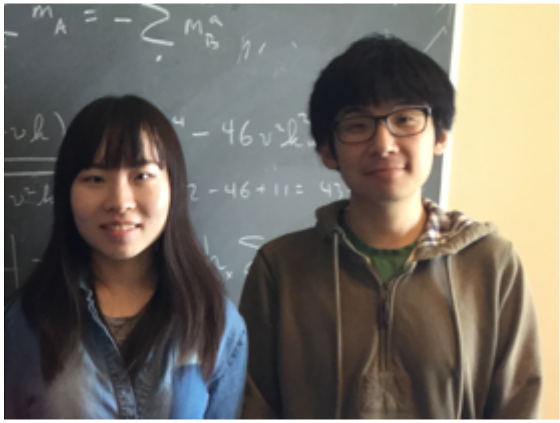

Quantum Spin
Liquids
Leon Balents, KITP
LT28, Götheborg

Collaborators (whose work I'll mention)



Xue-Yang Song
Yi-Zhuang You

Chunxiao Liu

Lucile Savary

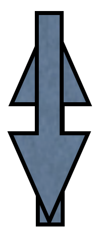
Gábor Halász

Jason Iaconis

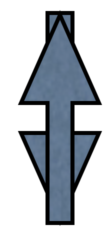
Xiao Chen

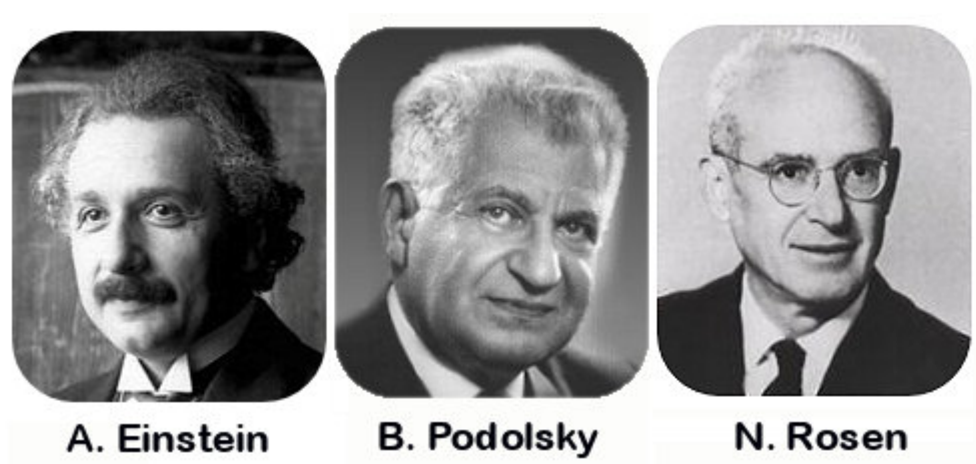
Quantum non-locality

EPR $$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

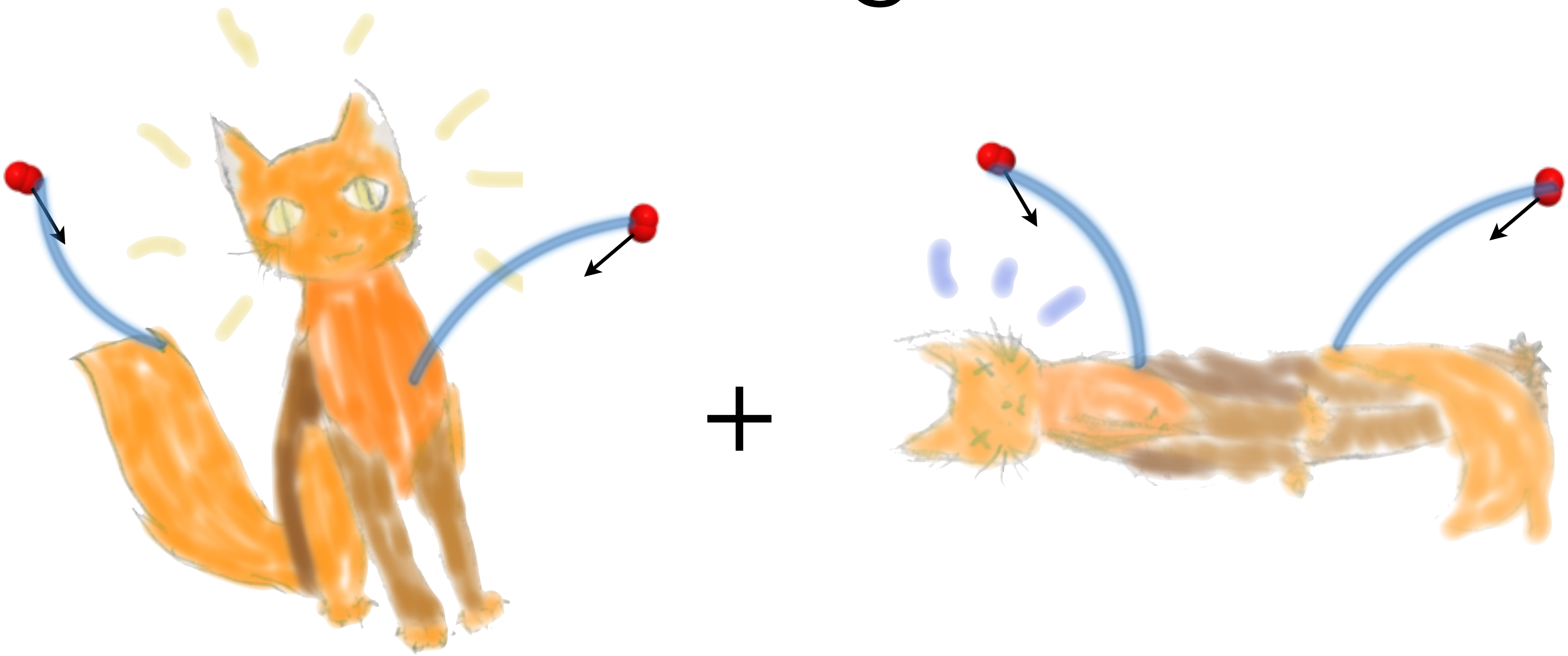


??where is the information??





Schrödinger's Cat

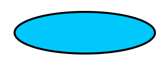


UNSTABLE to decoherence - uncontrolled entanglement with the environment

Strange Stuff

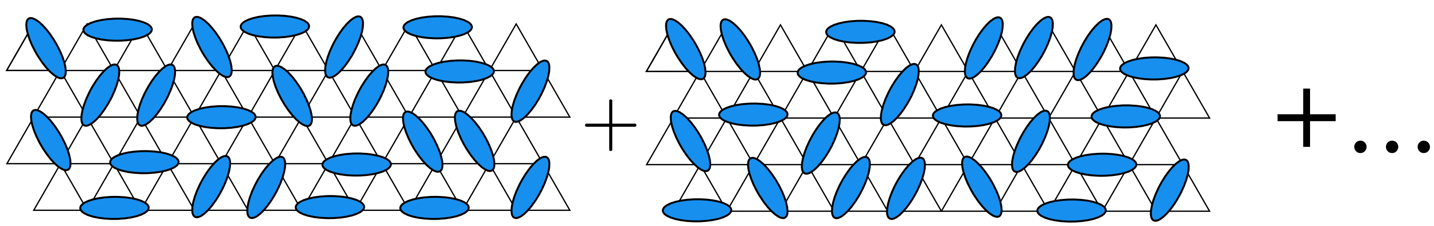
Phil Anderson, 1973

a "quantum liquid" of spins



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Ψ =

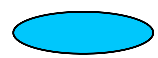


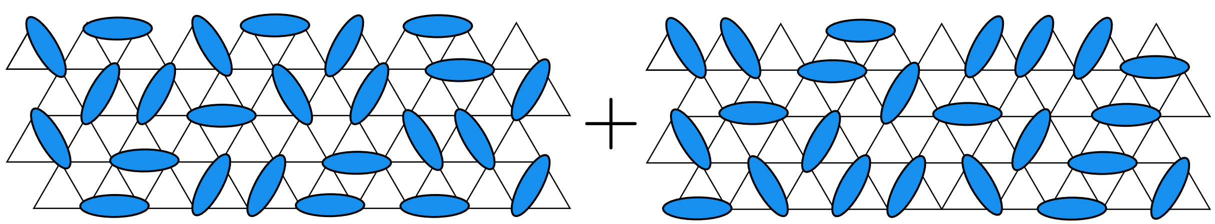
Resonating **V**alence **B**ond state

Strange Stuff

Phil Anderson, 1973

a "quantum liquid" of spins

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Ψ =  + ...

Resonating **V**alence **B**ond state

Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties

- Measurements far away do not affect one another
- From local measurements we can deduce the global state

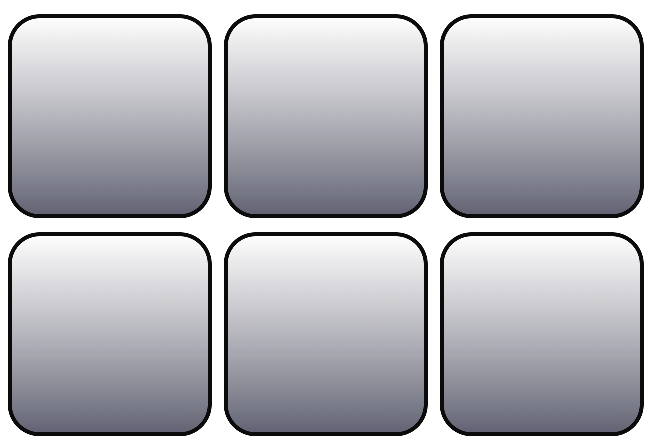
Ordinary (local) Matter

Hamiltonian is local

$$H = \sum_{\mathbf{x}} \mathcal{H}(x) \qquad \mathcal{H}(x) \text{ has local support near x}$$

Ground state is "essentially" a product state

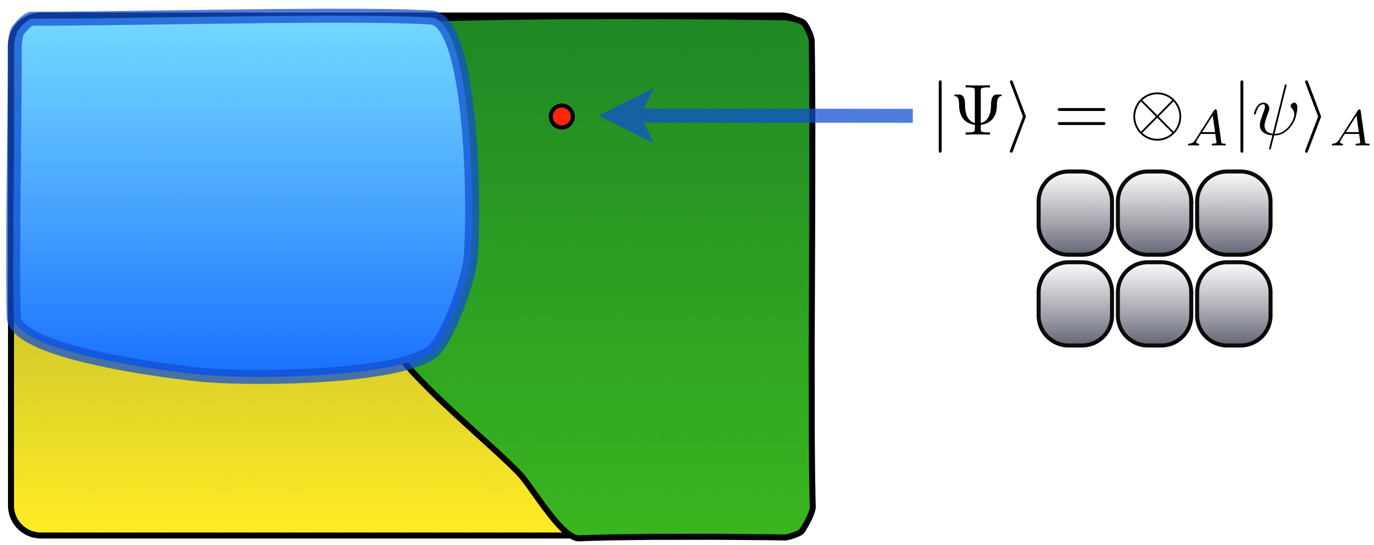
$$|\Psi\rangle = \otimes_A |\psi\rangle_A$$



no entanglement between blocks

“Essentially” a product state?

- Adiabatic continuity

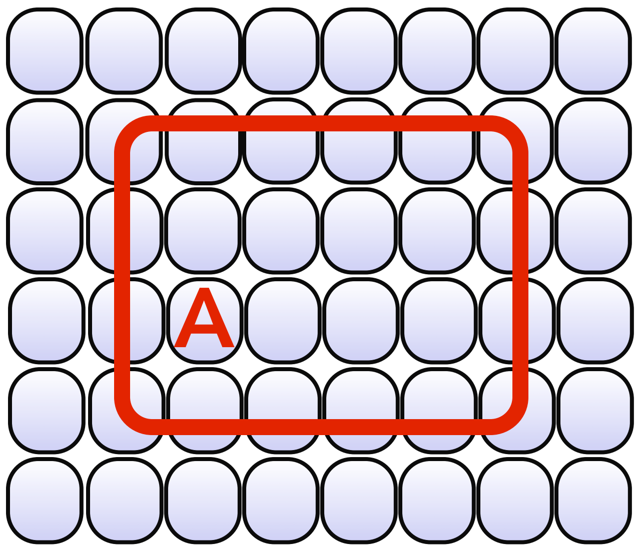


phase space

n.b. This is not true for gapless fermi systems

“Essentially” a product state?

- Entanglement scaling



$$\rho_A = \mathrm{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\mathrm{Tr}_A \left(\rho_A \ln \rho_A\right)$$

$S(A) \sim \sigma L^{d-1}$ area law

satisfied with exponentially small corrections

Best example: ordered magnet

Hamiltonian

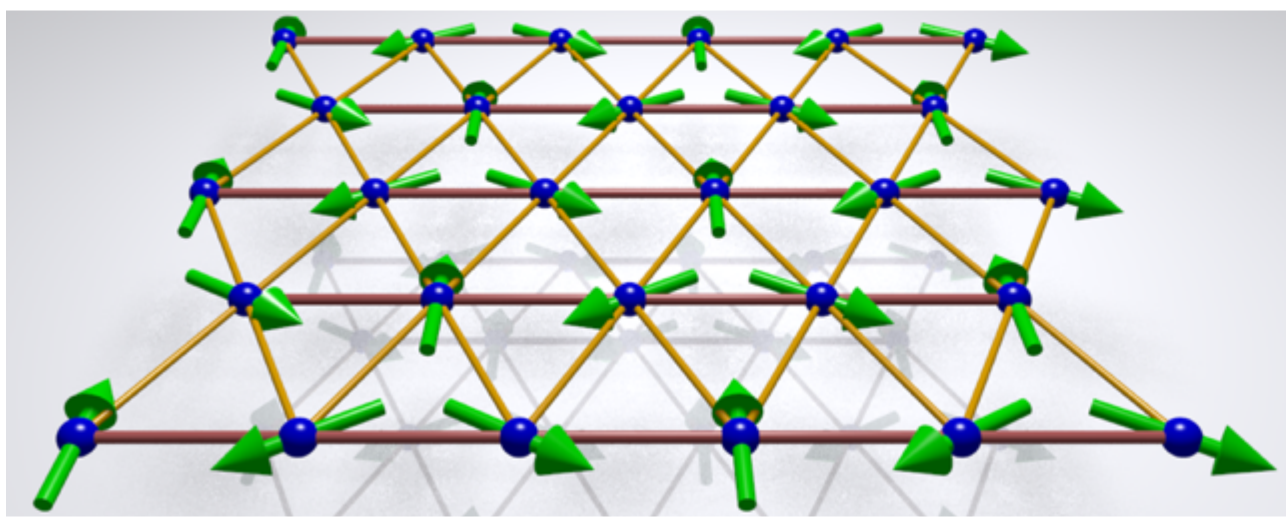
$$H = \sum_{(ij)} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

exchange is short-range: local

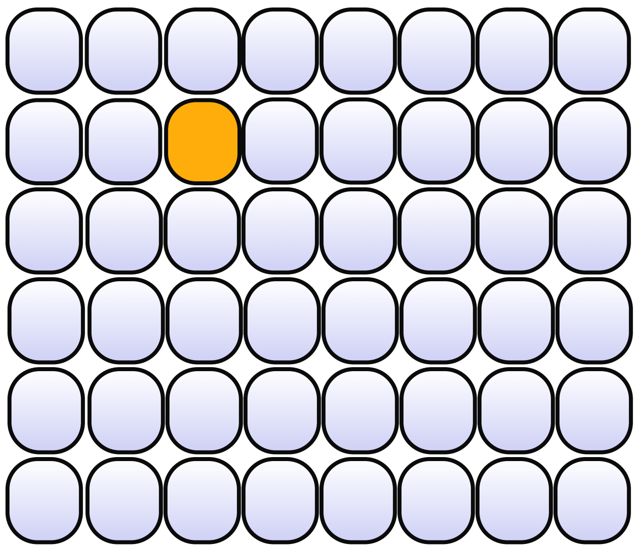
ordered state

$$|\Psi\rangle \approx \bigotimes_i |\boldsymbol{S}_i \cdot \hat{n}_i = +S\rangle$$

block is a single spin



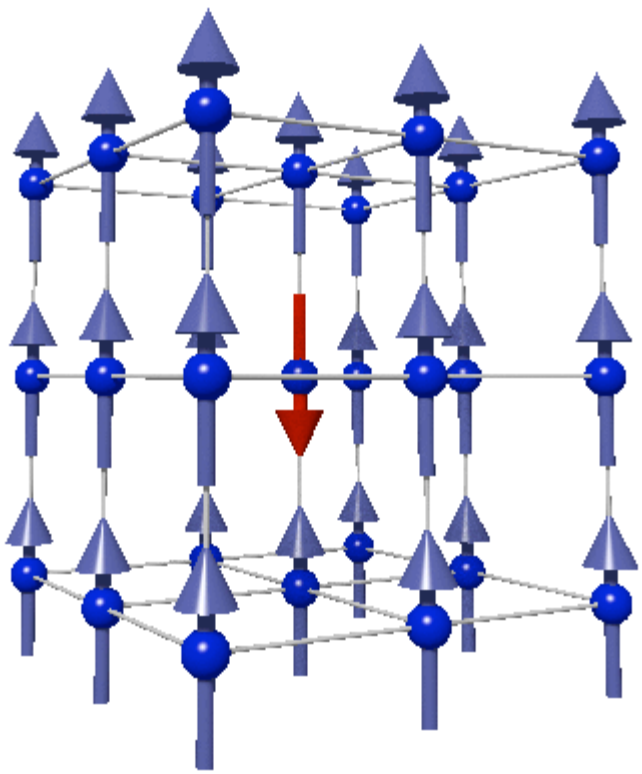
Quasiparticles



excited states ~ excited levels of one block

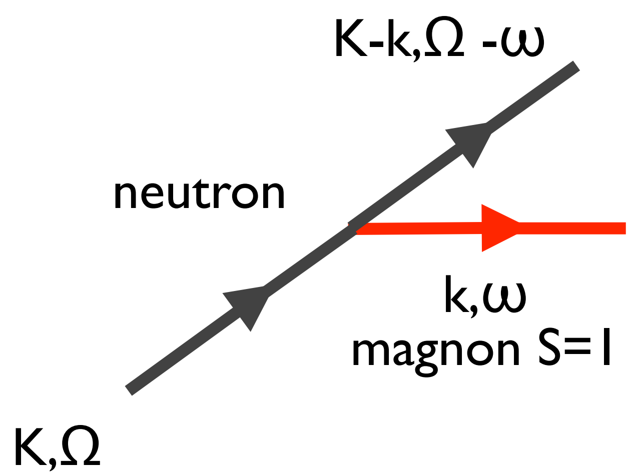
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

