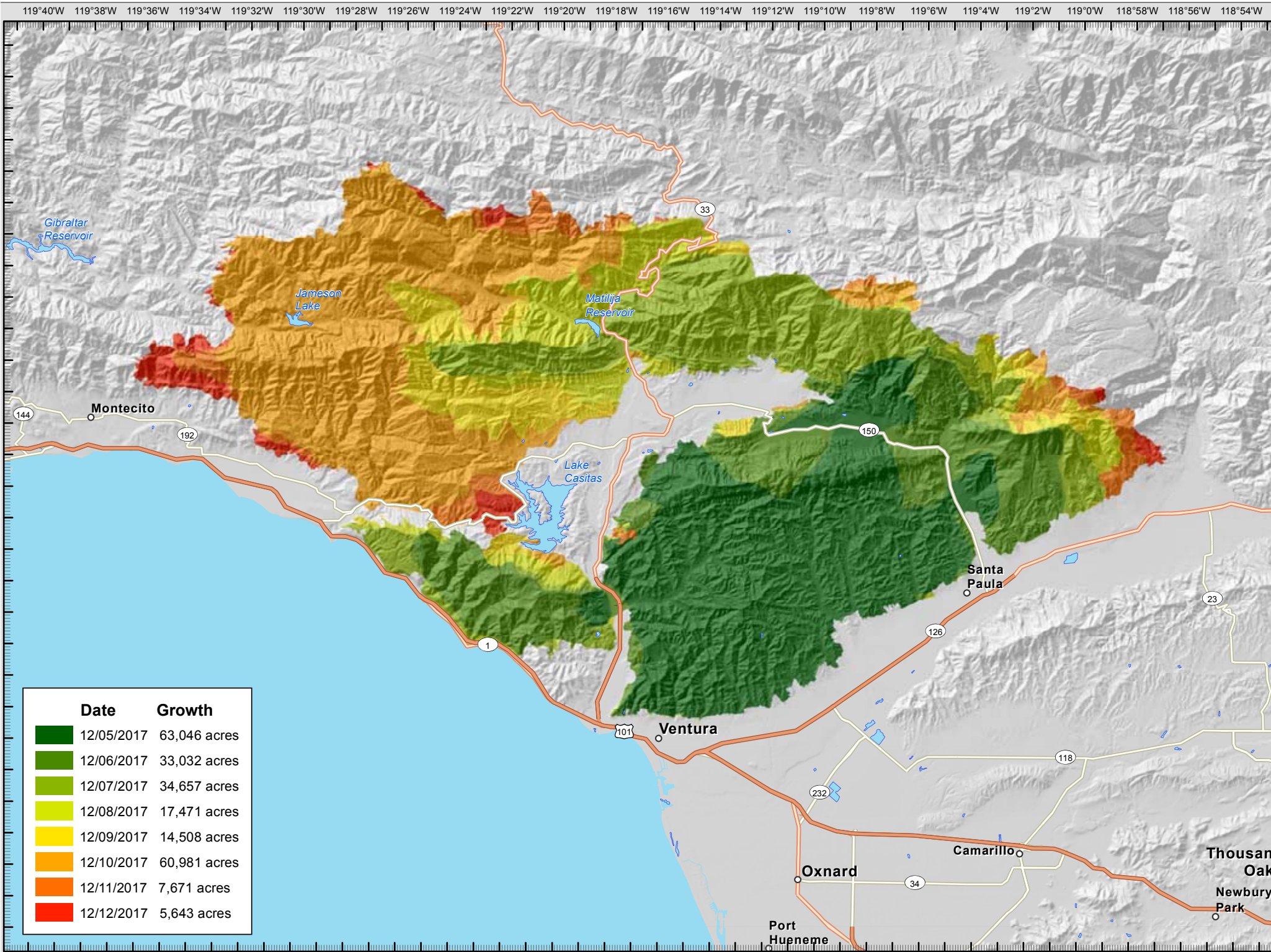




not necessarily

Magnetic ~~topological~~ semimetals

Leon Balents, KITP





Collaborators



Jianpeng
Liu

KITP

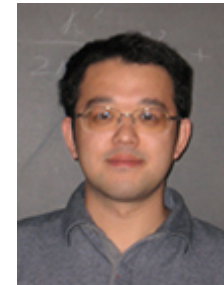


Lucile
Savary

ENS Lyon



Joe
Checkelsky



Takehito
Suzuki

MIT

Topological semimetals

- Band touching at some points/lines in momentum space
- May be protected by topology and/or symmetry
- 2d: graphene
- 3d: Weyl points, Dirac points, nodal lines...

Weyl semimetal

937

PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING

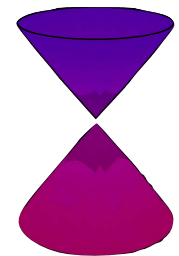
Princeton University, Princeton, New Jersey

(Received June 16, 1937)



For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neighborhood of a point \mathbf{k} where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa \rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

$$H = v \vec{\sigma} \cdot \vec{k}$$



A two-component spinor in three dimensions: "half" of a Dirac fermion.
Weyl fermions have a chirality and
must be massless

Weyl semimetal

937

PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING

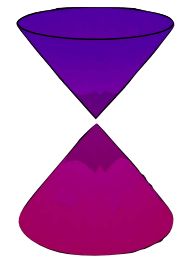
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$$H = v\vec{\sigma} \cdot \vec{k}$$



A Weyl point is a “topological defect” in momentum space: a monopole for the Berry curvature

$$\nabla_{\mathbf{k}} \cdot \mathcal{B} = \pm 2\pi q$$

Weyl semimetal

937

PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING

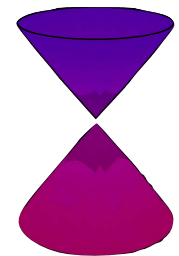
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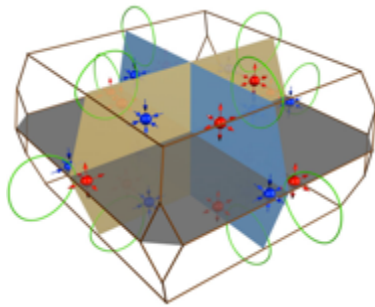
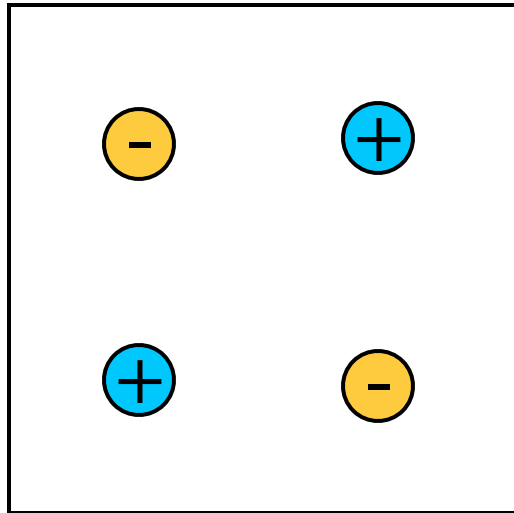
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$$H = v \vec{\sigma} \cdot \vec{k}$$



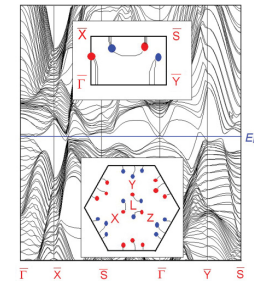
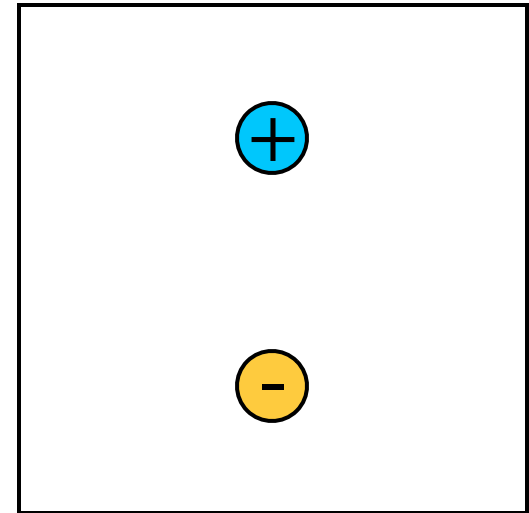
Weyl point is "symmetry prevented":
needs *broken* inversion and/or time-reversal

I-breaking Weyls



TaAs, Na₃Bi, TaP, WTe₂,...

TR-breaking Weyls



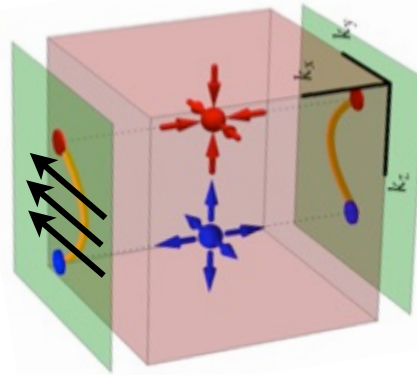
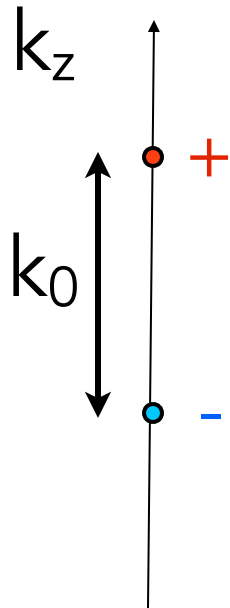
R₂Ir₂O₇?, Mn₃(Sn/Ge), RAlGe

Why magnetic Weyls?

- Possibility to observe AHE
- Interesting correlation physics of magnetism
- Ability to affect electrons *in situ* by modifying magnetic configuration
- Probe static and dynamical effects of *real space* topological defects

Anomalous Hall Effect

Unique property of a magnetic Weyl semimetal

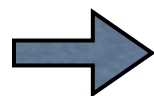


Fermi arc = chiral edge state

$$\sigma_{xy} = \frac{e^2}{h} \frac{k_0}{2\pi}$$

semi-quantum AHE

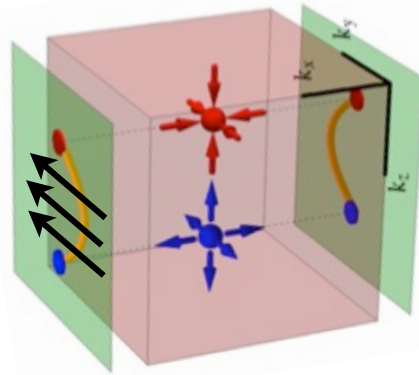
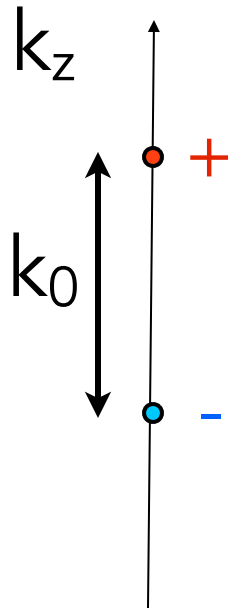
obviously breaks time-reversal symmetry



need a magnetic material

Anomalous Hall Effect

Unique property of a magnetic Weyl semimetal



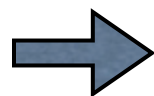
Fermi arc = chiral edge state

$$\sigma_{\mu\nu} = \frac{e^2}{2\pi h} \epsilon_{\mu\nu\lambda} Q_\lambda$$

$$\vec{Q} = \sum_i \vec{k}_i q_i + \vec{Q}_{RLV}$$

semi-quantum AHE

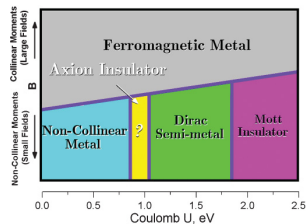
obviously breaks time-reversal symmetry



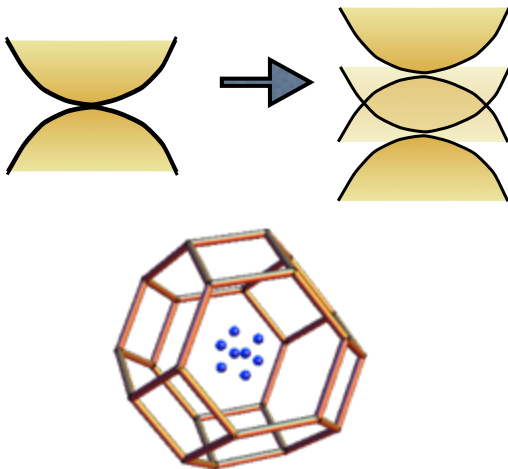
need a magnetic material

Antiferromagnetic Weyls

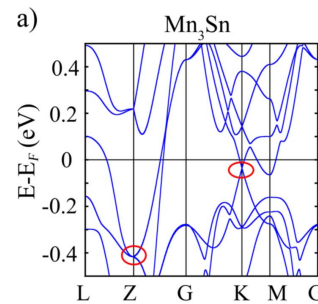
Pyrochlore
iridates



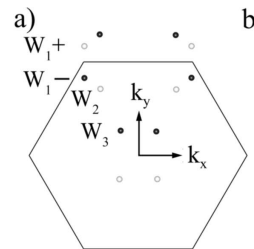
X. Wan et al, 2011



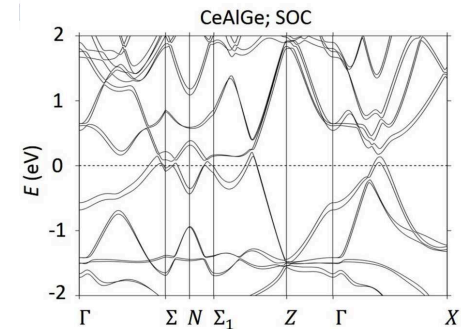
Mn_3Sn ,
 Mn_3Ge



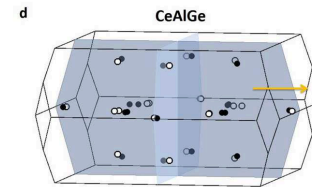
H. Yang et al, 2017



RAIGe



G. Chang et al, 2016



Quasiparticles

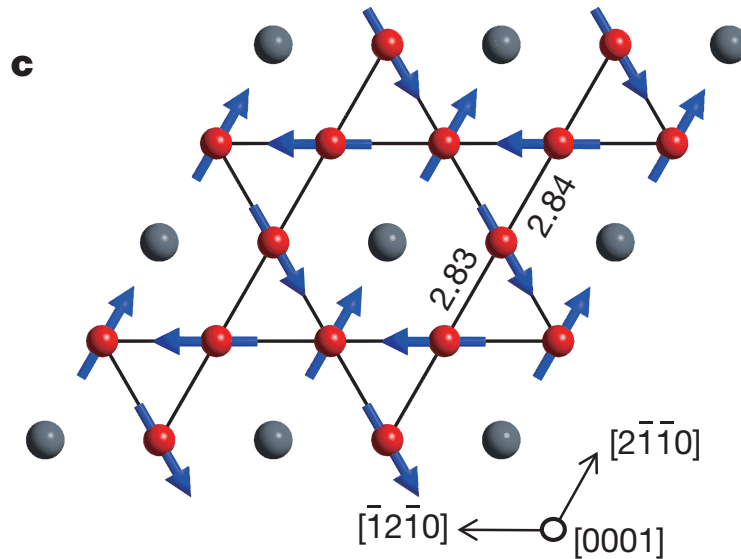
Expect that any magnetically ordered system is described at first order by mean-field quasiparticle Hamiltonian

