

An abstract, pixelated pattern of various colors including yellow, light blue, light green, and light purple, creating a textured, mosaic-like background.

Topological order and interplay with real space textures

Leon Balents, KITP

A wide-angle photograph of a tropical beach. In the foreground, there's a rocky shoreline with dark, wet stones. The water is a deep blue-grey color with gentle ripples. In the background, a large, green, forested island with a prominent peak rises from the sea. The sky is bright blue with large, white, fluffy clouds. On the right side, there are palm trees and some greenery on the shore.

EPIQS-TMS Trans-Pacific Conference on Topological Quantum Materials, Moorea, Dec. 2016

Collaborators



Jianpeng Liu

Interplay of band topology and topological magnetic textures in an antiferromagnet



Chaoming Jian

Intrinsic topological order and local measurements



Lucile Savary

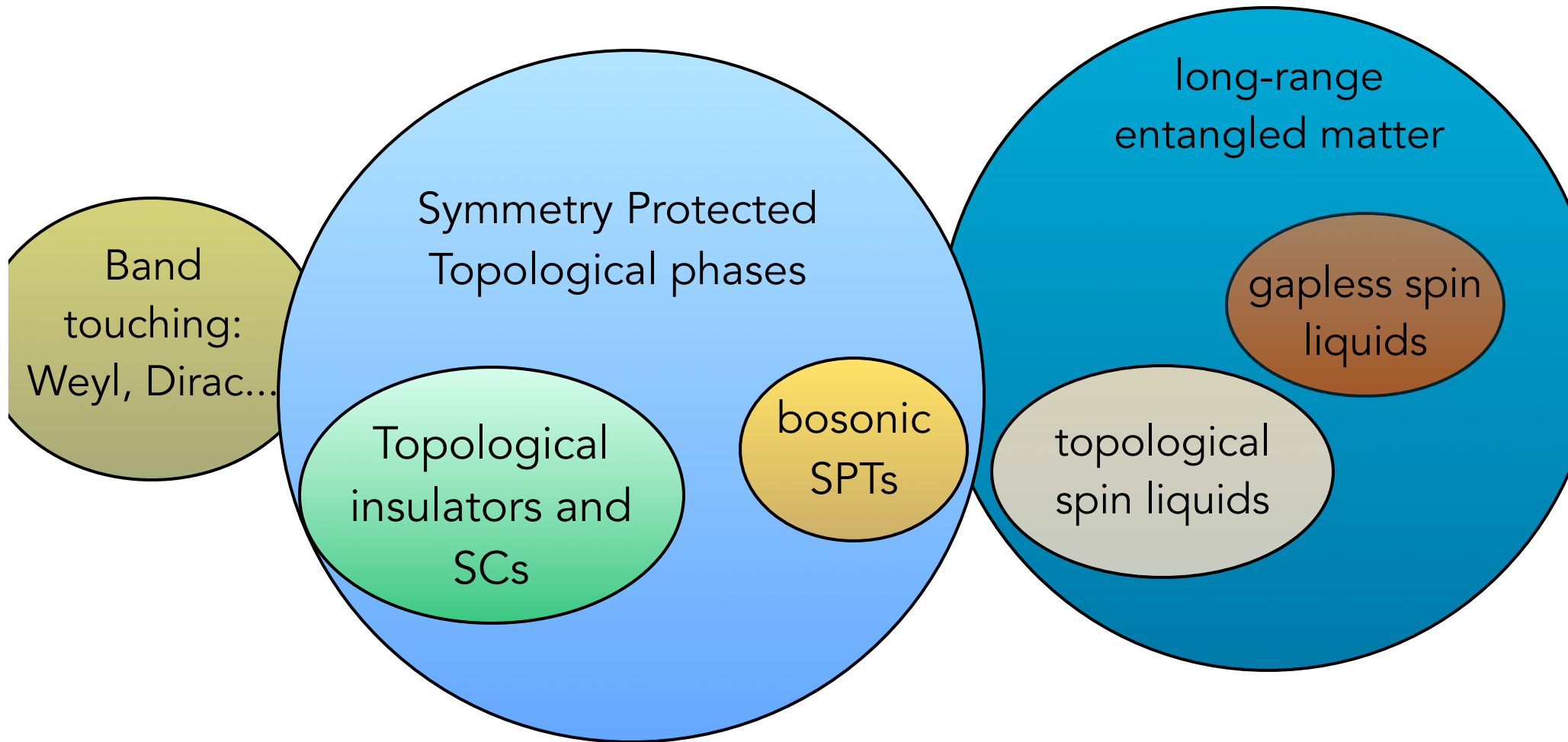
A large, light-brown, textured donut-shaped sign is mounted on a metal pole against a clear blue sky. The sign has a large circular hole in the center. The words "RANDY'S" are written in a bold, black, sans-serif font with a white outline, curving along the top inner edge of the donut. The word "DONUTS" is written in the same style, curving along the bottom inner edge. In the center of the hole, the text "Topology is big" is written in a black, sans-serif font.

