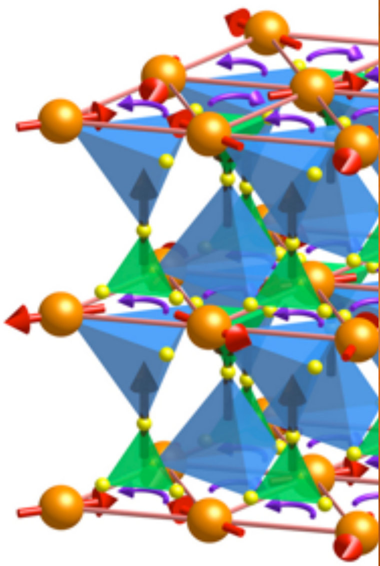
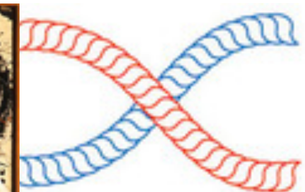
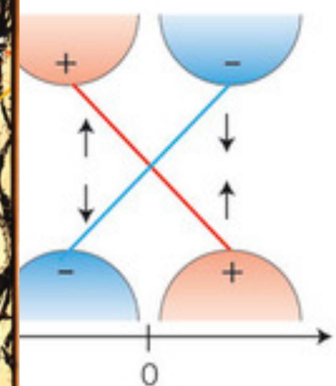
When topology meets SCES

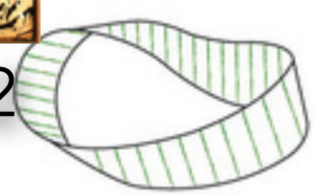
Leon Balents, KITP

Seeking a convergence?







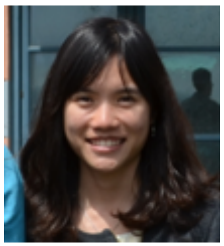


Jackson Pollack, Convergence, 1952

SCES

Topology

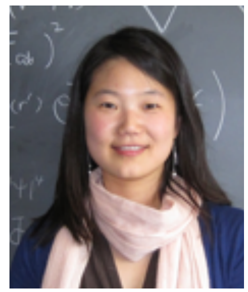
People



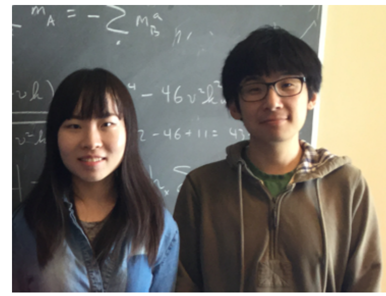
Ru Chen

Lucile Savary

Eun-Gook Moon

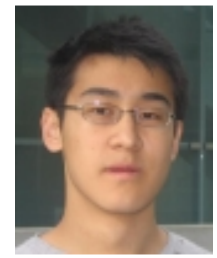


Sung-Bin Lee



宋雪洋 尤亦庄

Xue-Yang Song Yi-Zhuang You



T. Hsieh

H. Ishizuka

Jianpeng Liu

S. Nakatsuji
T. Kondo
Z. Tian
S. Shin
Yong-Baek Kim
Shigeki Onoda

Jay-Z

Outline

- Where can correlations enrich topology?
- Three types of topology: band topology, Berry phase topology, intrinsic topological order
- Correlations in two of three:
  - Correlated Weyl semimetals
  - Quantum spin liquids

Three types of topology

Topological Insulator

topology of filled bands

"symmetry protected topological order"

Topological Semimetal

topology of k-surfaces

"Berry phase topology"

Topological Spin Liquid

topology of entanglement

"intrinsic topological order"

Three types of topology

Topological Insulator	Topological Semimetal	Topological Spin Liquid
topology of filled bands	topology of k-surfaces	topology of entanglement
“symmetry protected topological order”	“Berry phase topology”	“intrinsic topological order”

+ Correlations??

Three types of topology

Topological Insulator

topology of filled bands

"symmetry protected topological order"

+ Correlations:

✦Topological Kondo Insulator SmB_6?

C. Broholm
Lu Li
S. Sebastian
S. Wirth
J. Denlinger

Three types of topology

Topological Insulator

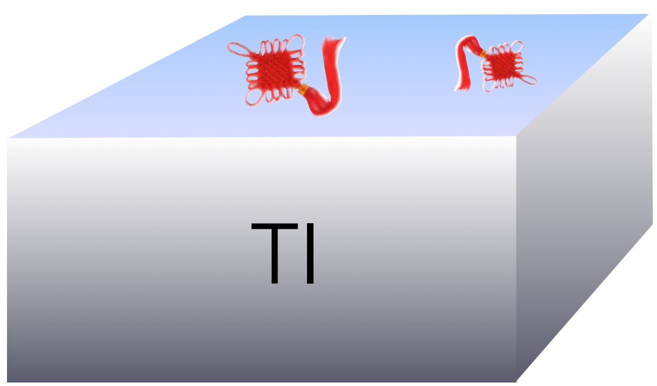
topology of filled bands

"symmetry protected topological order"

+ Correlations:

- ✦ Topological Kondo Insulator SmB_6?
- ✦ Surface states may be more correlated than bulk

surface state with gapped non-abelian anyons



C. Wang *et al*, 2013
L. Fidkowski *et al*, 2013
M.A. Metlitski *et al*, 2013
P. Bonderson *et al*, 2013

Three types of topology

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“symmetry protected topological order”

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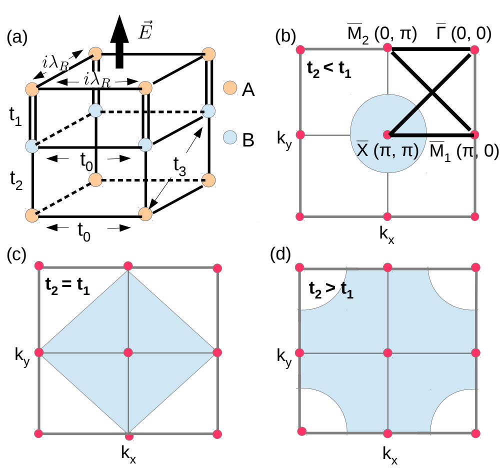
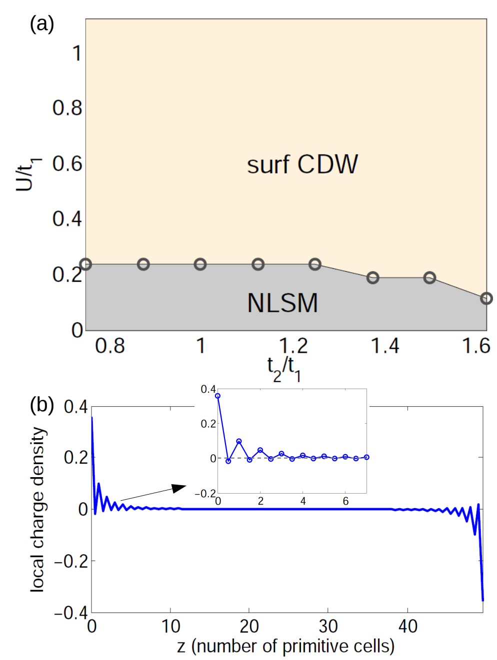
- ✦Surface states may be more correlated than bulk

nodal-loop semimetals w/ quasi-flat surface bands



c.f. ZrSiS, $PbTaSe_2$

surface instabilities at relatively small U



Jianpeng Liu +LB, in preparation

Three types of topology

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"symmetry protected topological order"

+ Correlations:

- ✦ Topological Kondo Insulator SmB_6?
- ✦ Surface states may be more correlated than bulk
- ✦ Bosonic SPT states

Symmetry protected topological order

From Wikipedia, the free encyclopedia

Symmetry Protected Topological order (**SPT** order)[1] is a kind of order in zero-temperature quantum-mechanical states of matter that have a symmetry and a finite energy gap.

To derive the results in a most-invariant way, renormalization group methods are used (leading to equivalence classes corresponding to certain fixed points).[1] The SPT order has the following defining properties:

(a) *distinct SPT states with a given symmetry cannot be smoothly deformed into each other without a phase transition, if the deformation preserves the symmetry.*
(b) *however, they all can be smoothly deformed into the same trivial product state without a phase transition, if the symmetry is broken during the deformation.*

Using the notion of quantum entanglement, we can say that SPT states are short-range entangled states *with a symmetry* (by contrast: for long-range entanglement see topological order, which is not related to the famous EPR paradox). Since short-range entangled states have only trivial topological orders we may also refer the SPT order as Symmetry Protected "Trivial" order.

Contents [hide]

A big subject for theorists

Three types of topology

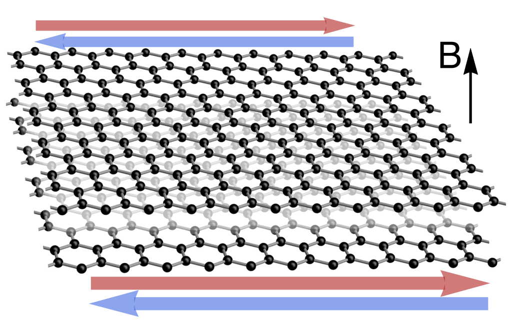
Topological Insulator

topology of filled bands

"symmetry protected topological order"

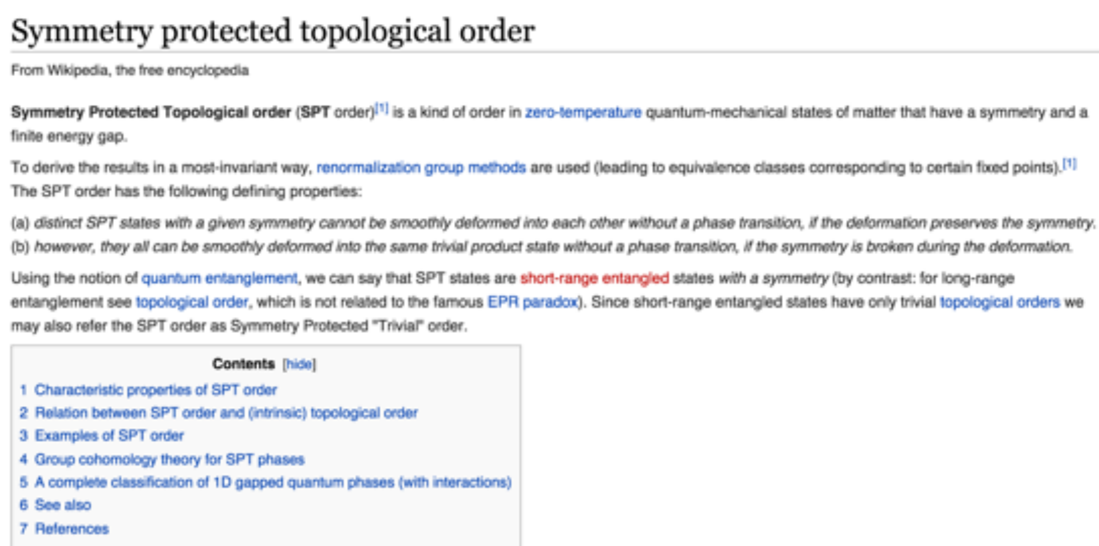
+ Correlations:

- ✦Topological Kondo Insulator SmB_6?
- ✦Surface states may be more correlated than bulk
- ✦Bosonic SPT states



Zhen Bi *et al*, arXiv:1602.03190

T. Yoshida Mo-S4-5



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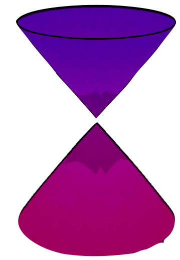
Weyl semimetal

Topological Semimetal

topology of k-surfaces

"Berry phase topology"

$$H = v\vec{\sigma}\cdot\vec{k}$$



937 PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING
Princeton University, Princeton, New Jersey
(Received June 16, 1937)

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neigborhood of a point $\mathbf{k}$ where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa\rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

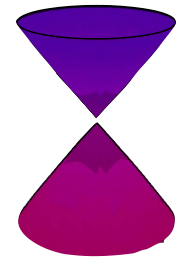
Weyl semimetal

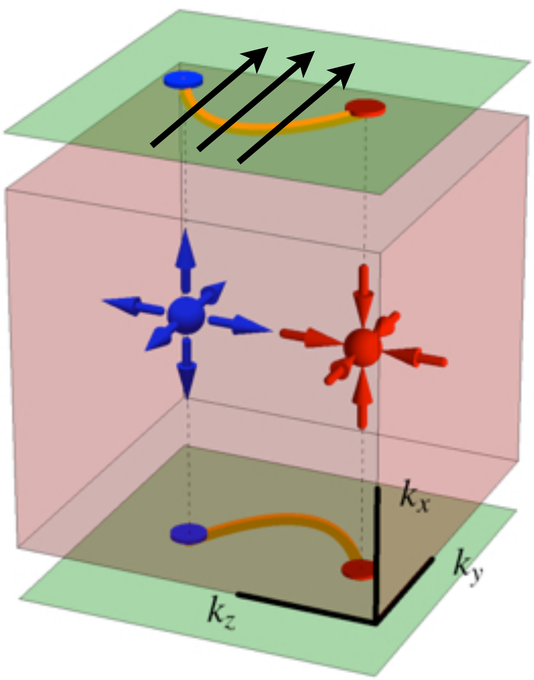
Topological Semimetal

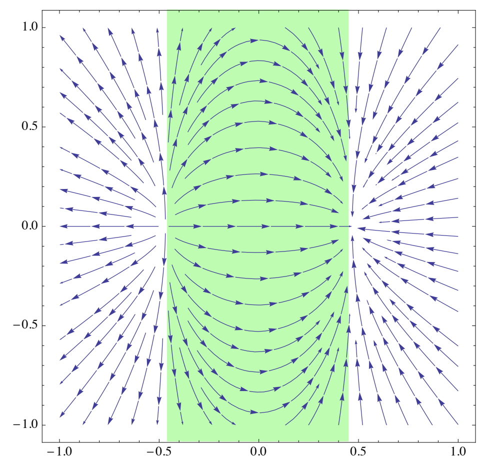
topology of k-surfaces

"Berry phase topology"

$$H = v\vec{\sigma}\cdot\vec{k}$$





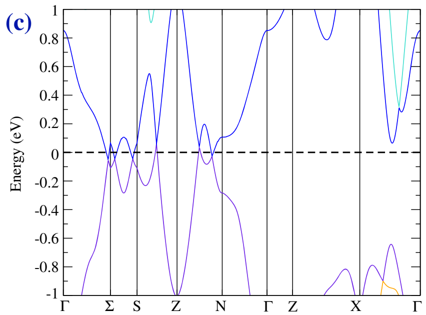


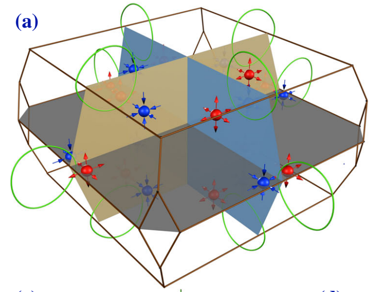
S. Murakami, 2007
X. Wan *et al*, 2011
A. Burkov+LB, 2011

Weyl points are "monopoles" of Berry curvature: topology in k-space!

