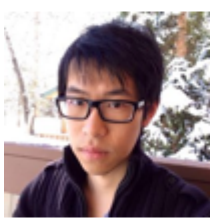
Strongly interacting Fermi liquids and the SYK model

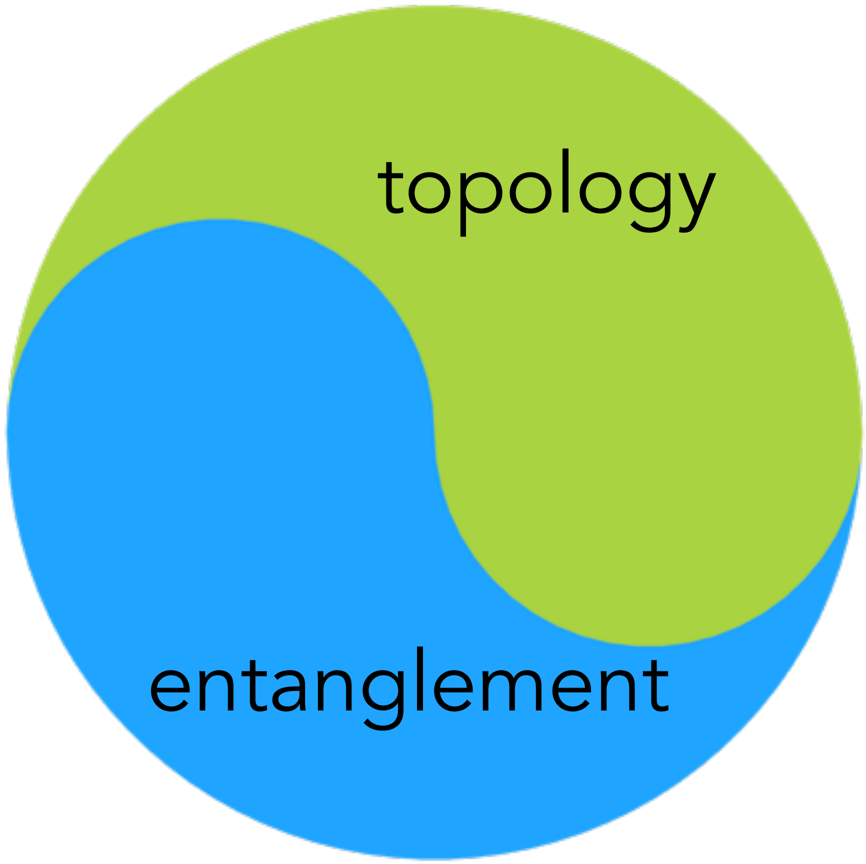
Leon Balents, KITP, UCSB

Xue-Yang Song
Beijing -> Harvard

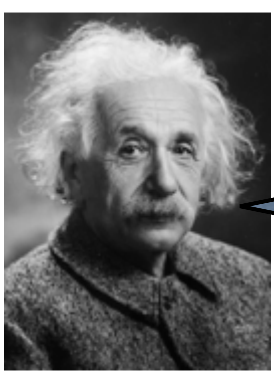


Chao-Ming Jian

KITP/Microsoft Q


topology
entanglement

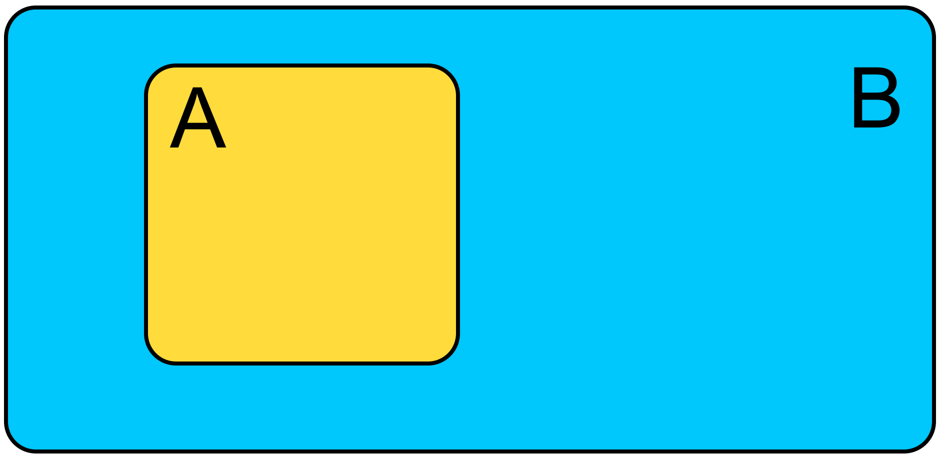
Quantum Entanglement



spukhafte Fernwirkung!

Degrees of Entanglement

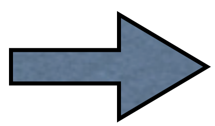
How much does A depend on B?



$$\rho_A = \mathrm{Tr}_B \, |\psi\rangle\langle\psi|$$

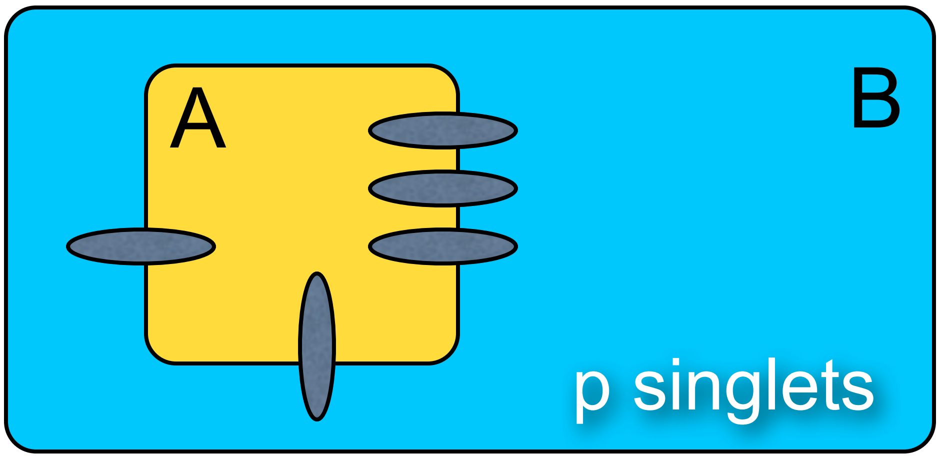
von Neumann "entanglement entropy"

$$S(A) = -\mathrm{Tr}\left[\rho_A \ln \rho_A\right]$$

$S(A) > 0$  $|\psi\rangle \neq |\psi\rangle_A \otimes |\psi\rangle_B$

Degrees of Entanglement

How much does A depend on B?



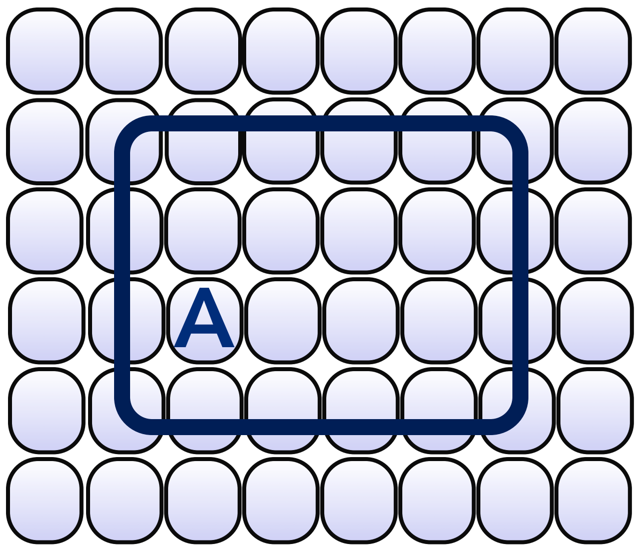
$$\rho_A = \mathrm{Tr}_B \, |\psi\rangle\langle\psi|$$

von Neumann "entanglement entropy"

$$S(A) = -\mathrm{Tr}\left[\rho_A \ln \rho_A\right]$$
$$= p \ln 2$$

Degrees of Entanglement

- Extensive system



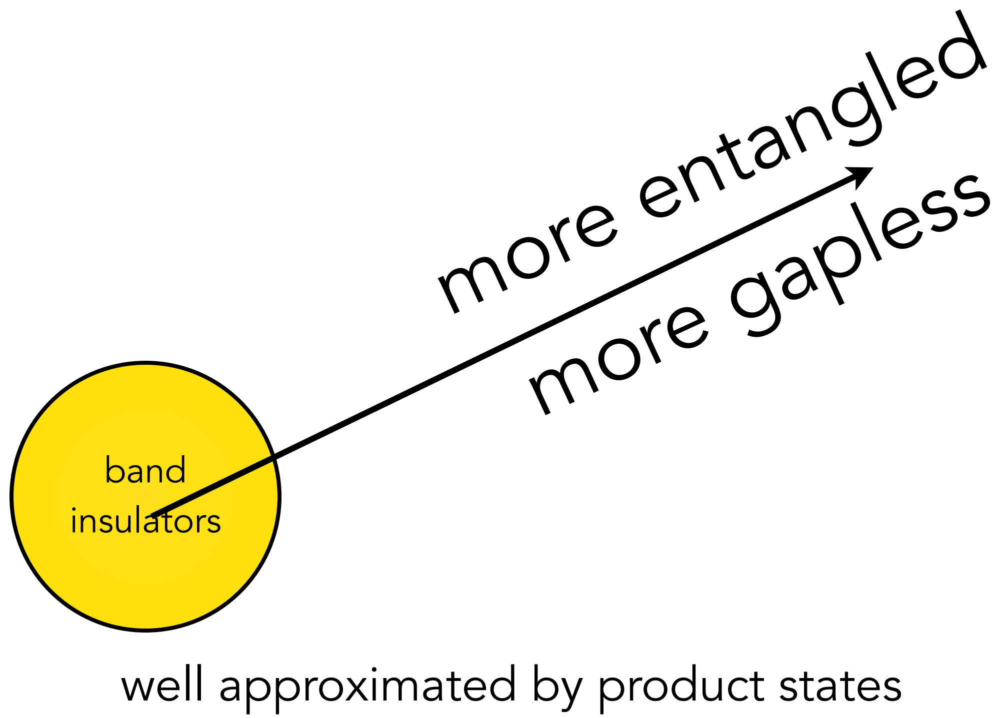
$$\rho_A = \mathrm{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\mathrm{Tr}_A \left(\rho_A \ln \rho_A\right)$$

