

Quantum Spin Liquids



Influenced by

Lucile Savary

SungBin Lee

Shigeki Onoda

Gang Chen

Oleg Starykh

Hong-Chen Jiang

Yi-Zhuang You

Xueyang Song

Bruce Gaulin

Kate Ross



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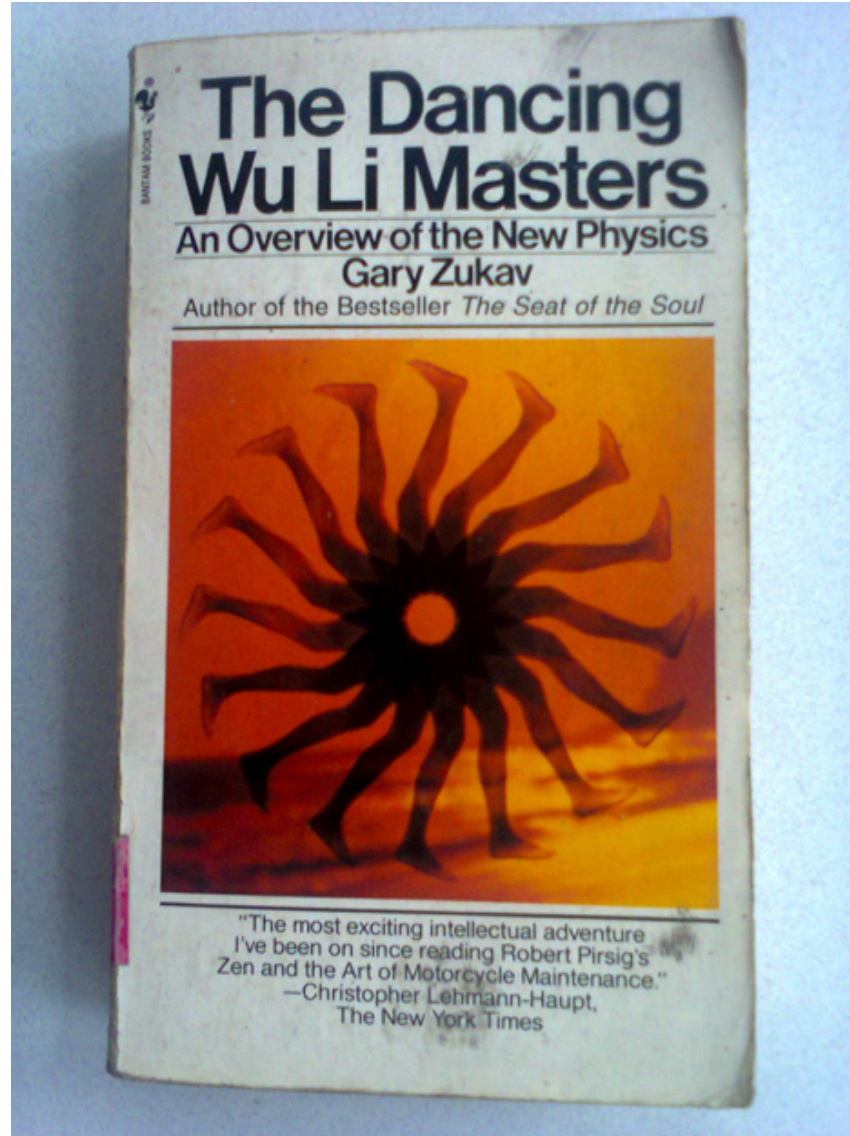
Xueyang Song

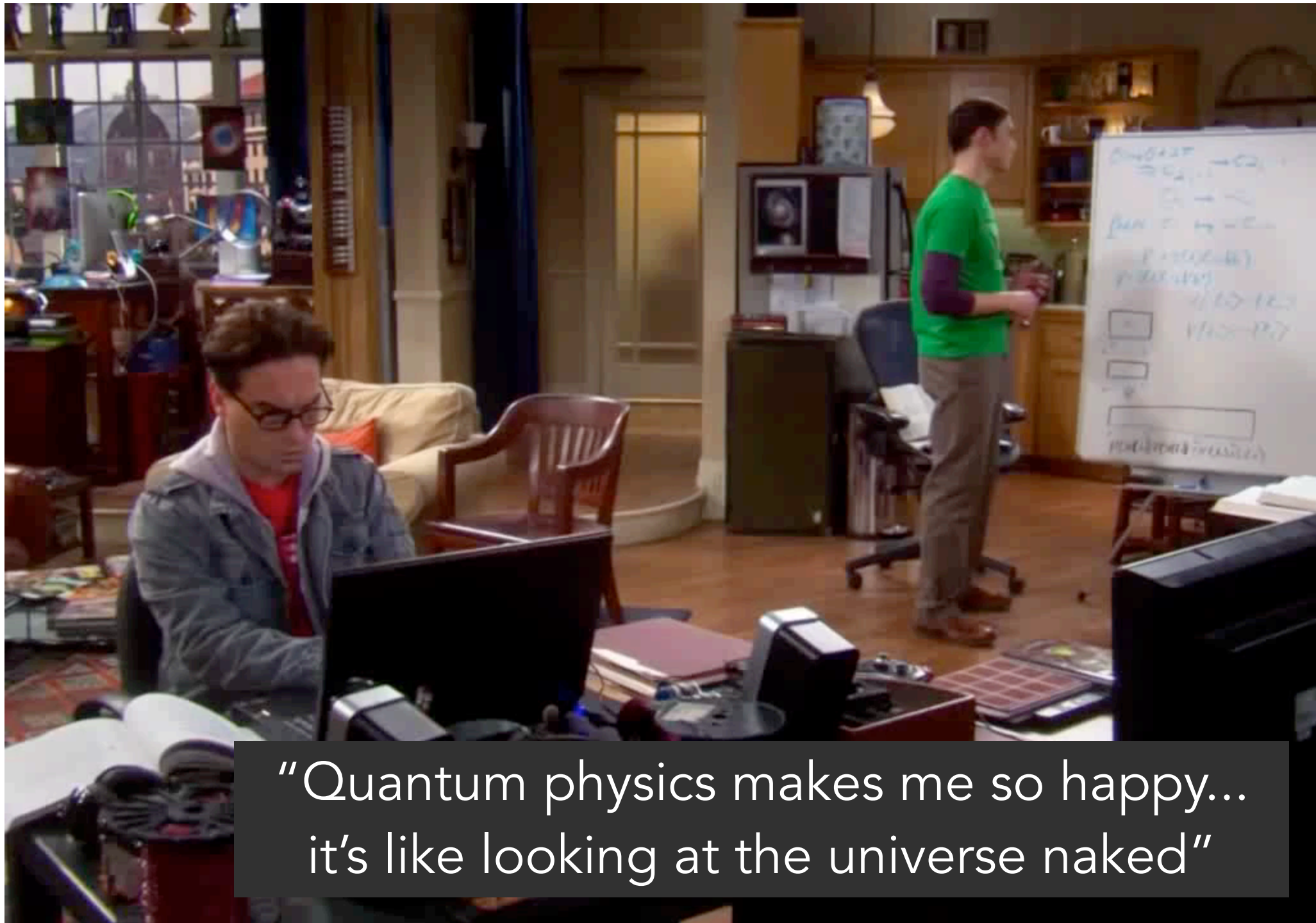
Bruce Gaulin

Kate Ross



Quantum Spin Liquid





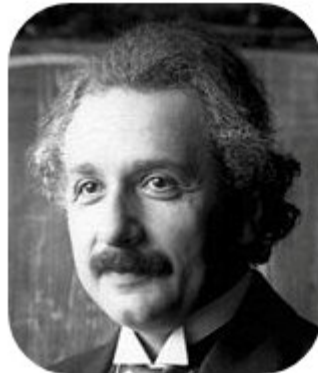
“Quantum physics makes me so happy...
it's like looking at the universe naked”

Quantum non-locality

EPR $|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$



??where is the information??