Experiment

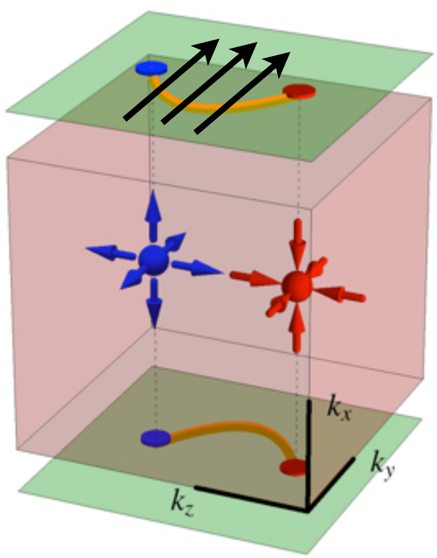
TaAs

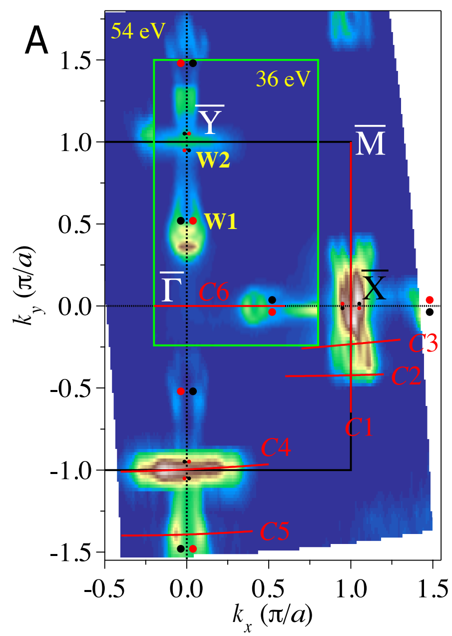


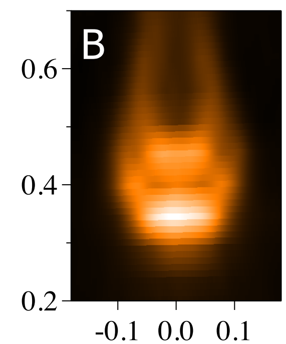


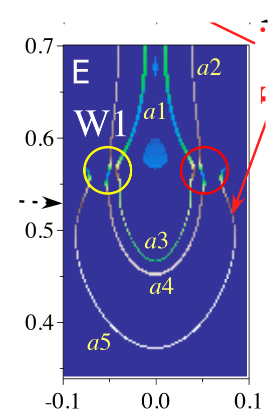
Prediction:
Hongmin Weng *et al*, 2015

- Striking properties:
  - Surface Fermi arcs





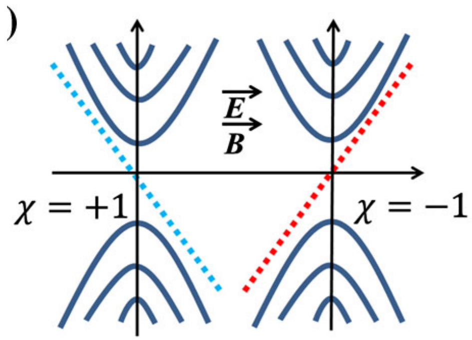




B.Q. Lv *et al*, 2015

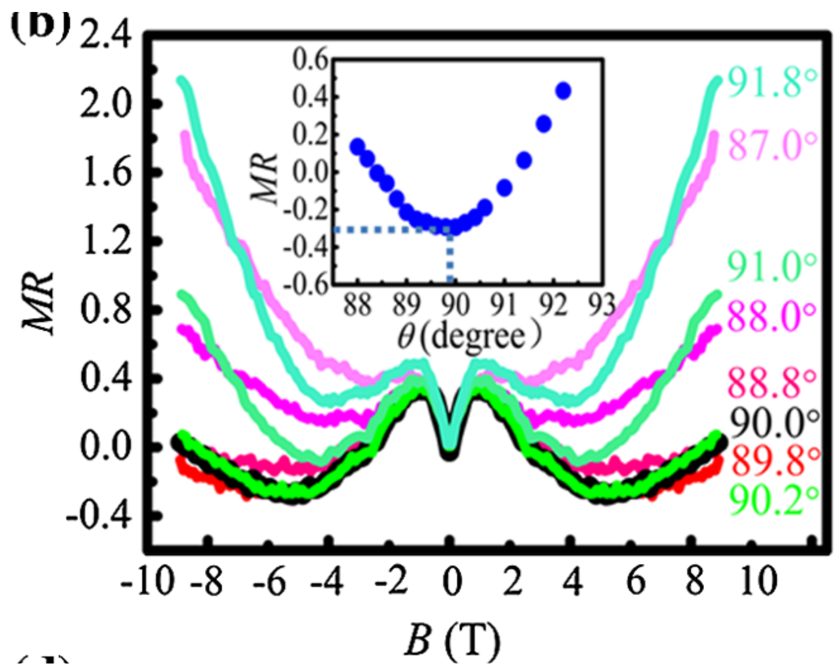
Experiment

- Striking properties:
  - Surface Fermi arcs
  - ABJ "anomaly": strong negative MR for I ∥ B



0[th] LL connects
Weyl points

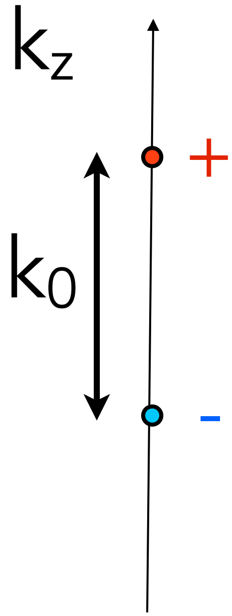
Nielson + Ninomiya ,1983
Zyuzin+Burkov, 2012
Son+Spivak, 2013

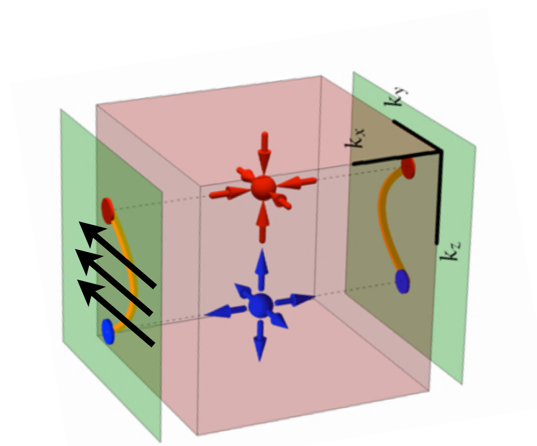


Xiaochun Huang *et al*, 2015

Anomalous Hall Effect

The third striking property of a Weyl semimetal





Fermi arc = chiral edge state

$$\sigma_{xy} = \frac{e^2}{h}\frac{k_0}{2\pi}$$

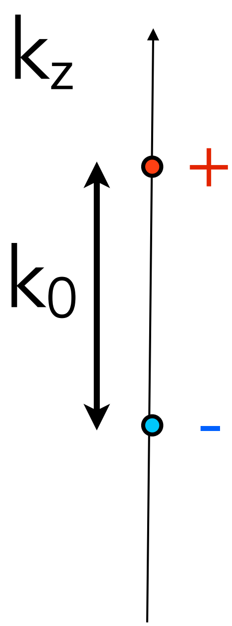
semi-quantum AHE

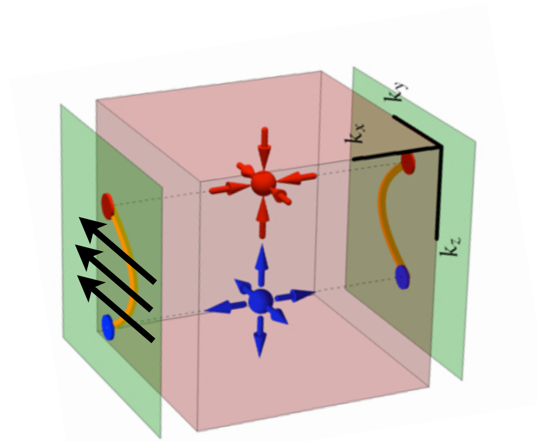
obviously breaks time-reversal symmetry

need a magnetic material

Anomalous Hall Effect

The third striking property of a Weyl semimetal





Fermi arc = chiral edge state

$$\sigma_{\mu\nu} = \frac{e^2}{2\pi h}\epsilon_{\mu\nu\lambda}Q_\lambda$$

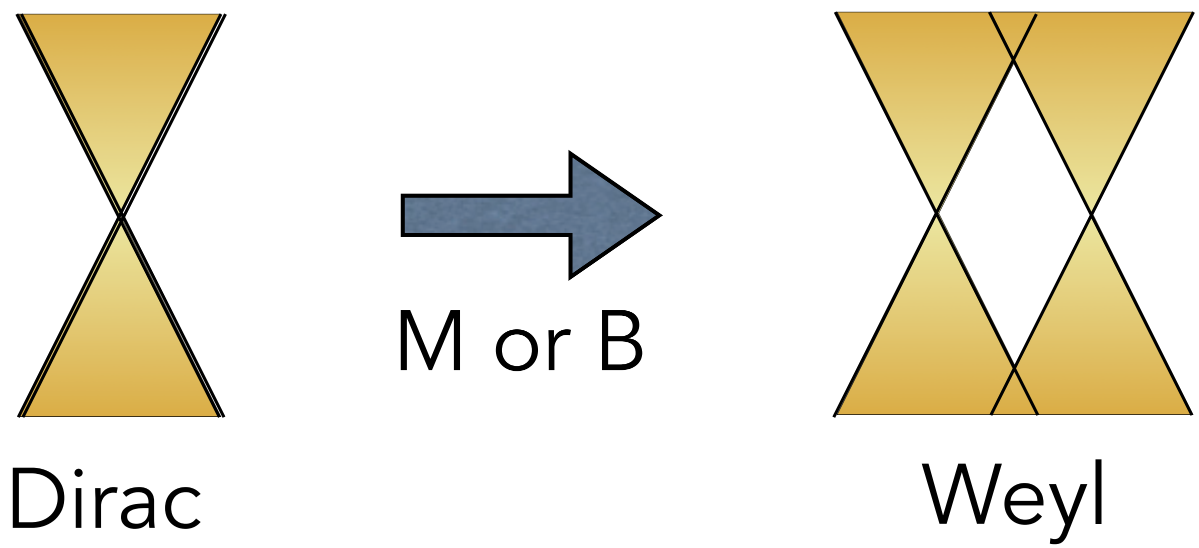
$$\vec{Q} = \sum_i \vec{k}_i q_i + \vec{Q}_{RLV}$$

semi-quantum AHE

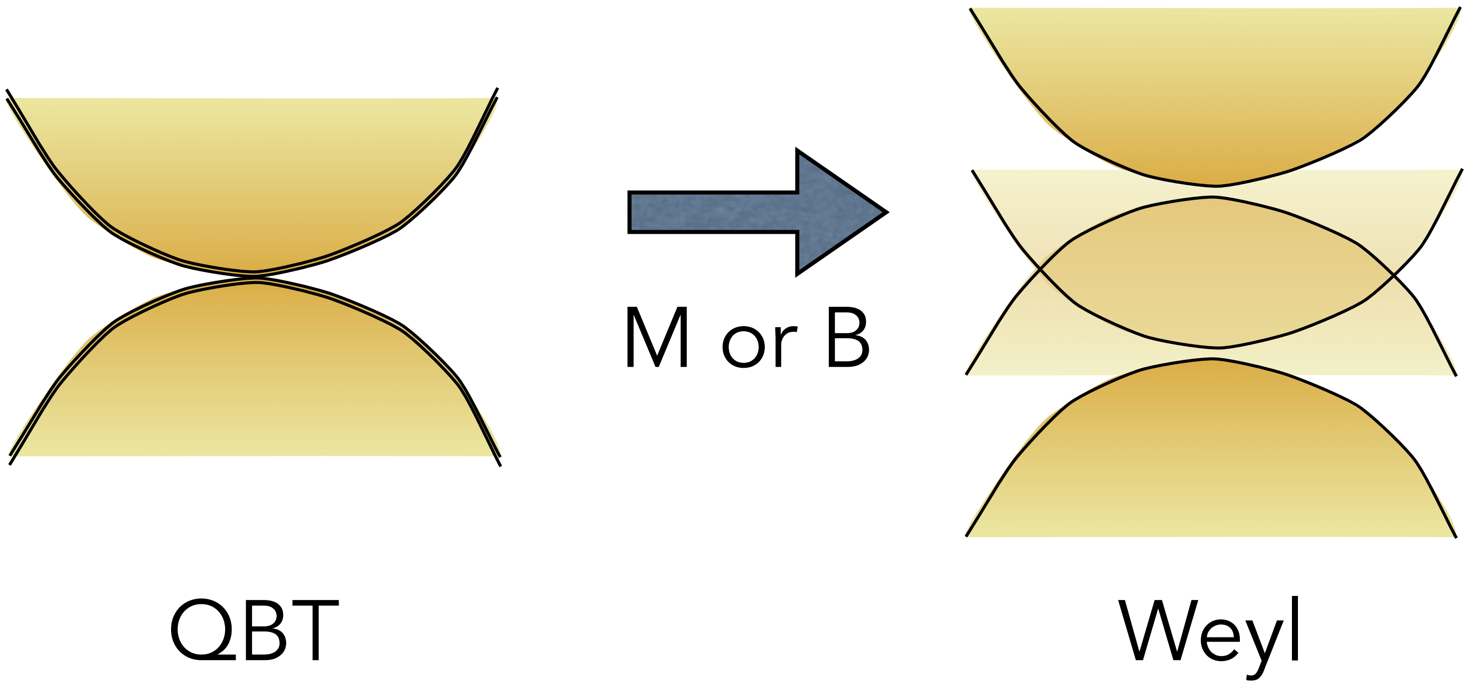
obviously breaks time-reversal symmetry

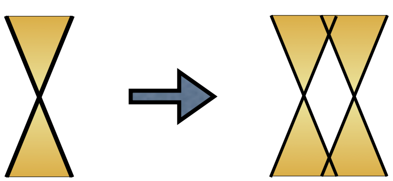
→ need a magnetic material

Magnetic Weyl semimetals



Magnetic Weyl semimetals





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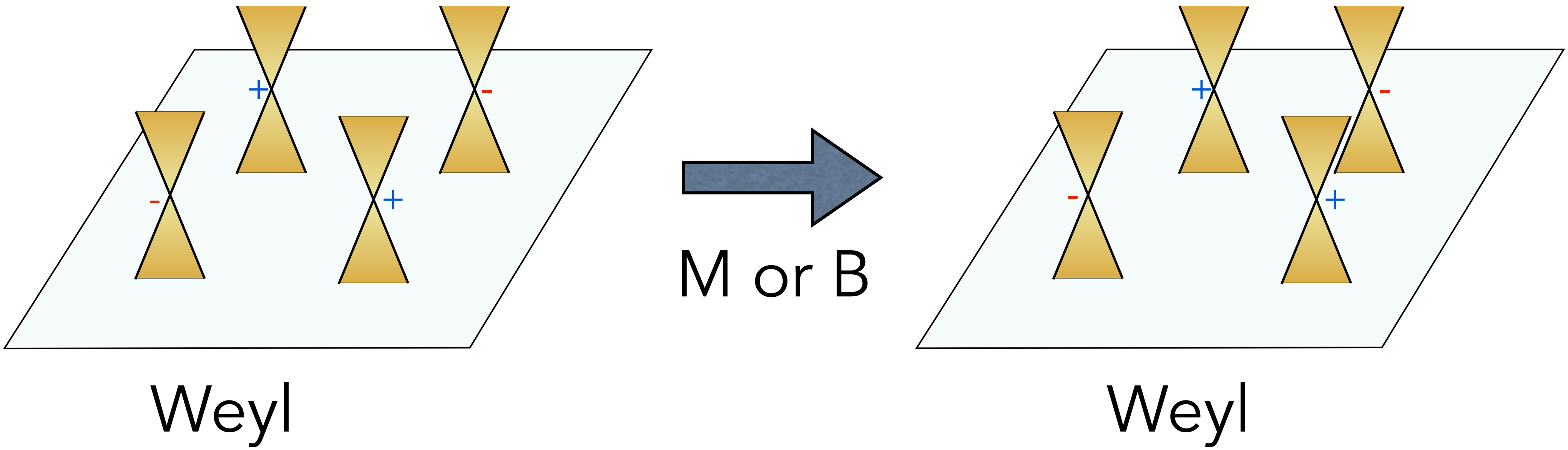
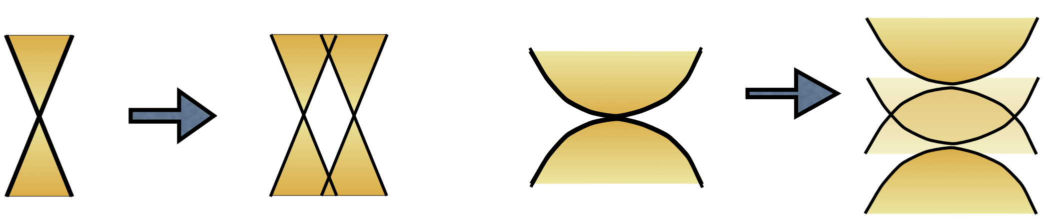