$$H = H_{\text{band}} - \sum_i \underset{\uparrow}{\mathbf{h}_i} \cdot \mathbf{c}_i^\dagger \frac{\boldsymbol{\sigma}}{2} \mathbf{c}_i$$

effective Zeeman “exchange” field
due to local ordered moment

Think of free-electron structure associated with each magnetic configuration

Mn₃Sn family



two kagomé layers of
Mn, related by inversion

large ordered
antiferromagnetic
moment

$$\sim 2 \mu_B / \text{Mn}$$

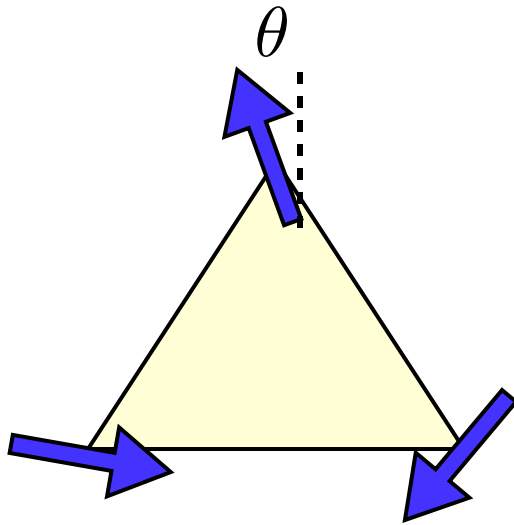
tiny FM moment:

$$.002 \mu_B / \text{Mn}$$

$$T_N \sim 420\text{K}$$

Nagamiya et al, 1982

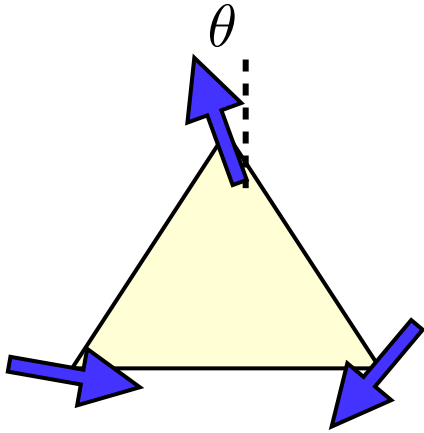
Energetics: triangle



$$E = J (\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_1) \\ + D \hat{\mathbf{z}} \cdot (\mathbf{S}_1 \times \mathbf{S}_2 + \mathbf{S}_2 \times \mathbf{S}_3 + \mathbf{S}_3 \times \mathbf{S}_1) \\ - K \sum_i (\hat{\mathbf{n}}_i \cdot \mathbf{S}_i)^2$$

$J \gg D \gg K$ **Hierarchy of interactions**

- J: spins at 120° angles and $M=0$
- D: spins are "anti-chiral" in XY plane
- K: weak canting toward easy axes creates tiny moment and fixes in-plane angle



Textures

$$\psi = |\psi|e^{i\theta}$$

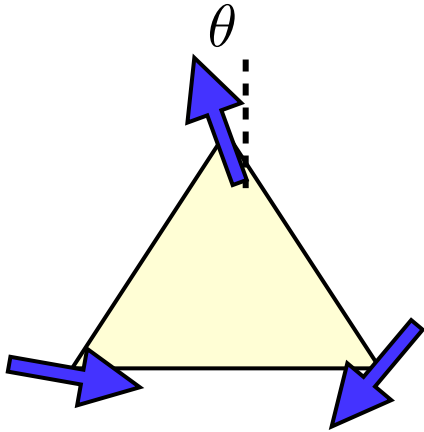
$$F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla \theta)^2 - \lambda \cos 6\theta \right\}$$

sine-Gordon model with 6-fold anisotropy

$$\rho \sim \frac{J}{a}$$

$$\lambda \sim \frac{K^3}{J^2 a^3}$$

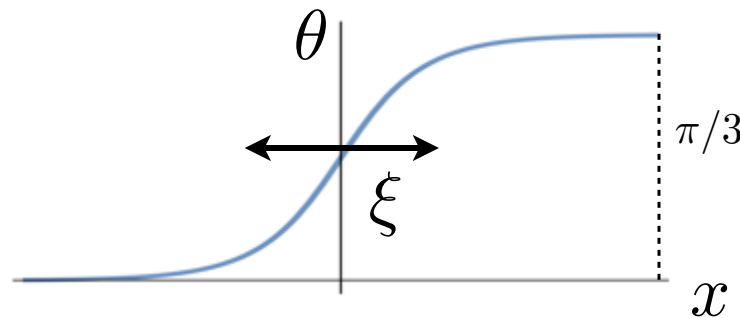
Textures



$$\psi = |\psi|e^{i\theta}$$

$$F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla \theta)^2 - \lambda \cos 6\theta \right\}$$