RANDY'S

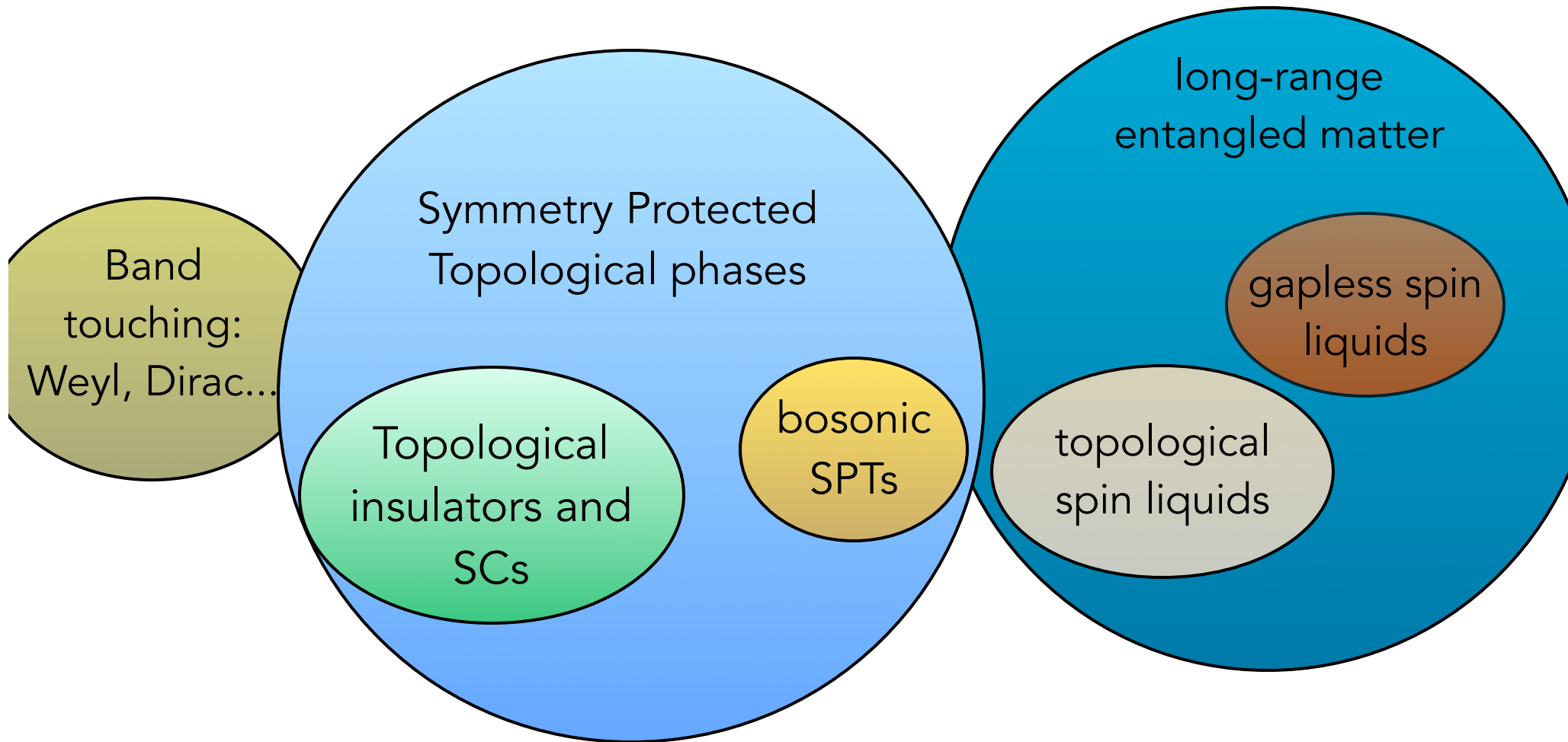
Topology is big

DONUTS

Topology++

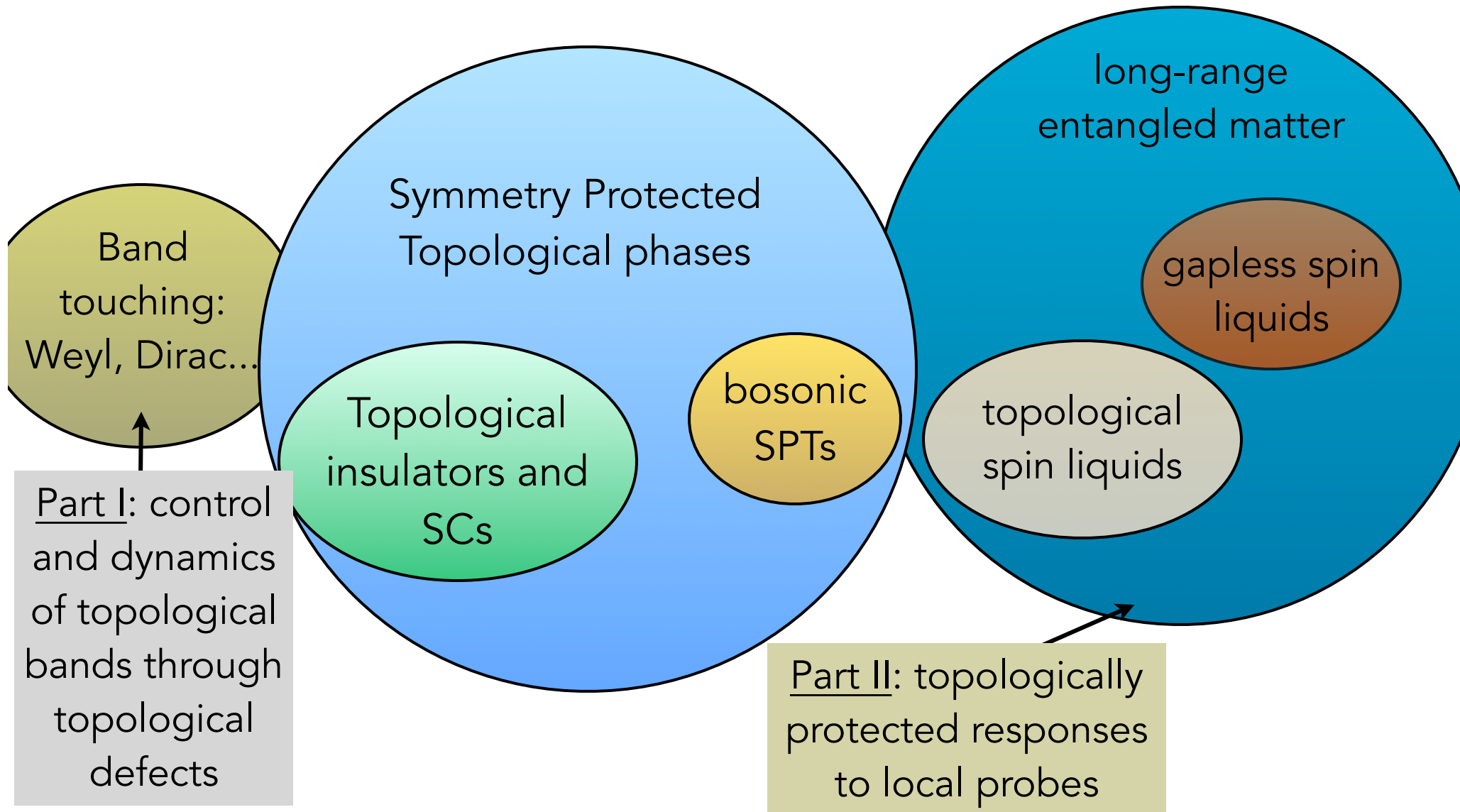


Topology++



This talk: theoretical speculations on new experimental prospects

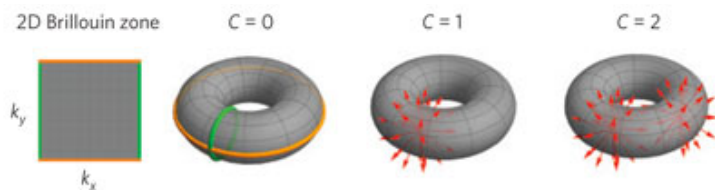
Topology++



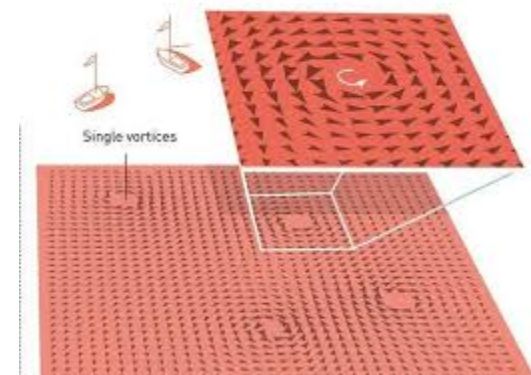


Thouless:
Chern number

$$q_n = \frac{1}{2\pi} \int d^2k \mathcal{B}_n^z$$



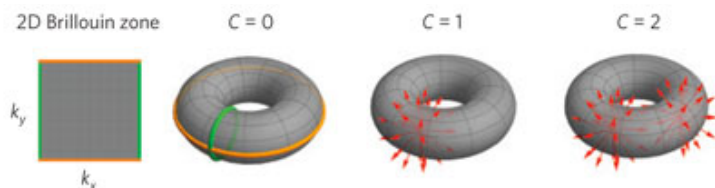
Kosterlitz+ Thouless:
Vortices



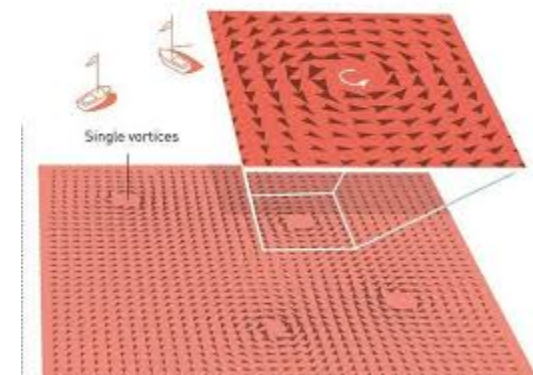


Thouless:
Chern number

$$q_n = \frac{1}{2\pi} \int d^2k \mathcal{B}_n^z$$

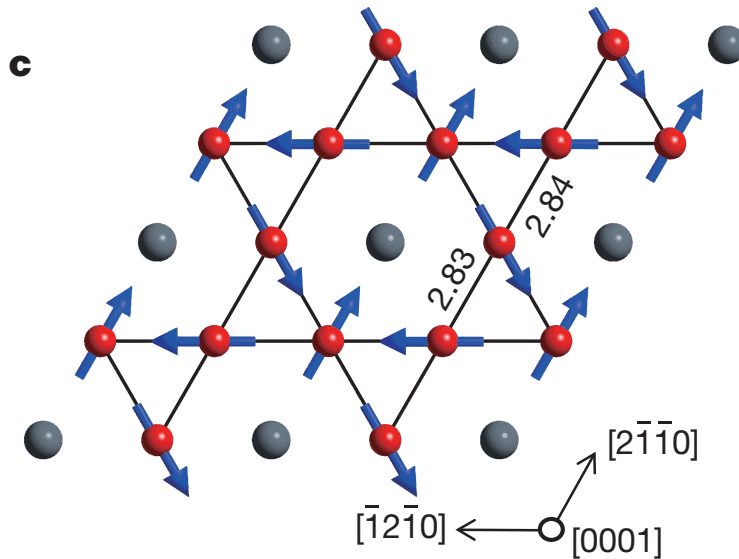


Kosterlitz+ Thouless:
Vortices



To control the interplay, we want a soft magnet

Mn₃Sn family



two kagomé layers of
Mn, related by inversion

large ordered
antiferromagnetic
moment

$$\sim 3 \mu_B / \text{Mn}$$

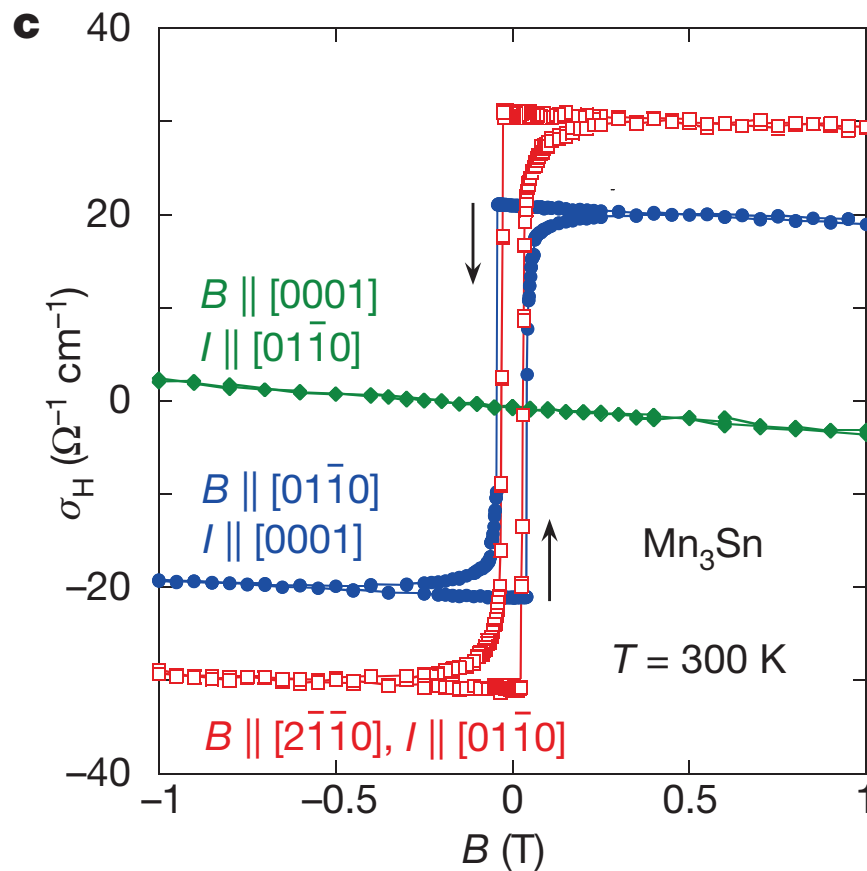
tiny canting moment:

$$.002 \mu_B / \text{Mn}$$

$$T_N \sim 420\text{K}$$

Nagamiya et al, 1982

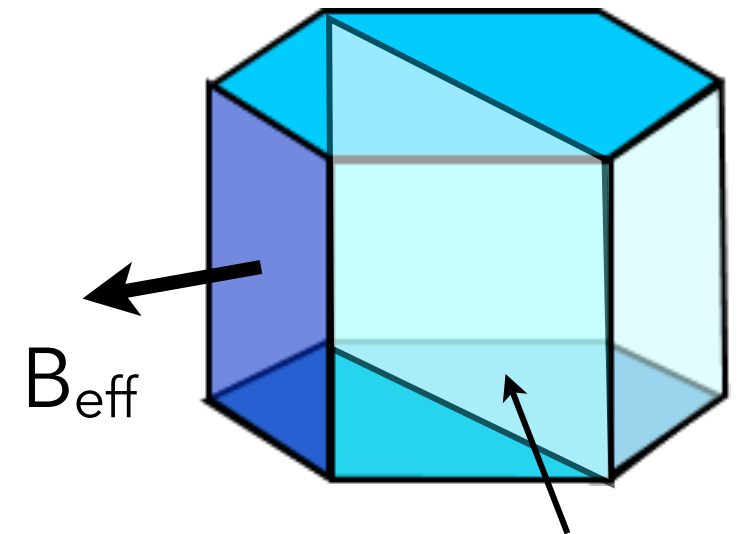
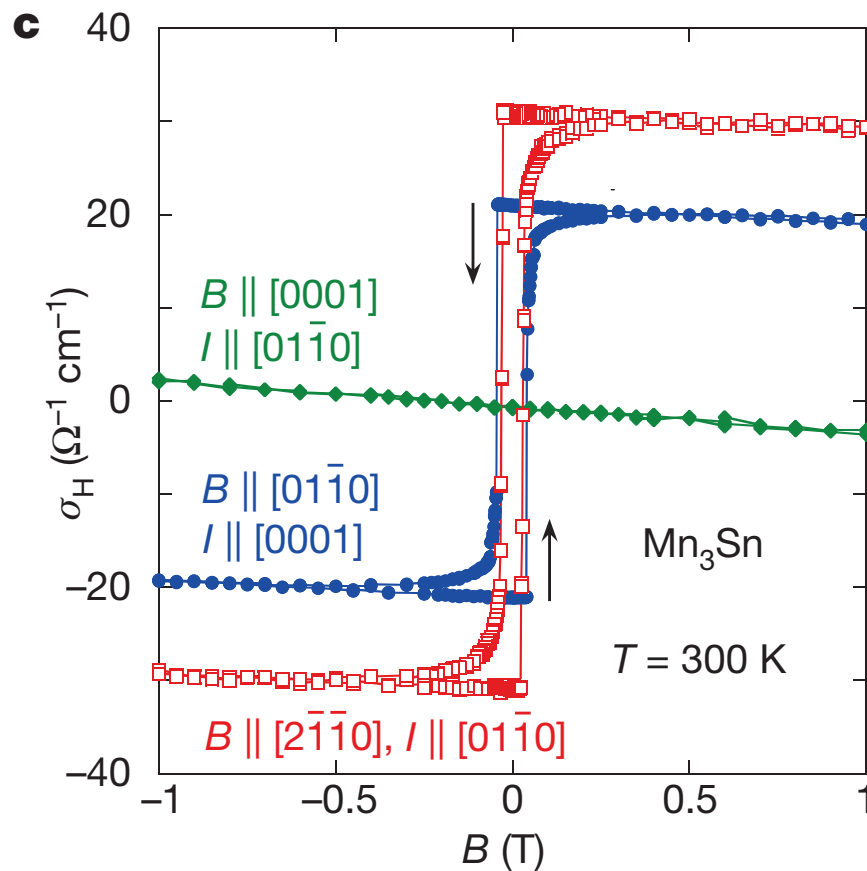
anomalous Hall effect