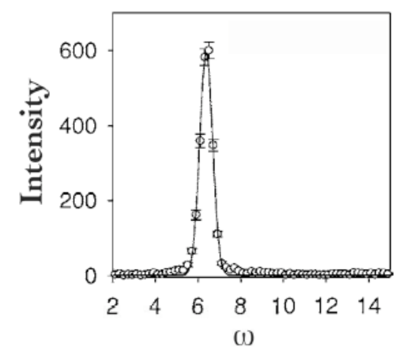
Spin wave



$$\omega(k) \approx \Delta - 2t\cos k_x a - \cdots$$

$$|f\rangle = S_k^+|i\rangle$$

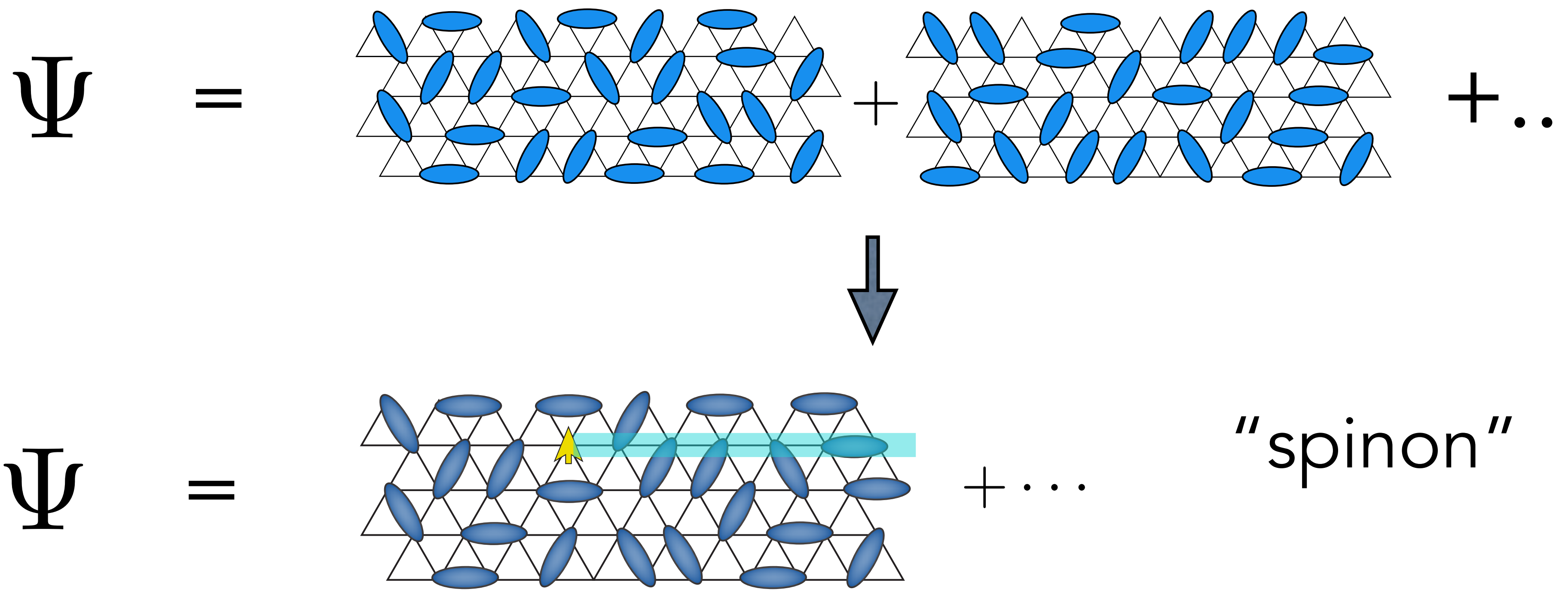




Line shape in Rb_2MnF_4

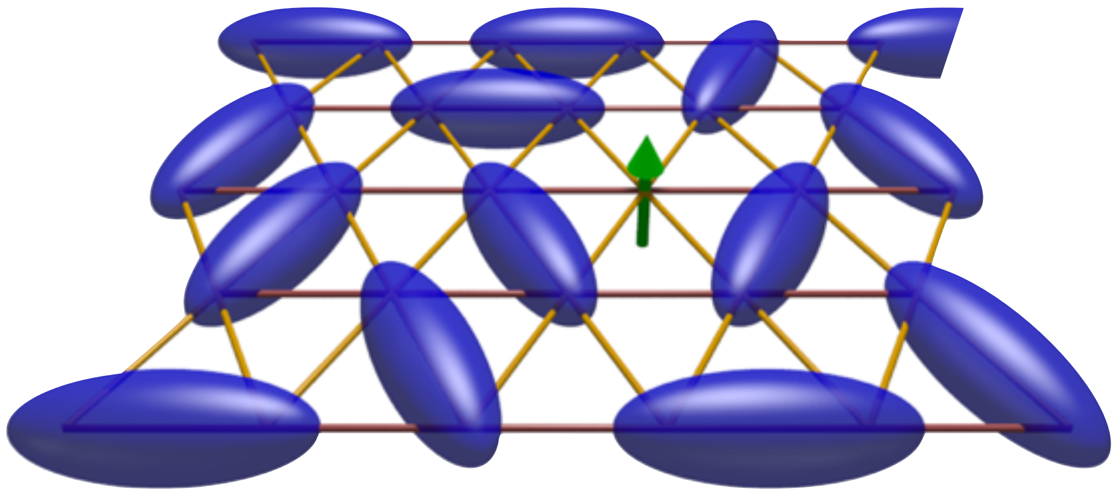
Quantum spin liquid

Entanglement -> non-local excitation



"quasiparticle" above a non-zero gap

Fractional quantum number

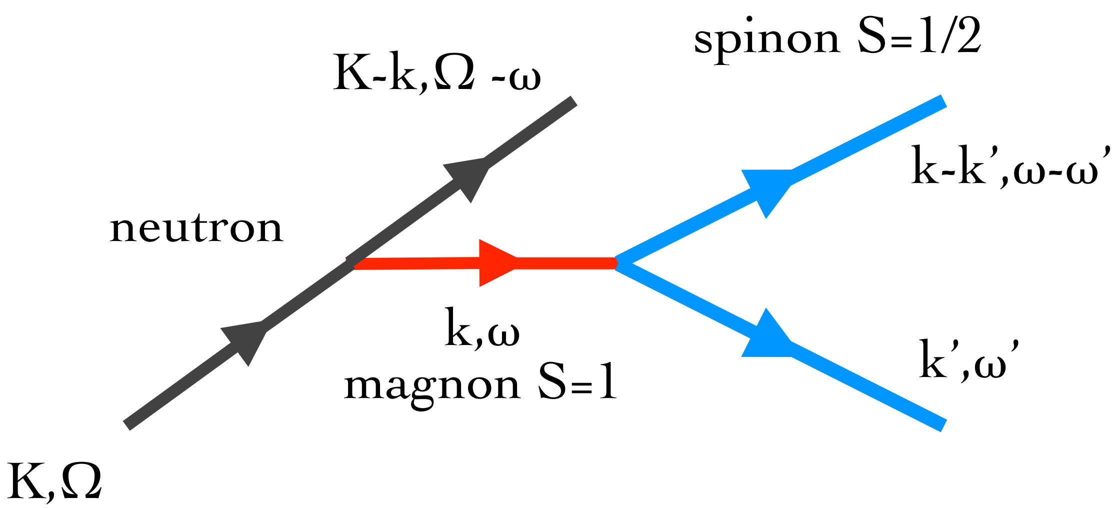


excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins

always created in pairs by any
local operator

No spin waves

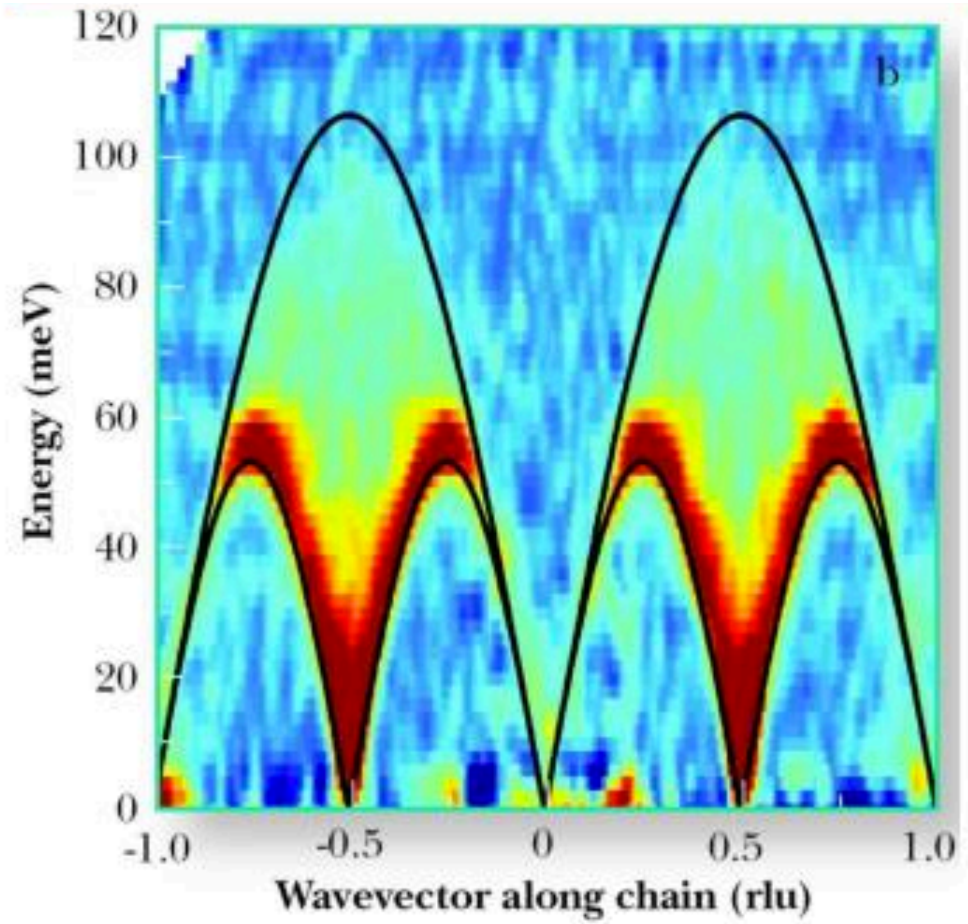
- Magnon is not elementary: decays into two spinons



broad peak with
$\omega=\varepsilon(k')+\varepsilon(k-k')$

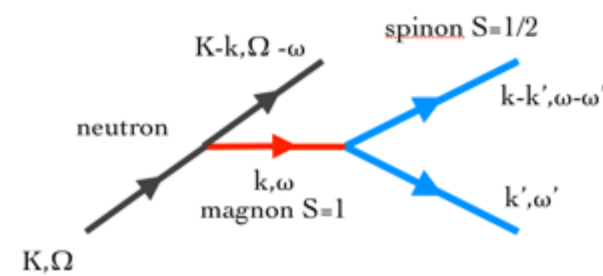
- Sharp peaks should be reduced or absent in the spin structure factor

c.f. One dimension

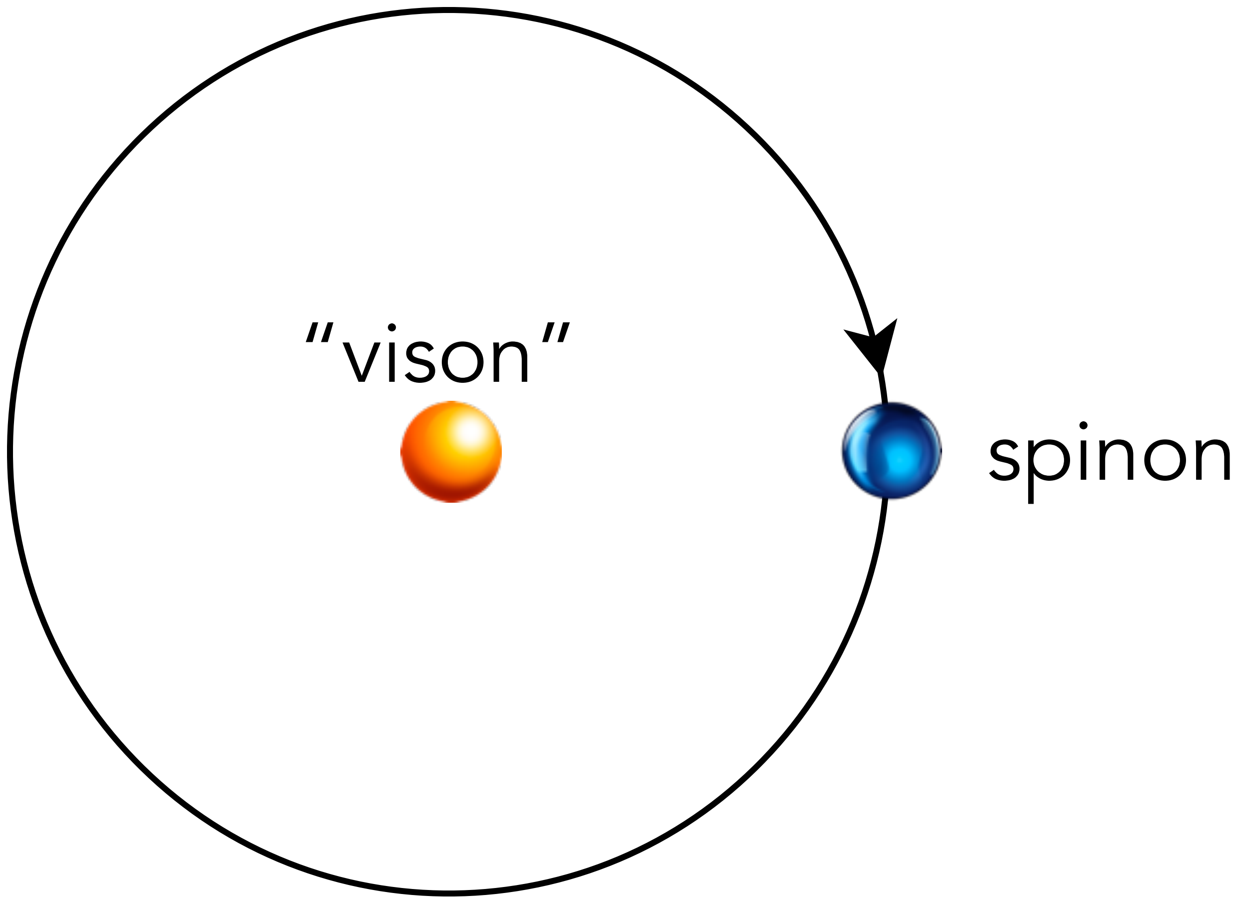


A. Tennant *et al*, 2001

$KCuF_3$



Anyons



$$\Psi \to -\Psi$$

“mutual semions”

X.-G. Wen

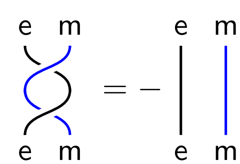
Topological phases

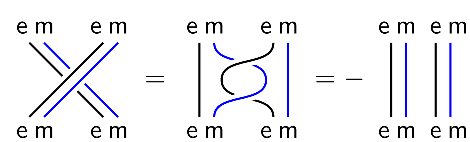
A. Kitaev

The American Physical Society
OLIVER E. BUCKLEY
SOLID STATE PHYSICS PRIZE

Anderson's RVB state is thus an example of a "topological phase" - the best understood sort of QSL

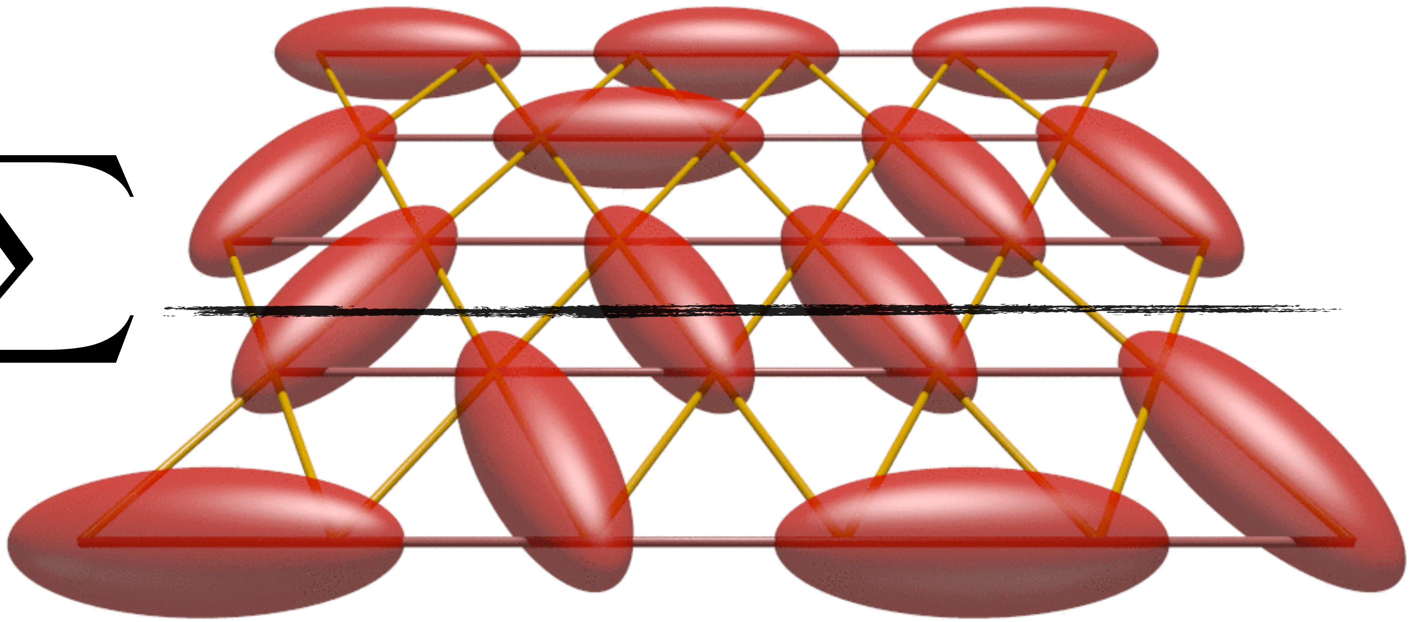
Understood and classified by anyons and their braiding rules in 2d





Stability

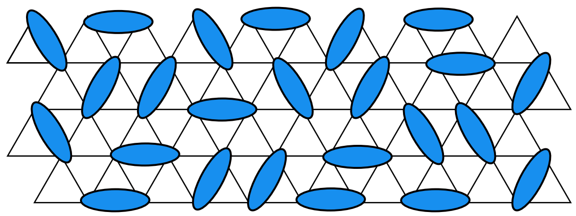
$$\Psi = \sum$$



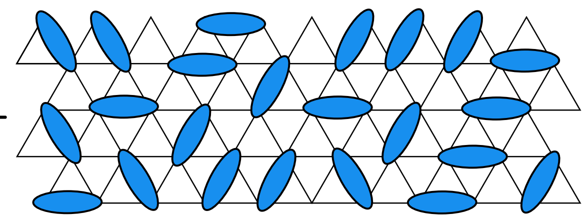
Robustness arises from topology: a QSL is a stable *phase* of matter (at T=0)

Quantum spin liquid

Ψ =



$+$



$+ \ldots$

For ~500 spins, there are more amplitudes than there are atoms in the visible universe!