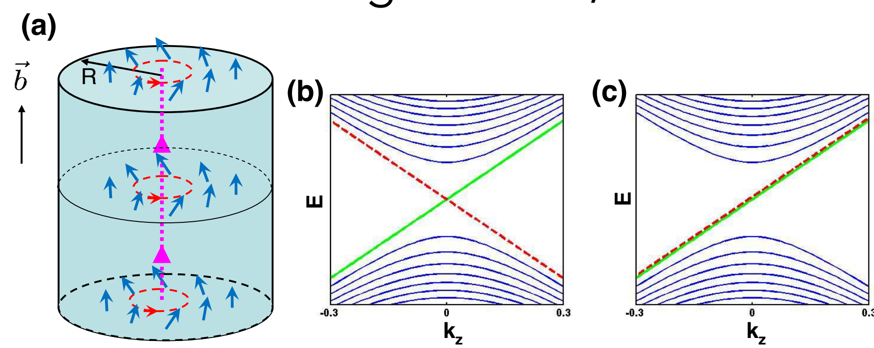


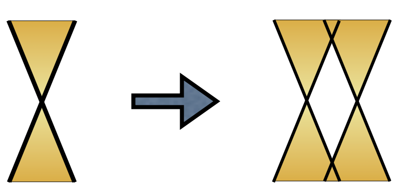
Magnetic Weyl semimetals

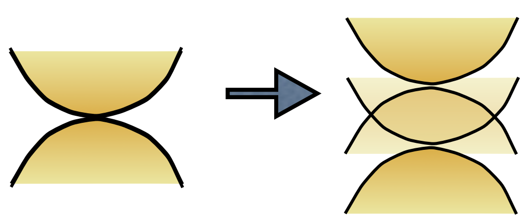
Movable Weyl points have their own interest

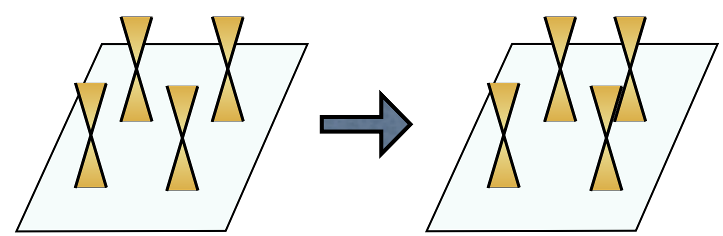
magnetic fluctuations generate a *dynamical chiral gauge field* for Weyl fermions, by shifting Weyl points

Chao-Xing Liu *et al*, 2012









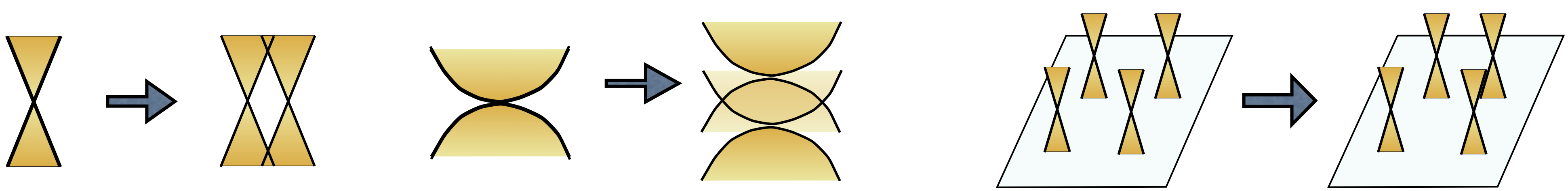
Magnetic Weyl semimetals

Exercise: how far can we move Weyl points?

We'll see that this is why we need SCES

Dirac/Weyl: $\hbar v_F \Delta k \approx E_Z$

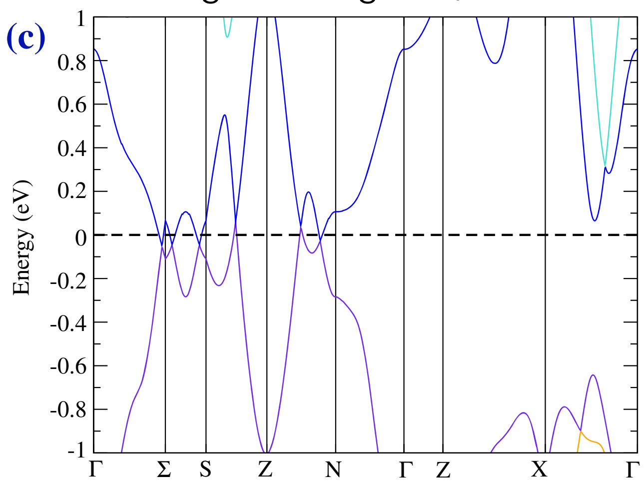
QBT: $\frac{\hbar^2 (\Delta k)^2}{2m^*} \approx E_Z$



Magnetic Weyl semimetals

Suppose you modify, e.g. magnetically dope, TaAs

Hongmin Weng *et al*, 2015

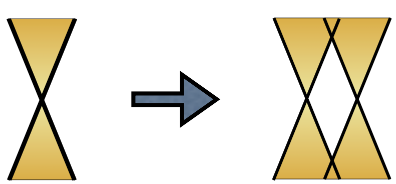


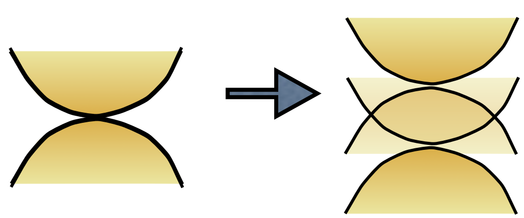
$$\Delta k \approx E_Z/(\hbar v_F)$$

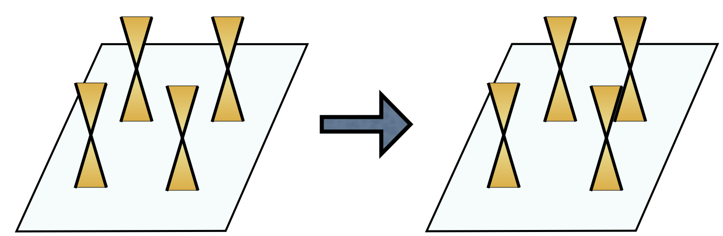
$$\hbar v_F \approx 2eV\mathring{A} \quad E_Z < 1000K$$

$$\Delta k < 0.04\mathring{A}^{-1}$$

Result: Weyl points move < 1/50th of the zone







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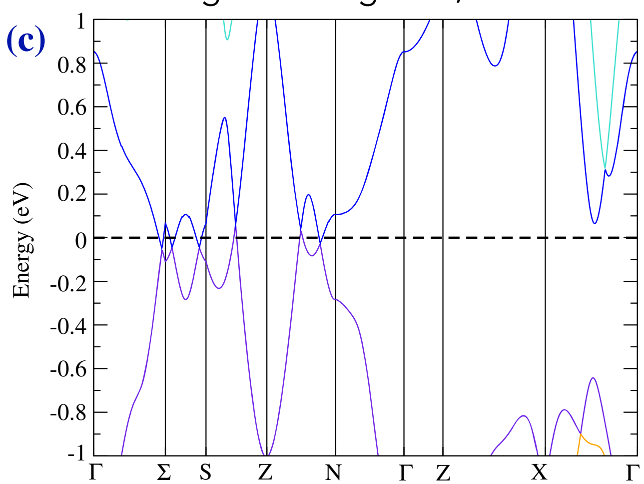

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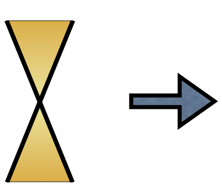
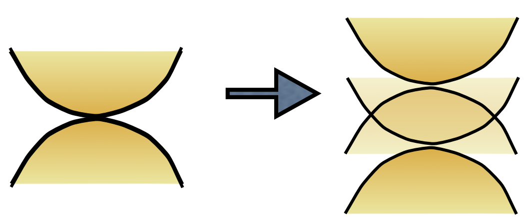
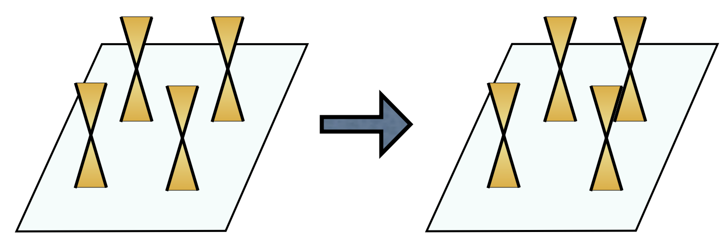
Result: Weyl points move < 1/50th of the zone

need a narrower band



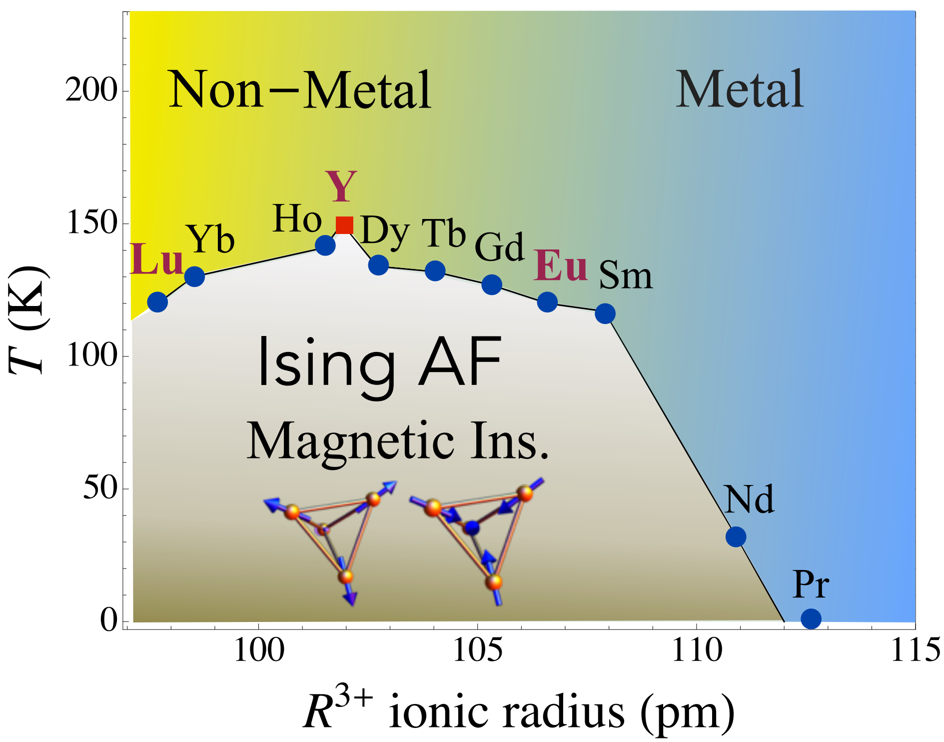




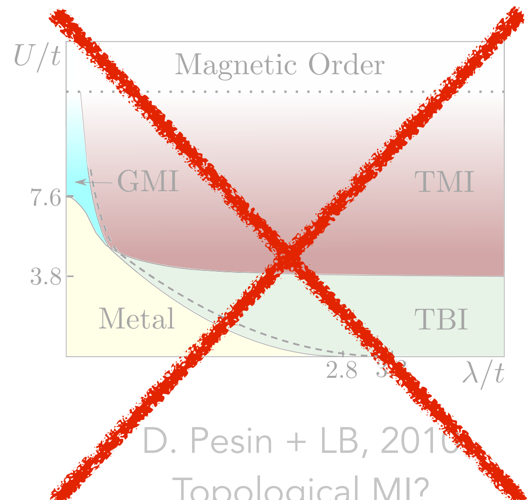
Pyrochlore iridates

$A_2Ir_2O_7$

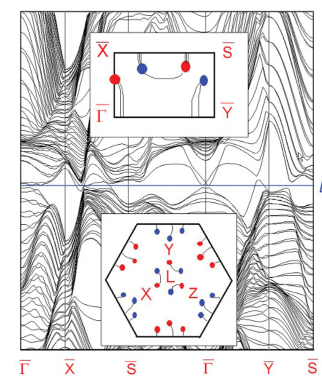
- A good place to look for correlated Weyls



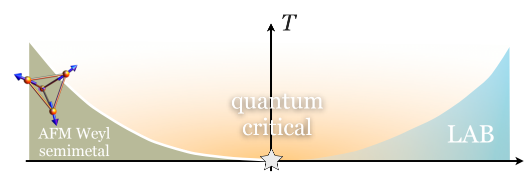
Yanagashima+Maeno, JPSJ 2001
K. Matsuhira et al, JPSJ 2011
W. Witczak-Krempa et al, ARCMP 2013



D. Pesin + LB, 2010
Topological MI?



X. Wan *et al*, 2011
AF Weyl semimetal?

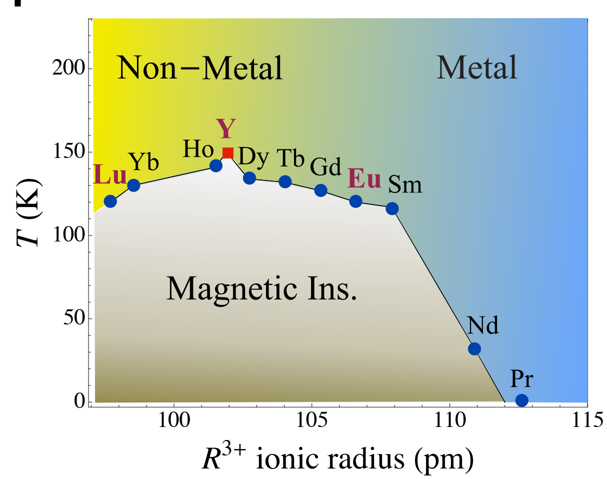


L. Savary *et al*, 2014 - topological QCP?

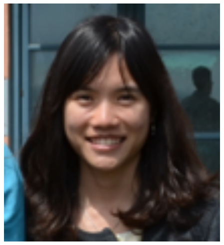
Ingredients

- Semimetallic electronic structure ✓
- Magnetism via Ir e-e Hubbard interactions ✓
- Rare earth moments?

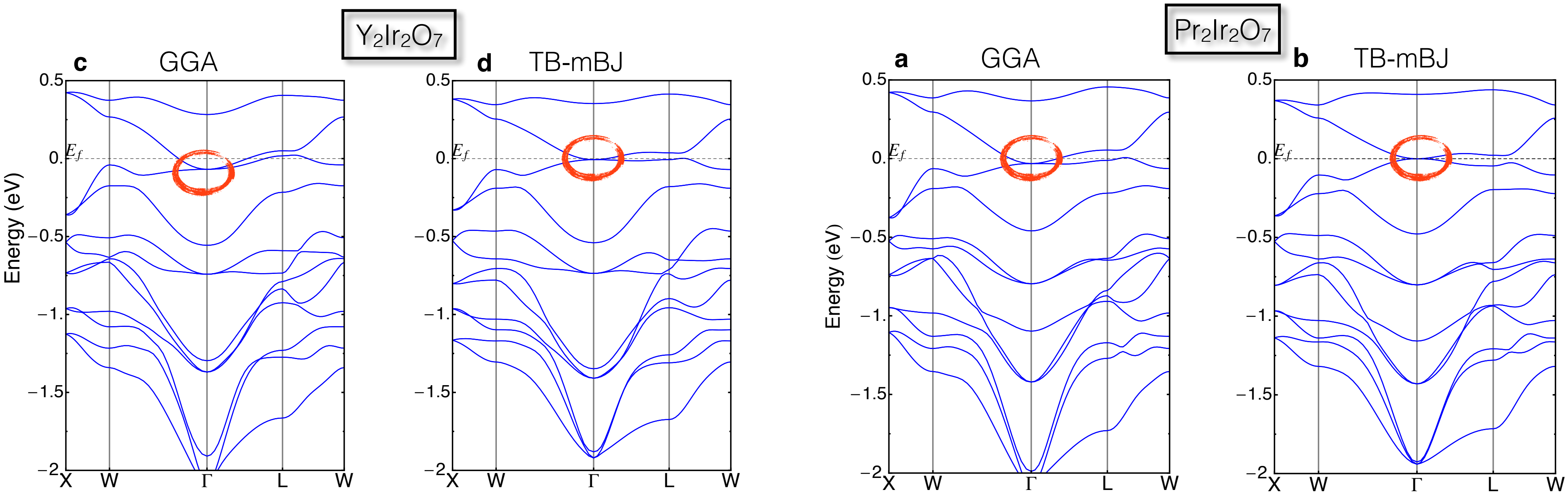
probably not important?