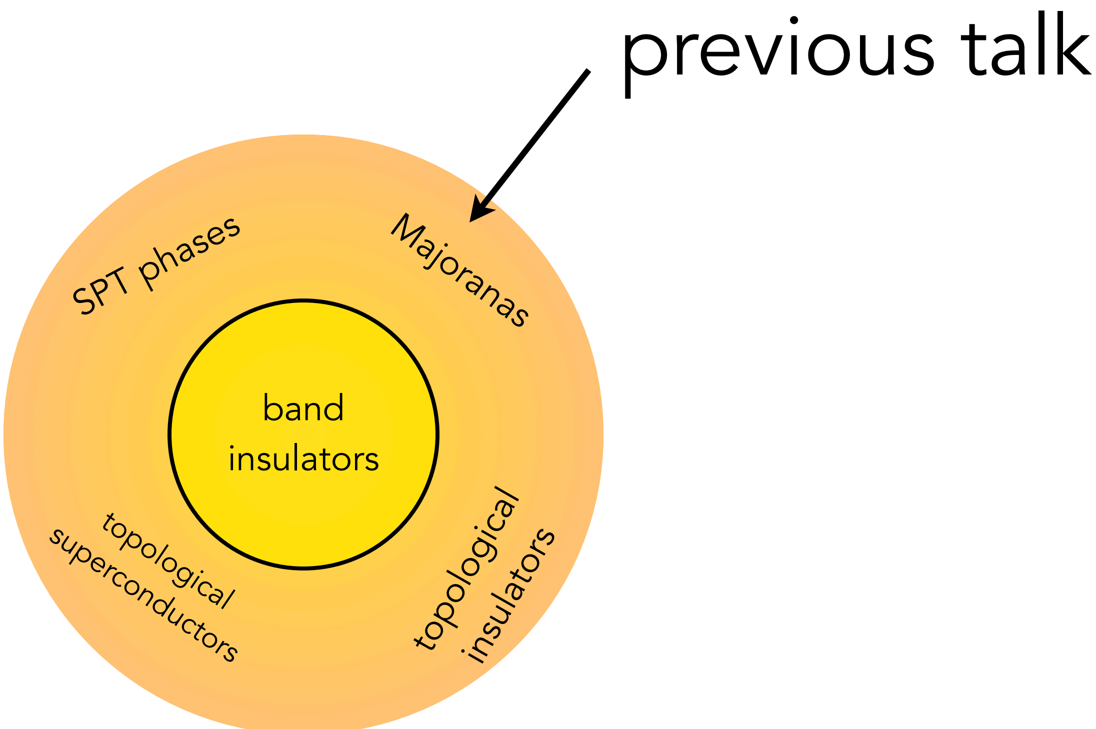
$S(A) \sim \sigma L^{d-1}$ area law

True for any “product-like” state

Degrees of Entanglement

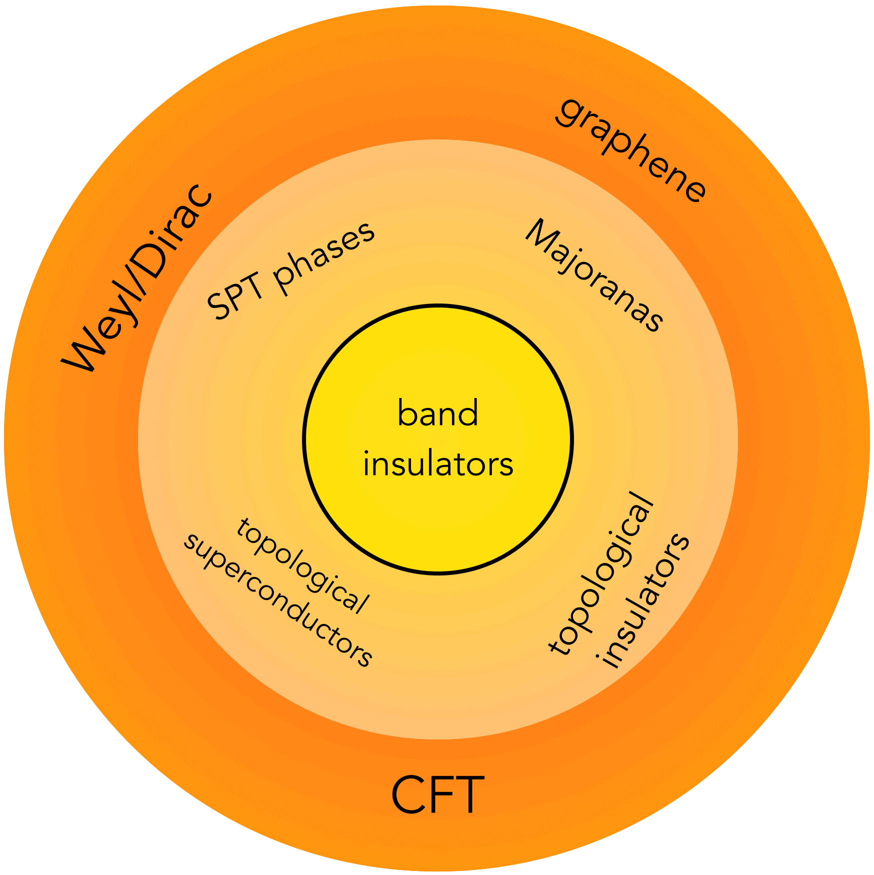


Degrees of Entanglement



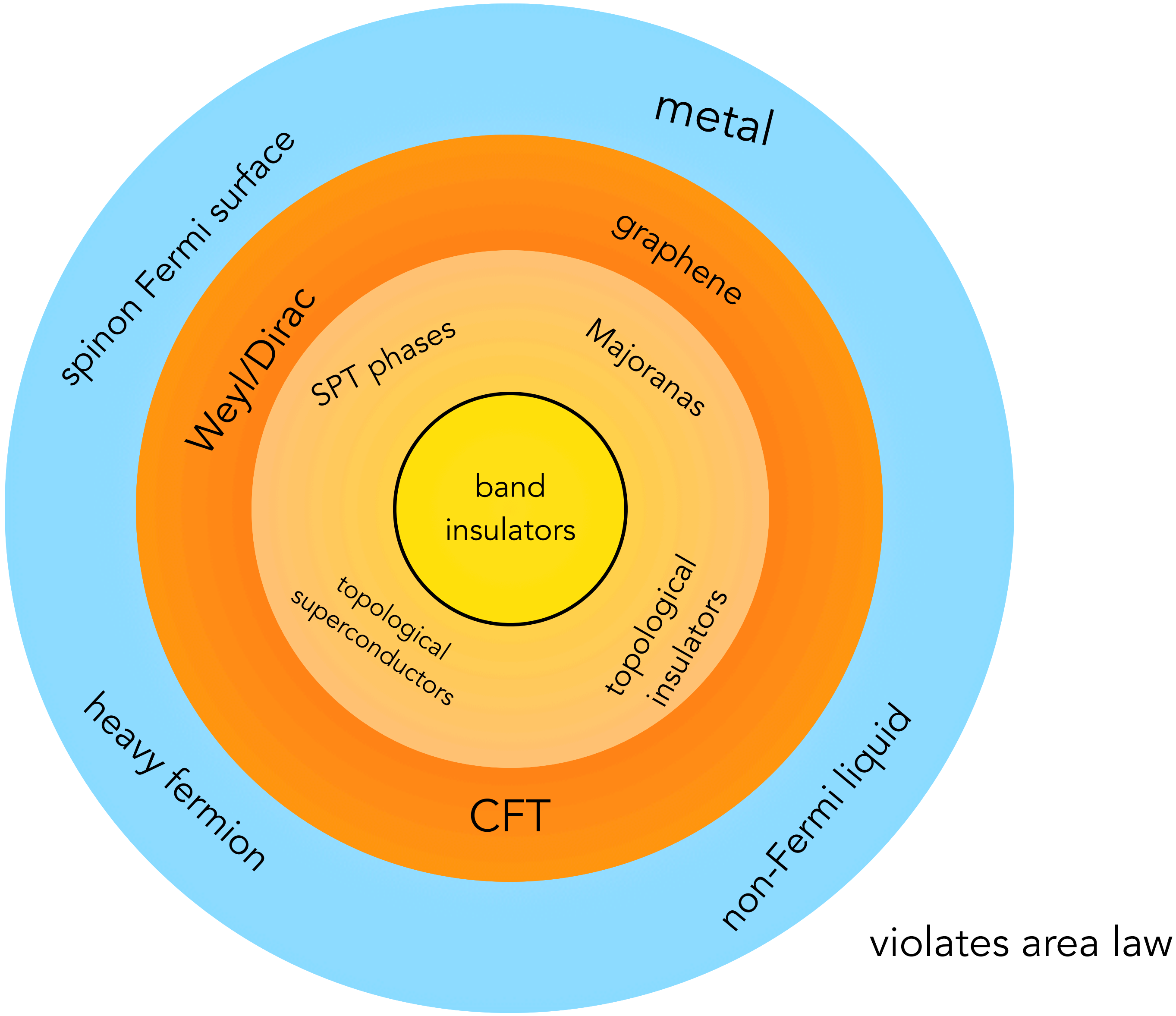
well approximated by product states (but some little errors are unavoidable)