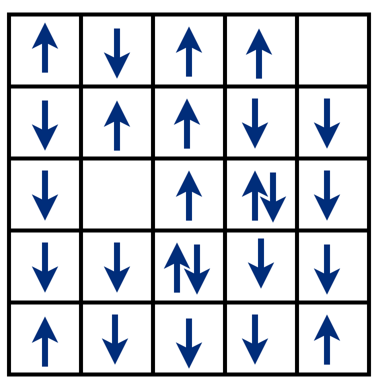
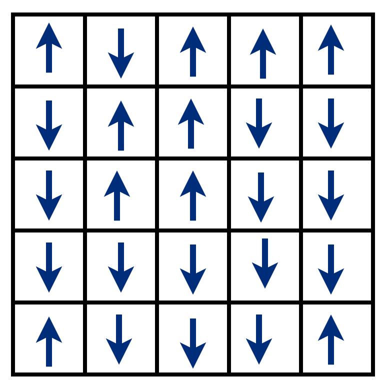
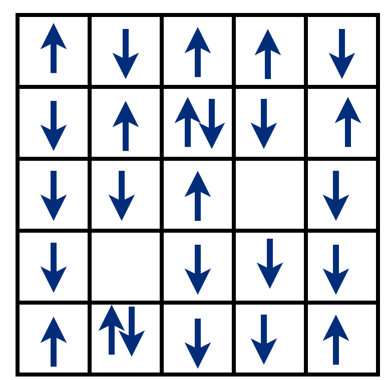
Different choices of amplitudes can realize different QSL phases of matter.

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site (S=0)

"partons"

$$|\Psi_0\rangle = \prod_{k\in FS} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow} |0\rangle$$

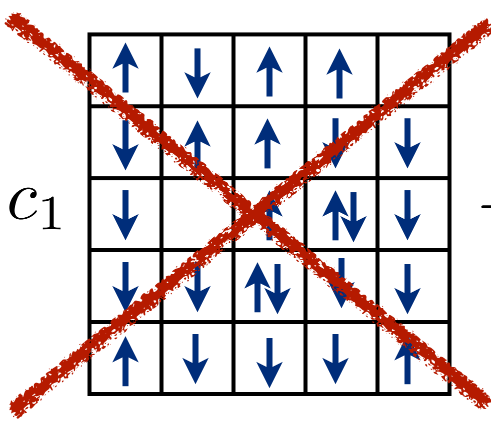
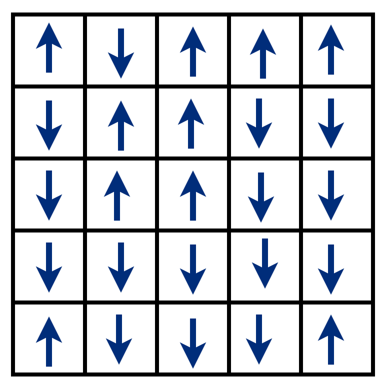
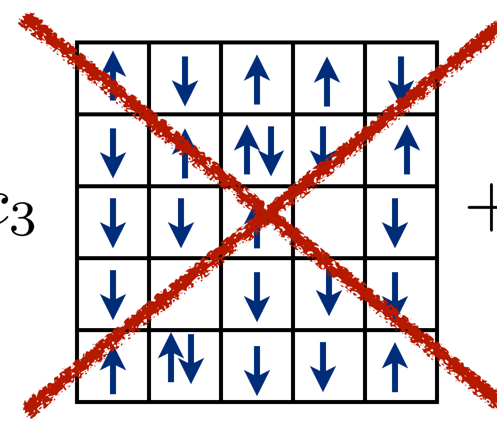
$= \; c_1$  $+c_2$  $+c_3$  $+\cdots$

Gutzwiller Construction

- Project out any components with empty or doubly occupied sites

“partons”

$$|\Psi\rangle = \hat{P}_G|\Psi_0\rangle$$

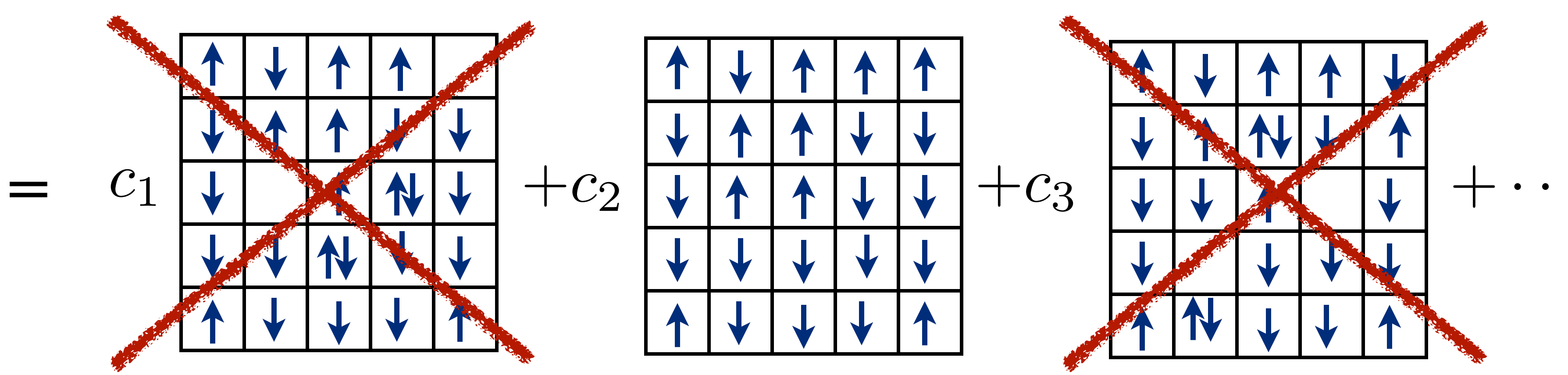
$= c_1$  $+c_2$  $+c_3$  $+\cdots$

Gutzwiller Construction

- Can build many QSL states by choosing different free fermion states

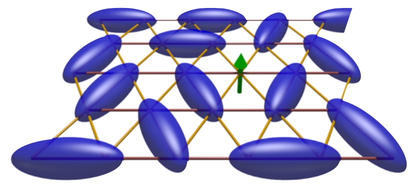
"partons"

$$|\Psi\rangle = \hat{P}_G|\Psi_0\rangle$$



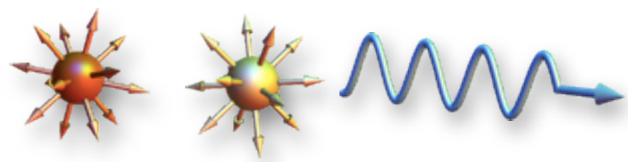
Classes of QSLs

- Topological QSLs



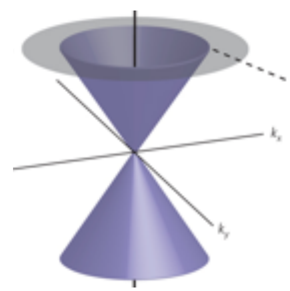
projected superconductor

- U(1) QSL



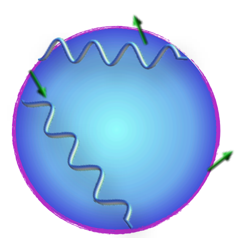
projected 3d band insulator

- Dirac QSLs



projected graphene

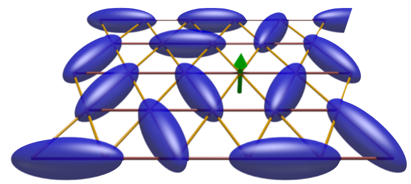
- Spinon Fermi surface



projected metal

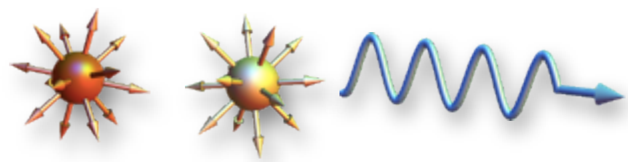
Classes of QSLs

- Topological QSLs



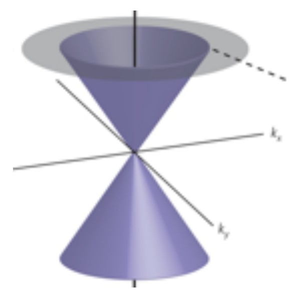
anyonic
spinons

- U(1) QSL



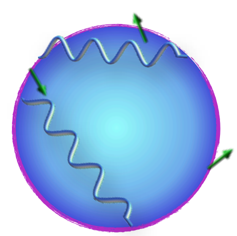
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

Strange stuff

where do we find it?

A rough guide to experiments on QSLs

Does it order?

- NMR line splitting
- muSR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/x-ray

Is there a gap?

- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of χ

Exotica

- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects

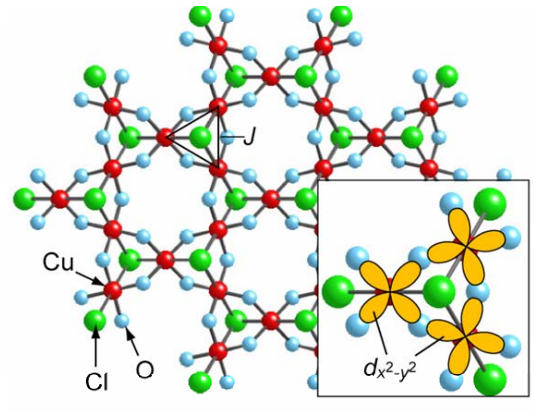
Delocalized excitations?

- thermal conductivity
- INS

Structure of excitations?

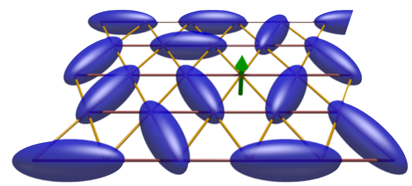
- E(k) from INS,RIXS
- optics, Raman

the new classics

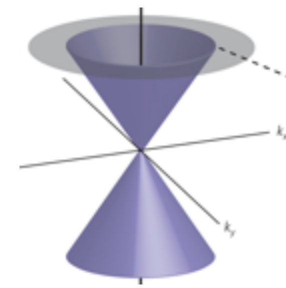


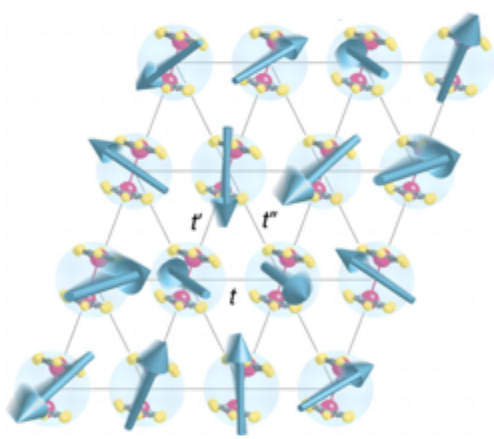
herbertsmithite

=



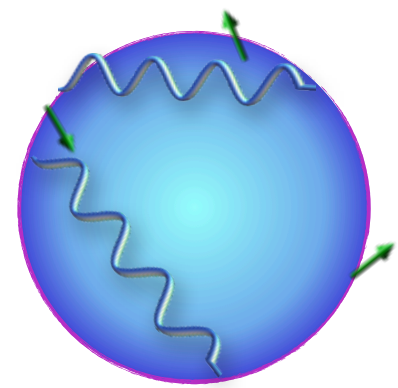
or



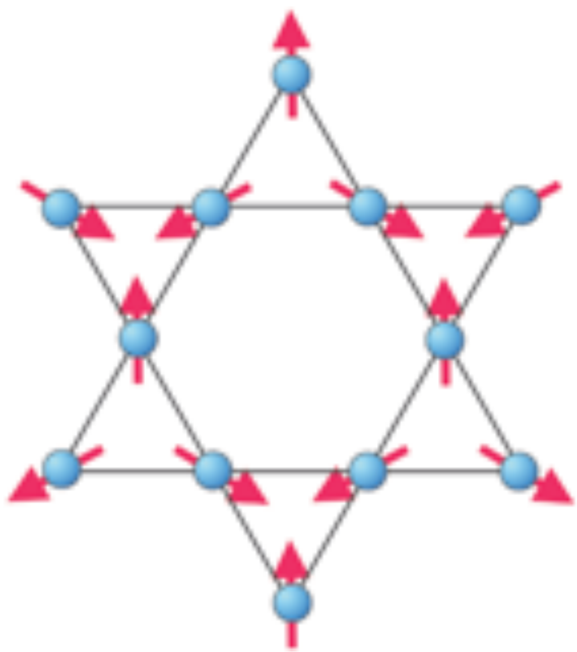


organics

=



Kagomé antiferromagnet



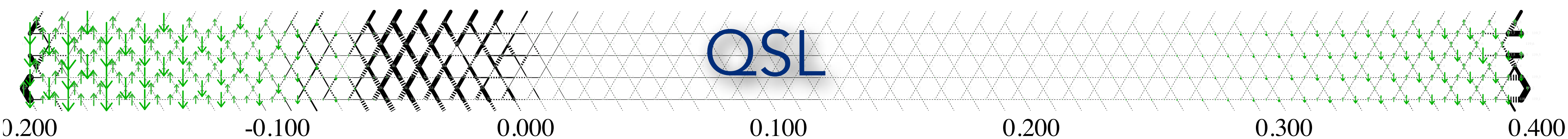
$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

Very large classical degeneracy → likely to be a QSL

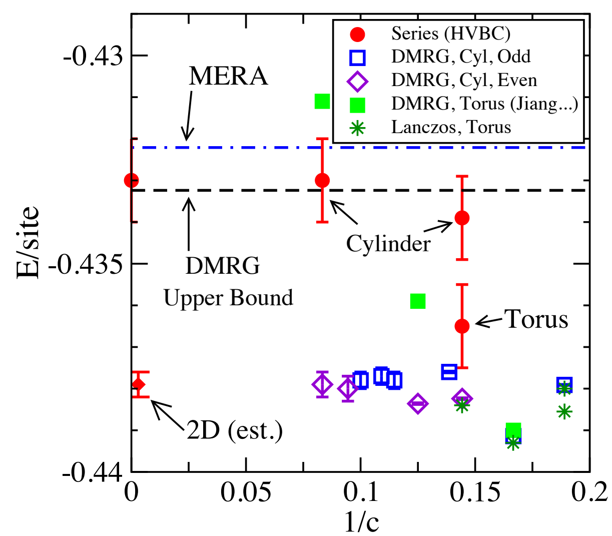
V. Elser, 1989 + many many others

S=1/2 kagomé AF

- Rather definitive evidence for QSL by DMRG



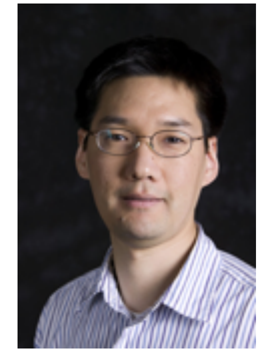
© Steve White



S. Yan *et al*, 2010

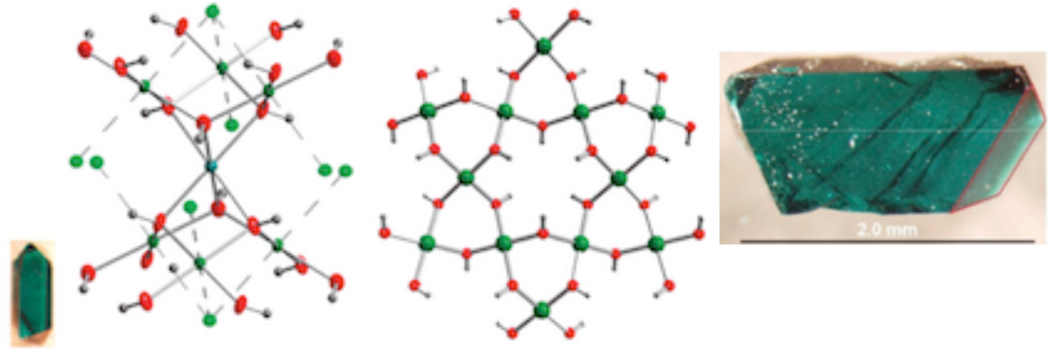
many other studies support existence of some QSL phase

Herbertsmithite



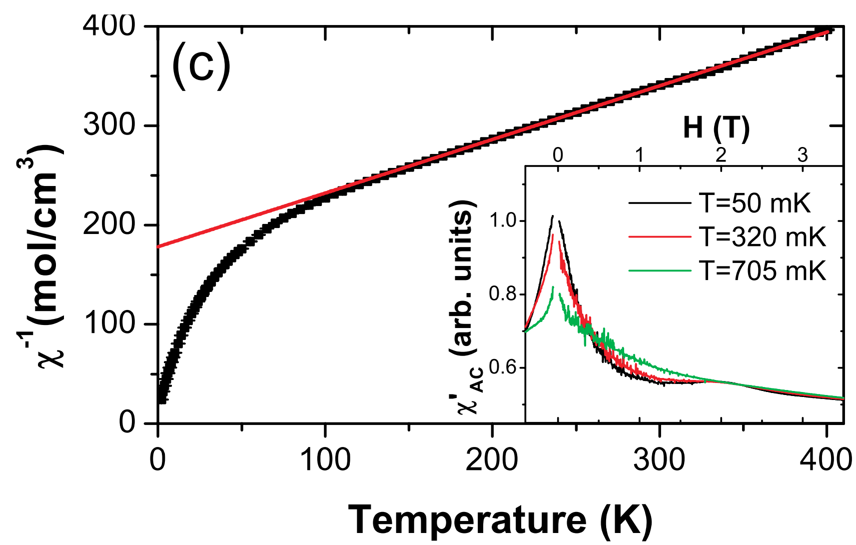
kagomé layers of Cu S=1/2 spins, separated by non-magnetic Zn

$ZnCu_3(OH)_6Cl_2$



Heisenberg-like with J ~ 200K

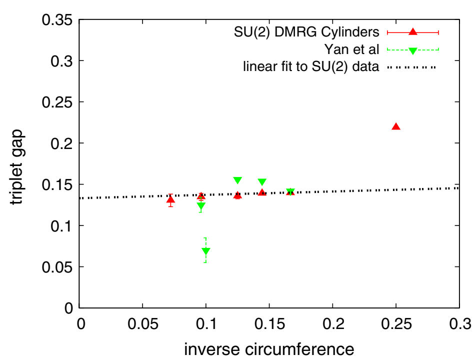
no order down to 50mK



Helton *et al*, 2007

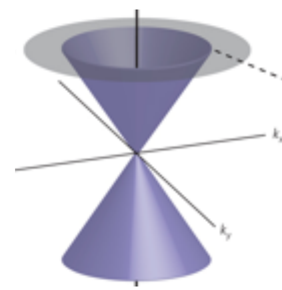
Theory

- What kind of QSL?



