

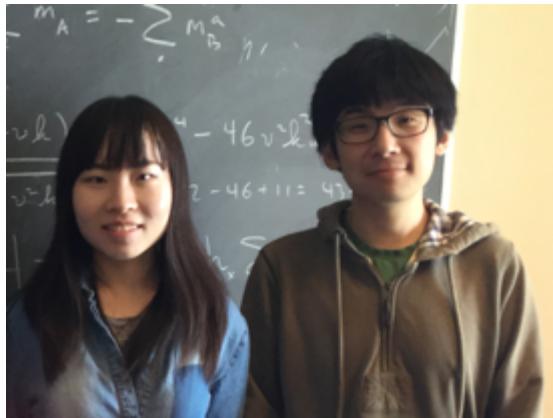
# Quantum Spin Liquids



Leon Balents, KITP

LT28, Götheborg

# Collaborators (whose work I'll mention)



Xue-Yang Song  
Yi-Zhuang You



Gábor Halász



Chunxiao Liu



Jason Iaconis



Lucile Savary



Xiao Chen

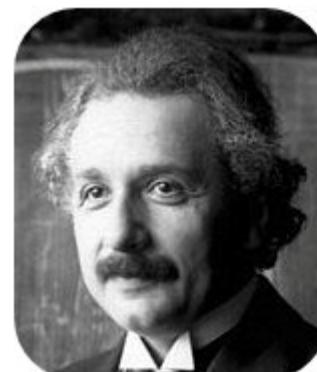
# Quantum non-locality

EPR

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



??where is the information??



A. Einstein

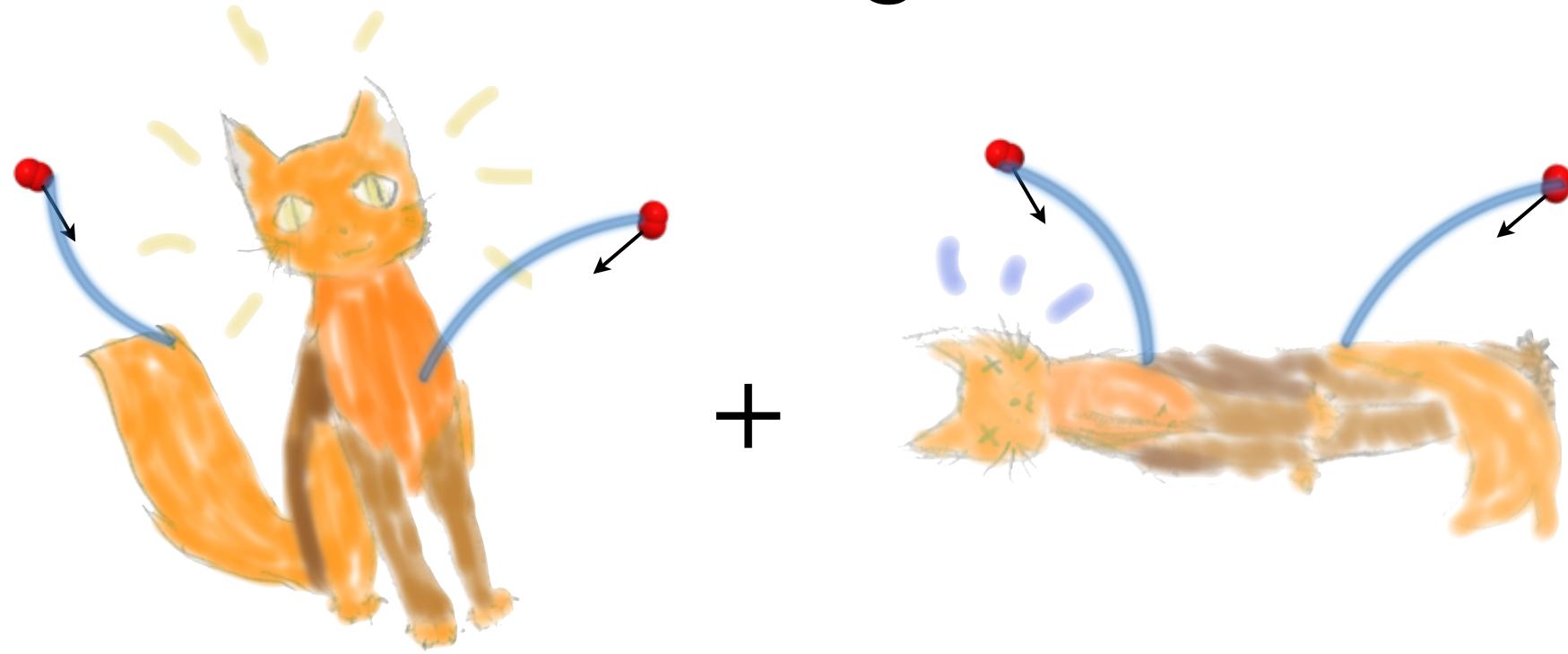


B. Podolsky



N. Rosen

# Schrödinger's Cat



UNSTABLE to decoherence - uncontrolled entanglement with the environment



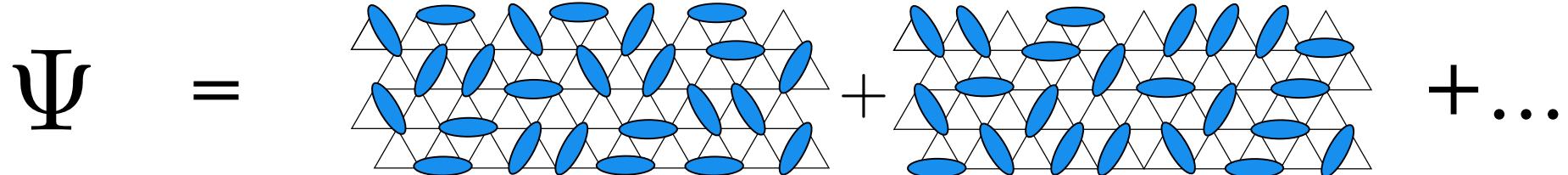
# Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Resonating Valence Bond state

# Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Psi = \text{Diagram 1} + \text{Diagram 2} + \dots$$

The equation shows the wavefunction  $\Psi$  as a sum of two diagrams representing different valence bond configurations in a lattice, plus a continuation symbol. Diagram 1 shows a horizontal chain of blue ovals (valence bonds) with alternating up and down spins. Diagram 2 shows a similar chain but with a different phase or arrangement.

Resonating Valence Bond state



# Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties



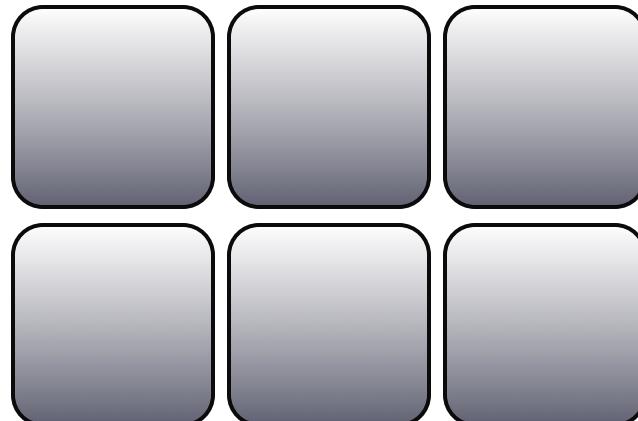
- Measurements far away do not affect one another
- From local measurements we can deduce the global state

# Ordinary (local) Matter

Hamiltonian is local

$$H = \sum_x \mathcal{H}(x) \quad \mathcal{H}(x) \text{ has local support near } x$$

Ground state is “essentially”  
a product state

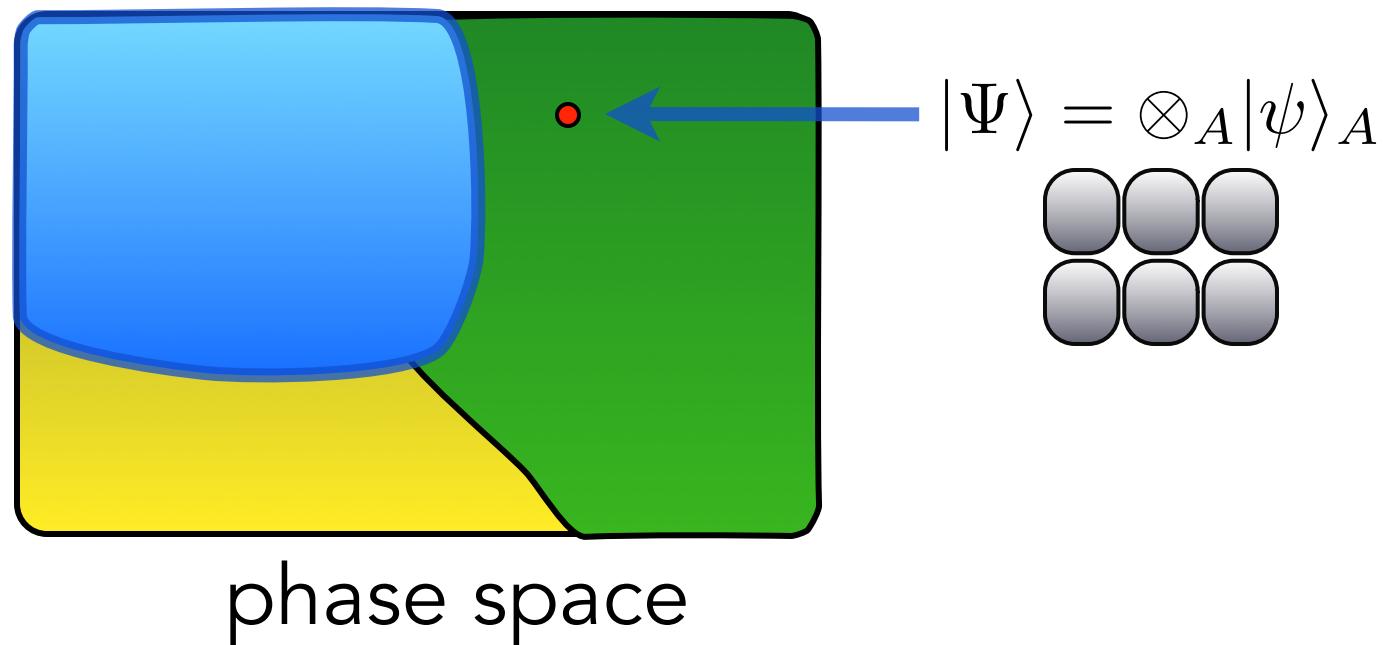


$$|\Psi\rangle = \otimes_A |\psi\rangle_A$$

no entanglement  
between blocks

# “Essentially” a product state?

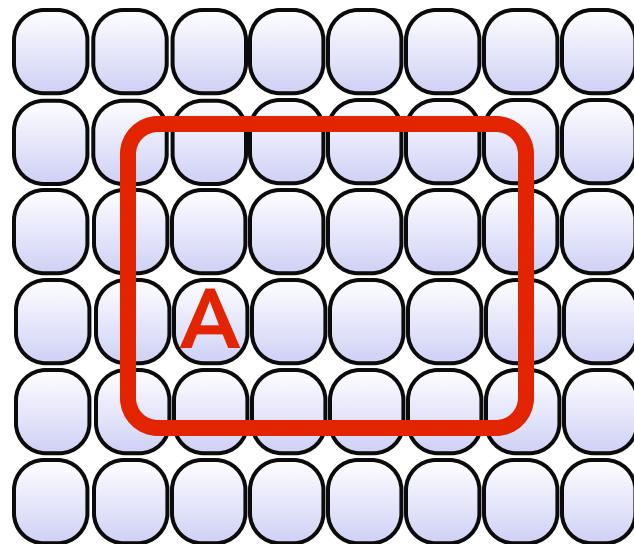
- Adiabatic continuity



n.b. This is not true for gapless fermi systems

# “Essentially” a product state?

- Entanglement scaling



$$\rho_A = \text{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\text{Tr}_A (\rho_A \ln \rho_A)$$

$$S(A) \sim \sigma L^{d-1} \quad \text{area law}$$

satisfied with exponentially small corrections

# Best example: ordered magnet

Hamiltonian

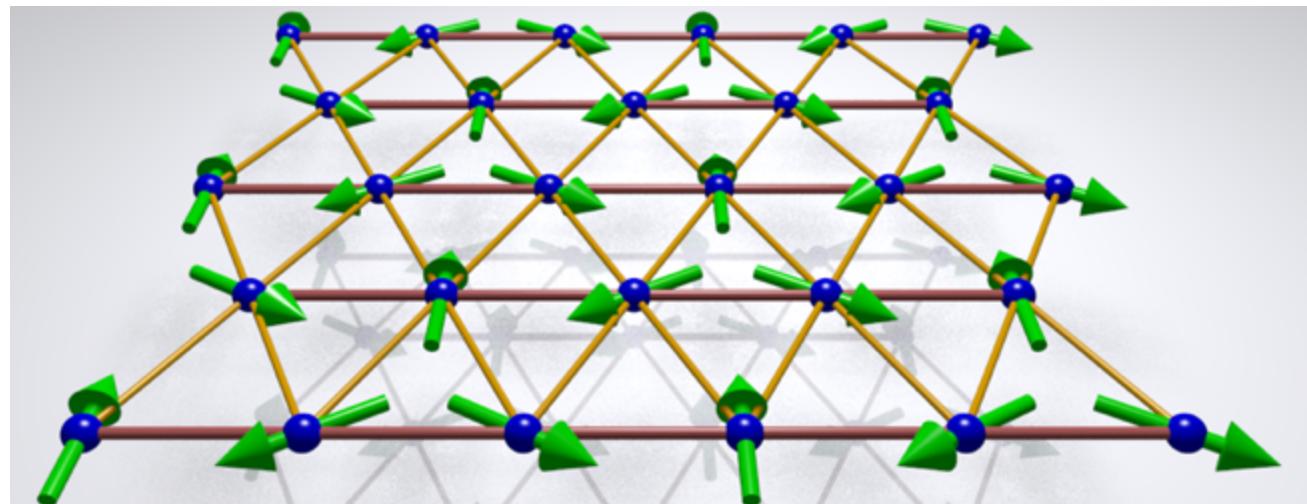
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-  
range: local

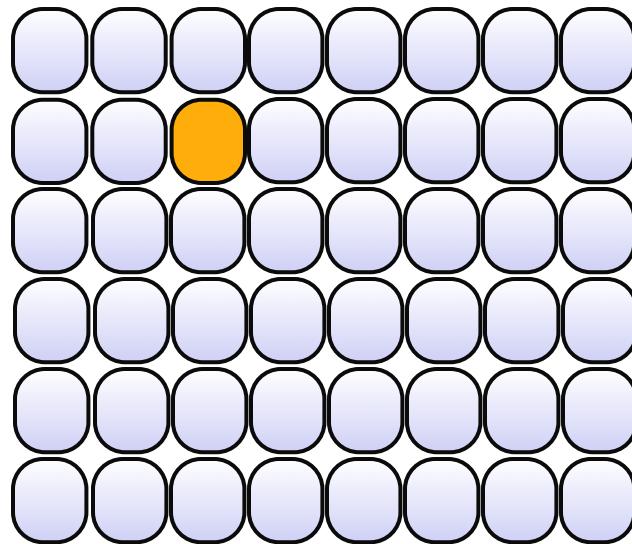
ordered state

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{\mathbf{n}}_i = +S\rangle$$

block is a single  
spin



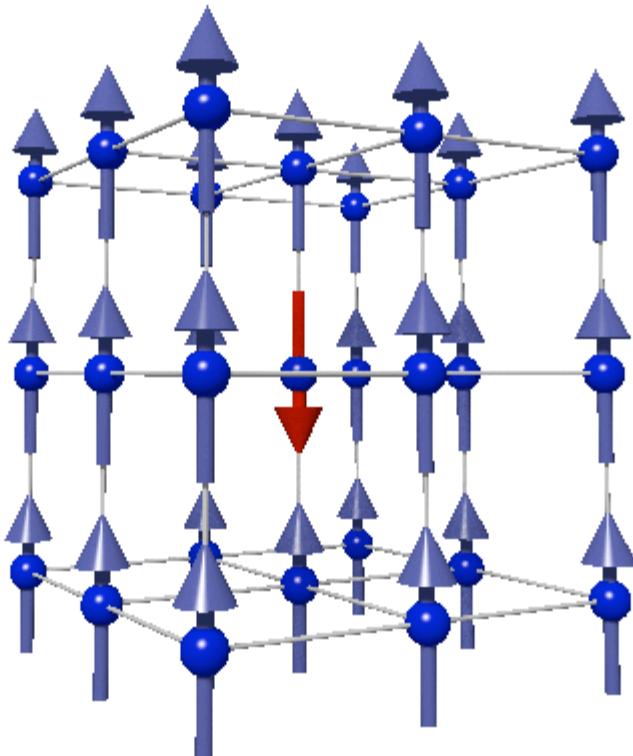
# Quasiparticles



excited states  $\sim$  excited  
levels of one block

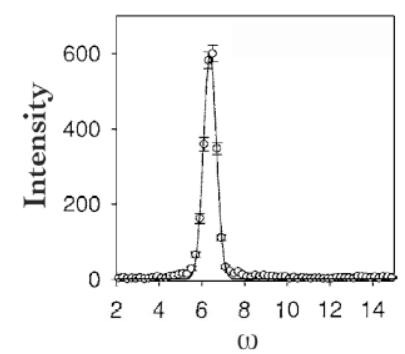
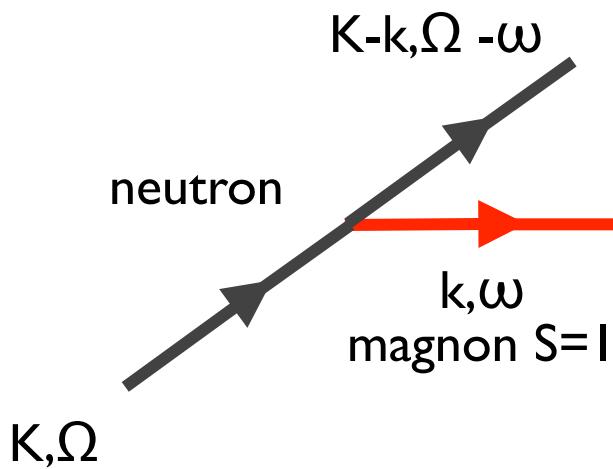
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

# Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

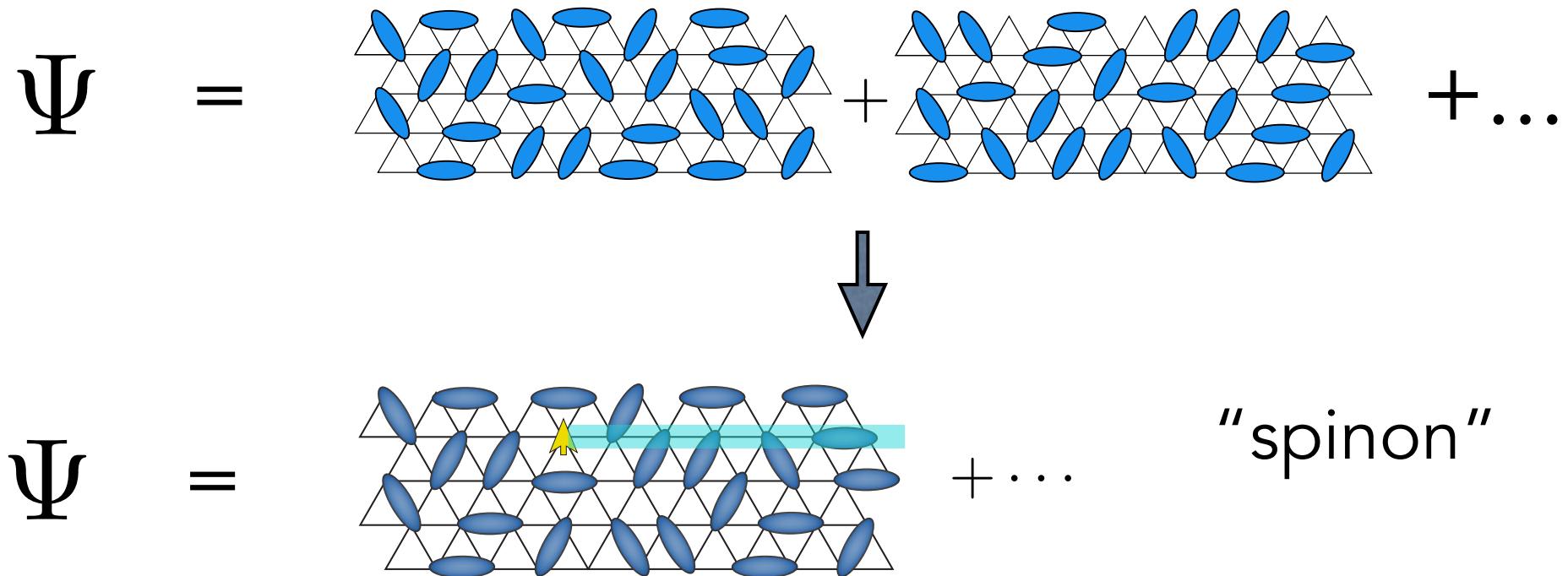
$$|f\rangle = S_k^+ |i\rangle$$



Line shape in  $\text{Rb}_2\text{MnF}_4$

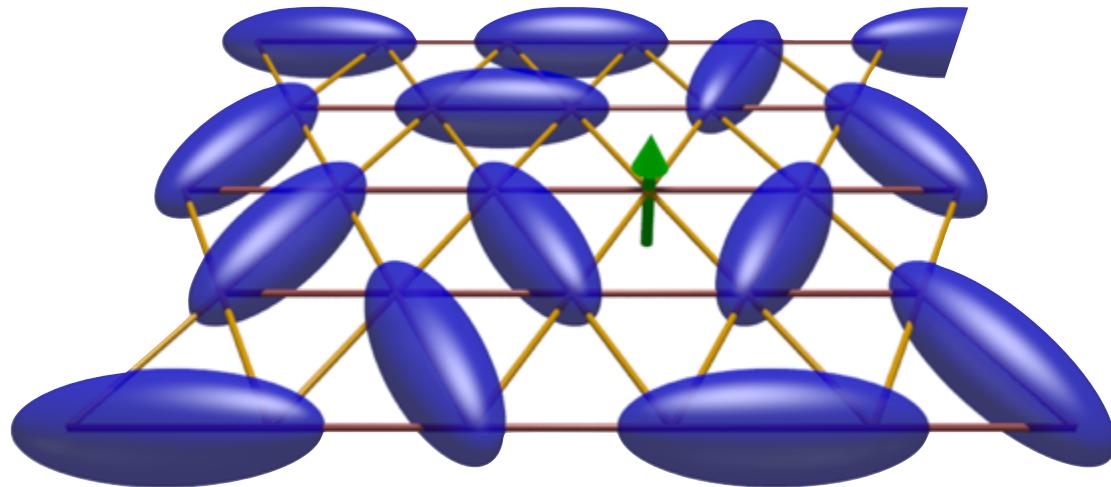
# Quantum spin liquid

Entanglement  $\rightarrow$  non-local excitation



“quasiparticle” above a non-zero gap

# Fractional quantum number

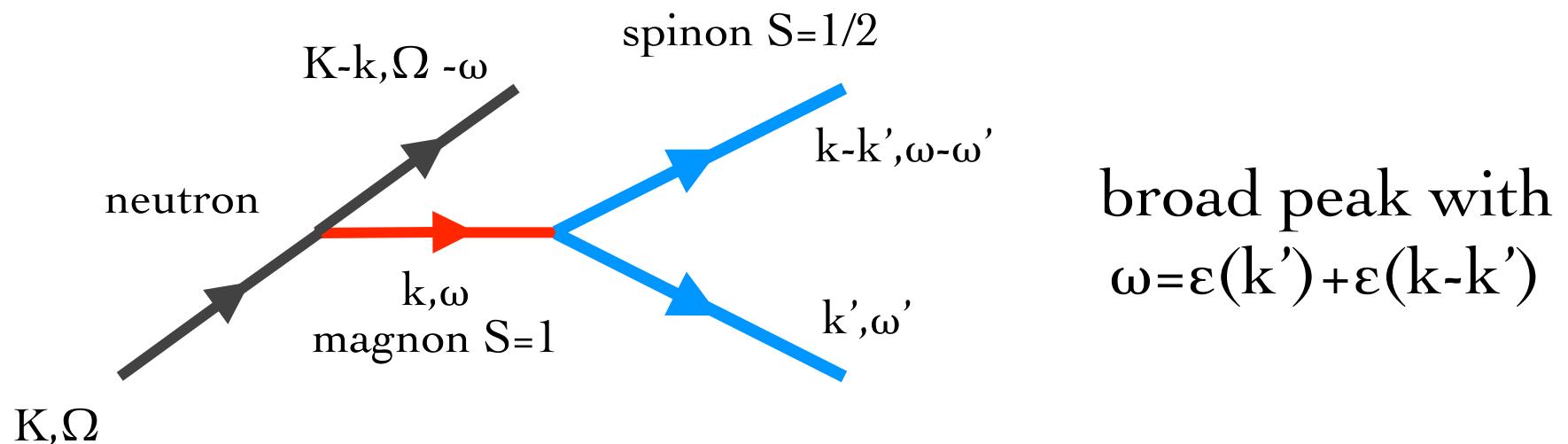


excitation with  $\Delta S = 1/2$   
not possible for any finite  
cluster of spins

always created in pairs by any  
local operator

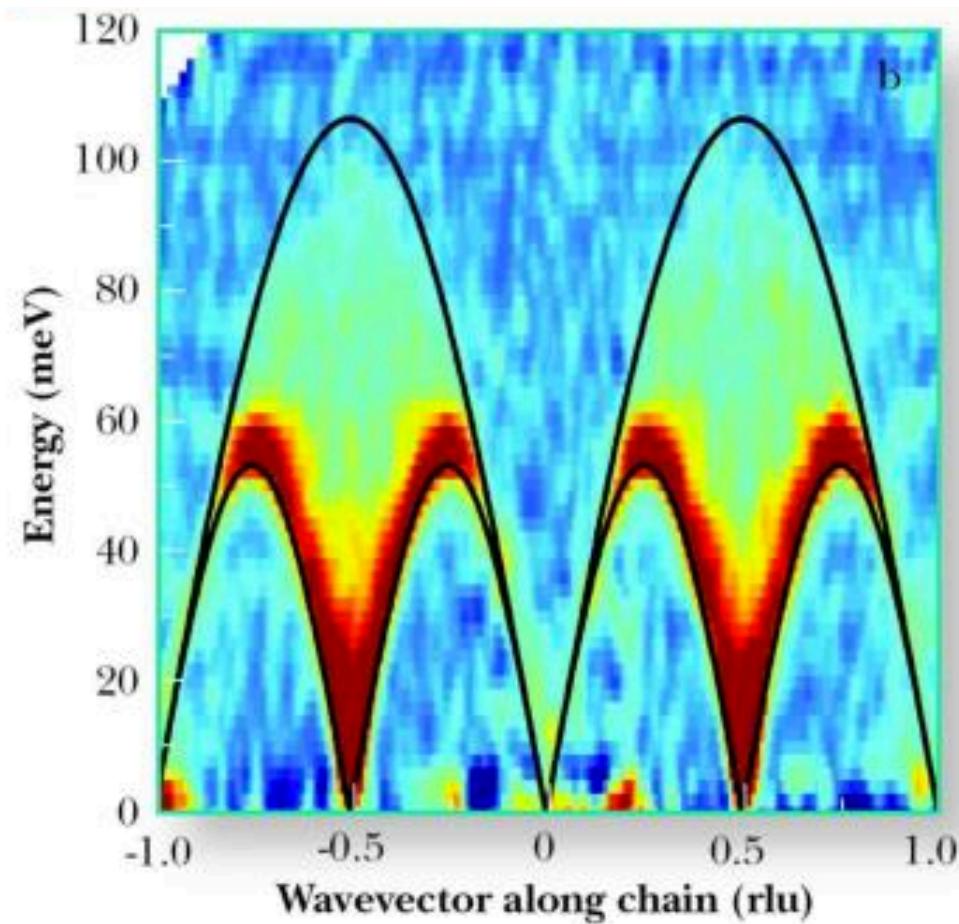
# No spin waves

- Magnon is not elementary: decays into two spinons



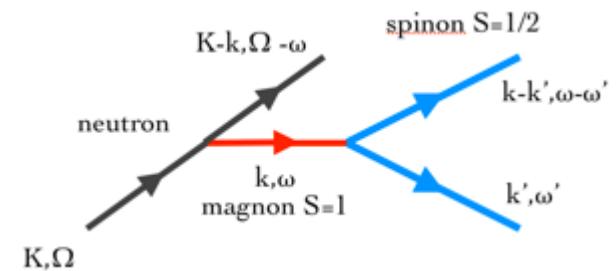
- Sharp peaks should be reduced or absent in the spin structure factor

# c.f. One dimension

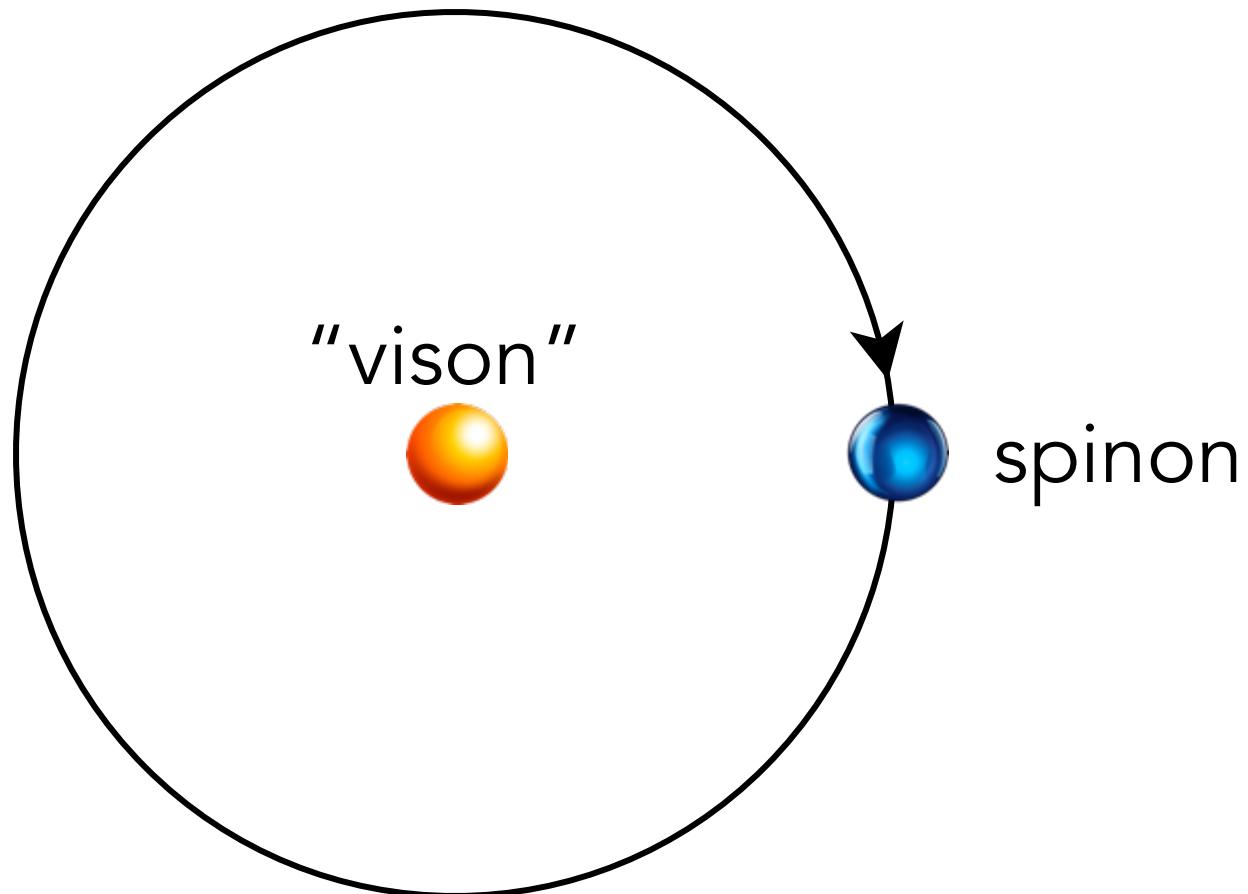


A. Tennant *et al*, 2001

KCuF<sub>3</sub>



# Anyons



$$\Psi \rightarrow -\Psi$$

"mutual semions"

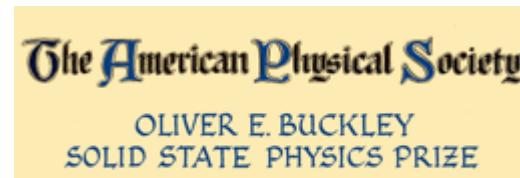


X.-G. Wen



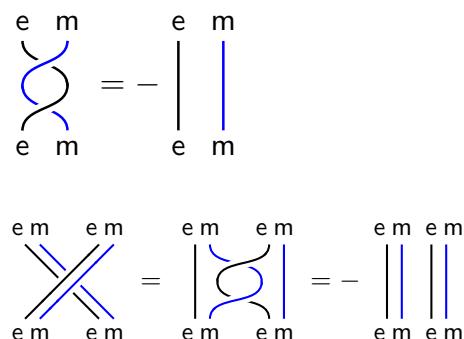
A. Kitaev

# Topological phases



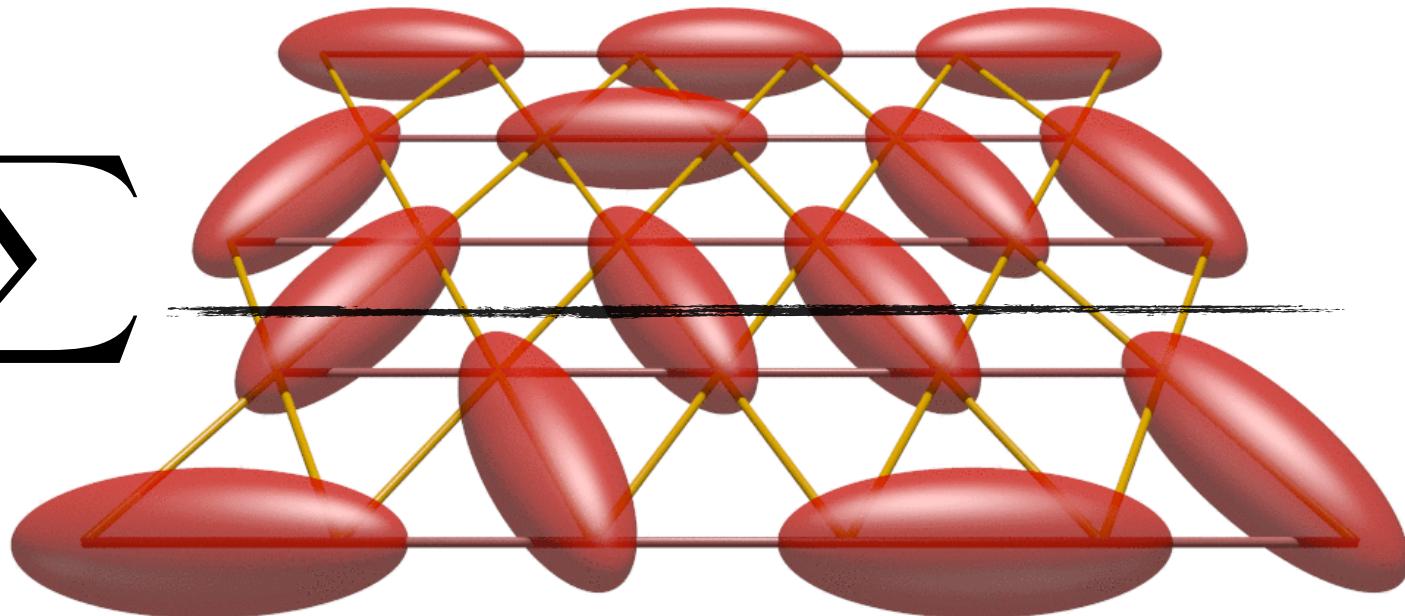
Anderson's RVB state is thus an example of a “topological phase” - the best understood sort of QSL

Understood and classified by anyons and their braiding rules in 2d



# Stability

$$\Psi = \sum$$



Robustness arises from topology: a QSL is a stable phase of matter (at T=0)

# Quantum spin liquid

$$\Psi = \begin{array}{c} \text{Diagram of a triangular lattice with blue ovals representing spins, showing two different local arrangements of spins.} \end{array} + \dots$$

For  $\sim 500$  spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.

# Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site ( $S=0$ )

$$|\Psi_0\rangle = \prod_{k \in FS} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$

"partons"

$$= c_1 \begin{array}{|c|c|c|c|c|c|} \hline & \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline & \downarrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline & \downarrow & & \uparrow & \uparrow \downarrow & \downarrow & \downarrow \\ \hline & \downarrow & \downarrow & \uparrow \downarrow & \downarrow & \downarrow & \downarrow \\ \hline & \uparrow & \downarrow & \downarrow & \downarrow & \uparrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|c|} \hline & \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline & \downarrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline & \downarrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline & \downarrow & \downarrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline & \uparrow & \downarrow & \downarrow & \downarrow & \uparrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|c|} \hline & \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow \\ \hline & \downarrow & \uparrow & \uparrow \downarrow & \downarrow & \downarrow & \uparrow \\ \hline & \downarrow & \downarrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline & \uparrow & \uparrow \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

# Gutzwiller Construction

- Project out any components with empty or doubly occupied sites

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

"partons"

$$= c_1 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + \dots$$

The equation illustrates the Gutzwiller construction. It starts with the state  $|\Psi_0\rangle$  and applies the operator  $\hat{P}_G$  to project out configurations with empty or doubly occupied sites. The first term  $c_1$  is crossed out because it has a doubly occupied site (up-up). The second term  $c_2$  is the Gutzwiller-Projected state, which has no empty or doubly occupied sites. Subsequent terms  $c_3$  and beyond are also crossed out for similar reasons.

# Gutzwiller Construction

- Can build many QSL states by choosing different free fermion states

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

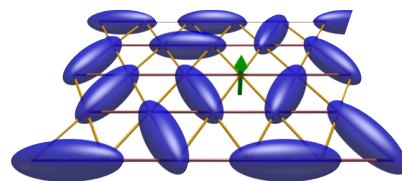
"partons"

$$= c_1 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + \dots$$

The equation illustrates the Gutzwiller construction of Quantum Spin Liquid (QSL) states. It shows a sum of terms, each representing a different configuration of "partons" (fermion states) on a 5x5 grid. The first term,  $c_1$ , is crossed out with a large red 'X'. The second term,  $c_2$ , shows a configuration where each site has a unique spin direction. The third term,  $c_3$ , is also crossed out with a red 'X'. The ellipsis at the end indicates that there are many more terms in the sum.

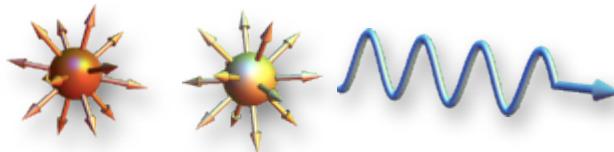
# Classes of QSLs

- Topological QSLs



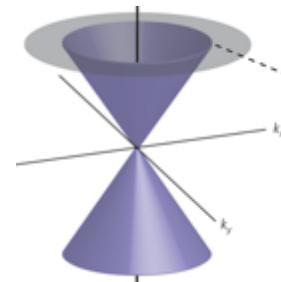
projected  
superconductor

- $U(1)$  QSL



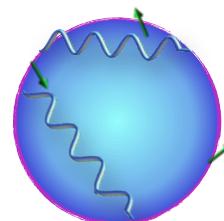
projected 3d band  
insulator

- Dirac QSLs



projected  
graphene

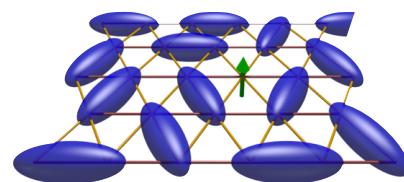
- Spinon Fermi surface



projected  
metal

# Classes of QSLs

- Topological QSLs



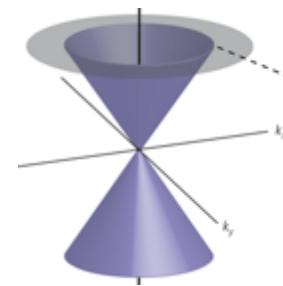
anyonic  
spinons

- $U(1)$  QSL



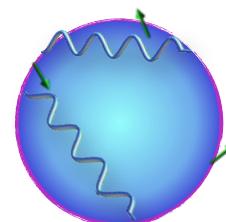
electric+magnetic  
monopoles, photon

- Dirac QSLs



strongly  
interacting  
Dirac fermions

- Spinon Fermi surface



non-Fermi  
liquid "spin  
metal"

# Strange stuff



where do we find it?

# A rough guide to experiments on QSLs

## Does it order?

- NMR line splitting
- muSR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/x-ray

## Delocalized excitations?

- thermal conductivity
- INS

## Is there a gap?

- Specific heat
- NMR  $1/T_1$
- Dynamic susceptibility
- T-dependence of  $\chi$

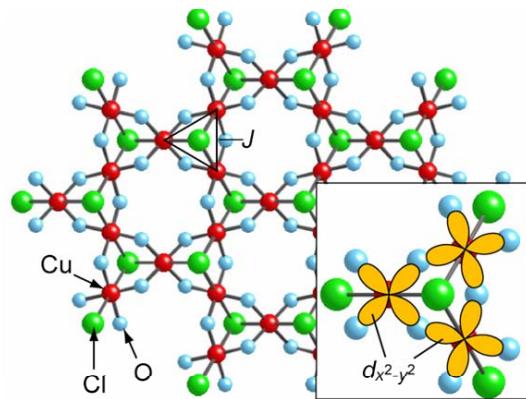
## Structure of excitations?

- $E(k)$  from INS, RIXS
- optics, Raman

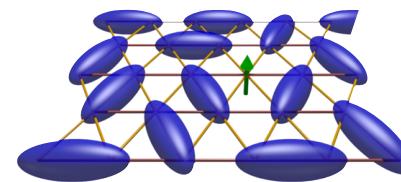
## Exotica

- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects

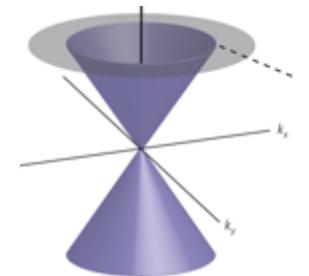
# the new classics



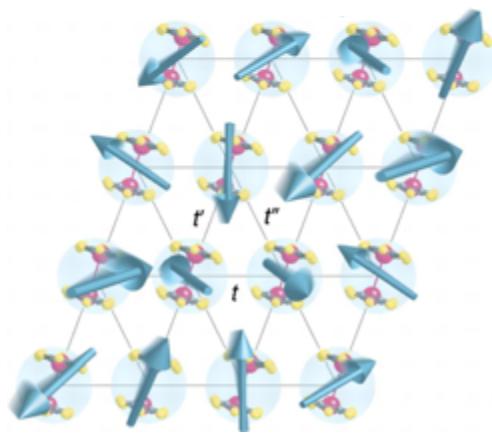
=



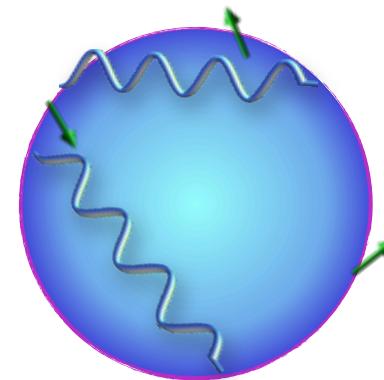
or



herbertsmithite

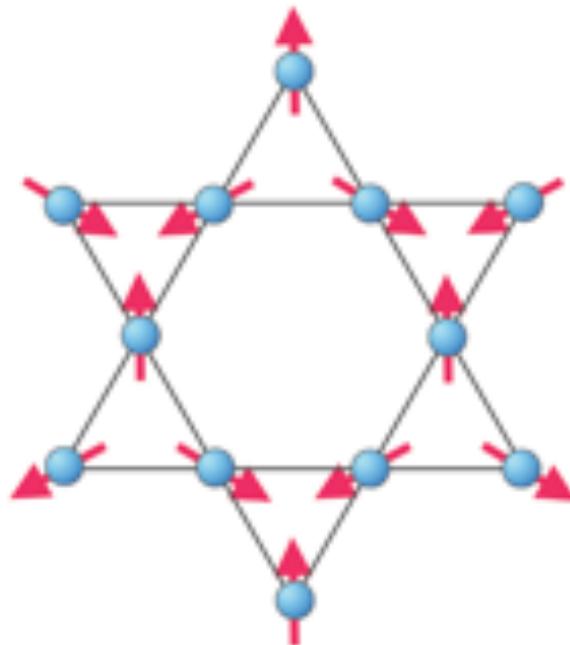


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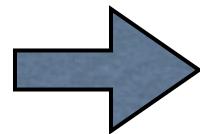
organics

# Kagomé antiferromagnet



$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \dots$$

Very large classical  
degeneracy

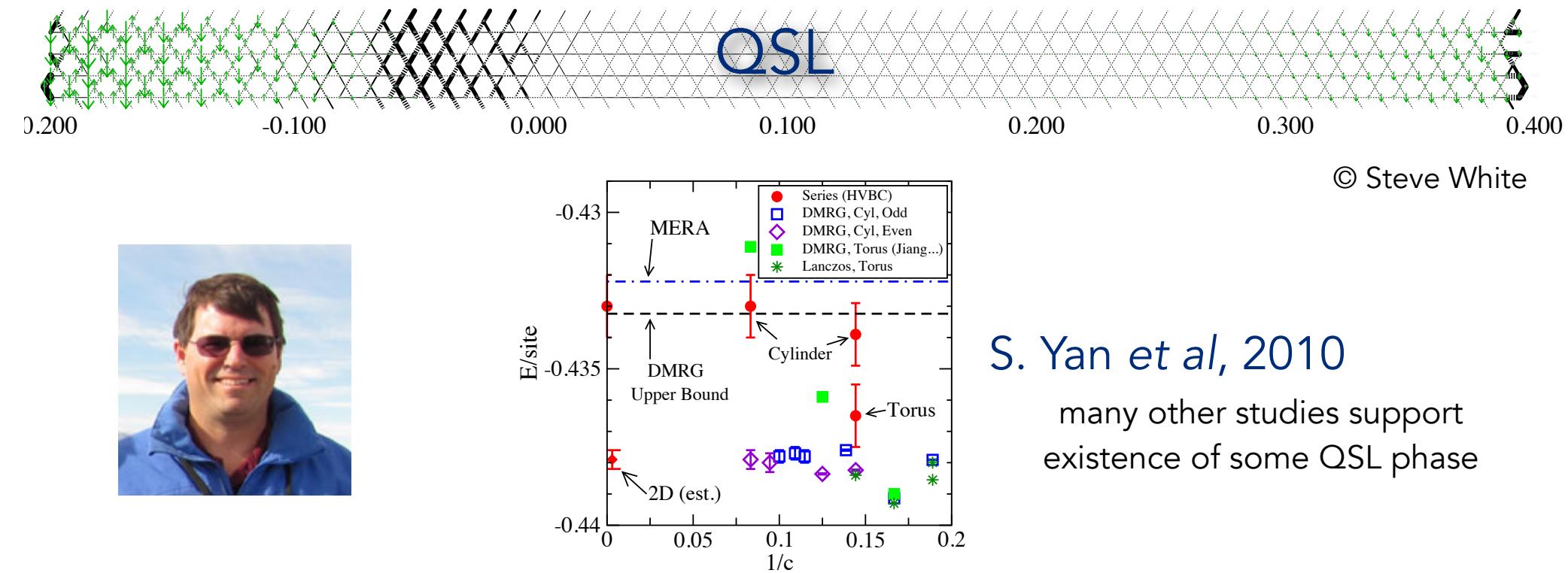


likely to be a QSL

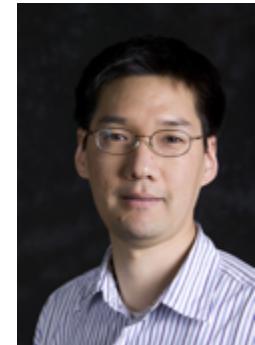
V. Elser, 1989 + many many others

# $S=1/2$ kagomé AF

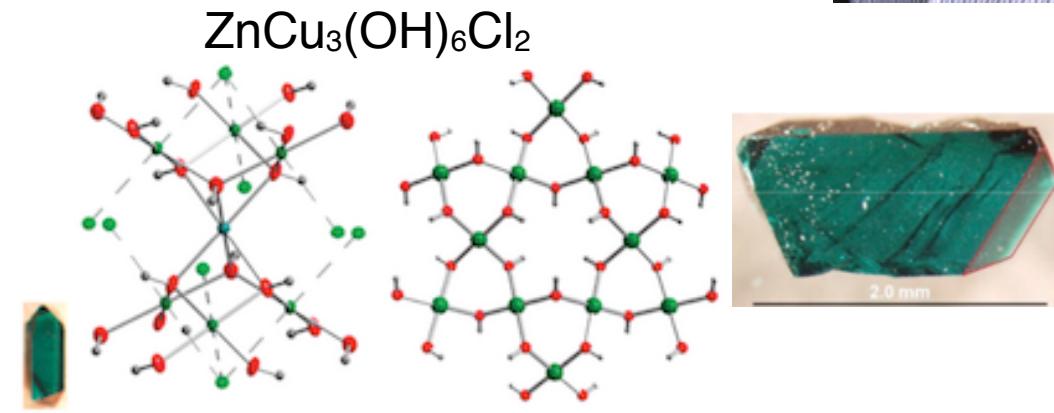
- Rather definitive evidence for QSL by DMRG



# Herbertsmithite

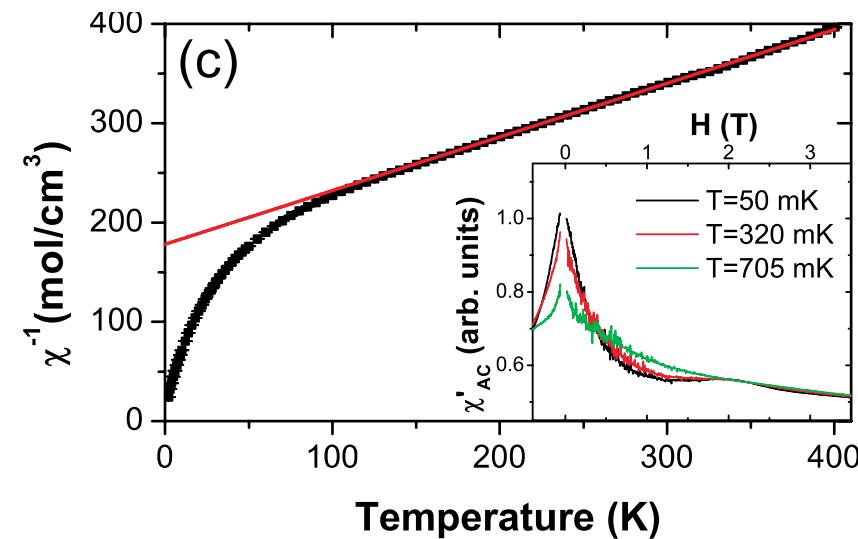


kagomé layers of Cu  
S=1/2 spins, separated  
by non-magnetic Zn



Heisenberg-like  
with  $J \sim 200\text{K}$

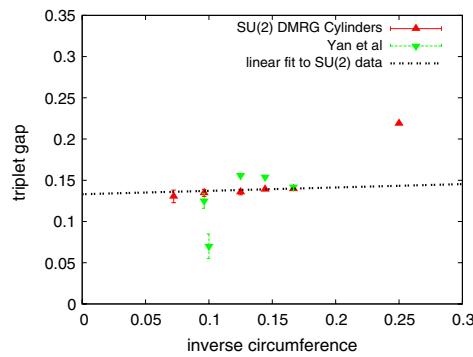
no order down to  
50mK



Helton et al, 2007

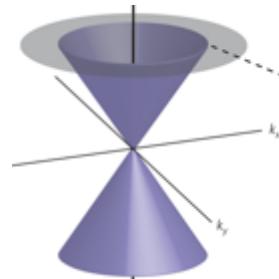
# Theory

- What kind of QSL?



