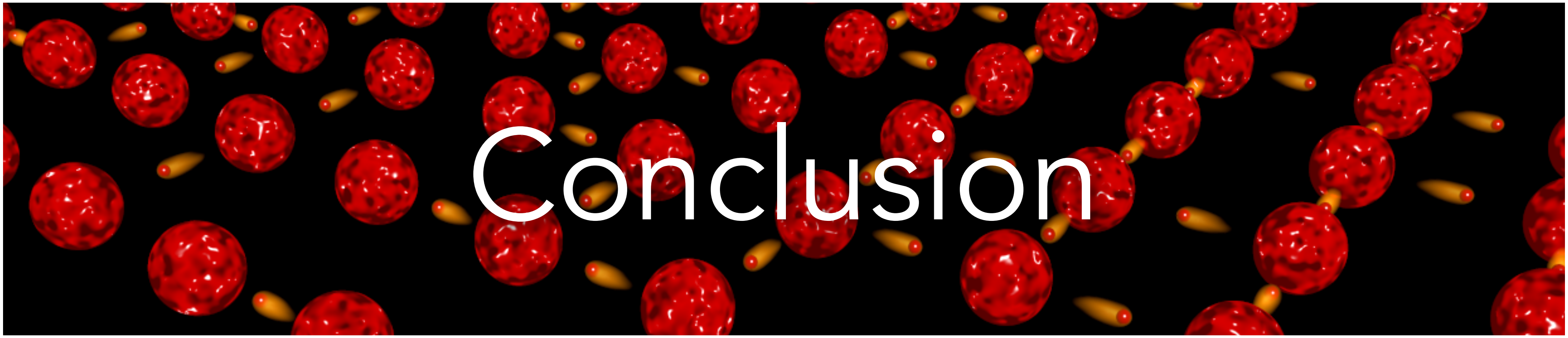
$$\Sigma =$$



$$S_{i;abcd} = f^{\dagger}_{i;Aab} f_{j;Acd}$$

$$H = \frac{J}{N^{3/2}} \sum_{\langle ij \rangle} S_{i;abcd} S_{j;adcb}$$

Conclusion



- Convergence of ideas suggests that maybe problematic metals are not "strange", "bad", or "incoherent" but rather they are **SYK**

GORDON AND BETTY MOORE FOUNDATION