S. Depenbrock *et al*, 2012

gapped, topological QSL



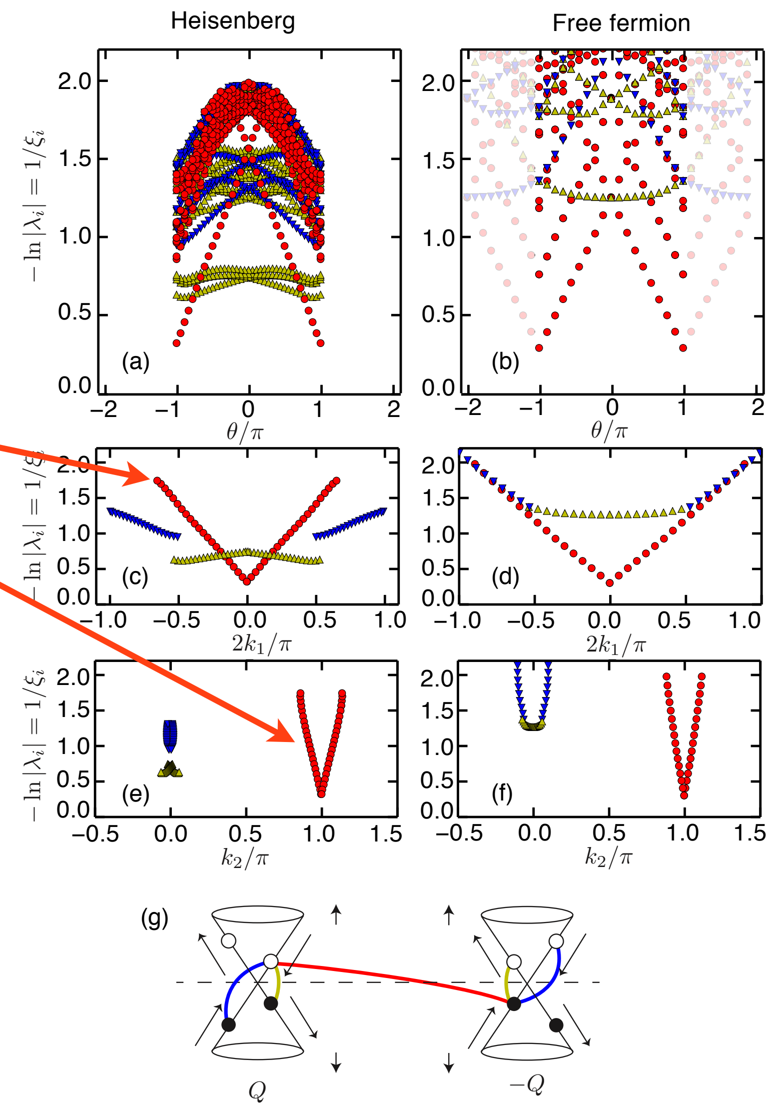
Y. Ran *et al*, 2007
F. Becca...

gapless Dirac QSL

+ various other proposals with weaker quantitative support

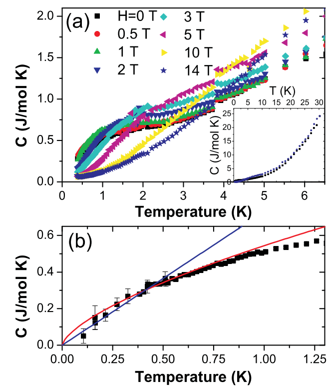
DMRG (2016)

Y.-C. He *et al*: evidence for Dirac QSL

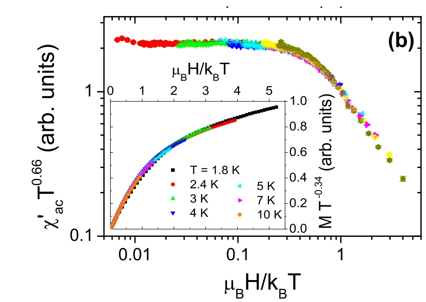


Herbertsmithite

Lots of early evidence for gaplessness



Helton *et al*, *2007*

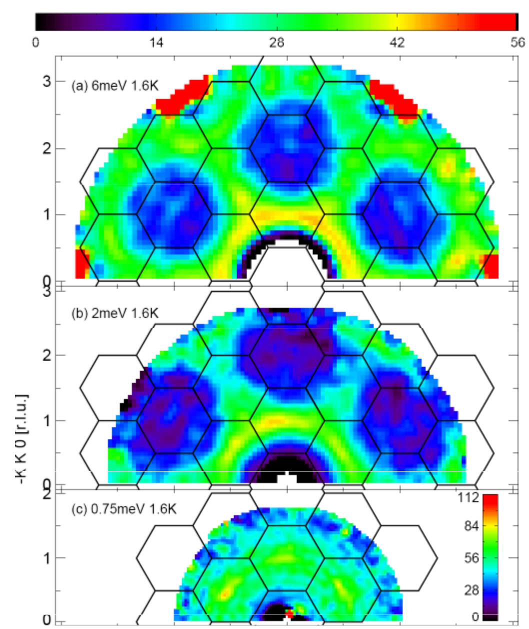


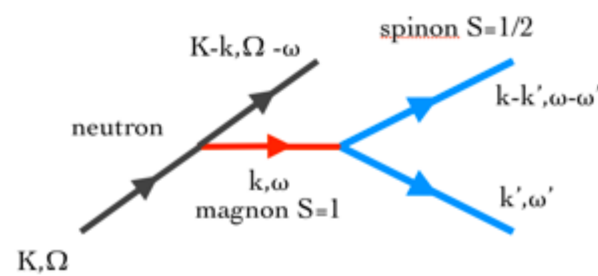
Helton *et al*, *2010*

Single crystal INS

smooth continuum scattering

T-H Han *et al*, 2012





continuum scattering expected

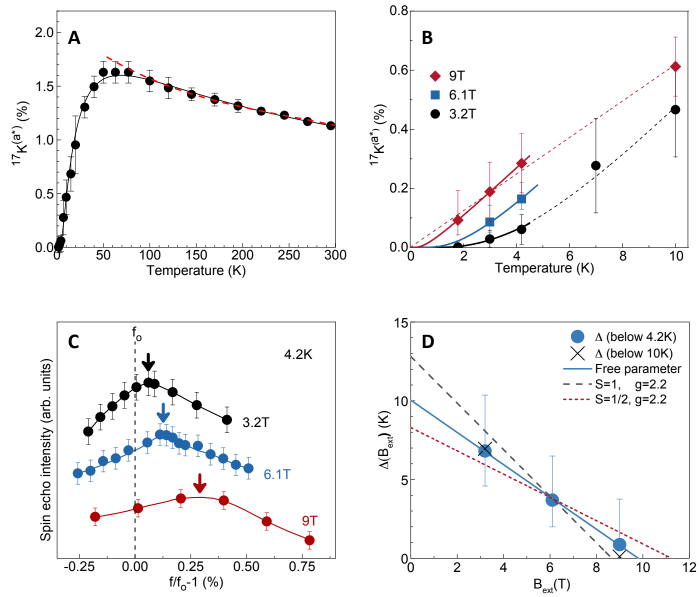
...but probably with more structure?

Herbertsmithite

Single crystal NMR

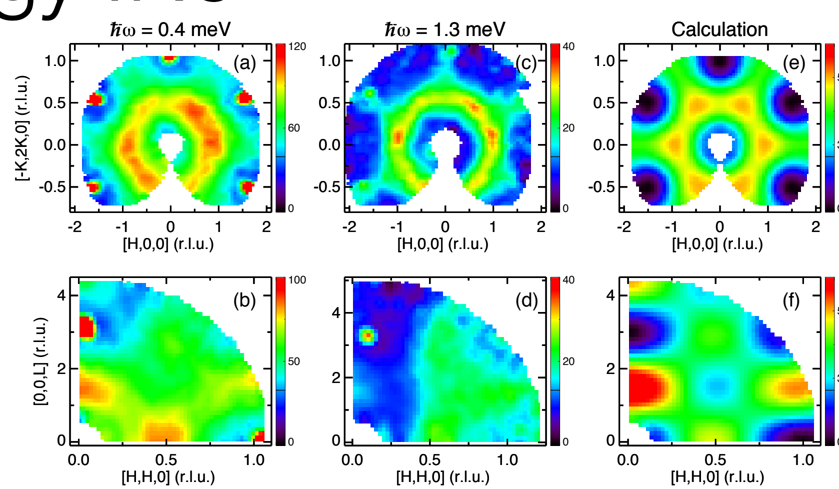
M. Fu *et al*, 2015

McMaster



estimate gap ~ 10K

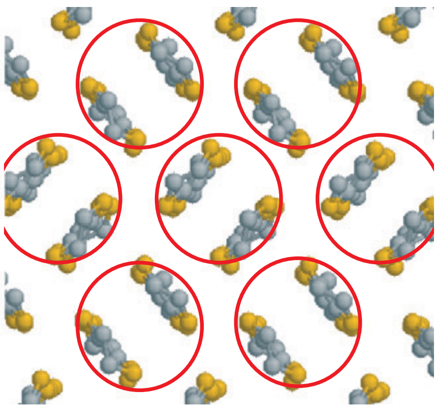
Low energy INS



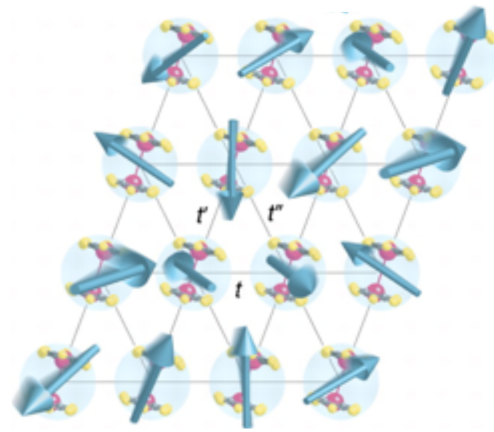
T-H Han *et al*, 2015

claim to separate impurity signal below 0.7meV

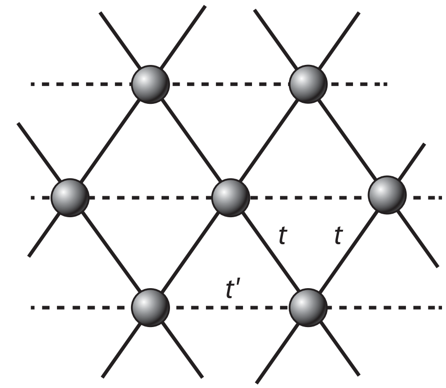
Organics



κ-(ET)$_2$X

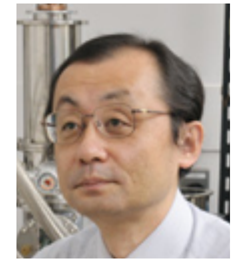


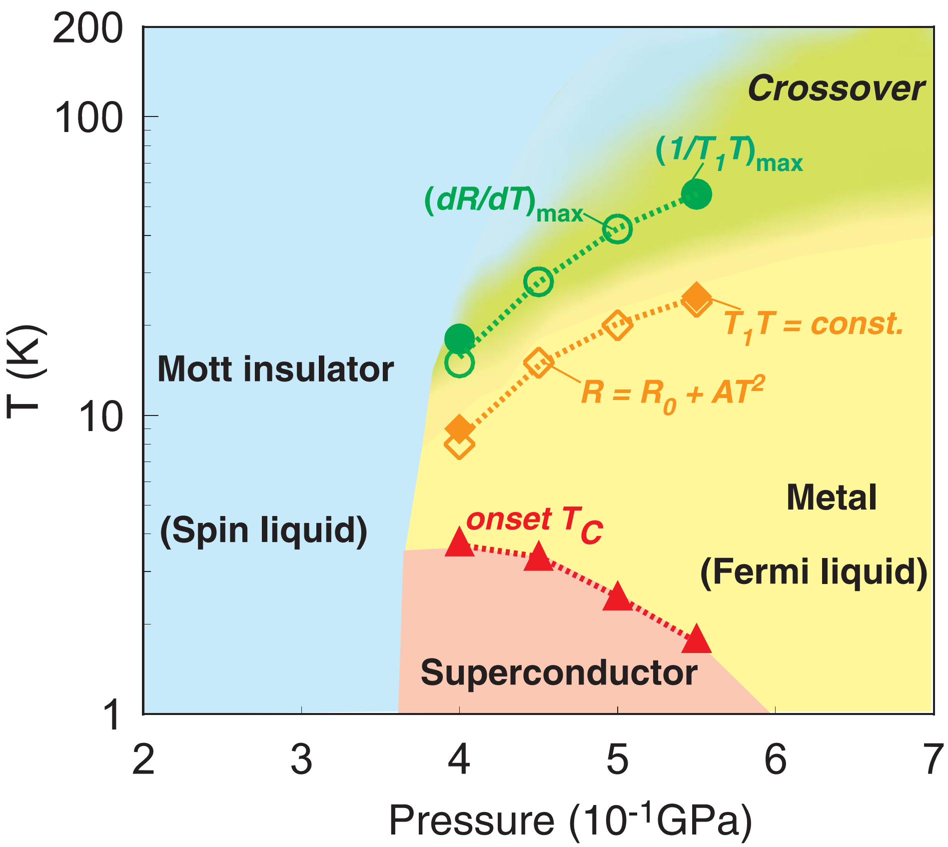
β′-Pd(dmit)$_2$

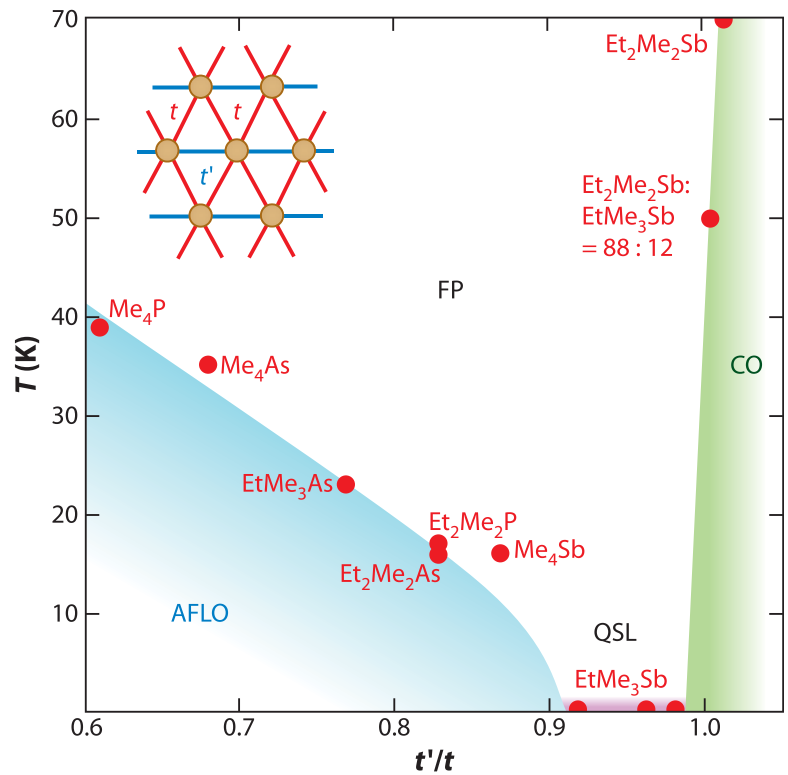


- Molecular materials which behave as effective triangular lattice S=1/2 antiferromagnets with J ~ 250K
  - significant charge fluctuations

Organics







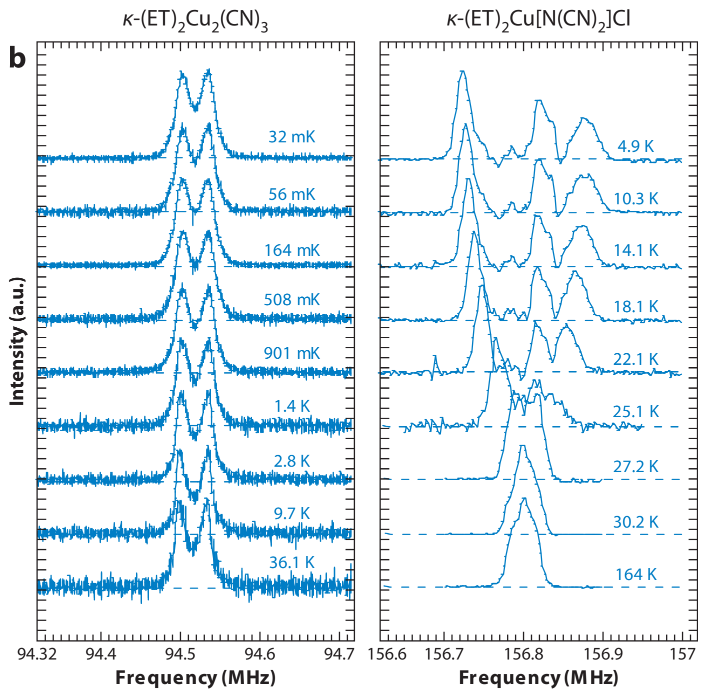
κ-$(ET)_2Cu_2(CN)_3$

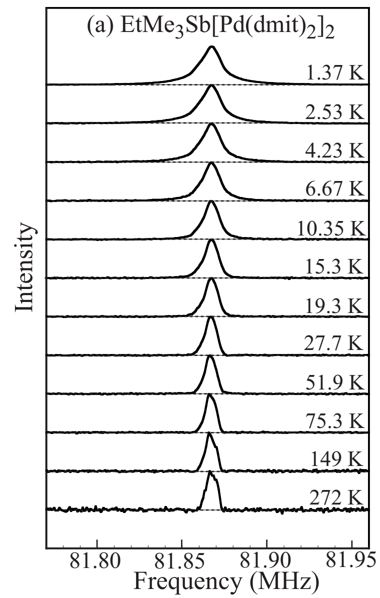
K. Kanoda group (2003-)