A. Einstein



B. Podolsky

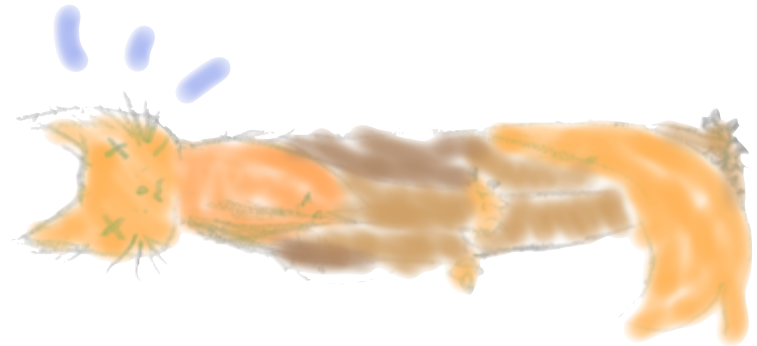


N. Rosen

Schrödinger's Cat

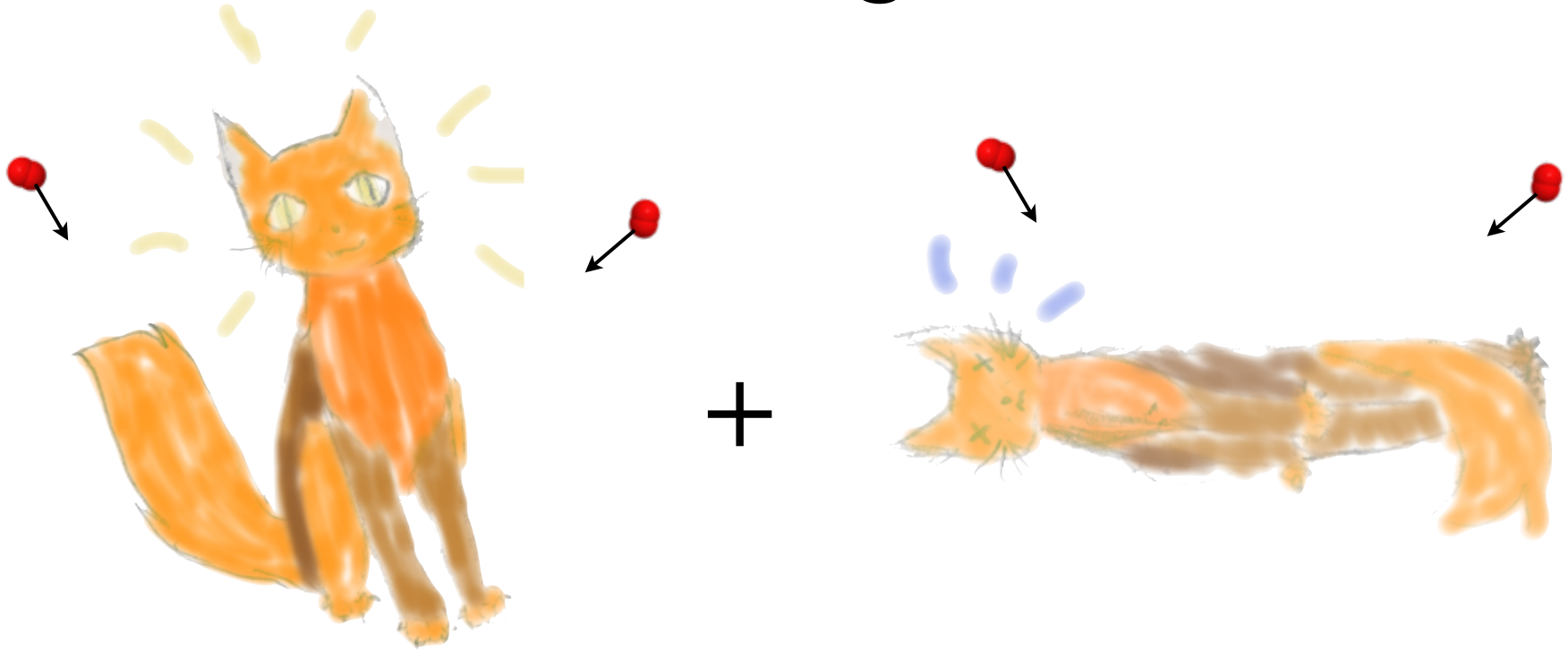


+



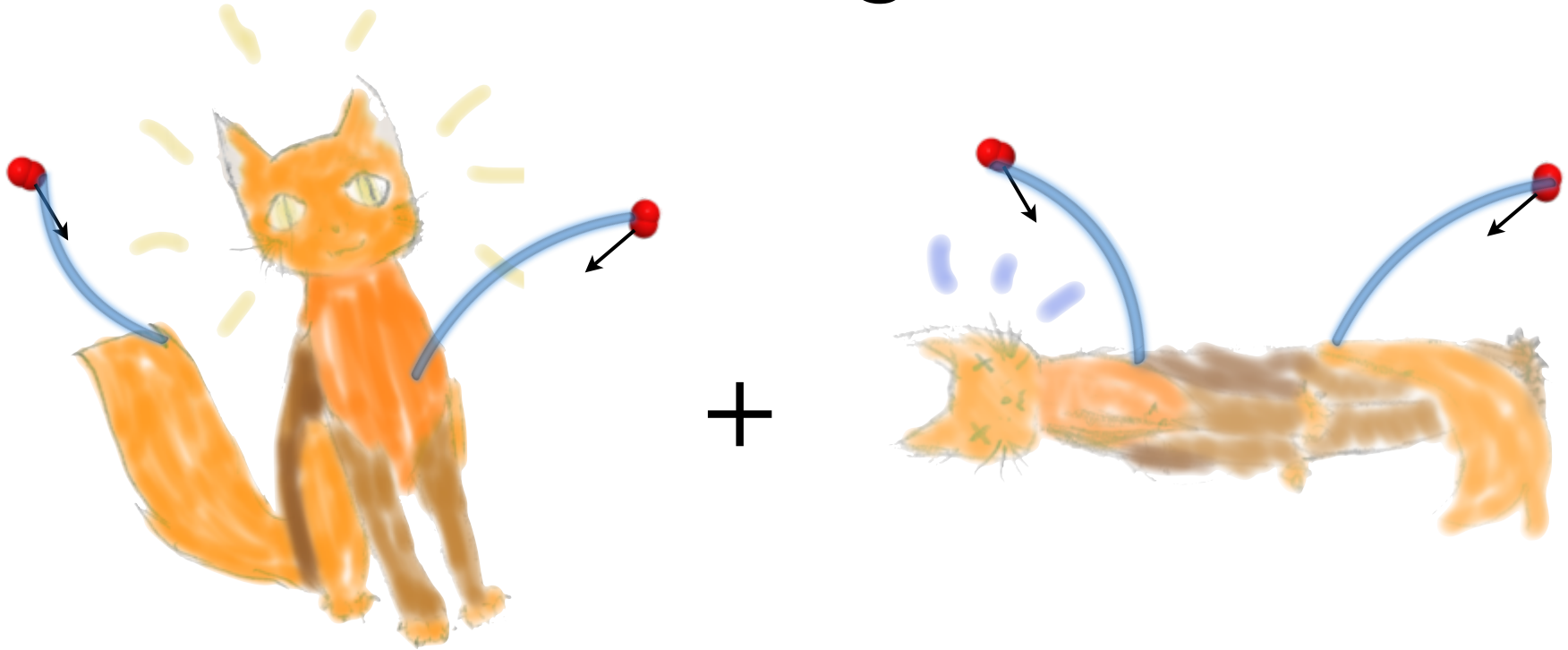
© Megan Balents

Schrödinger's Cat



measurement of any hair of the cat will collapse
the superposition

Schrödinger's Cat



UNSTABLE to decoherence - uncontrolled entanglement with the environment



Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Psi = \text{diagram 1} + \text{diagram 2} + \dots$$

The diagram shows two 4x4 grids of blue ovals on a triangular lattice. In the first grid, the ovals are arranged in a regular pattern. In the second grid, the ovals are arranged in a different pattern, representing a different state in the superposition.

Resonating **V**alence **B**ond state

Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Psi = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$

The diagram shows the wavefunction Ψ as a sum of states. Each state is represented by a triangular lattice of blue ovals, which represent spin singlets. In the first diagram, the ovals are arranged in a regular, repeating pattern. In the second diagram, the ovals are shifted or rearranged, representing a different configuration of the quantum liquid. The ellipsis indicates that there are many more such states in the sum.

Resonating Valence Bond state



Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties



- Measurements far away do not affect one another
- From local measurements we can deduce the global state

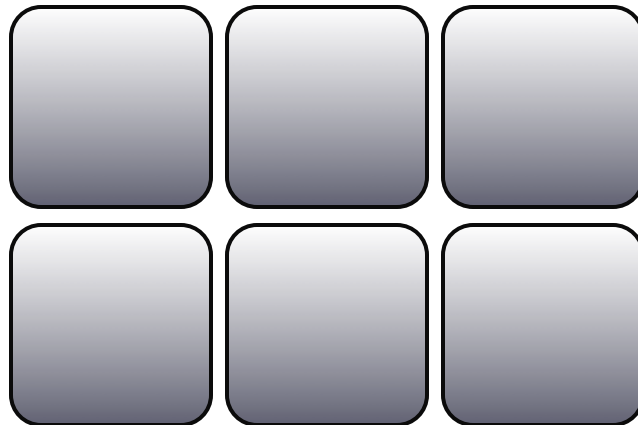
Ordinary (local) Matter

Hamiltonian is local

$$H = \sum_{\mathbf{x}} \mathcal{H}(x) \quad \mathcal{H}(x) \text{ has local support near } x$$

Ground state is "essentially"
a product state

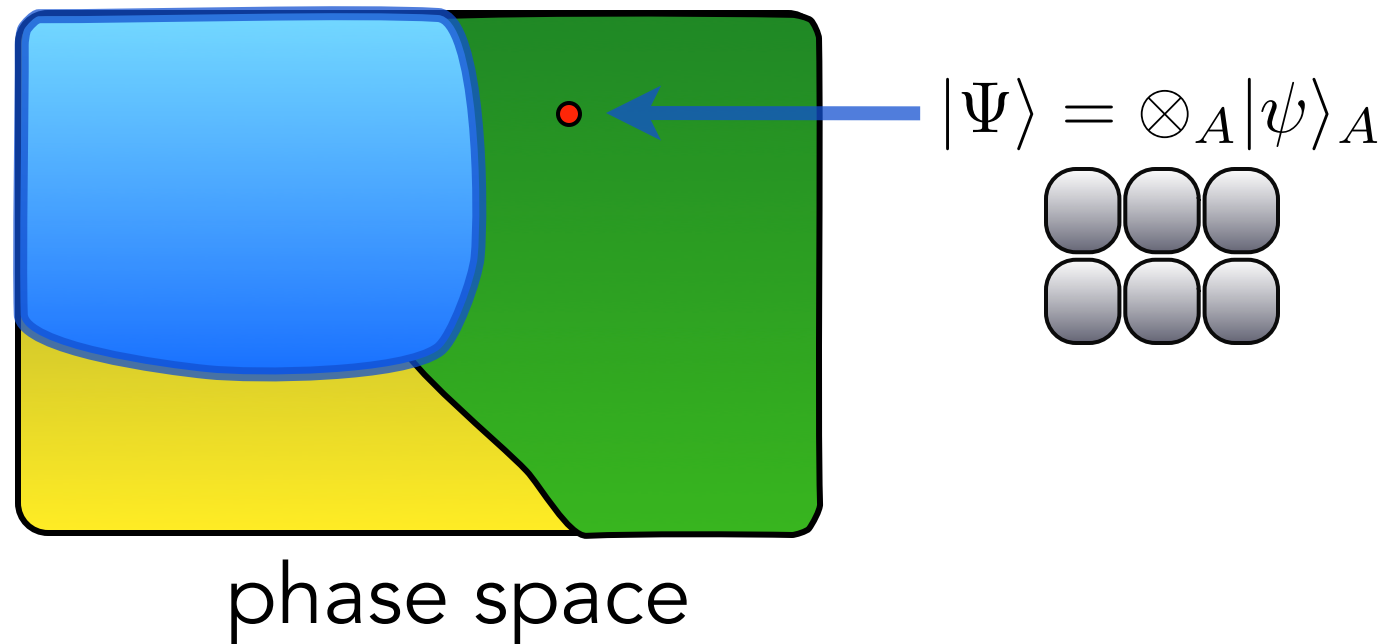
$$|\Psi\rangle = \otimes_A |\psi\rangle_A$$



no entanglement
between blocks

"Essentially" a product state?