soliton = domain wall connecting
neighboring minima of cosine

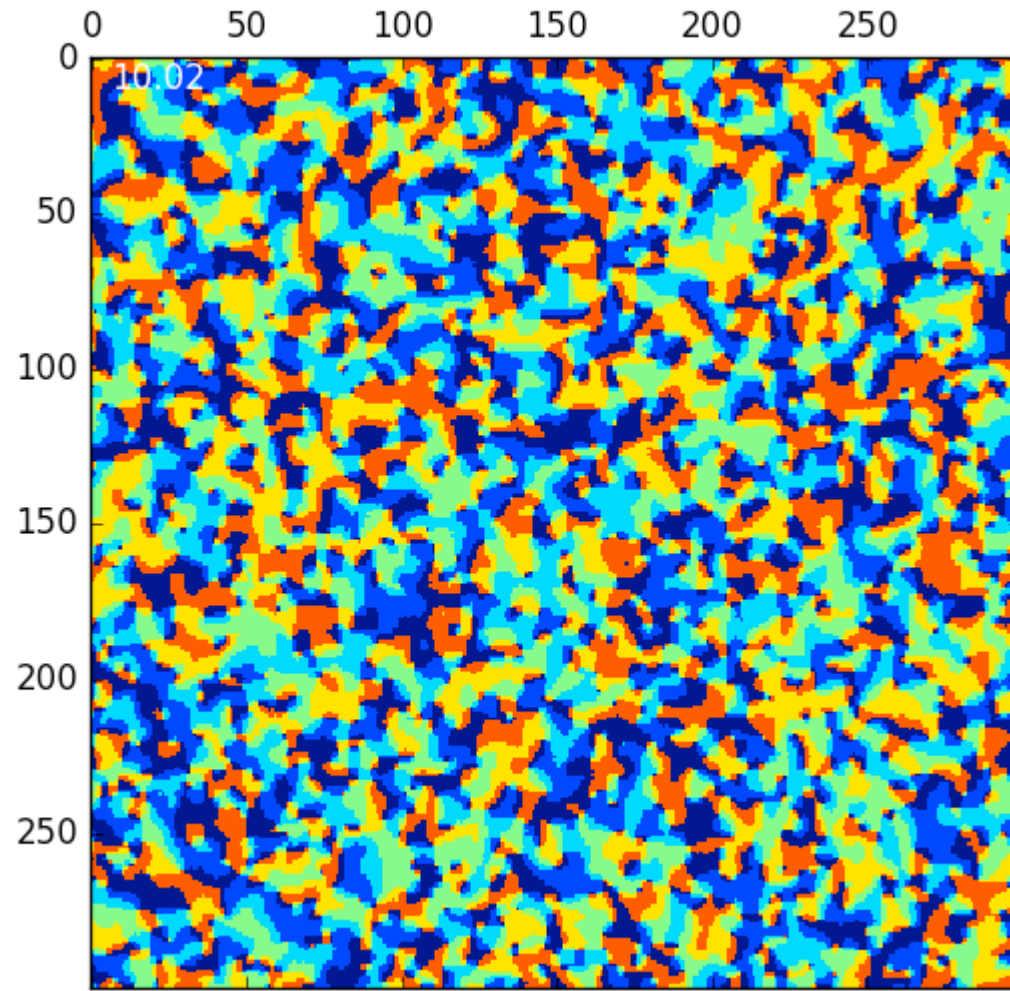


$$\theta(x) = \frac{2}{3} \tan^{-1} \exp(x/\xi)$$

$$\xi = \frac{1}{6} \sqrt{\frac{\rho}{\lambda}}$$

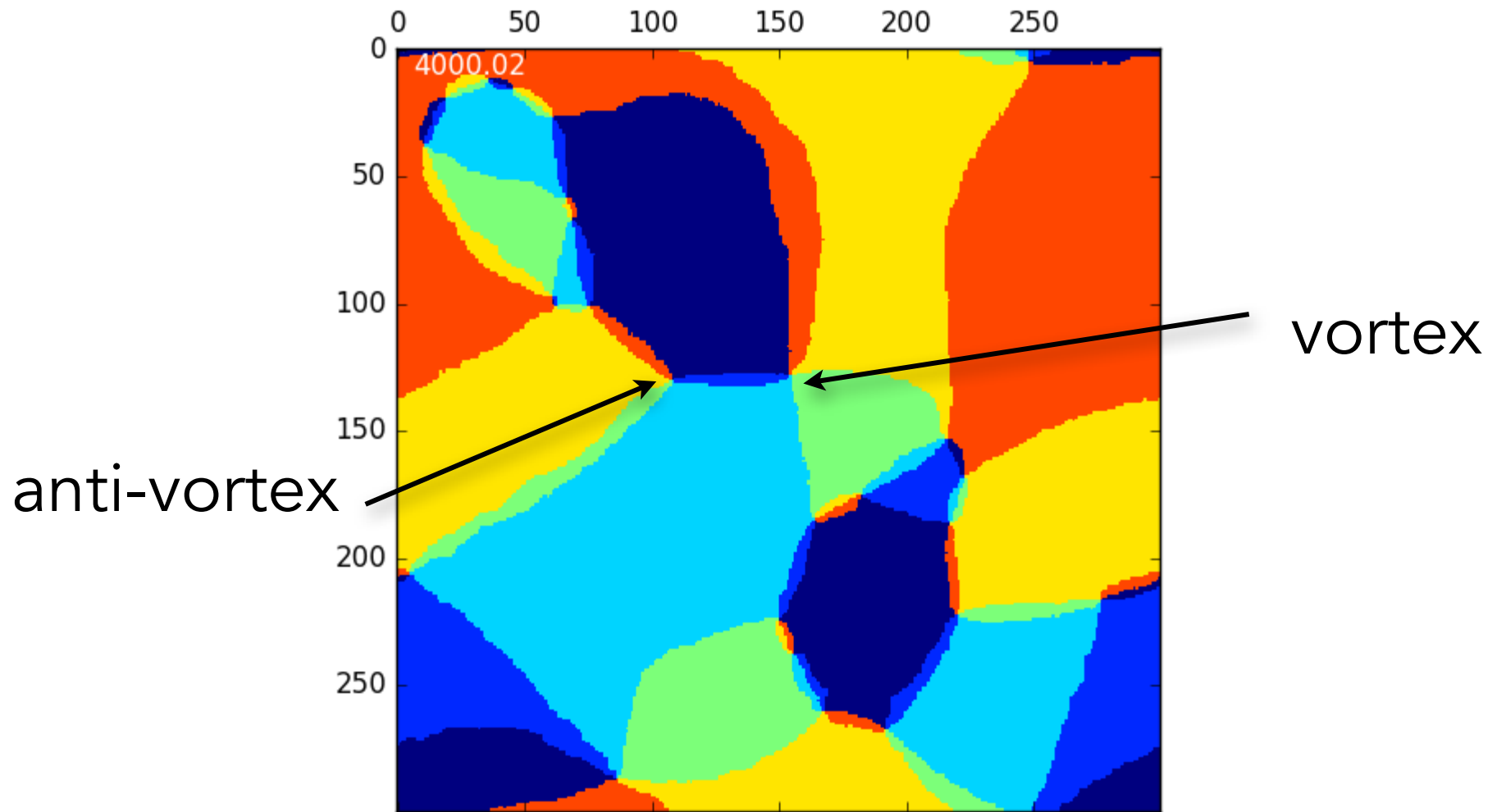
wide
DWs

Domain formation

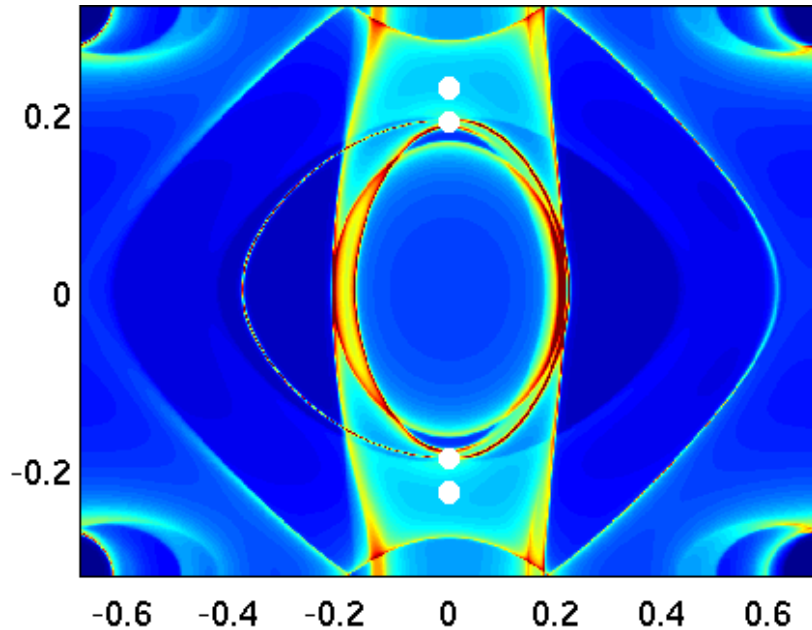


quench

Domain formation



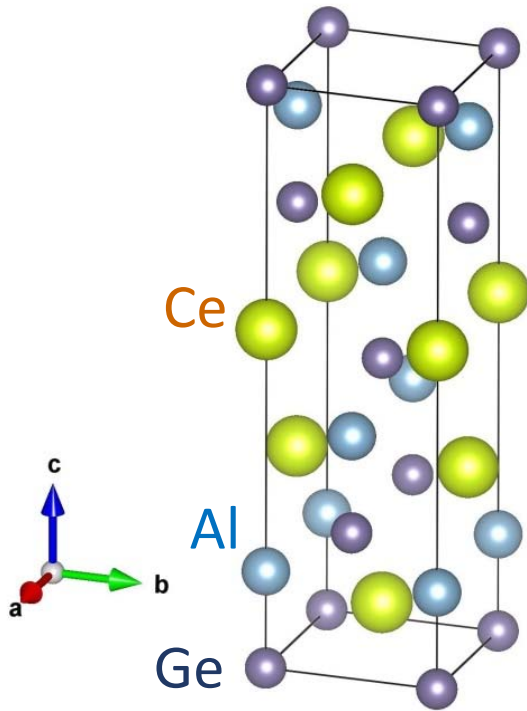
Domain wall bound states



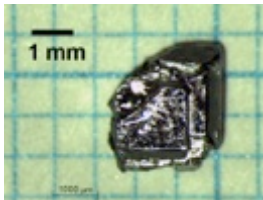
ARPES of domain wall
seems challenging to say
the least!

- Transport: enhanced intrinsic Hall conductivity within a DW?
- STM: signatures of bound states in LDOS?

CeAlGe



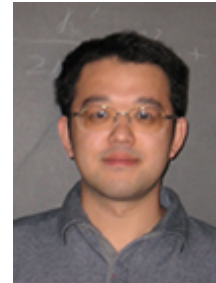
Space group: $I4_1md$



- tetragonal
- Ce $4f^1$ moments
- Semi-metallic band structure



Joe
Checkelsky



Takehito
Suzuki



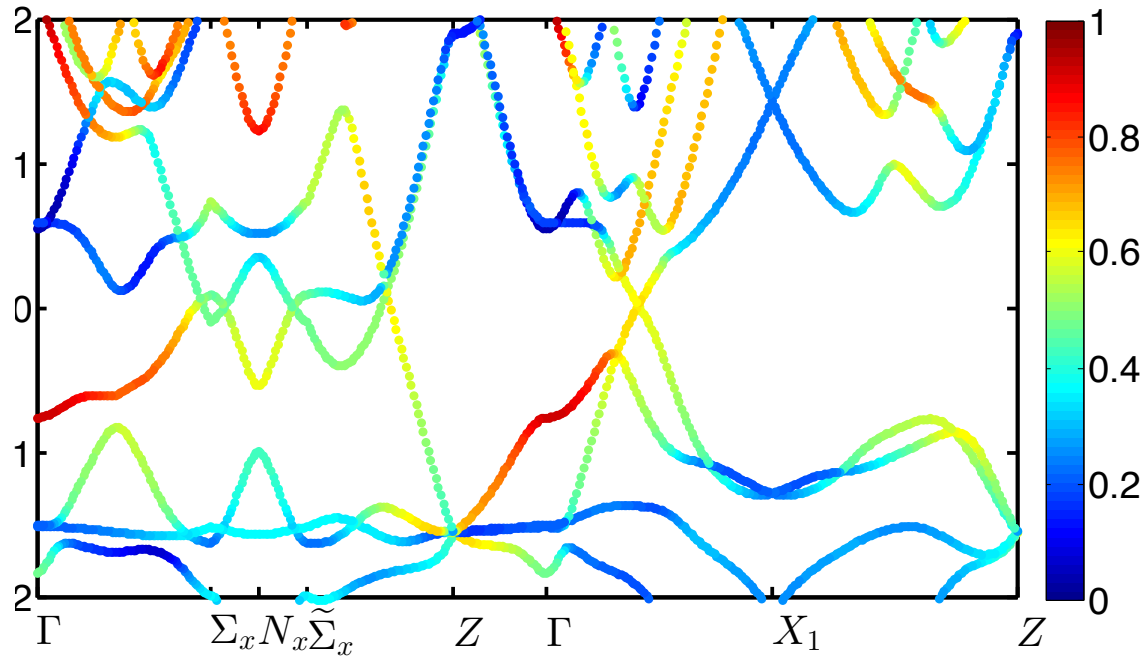
Lucile
Savary



Jianpeng
Liu

Band structure

(non-magnetic, no SOC)

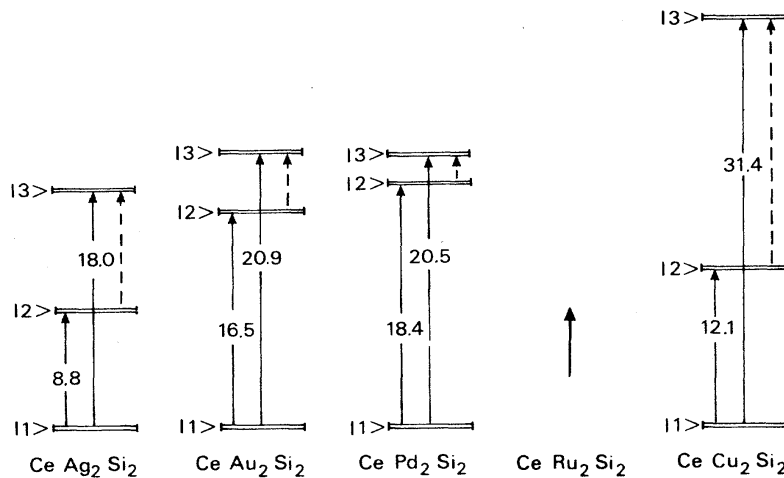


Ce d-orbital
content

- bandwidth $\sim 1\text{eV}$
- no large Fermi surface: true semi-metal
- large rare-earth d-orbital content: substantial coupling to rare earth moments

Ce moments

Ce^{3+} typically Ising-like Kramers doublet

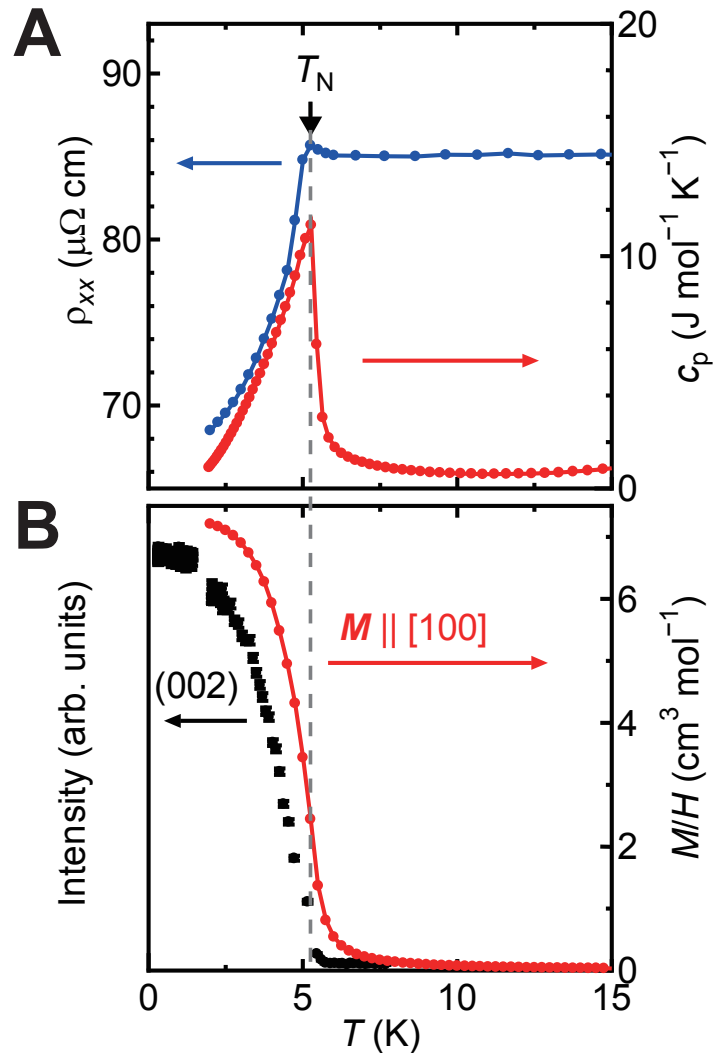


e.g. A. Severing *et al*, 1989

effective $S=1/2$ spin below
 $\sim 10\text{meV} \sim 100\text{K}$ energy scale

$4f^1$ configuration: large orbital
component and hence strong
magnetic anisotropy

Magnetic order

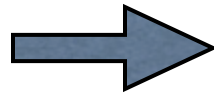


Magnetic
transition at 5K

2 Ce sublattices. Order
does not enlarge unit cell

Kondo lattice scales

$$H = H_{\text{band}} + J_K \sum_i \mathbf{S}_i \cdot c_i^\dagger \frac{\boldsymbol{\sigma}}{2} c_i$$



RKKY

$$J_{\text{RKKY}} \sim \frac{J_K^2}{E_F}$$

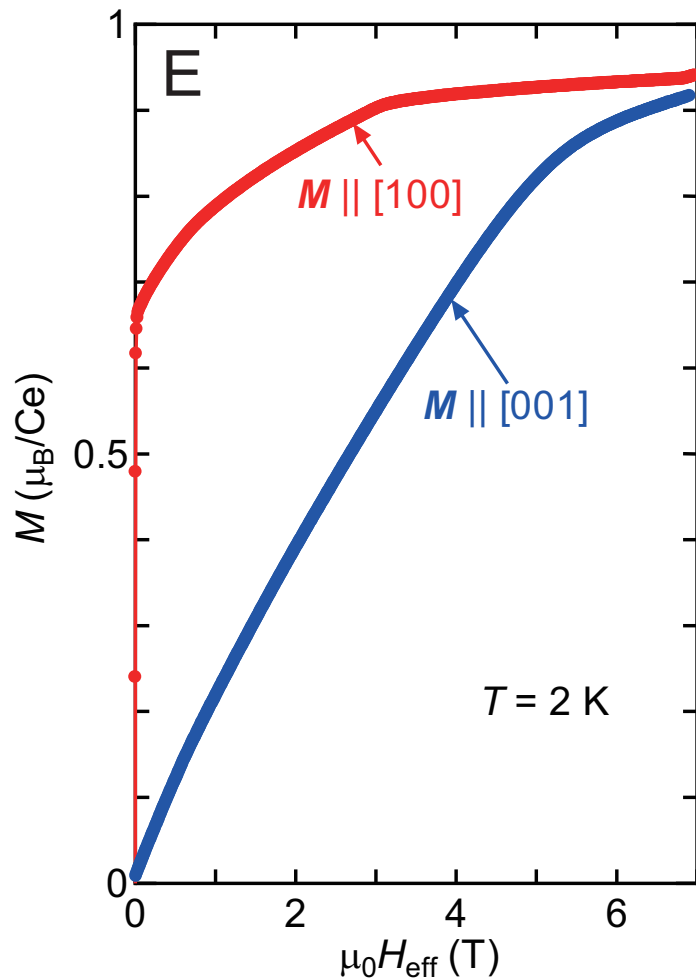
$$J_K \sim \sqrt{J_{\text{RKKY}} E_F} \quad \sim 100 \text{ meV?}$$

5K 1eV

Summary: key features

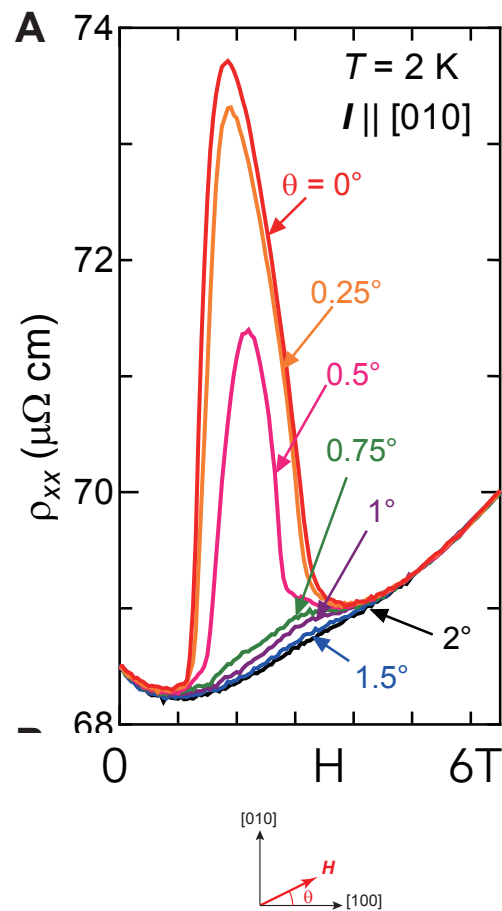
- Semi-metal
- Small bandwidth $\sim 1\text{eV}$
- Large $J_K \sim 100\text{meV}$
- Strong magnetic anisotropy/SOC
- Low $T_N \sim 5\text{K}$