comparable to
metallic FMs

switchable because
of small magnetic
moment and small
anisotropy

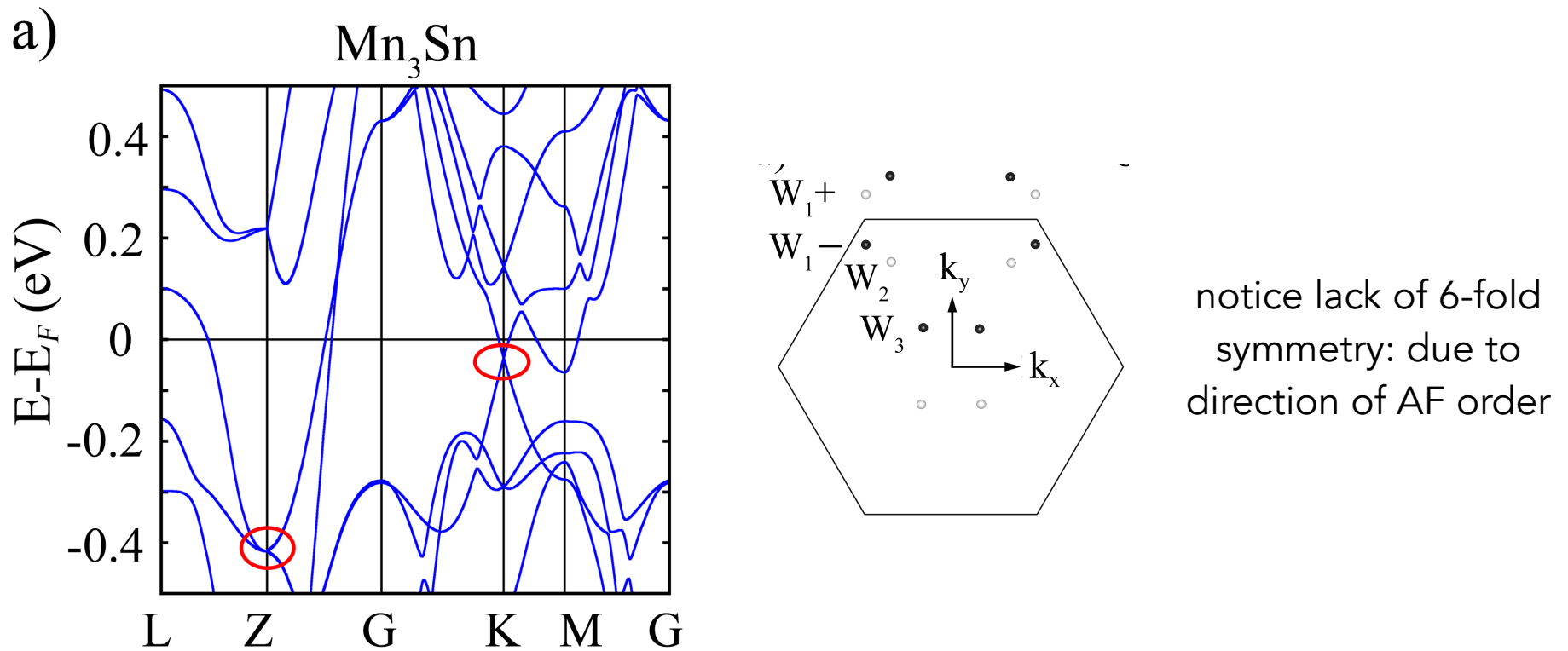
anomalous Hall effect



Hall effect in
vertical plane

Weyl

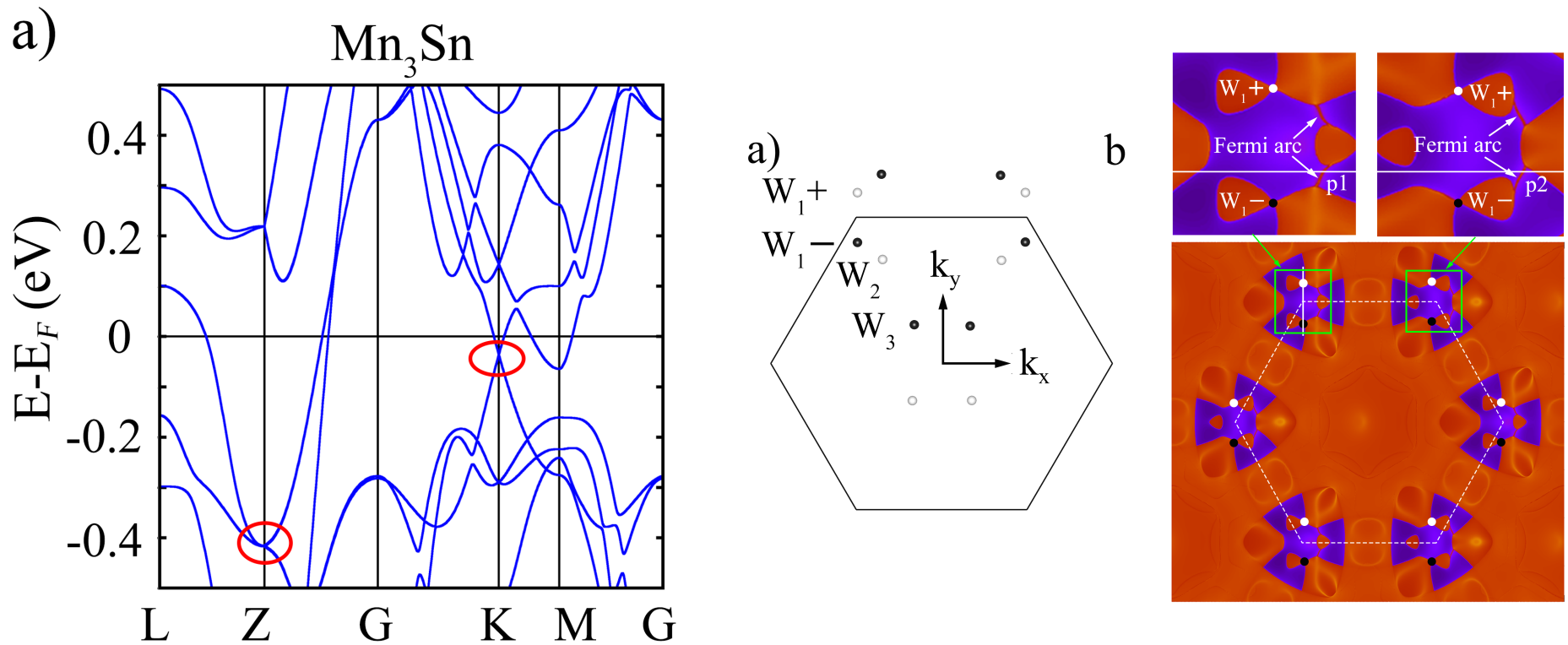
ab initio finds Weyl points and surface Fermi arcs



Hao Yang et al, arXiv:1608.03404

Weyl

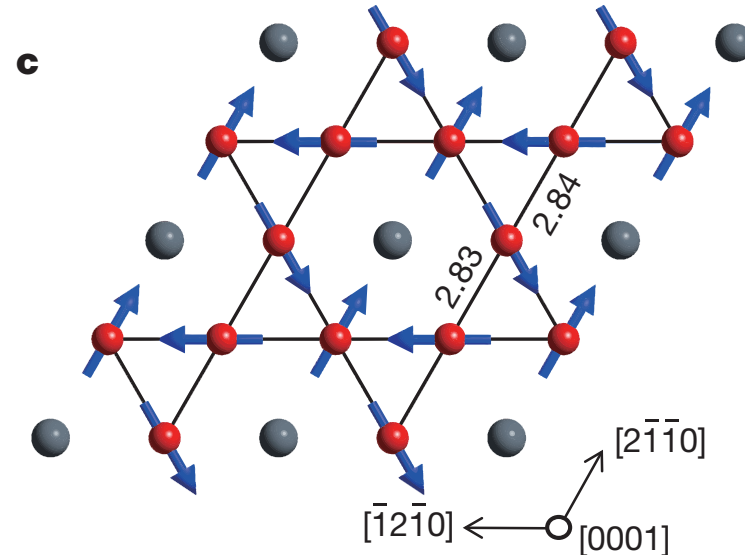
ab initio finds Weyl points and surface Fermi arcs



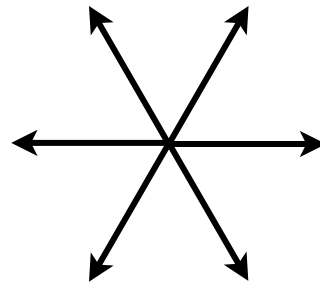
Hao Yang et al, arXiv:1608.03404

Textures

Magnetic order has Z_6 structure



direction of
inward-pointing
spin



$$\psi = |\psi| e^{2\pi i n / 6}$$

3 pairs of time-reversed domains

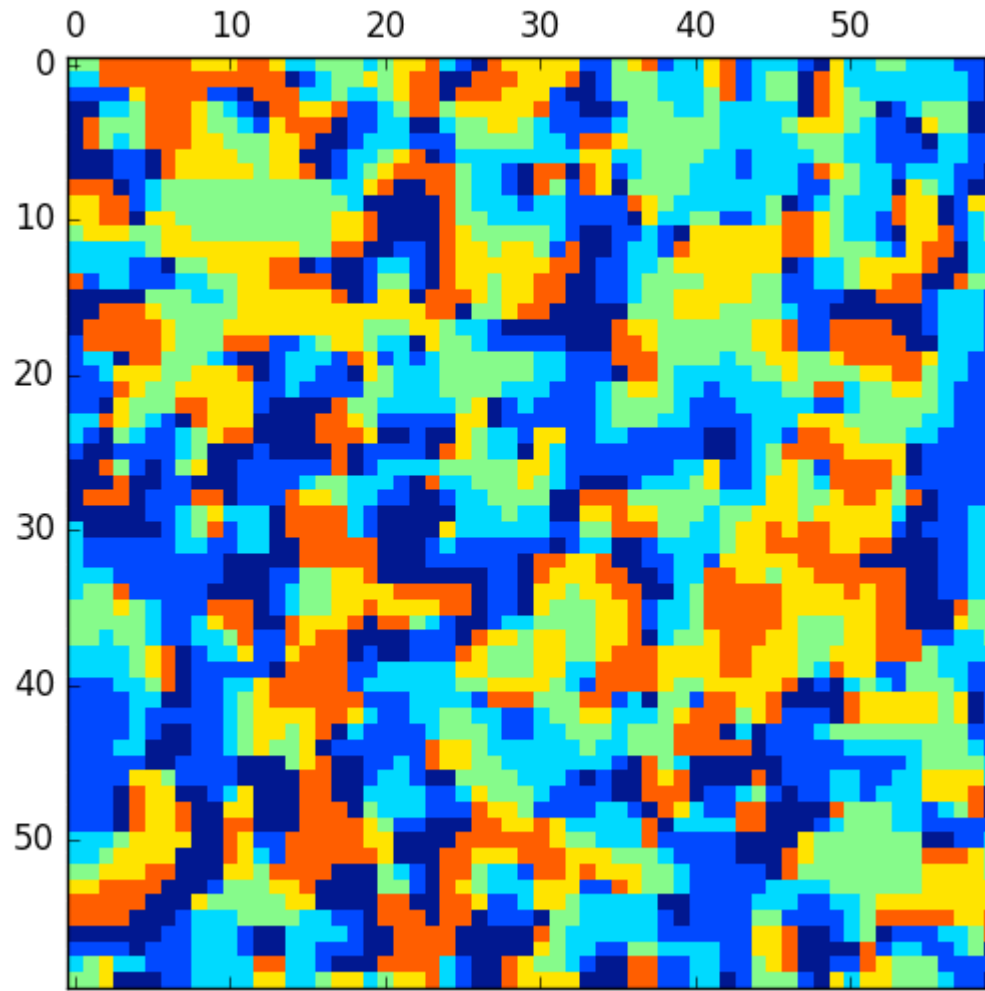
Textures

$$\psi = |\psi|e^{i\theta} \quad F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla \theta)^2 - \lambda \cos 6\theta \right\}$$

sine-Gordon model with 6-fold anisotropy

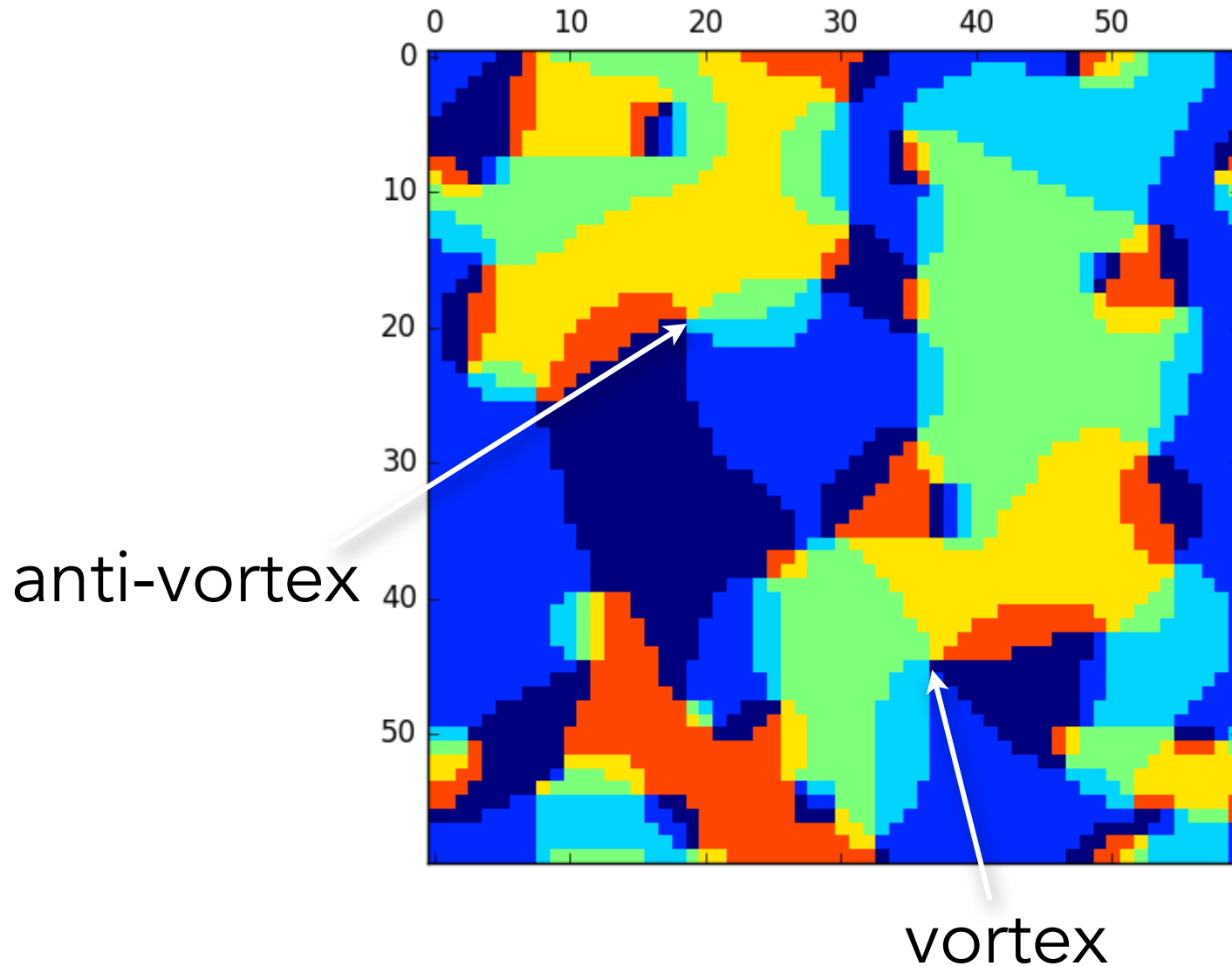
- Minimal energy domain walls are *not* between time-reversed states
- Magnetization, Hall vector, location of Weyl points are all determined by domain choice, not by field in general
- Stable Z_6 vortices exist