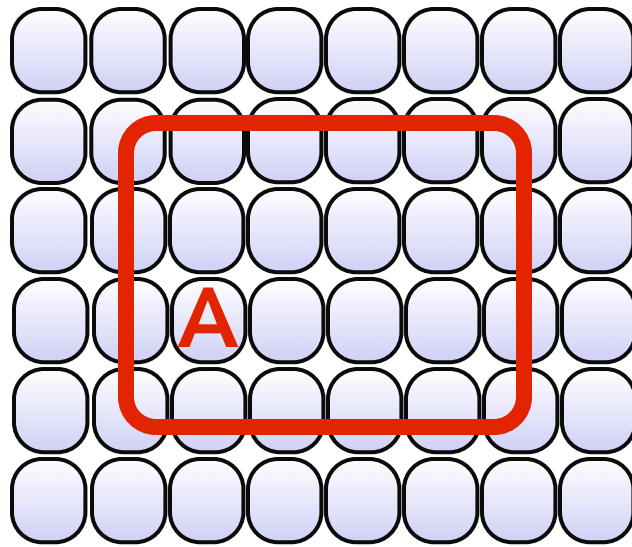
- Adiabatic continuity



n.b. This is not true for gapless fermi systems

“Essentially” a product state?

- Entanglement scaling



$$\rho_A = \text{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\text{Tr}_A (\rho_A \ln \rho_A)$$

$$S(A) \sim \sigma L^{d-1} \quad \text{area law}$$

satisfied with exponentially small corrections

Best example: ordered magnet

Hamiltonian

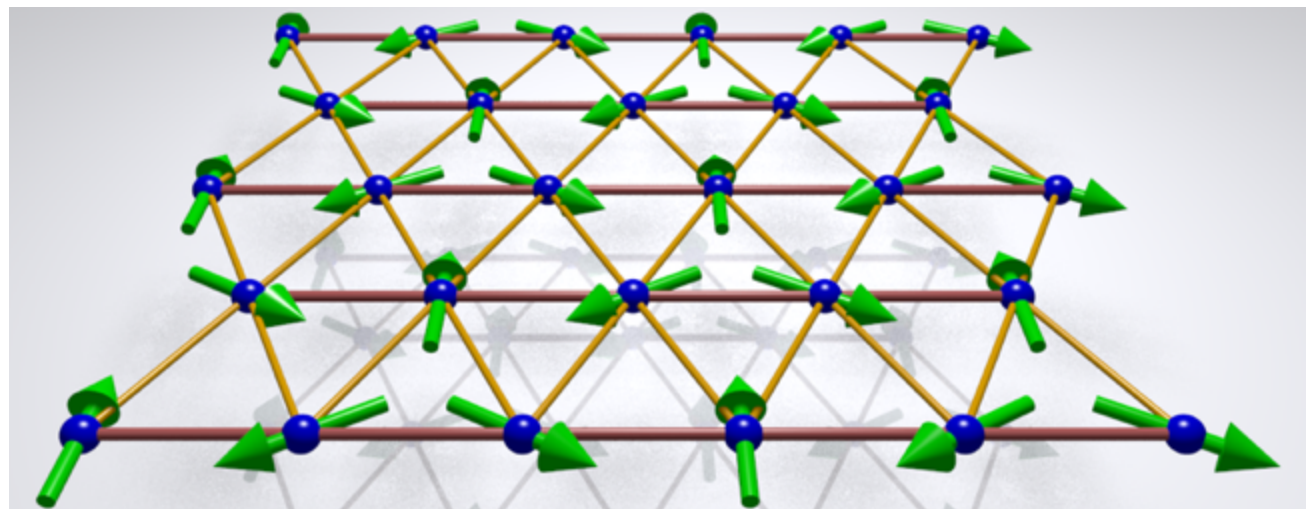
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-
range: local

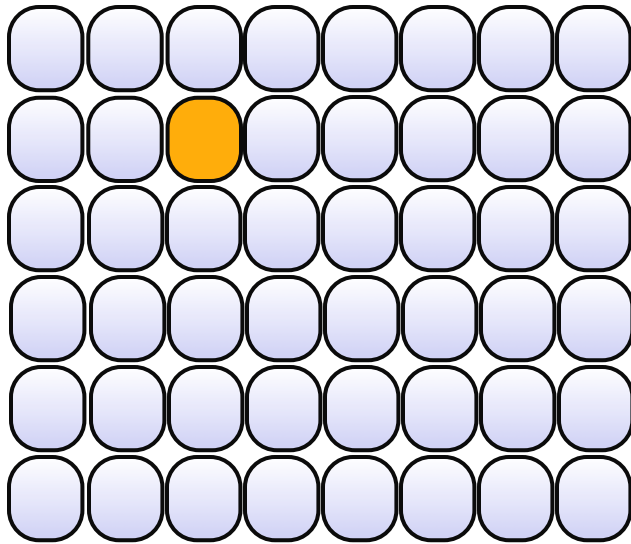
ordered state

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{n}_i = +S\rangle$$

block is a single
spin



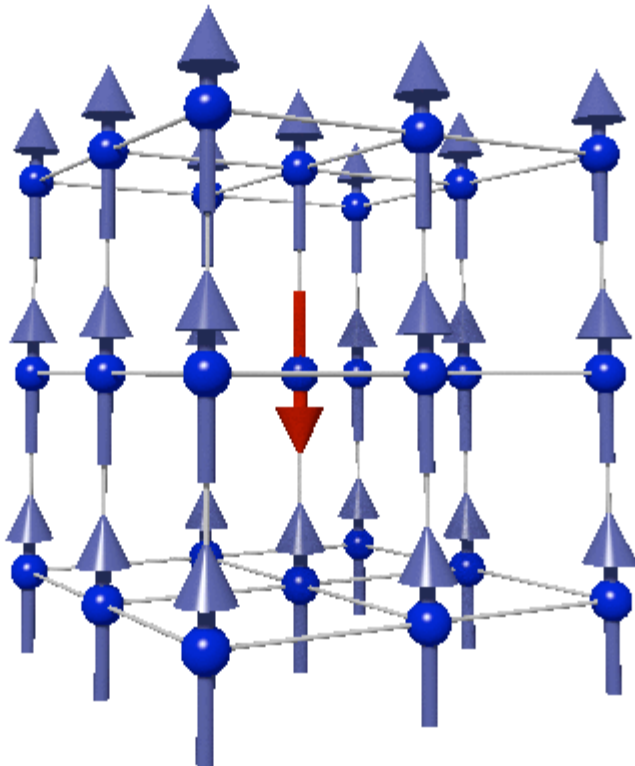
Quasiparticles



excited states $\sim$ excited levels of one block

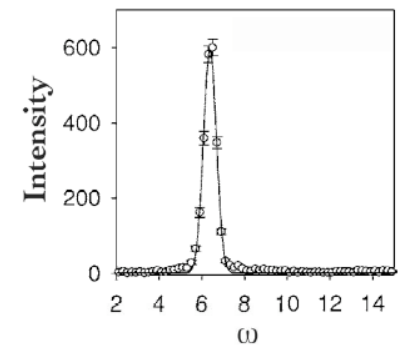
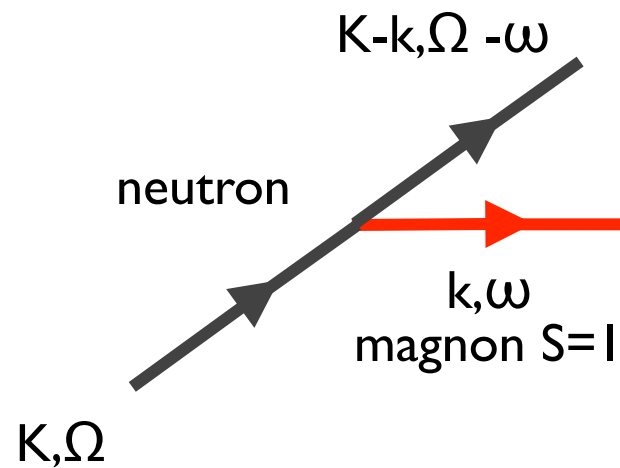
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