Magnetization



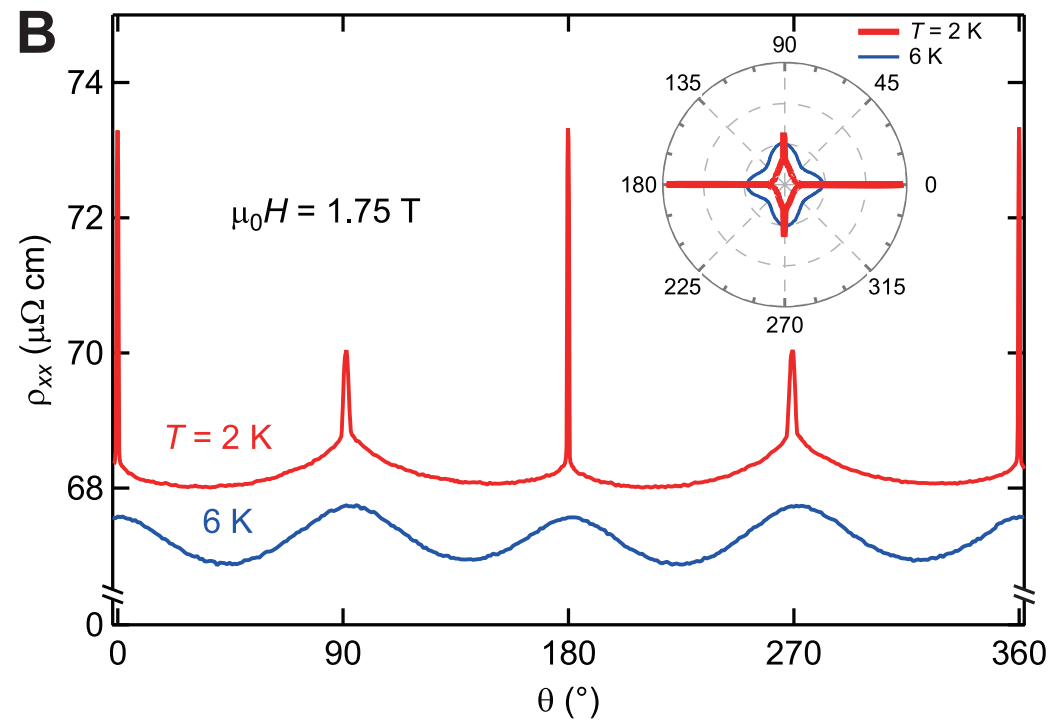
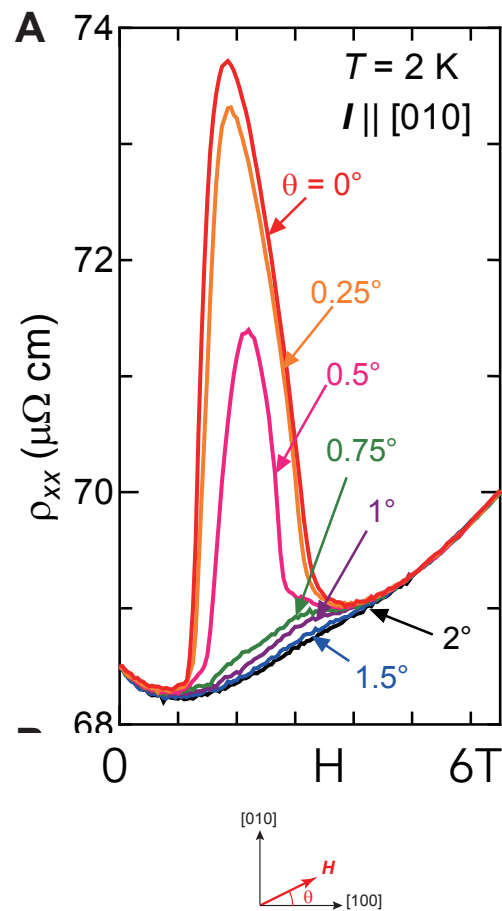
- In-plane field shows ferromagnetic component
- Out of plane field paramagnetic
- If you look carefully, hints of more transitions

Resistivity



resistivity
enhancement at
intermediate fields
and low T

Resistivity



very narrow angular dependence!

Suzuki Angular Magneto-Resistance

~~Suzuki~~ Angular Magneto-Resistance

Savary Angular Magneto-Resistance

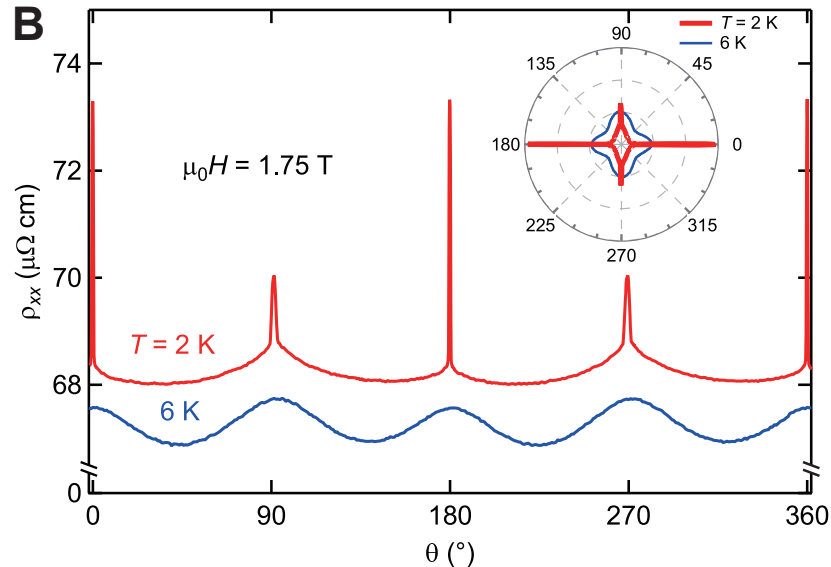
~~Suzuki~~ Angular Magneto-Resistance

~~Savary~~ Angular Magneto-Resistance

Singular Angular Magneto-Resistance

SAMR

Symmetry



Effect is tied to crystalline axes. Yet appears only below critical temperature.



Must be some effect of space group symmetry breaking. Unique to $\langle 100 \rangle$ axis?

Symmetry



symmetry	(h^x, h^y, h^z)	\mathbf{h} doesn't break sym. explicitly if	(N_x, N_y, N_z)	\mathbf{N} breaks spont. if
TR	$(-h_x, -h_y, -h_z)$	$\mathbf{h} = \mathbf{0}$	$(-N_x, -N_y, -N_z)$	$\mathbf{N} \neq \mathbf{0}$
C_2	$(-h_x, -h_y, h_z)$	$h_x = h_y = 0$	$(-N_x, -N_y, N_z)$	$N_x \neq 0$ or $N_y \neq 0$
m_{010}	$(-h_x, h_y, -h_z)$	$h_x = h_z = 0$	$(-N_x, N_y, -N_z)$	$N_x \neq 0$ or $N_z \neq 0$
$m_{100} \times \text{TR}$	$(-h_x, h_y, h_z)$	$h_x = 0$	$(-N_x, N_y, N_z)$	$N_x \neq 0$
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$m_{110}^* \times C_2$	$(-h_y, -h_x, -h_z)$	$h_y = -h_x$ and $h_z = 0$	(N_y, N_x, N_z)	$N_x \neq N_y$
$m_{110}^* \times \text{TR}$	$(-h_y, -h_x, h_z)$	$h_y = -h_x$	$(N_y, N_x, -N_z)$	$N_x \neq N_y$ or $N_z \neq 0$
$C_4 C_4 C_4^* \times \text{TR}$	$(-h_y, h_x, -h_z)$	$\mathbf{h} = \mathbf{0}$	$(N_y, -N_x, N_z)$	$N_x \neq 0$ or $N_y \neq 0$
C_4^*	$(-h_y, h_x, h_z)$	$h_x = h_y = 0$	$(N_y, -N_x, -N_z)$	$\mathbf{N} \neq \mathbf{0}$
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$C_4 C_4 C_4^*$	$(h_y, -h_x, h_z)$	$h_x = h_y = 0$	$(-N_y, N_x, -N_z)$	$\mathbf{N} \neq \mathbf{0}$
m_{110}^*	$(h_y, h_x, -h_z)$	$h_x = h_y$ and $h_z = 0$	$(-N_y, -N_x, N_z)$	$N_y \neq -N_x$
$m_{110}^* \times C_2 \times \text{TR}$	(h_y, h_x, h_z)	$h_x = h_y$	$(-N_y, -N_x, -N_z)$	$N_y \neq -N_x$ or $N_z \neq 0$

Table 2: All transformations for \mathbf{h} .

Symmetry

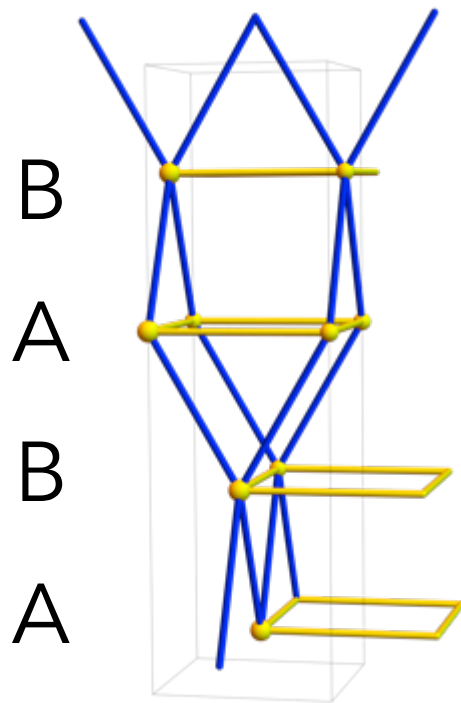


symmetry	(h^x, h^y, h^z)	\mathbf{h} doesn't break sym. explicitly if	(N_x, N_y, N_z)	\mathbf{N} breaks spont. if
TR	$(-h_x, -h_y, -h_z)$	$\mathbf{h} = \mathbf{0}$	$(-N_x, -N_y, -N_z)$	$\mathbf{N} \neq \mathbf{0}$
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$m_{100} \times \text{TR}$	$(-h_x, h_y, h_z)$	$h_x = 0$	$(-N_x, N_y, N_z)$	$N_x \neq 0$
m_{100}	$(h_x, -h_y, -h_z)$	$h_y = h_z = 0$	$(N_x, -N_y, -N_z)$	$N_y \neq 0$ or $N_z \neq 0$
$m_{010} \times \text{TR}$	$(h_x, -h_y, h_z)$	$h_y = 0$	$(N_x, -N_y, N_z)$	$N_y \neq 0$
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$C_4 C_4 C_4^* \times \text{TR}$	$(-h_y, h_x, -h_z)$	$\mathbf{h} = \mathbf{0}$	$(N_y, -N_x, N_z)$	$N_x \neq 0$ or $N_y \neq 0$
C_4^*	$(-h_y, h_x, h_z)$	$h_x = h_y = 0$	$(N_y, -N_x, -N_z)$	$\mathbf{N} \neq \mathbf{0}$
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$C_4 C_4 C_4^*$	$(h_y, -h_x, h_z)$	$h_x = h_y = 0$	$(-N_y, N_x, -N_z)$	$\mathbf{N} \neq \mathbf{0}$
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Table 2: All transformations for \mathbf{h} .