Domain formation



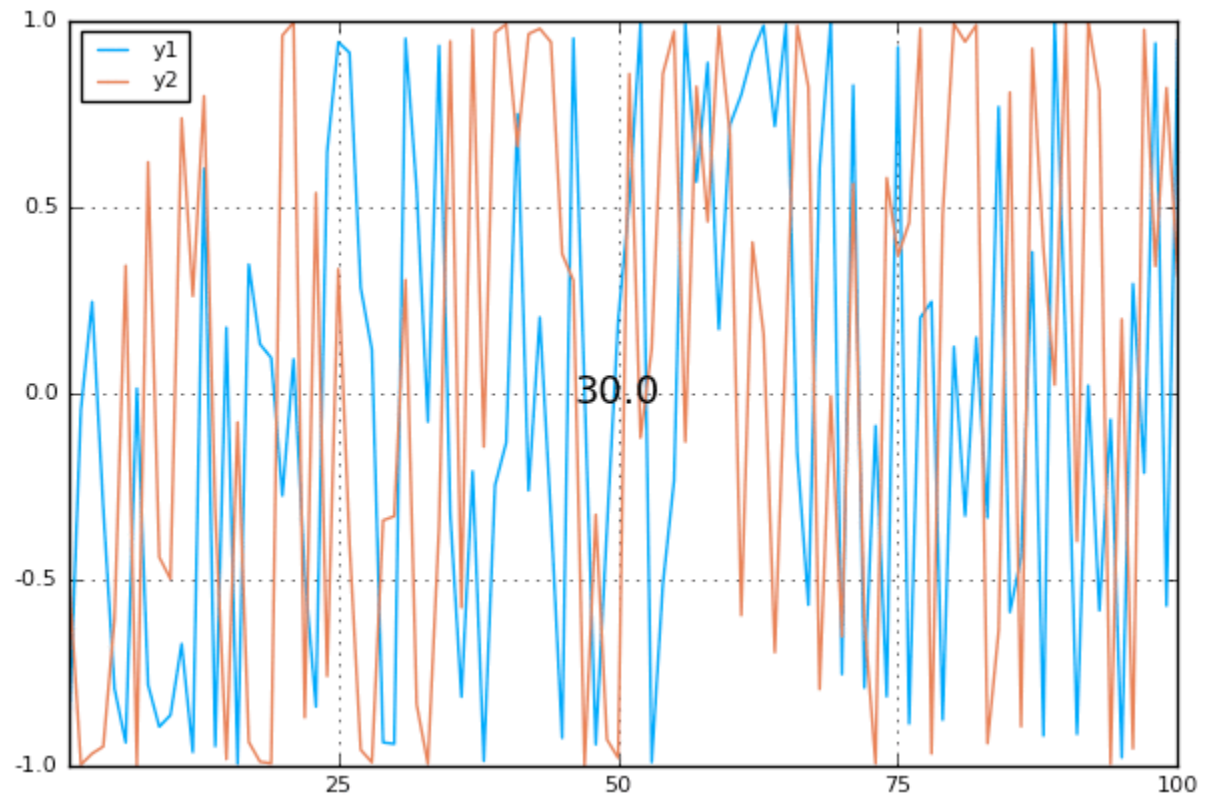
quench
(model A dynamics)

Domain formation

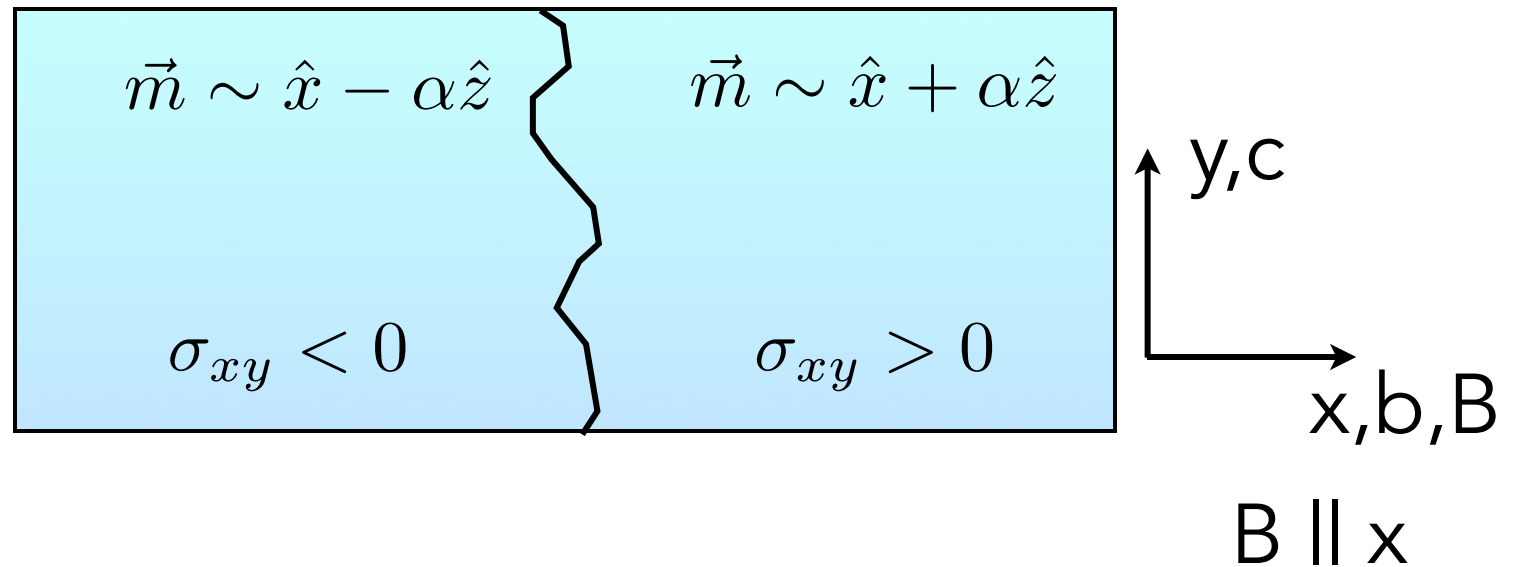


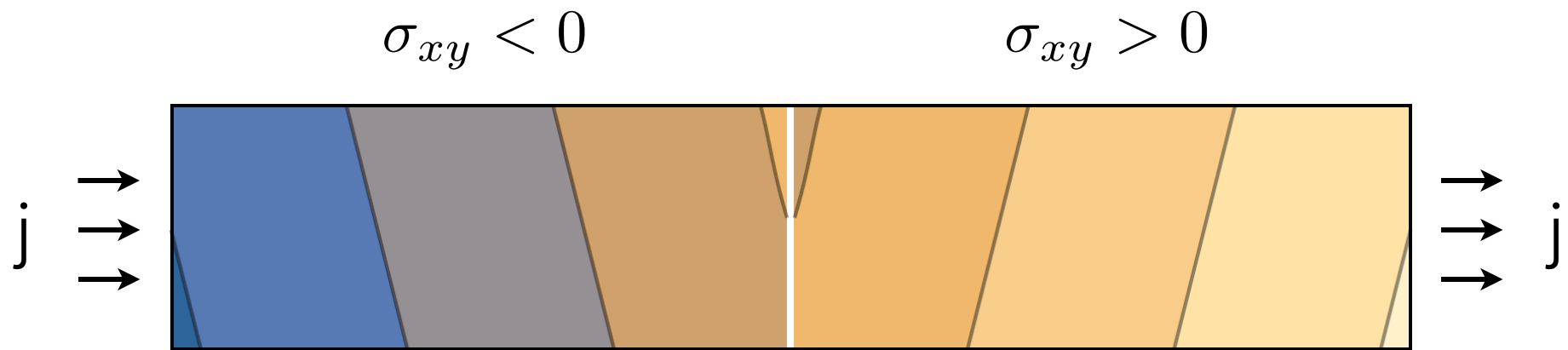
controllable
domain walls
in narrow
Hall bar

(triangle wave of $B_z(t)$)

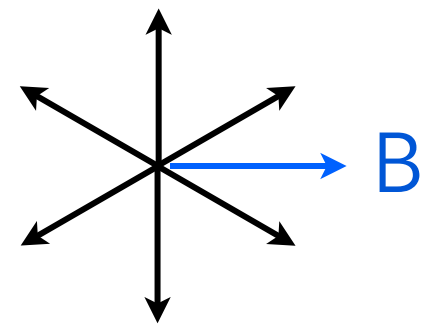


$\cos \theta$
 $\sin \theta$



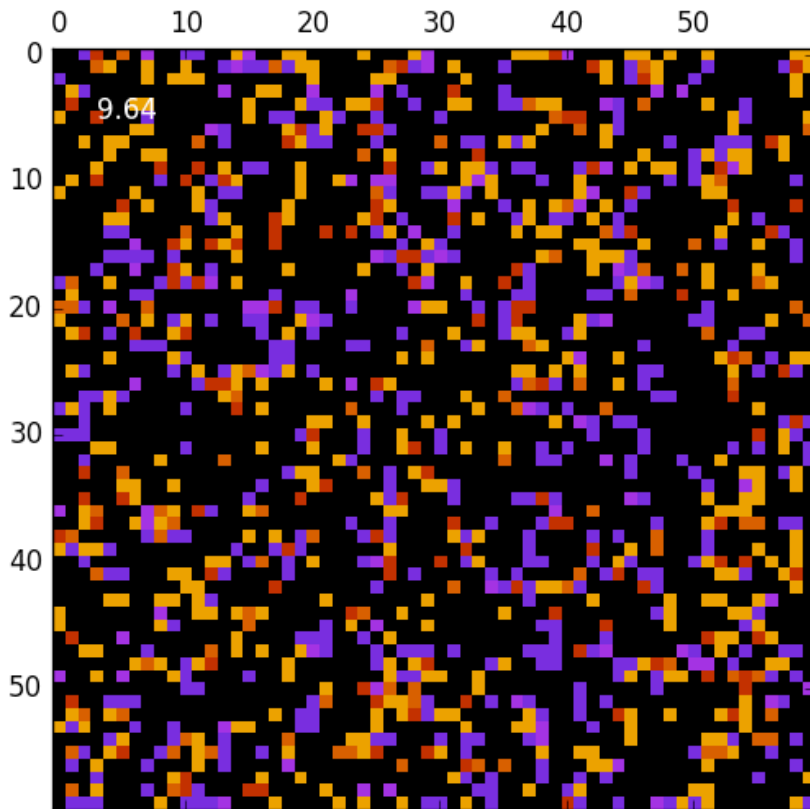
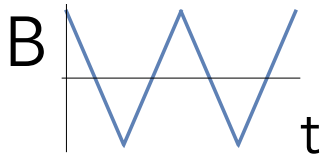


equipotentials from
solution of Laplace's
equation for a Hall bar
with two domains

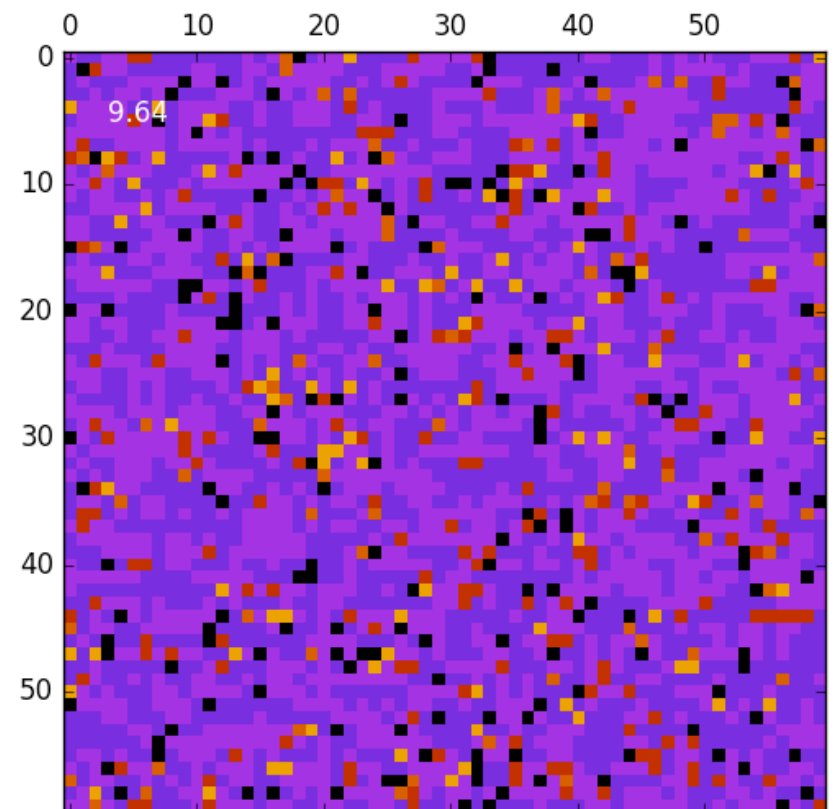


Could use this DW as a switch??