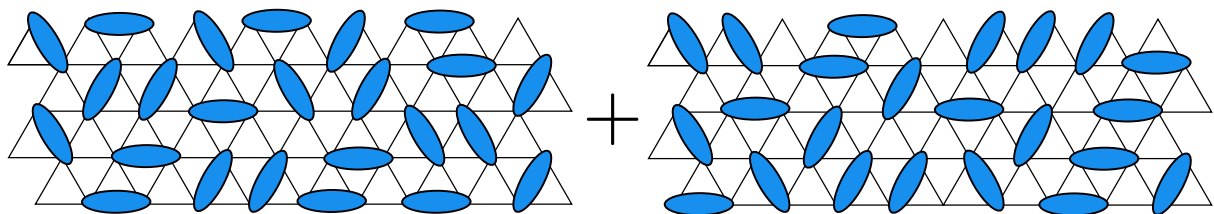
$$|f\rangle = S_k^+ |i\rangle$$



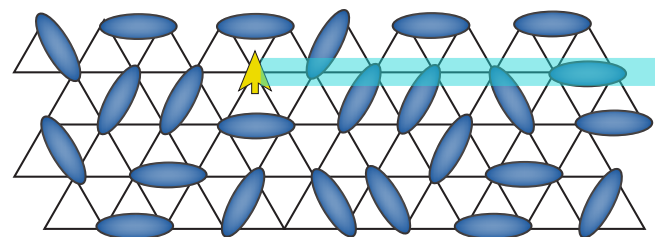
Line shape in Rb_2MnF_4

Quantum spin liquid

Entanglement $\rightarrow$ non-local excitation

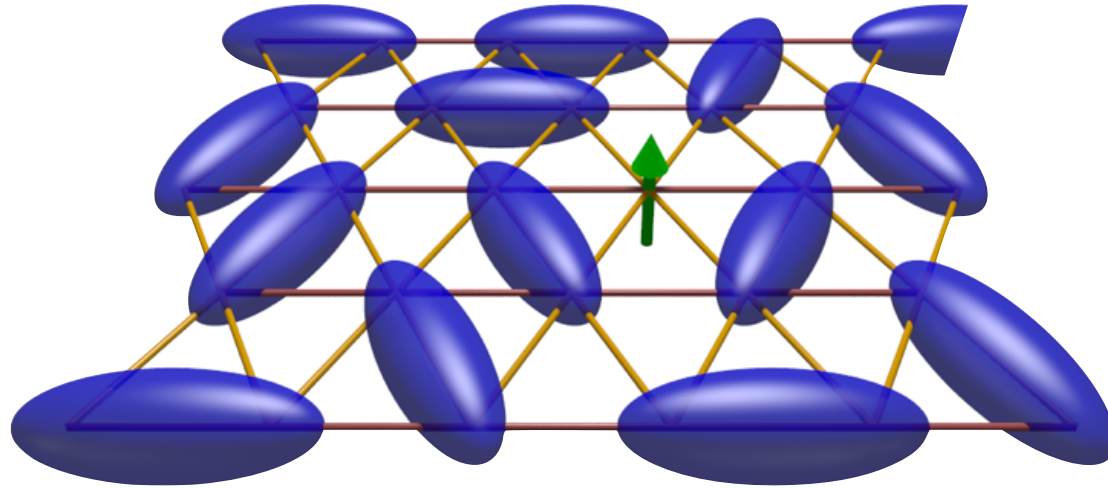
$$\Psi = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$




$$\Psi = \text{[Diagram 3]} + \dots \quad \text{"spinon"}$$


"quasiparticle" above a non-zero gap

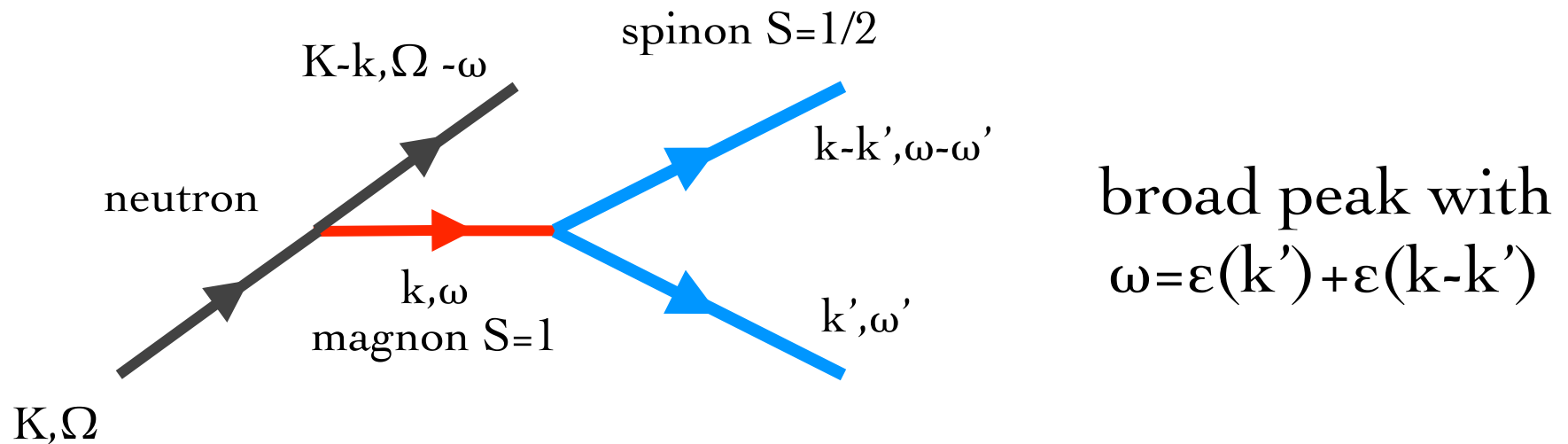
Fractional quantum number



excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins
always created in pairs by any
local operator

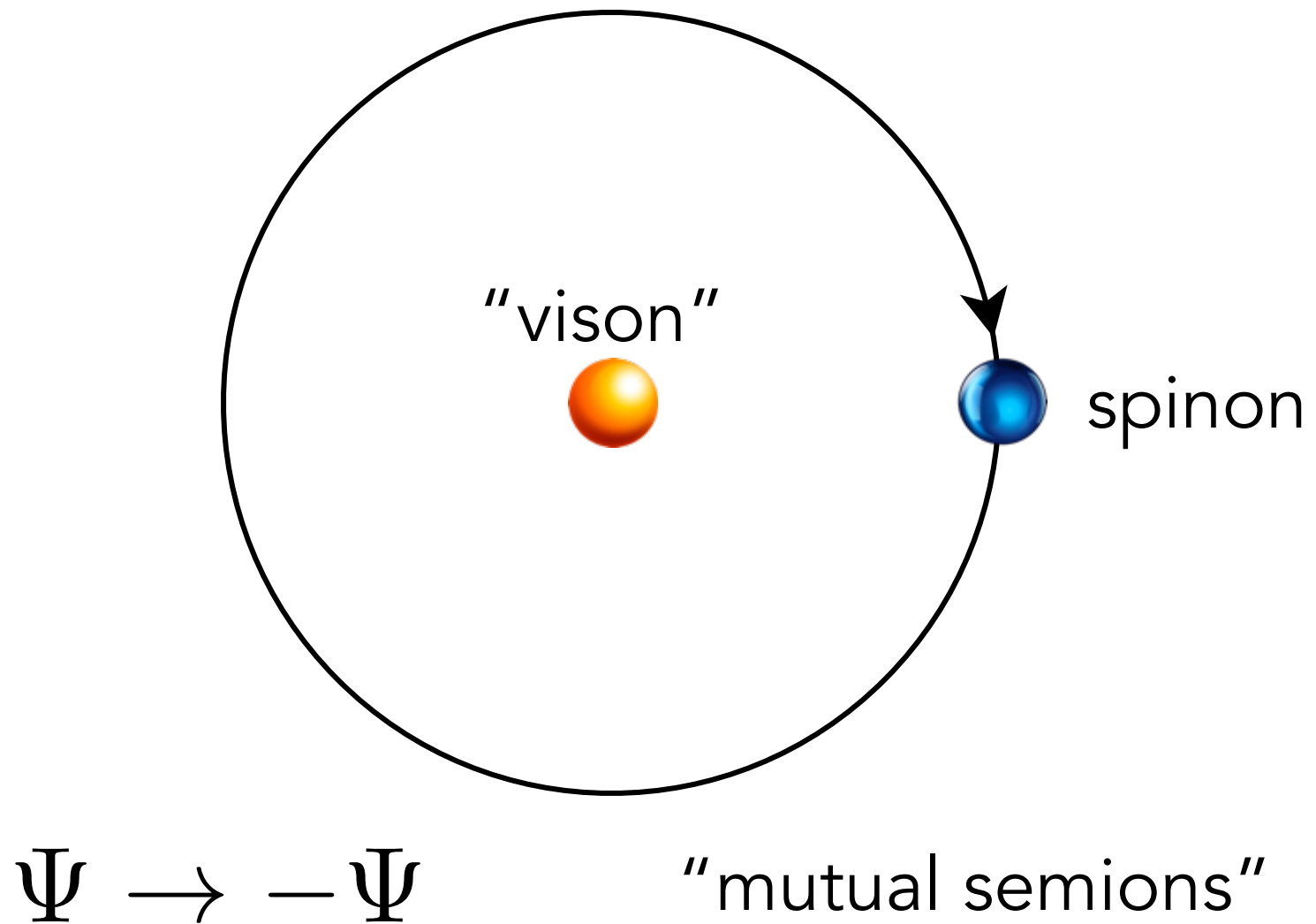
No spin waves

- In a quantum spin liquid, the elementary spin excitations are *fractional*, $S=1/2$ spinons