Degrees of Entanglement

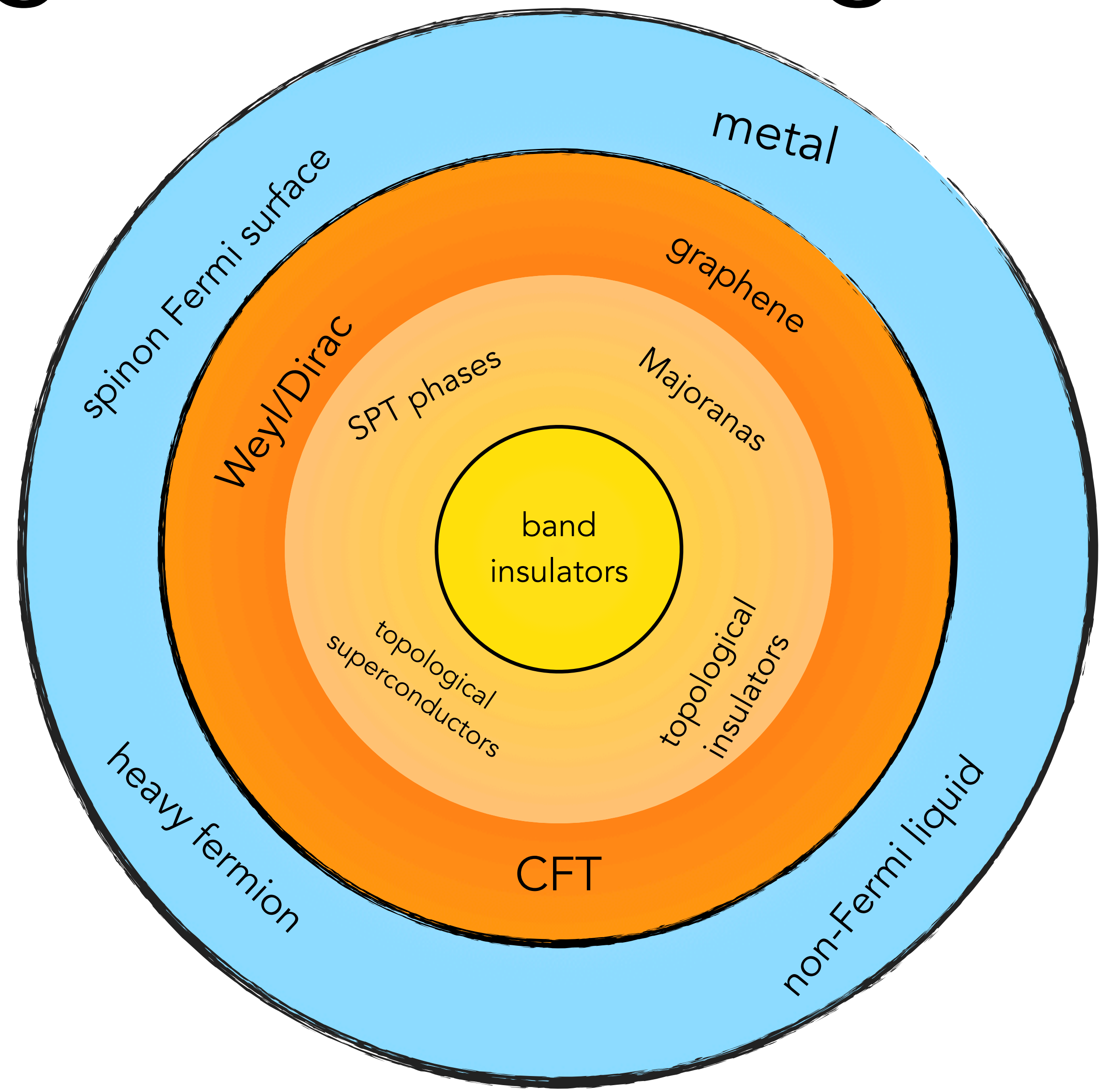


very badly approximated by product states
but still obeys area law

Degrees of Entanglement



Degrees of Entanglement



This talk

Metals Conduct

$$\boldsymbol{j} = \underline{\sigma}\boldsymbol{E} \qquad \boldsymbol{j}_e = -\underline{\kappa}\boldsymbol{\nabla}T$$

- Arguably most important aspect of quantum materials: electrical and thermal conductivity (and crossed coefficients)
- Sensitive, versatile
- Probes extreme long wavelength, low frequency

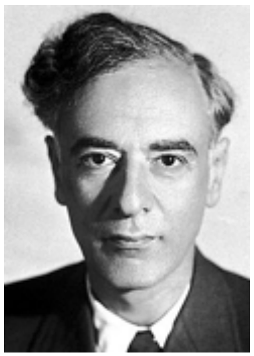
Theory

- *Understanding* of transport mainly through **quasiparticle** picture
  - Boltzmann equation:

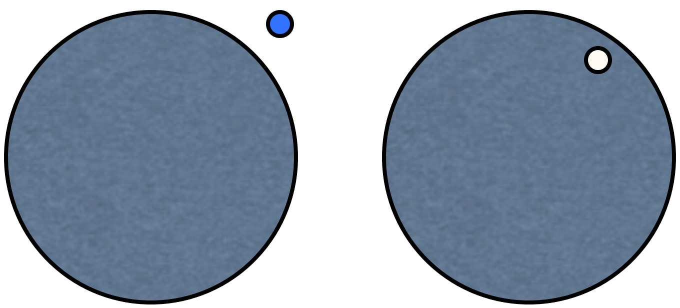
$$[\partial_t + \boldsymbol{v}_n(\boldsymbol{k}) \cdot \boldsymbol{\nabla}_r - e\boldsymbol{E} \cdot \boldsymbol{\nabla}_k] \, f_n = \left.\frac{\partial f_n}{\partial t}\right|_{\text{collision}}$$

Linearizing this around equilibrium gives conductivities in terms of band velocities and scattering rates

Fermi Liquid Theory



Landau provided justification for quasiparticle picture in metals when $T << E_F$

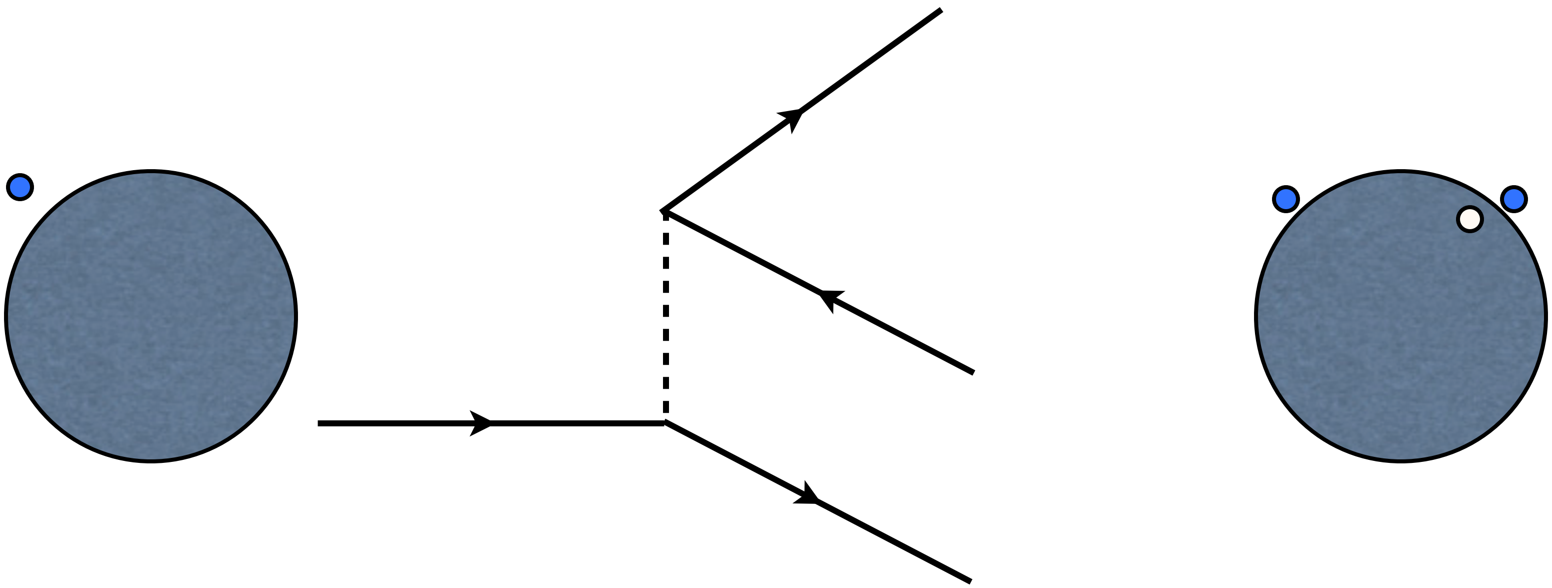


Low energy excitations act like electrons and holes but with wavefunction dressing (Z<1), effective mass, and Landau interactions

$$E = \sum_k \epsilon_k \delta n_k + \frac{1}{2V} \sum_{k,k'} U_{k,k'} \delta n_k \delta n_{k'}$$

scattering is weak because not so many low energy qp states to scatter to

Scattering



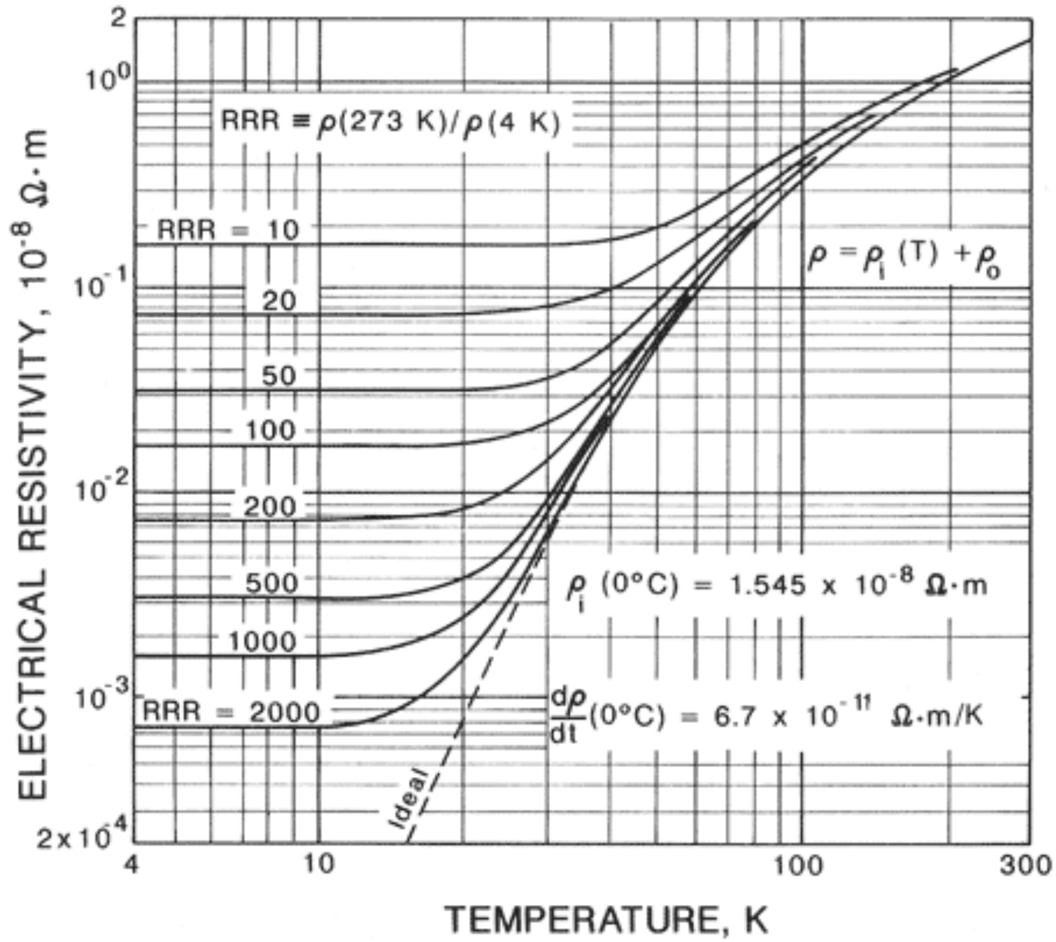
phase space ~ $(T/E_F)^2$

Typical metal?