Fields along $\langle 100 \rangle$ axes preserve this
 "magnetic mirror" symmetry

Minimal model

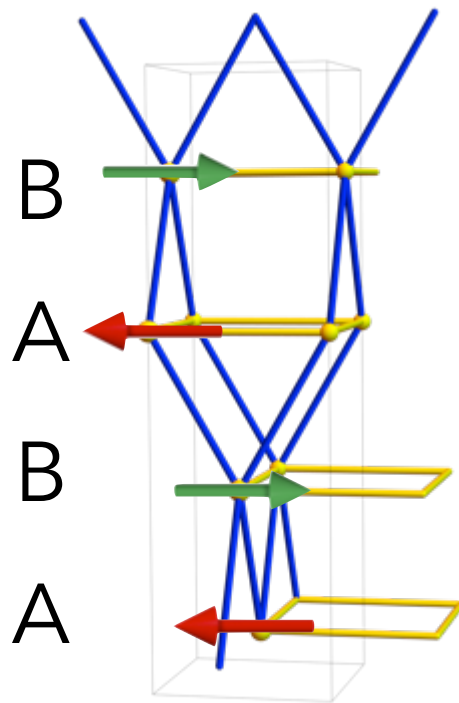


Two Ce sublattices

"intra-unit cell antiferromagnet"

$$E = J_{\perp}(S_A^x S_B^x + S_A^y S_B^y) + J_z S_A^z S_B^z + \sum_{\alpha} [D (S_{\alpha}^z)^2]$$

Minimal model



Two Ce sublattices

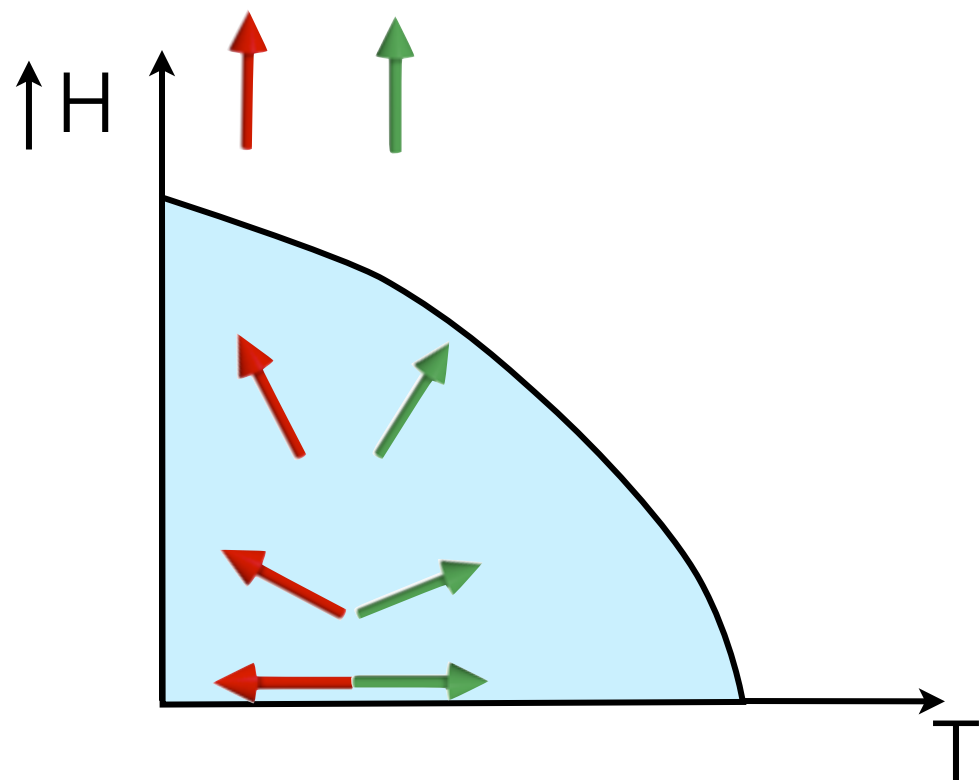
"intra-unit cell antiferromagnet"

$$E = J_{\perp}(S_A^x S_B^x + S_A^y S_B^y) + J_z S_A^z S_B^z + \sum_{\alpha} [D (S_{\alpha}^z)^2]$$

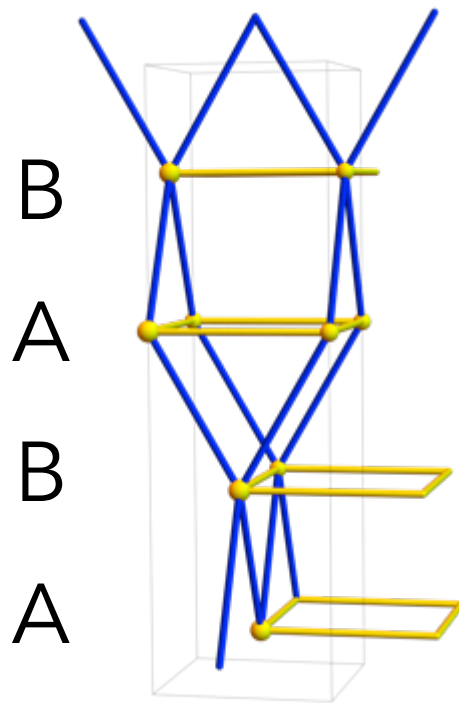
$2D > J_z - J_p \Rightarrow$ in-plane (XY) spins

Spin Flop

Standard
Heisenberg or XY
antiferromagnet



Minimal model



Two Ce sublattices

"intra-unit cell antiferromagnet"

$$E = J_{\perp}(S_A^x S_B^x + S_A^y S_B^y) + J_z S_A^z S_B^z + \sum_{\alpha} [D (S_{\alpha}^z)^2 - \mathbf{H} \cdot \mathbf{m}_{\alpha}] ,$$

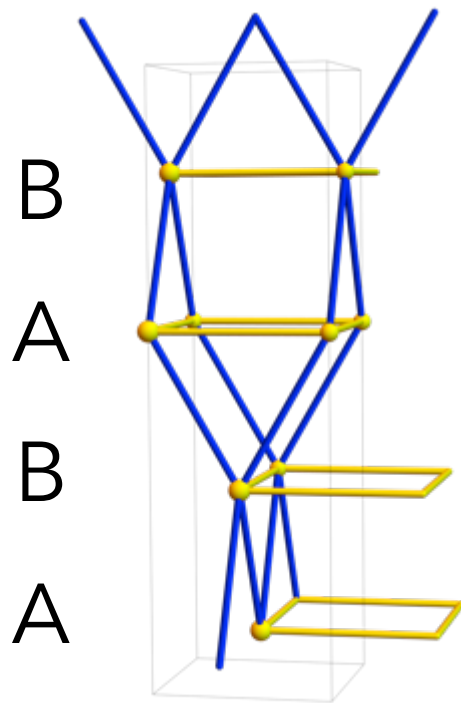
g-tensor anisotropy

$$\mathbf{m}_{\alpha} = g_{\alpha} \mathbf{S}_{\alpha}$$

$$g_A = \begin{pmatrix} g_x & & \\ & g_y & \\ & & g_z \end{pmatrix}$$

$$g_B = \begin{pmatrix} g_y & & \\ & g_x & \\ & & g_z \end{pmatrix}$$

Minimal model



Two Ce sublattices

"intra-unit cell antiferromagnet"

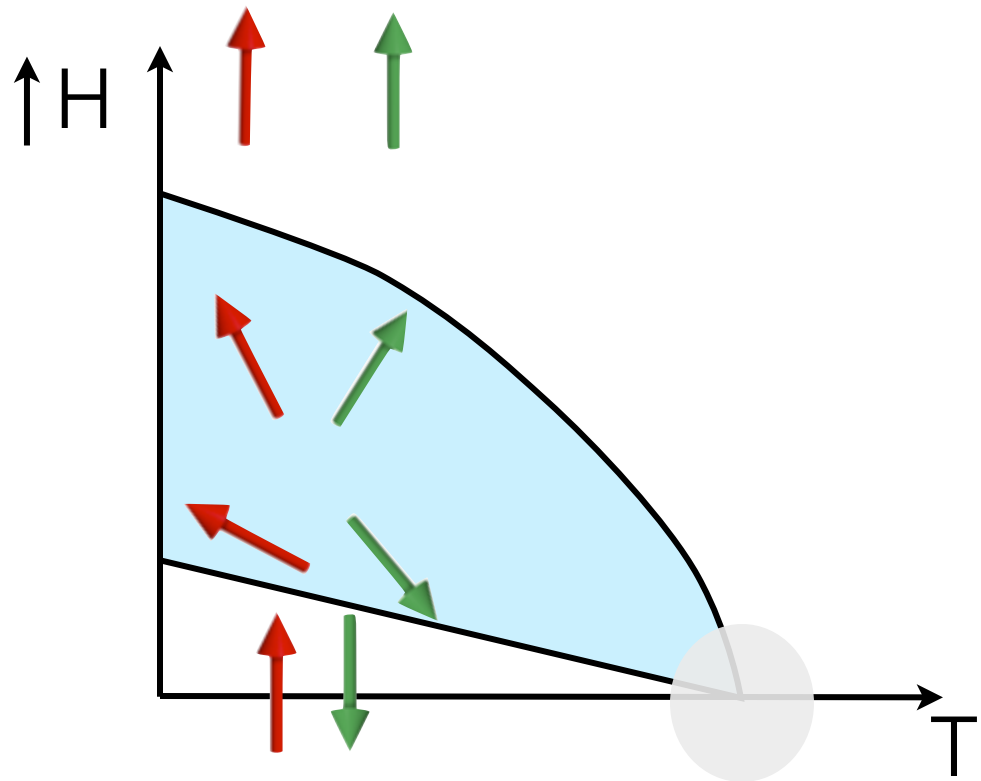
$$E = J_{\perp}(S_A^x S_B^x + S_A^y S_B^y) + J_z S_A^z S_B^z + \sum_{\alpha} [D (S_{\alpha}^z)^2 - \mathbf{H} \cdot \mathbf{m}_{\alpha}] ,$$

$$\mathbf{H} = (H, 0, 0)$$

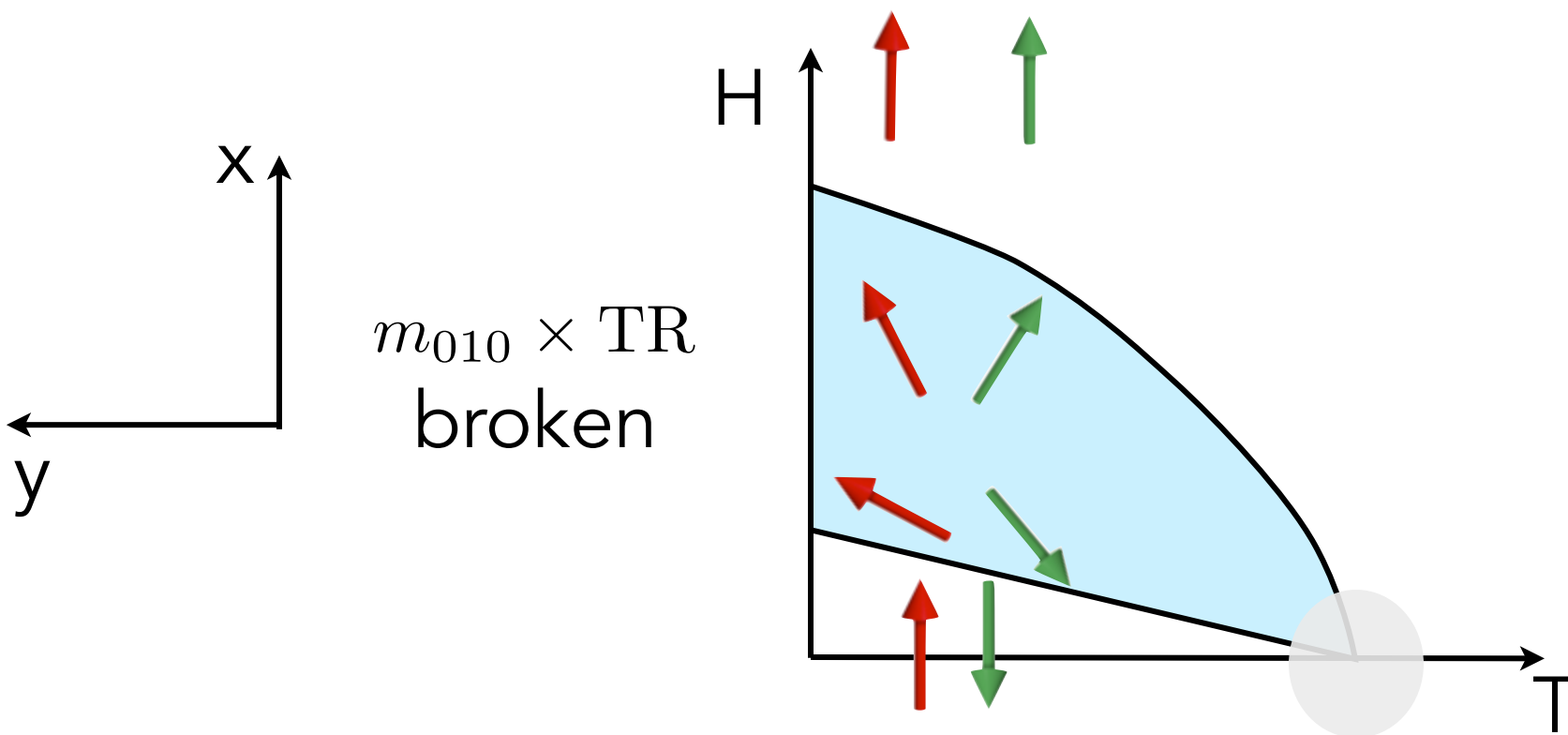
$$E = J_{\perp}(S_A^x S_B^x + S_A^y S_B^y) - H(g_x S_A^x + g_y S_B^x)$$