- Sharp peaks should be reduced or absent in the spin structure factor

Anyons





X.-G. Wen

Topological phases



A. Kitaev



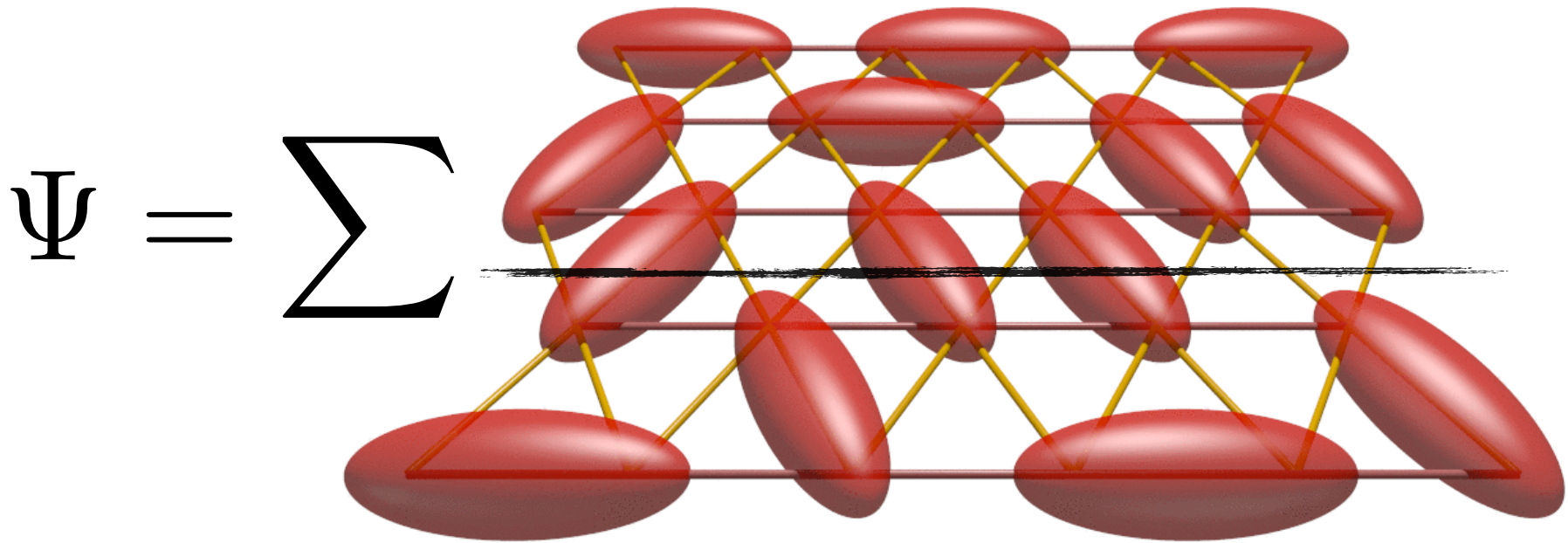
Anderson's RVB state is thus an example of a "topological phase" - the best understood sort of QSL

Understood and classified by anyons and their braiding rules in 2d

$$\begin{array}{c} e \quad m \\ \diagdown \quad \diagup \\ \text{---} \\ \diagup \quad \diagdown \\ e \quad m \end{array} = - \begin{array}{cc} e & m \\ | & | \\ e & m \end{array}$$

$$\begin{array}{cc} e m & e m \\ \diagdown & \diagup \\ \text{---} & \text{---} \\ \diagup & \diagdown \\ e m & e m \end{array} = \begin{array}{cc} e m & e m \\ | & | \\ \text{---} & \text{---} \\ | & | \\ e m & e m \end{array} = - \begin{array}{ccc} e m & e m & e m \\ | & | & | \\ e m & e m & e m \end{array}$$

Stability

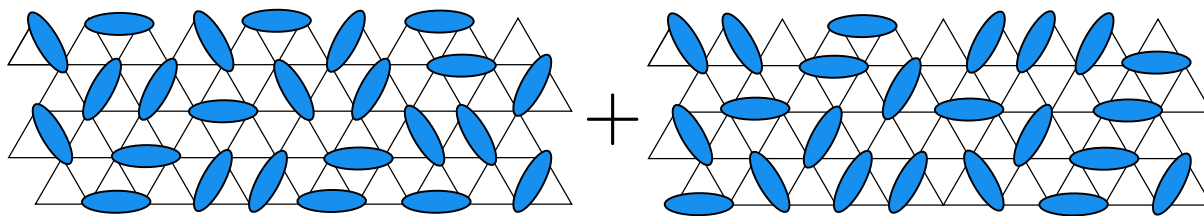


Robustness arises from topology: a QSL is a stable *phase* of matter (at $T=0$)

There is not just one kind
of QSL



Quantum spin liquid

$$\Psi = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$


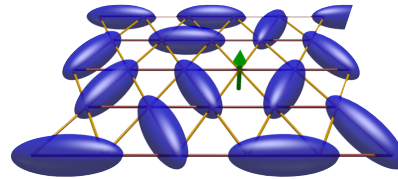
The diagram illustrates the wavefunction Ψ of a quantum spin liquid as a superposition of different spin configurations. It shows two triangular lattices, each with blue ellipses representing spins. The first lattice shows a specific arrangement of spins, and the second lattice shows a different arrangement. The two lattices are added together, followed by an ellipsis indicating that there are many more such configurations in the sum.

For ~ 500 spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.

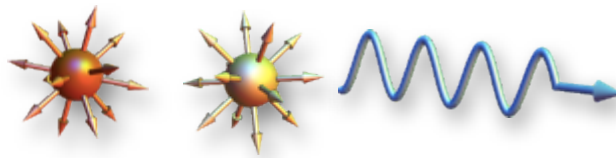
Classes of QSLs

- Topological QSLs



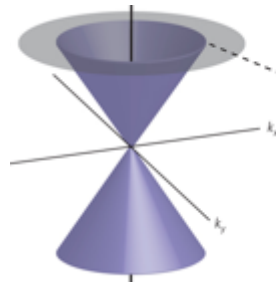
anyonic
spinons

- U(1) QSL



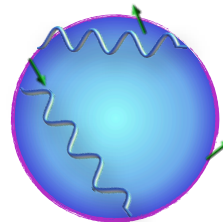
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

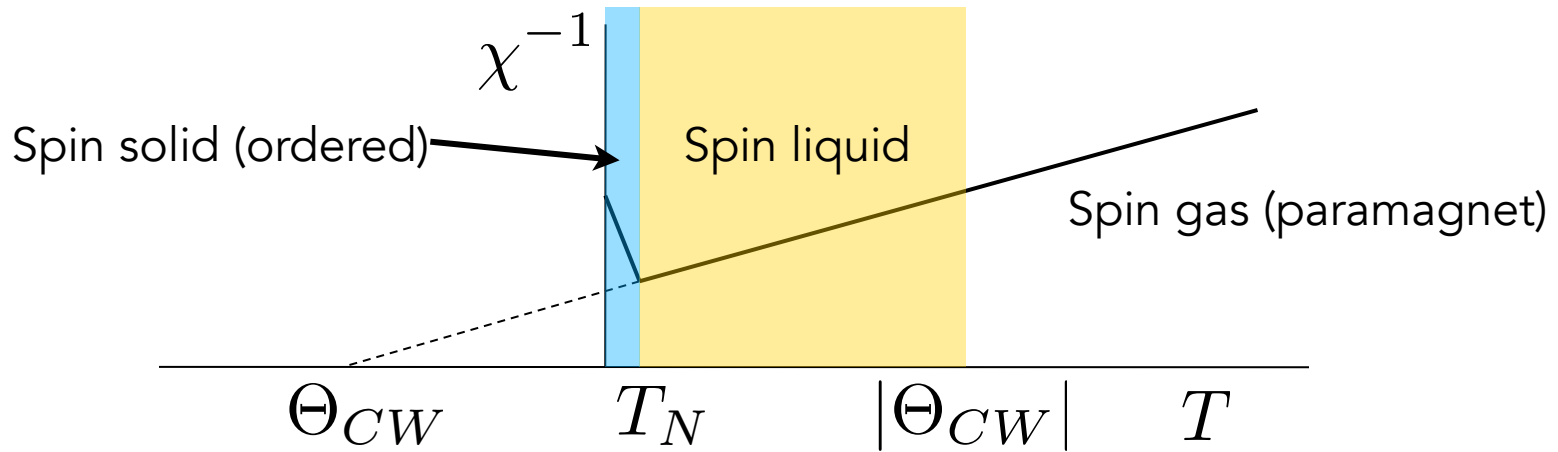
Strange stuff



where do we find it?