Wider sample



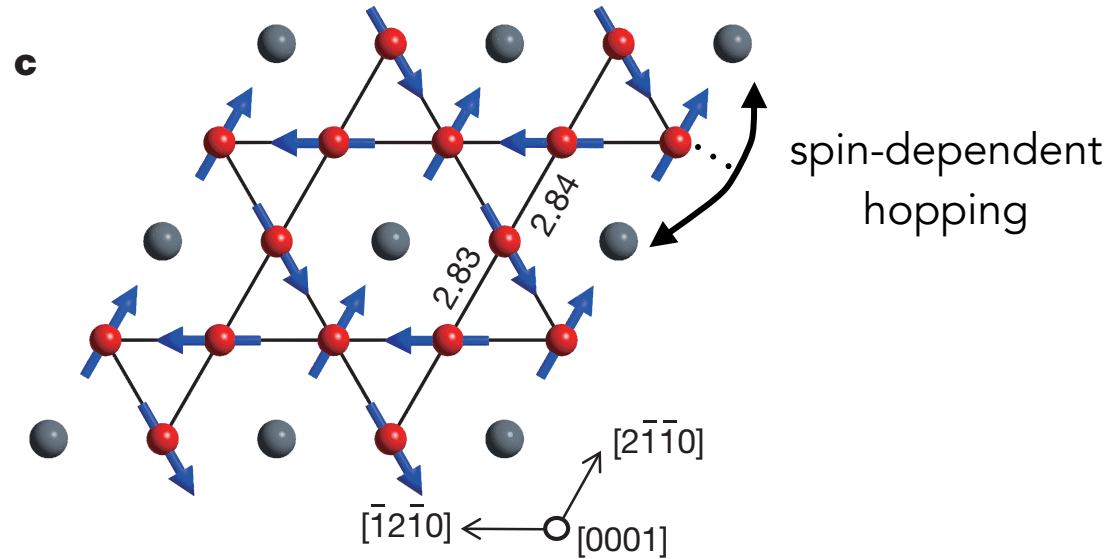
$B \parallel$ easy axis



$B \perp$ easy axis

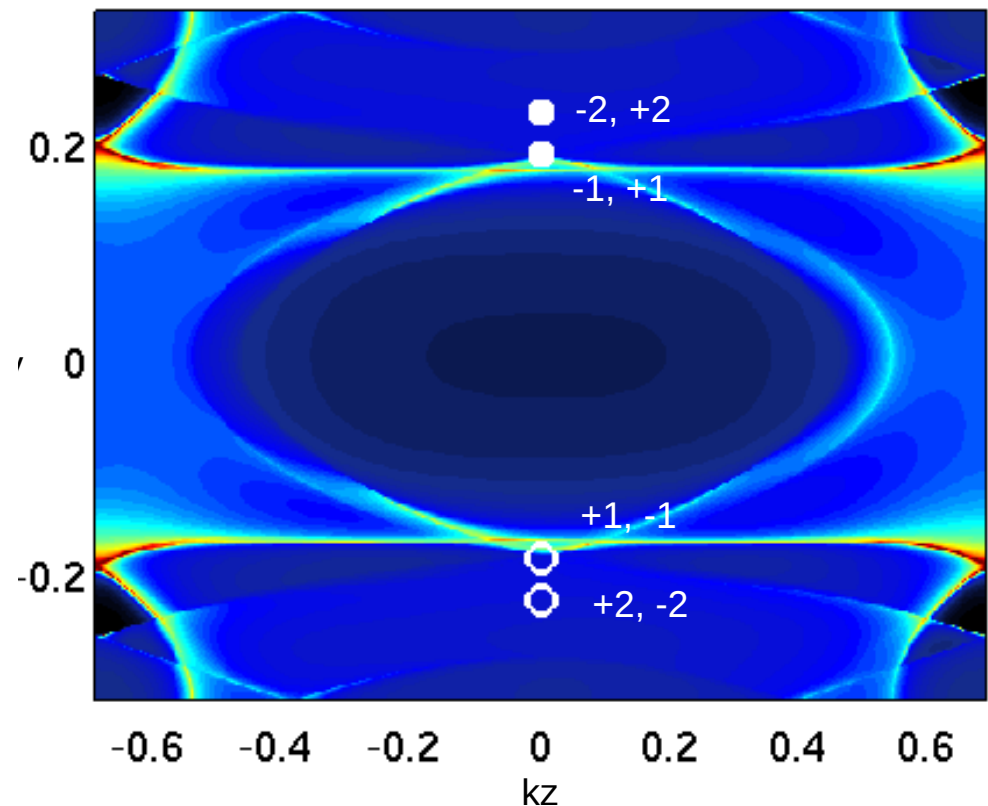
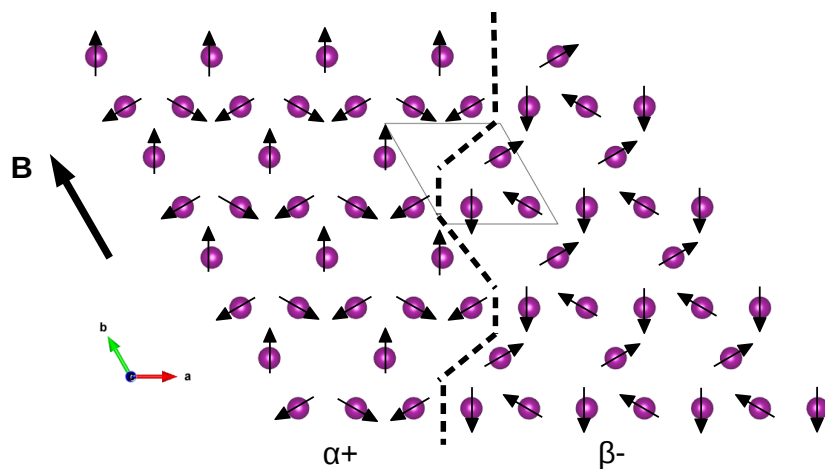
A model

tight-binding
of single
orbital on Sn
sites: a 4 band
model



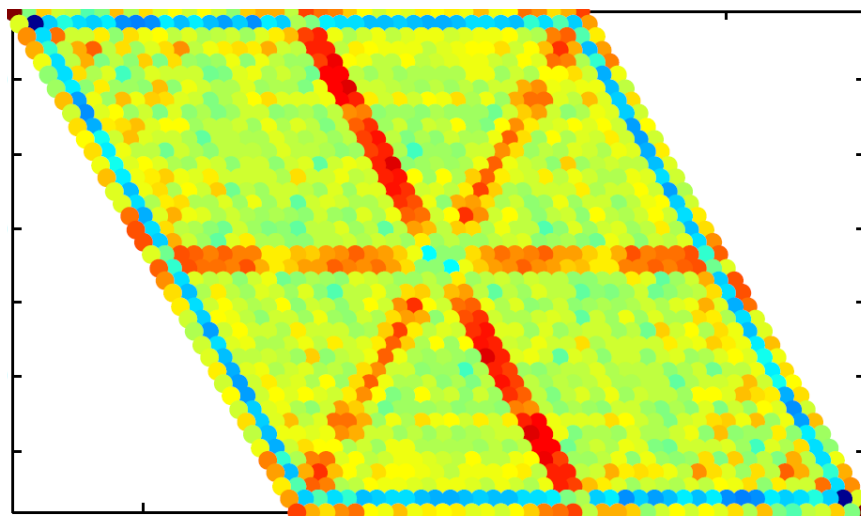
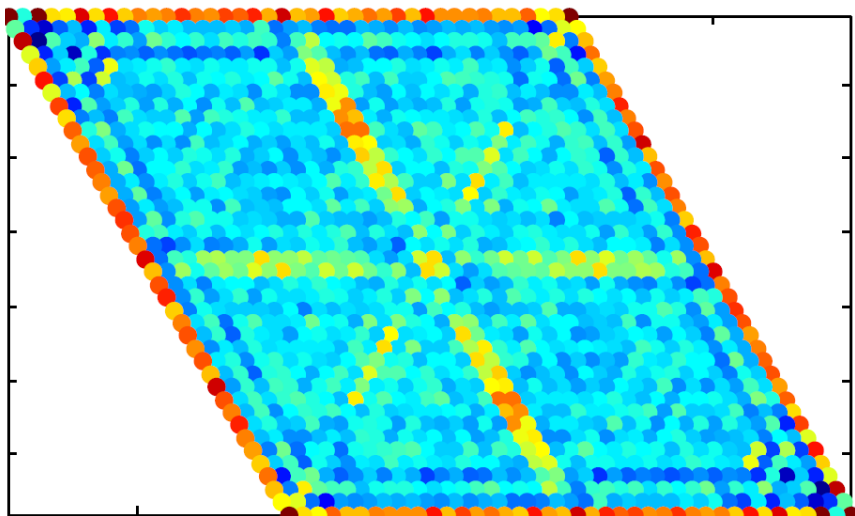
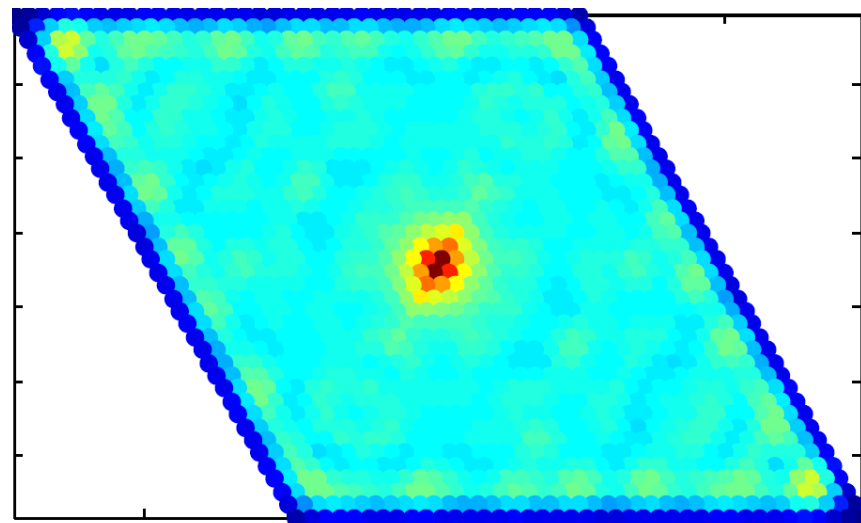
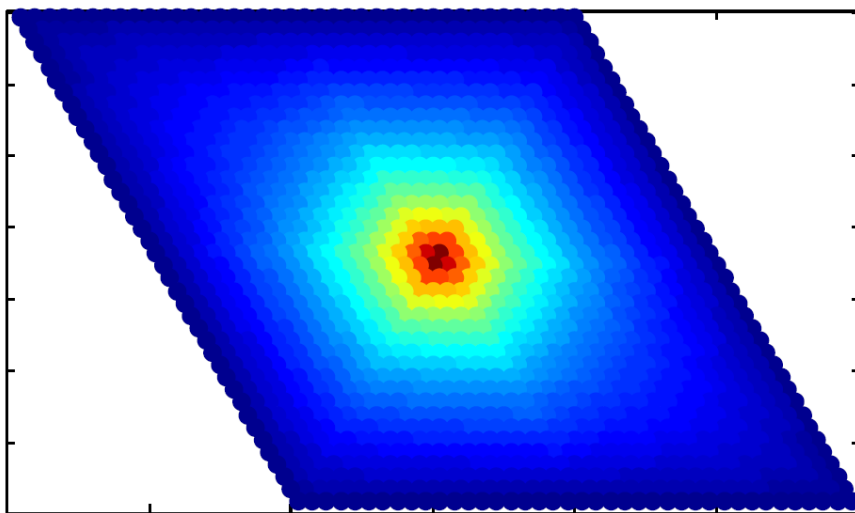
Enables efficient study of domain walls, vortices
etc.