S. Depenbrock *et al*, 2012

gapped,  
topological QSL



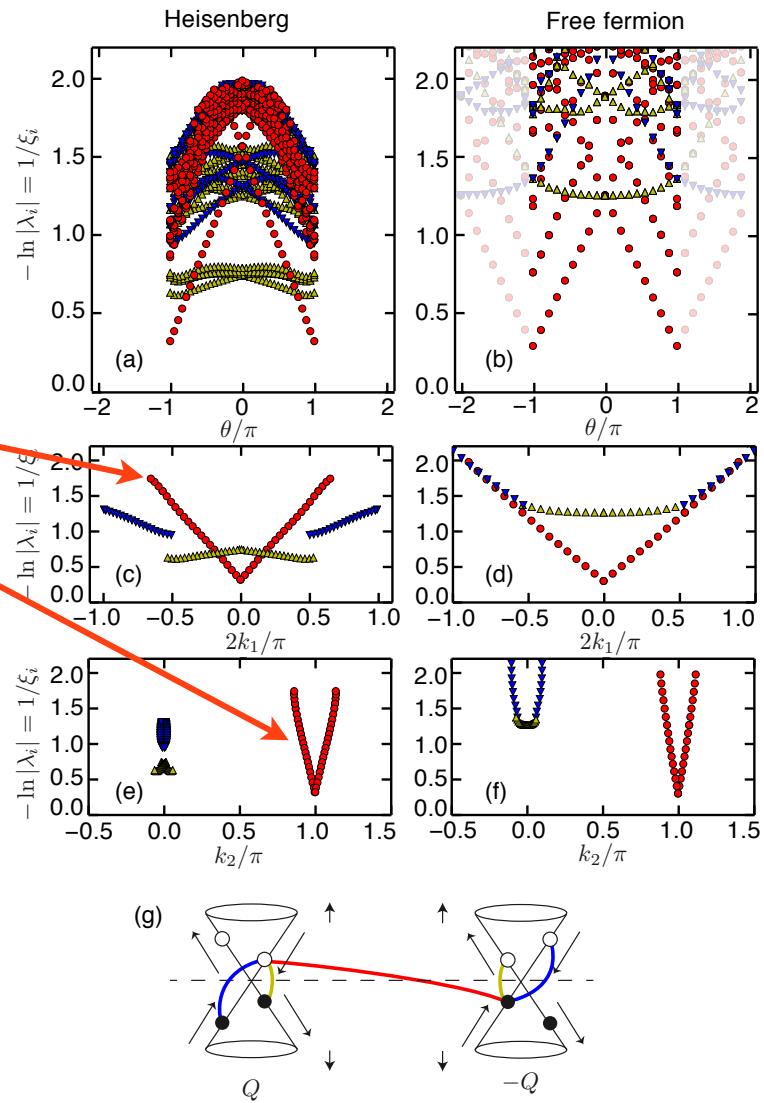
Y. Ran *et al*, 2007  
F. Becca...

gapless  
Dirac QSL

+ various other  
proposals with  
weaker  
quantitative  
support

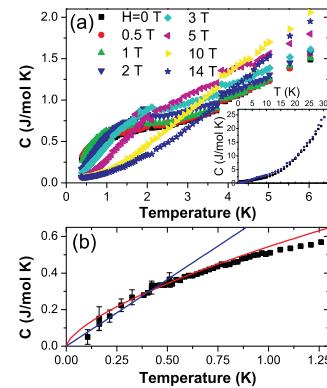
# DMRG (2016)

Y.-C. He *et al*:  
evidence for  
Dirac QSL

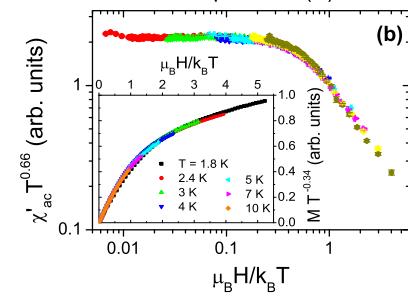


# Herbertsmithite

Lots of early evidence  
for gaplessness



Helton *et al*, 2007

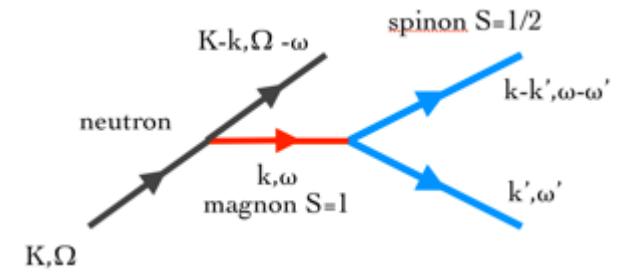
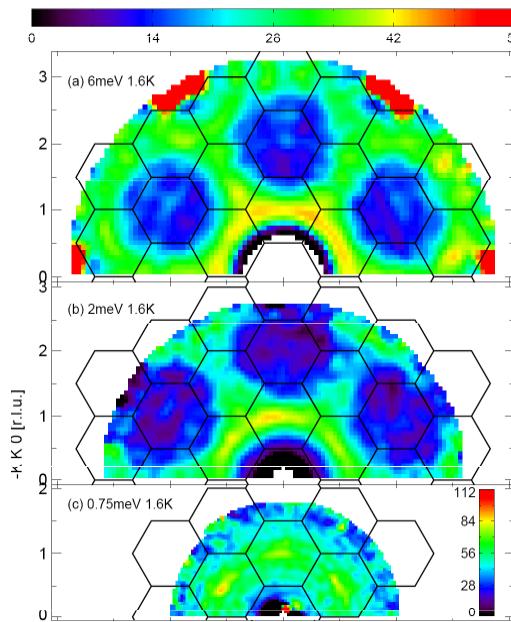


Helton *et al*, 2010

Single crystal INS

smooth continuum  
scattering

T-H Han *et al*, 2012



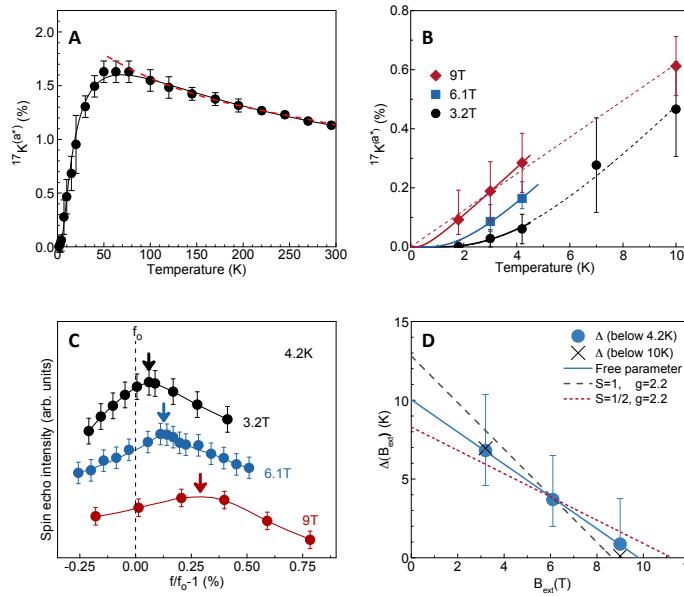
continuum scattering  
expected  
...but probably with more  
structure?

# Herbertsmithite

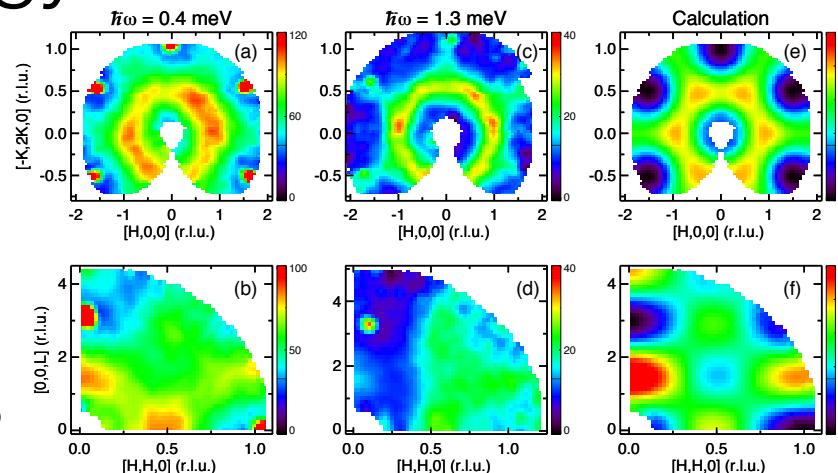
## Single crystal NMR

M. Fu *et al*, 2015

McMaster



## Low energy INS

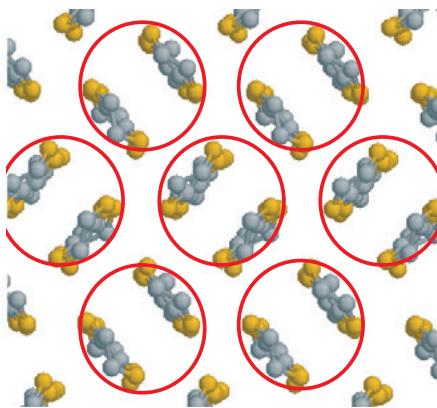


T-H Han *et al*, 2015

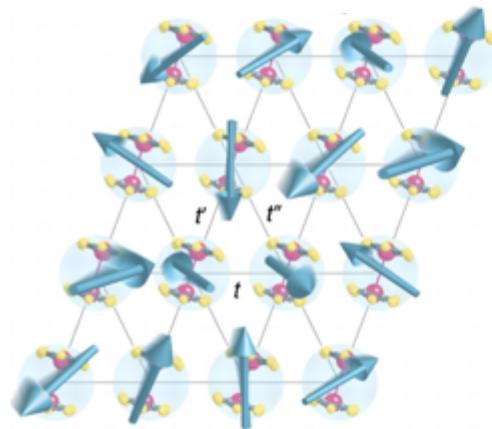
estimate gap ~  
10K

claim to separate  
impurity signal  
below 0.7meV

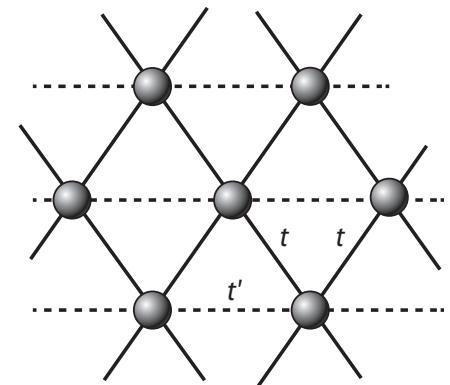
# Organics



$\kappa$ -(ET)<sub>2</sub>X

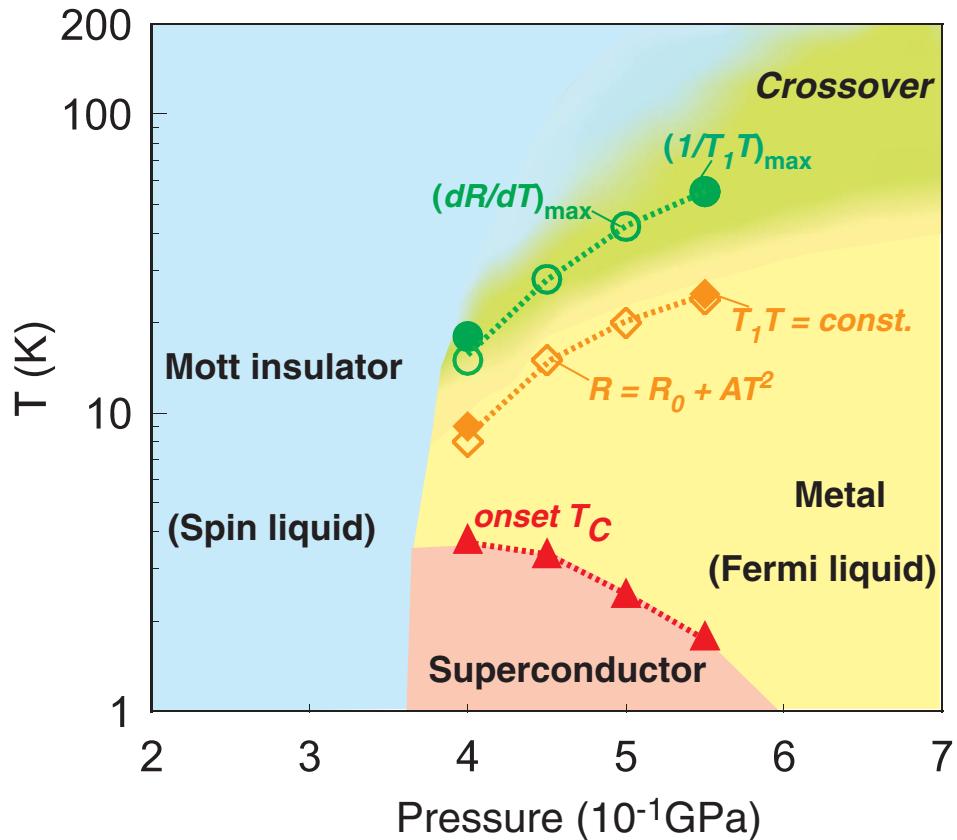


$\beta'$ -Pd(dmit)<sub>2</sub>



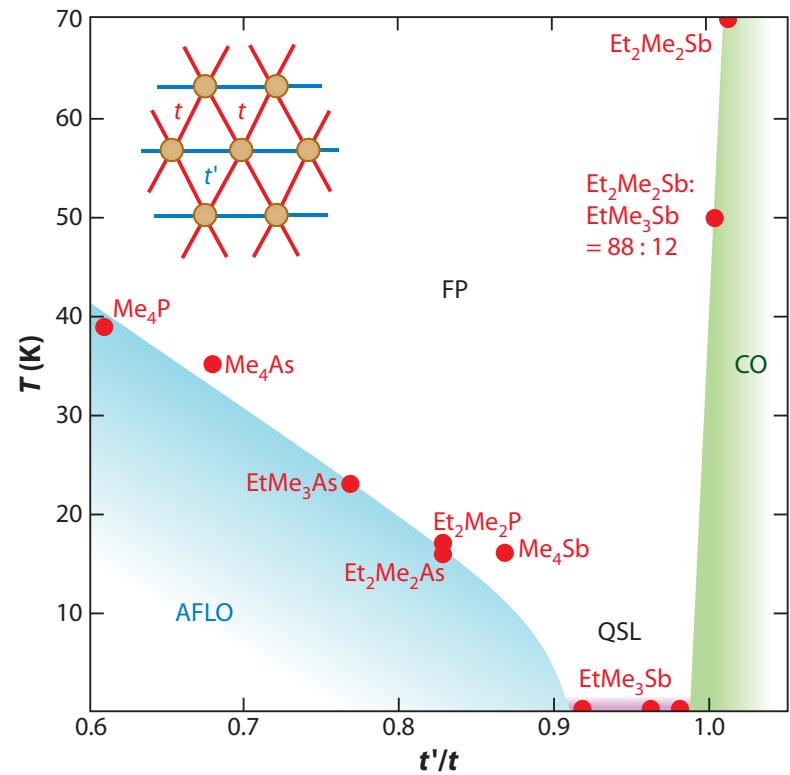
- Molecular materials which behave as effective triangular lattice  $S=1/2$  antiferromagnets with  $J \sim 250\text{K}$
- significant charge fluctuations

# Organics



$\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub>

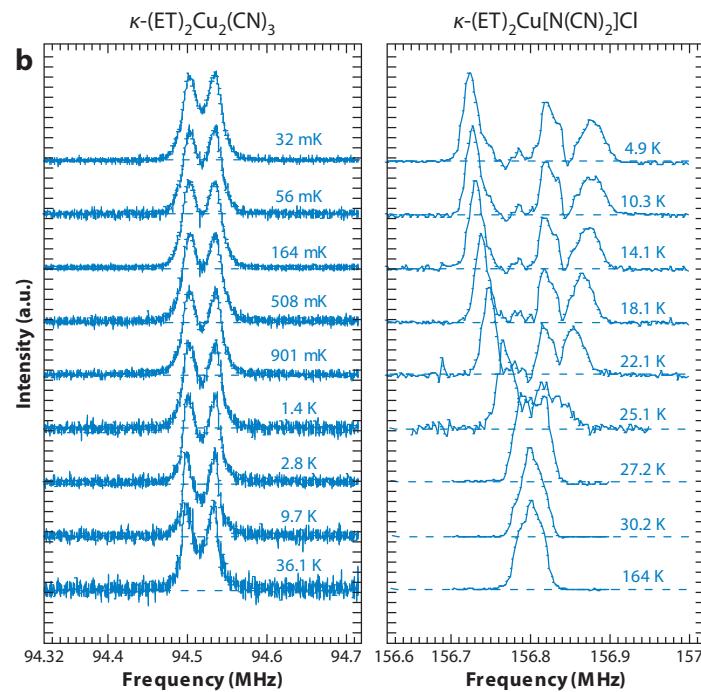
K. Kanoda group (2003-)



$\beta'$ -Pd(dmit)<sub>2</sub>

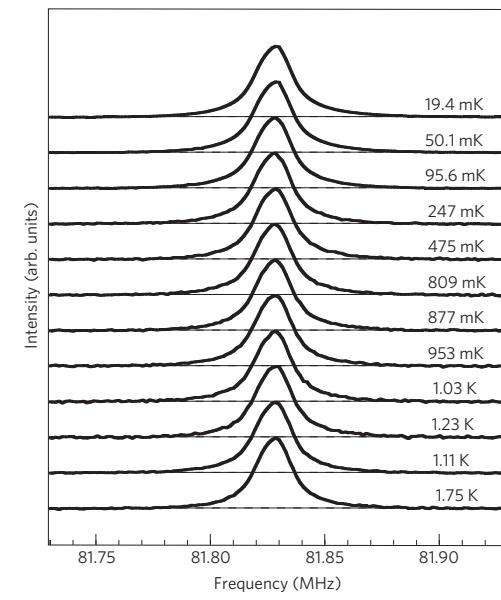
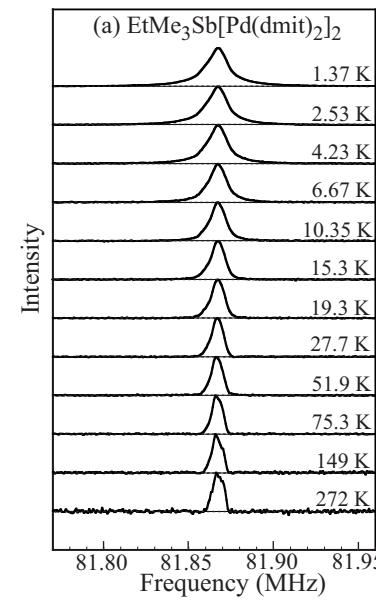
R. Kato group (2008-)