$E_F \sim 10^4$ K

$\rho(T) - \rho(0) \sim AT^2$

for $T \ll E_F$



(Cu)

Heavy Fermi Liquids

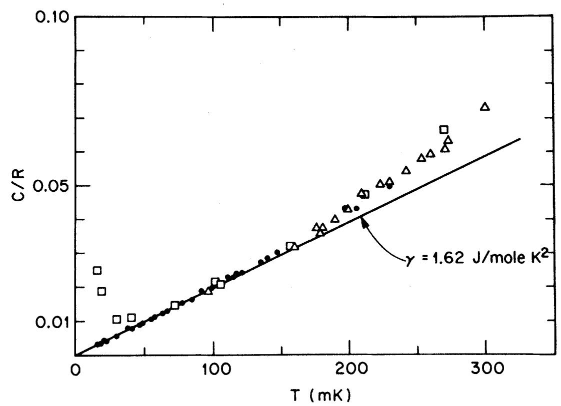


FIG. 1. Specific heat of $CeAl_3$ at very low temperatures in zero field (●, Δ) and in 10 kOe (□).

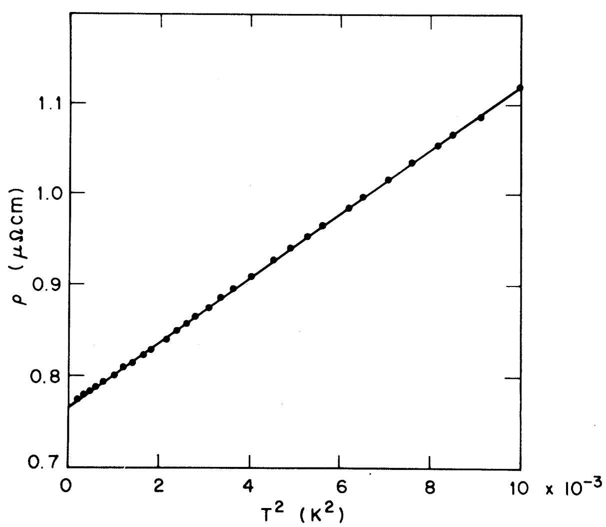


FIG. 3. Electrical resistivity of $CeAl_3$ below 100 mK, plotted against T^2.

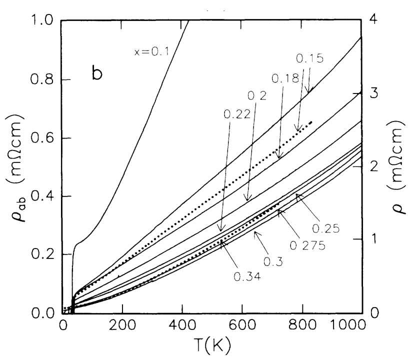
$$C \sim \gamma T$$

$$\rho(T) - \rho(0) \sim AT^2$$

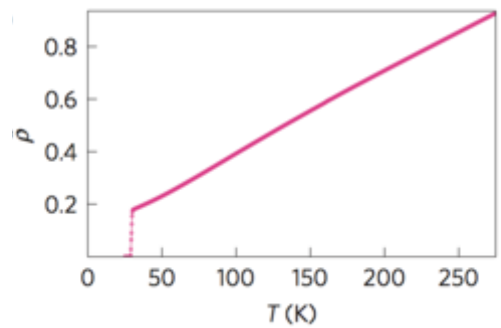
Both γ and A **huge**

Behave like Fermi liquid with tiny E_F and large electron mass, but only for $T \ll E_F$

Non-Fermi Liquids



LSCO Takagi *et al, 1992*



$BaFe_2(As_{1-x}P_x)_2$, Hayes *et al, 2016*

$$\frac{1}{\tau} \sim T\,?$$

T-linear resistivity:

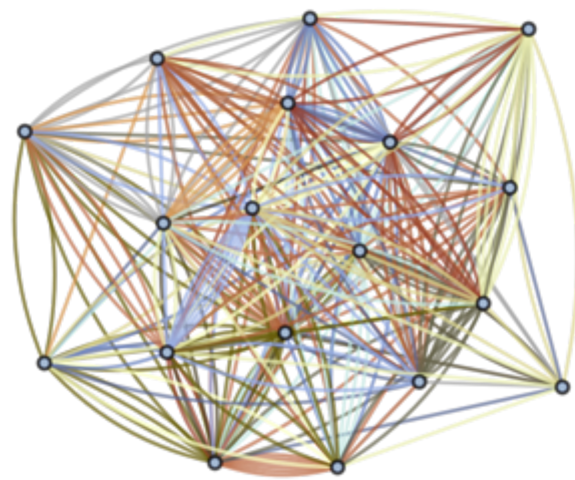
- Many materials
- Often nearby to uncoventional superconductivity
- Symptom of a different type of metal?
- If no quasiparticles exist, what is the starting point?

Sachdev-Ye-Kitaev model

A toy *exactly soluble* model
of a non-Fermi liquid

$$H = \sum_{i<j,k<l} U_{ijkl} \, c_i^\dagger c_j^\dagger c_k c_l$$

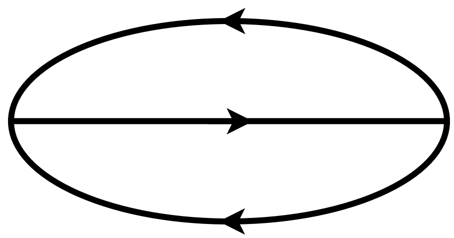
$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3}$$



Like a strongly interacting quantum dot
or atom with complicated Kanamori
interactions between many "orbitals"

SYK Model

Sachdev-Ye, 1993: Model has a soluble large-N limit

$\Sigma =$  $+O(1/N)$

In equations: very similar to DMFT:

$$G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)} \qquad \Sigma(\tau) = -U^2 G(\tau)^2 G(-\tau)$$

Solution:

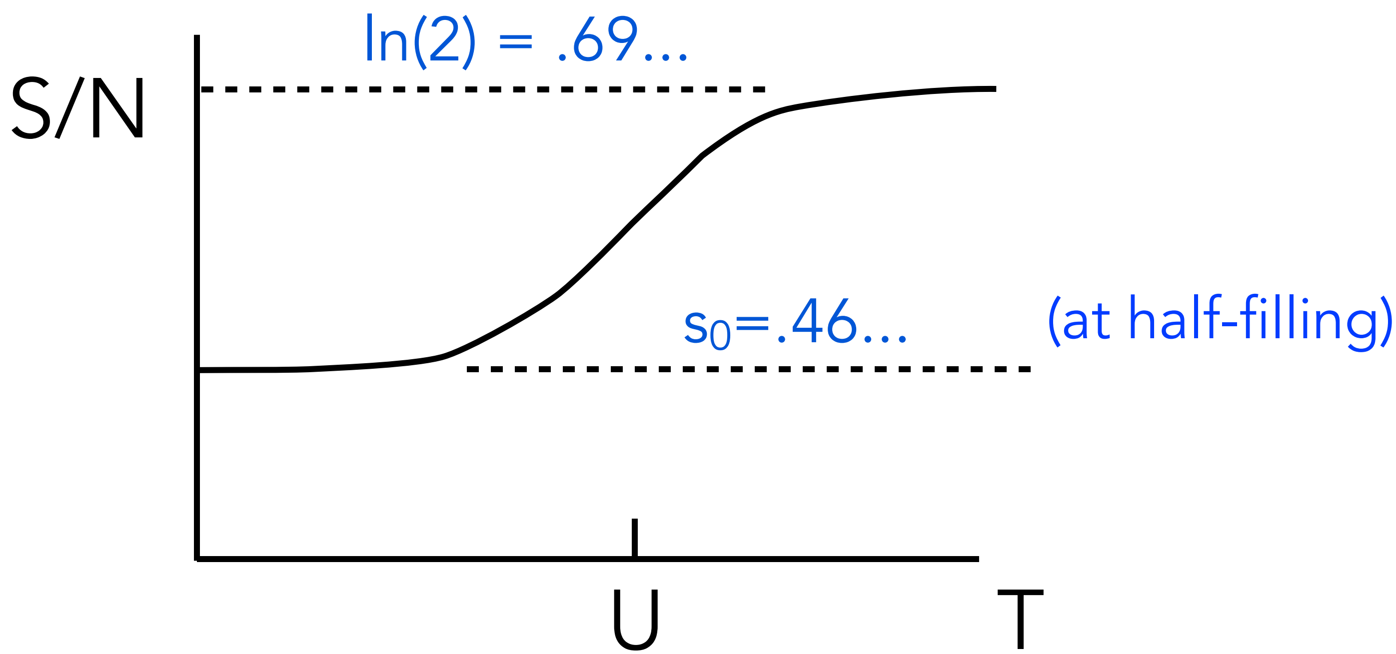
$$G(i\omega) \sim 1/\sqrt{\omega}$$

not a pole: non-Fermi liquid

SYK Model

Why not quasiparticles?