e.g.: Domain wall



twice as many Fermi arcs as a surface

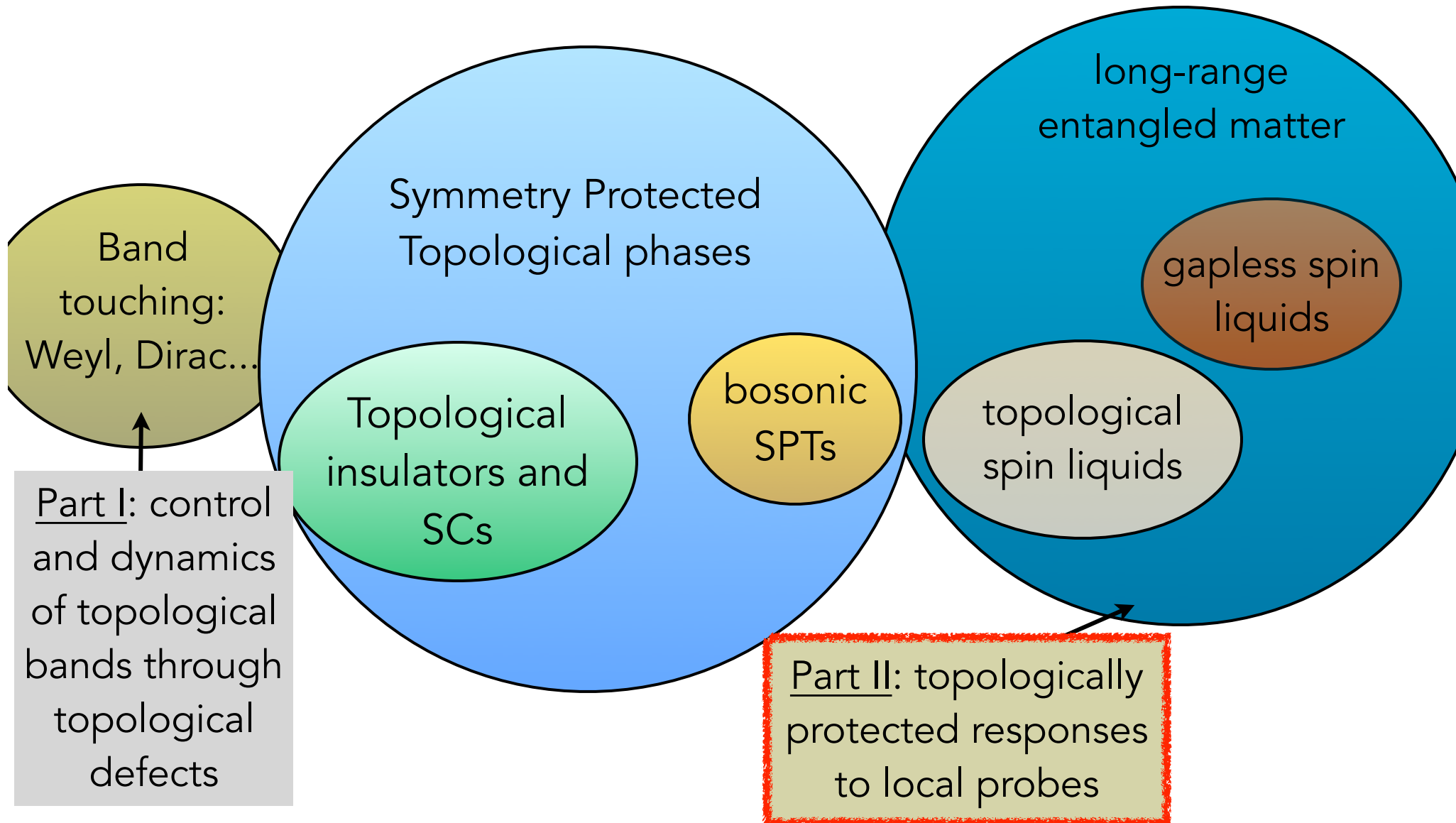
Z_6 vortex



Summary of Part I

- Ingredients are in place for nano-scale transport studies of non-trivial domain walls (conformational modeling and spatially resolved measurements crucial)
- Q: Are there interesting electronic features of Z_6 vortices?
- Q: What is the role of *fluctuations* of magnetic textures?

Topology++



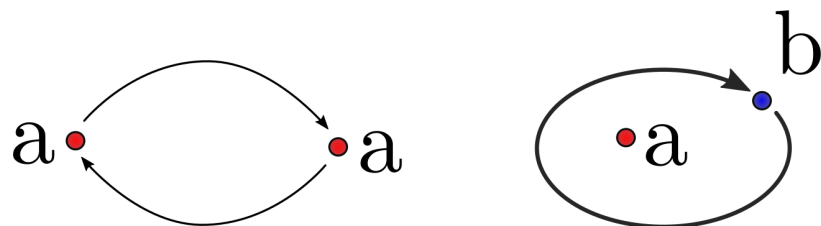
Quantum Spin Liquids



$$|\Psi\rangle = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$

The diagram shows a sum of two states. Each state is represented by a triangular lattice of blue ovals, which represent spins. In the first state, the ovals are arranged in a regular pattern. In the second state, the ovals are shifted or rotated, representing a different configuration of the spin liquid. The ellipsis indicates that there are many more such configurations in the full wavefunction.

Simplest examples: *topological* spin liquids



excitations are
anyons

$$\begin{array}{c} e \quad m \\ \diagdown \quad \diagup \\ e \quad m \end{array} = - \begin{array}{c} e \quad m \\ | \quad | \\ e \quad m \end{array}$$

$$\begin{array}{c} e \quad m \quad e \quad m \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ e \quad m \quad e \quad m \end{array} = \begin{array}{c} e \quad m \quad e \quad m \\ | \quad \diagdown \quad | \quad \diagup \\ e \quad m \quad e \quad m \end{array} = - \begin{array}{c} e \quad m \quad e \quad m \\ | \quad | \quad | \quad | \\ e \quad m \quad e \quad m \end{array}$$

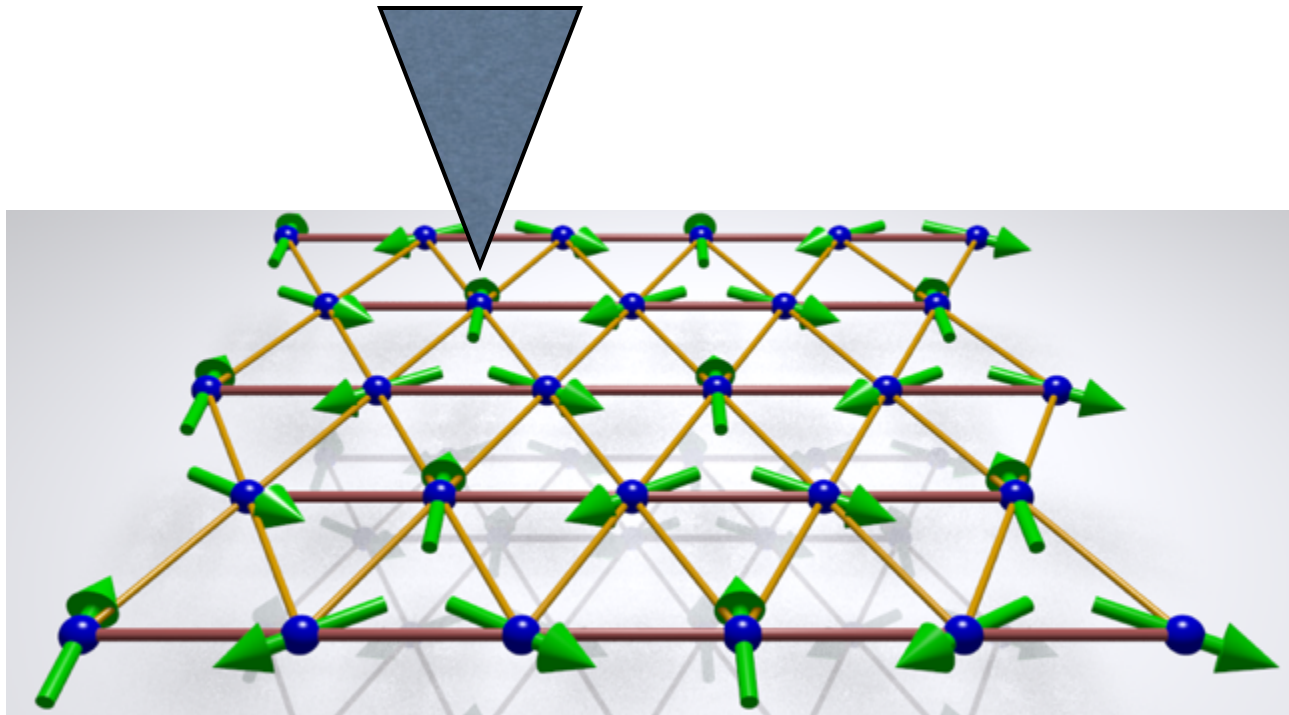
toric code/ \mathbb{Z}_2 QSL

How do we probe them?

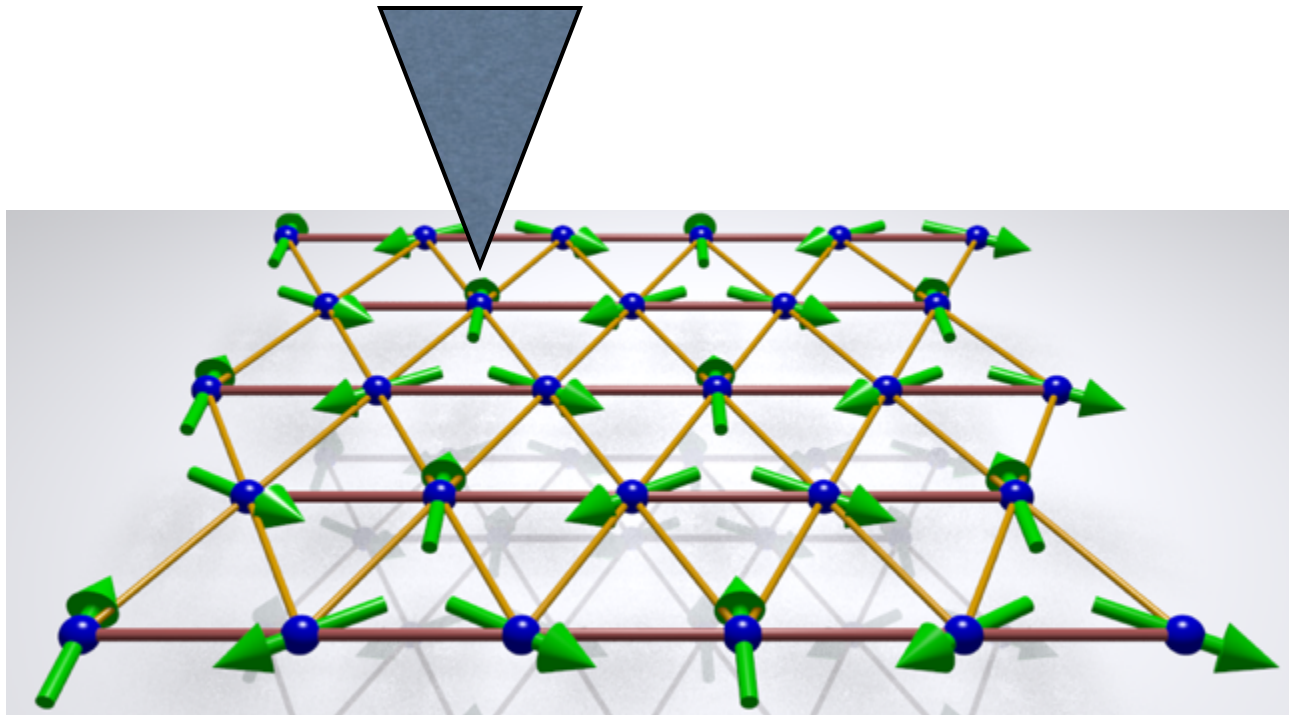
Experimentalist: "I've got this great scanning XXX*"

Theorist: "Pretty picture...but how can a local measurement tell us anything about topology?"

*STM, AFM, SQUID, terahertz STM, SC STM, NV magnetometer, banana, quantum dot, microwave detector,...

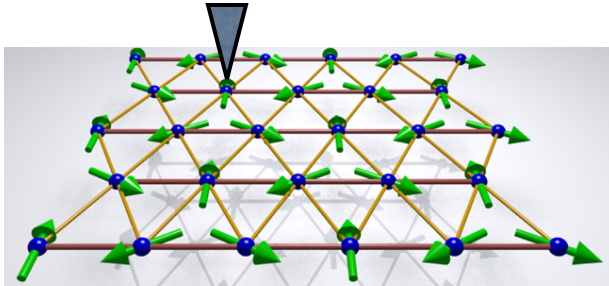


What happens when you apply a local perturbation to a topological system?