Ramirez Plot



- Local moments: Curie-Weiss law at high T

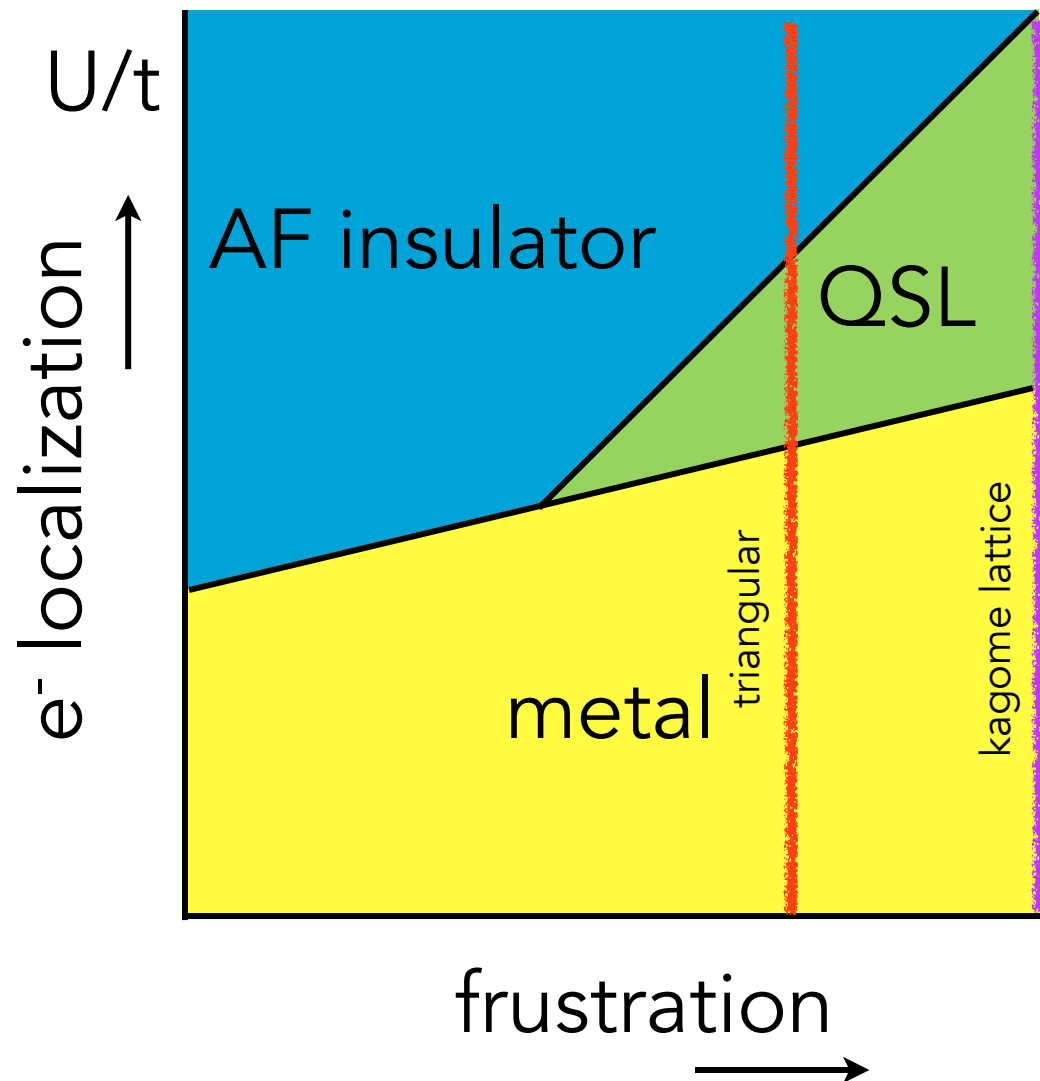
$$\chi \sim \frac{A}{T - \Theta_{CW}}$$

- Frustration parameter: $f = |\Theta_{CW}|/T_N$
- Larger $f \gg 1$ is better. $f = \infty$ for true QSL

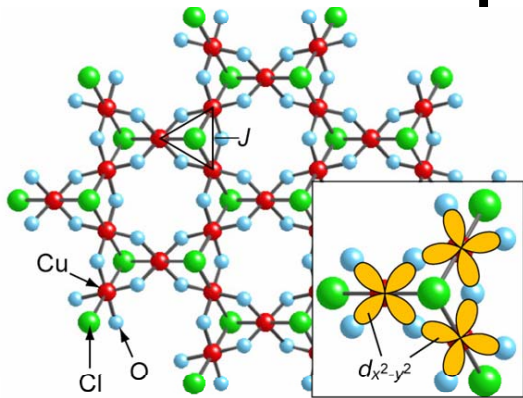
Materials criteria

- $S=1/2$ spins
- Geometrical or exchange frustration
- Significant charge fluctuations
- Exotic interactions (c.f. Spin-orbit coupling)

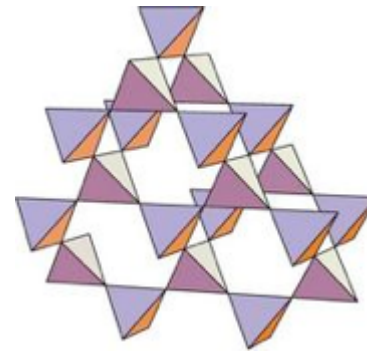
Where to look?