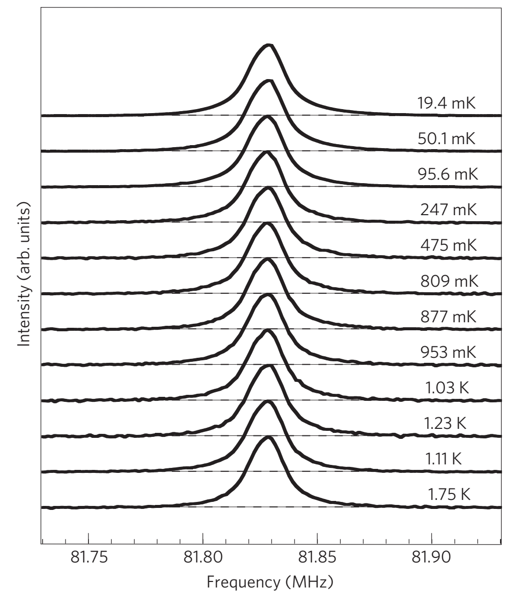
β′-$Pd(dmit)_2$

R. Kato group (2008-)

NMR lineshapes







κ-$(ET)_2Cu_2(CN)_3$

β′-$Pd(dmit)_2$

Y. Shimizu *et al*, 2003

^{1}H NMR

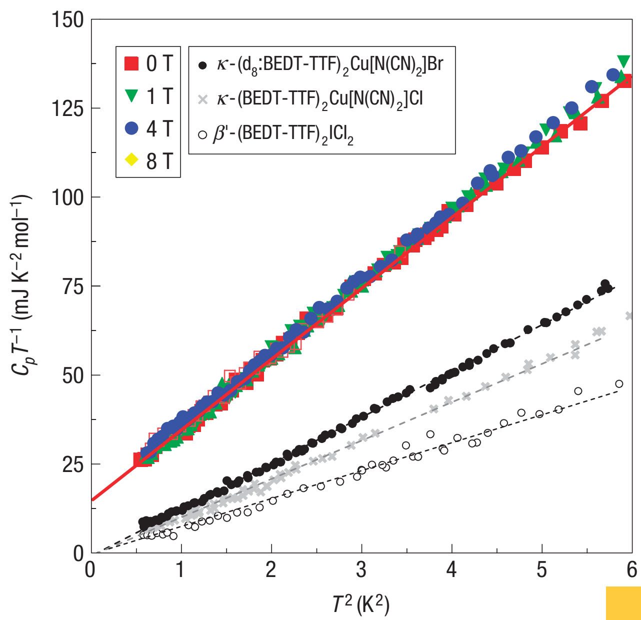
T. Itou *et al*, 2008,2010

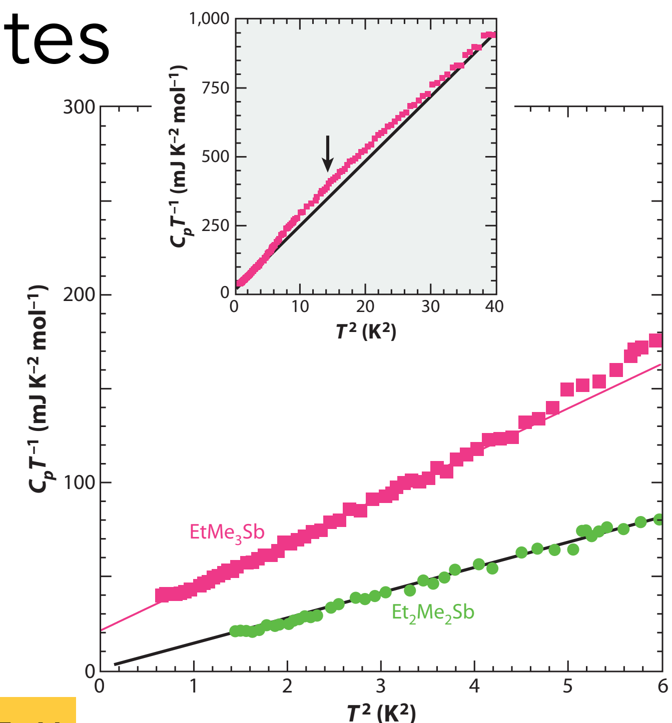
^{13}Cs NMR

Evidence for lack of static moments: f > 1000!

Specific Heat

- C ~ γT indicates gapless behavior with large density of states





γ_{Cu} ~ 0.7 !!

κ-$(ET)_2Cu_2(CN)_3$

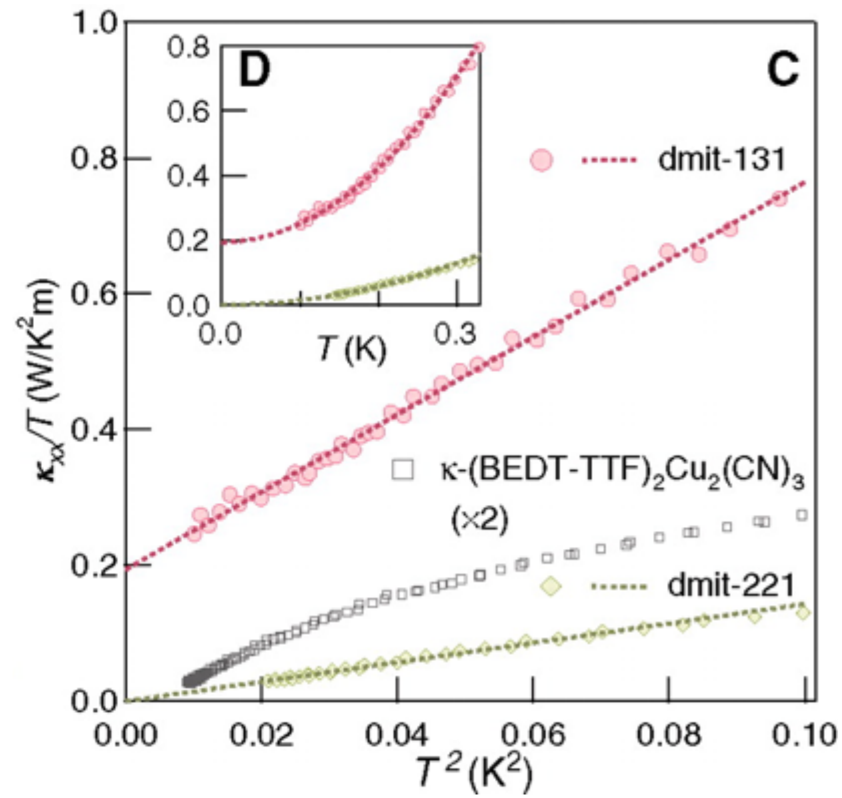
β′-Pd$(dmit)_2$

S. Yamashita *et al*, 2008

Thermal conductivity

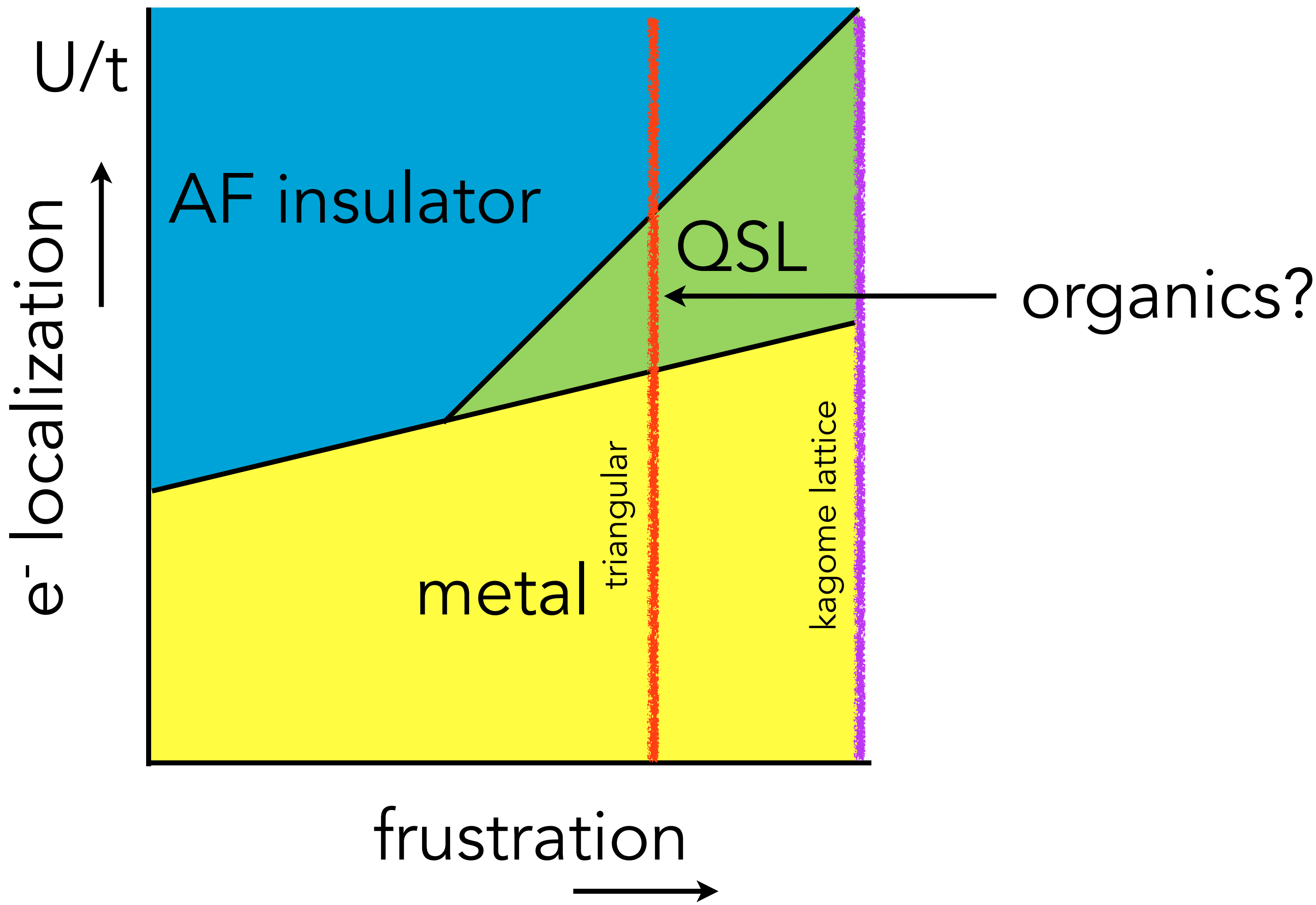
- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit

- Estimate for a *metal* would correspond to a mean free path *l* ~ 1 μm ≈1000 a !



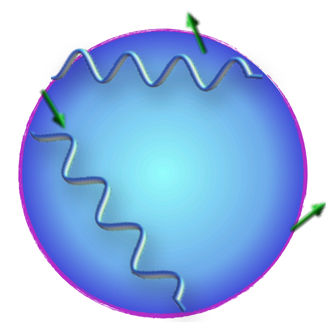
M. Yamashita *et al*, 2010

Charge fluctuations

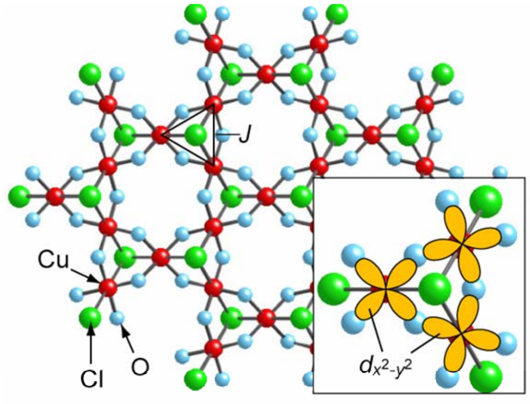


Organics - Theory

- RVB/QSL state:
  - Motrunich, Lee+Lee: (2005) “uniform RVB”
  - It is described by a **“Fermi sea” of spinons** coupled to a U(1) gauge field
- The anomalous thermal conductivity may be a window into an emergent fermi surface in an insulator!

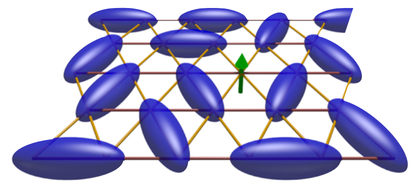


the new classics

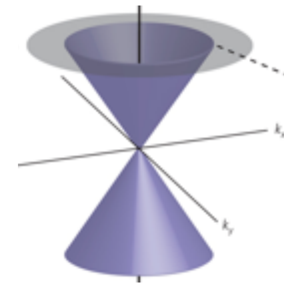


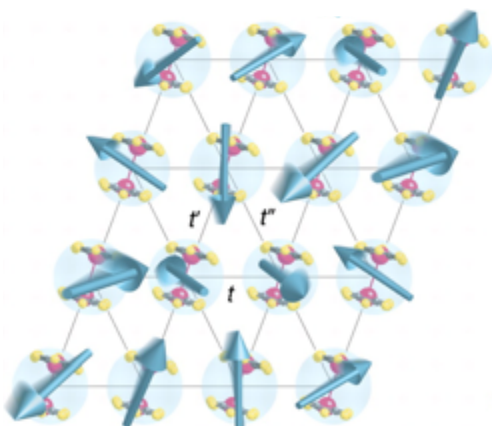
herbertsmithite

=



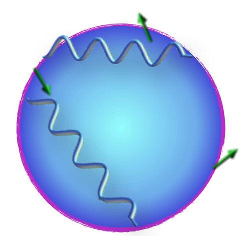
or



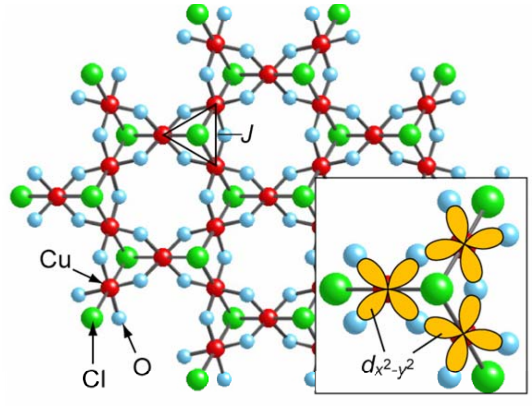


dmit

=



the new classics



herbertsmithite

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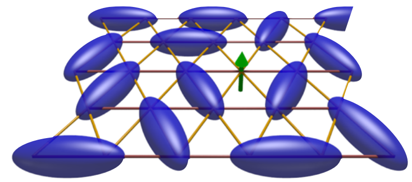
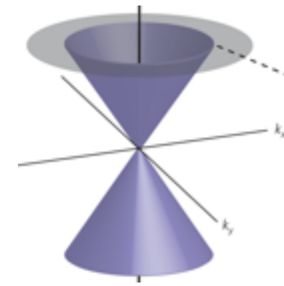
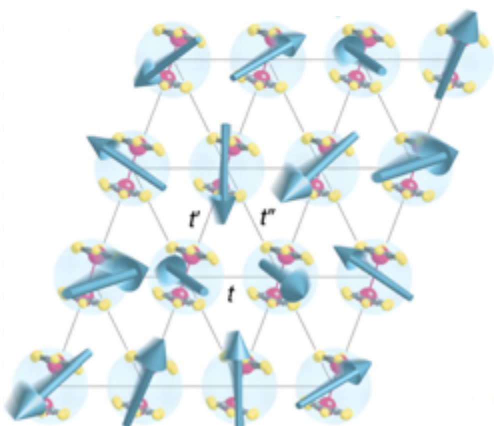


or

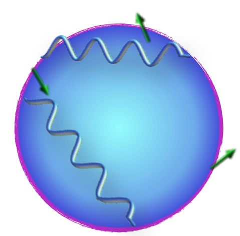


disorder



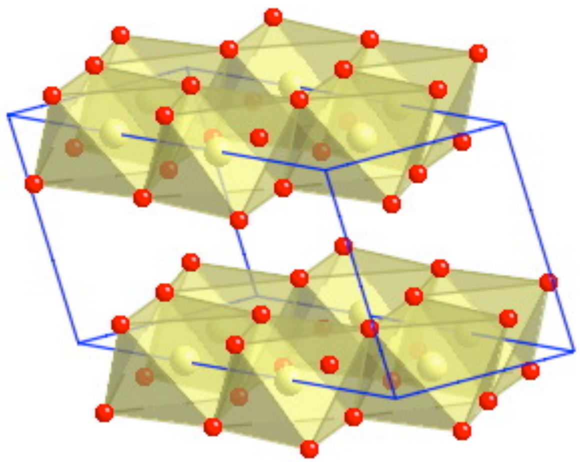
dmit

=

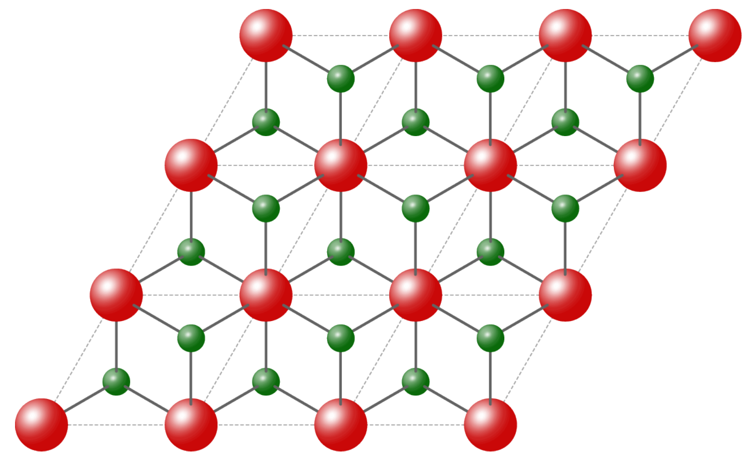


large lattice contribution

New direction: strong anisotropy



Kitaev materials

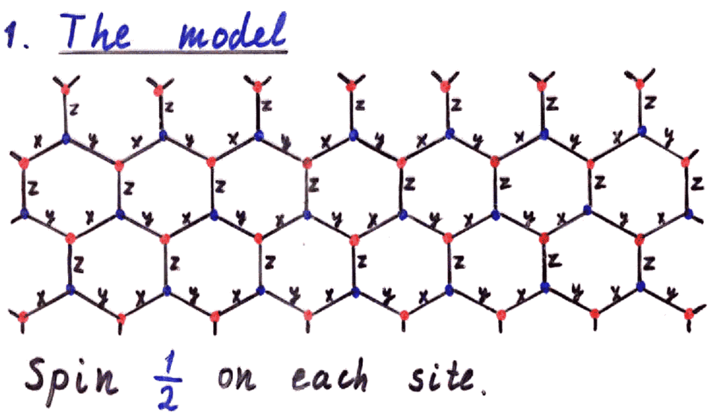


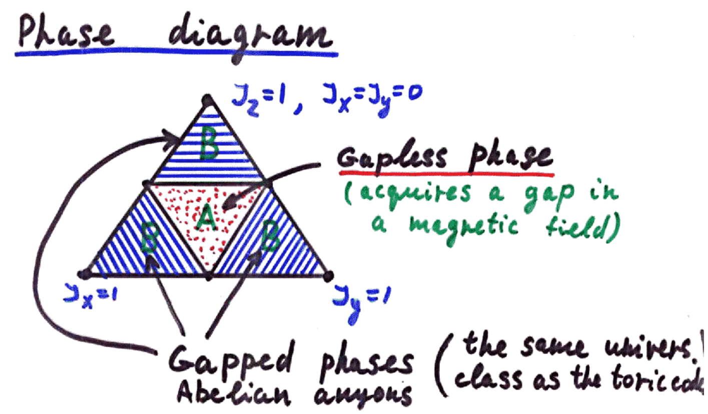
$YbMgGaO_4$

Kitaev model

Kitaev's honeycomb model $H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$