Georges, Parcollet, Sachdev, 2001: **ground state entropy!**



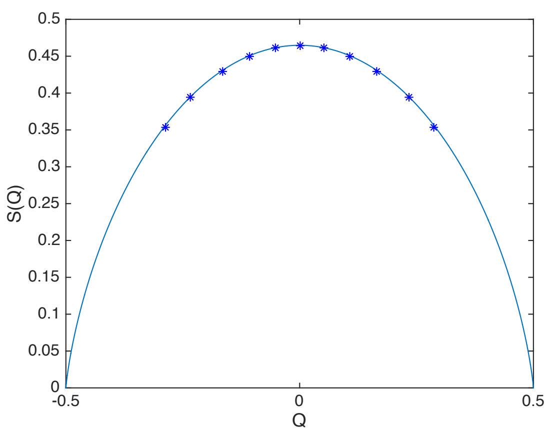
Many states available for scattering

"level spacing" ~ U exp($-Ns_0$)

Density dependence

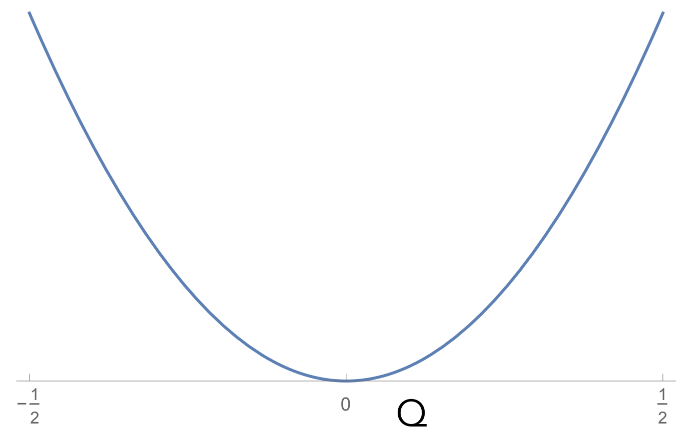
$$H \to H - \mu\mathcal{N} \qquad \mathcal{N} = \sum_i c_i^\dagger c_i \qquad \mathcal{Q} = \frac{\mathcal{N}}{N} - \frac{1}{2}$$

Entropy



Davison *et al*, arXiv:1612.00849

Energy

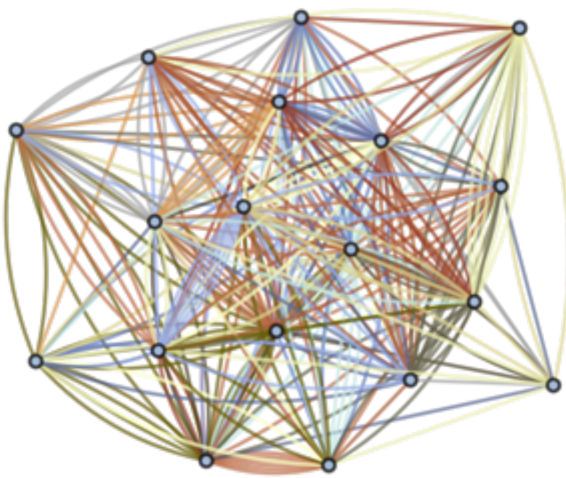


schematic

- Compressibility is constant at T=0

$$K = \left.\frac{\partial\mathcal{Q}}{\partial\mu}\right|_{\mu=0} = \frac{1.04}{U}$$

SYK Summary



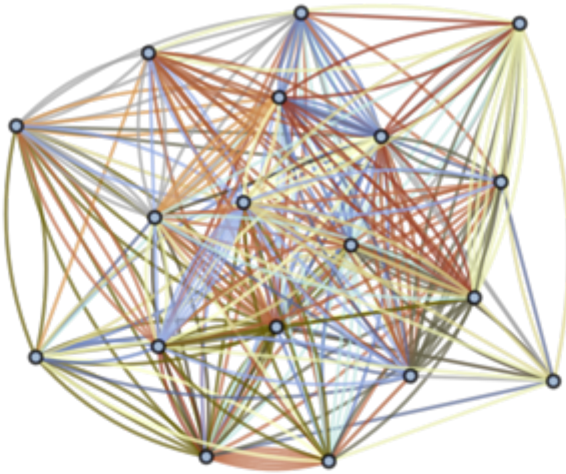
- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T=0) = \frac{1.04}{U}$$

$$S(T=0)/N = .46\ldots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

SYK Summary



- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T=0) = \frac{1.04}{U}$$

$$S(T=0)/N = .46\ldots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

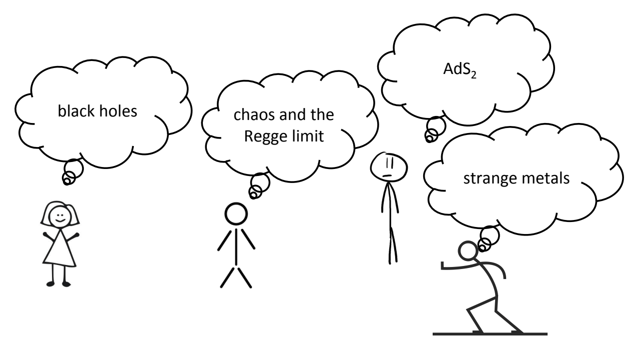
circa
2015

Chaos

$$\left\langle [\mathcal{O}(0), \tilde{\mathcal{O}}^\dagger(t)]^2 \right\rangle \sim \frac{1}{N} e^{\lambda_L t}$$

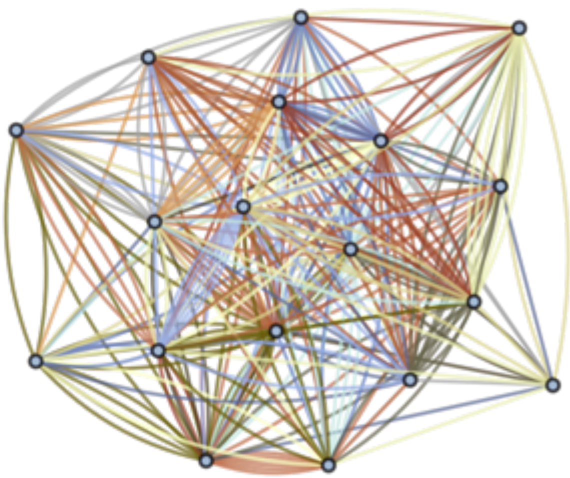
$$\lambda_L = \frac{2\pi k_B T}{\hbar}$$

Holography



slide from D. Stanford, IAS, 2017

SYK Summary



- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T=0) = \frac{1.04}{U}$$

$$S(T=0)/N = .46\ldots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

See David Gross' and Vlad Rosenhaus' lectures next week

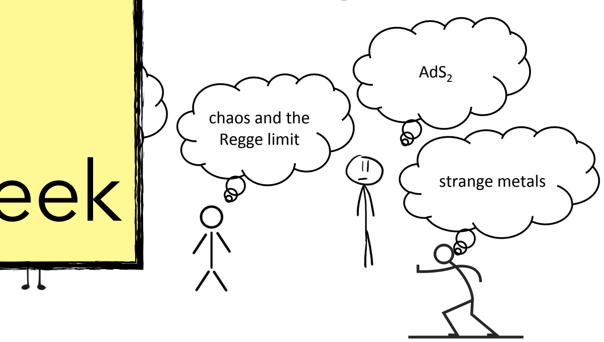
circa 2015

Chaos

$$\left\langle [\mathcal{O}(0), \tilde{\mathcal{O}}^\dagger(t)]^2 \right\rangle$$

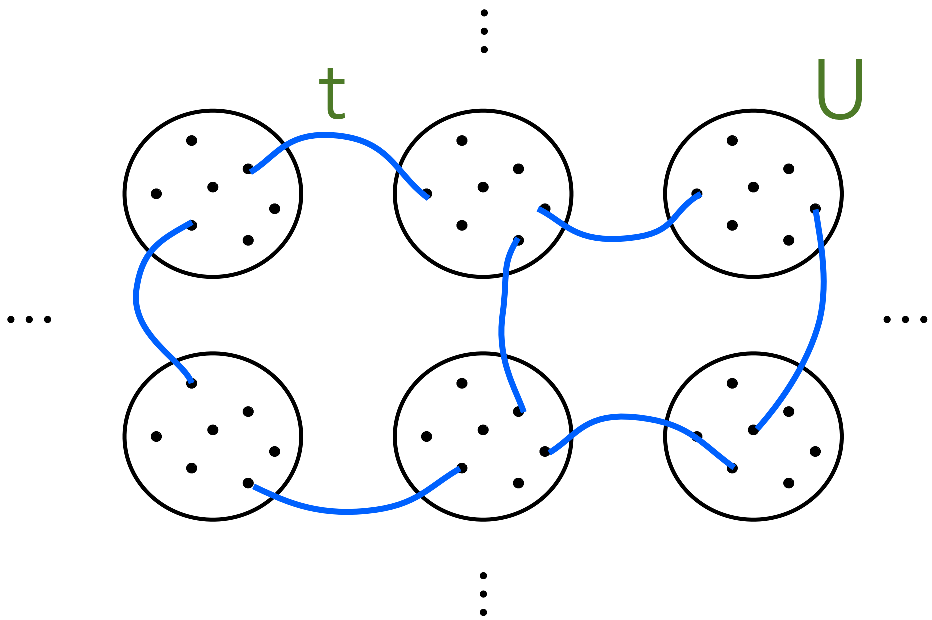
$$\lambda_L = \frac{2\pi k_B T}{\hbar}$$