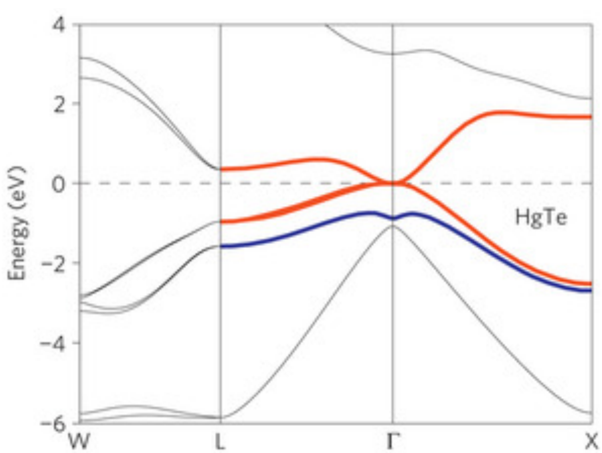


T. Kondo *et al*, Nat. Comm., 2015



Paramagnetic electronic structure



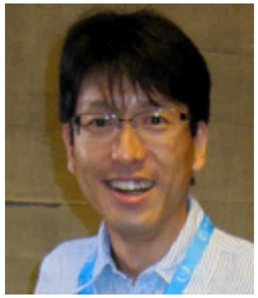


Γ_8 4-dimensional irrep guaranteed by cubic symmetry and time reversal

ARPES

T. Kondo *et al*, Nat. Comm., 2015

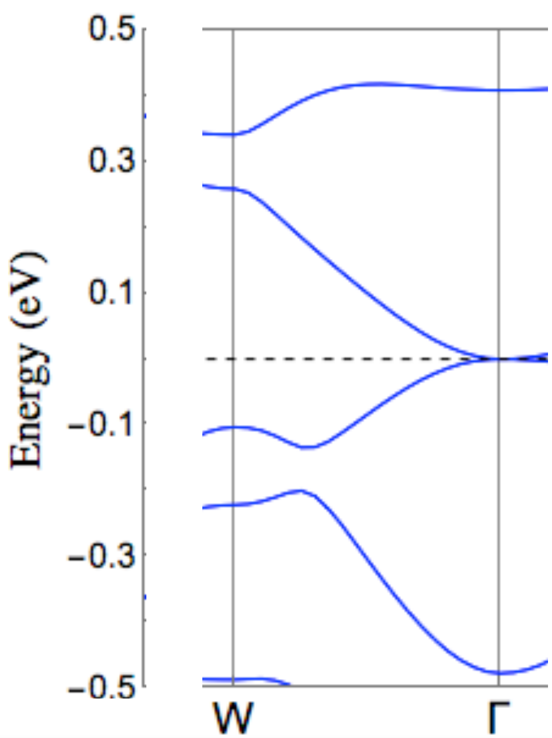
$Pr_2Ir_2O_7$

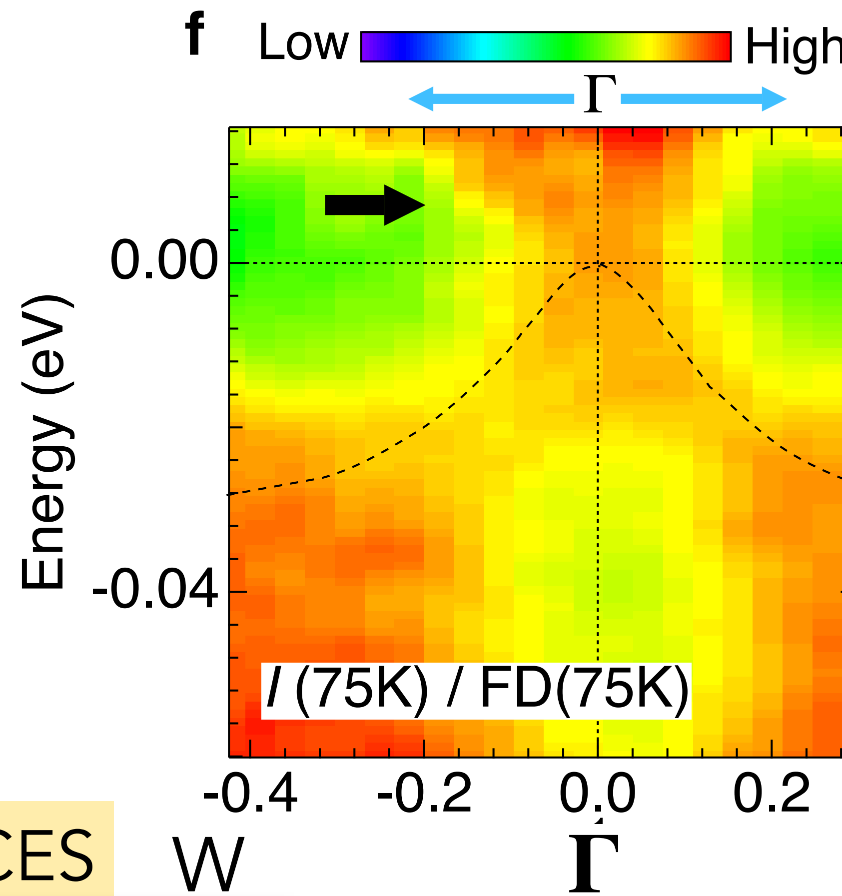


S. Nakatsuji

T. Kondo

S. Shin

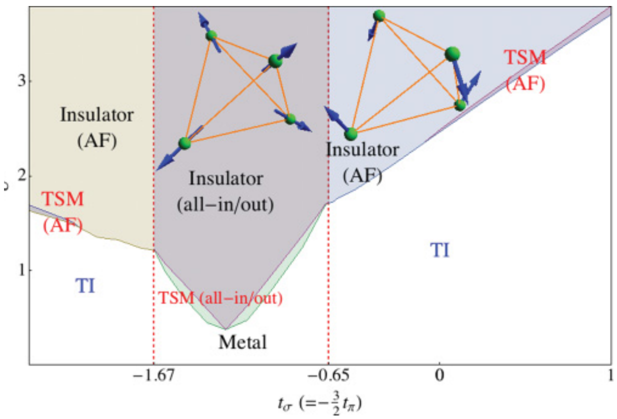




SCES

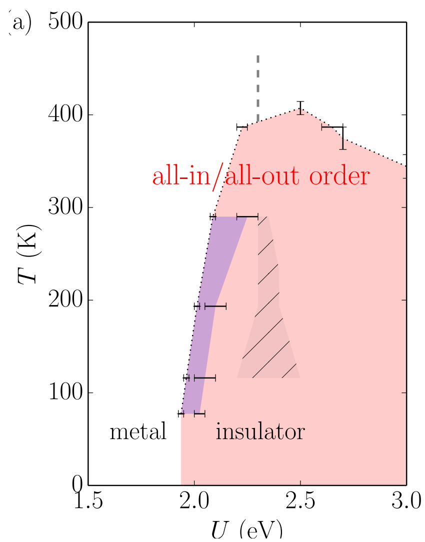
✓Bandwidth reduced by 3-5 from DFT

Magnetism: theory



W. Witzak-Krempa + YB Kim, 2012

Hartree-Fock

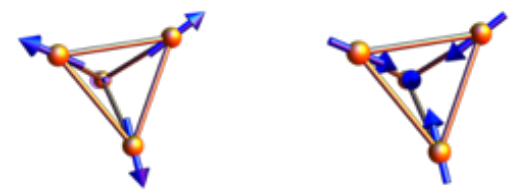


H. Shinaoka *et al*, 2015

DMFT

Jay-Z model

$$H = J_z \sum_{\langle i,j \rangle} S_i^z S_j^z$$

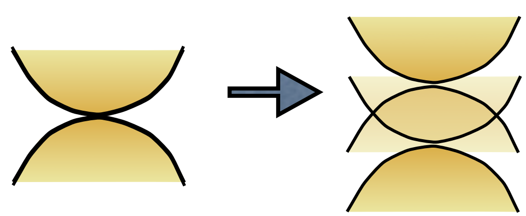


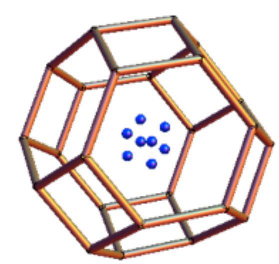
$J_z < 0$: all-in/all-out order

superexchange

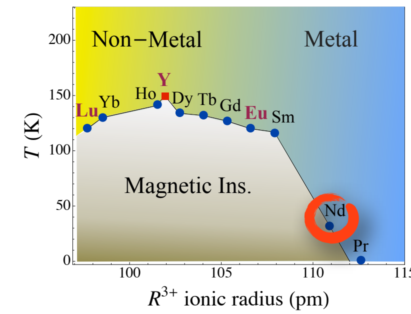
- General agreement: transition to AIAO Ising AF order

We expect this to lead to Weyl points

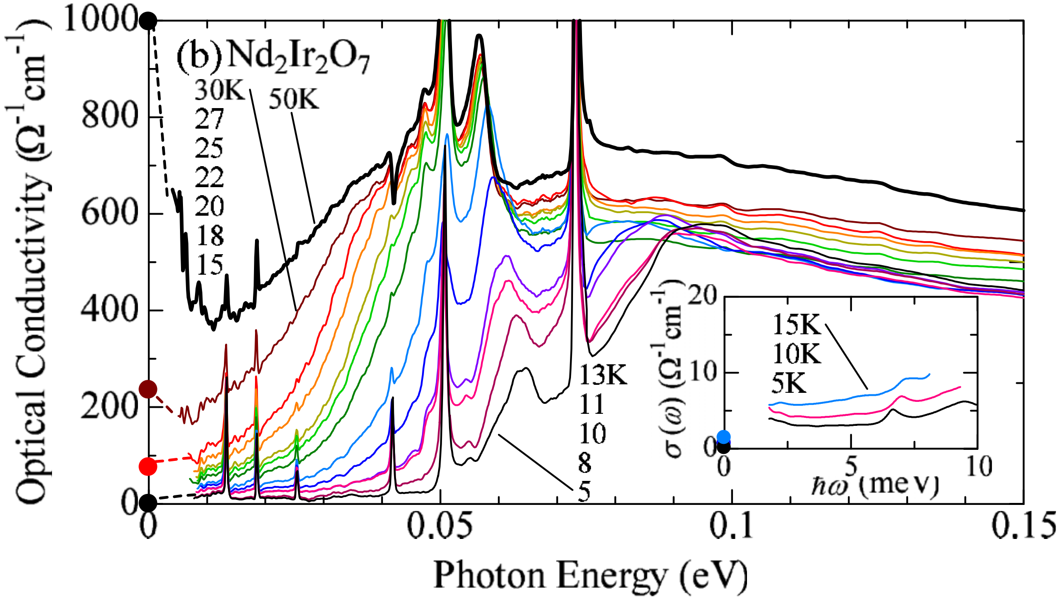




Weyl not?

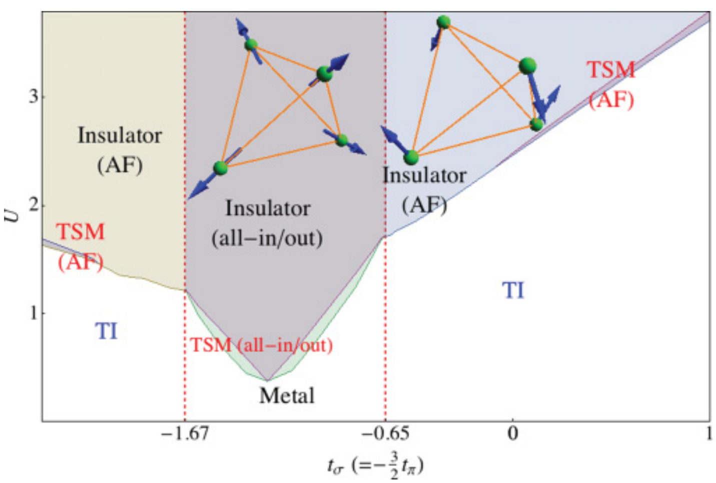


K. Ueda *et al*, 2012



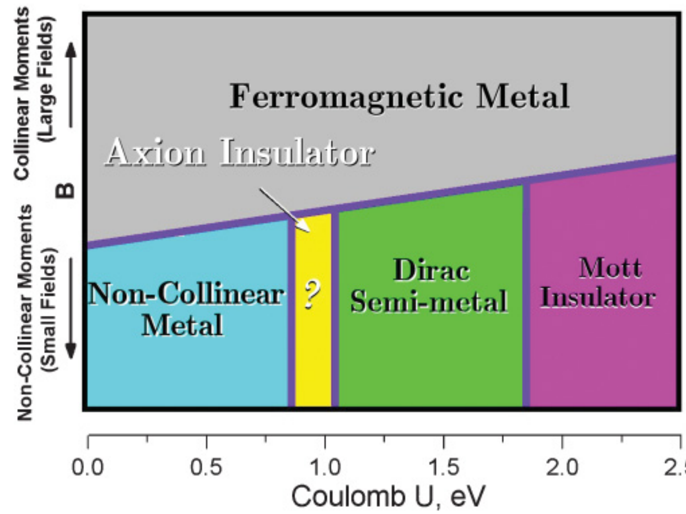
charge gap ~ 45meV

Moving Weyl Points



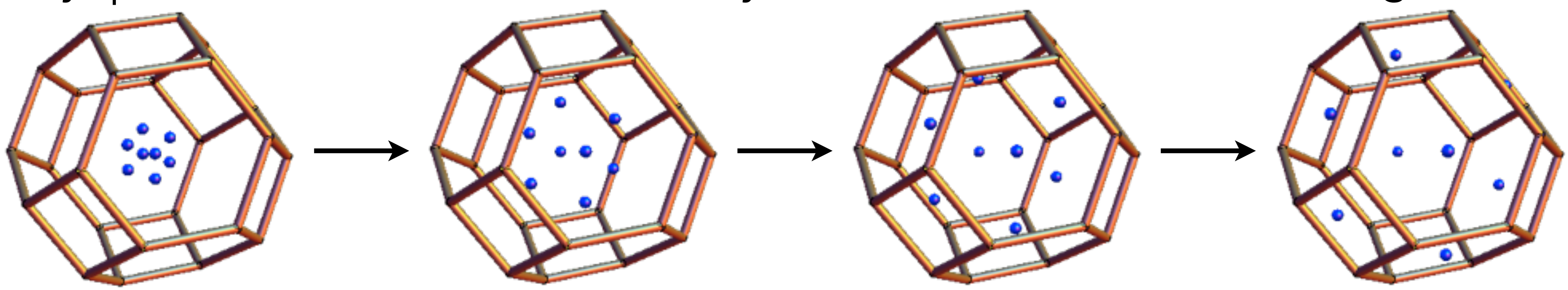
W. Witzak-Krempa + YB Kim, 2012

vs



X. Wan *et al*, 2011

Weyl points move to zone boundary and annihilate with increasing order?

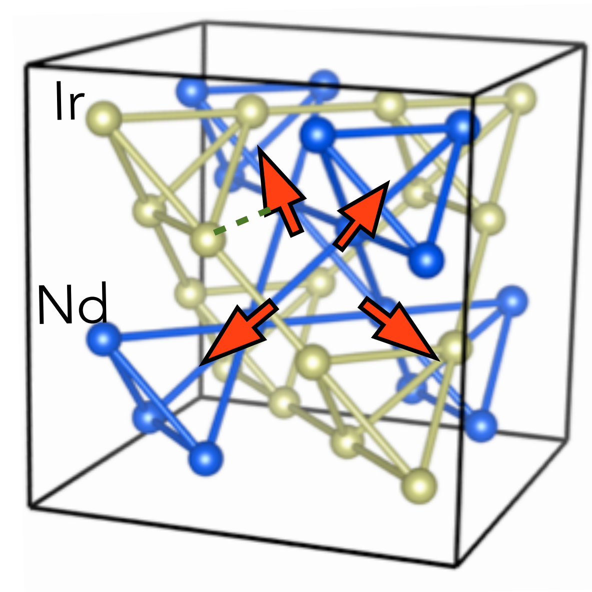


If so, Weyl points are *too* mobile!