Spin Flop

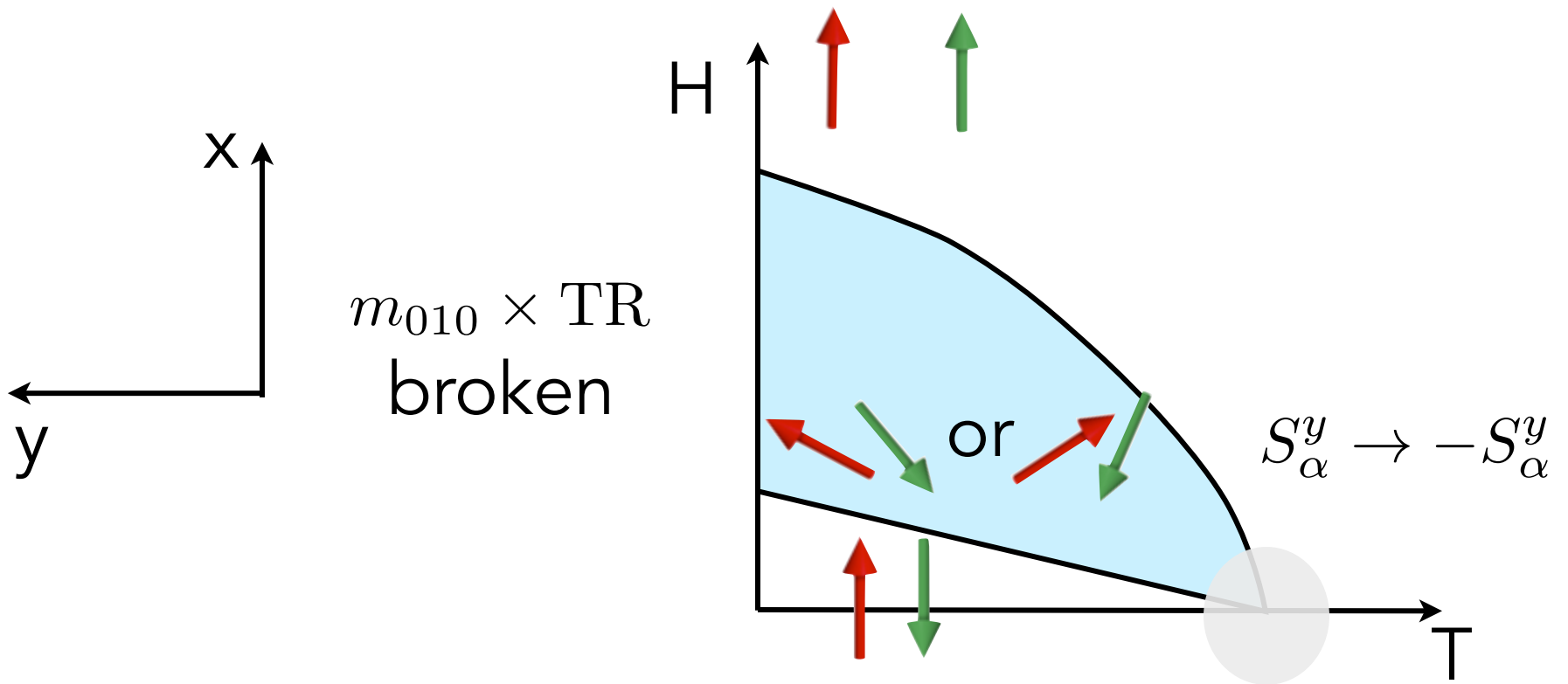
With g-factor
anisotropy *and* H
along (100)



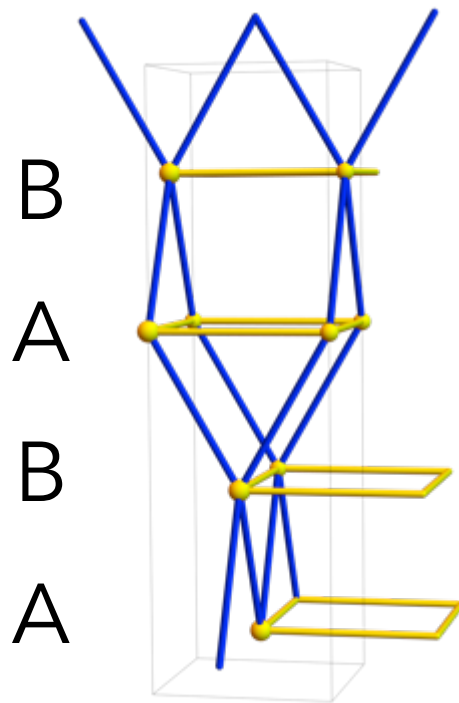
Spin Flop



Domains



Minimal model



Two Ce sublattices

"intra-unit cell antiferromagnet"

$$E = J_{\perp}(S_A^x S_B^x + S_A^y S_B^y) + J_z S_A^z S_B^z + \sum_{\alpha} [D (S_{\alpha}^z)^2 - \mathbf{H} \cdot \mathbf{m}_{\alpha}] ,$$

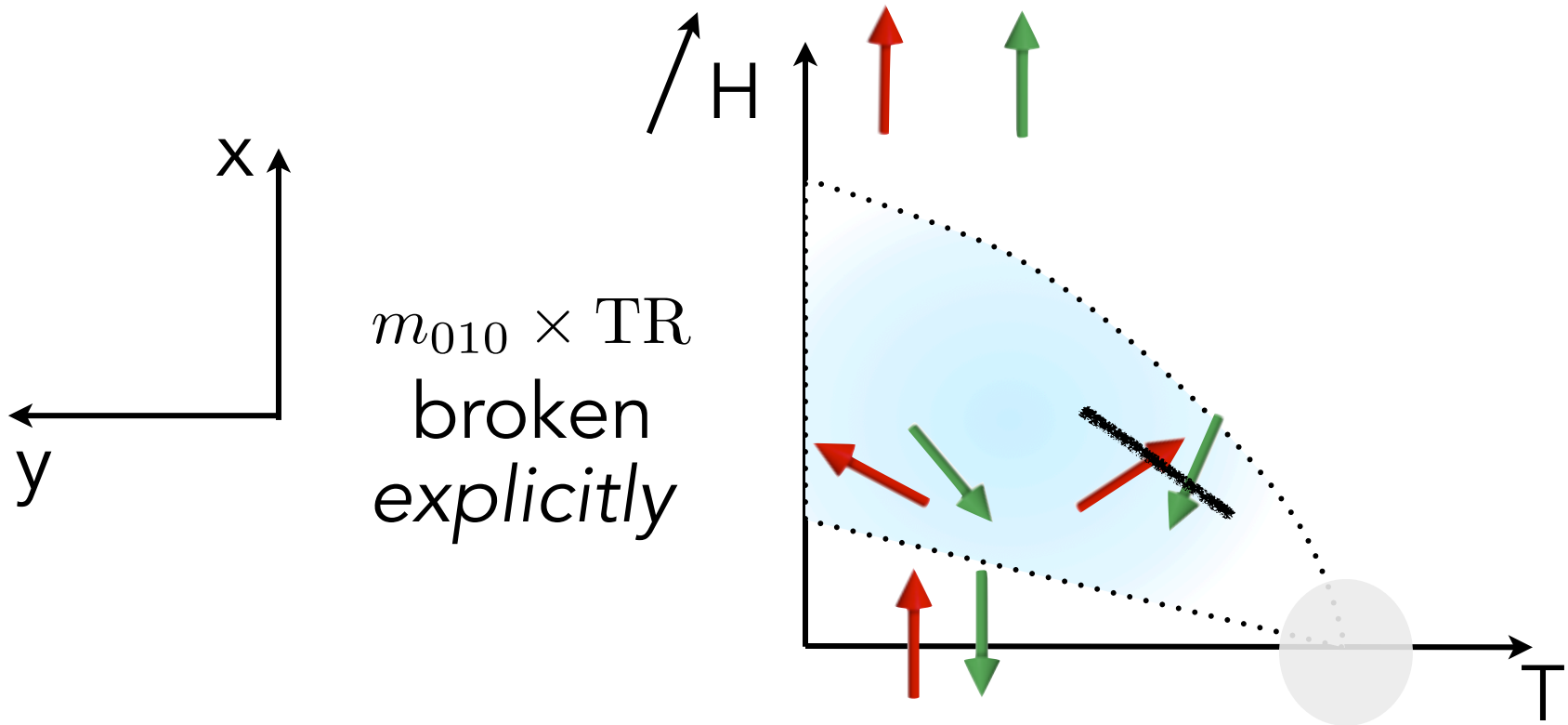
g-tensor anisotropy

$$\mathbf{m}_{\alpha} = g_{\alpha} \mathbf{S}_{\alpha}$$

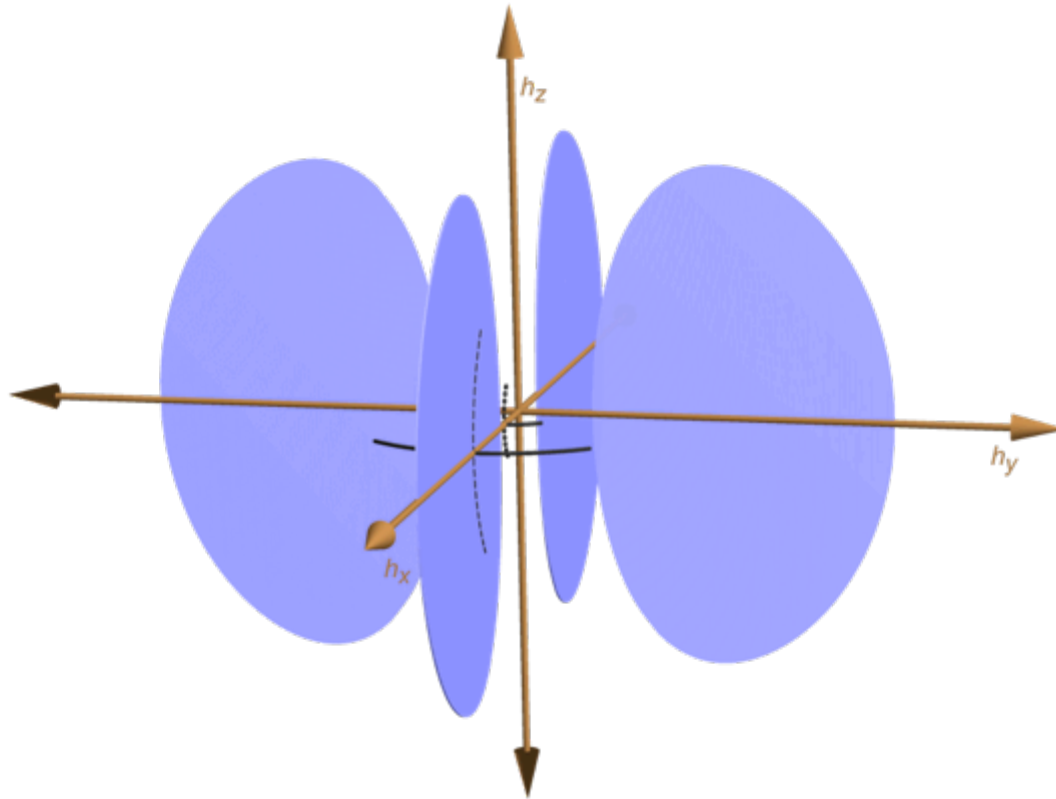
$$g_A = \begin{pmatrix} g_x & & \\ & g_y & \\ & & g_z \end{pmatrix}$$

$$g_B = \begin{pmatrix} g_y & & \\ & g_x & \\ & & g_z \end{pmatrix}$$

Domains

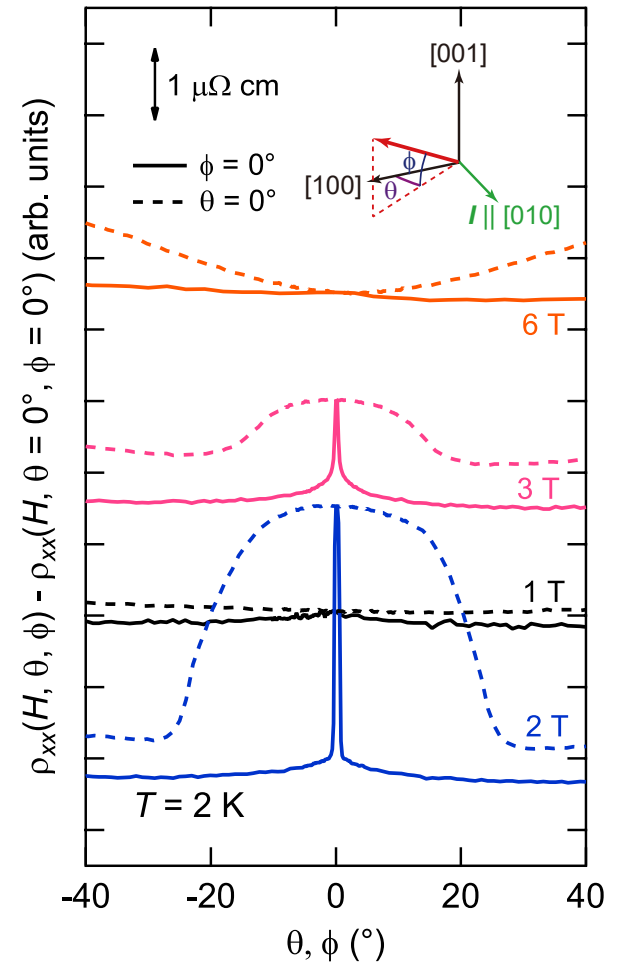
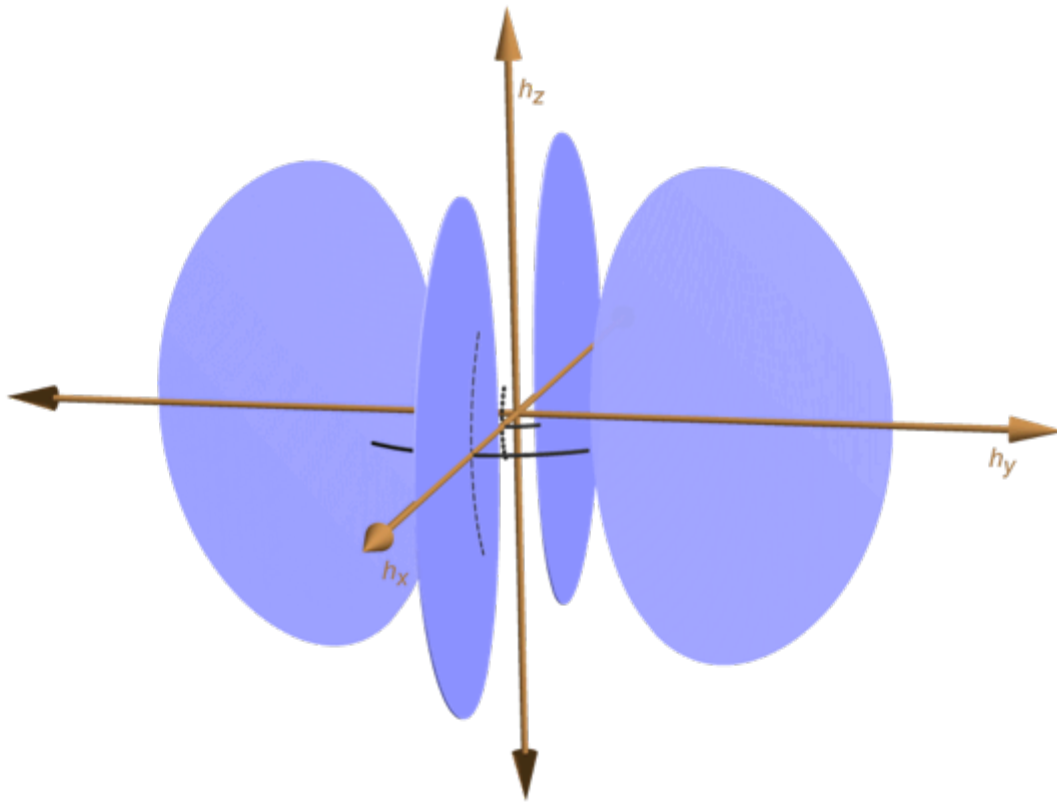