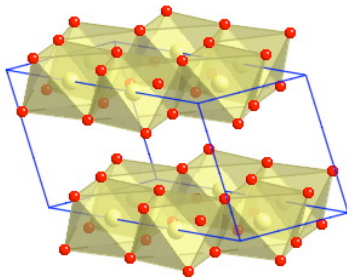
Top experimental platforms



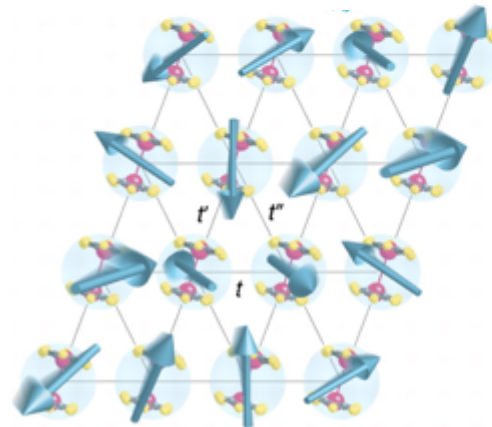
kagomé



Quantum spin ice

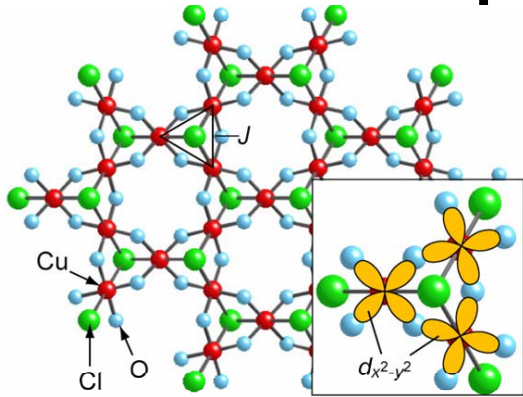


Kitaev materials

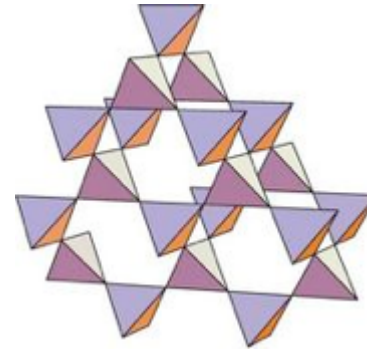


organics

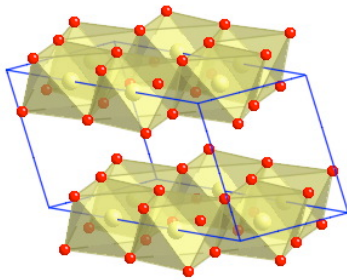
Top experimental platforms



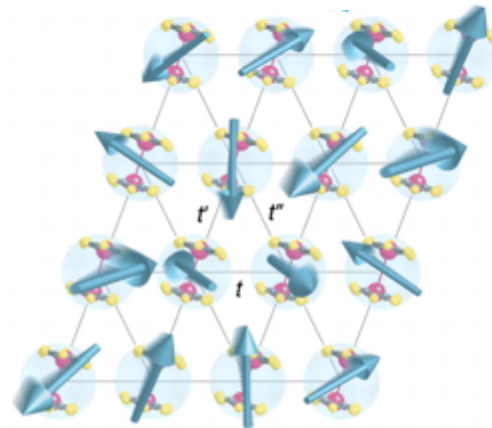
kagomé



Quantum spin ice

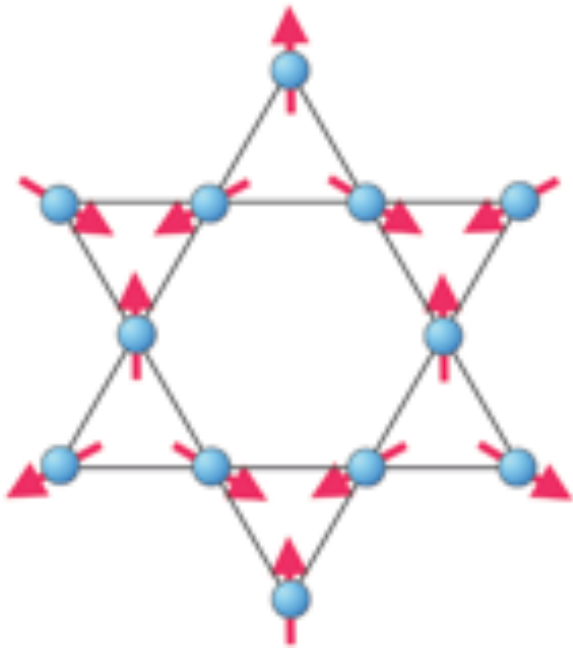


Kitaev materials



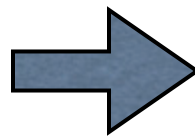
organics

Kagomé antiferromagnet



$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \dots$$

Very large classical
degeneracy

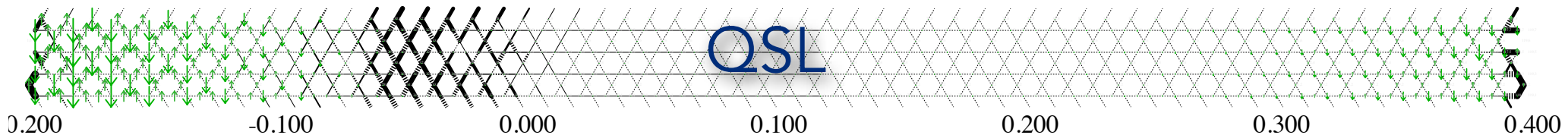


likely to be a QSL

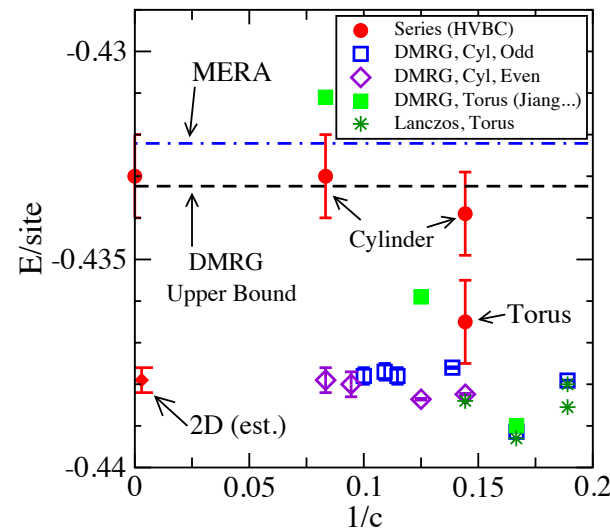
V. Elser, 1989 + many many others

$S=1/2$ kagomé AF

- Rather definitive evidence for QSL by DMRG



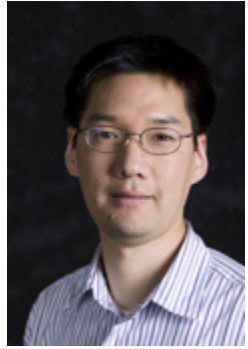
© Steve White



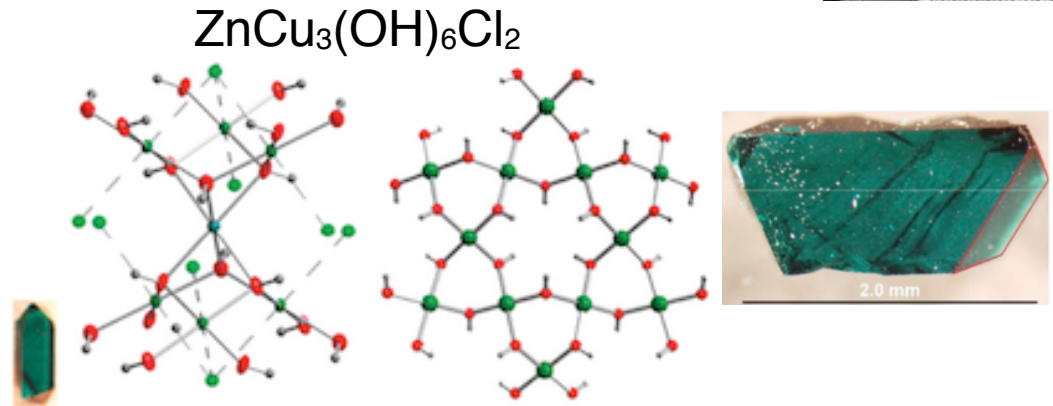
S. Yan et al, 2010

many other studies support
existence of some QSL phase

Herbertsmithite

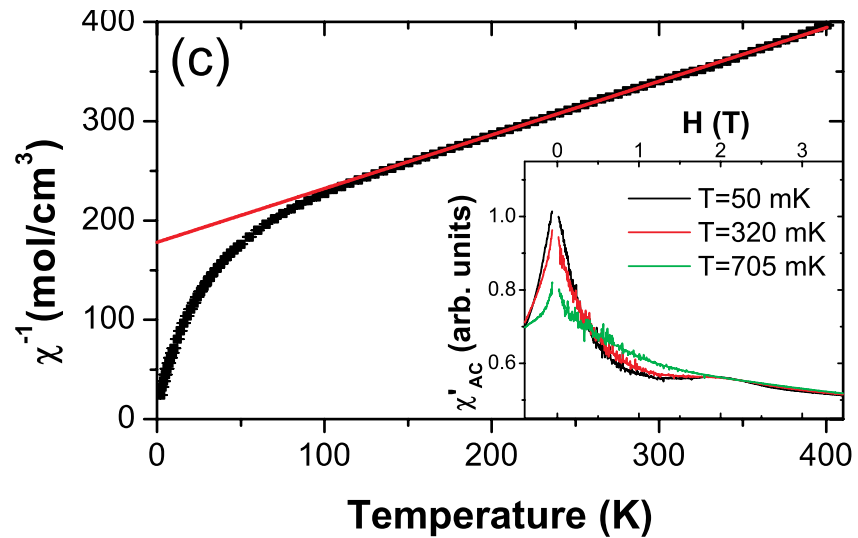


kagomé layers of Cu
 $S=1/2$ spins, separated
by non-magnetic Zn



Heisenberg-like
with $J \sim 200\text{K}$

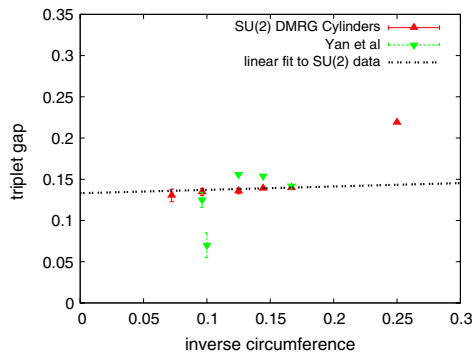
no order down to
50mK



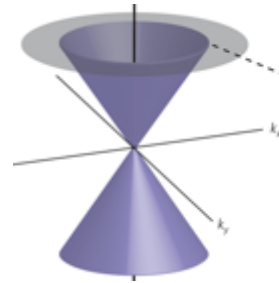
Helton *et al*, 2007

Theory

- What kind of QSL?



S. Depenbrock *et al*, 2012



Y. Ran *et al*, 2007
F. Becca...

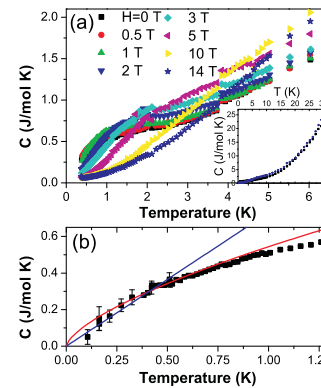
gapped,
topological QSL

gapless
Dirac QSL

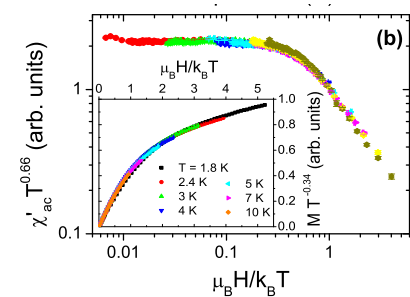
+ various other
proposals with
weaker
quantitative
support

Herbertsmithite

Lots of early evidence
for gaplessness



Helton et al, 2007

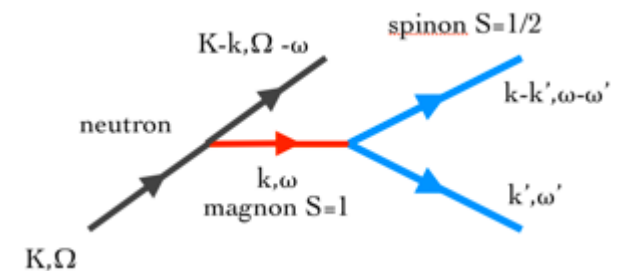
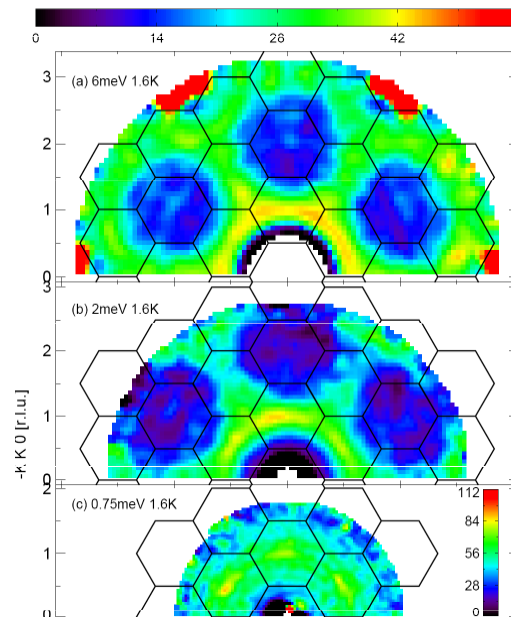


Helton et al, 2010

Single crystal INS

smooth continuum
scattering

T-H Han et al, 2012

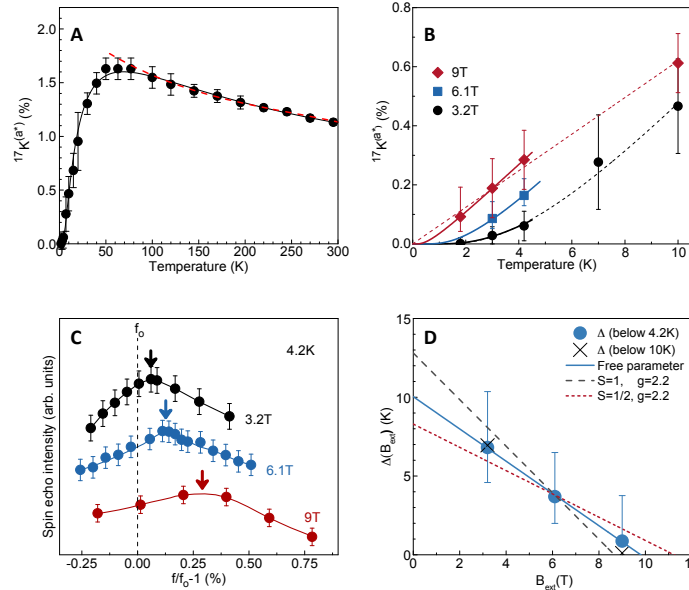


continuum scattering
expected
...but probably with more
structure?