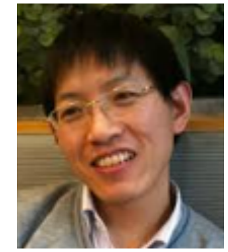
Nd physics

Nd spins provide anisotropic "exchange enhancement"

- Large moment couples strongly to field
- Polarized Nd act back on Ir via $J_{Ir\text{-}Nd} \sim 10$ meV
- Maximum effect: B || (100) - aligns all Nd moments



Nd physics



Nd spins provide anisotropic "exchange enhancement"

- Large moment couples strongly to field
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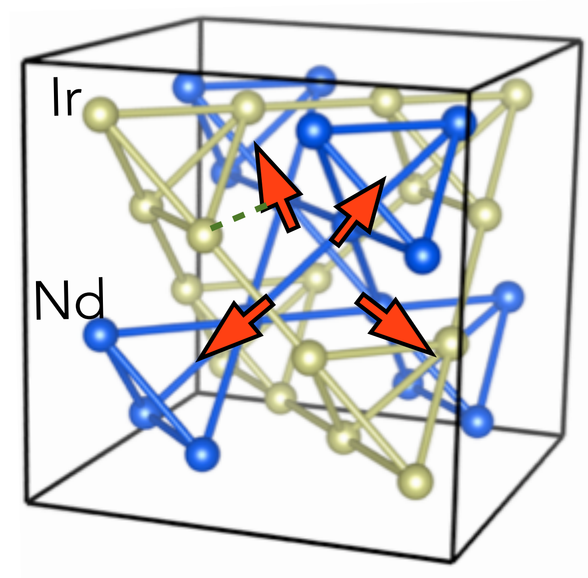
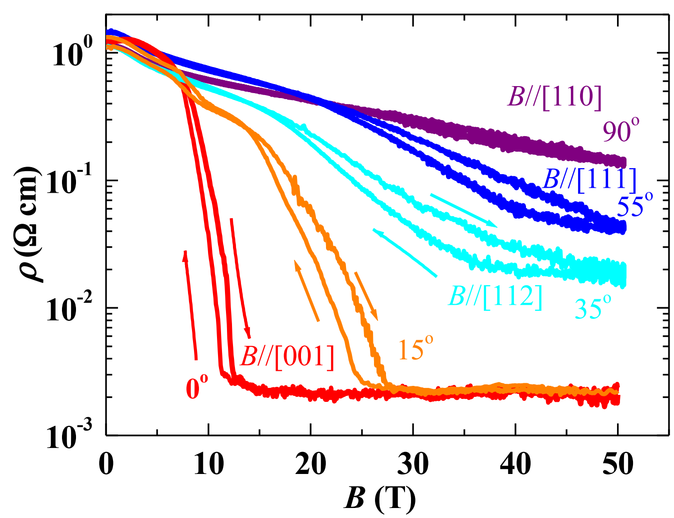
Result: MIT



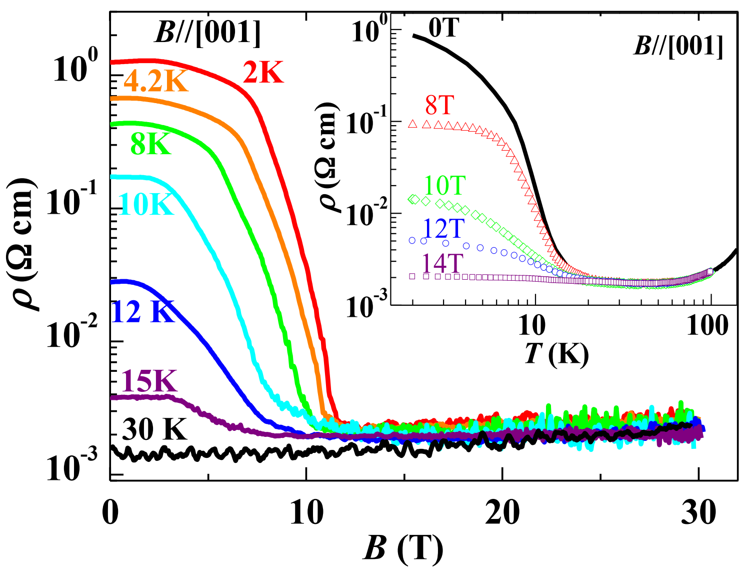
Zhaoming Tian *et al*, Nature Physics, 2015

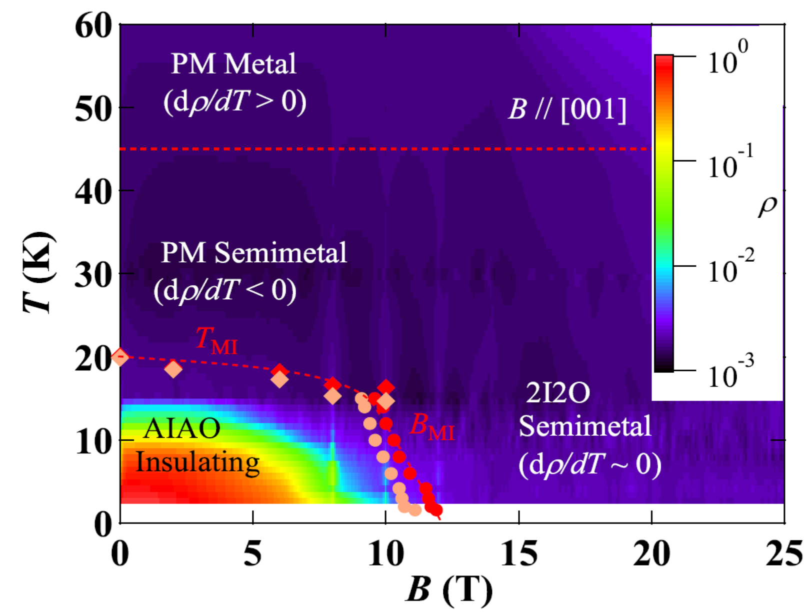


also K. Ueda *et al*, 2015

Zhaoming Tian *et al*, Nature Physics, 2015

Metal-Insulator Transition

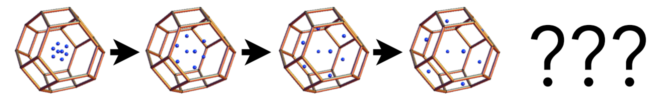




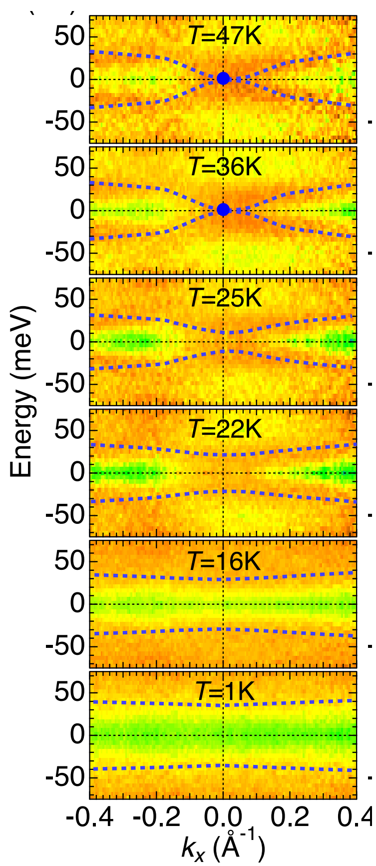
- Seems that the antiferromagnetic phase forms a closed region at small B and T.
- Not known: what is the nature of the high-field semimetal? Maybe a magnetic Weyl state?

M. Nakayama *et al*, arXiv:1603.06095

ARPES



- Is the absence of Weyl points really due to this mean-field picture of moving nodes?



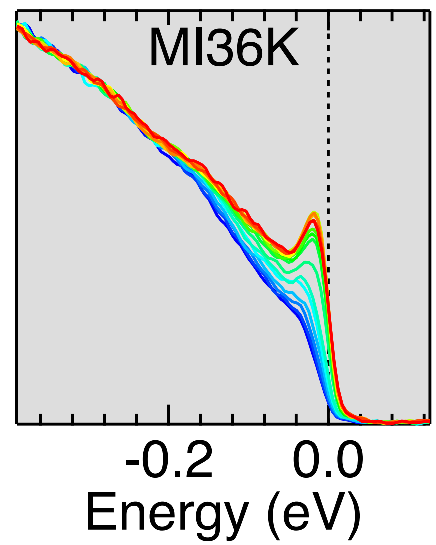
$T>T_c$: like Pr

$T=T_c$: no precursor - Slater

$T < T_c$: gap developing

Slater to Mott crossover?

$T << T_c$: remarkably flat

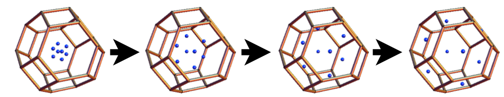


loss of quasiparticle peak

Rather than moving nodes, we observe loss of quasiparticle as gap opens

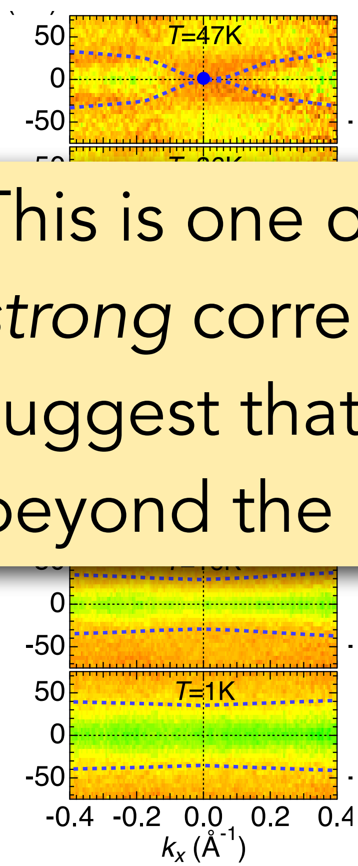
M. Nakayama *et al*, arXiv:1603.06095

ARPES



???

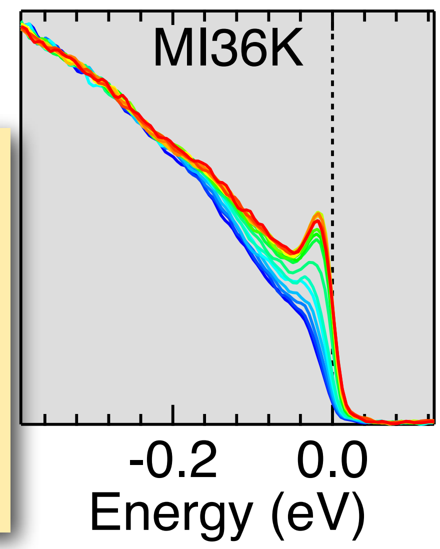
- Is the absence of Weyl points really due to this mean-field picture of moving nodes?



$T > T_c$: like Pr

This is one of several indications of *strong* correlation effects, that suggest that there may be subtleties beyond the mean field picture

$T << T_c$: remarkably flat

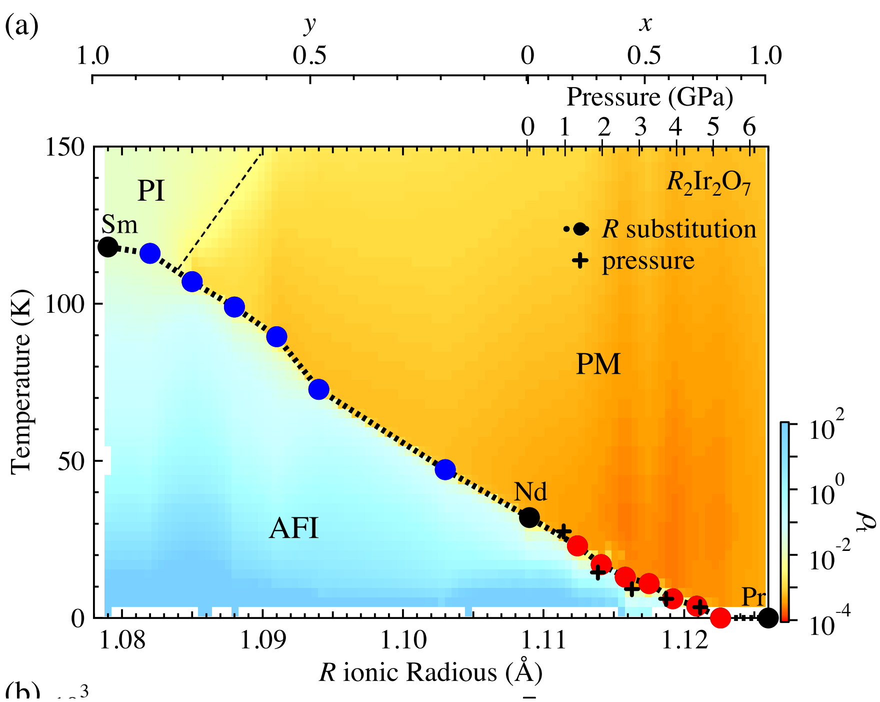


loss of quasiparticle peak

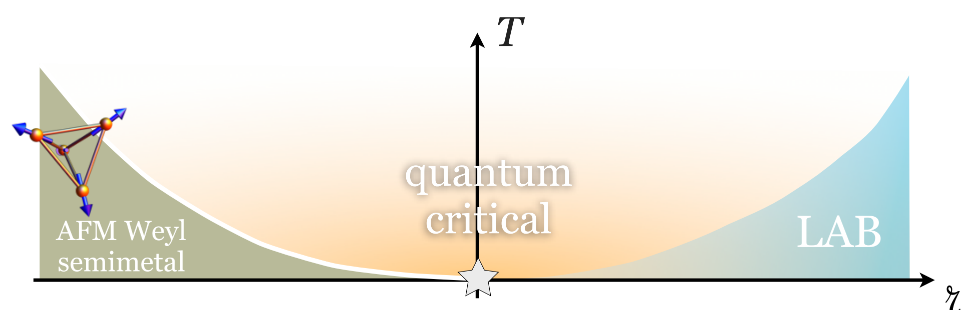
Rather than moving nodes, we observe loss of quasiparticle as gap opens

Prospects

It may be possible to weaken the order sufficiently to expose the Weyl points, and perhaps also explore quantum criticality



K. Ueda *et al*, 2015



L. Savary *et al*, 2014

nodal quantum criticality distinct from the Hertz problem

Three types of topology

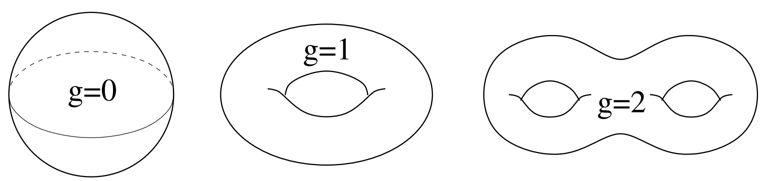
Topological Spin Liquid

topology of entanglement

"intrinsic topological order"

This type of topological phase can *only* exist with strong correlations. It reflects extreme entanglement of the many-body states

- Wen (1989): some many-body systems exhibit an "order" which is sensitive to the topology of the *spatial* manifold

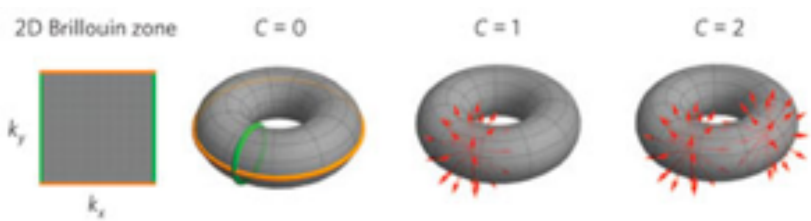


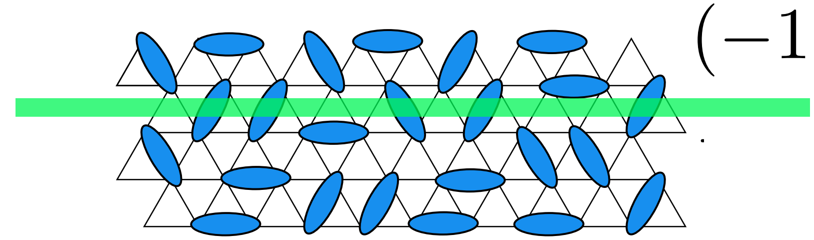
Deg.=1 Deg.=D_1 Deg.=D_2

- This type of order is *completely robust*: does not need any symmetry

TI versus iTO

Topological invariants: a non-local integral over an extended manifold





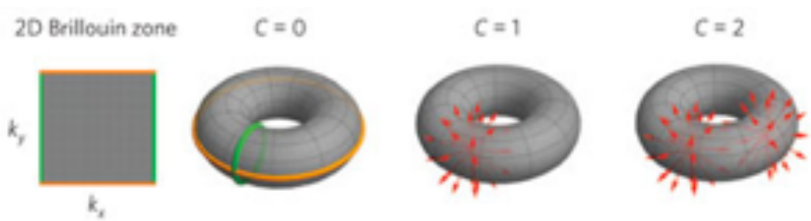
$$(-1)^{N_{\mathrm{vb}}}$$

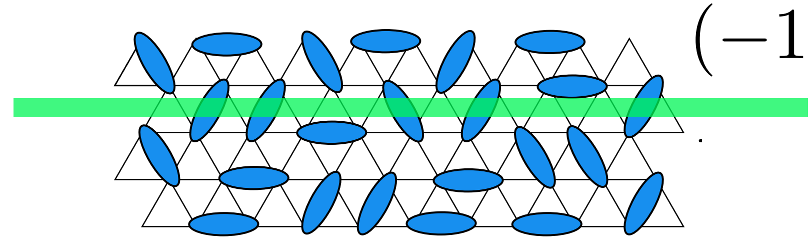
Chern number: integral over **2d k space** whose value differentiates **phases**

Wilson loop: integral over a **1d real space** curve whose value differentiates **states** in the same phase

TI versus iTO

Topological invariants: a non-local integral over an extended manifold





$$(-1)^{N_{\mathrm{vb}}}$$

Chern number: integral over **2d k space** whose value differentiates **phases**

Wilson loop: integral over a **1d real space** curve whose value differentiates **states** in the same phase

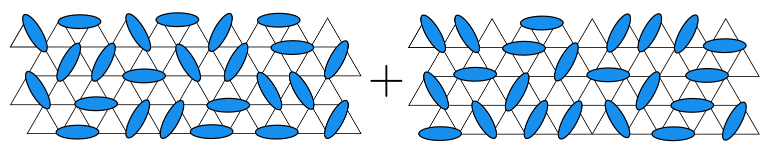
break the 2d space: forms a **gapless edge**

break the 1d curve: forms a **gapped exotic quasiparticle**

Where is iTO?