Phase diagram

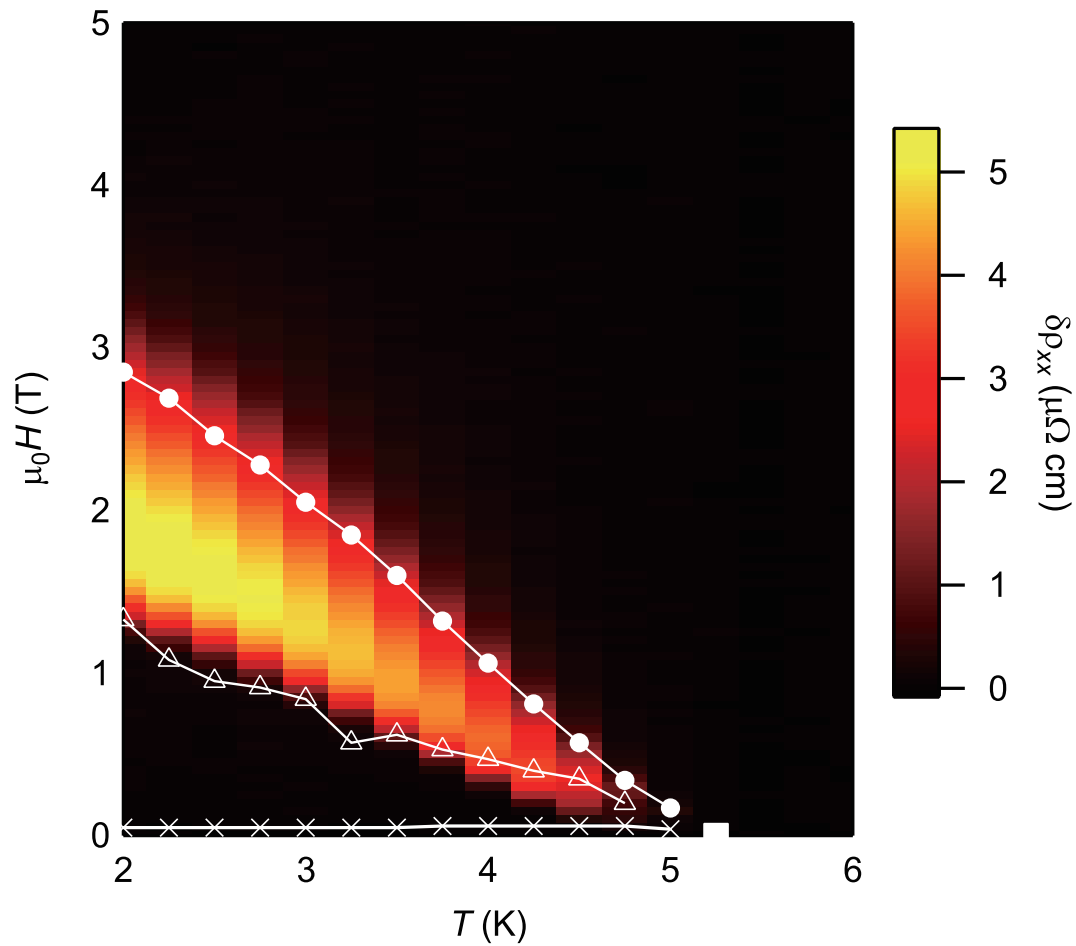


Canted phase forms 4 "infinitely thin" wedges

Phase diagram

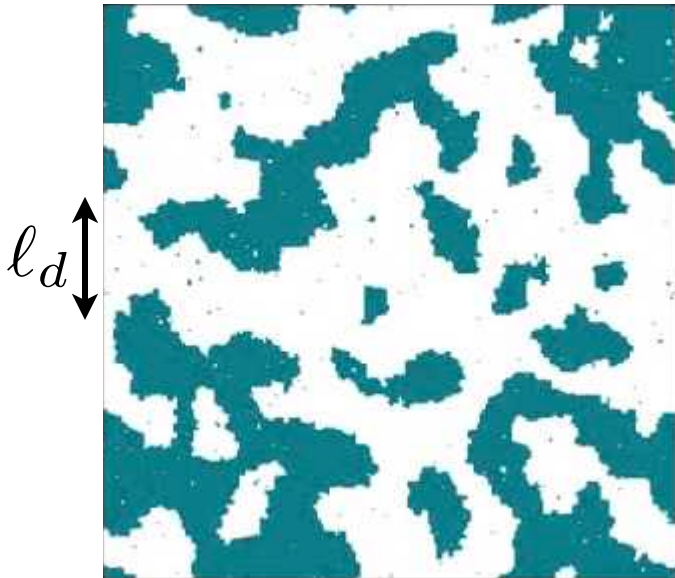


Phase Diagram



Resistivity

Extra resistance comes from *domain walls*



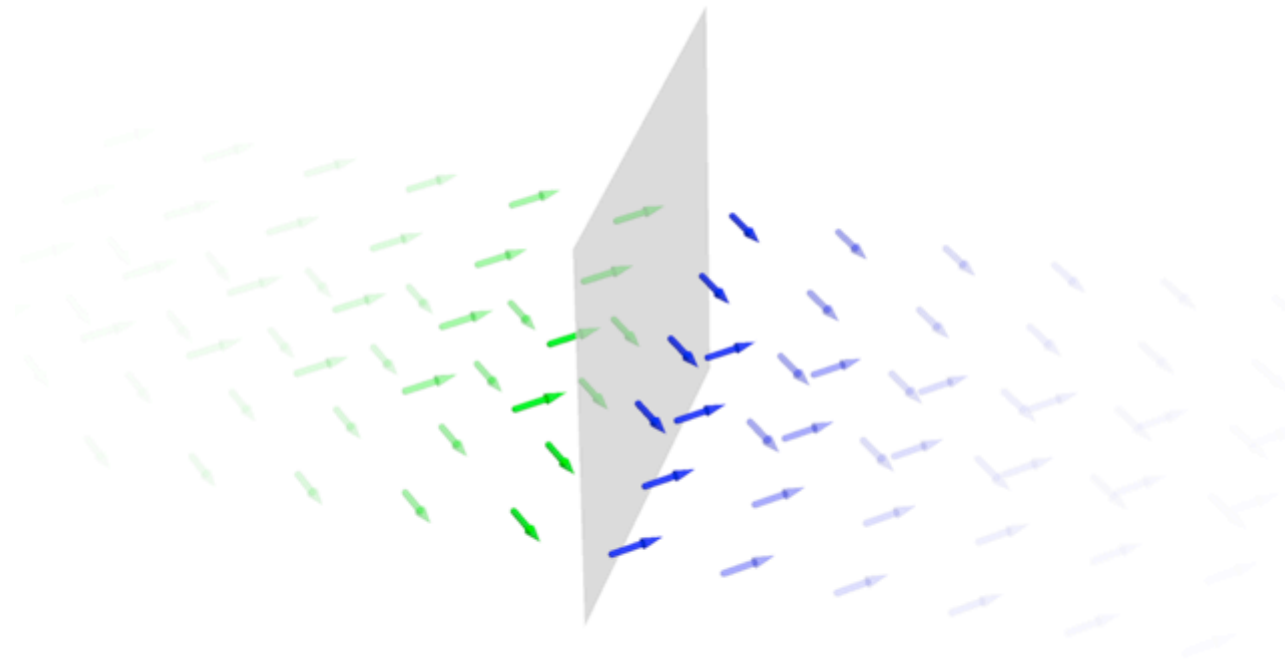
$$\rho_{\text{eff}} = \rho + \frac{\tilde{\rho}_{\text{dw}}}{\ell_d}$$

$$V_{\text{dw}} = \tilde{\rho}_{\text{dw}} j$$

Size of the effect depends on size of $\tilde{\rho}_{dw}$

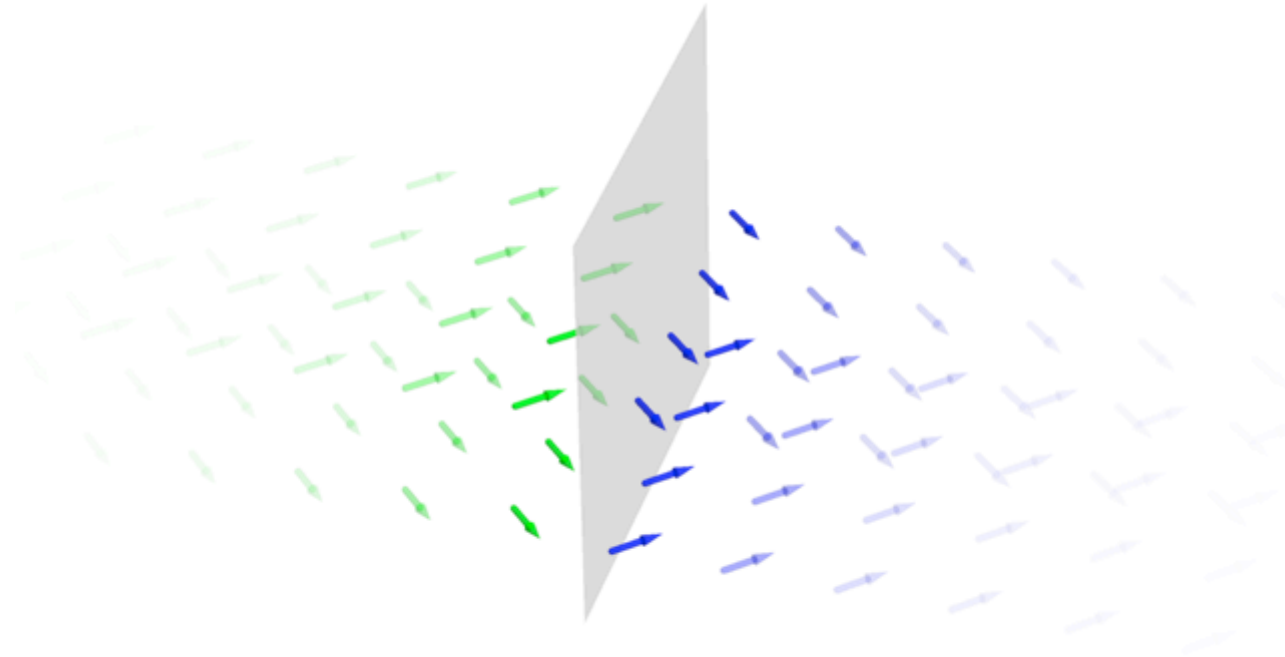
Domain wall

Strong anisotropy/Ising order:
narrow domain walls



Crudely, effective potential for electrons is
"abrupt": strong scattering if Fermi energy is low