Warm-up: non-topological system

Transverse-field Ising model

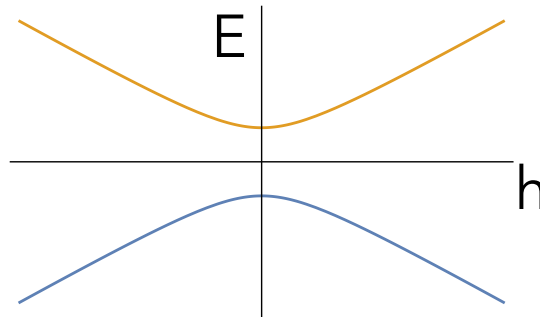


$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h_{\perp} \sum_i \sigma_i^x$$

Apply local field $H' = -h\sigma_n^z$

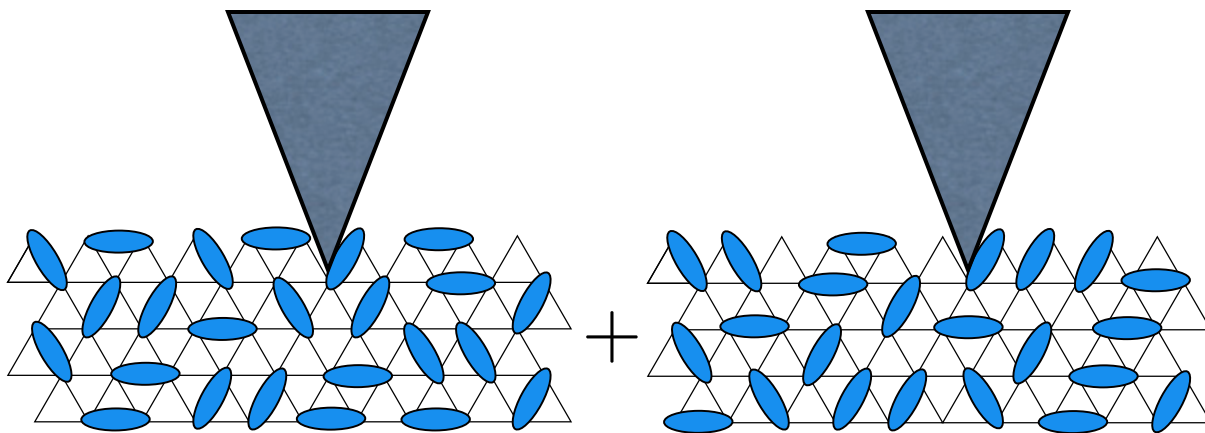
$$h_{\perp} \gg J$$

$$H_n \approx -h_{\perp} \sigma_n^x - h \sigma_n^z$$



avoided crossing: *smooth response* to applied force.

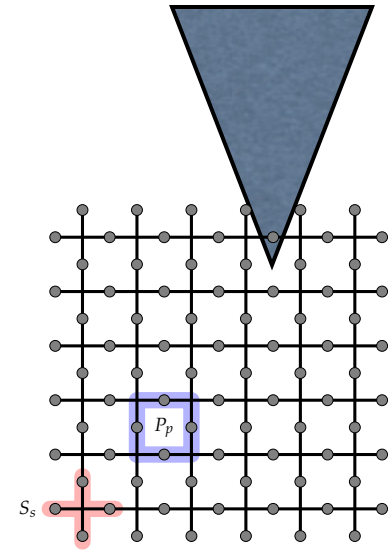
This is expected for bosonic/spin system whenever local force breaks all symmetries

$$|\Psi\rangle =$$


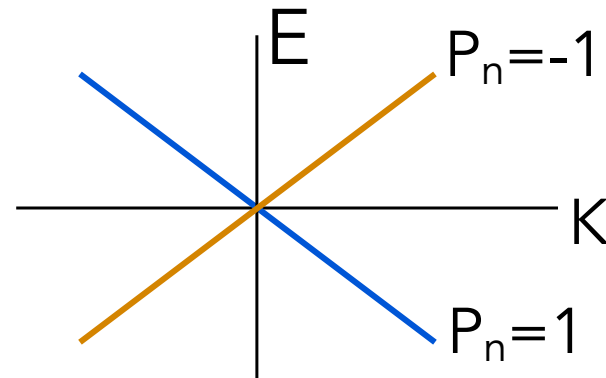
The diagram illustrates a quantum state $|\Psi\rangle$ as a superposition of different spin configurations on a triangular lattice. Each configuration is represented by a triangular lattice of blue ovals (spins) with a dark blue triangle pointing to a specific site. The first term shows a specific spin configuration, and the second term shows a different configuration, with an ellipsis indicating further terms.

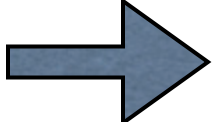
What happens in a spin liquid?

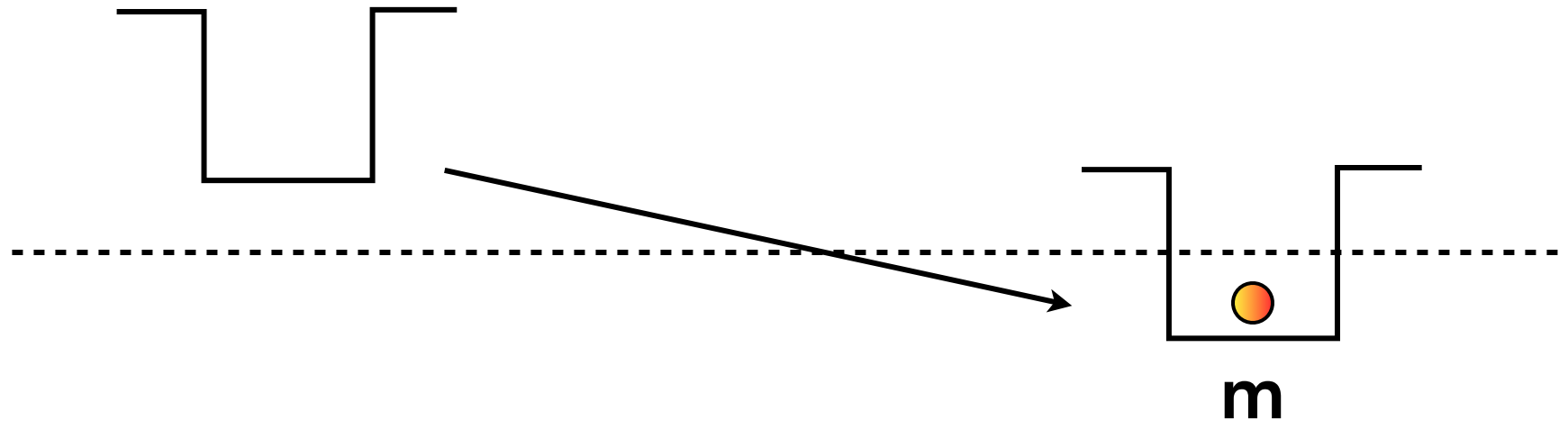
A model topological spin liquid: Kitaev's toric code



$$H_{\text{tc}} = -K \sum_p P_p - K' \sum_s S_s,$$

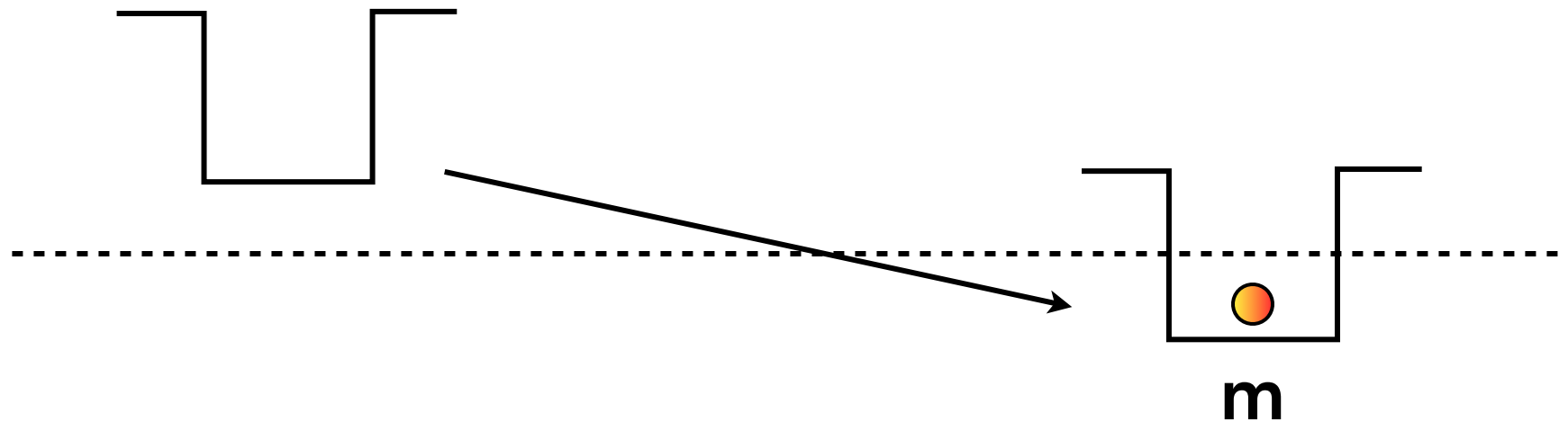


vary one K_n  level crossing



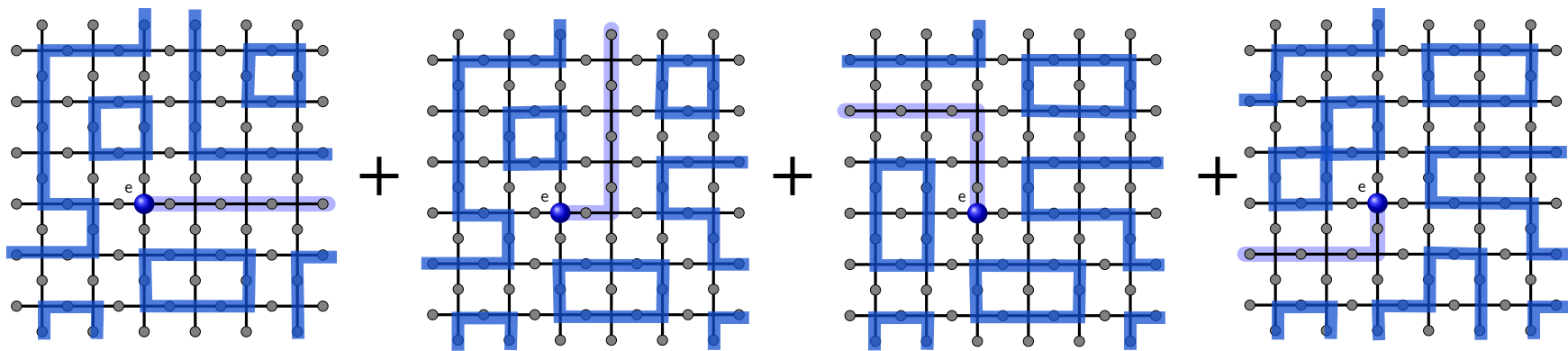
local potential binds an anyon

Level repulsion between state with and without **m** particle is *topologically* forbidden: no local operators can create/annihilate a single anyon. The crossing persists under arbitrary perturbations.