- Fractional quantum Hall effect is *both* an iTO state *and* a TI (Chern insulator)
- Other main candidates are *quantum spin liquids*

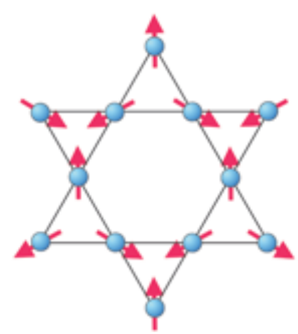
$$|\Psi\rangle =$$



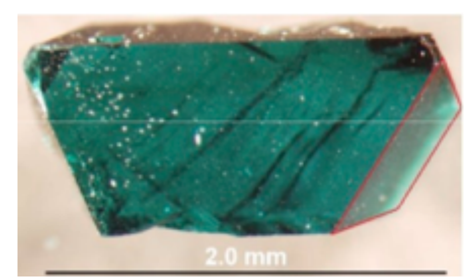
$+ \ldots$

"RVB" state on kagomé lattice?

still seeking definitive id



$ZnCu_3(OH)_6Cl_2$

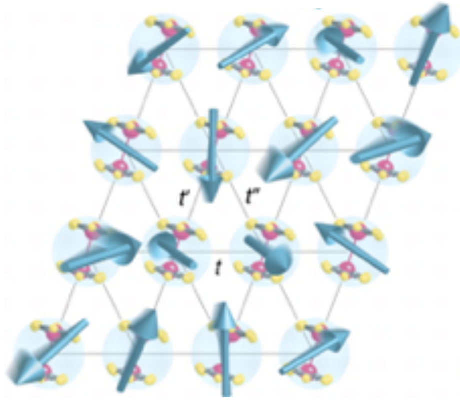


Young Lee, Takashi Imai,...

Spin liquid candidates?

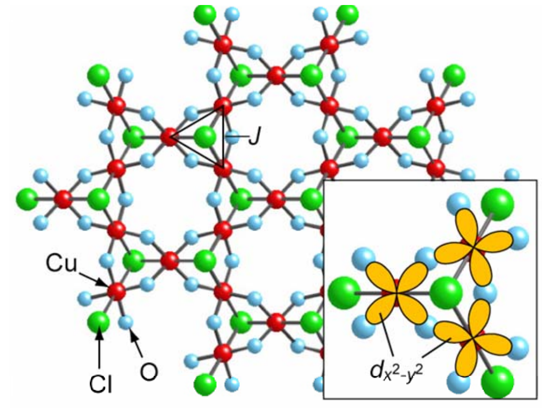
Top experimental platforms

K. Kanoda

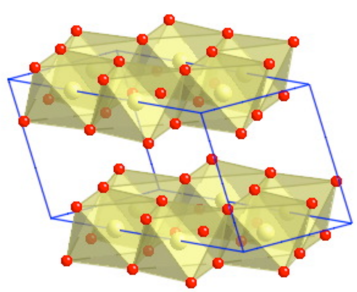


Organics

M. Fu



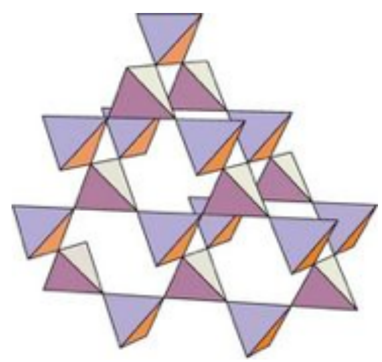
Herbertsmithite



Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3
α-$RuCl_3$

Y.-B. Kim

Kitaev materials



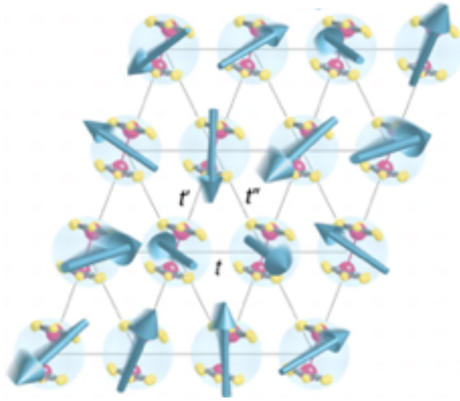
$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$
...

G. Chen
Y. Tokiwa

Quantum spin ice

Top experimental platforms

K. Kanoda



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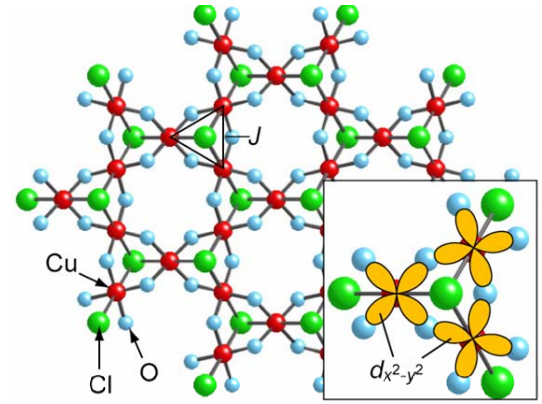

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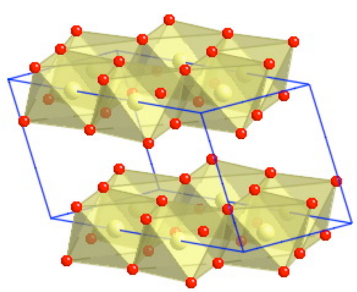

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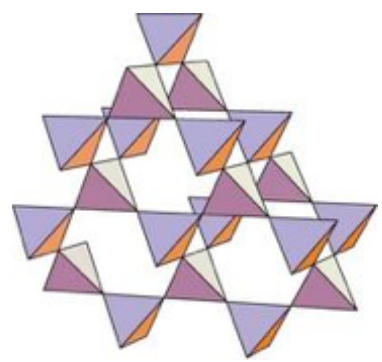


$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$
...

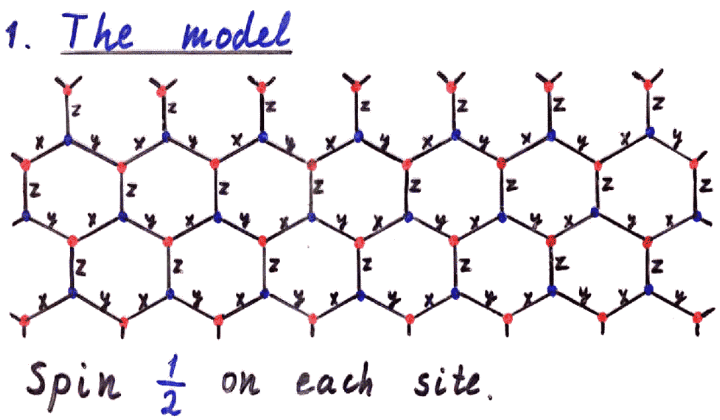
G. Chen
Y. Tokiwa

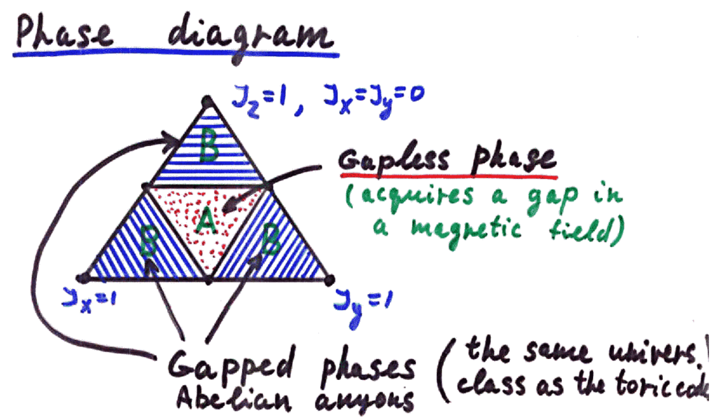
Quantum spin ice

Kitaev model

Kitaev's honeycomb model $H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$

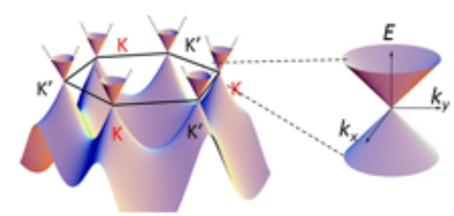
KITP, 2003



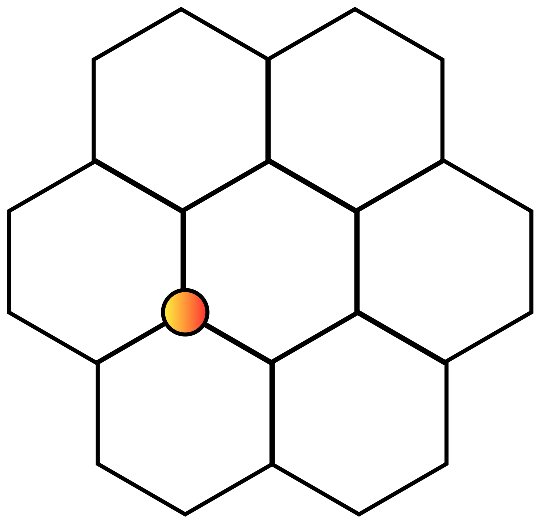


exact parton construction $\sigma_i^\mu = i c_i c_i^\mu \quad c_i c_i^x c_i^y c_i^z = 1$

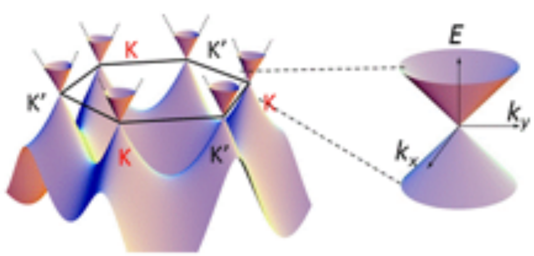
physical Majoranas $H_\mathrm{m} = K \sum_{\langle ij \rangle} i c_i c_j$



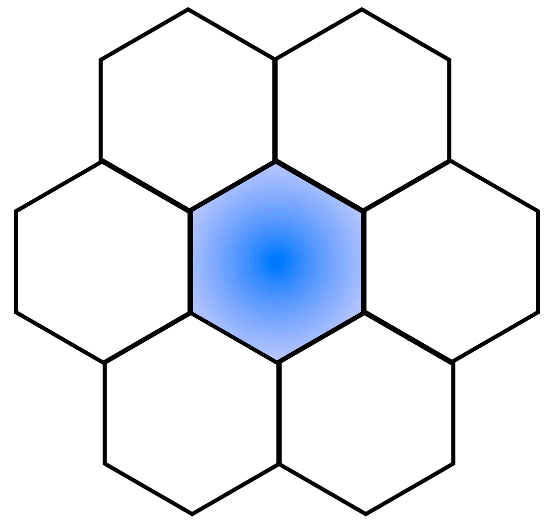
Non-local excitations



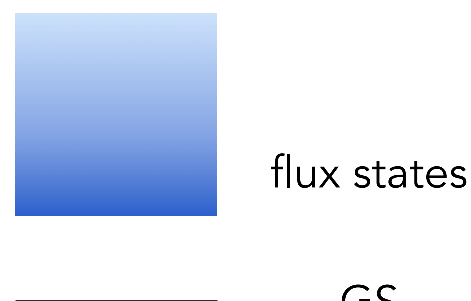
Majorana ε



gapless Dirac



Flux e, m

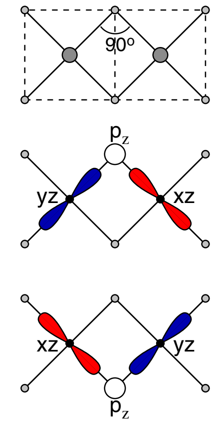


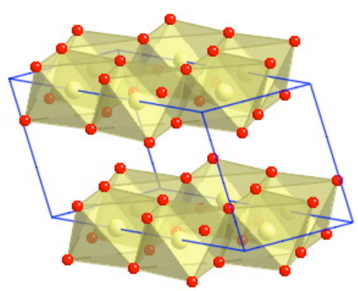
gapped

Kitaev Materials

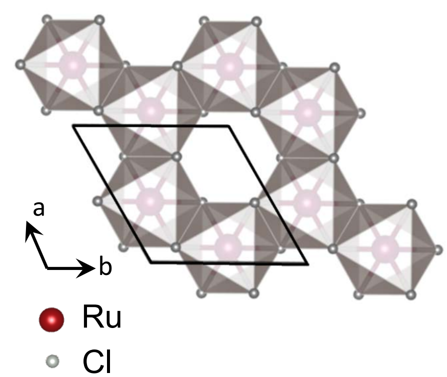
Jackeli, Khaliullin

Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling





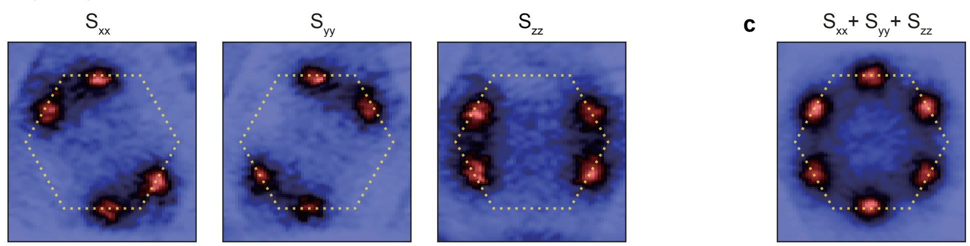
Na_2IrO_3, (α,β,γ)-Li_2IrO_3



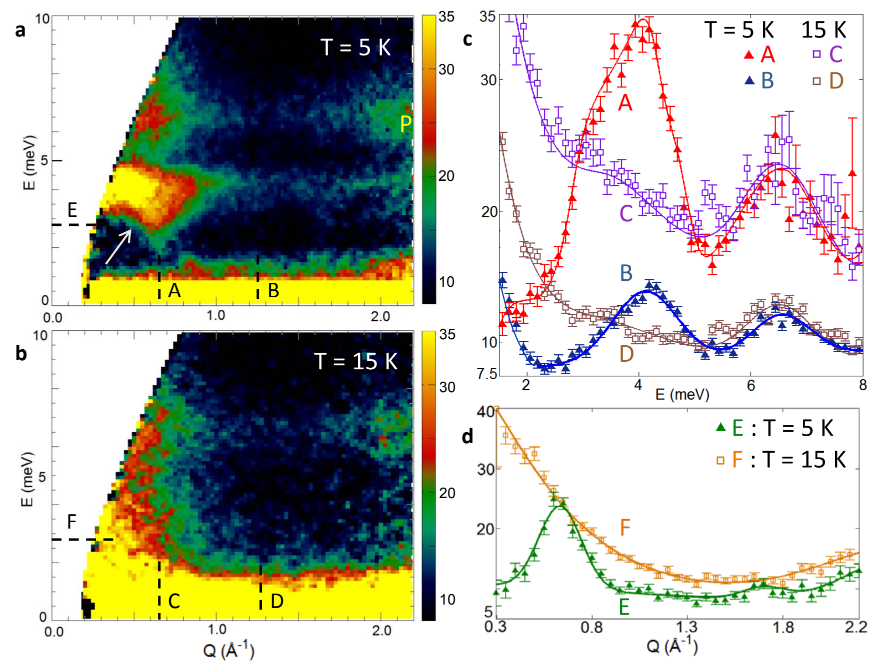
α-$RuCl_3$

Honeycomb and hyper-honeycomb structures

Kitaev Materials



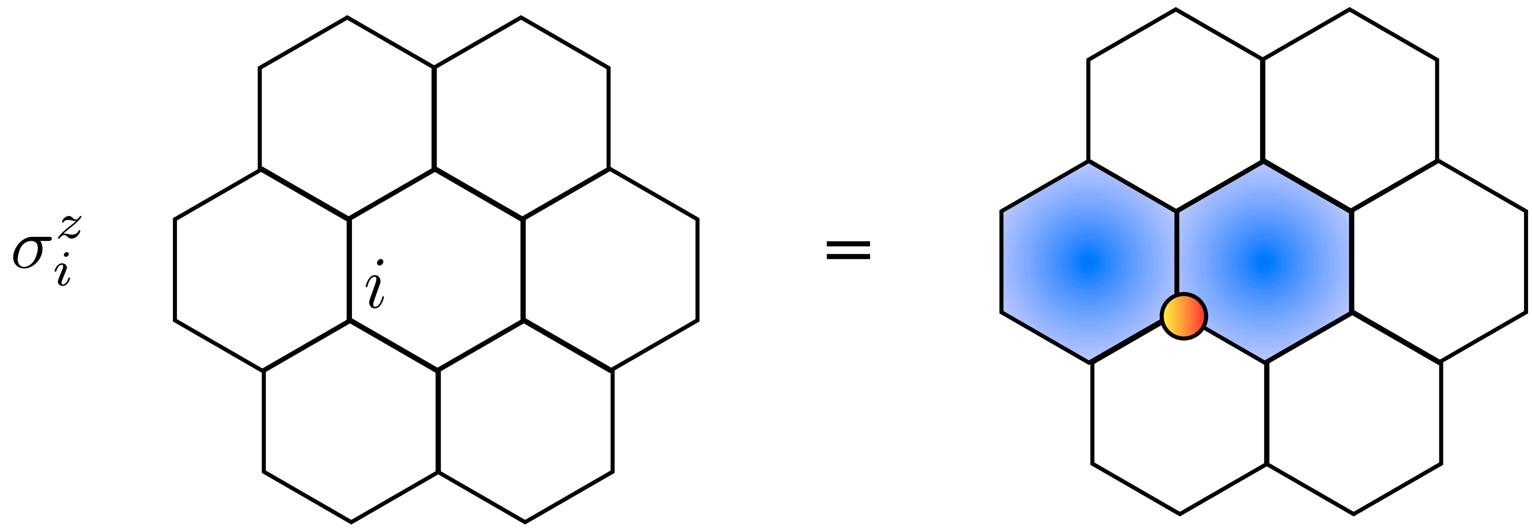
direct evidence for direction-dependent anisotropic exchange from diffuse magnetic x-ray scattering in Na_2IrO_3 (BJ Kim group)



there is pretty strong evidence of substantial Kitaev exchange in quite a few materials