lography



slide from D. Stanford, IAS, 2017

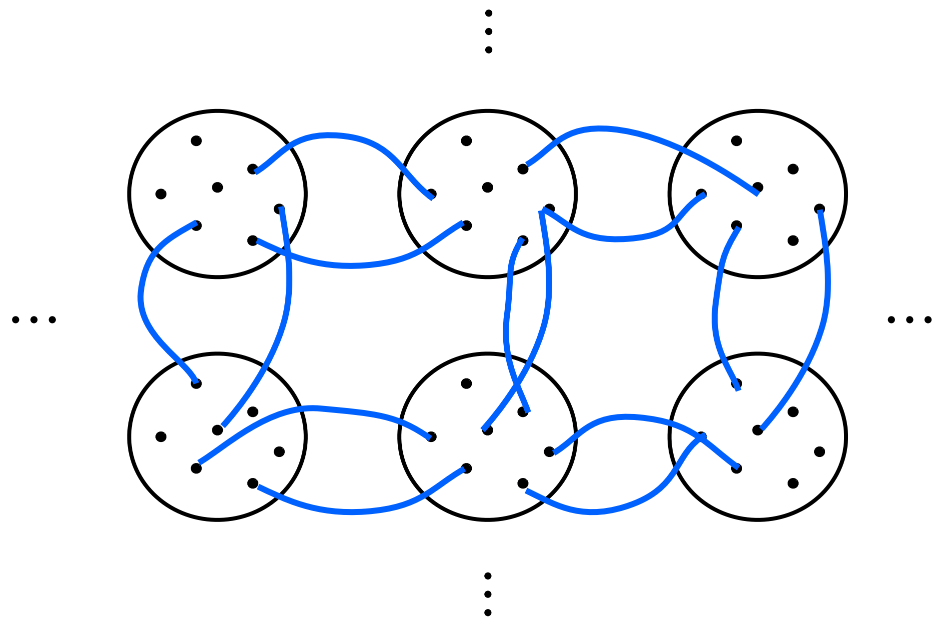
Building a metal



$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|t_{ij,x,x'}|^2} = t_0^2/N.$$

Building a metal



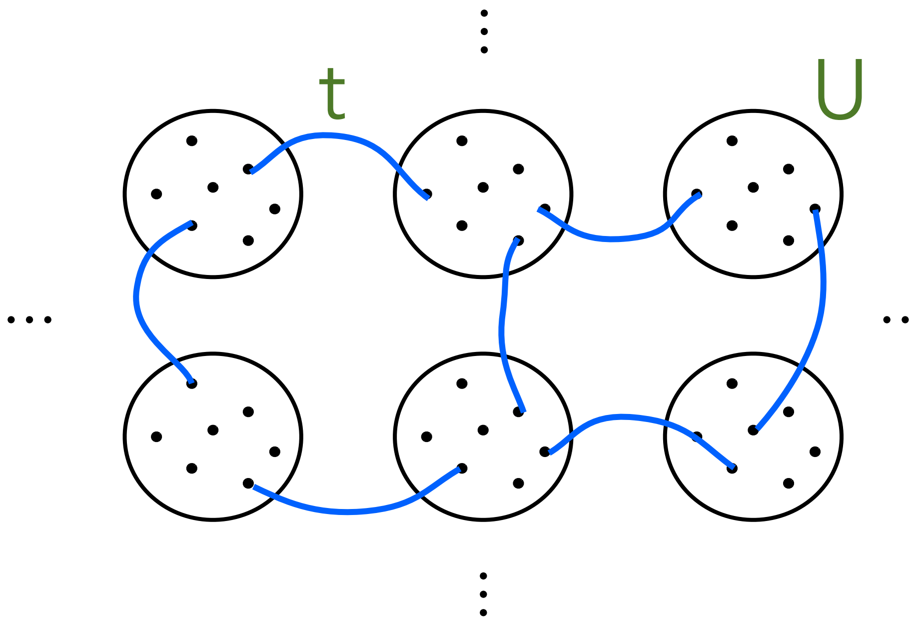
Other work: 2-electron hopping

Y.Gu *et al*, arXiv:1609.07832
R. Davison *et al*, arXiv:1612.00849

$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ijkl,xx'} c^\dagger_{i,x} c^\dagger_{j,x} c_{k,x'} c_{l,x'} + \text{h.c.}$$

Omitting *relevant* 1-electron hopping leaves system NFL even at T=0

Building a metal

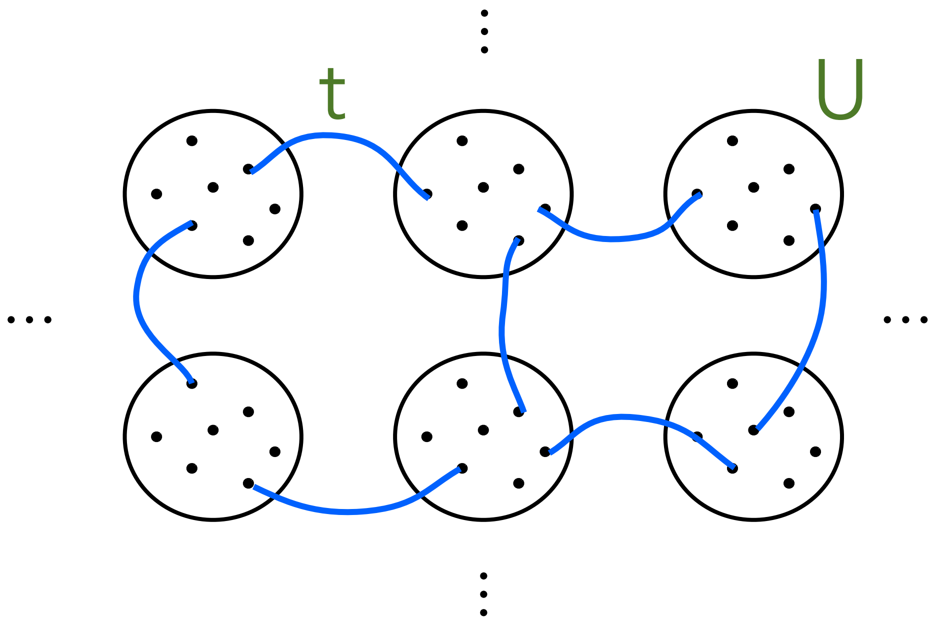


$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

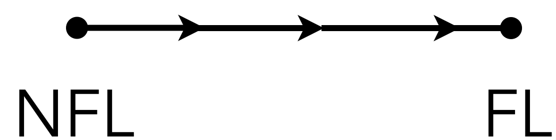
competition!

$t/U \ll 1$ interesting

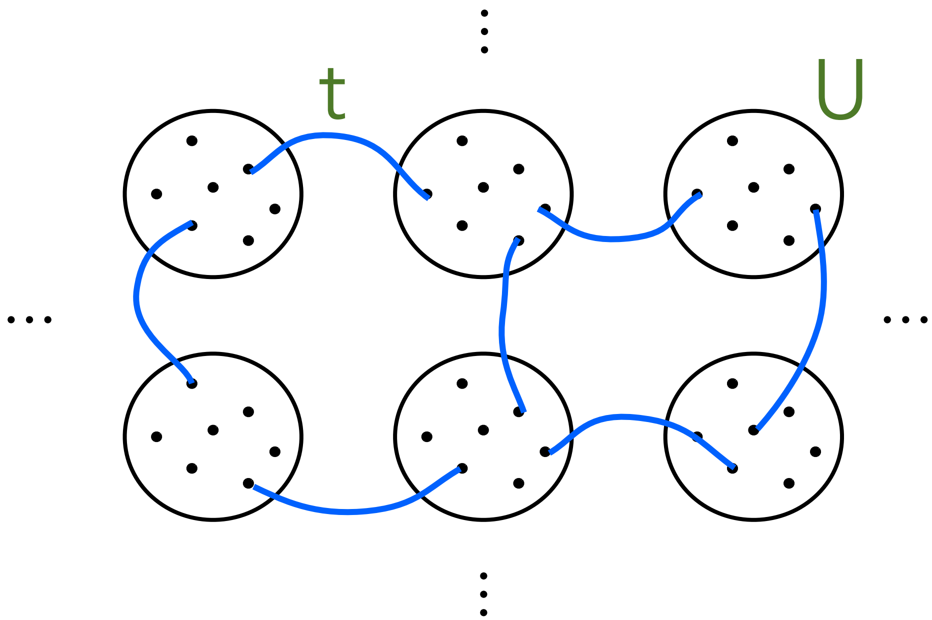
Building a metal



$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x'}$$

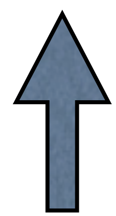


Self-consistent equations



$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$
$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

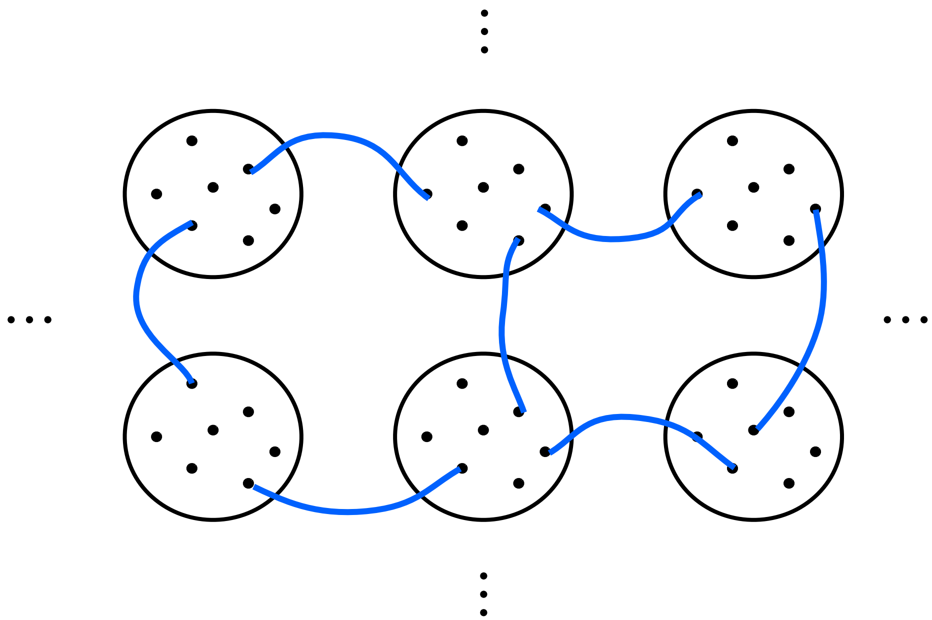
$$\bar{G}(i\bar{\omega}) = \tilde{t}G(i\omega)$$



strong similarities to DMFT equations

mathematical structure appeared in early study of doped t-J model with *double* large N and infinite dimension limits: O. Parcollet+A. Georges, 1999