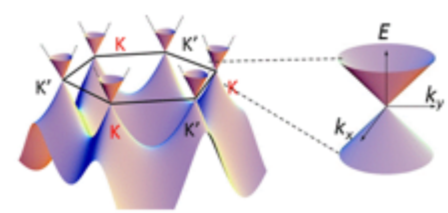
KITP, 2003



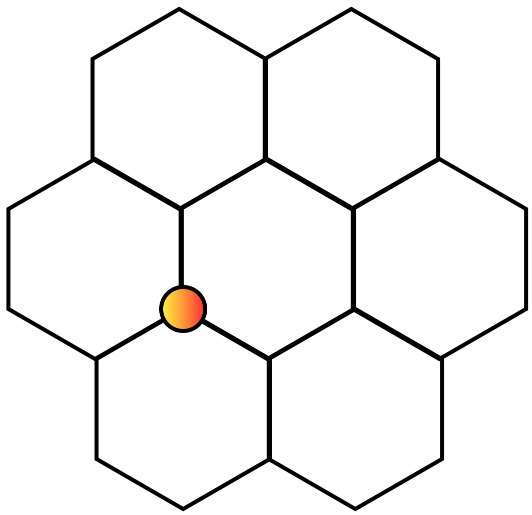


exact parton construction $\sigma_i^\mu = i c_i c_i^\mu \quad c_i c_i^x c_i^y c_i^z = 1$

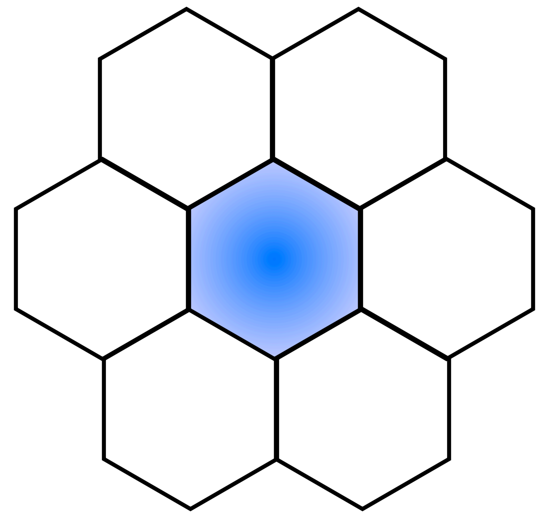
physical Majoranas $H_{\mathrm{m}} = K \sum_{\langle ij \rangle} i c_i c_j$



Non-local excitations



Majorana ε



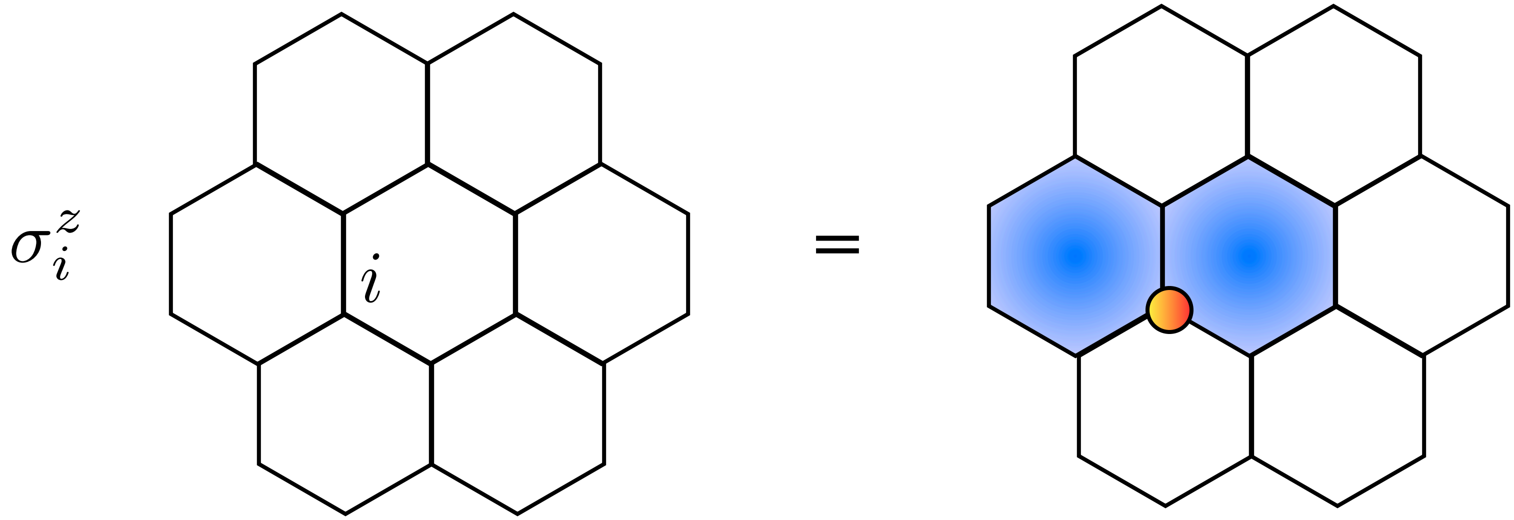
Flux e, m

In Kitaev's model:

- Majorana's dispersion ~ K and Dirac-like
- Fluxes are localized with small gap

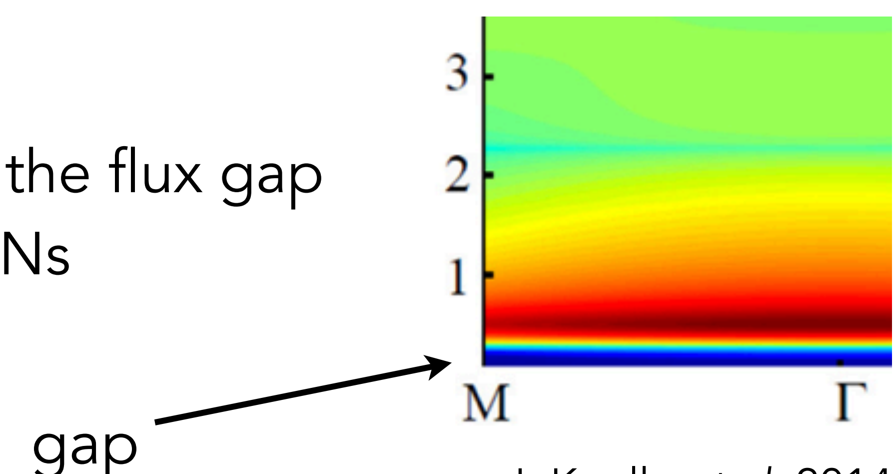
Fractionalization

Spin flip produces a free Majorana fermion and two immobile fluxes



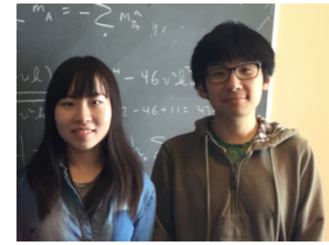
Because fluxes are created

- Spectral weight is zero below the flux gap
- Correlations vanish beyond NNs



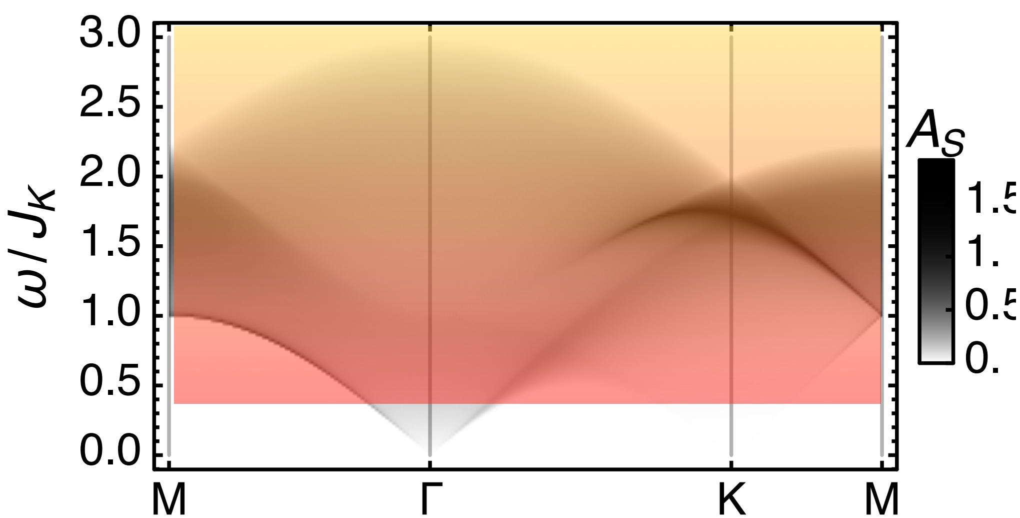
J. Knolle *et al*, 2014

Fractionalization



- Another process: fluxes recombine into a second Majorana fermion
- This gives rise to an excitation branch of power-law Dirac "fans"

What to look for if we ever discover a true Kitaev QSL - sharp low energy structure

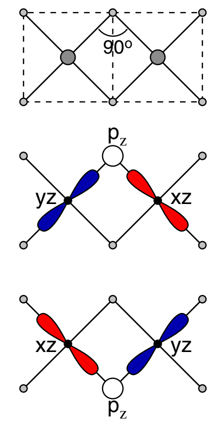


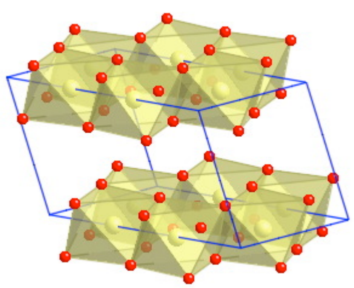
Xueyang Song, Yi-Zhuang You + LB, PRL 2016

Kitaev Materials

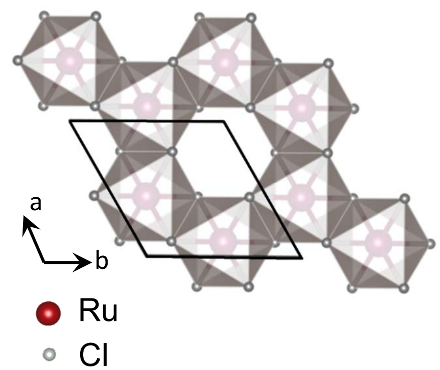
Jackeli, Khaliullin
2009

Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling





Na_2IrO_3, (α,β,γ)-Li_2IrO_3



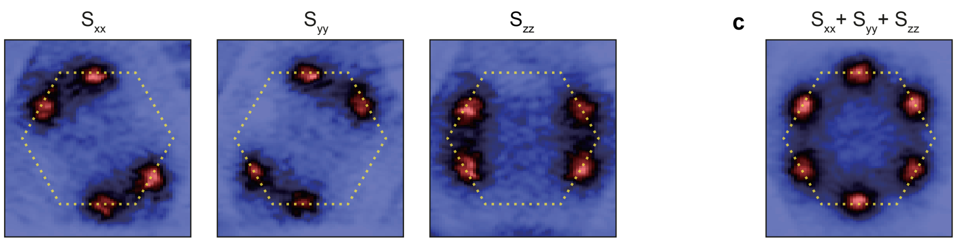
α-$RuCl_3$

Y.-J. Kim...

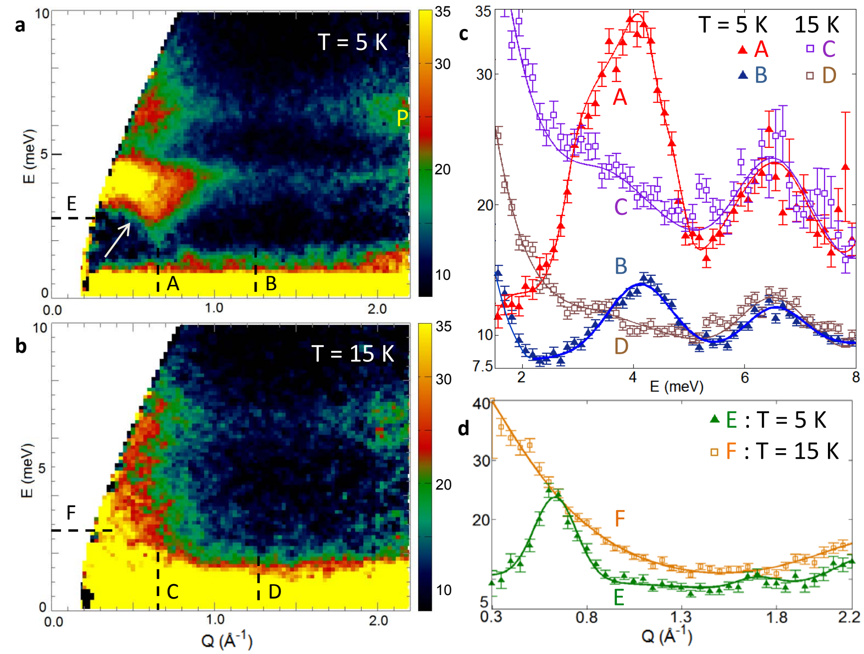
P.Gegenwart
H. Takagi
...

Honeycomb and hyper-honeycomb structures

Kitaev Materials



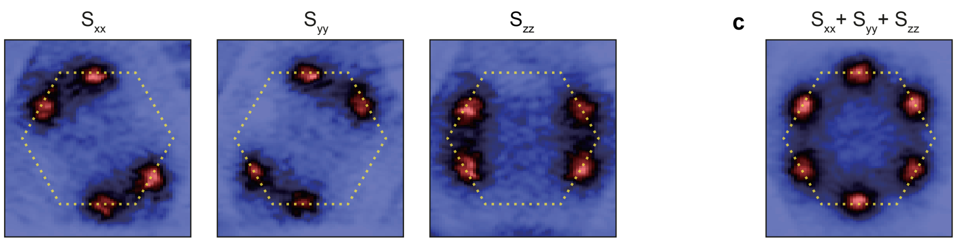
direct evidence for direction-dependent anisotropic exchange from diffuse magnetic x-ray scattering in Na_2IrO_3 (BJ Kim group)



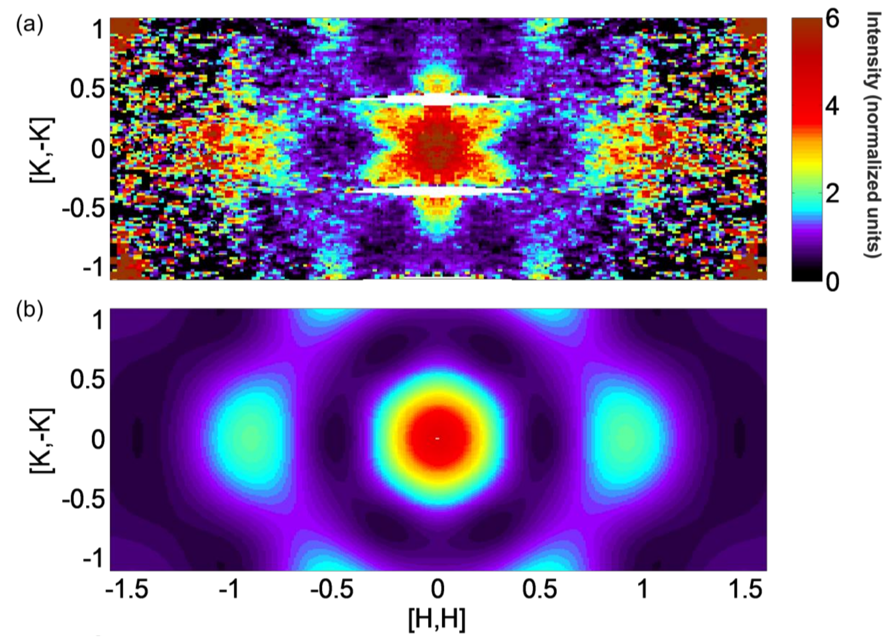
there is pretty strong evidence of substantial Kitaev exchange in quite a few materials

Observation of gapped continuum mode persisting above T_N in α-$RuCl_3$ consistent with Majoranas (A. Banerjee *et al)*

Kitaev Materials



direct evidence for direction-dependent anisotropic exchange from diffuse magnetic x-ray scattering in Na_2IrO_3 (BJ Kim group)



single-crystal datain α-$RuCl_3$ compared to Kitaev's soluble model (A. Banerjee *et al)*