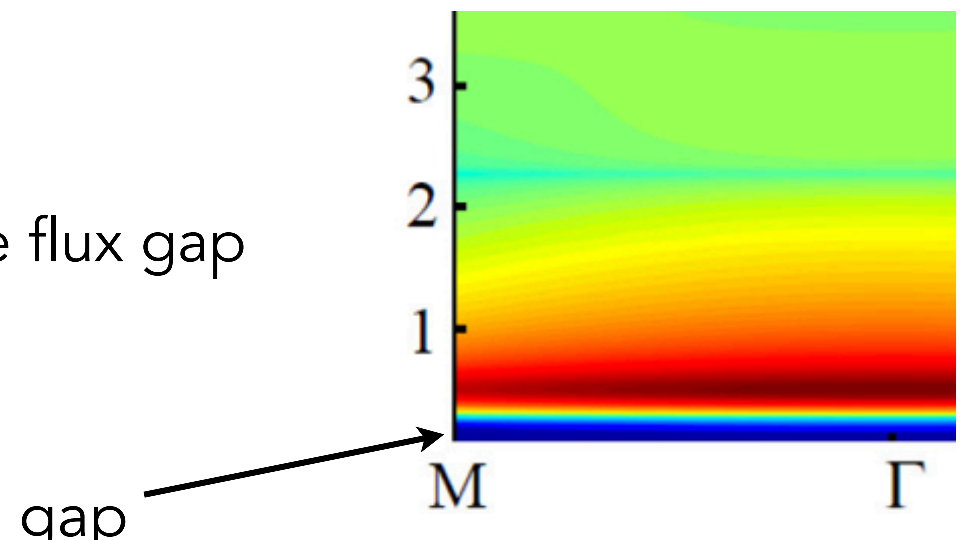
Observation of gapped continuum mode persisting above T_N in α-$RuCl_3$ consistent with Majoranas (A. Banerjee *et al*)

Exact spin correlations

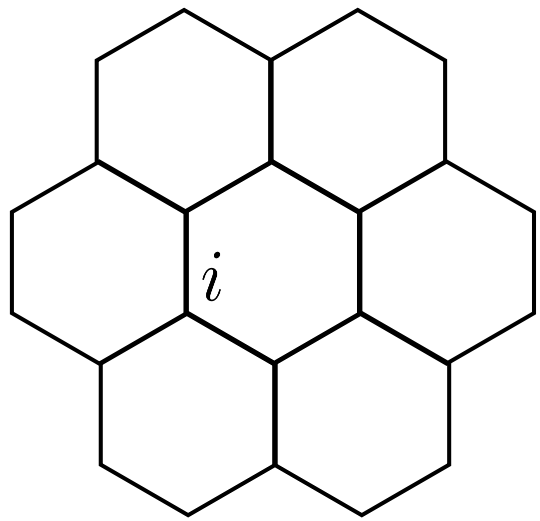
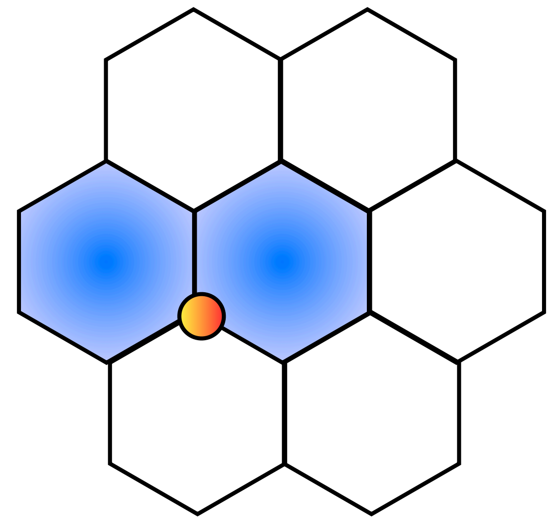


In the soluble model:

- The spin creates two fluxes
- Spectral weight is zero below the flux gap
- Correlations vanish beyond NNs



Exact spin correlations

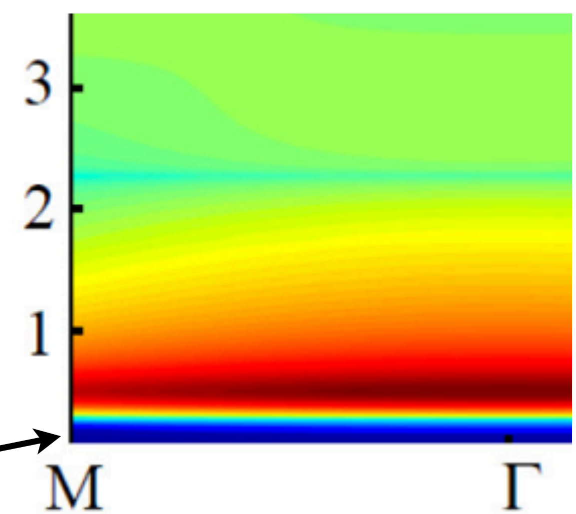
σ_i^z  $=$ 

In the soluble model:

- The spin creates two fluxes
- Spectra … gap
- Correla…

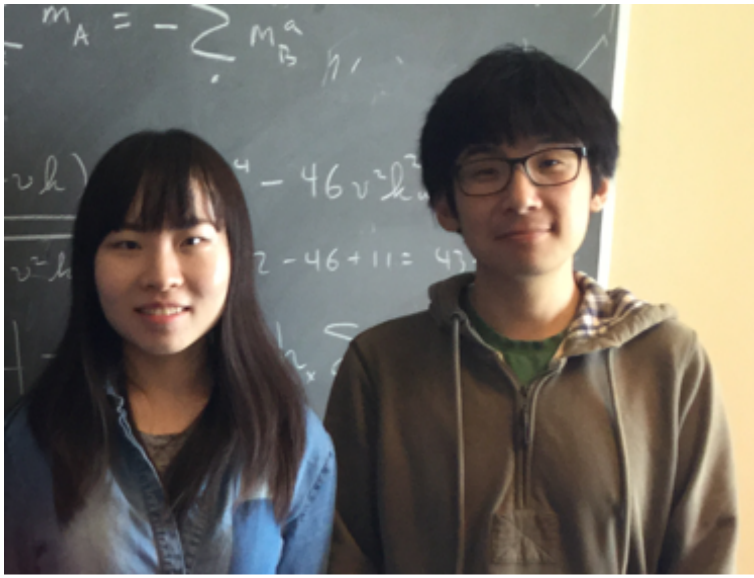
very boring

But fortunately it is not physical



J. Knolle *et al*, 2014

Inexact but correct (universal) answer



宋雪洋
Xue-Yang Song

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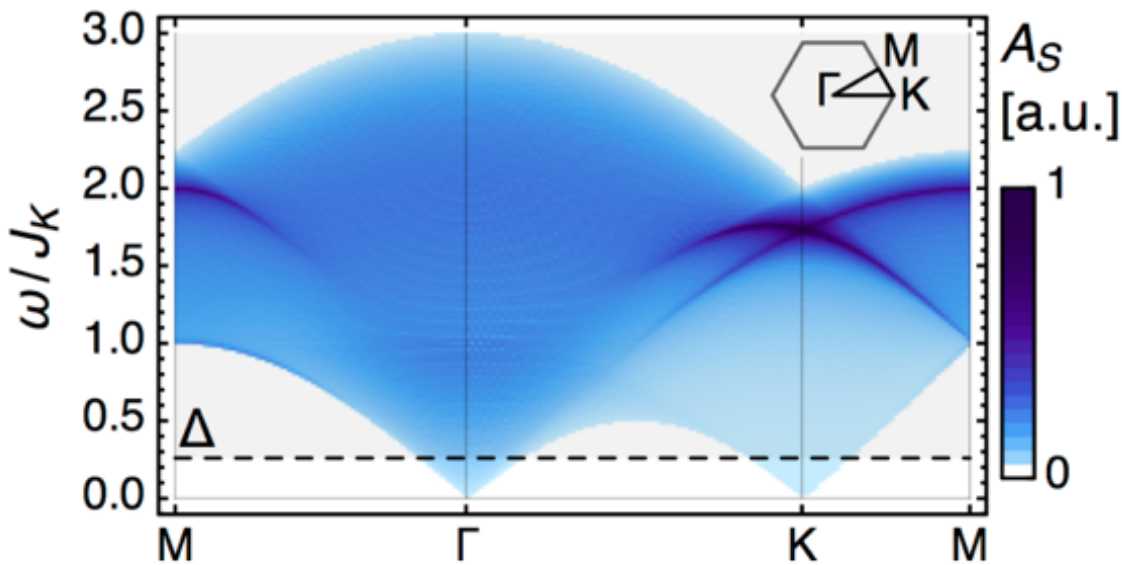
尤亦庄
Yi-Zhuang You

postdoc
UCSB

Generically: spin correlations are *gapless* and structured

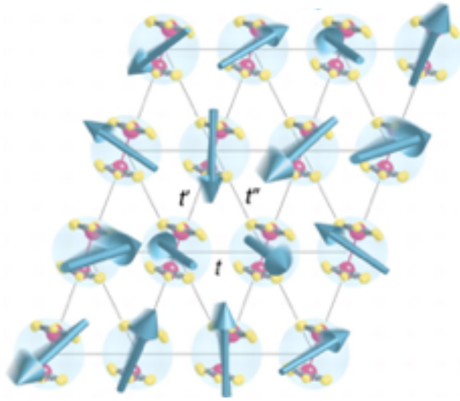
(gapless contribution should be added to the other one, which is like the "incoherent" part in a Fermi liquid)



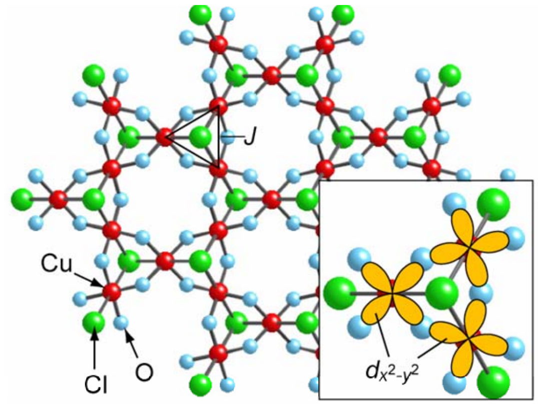
Top experimental platforms

K. Kanoda

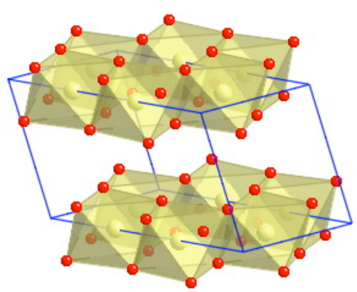
M. Fu



Organics



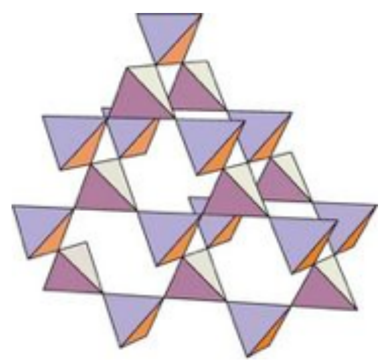
Herbertsmithite



Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3
α-$RuCl_3$

Y.-B. Kim

Kitaev materials



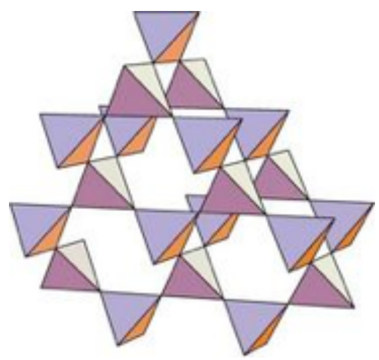
$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$
…

G. Chen
Y. Tokiwa

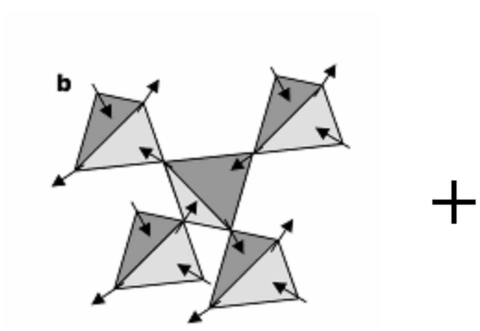
Quantum spin ice

Quantum spin ice

- Quantum S=1/2 spins on pyrochlore lattice



$$H = J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z + H_Q$$



+ Quantum fluctuations

2in-2out states

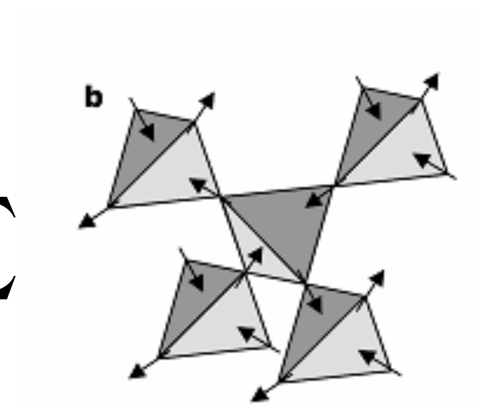
Quantum spin ice

- For non-Kramers ions, e.g. Pr^{3+}

$$H_Q = \sum_{\langle ij \rangle} \underbrace{\left[-J_{\pm} S_i^+ S_j^- + \text{h.c.} - J_{\pm\pm}\gamma_{ij} S_i^+ S_j^+ + \text{h.c.}\right]}_{\text{intrinsic exchange}} - \sum_i \underbrace{\left[h_i S_i^+ + \text{h.c.}\right]}_{\text{extrinsic crystal fields}}$$

With a lot more work one can show that *both* types of quantum terms favor a massive superposition: a quantum spin liquid state