# NMR lineshapes



$\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub>

Y. Shimizu  
et al, 2003 <sup>1</sup>H NMR



$\beta'$ -Pd(dmit)<sub>2</sub>

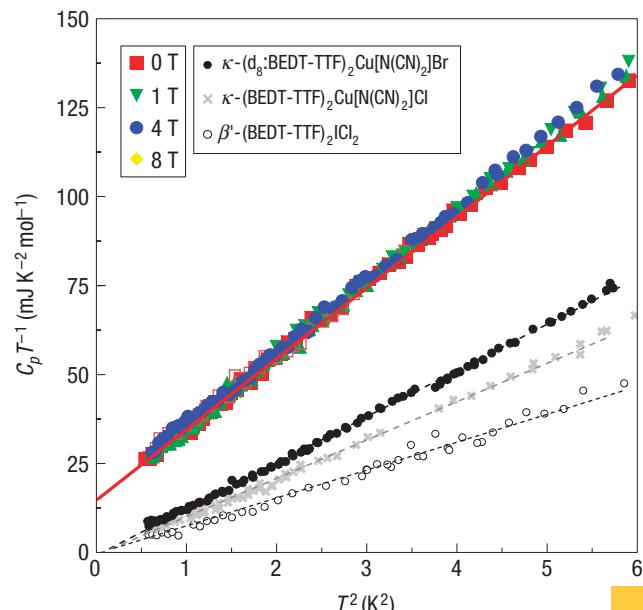
T. Itou et al,  
2008,2010

<sup>13</sup>Cs NMR

Evidence for lack of static moments:  $f > 1000!$

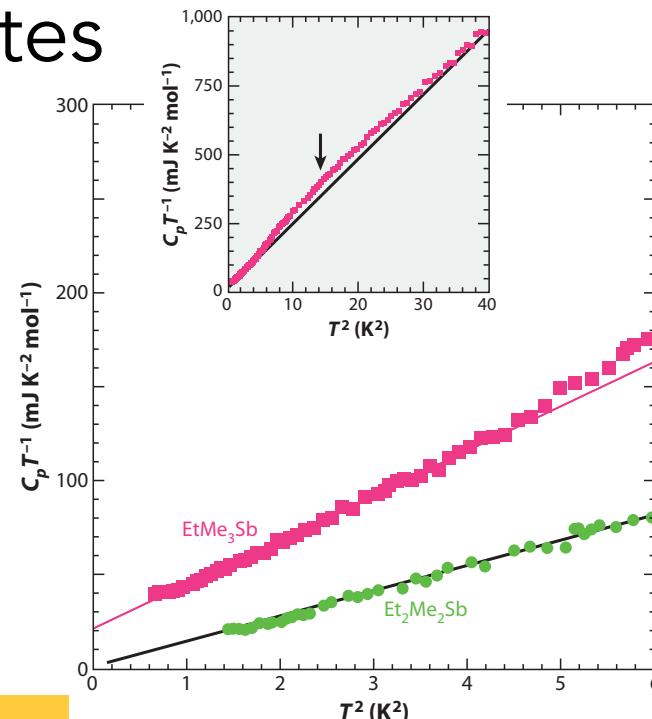
# Specific Heat

- $C \sim \gamma T$  indicates gapless behavior with large density of states



$$\gamma_{\text{Cu}} \sim 0.7 !!$$

$\kappa$ - $(\text{ET})_2\text{Cu}_2(\text{CN})_3$

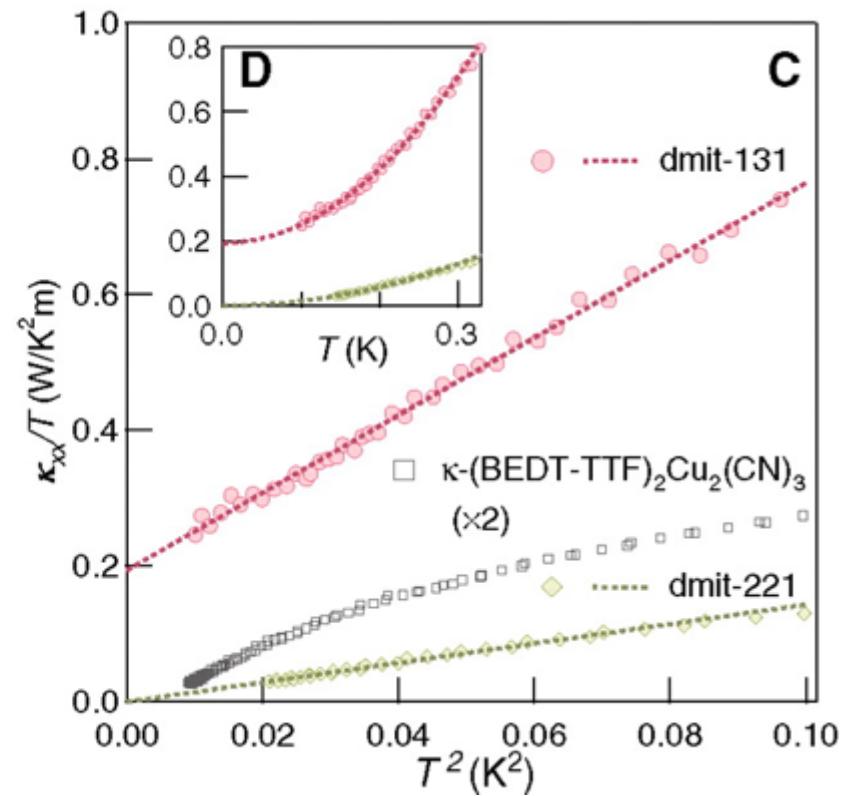


$\beta'$ - $\text{Pd}(\text{dmit})_2$

S. Yamashita *et al*, 2008

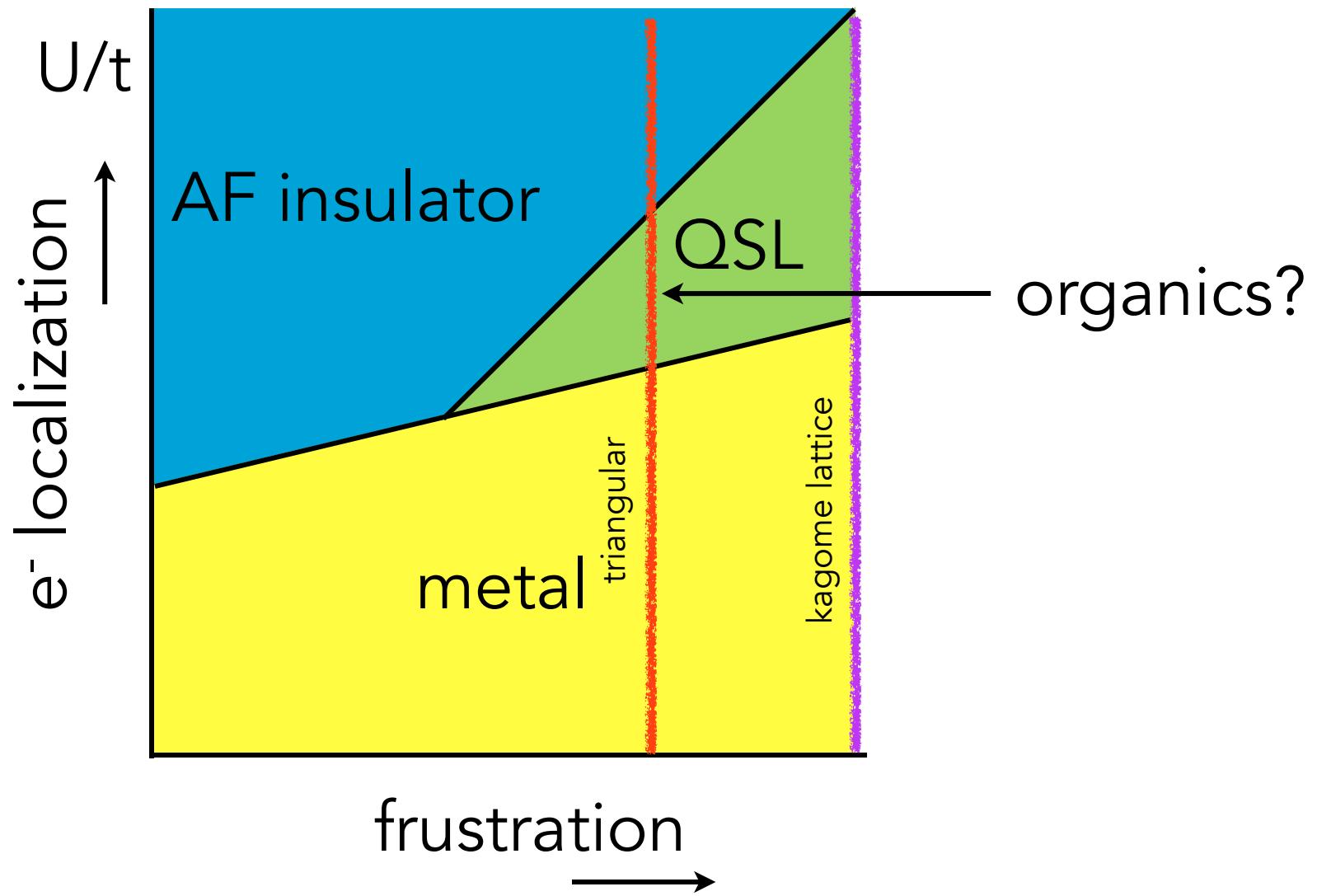
# Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit
- Estimate for a *metal* would correspond to a mean free path  $l \sim 1 \mu\text{m} \approx 1000 \text{ \AA}$  !

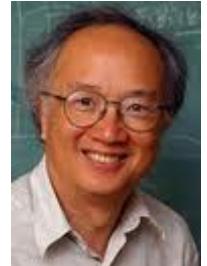


M. Yamashita *et al*, 2010

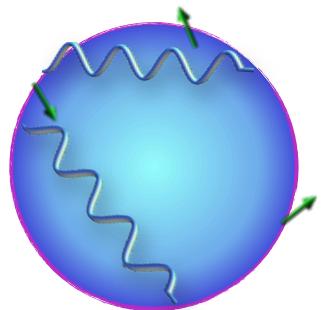
# Charge fluctuations



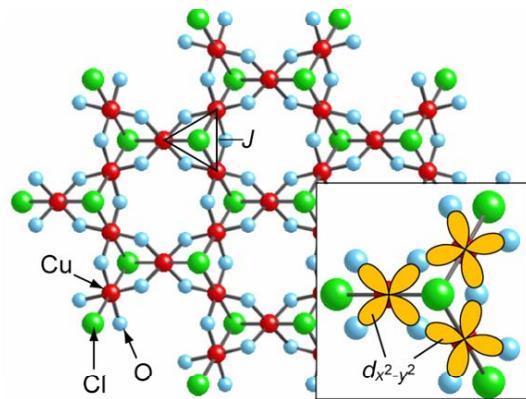
# Organics - Theory



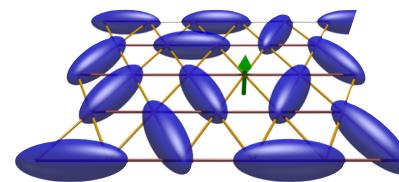
- RVB/QSL state:
  - Motrunich, Lee+Lee: (2005) “uniform RVB”
  - It is described by a **“Fermi sea” of spinons** coupled to a  $U(1)$  gauge field
  - The anomalous thermal conductivity may be a window into an emergent fermi surface in an insulator!



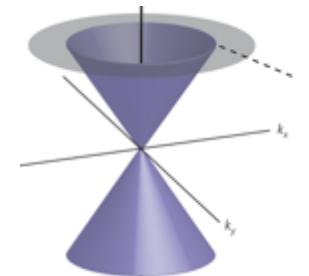
# the new classics



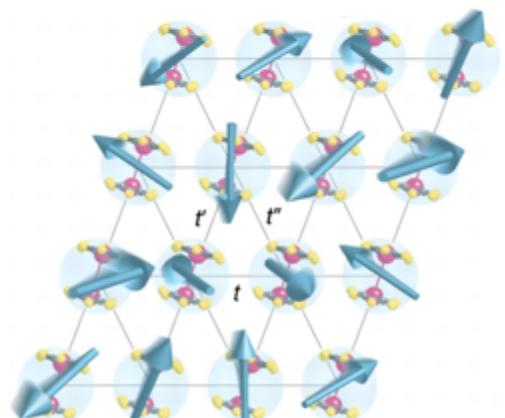
=



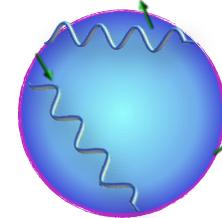
or



herbertsmithite

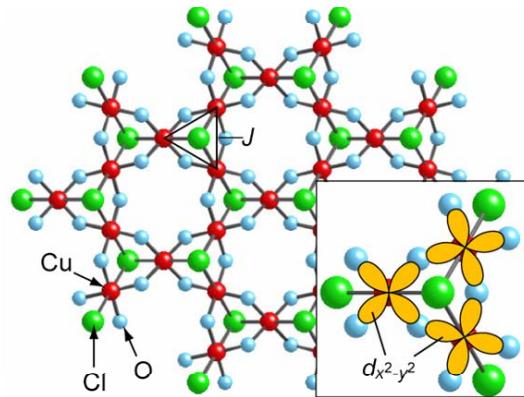


=



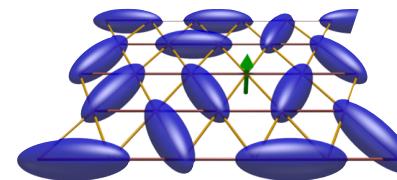
dmit

# the new classics

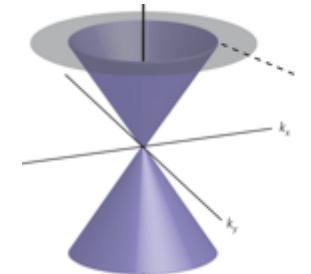


herbertsmithite

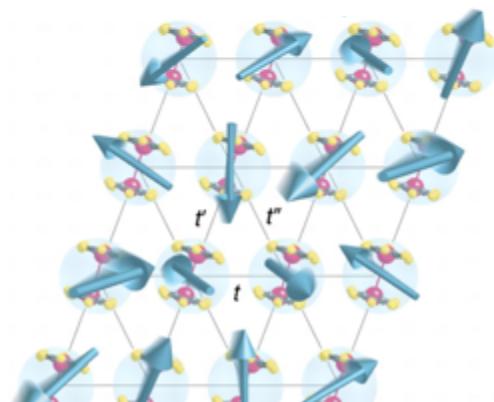
=



or

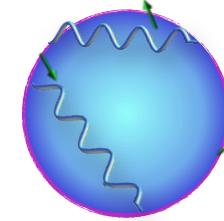


disorder



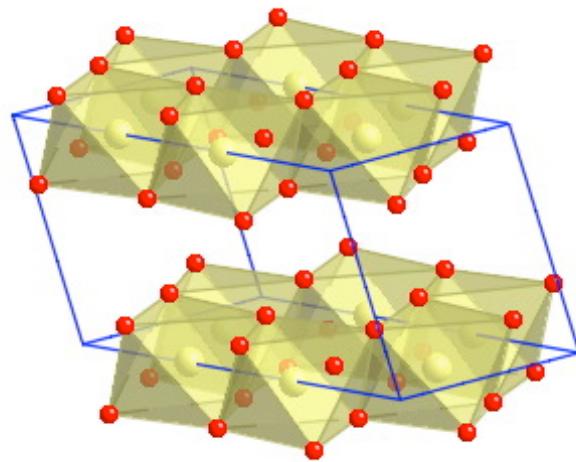
dmit

=

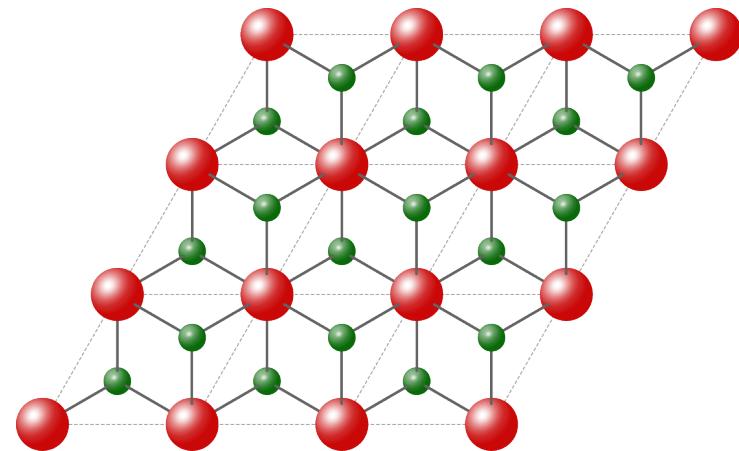


large lattice  
contribution

# New direction: strong anisotropy



Kitaev materials



$\text{YbMgGaO}_4$



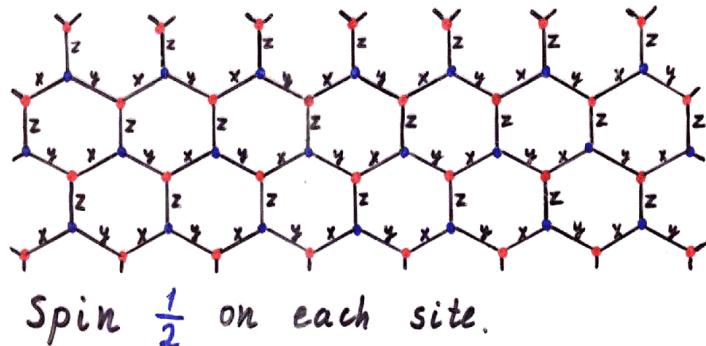
# Kitaev model

Kitaev's honeycomb model

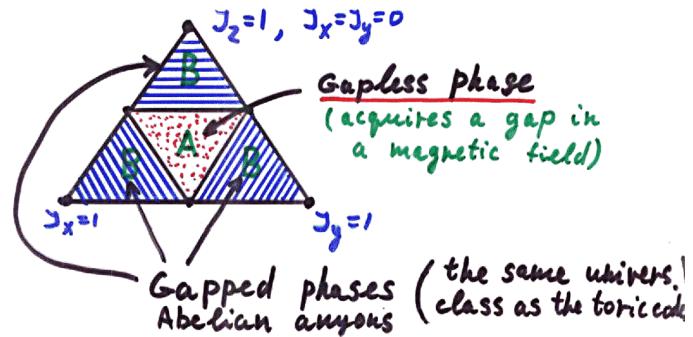
$$H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$$

KITP, 2003

## 1. The model

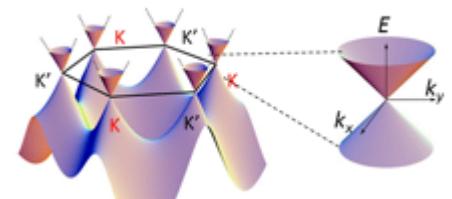


## Phase diagram

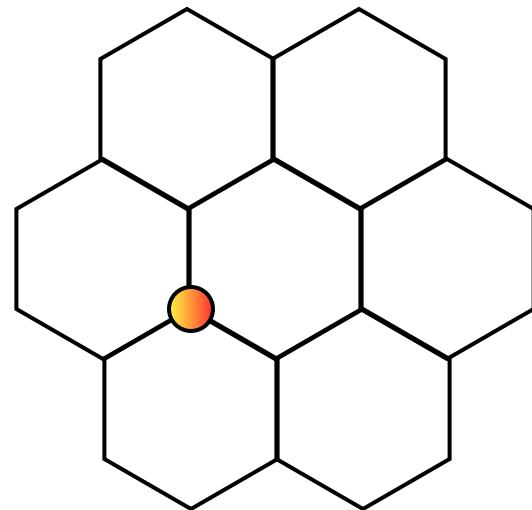


exact parton construction  $\sigma_i^\mu = i c_i c_i^\mu$   $c_i c_i^x c_i^y c_i^z = 1$