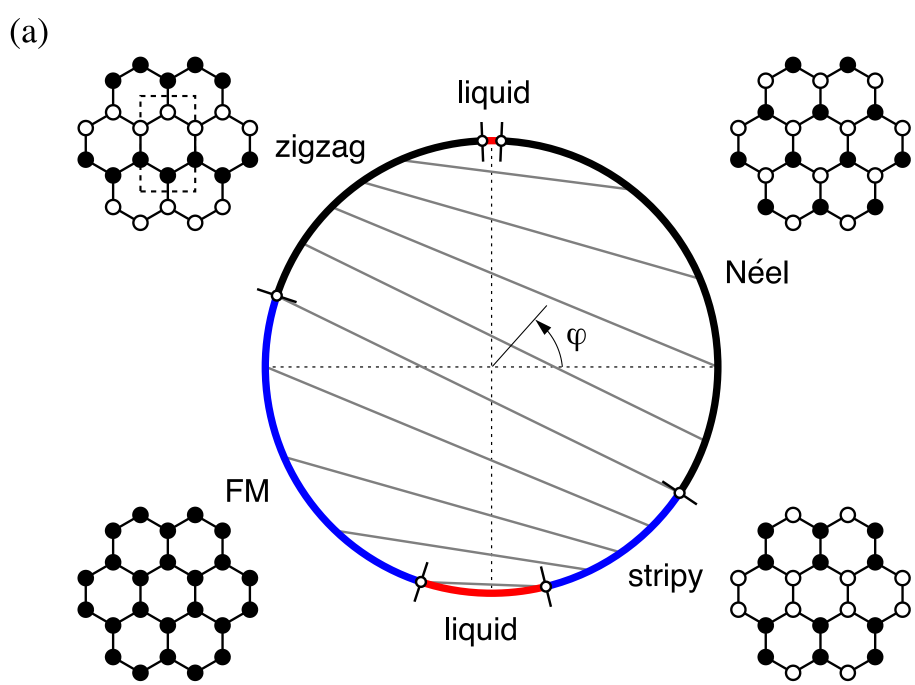
there is pretty strong evidence of substantial Kitaev exchange in quite a few materials

Magnetism

- Na_2IrO_3, Li_2IrO_3, α-$RuCl_3$ all order

due to additional interactions,

e.g. Heisenberg

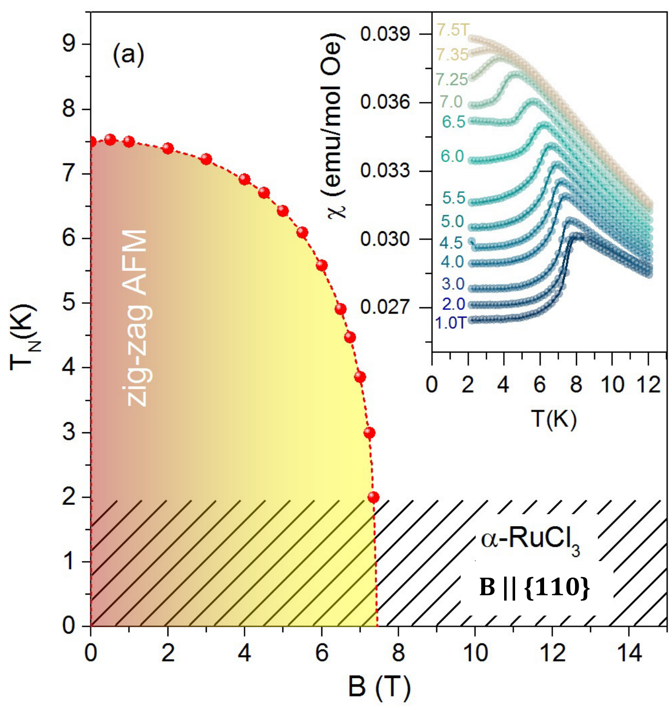


$$\mathcal{H} = \sum_{\langle i,j \rangle} J_1 \, \mathbf{S}_i \cdot \mathbf{S}_j + K_1 S_i^\gamma S_j^\gamma$$
$$+ \Gamma_1 (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha)$$
$$+ \sum J_3 \, \mathbf{S}_i \cdot \mathbf{S}_j$$

Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017

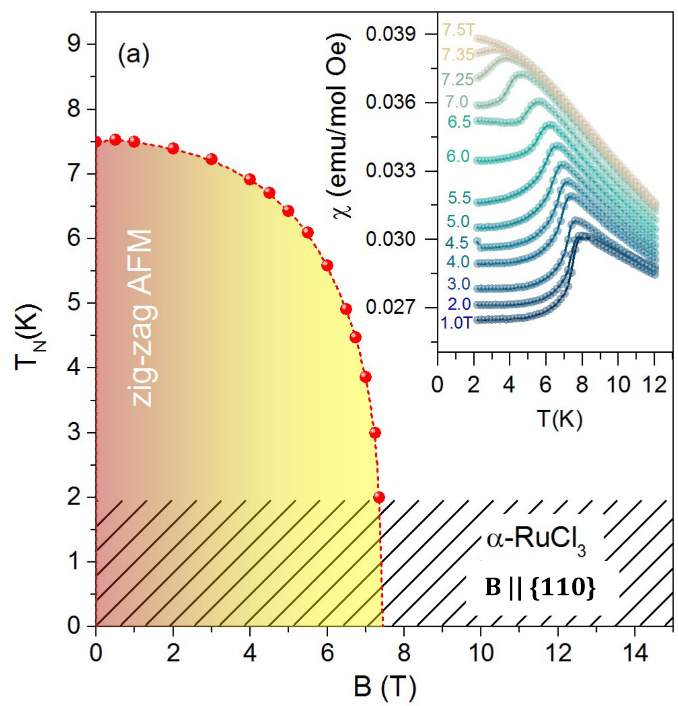


spin liquid or boring paramagnet?

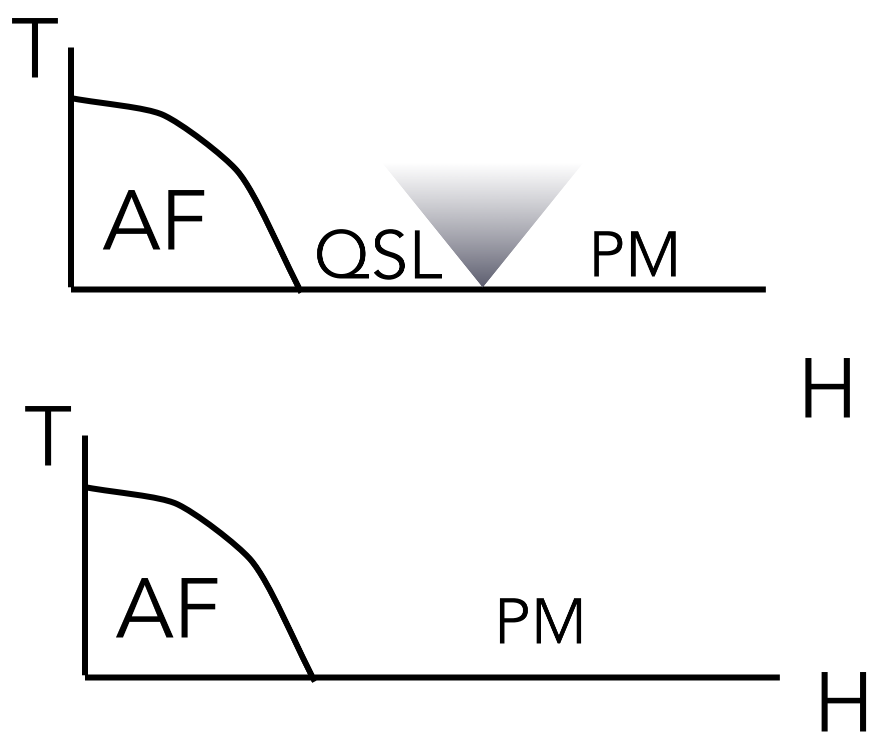
Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017



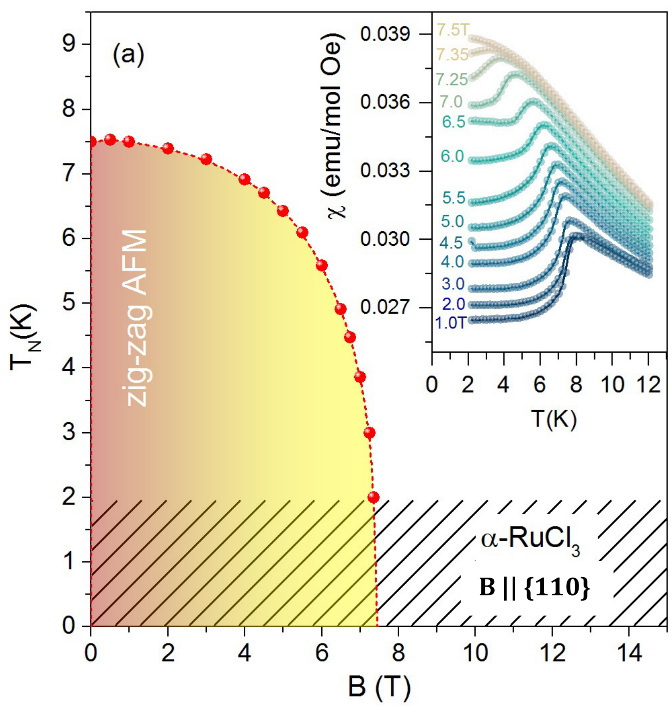
theoretical possibilities

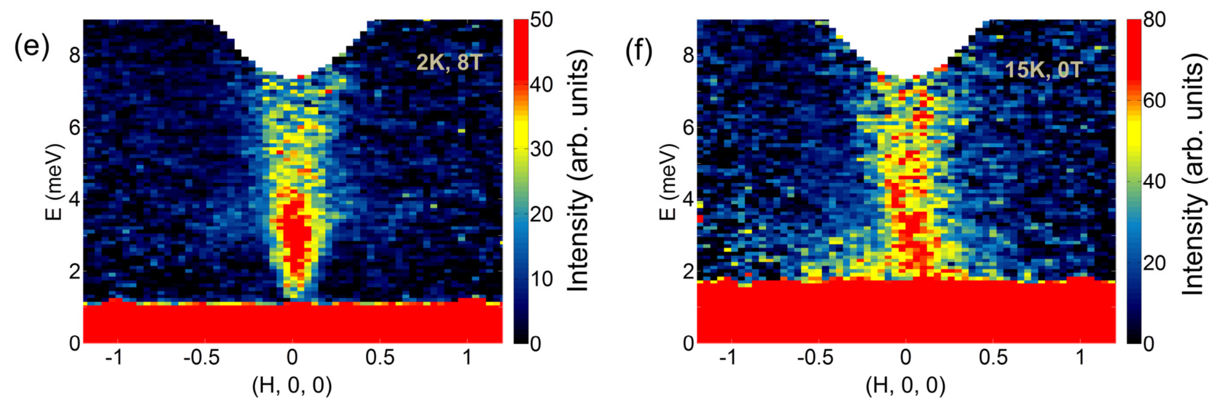


Suppressing order

With a magnetic field:

A. Banerjee *et al*, 2017



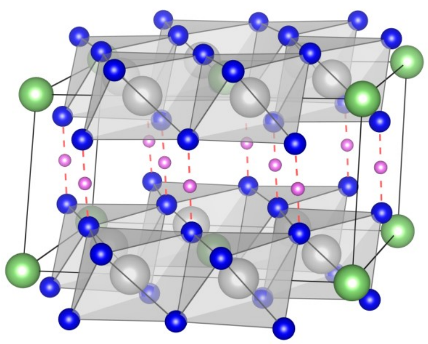


Lots of current studies with many probes...stay tuned!

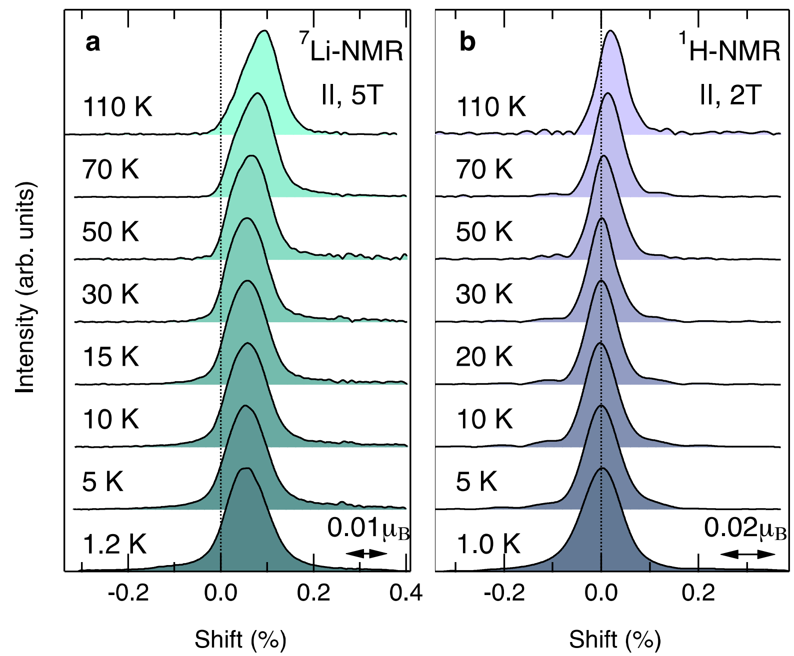
Suppressing order

With a new material

K. Kitagawa *et al*, unpublished



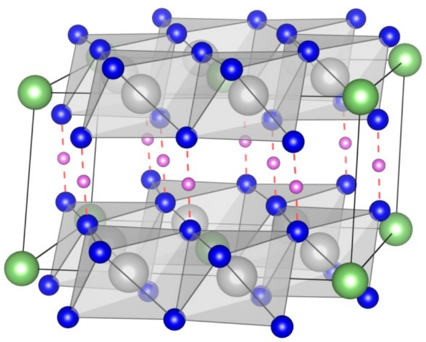
$H_3LiIr_2O_6$



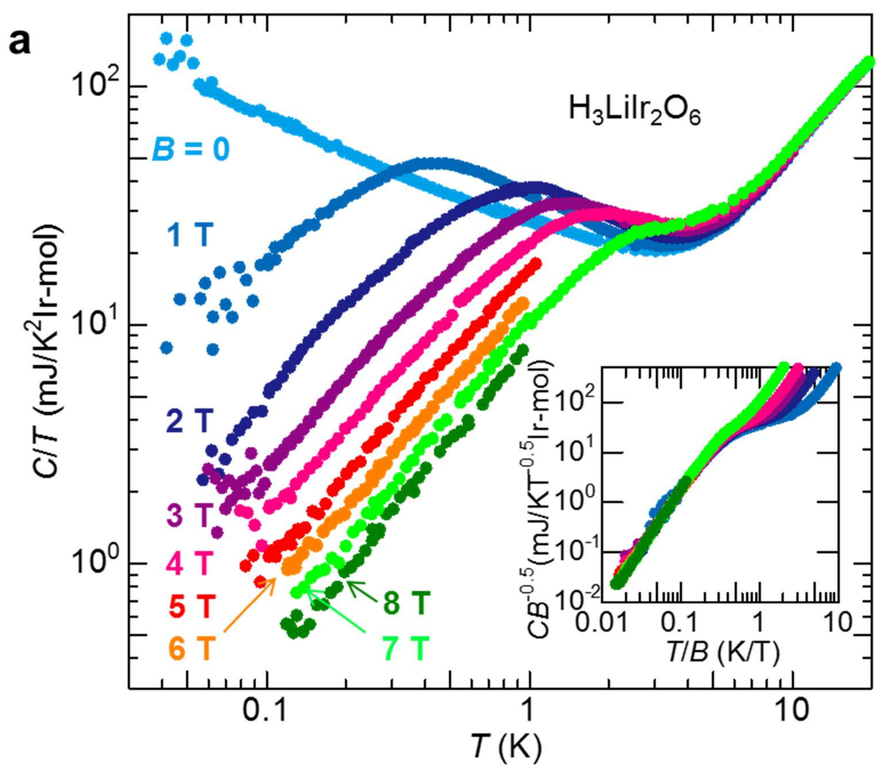
Suppressing order

With a new material

K. Kitagawa *et al*, unpublished

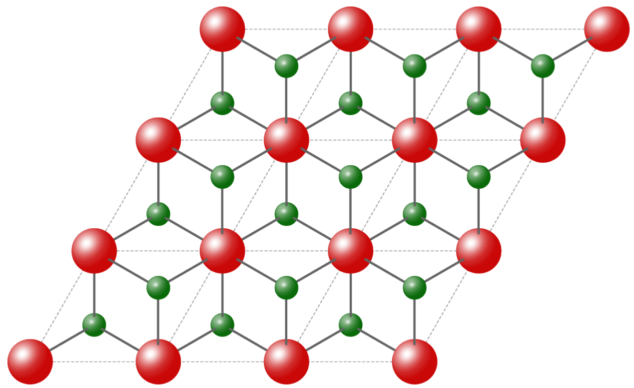


$H_3LiIr_2O_6$

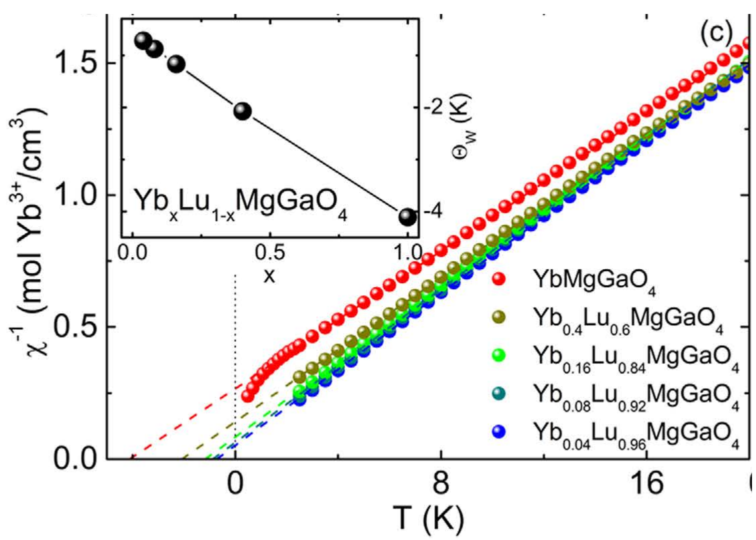


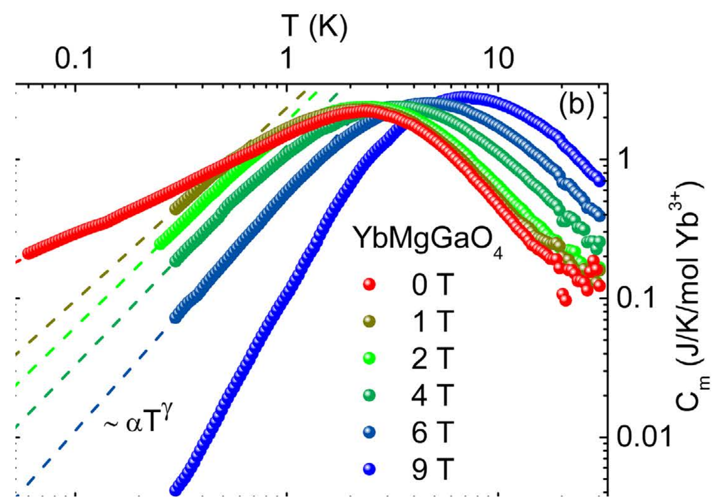
intriguing behavior *not* expected from Kitaev's solution