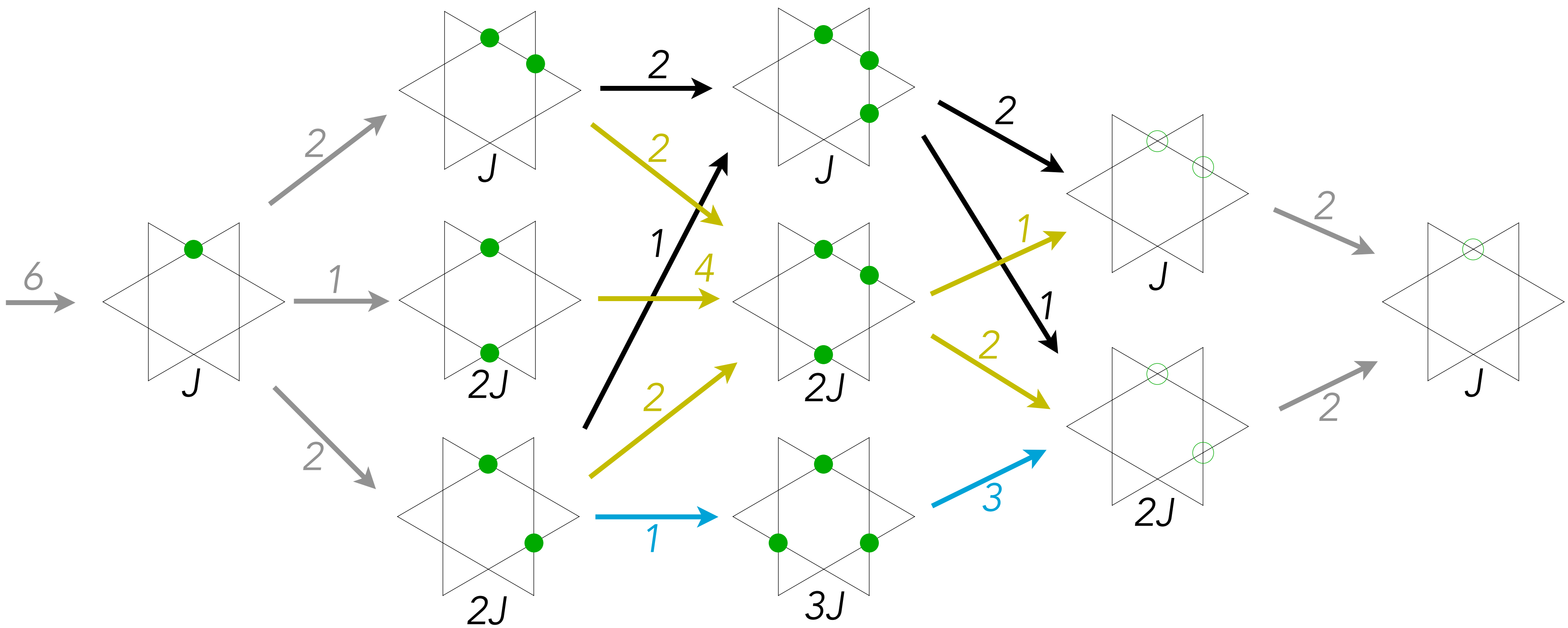
$$|\Psi\rangle = \sum$$



M. Hermele, MPA Fisher, L.B., 2004; A. Banerjee *et al*, 2008; L. Savary + LB, 2012; SB Lee *et al*, 2012...

Perturbation theory

e.g. random crystal fields h_i



Leading result of disorder is to induce quantum "ring exchange"

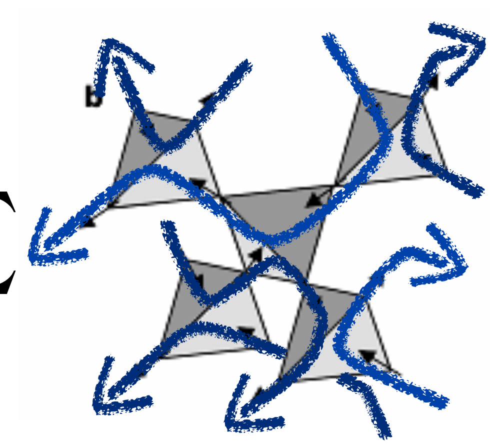
"dynamics from disorder"

Quantum spin ice

- For non-Kramers ions, e.g. Pr^{3+}

$$H_Q = \sum_{\langle ij \rangle} \underbrace{\left[-J_{\pm} S_i^+ S_j^- + \text{h.c.} - J_{\pm\pm} \gamma_{ij} S_i^+ S_j^+ + \text{h.c.} \right]}_{\text{intrinsic exchange}} - \sum_i \underbrace{\left[h_i S_i^+ + \text{h.c.} \right]}_{\text{extrinsic crystal fields}}$$

U(1) QSL: fluctuating "field lines" and emergent electromagnetism

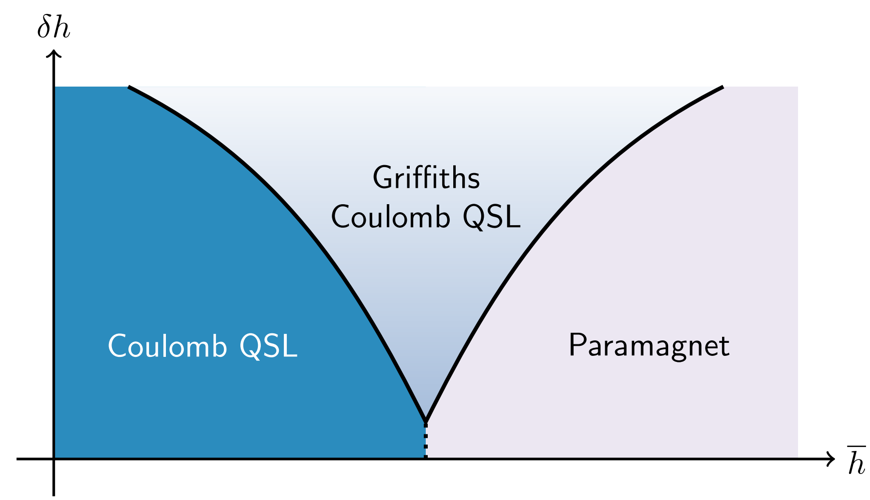
$$|\Psi\rangle = \sum$$ 

M. Hermele, MPA Fisher, L.B., 2004; A. Banerjee *et al*, 2008; L. Savary + LB, 2012; SB Lee *et al*, 2012...

Quantum spin ice

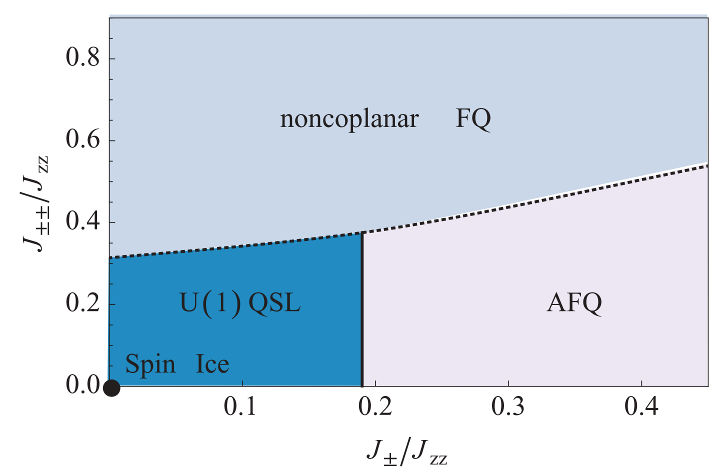
a surprise: in the non-Kramers case, **disorder alone** can generate the QSL

L. Savary + LB, arXiv:1604.04630

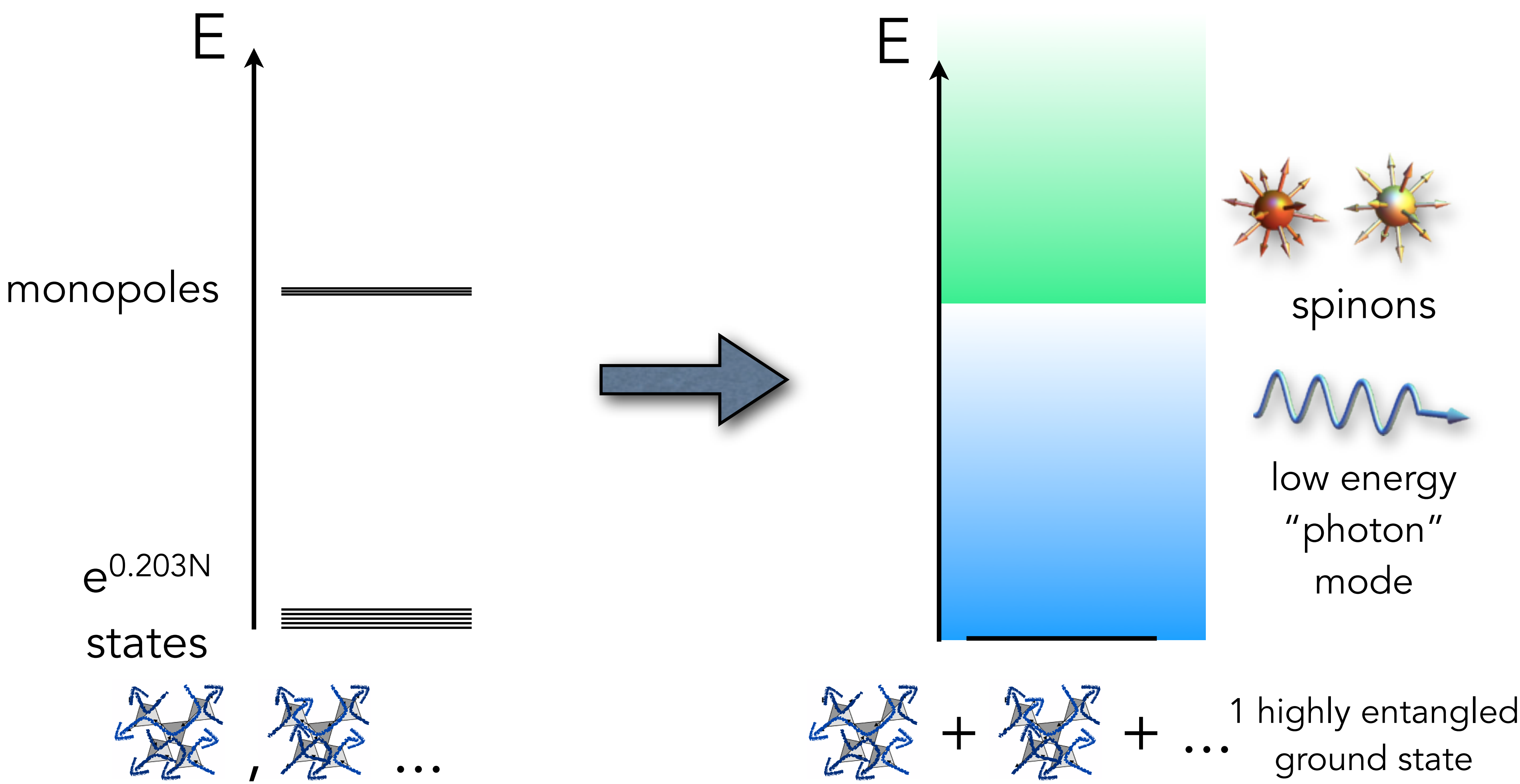


in general, we expect it to assist the intrinsic quantum exchange terms.

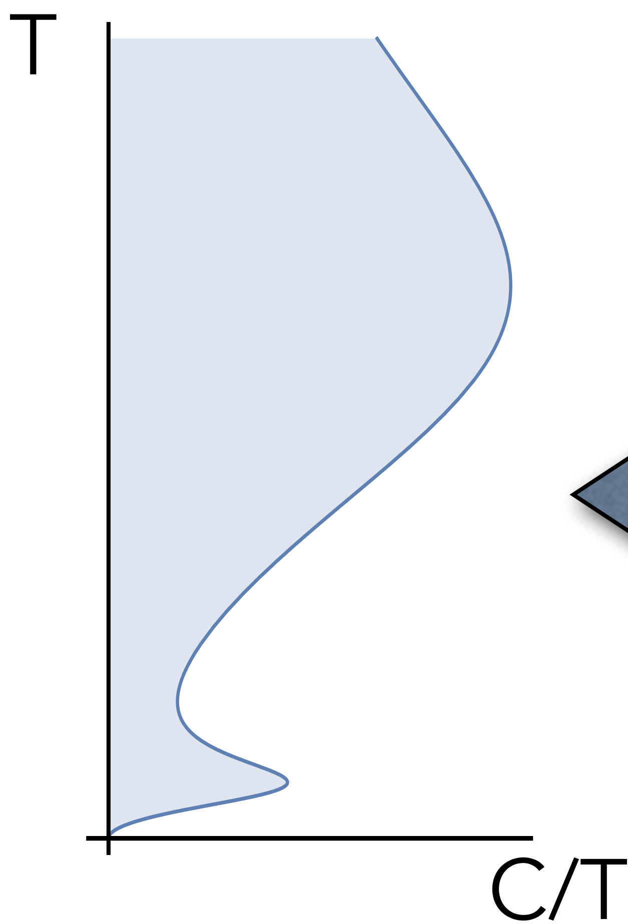
SB Lee *et al*, 2012

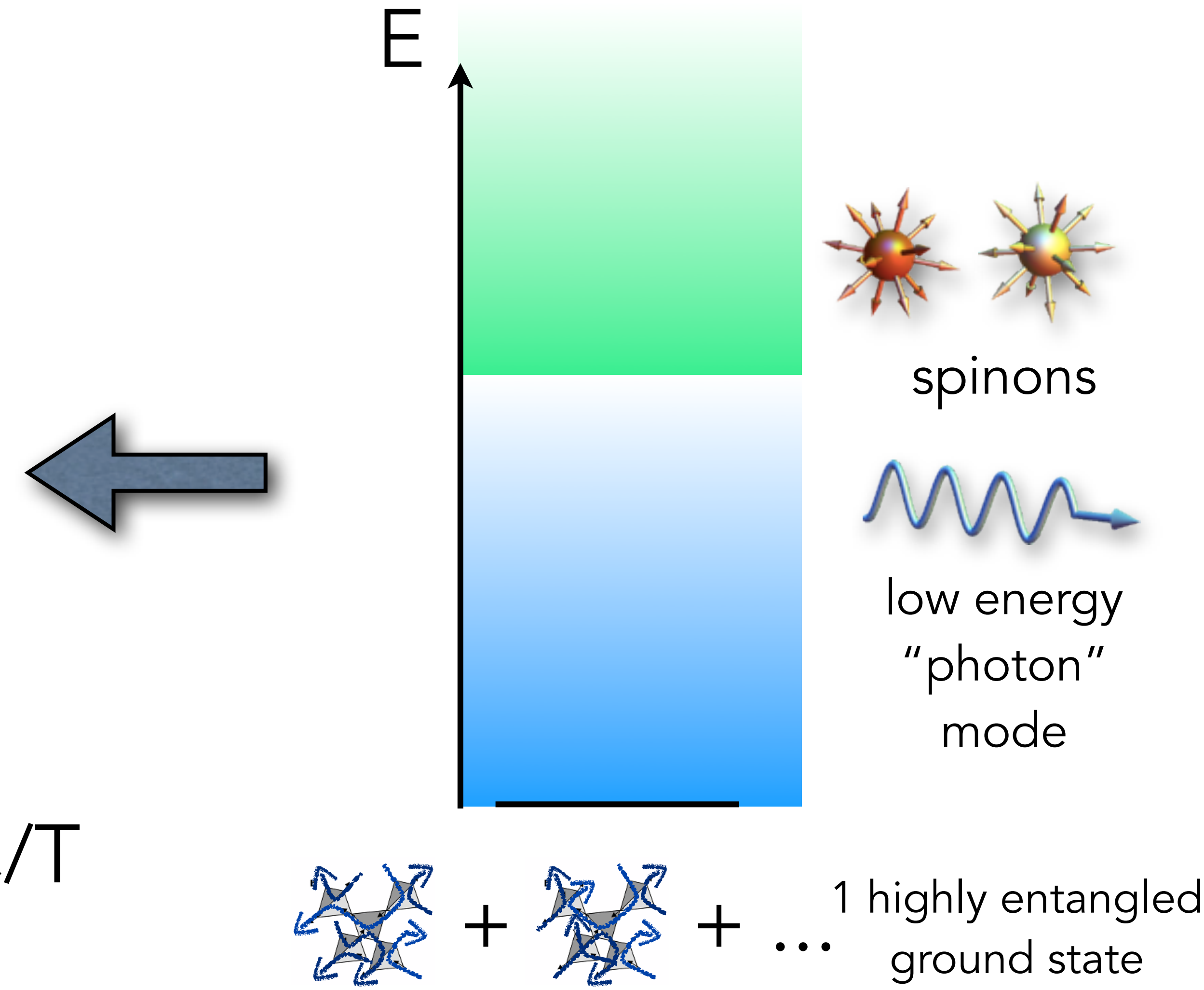


Quantum spin ice



Quantum spin ice





c.f. Y. Tokiwa We-S12-5

Summary

- Together, topology and SCES can help to achieve enlightenment.