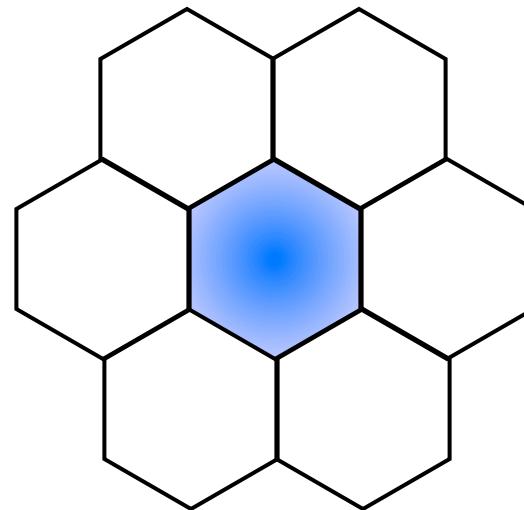
physical Majoranas  $H_m = K \sum_{\langle ij \rangle} i c_i c_j$



# Non-local excitations



Majorana  $\varepsilon$



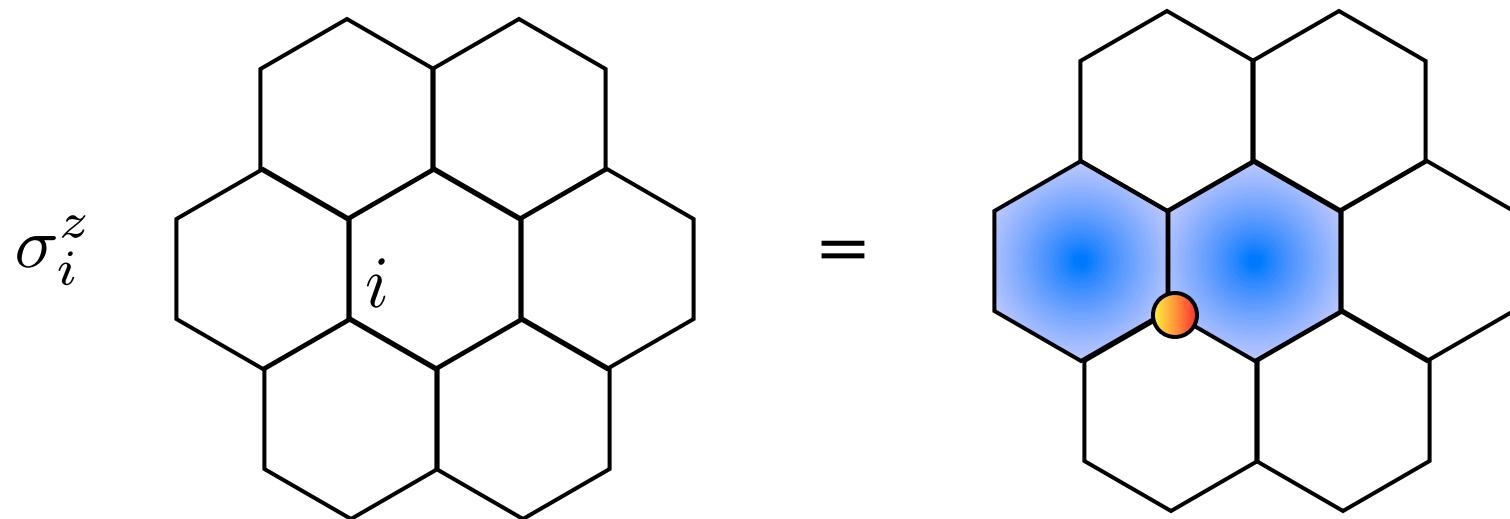
Flux  $e, m$

In Kitaev's model:

- Majorana's dispersion  $\sim K$  and Dirac-like
- Fluxes are localized with small gap

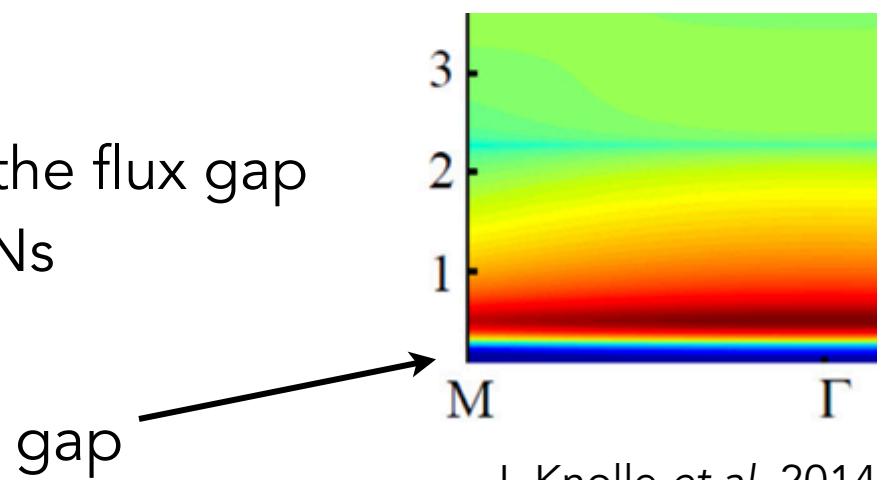
# Fractionalization

Spin flip produces a free Majorana fermion and two immobile fluxes



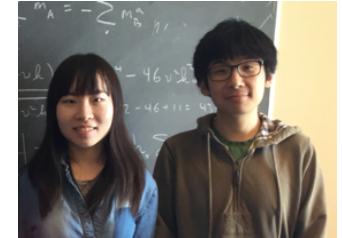
Because fluxes are created

- Spectral weight is zero below the flux gap
- Correlations vanish beyond NNs



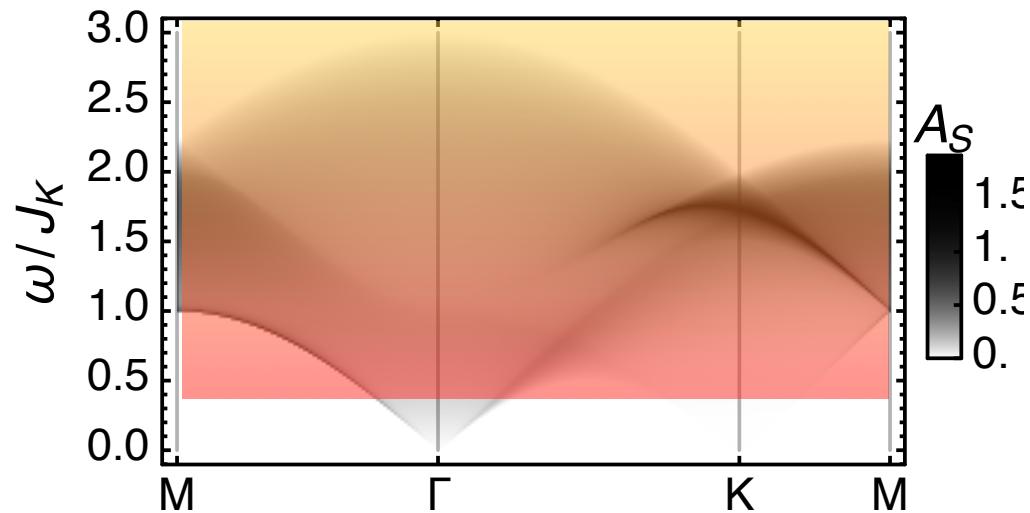
J. Knolle et al, 2014

# Fractionalization



- Another process: fluxes recombine into a second Majorana fermion
- This gives rise to an excitation branch of power-law Dirac “fans”

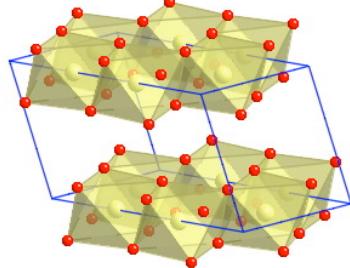
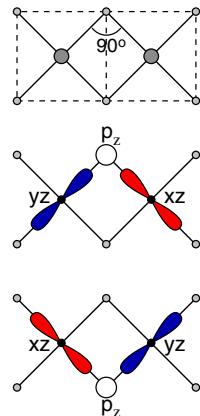
What to look for if we ever discover a true Kitaev QSL - sharp low energy structure



# Kitaev Materials

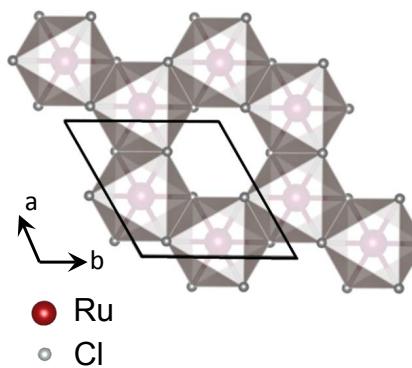
Jackeli, Khaliullin  
2009

Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling



P.Gegenwart  
H. Takagi  
...

$\text{Na}_2\text{IrO}_3$ ,  
 $(\alpha, \beta, \gamma)$ -  
 $\text{Li}_2\text{IrO}_3$

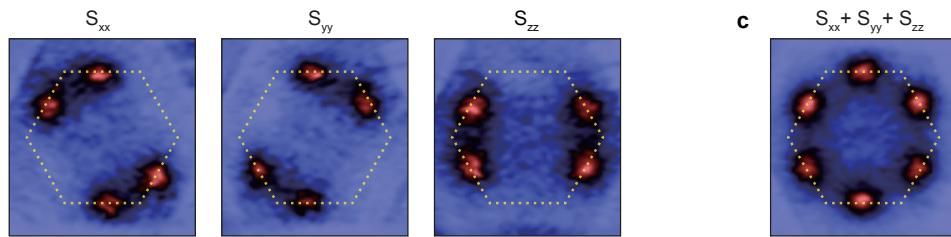


$\alpha\text{-RuCl}_3$

Y.-J. Kim...

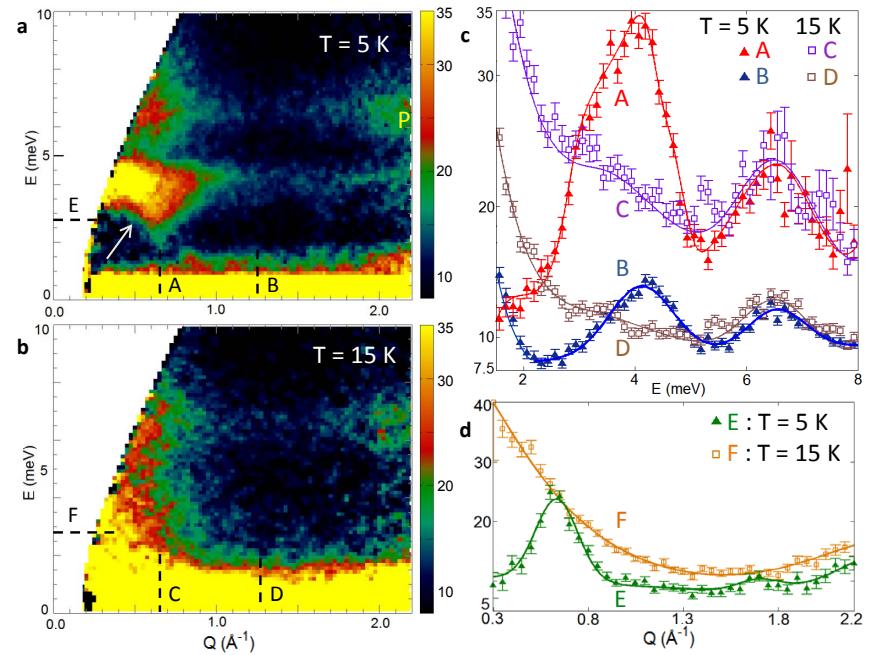
Honeycomb and hyper-honeycomb structures

# Kitaev Materials



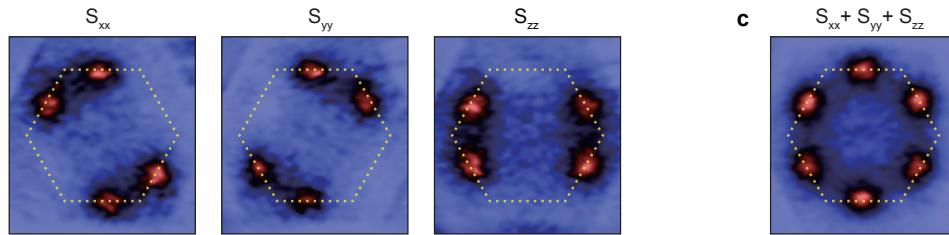
direct evidence for  
direction-dependent  
anisotropic exchange  
from diffuse magnetic  
x-ray scattering in  
 $\text{Na}_2\text{IrO}_3$  (BJ Kim group)

there is pretty strong evidence  
of substantial Kitaev exchange  
in quite a few materials



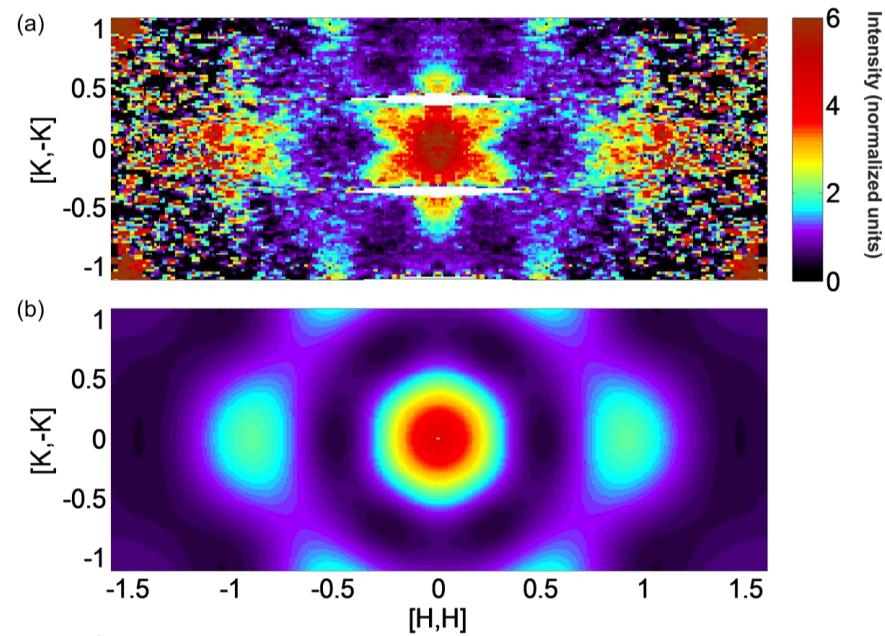
Observation of gapped  
continuum mode persisting  
above  $T_N$  in  $\alpha\text{-RuCl}_3$   
consistent with Majoranas  
(A. Banerjee *et al*)

# Kitaev Materials



direct evidence for  
direction-dependent  
anisotropic exchange  
from diffuse magnetic  
x-ray scattering in  
 $\text{Na}_2\text{IrO}_3$  (BJ Kim group)

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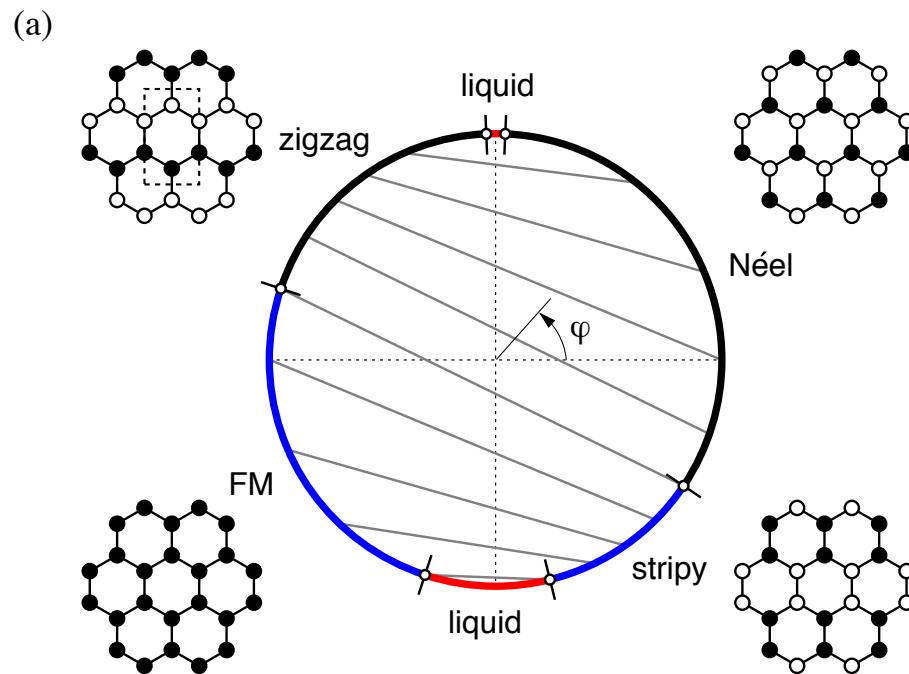


single-crystal data in  $\alpha\text{-RuCl}_3$   
compared to Kitaev's soluble  
model (A. Banerjee *et al*)