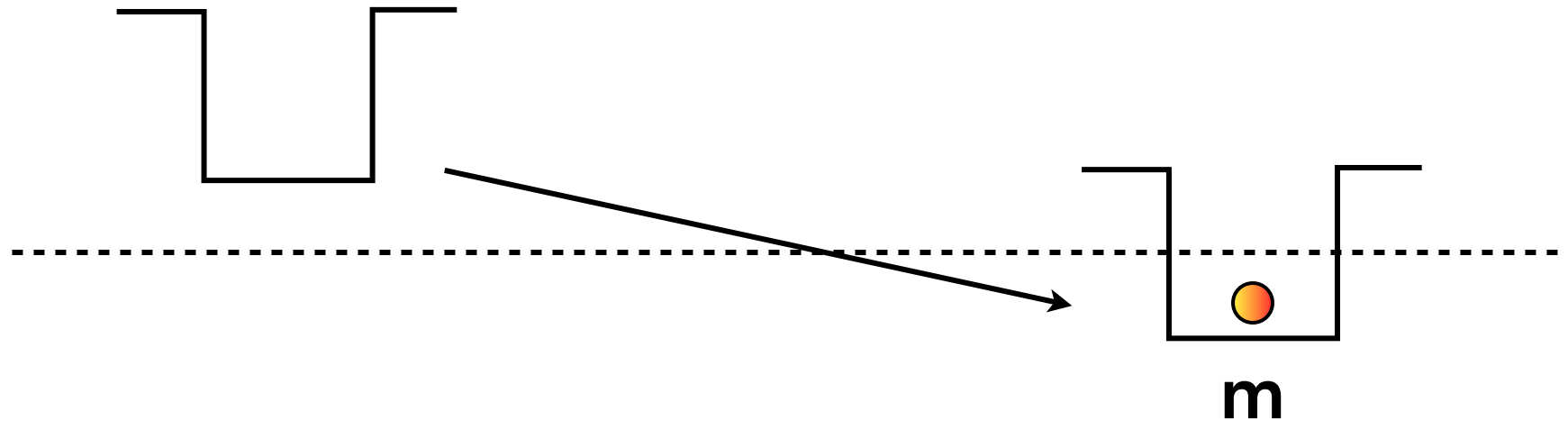


local potential binds an anyon

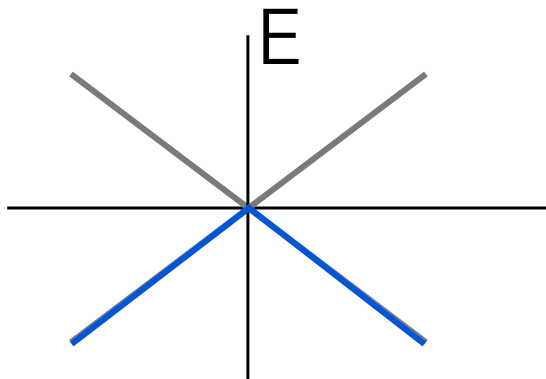
$$|e_s\rangle =$$



robustness due to "field line" emanating from anyon

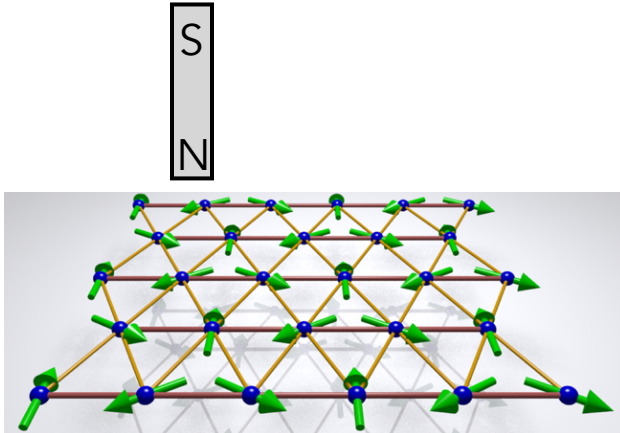


local potential binds an anyon



cusp in ground state energy:
 a topologically protected local
quantum phase transition with
 a discontinuity in local
 susceptibility

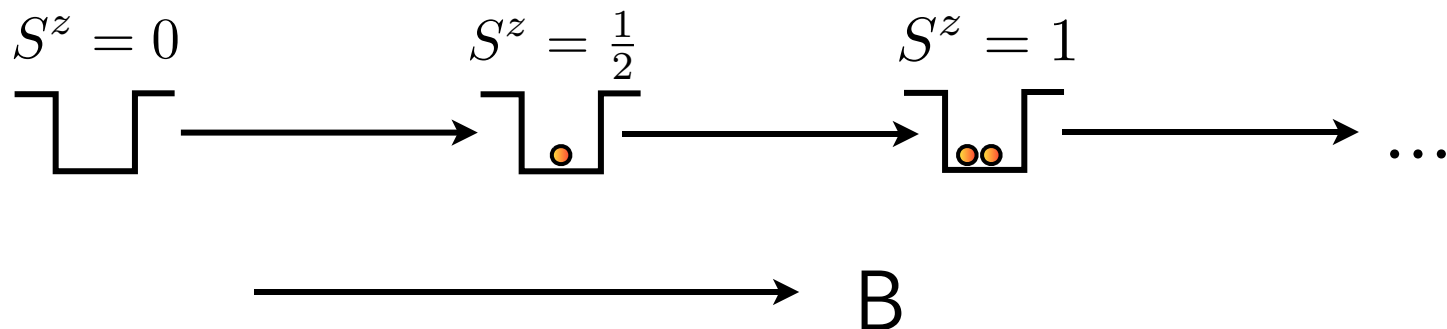
Weak spin-orbit-coupling



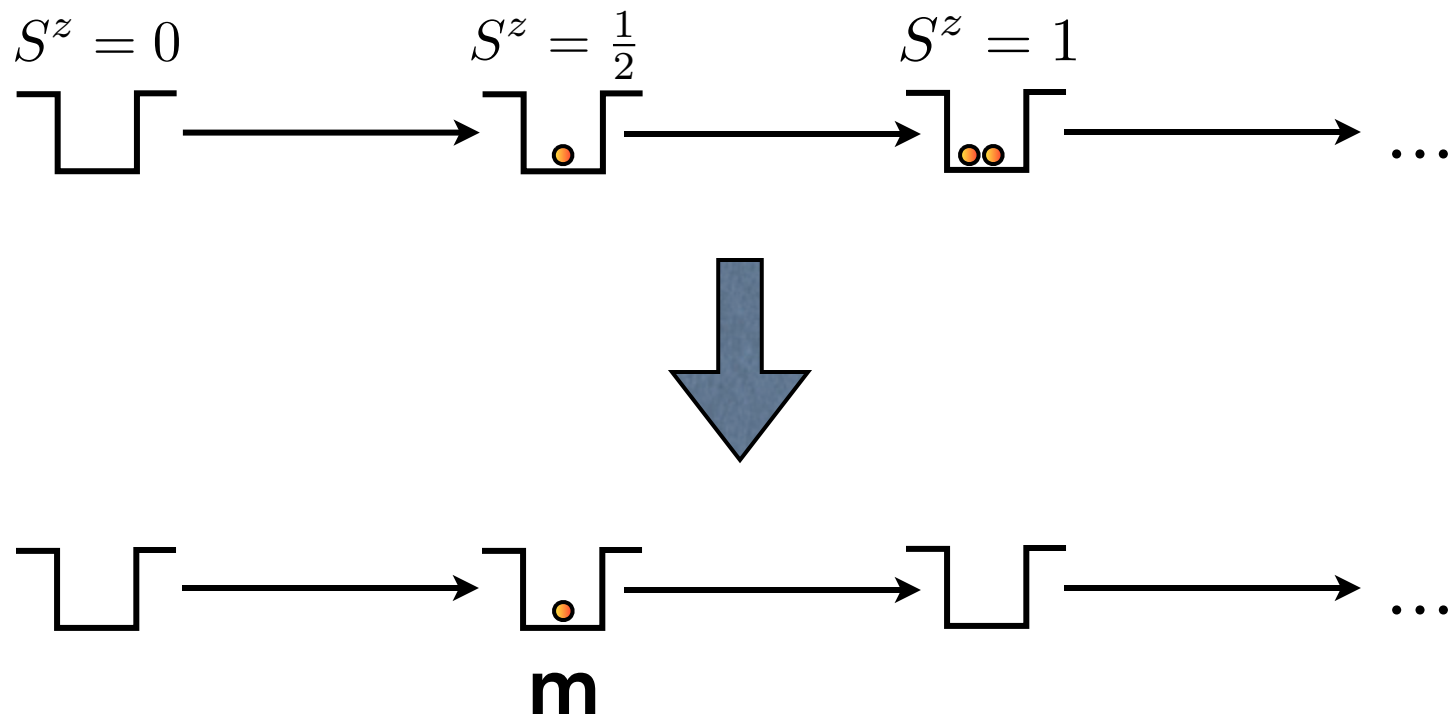
local magnetic field

$$H' = -B \sum_i f_i S_i^z \quad \text{lowers energy of excitations with spin}$$

Spinons: anyons of QSL carry spin-1/2

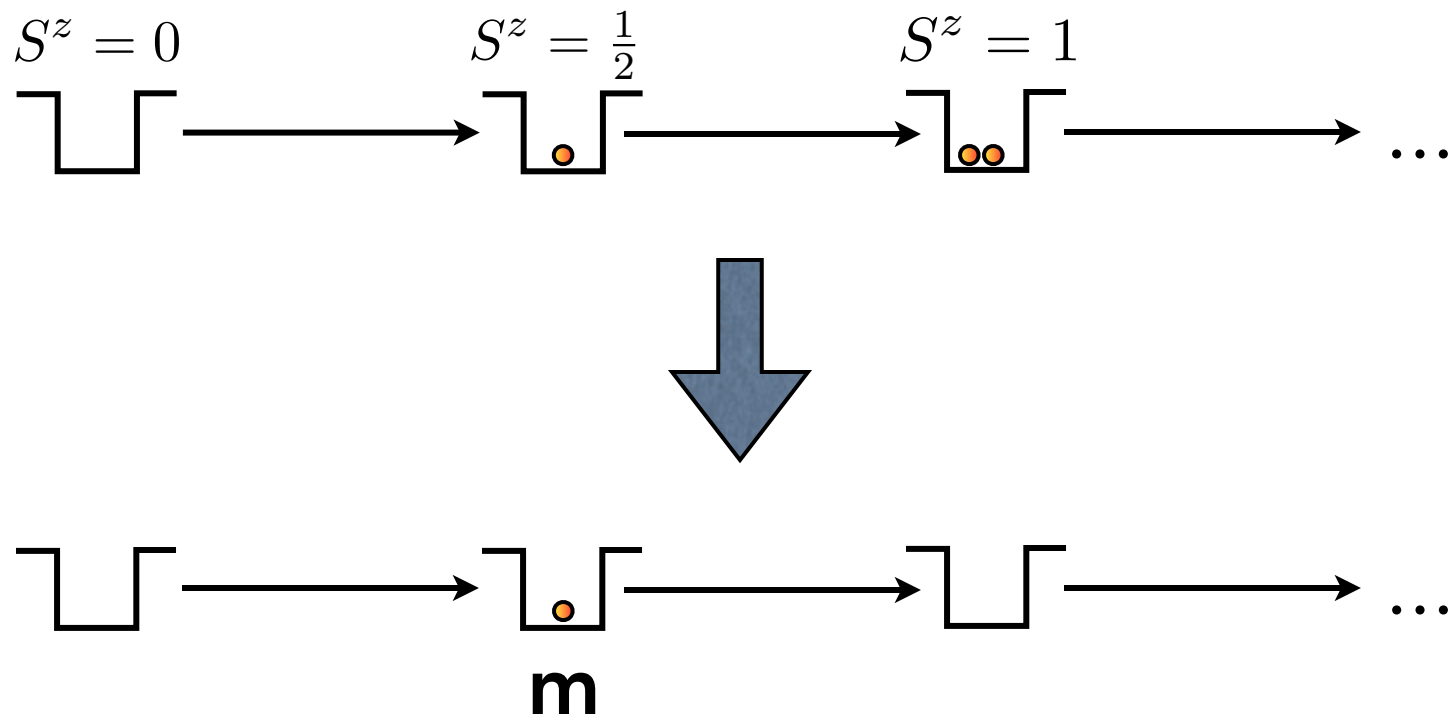


Spin non-conservation



Periodic level crossings remain robust,
and should be expected generically in
gapped spin liquids with weak SOC!

Spin non-conservation



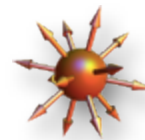
Strong SOC does not affect the protection,
but it makes it less obvious what local
perturbation creates the level crossing

Gapless spin liquids

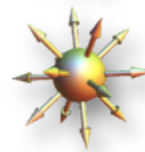
“non-local” excitations are characteristic of most QSLs: so we expect that the local QPT is very general

e.g. Coulomb 3d QSL (quantum spin ice)

- anyons replaced by non-local quasiparticles



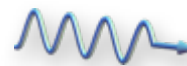
magnetic
monopoles



electric
monopoles



cannot be
created/
destroyed
locally



“photon”

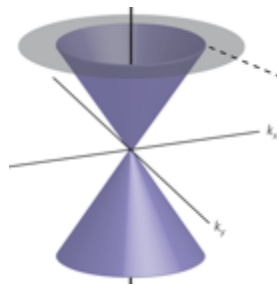
- protected level crossing ✓

Gapless spin liquids

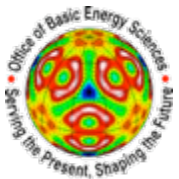
“non-local” excitations are characteristic of most QSLs: so we expect that the local QPT is very general

e.g. Kitaev honeycomb QSL ($\text{Na}_2\text{IrO}_3, \text{RuCl}_3, \dots$)

- “Fluxes” are non-local and can undergo binding
- Binding becomes non-trivial edge singularity due to gapless Majorana fermions
- But a local QPT persists ✓



Summary



- Rich interplay of band topology and AHE with real-space topological defects is possible in real AFs. Propose studies of domain walls and vortices in Mn_3Sn
- Local studies of QSLs can reveal topological anyons or even more exotic excitations through protected local QPTs