$YbMgGaO_4$



effective S=1/2 triangular lattice





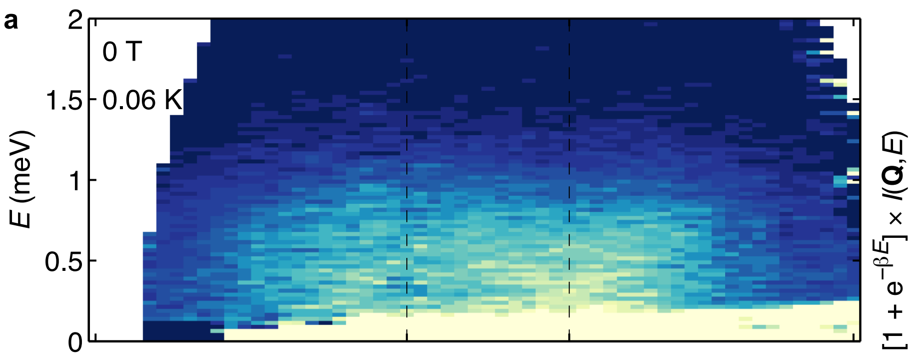
$f \approx 4K/50mK \geq 80$

$C \sim T^{0.7}$

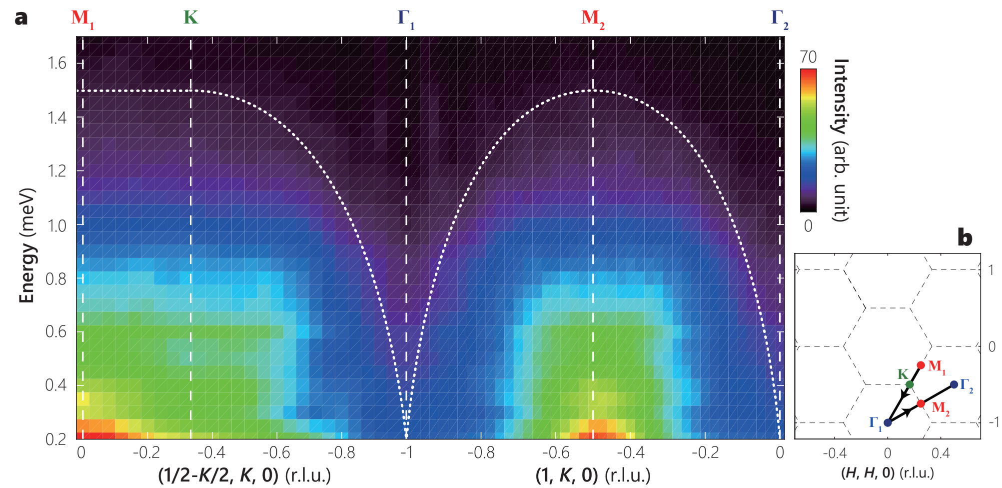
Y. Li *et al*, 2015

$YbMgGaO_4$

Neutron scattering studies



J. Paddison *et al*, arXiv:1607.3231

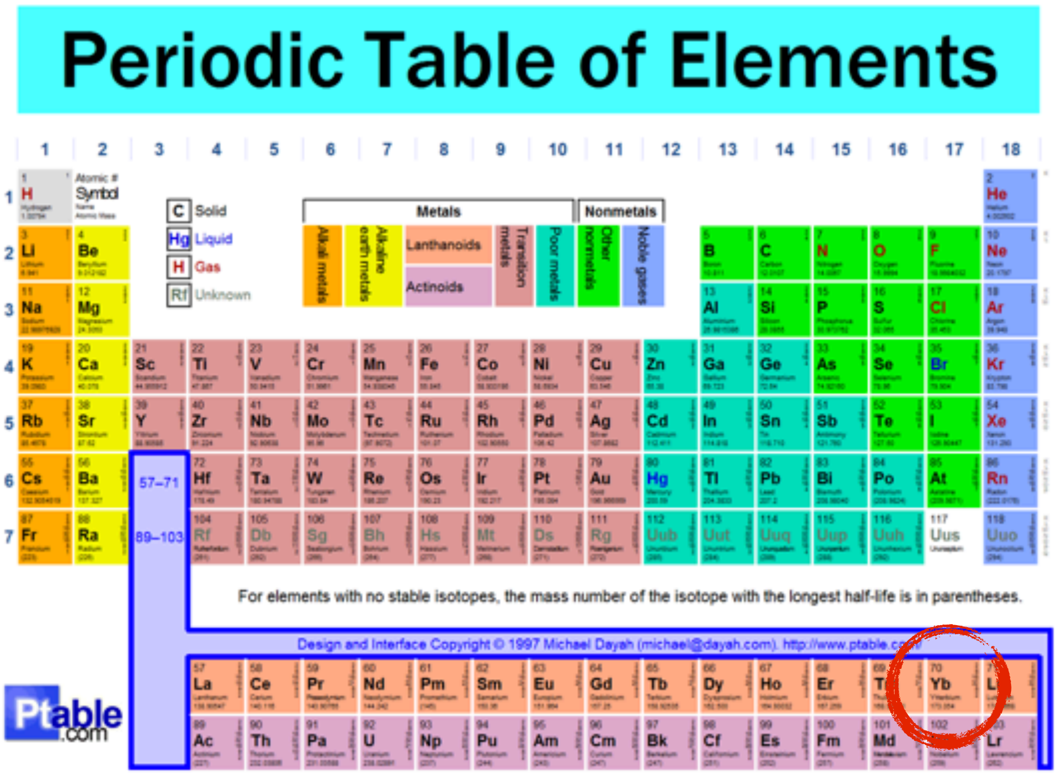


Y. Shen *et al*, arXiv:1607.02615

Shen *et al suggest* this is the structure factor of a spinon Fermi surface

a rare earth version of organics?

$YbMgGaO_4$



Interesting difference from organics: **strong spin-orbit coupling**

Anisotropic interactions like Kitaev but no exact solution. What do we do? Can QSLs compete? Is spinon Fermi surface favored?

SOC triangular

Generic model for "flat" triangular lattice

$$H = \sum_{\langle ij \rangle} \Big[J_{\pm} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) + J_z S_i^z S_j^z$$

$$+ J_{\pm\pm} \left(\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^- \right)$$

$$+ i J_{\pm z} \left(\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j) \right) \Big]$$

XXZ

bond-dependent couplings

Y. Li *et al*, 2015

Tool: variational wavefunctions

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

Gutzwiller projection of *spin-orbit coupled* free fermions

SOC triangular

Generic model for "flat" triangular lattice

$$H = \sum_{\langle ij \rangle} \Big[J_{\pm} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) + J_z S_i^z S_j^z$$

XXZ

$$+ J_{\pm\pm} \left(\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^- \right)$$

$$+ i J_{\pm z} \left(\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j) \right) \Big]$$

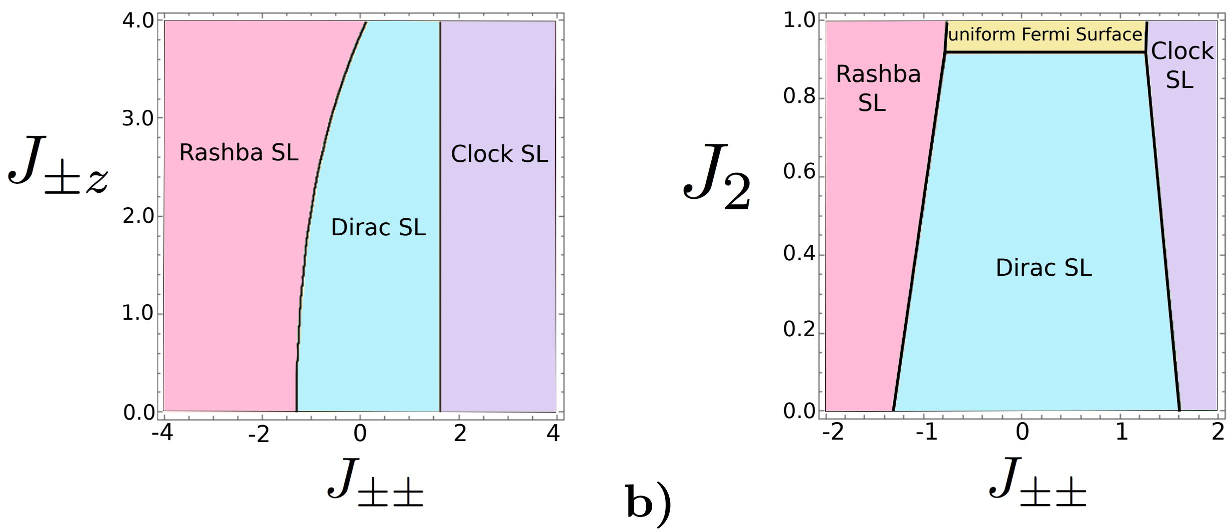
bond-dependent couplings

Tool: variational wavefunctions w/ VMC

- classify and **compare energetics** of all QSLs with triangular symmetry and their stability against magnetic order

SOC triangular

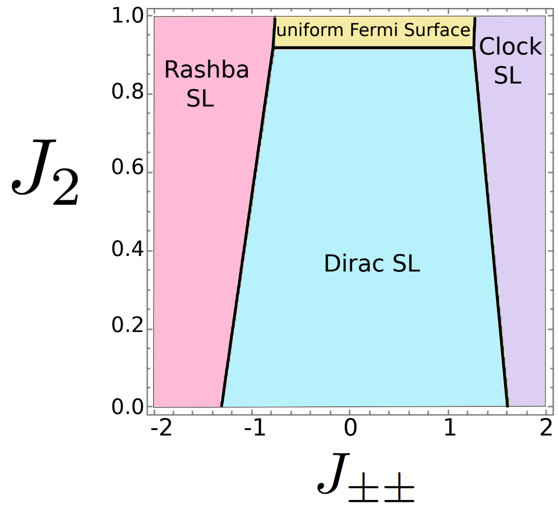
Comparing QSLs

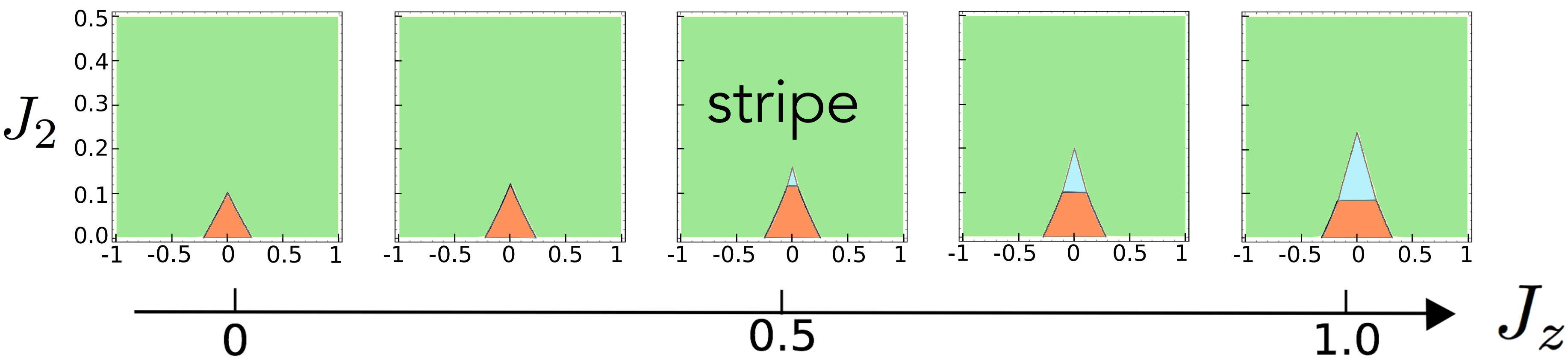


Dirac QSL dominates unless anisotropy is very strong

SOC triangular

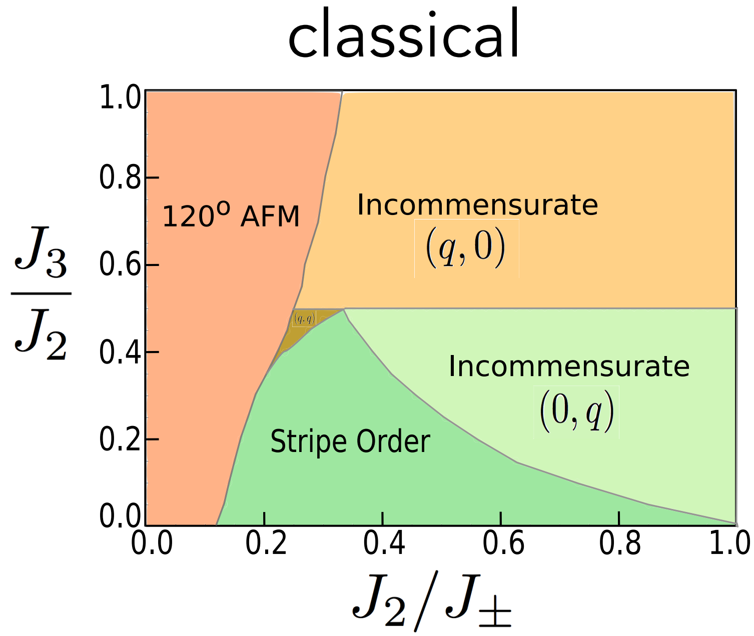
Allow magnetic order

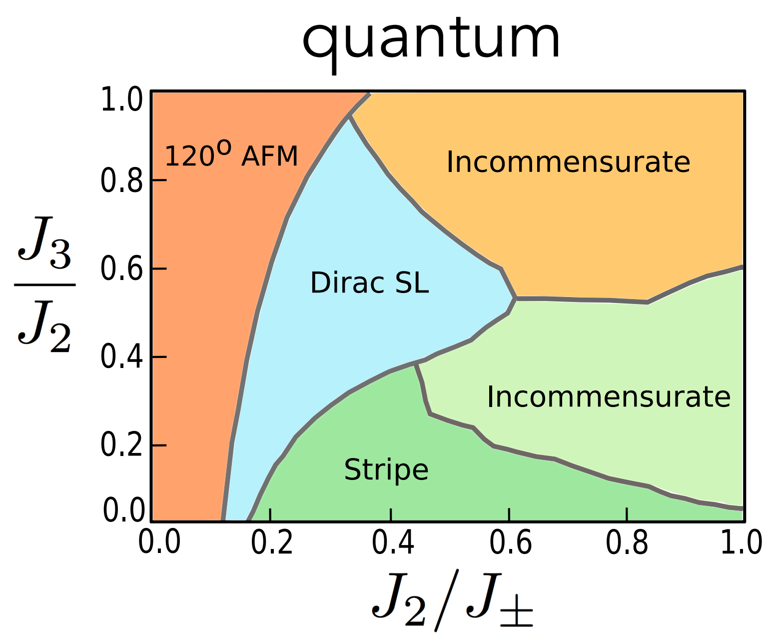




SOC triangular

QSLs versus magnetic order





Ordered states outcompete all but the Dirac QSL according to standard Gutzwiller method

SOC triangular

Any hope for the Fermi surface?

$$|\Psi\rangle = e^{-\alpha H_{\pm\pm}} \hat{P}_G |\psi_{FF}\rangle$$

beyond Gutzwiller:
qualitative effects
due to SOC

$$\mathrm{E}_{uFS} = -0.4693(1 + J_z/4) - \frac{0.39\, J_{\pm\pm}^2}{J_\pm + 1.42 J_z},$$

$$\mathrm{E}_{Dirac} = -0.7054(1 + J_z/4) - \frac{0.21\, J_{\pm\pm}^2}{J_\pm + 0.87 J_z}.$$

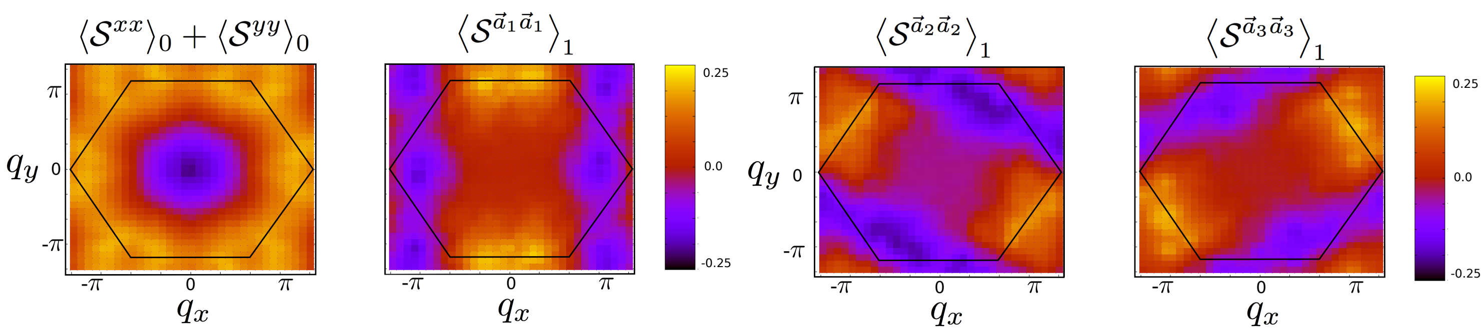
FS might
compete with
larger anisotropy,
and if stripy order
is removed

SOC Fermi surface

$$|\Psi\rangle = e^{-\alpha H_{\pm\pm}} \hat{P}_G |\psi_{FF}\rangle$$

Physical effects!

1. Spin correlations



2. Anomalous thermal Hall effect

field-induced Berry curvature

$$\kappa_{xy} \sim \frac{T h^3 J_{\pm z}^2}{J^5}$$

Summary

- Strongly SOC magnets are a new arena for QSLs
- VMC techniques are a systematic way to study their complex phases and fairly check the competition between QSLs and ordering
  - interesting to apply to Kitaev materials
- New physical effects: anisotropic spin correlations and thermal Hall effect appear through SOC's influence.

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A future history of magnetism (from 2014)

time

~500BC: Ferromagnetism documented in Greece, India, used in China

sinan, ~200BC

1949AD: Antiferromagnetism proven experimentally

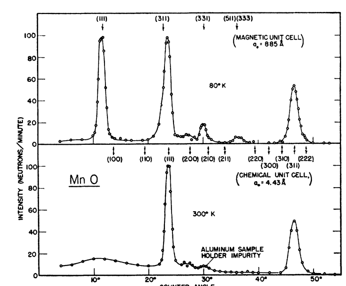

Fig. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

~2016AD: Conclusive experiments on quantum spin liquids?

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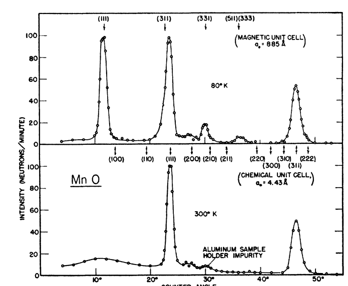

FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

~2019AD?: Conclusive experiments on quantum spin liquids?

Thanks for your attention

References here:

- https://spinsandelectrons.com/
- https://spinsandelectrons.com/pedagogy/

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