# Magnetism

- $\text{Na}_2\text{IrO}_3, \text{Li}_2\text{IrO}_3, \alpha\text{-RuCl}_3$  all order

due to additional interactions,  
e.g. Heisenberg

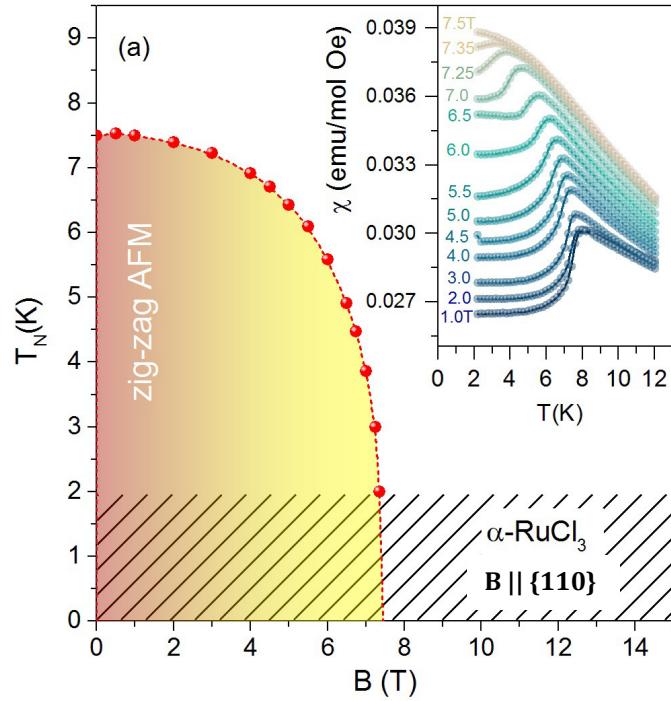


$$\begin{aligned}\mathcal{H} = & \sum_{\langle i,j \rangle} J_1 \mathbf{S}_i \cdot \mathbf{S}_j + K_1 S_i^\gamma S_j^\gamma \\ & + \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \\ & + \sum J_3 \mathbf{S}_i \cdot \mathbf{S}_j\end{aligned}$$

# Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017

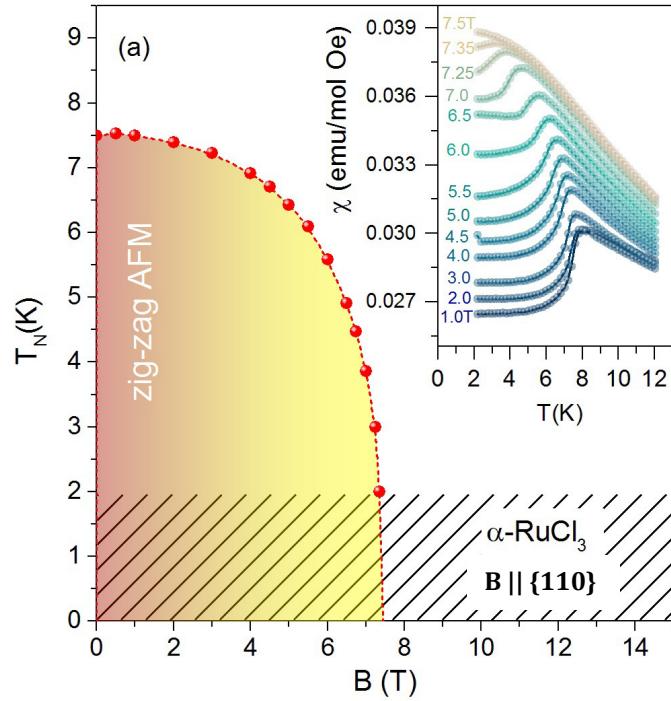


spin liquid or boring  
paramagnet?

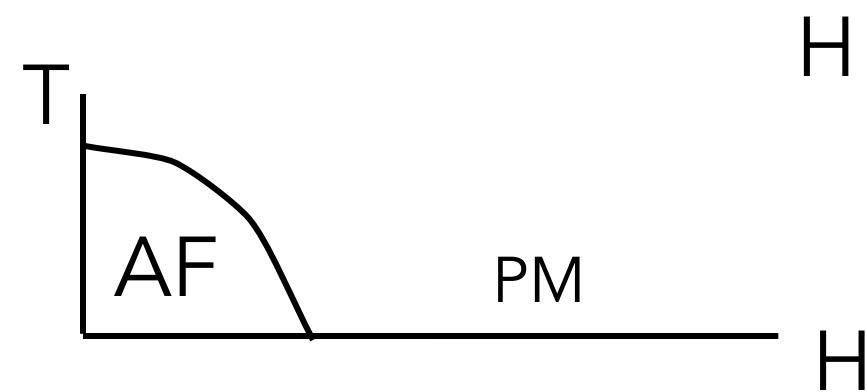
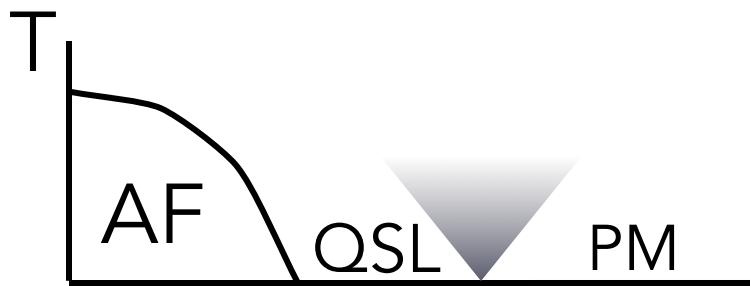
# Suppressing order

With a magnetic field:

A. Banerjee et al, 2017



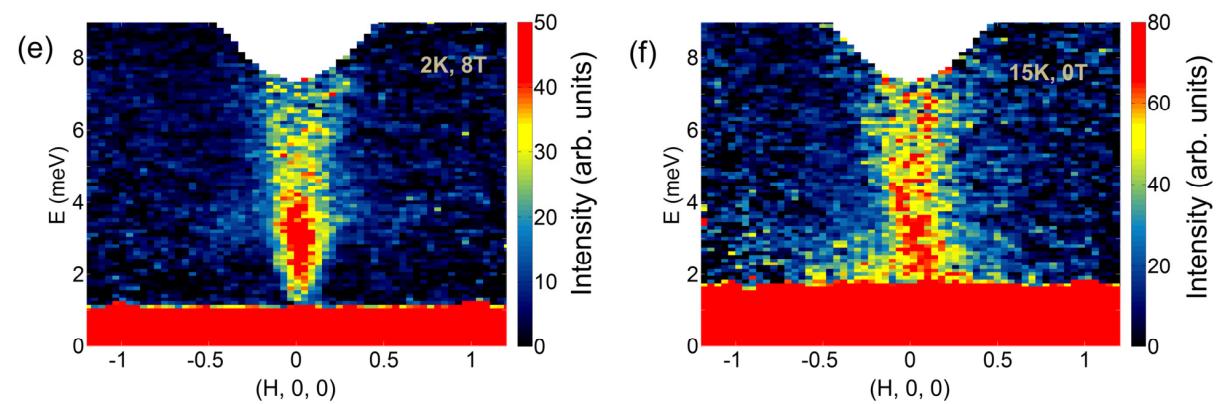
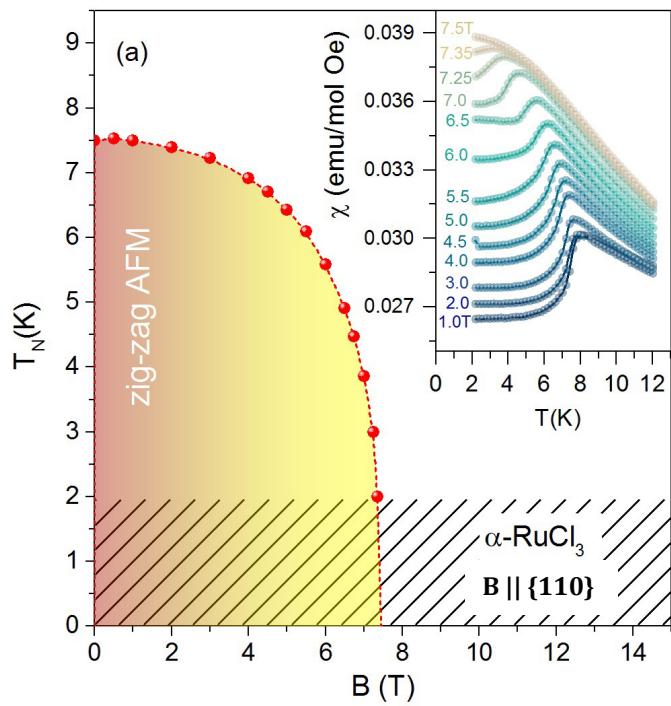
theoretical possibilities



# Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017



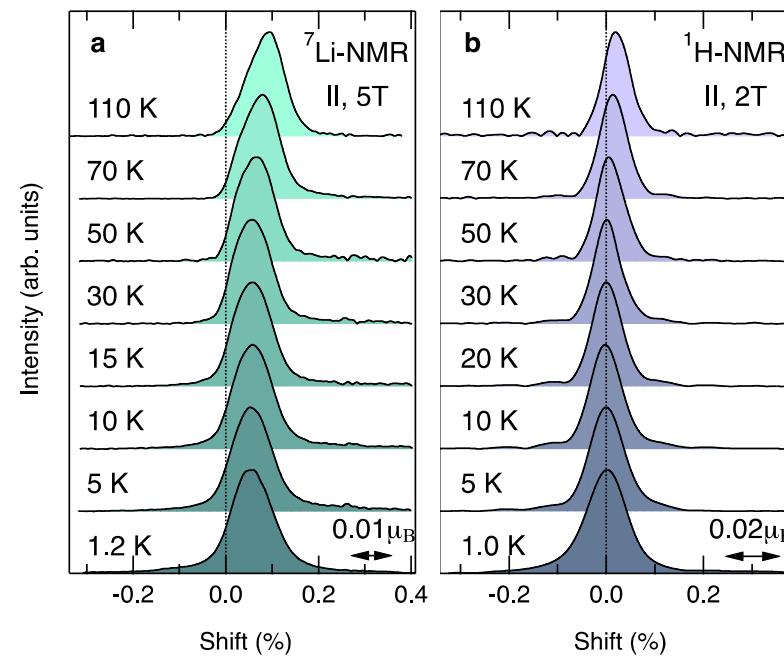
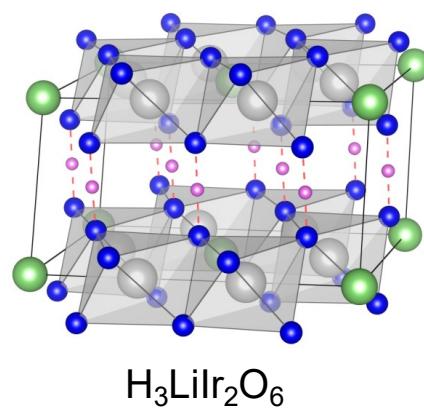
Lots of current studies with many probes...stay tuned!



# Suppressing order

With a new material

K. Kitagawa *et al*, unpublished

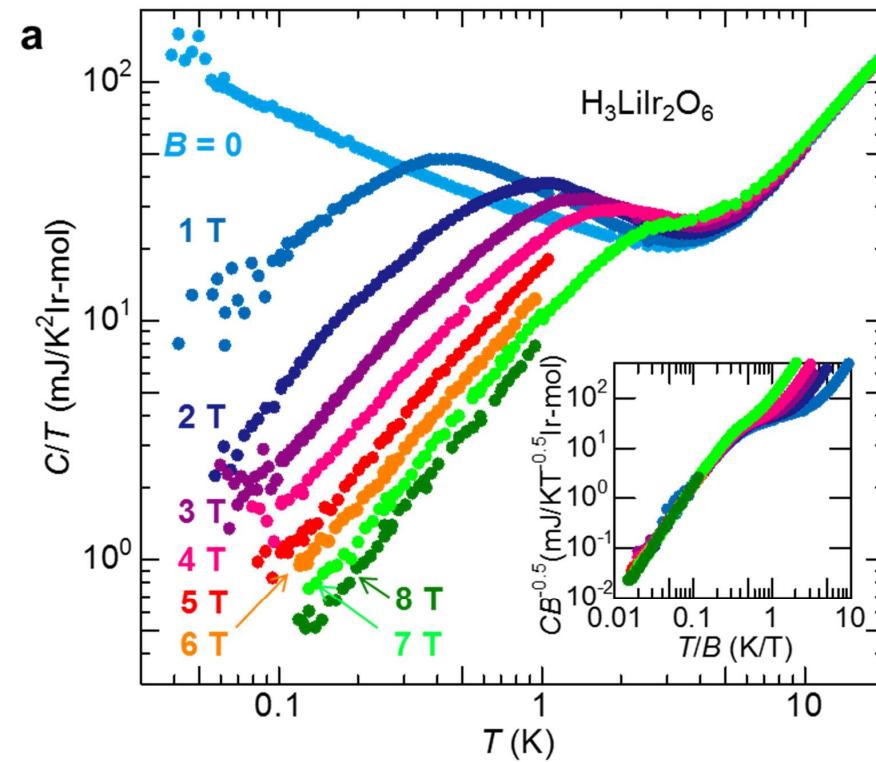
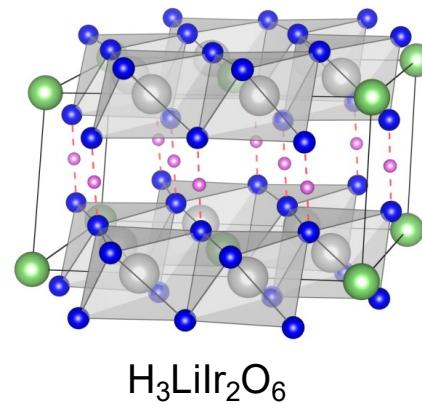




# Suppressing order

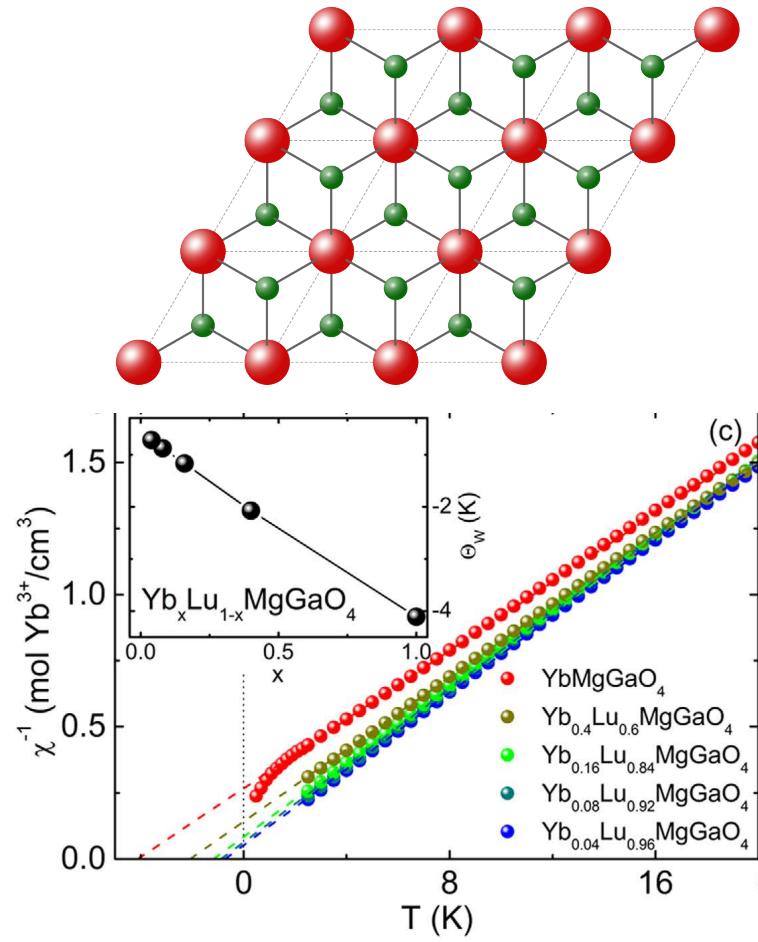
With a new material

K. Kitagawa *et al*, unpublished



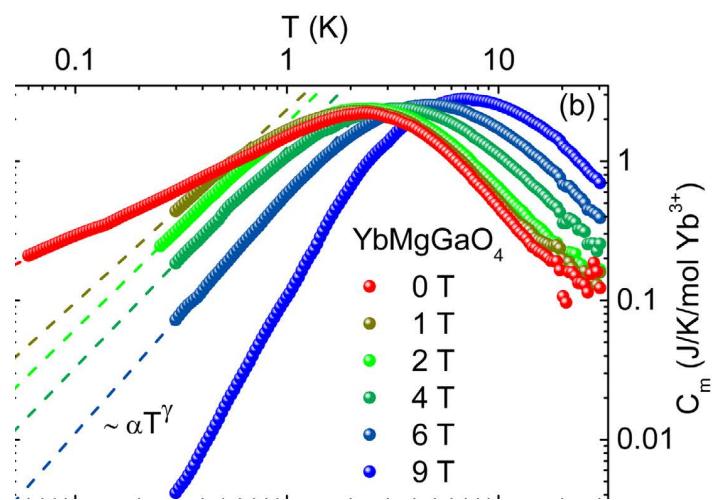
intriguing behavior not expected from Kitaev's solution

# YbMgGaO<sub>4</sub>



$$f \approx 4K/50mK \geq 80$$

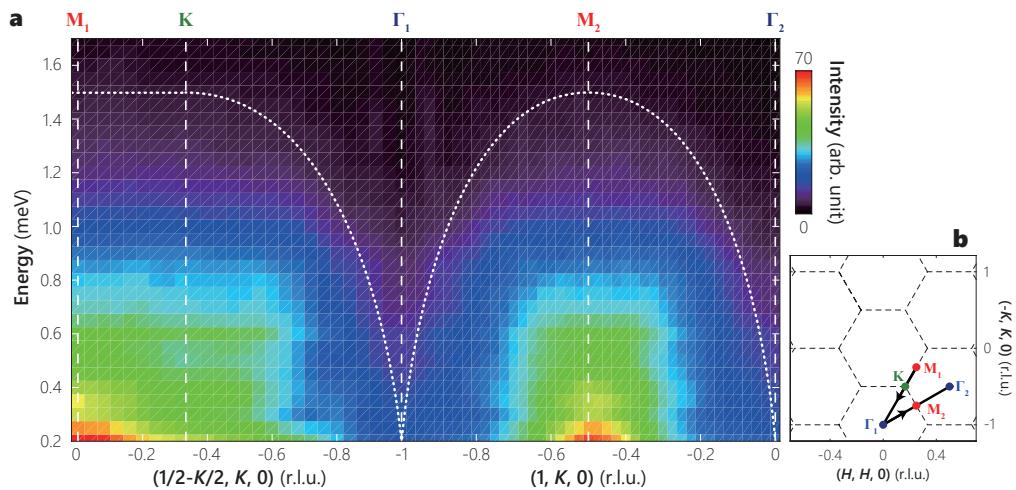
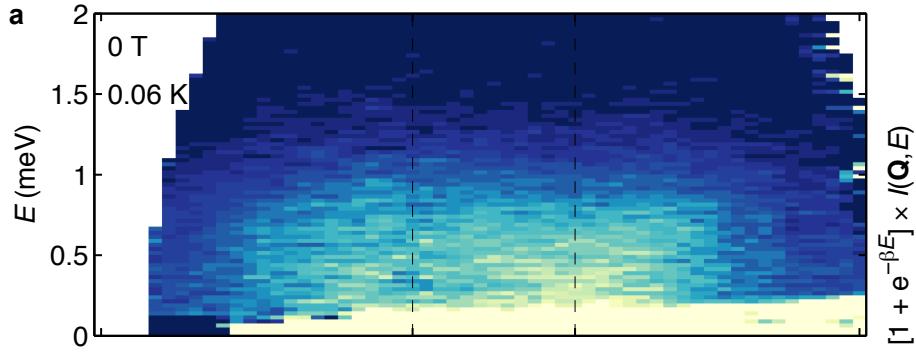
effective S=1/2 triangular lattice



$$C \sim T^{0.7}$$

# YbMgGaO<sub>4</sub>

## Neutron scattering studies



J. Paddison *et al*, arXiv:1607.3231

Y. Shen *et al*, arXiv:1607.02615

Shen *et al* suggest this is  
the structure factor of a  
spinon Fermi surface

a rare earth version  
of organics?