Phase space

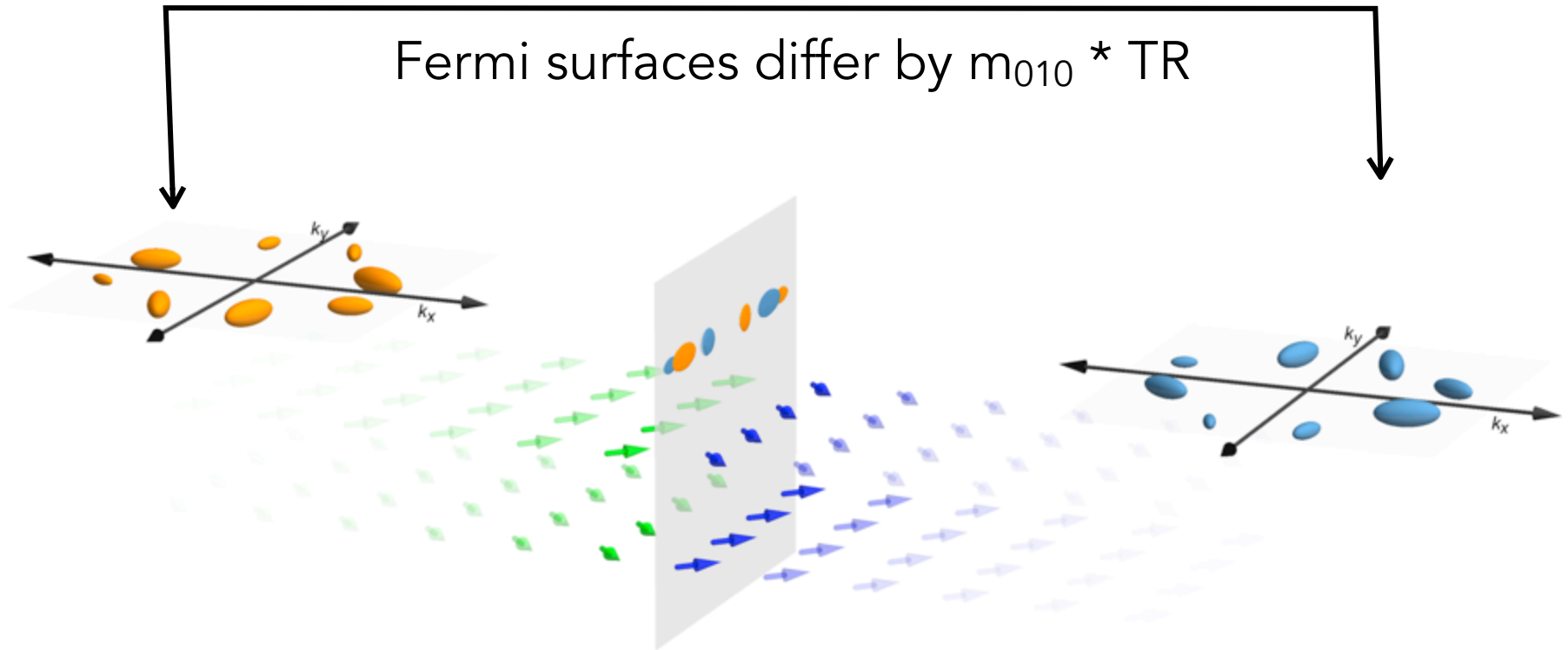


Landauer-Büttiker: one channel for each transverse momenta

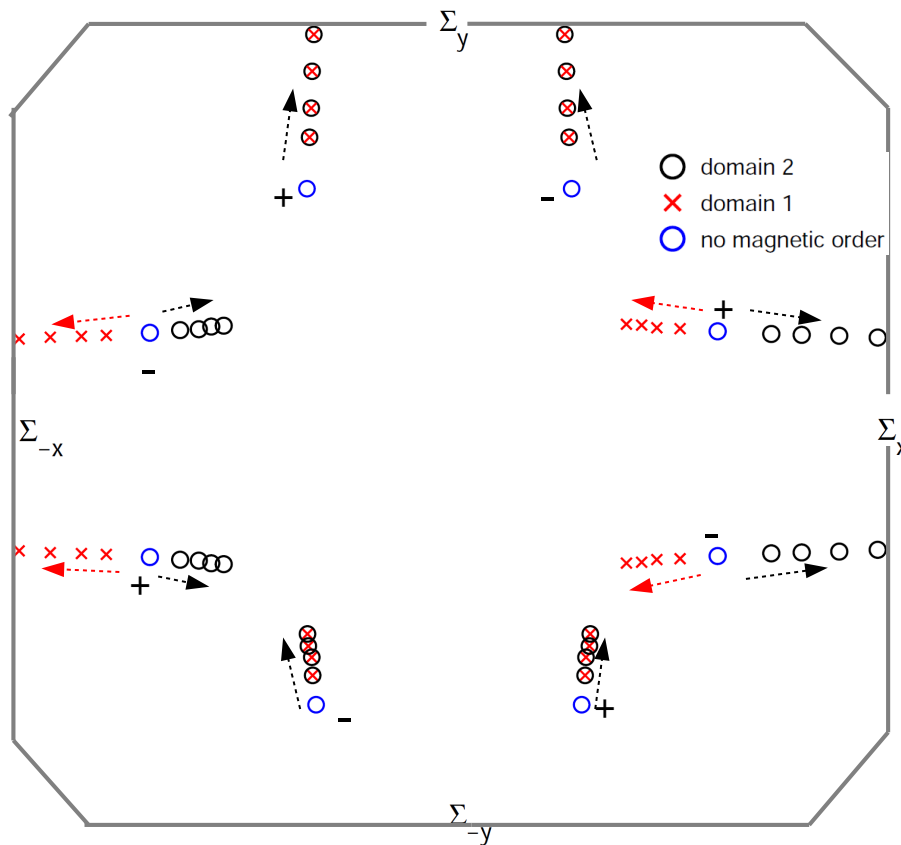
$$G = \sum_{n=1}^{L^2} g_n \frac{e^2}{h}, \quad \text{BUT sum only includes modes that exist at Fermi energy on both sides}$$

Phase space

Fermi surfaces differ by $m_{010} * \text{TR}$



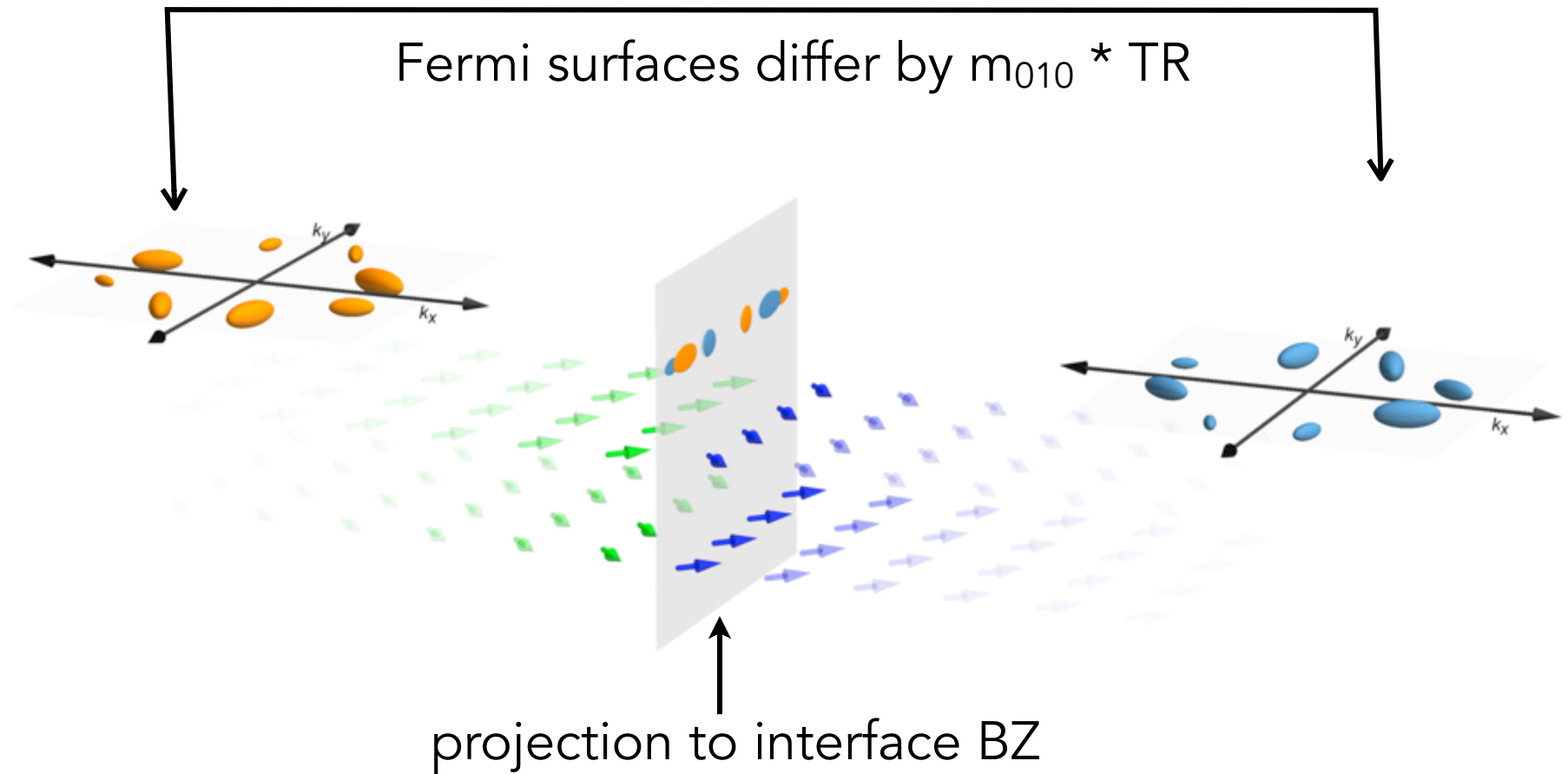
Weyl points



Low symmetry,
SOC: Weyl point
locations depend
on domain

(from DFT-fit tight-binding model)

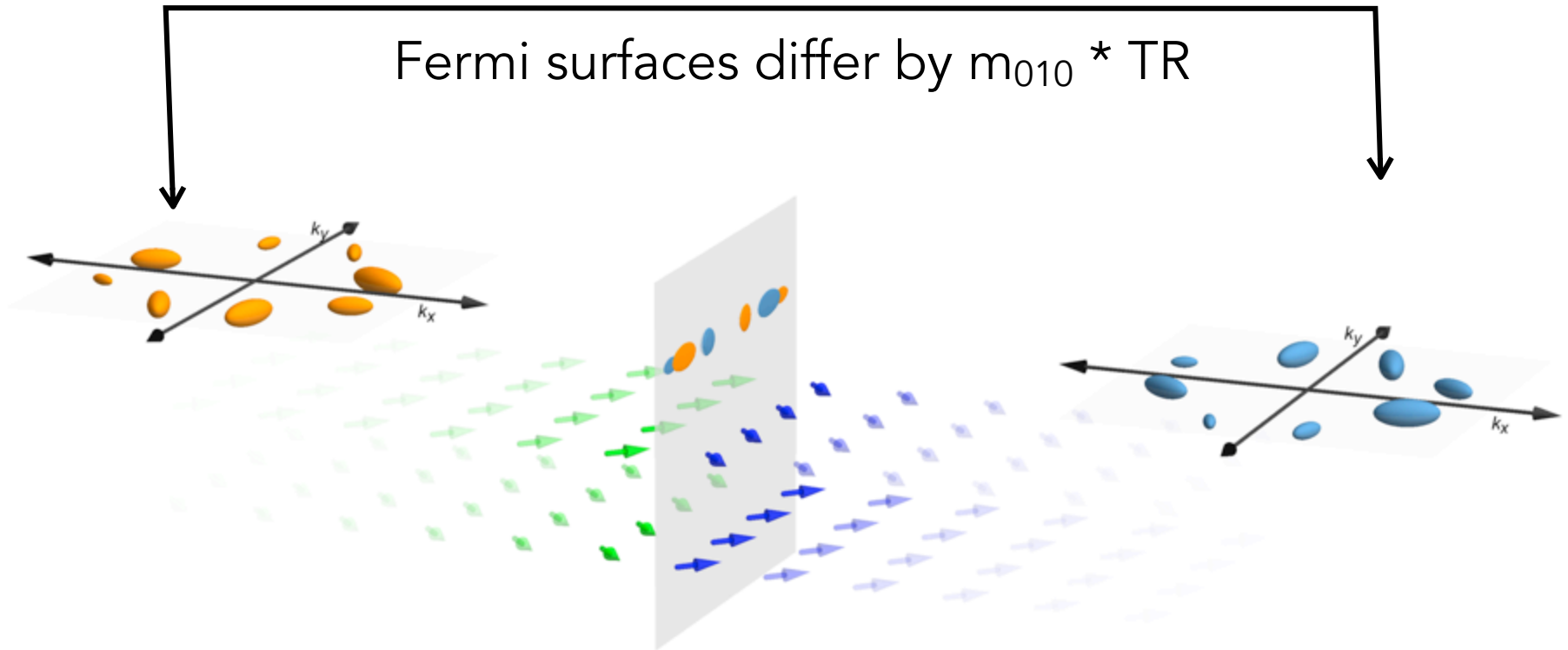
Phase space



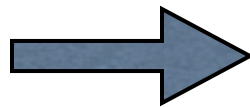
Only overlapping portions contribute!

Phase space

Fermi surfaces differ by $m_{010} * TR$

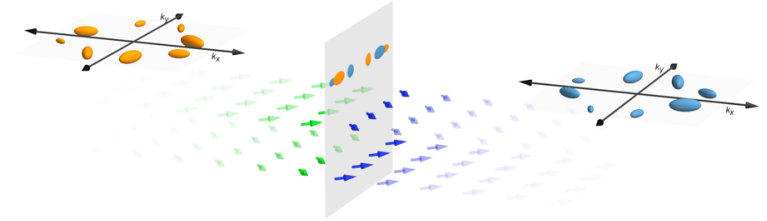
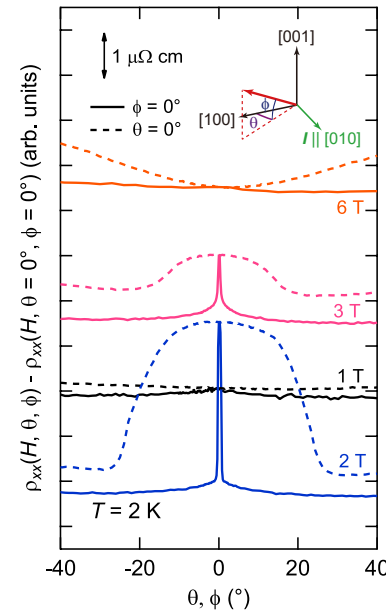
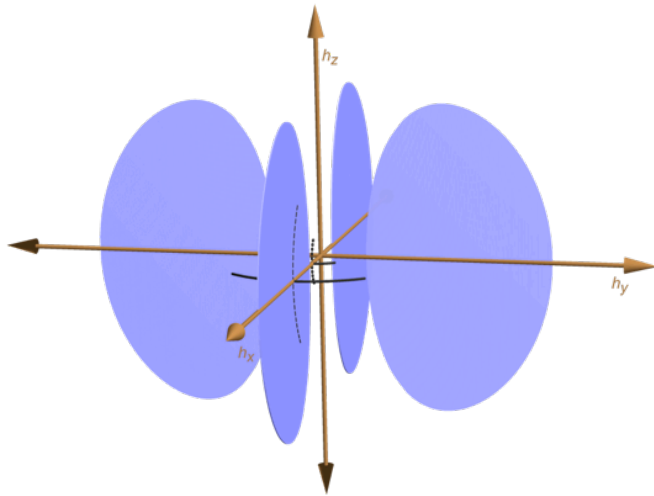


$$G = L^2 \frac{\mathcal{A}_{int}}{(2\pi)^2} T \frac{e^2}{h}$$



$$\frac{\rho_{eff} - \rho}{\rho} \sim \frac{1}{k_F^2 \mathcal{A}_{int}} \frac{\ell}{\ell_d} T^{-1}$$

End



Super Amazing Magneto-Resistance: a new effect in a SOC semimetal
 ?One of many new effects related to topological defects in semimetals?