Coherence scale



$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$
$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

$$\bar{G}(i\bar{\omega}) = \tilde{t}G(i\omega)$$

Rescaling

$$\bar{\omega} = \frac{\omega}{\tilde{E}_c}, \quad \bar{\tau} = \tau\tilde{E}_c, \qquad \bar{G}(i\bar{\omega}) = \tilde{t}G(i\omega) \quad \bar{\Sigma}(i\bar{\omega}) = \Sigma(i\omega)/\tilde{t} \qquad \tilde{t} = \left(\frac{z}{2}\right)^{\frac{1}{2}} t$$

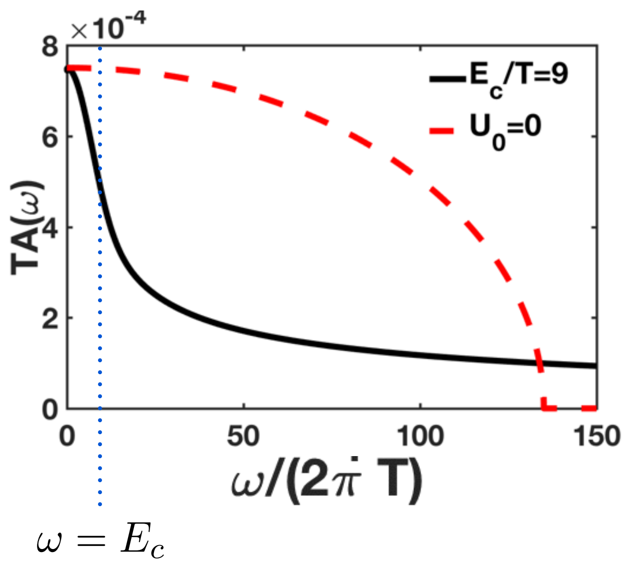
$$\bar{G}(i\bar{\omega}) = \frac{\tilde{t}}{U} i\bar{\omega} - \bar{\Sigma}(i\bar{\omega}) \qquad \bar{\Sigma}(\bar{\tau}) = -\bar{G}(\bar{\tau})^2\bar{G}(-\bar{\tau}) + 2\bar{G}(\bar{\tau}),$$

For t≪U, a single universal coherence scale appears

$$\tilde{E}_c = \frac{\tilde{t}^2}{U}$$

Coherence scale

We solve these equations in a real time Keldysh formulation numerically and determine asymptotics analytically.

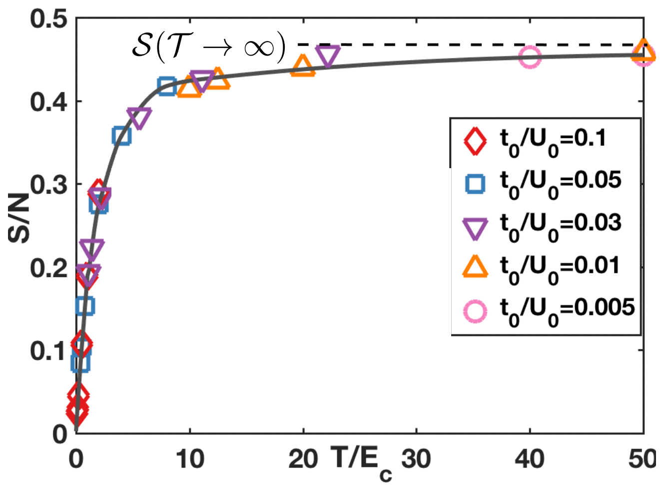


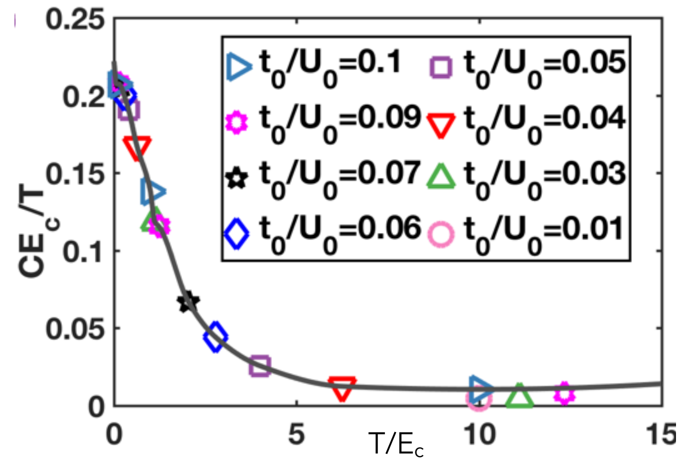
Narrow “coherence peak” appears in spectral function: heavy quasiparticles form for $T \ll E_c$

Quasiparticle weight $Z \sim t/U$

Entropy

Level repulsion: entropy is released for $T<E_c$!





Universal scaling forms

$$S/N \ = \ \mathcal{S}(T/E_c)$$

$$C/N = T/E_c\mathcal{S}'(T/E_c)$$

$$\gamma \equiv \lim_{T\to 0}\frac{C}{T} = \frac{\mathcal{S}'(0)}{E_c}$$

Sommerfeld enhancement

$$m^*/m \sim U/t$$

Compressibility

For t≪U, compressibility is almost unaffected by hopping

$$K = \frac{\partial \mathcal{Q}}{\partial \mu}\bigg|_{\mu=0} = \frac{1.04}{U} \ll \gamma \sim \frac{U}{t^2}$$

??How to reconcile with Sommerfeld enhancement??

- Fermi liquid theory: compressibility is renormalized by Fermi liquid parameter $F = g(E_F)\, U_{FL}$

$$\gamma / K = \frac{\pi^2}{3}(1 + F)$$

$$\gamma / K \sim (U/t)^2$$

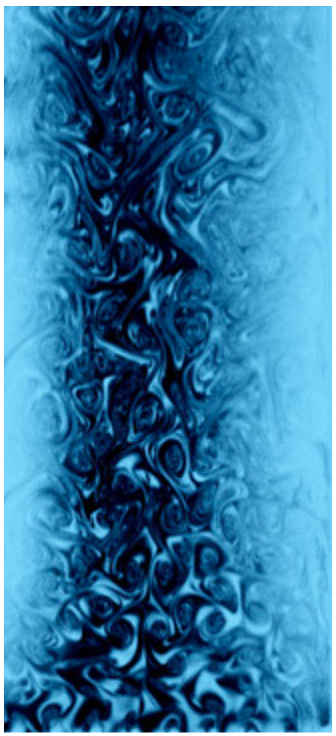
→

$$F \sim \left(\frac{U}{t}\right)^2 \gg 1$$

Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics



$$\sigma = \lim_{\omega \to 0} \lim_{p \to 0} \frac{-i\omega}{p^2} D_{Rn}(p,\omega)$$

- ✦ Calculate density response using Keldysh method.
- ✦ Do analogously for thermal conductivity

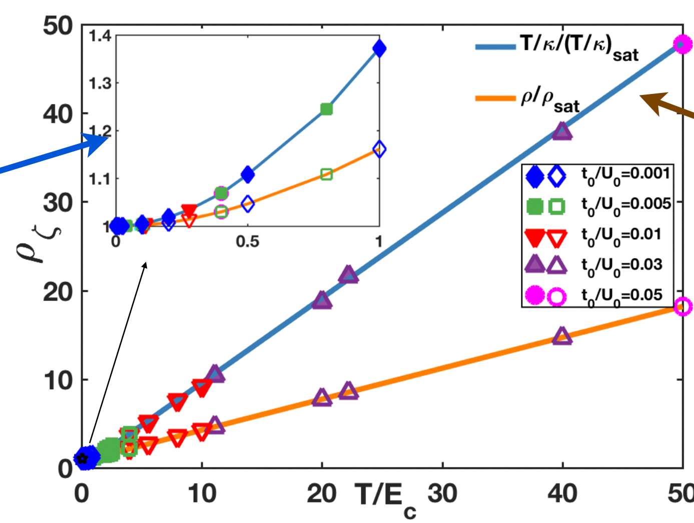
N.B. Because of randomness, momentum is not a hydrodynamic variable

Transport

Generalized resistivity $\rho_c = 1/\sigma$ $\rho_e = T/\kappa$

scaling $\rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta(\frac{T}{E_c})$

Fermi liquid $R=R_0+AT^2$ for $T \ll E_c$



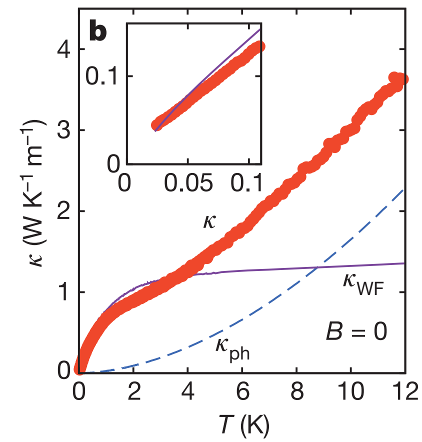
Linear in T for $E_c \ll T \ll U$

Crossover from heavy FL to strange metal

Wiedemann-Franz ratio

<u>Lorenz</u> $L = \dfrac{\kappa}{\sigma T}$

$= \pi^2/3$ for a Fermi liquid

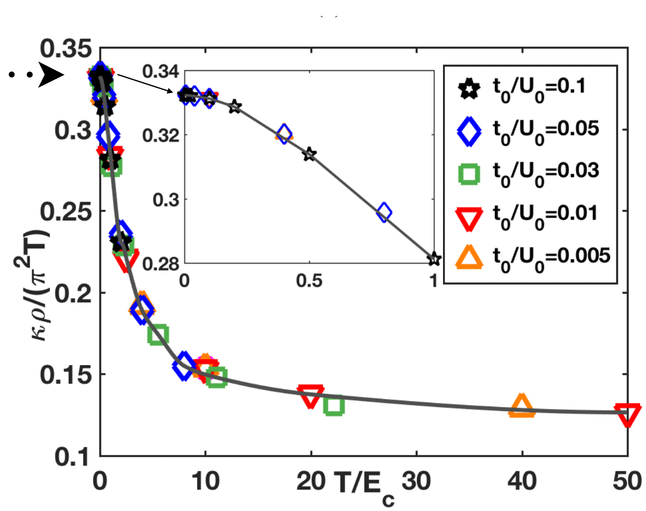


YbRh$_2$Si$_2$, Pfau *et al* (2012)