# YbMgGaO<sub>4</sub>

## Periodic Table of Elements

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H Hydrogen 1.00794	2 He Helium 3.99484	3 Li Lithium 6.941	4 Be Boron 9.01218	5 B Boron 10.81	6 C Carbon 12.0107	7 N Nitrogen 14.0067	8 O Oxygen 15.9994	9 F Fluorine 18.9984	10 Ne Neon 20.1797	11 Na Sodium 22.98977	12 Mg Magnesium 24.305	13 Al Aluminum 26.98153	14 Si Silicon 28.0855	15 P Phosphorus 30.97376	16 S Sulfur 32.0655	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.95552	22 Ti Titanium 47.87	23 V Vanadium 50.9415	24 Cr Chromium 51.981	25 Mn Manganese 54.93804	26 Fe Iron 55.845	27 Co Cobalt 58.93198	28 Ni Nickel 58.9345	29 Cu Copper 63.546	30 Zn Zinc 65.401	31 Ga Gallium 69.723	32 Ge Germanium 72.610	33 As Arsenic 74.947	34 Se Selenium 78.96	35 Br Bromine 80.916	36 Kr Krypton 83.798
37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.9068	40 Zr Zirconium 89.824	41 Nb Niobium 91.9053	42 Mo Molybdenum 95.94	43 Tc Technetium 97.912	44 Ru Ruthenium 98.917	45 Rh Rhodium 102.9055	46 Pd Palladium 102.942	47 Ag Silver 107.863	48 Cd Cadmium 112.491	49 In Indium 114.816	50 Sn Tin 118.719	51 Sb Antimony 121.769	52 Te Tellurium 127.604	53 I Iodine 126.9046	54 Xe Xenon 131.902
55 Cs Cesium 132.91084	56 Ba Barium 137.327	57-71	72 Hf Hafnium 178.49	73 Ta Tantalum 180.9479	74 W Tungsten 183.85	75 Re Rhenium 185.227	76 Os Osmium 190.221	77 Ir Iridium 192.217	78 Pt Platinum 190.9989	79 Au Gold 196.9665	80 Hg Mercury 200.532	81 Tl Thallium 204.2	82 Pb Lead 207.2	83 Bi Bismuth 208.9840	84 Po Polonium 209.9870	85 At Astatine 212.0270	86 Rn Radium 222.0770
87 Fr Francium 223	88 Ra Radium 226	89-103	104 Rf Rutherfordium 261	105 Db Dubnium 262	106 Sg Sg Sergoron 263	107 Bh Bh Bohrium 264	108 Hs Hs Hassium 265	109 Mt Mt Moscovium 266	110 Ds Ds Darmstadtium 268	111 Rg Rg Roentgenium 269	112 Db Db Darmstadtium 269	113 Uut Uut Uutonium 270	114 Uup Uup Uuponium 270	115 Uuh Uuh Uuhonium 270	116 Uuo Uuo Uuoonium 270	117 Uus Uus Uusonium 270	118 Uuo Uuo Uuoonium 270

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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Yb

Interesting difference  
from organics:  
**strong spin-orbit  
coupling**

Anisotropic interactions like Kitaev but no exact solution. What do we do? Can QSLs compete? Is spinon Fermi surface favored?



# SOC triangular

Generic model for “flat” triangular lattice

$$H = \sum_{\langle ij \rangle} \left[ J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_z S_i^z S_j^z \right. \\ \left. + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \right. \\ \left. + i J_{\pm z} (\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j)) \right]$$

XXZ  
bond-dependent  
couplings

Y. Li *et al*, 2015

Tool: variational wavefunctions

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

Gutzwiller projection of  
*spin-orbit coupled free*  
fermions



# SOC triangular

Generic model for “flat” triangular lattice

$$H = \sum_{\langle ij \rangle} \left[ J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_z S_i^z S_j^z \right. \quad \text{XXZ}$$

$$+ J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)$$
$$+ i J_{\pm z} (\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j)) \right] \quad \text{bond-dependent}$$

couplings

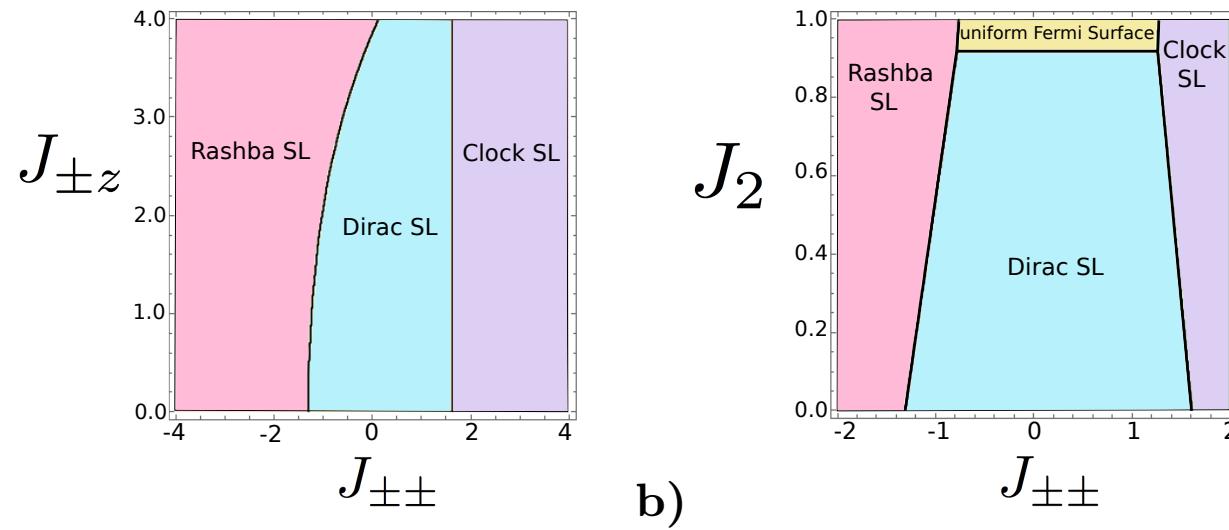
Tool: variational wavefunctions w/ VMC

- classify and compare energetics of all QSLs with triangular symmetry and their stability against magnetic order

# SOC triangular



## Comparing QSLs



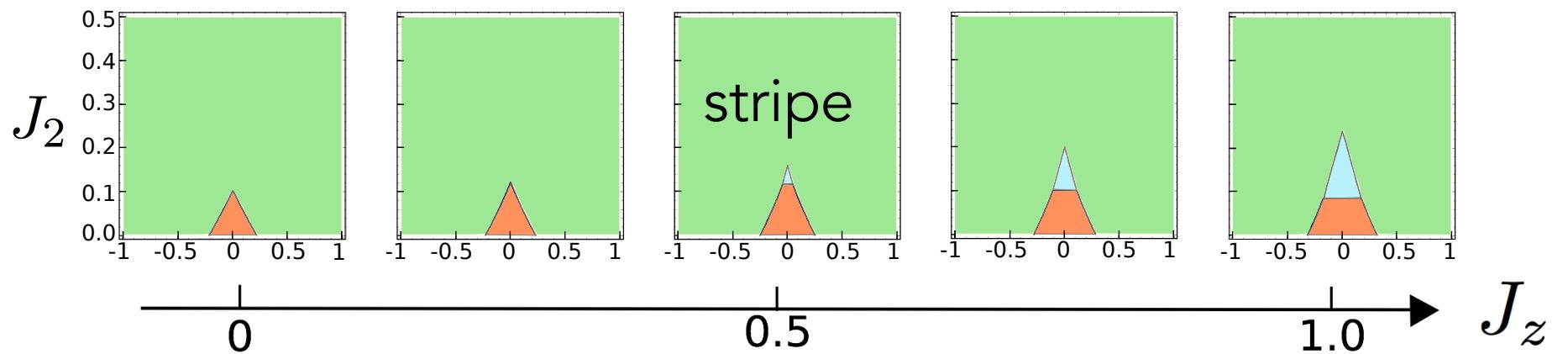
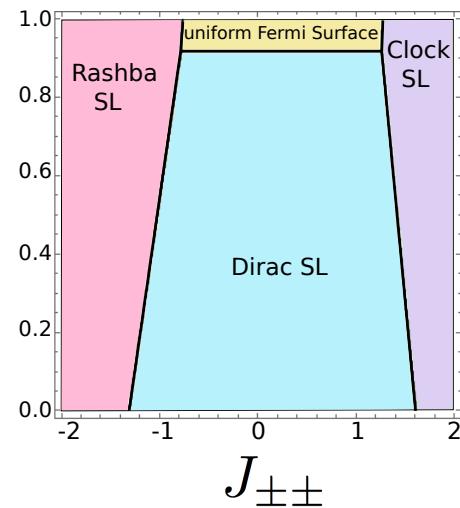
Dirac QSL dominates unless anisotropy is  
very strong

# SOC triangular



Allow magnetic  
order

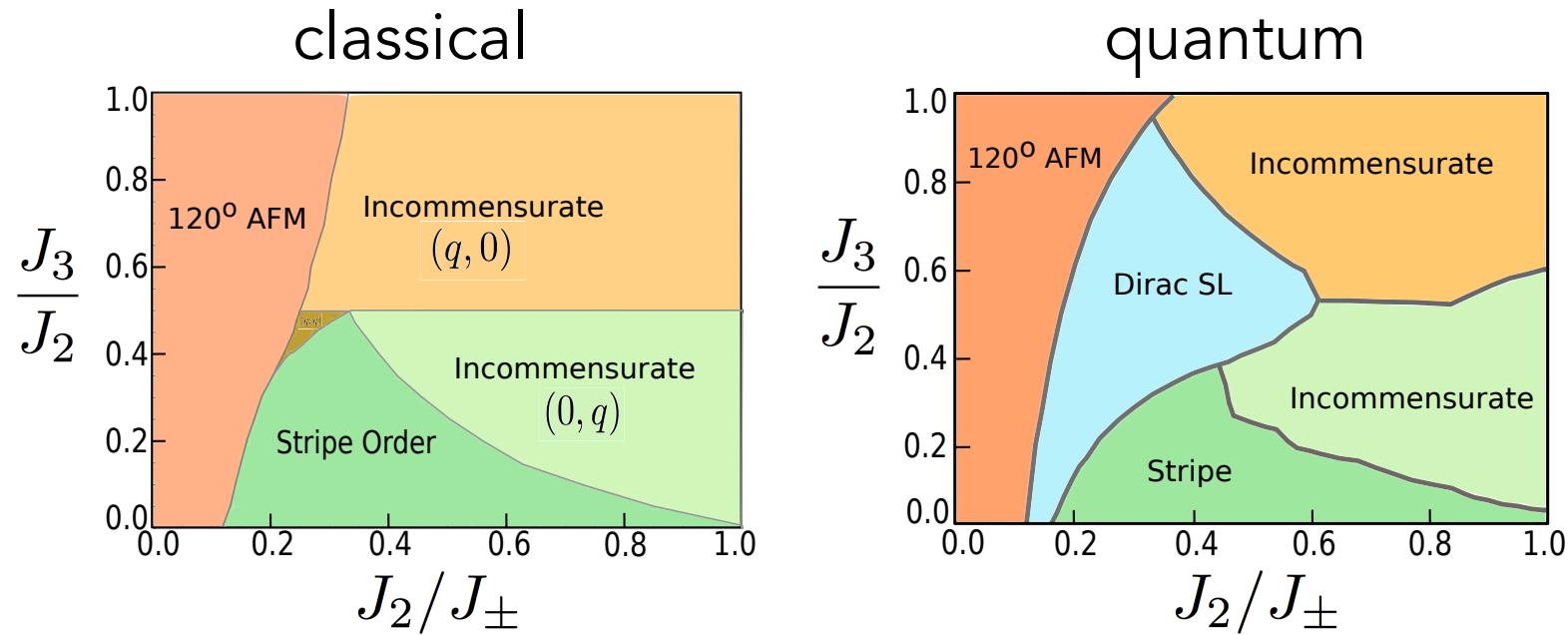
$J_2$



# SOC triangular



## QSLs versus magnetic order



Ordered states outcompete all but the Dirac QSL according to standard Gutzwiller method



# SOC triangular

Any hope for the Fermi surface?

$$|\Psi\rangle = e^{-\alpha H_{\pm\pm}} \hat{P}_G |\psi_{FF}\rangle$$

*beyond Gutzwiller:  
qualitative effects  
due to SOC*

$$E_{uFS} = -0.4693(1 + J_z/4) - \frac{0.39 J_{\pm\pm}^2}{J_{\pm} + 1.42 J_z},$$

$$E_{Dirac} = -0.7054(1 + J_z/4) - \frac{0.21 J_{\pm\pm}^2}{J_{\pm} + 0.87 J_z}.$$

FS might  
compete with  
larger anisotropy,  
and if stripy order  
is removed

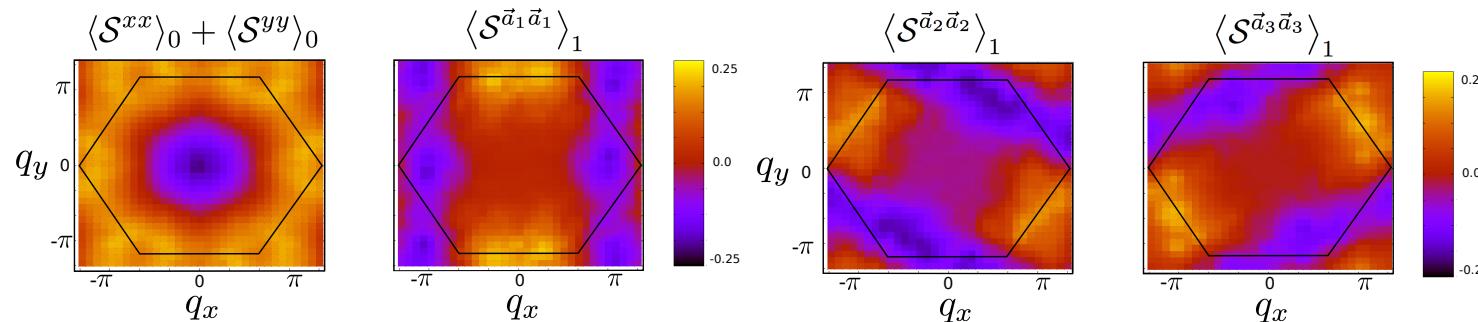
# SOC Fermi surface



$$|\Psi\rangle = e^{-\alpha H_{\pm\pm}} \hat{P}_G |\psi_{FF}\rangle$$

Physical effects!

## 1. Spin correlations



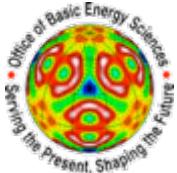
## 2. Anomalous thermal Hall effect

field-induced Berry  
curvature

$$\kappa_{xy} \sim \frac{Th^3 J_{\pm z}^2}{J^5}$$

# Summary

- Strongly SOC magnets are a new arena for QSLs
- VMC techniques are a systematic way to study their complex phases and fairly check the competition between QSLs and ordering
  - interesting to apply to Kitaev materials
- New physical effects: anisotropic spin correlations and thermal Hall effect appear through SOC's influence.

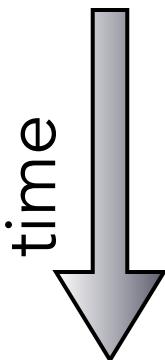


# A future history of magnetism (from 2014)

~500BC: Ferromagnetism  
documented in Greece,  
India, used in China



sinan, ~200BC



1949AD: Antiferromagnetism  
proven experimentally

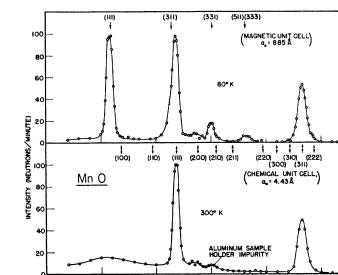
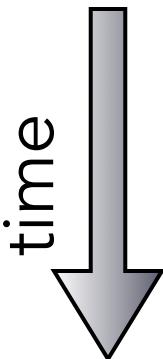


FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

~2016AD: Conclusive experiments on  
quantum spin liquids?

# A future history of magnetism ?

~500BC: Ferromagnetism  
documented in Greece,  
India, used in China



1949AD: Antiferromagnetism  
proven experimentally

~2019AD?: Conclusive experiments on  
quantum spin liquids?

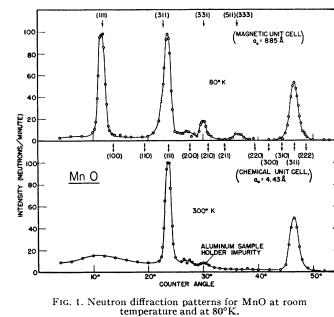


FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

# Thanks for your attention

References here:

<https://spinsandelectrons.com/>

<https://spinsandelectrons.com/pedagogy/>