<u>SYK lattice:</u>

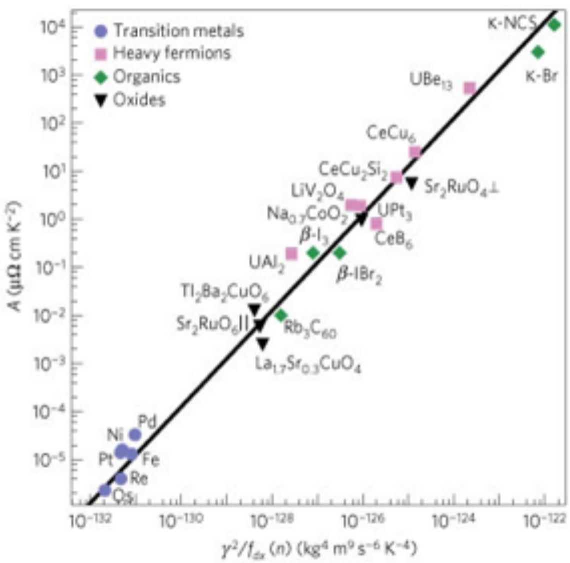
$L = \mathcal{L}(T/E_c)$

FL $\dfrac{\pi^2}{3}$



$\dfrac{\pi^2}{8}$ NFL

Kadowaki Woods ratio



$$\rho_\zeta(T \ll E_c) \approx \rho_\zeta(0) + A_\zeta T^2,$$

$$KW = A/\gamma^2$$

approximately constant for many metals

Scaling implies:

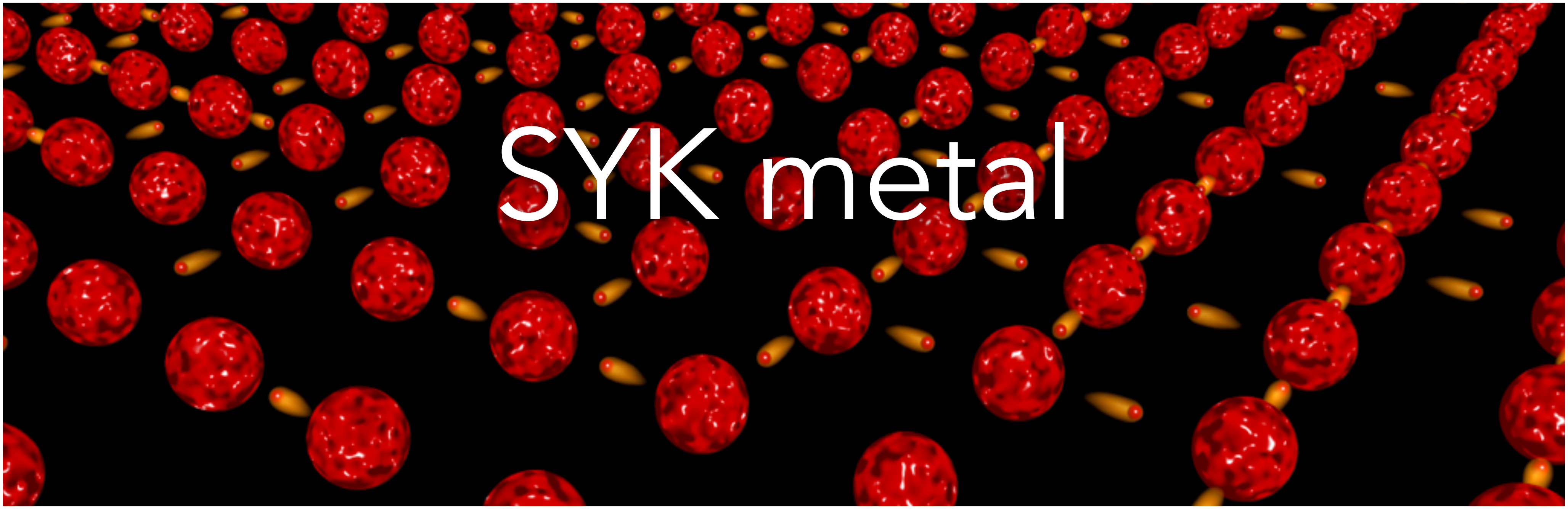
$$A \sim 1/(NE_c^2)$$

recall $\gamma \sim 1/E_c$

$$KW = A/(N\gamma)^2 \sim 1/N^3$$

independent of t,U!

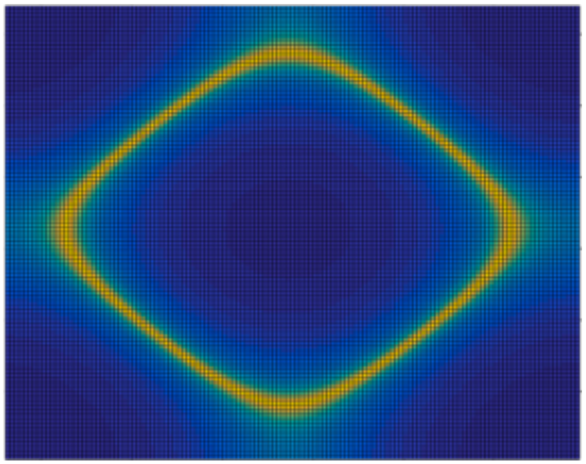
SYK metal



- Small coherence scale $E_c = t^2/U$
- Heavy mass $\gamma \sim m^*/m \sim U/t$
- Small QP weight $Z \sim t/U$
- Kadowaki-Woods A/γ^2 = constant
- Linear in T resistivity and T/κ
- Lorenz ratio crosses over from FL to NFL value

Future

- Extension to translationally invariant systems?



Fermi surface emerging in interacting tensor model by Aavishkar Patel

- SYK lattice, tensor models,...
- Momentum space differentiation and realistic applications?
- Relation to methods like DCA, cluster DMFT?

Future

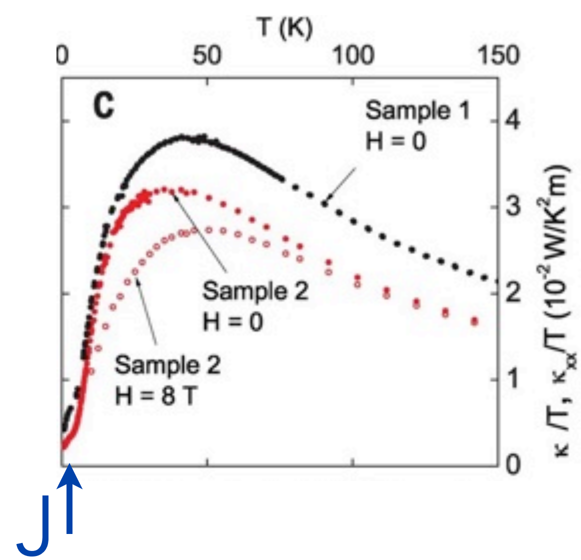
- Bosons/spins

$$H = \frac{1}{2} \sum_{\langle ij \rangle} \sum_{\mu\nu} J_{ij}^{\mu\nu} S_i^{\mu} S_j^{\nu}$$

  - no obvious free particle starting point
  - $E_F \to J$ much smaller

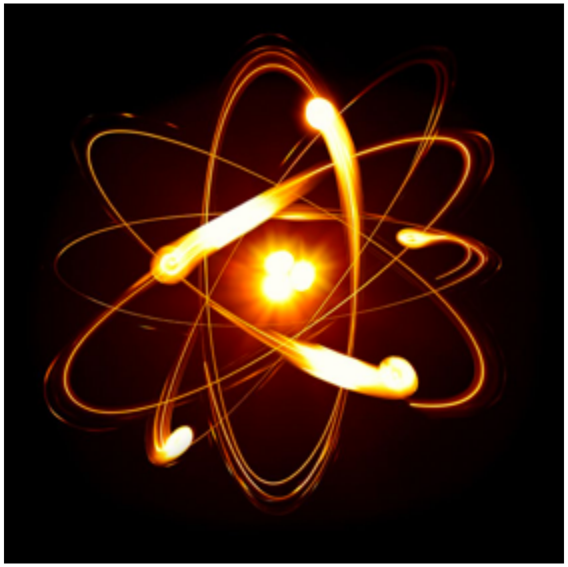
Many (most?) measurements have T>J
and are not in the QP regime

e.g. $Tb_2Ti_2O_7$



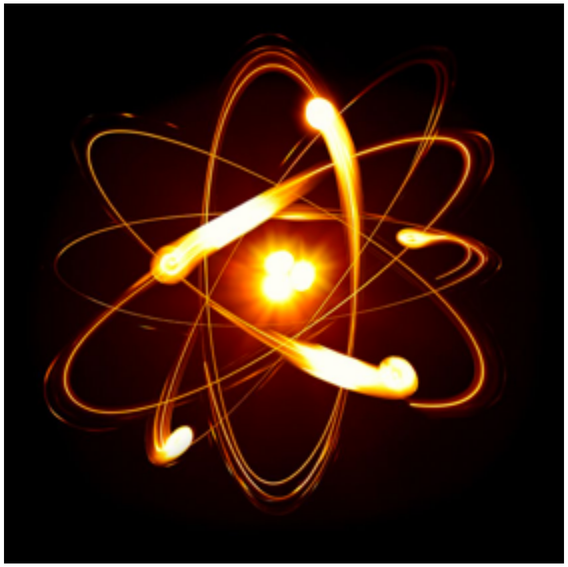
Future

- Deeper implications for many-body physics
  - Entropy and non-Fermi liquid behavior?
  - Many body chaos and dynamics?
  - Really learning something about correlated electrons from black holes?



Electrons

Black holes



Electrons

.0000000000000000000000000000009kg

Black holes

60000000000000000000000000000000kg

Thanks

GORDON AND BETTY MOORE FOUNDATION

X.-Y. Song, C.-M. Jian, and L. Balents, Phys. Rev. Lett., 119 (2017)