Herbertsmithite

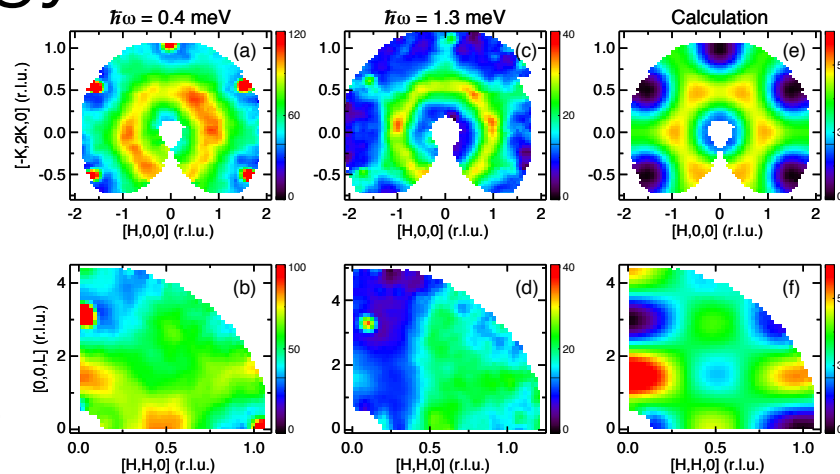
Single crystal NMR

M. Fu *et al*, 2015



estimate gap $\sim$
10K

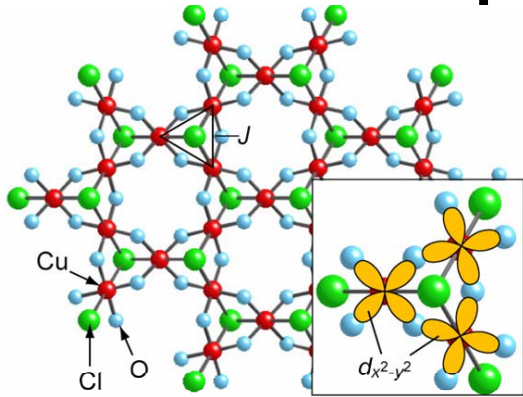
Low energy INS



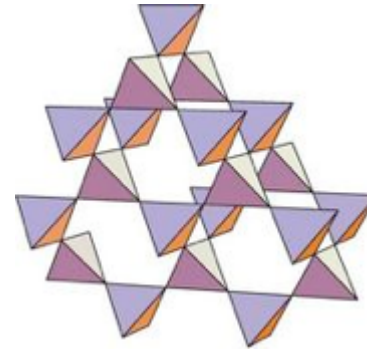
claim to separate
impurity signal
below 0.7 meV

T-H Han *et al*, 2015

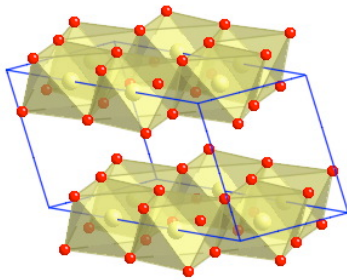
Top experimental platforms



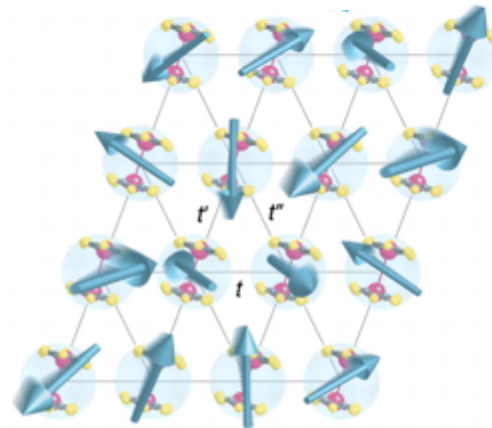
kagomé



Quantum spin ice



Kitaev materials



organics

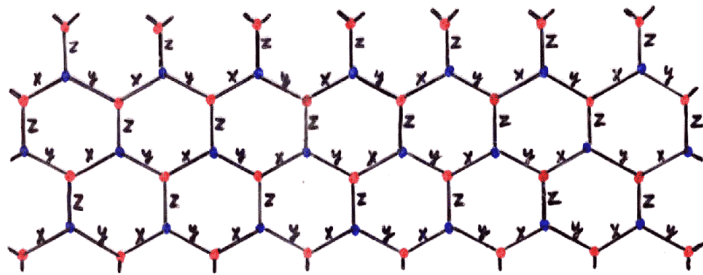


Kitaev model

Kitaev's honeycomb model

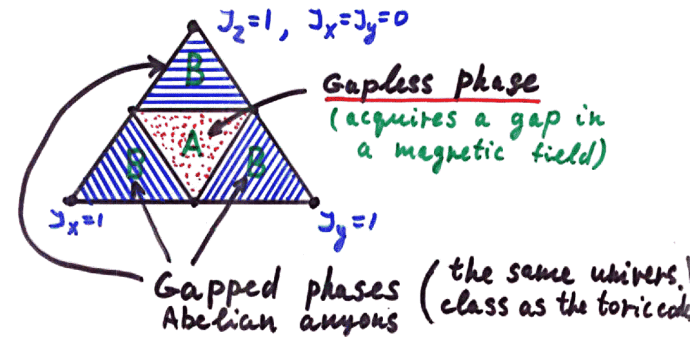
$$H = \sum_{i,\mu} K_{\mu} \sigma_i^{\mu} \sigma_{i+\mu}^{\mu}$$

1. The model



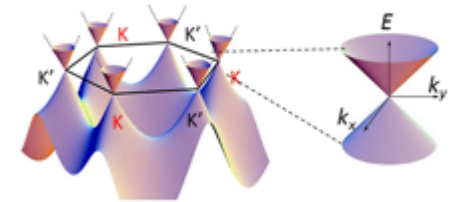
Spin $\frac{1}{2}$ on each site.

Phase diagram

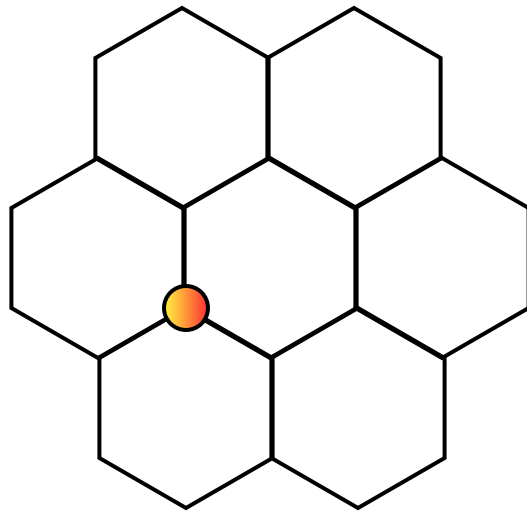


exact parton construction $\sigma_i^{\mu} = i c_i c_i^{\mu}$ $c_i c_i^x c_i^y c_i^z = 1$

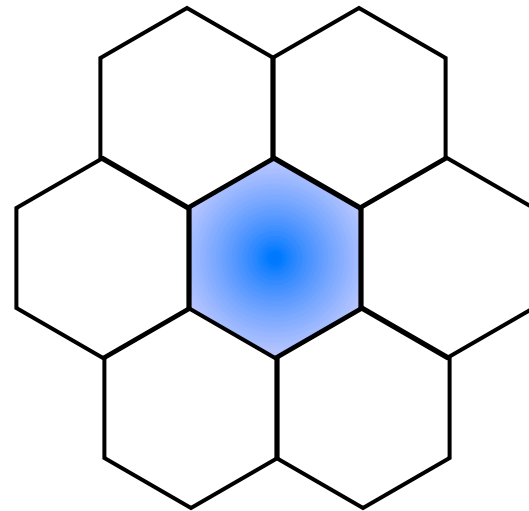
physical Majoranas $H_m = K \sum_{\langle ij \rangle} i c_i c_j$



Non-local excitations



Majorana ε



Flux e, m

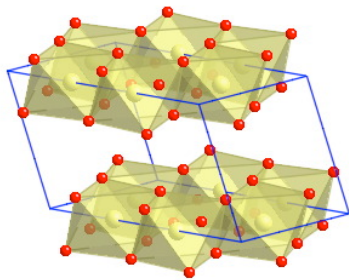
In Kitaev's model:

- Majorana's dispersion $\sim k$ and Dirac-like
- Fluxes are localized and gapped

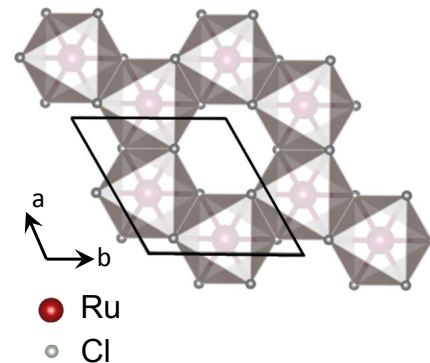
Kitaev Materials

Jackeli, Khaliullin
2009

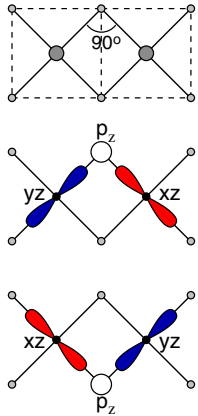
Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling



Na_2IrO_3 ,
 (α, β, γ) -
 Li_2IrO_3

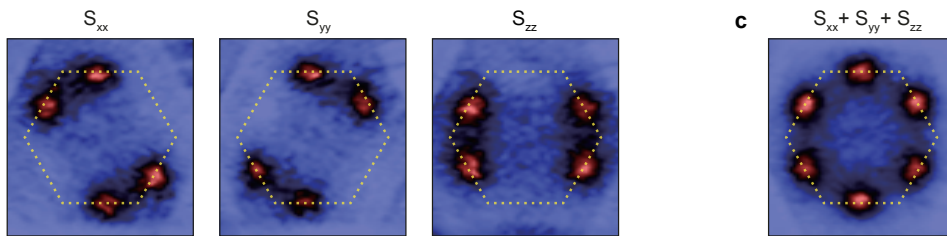


$\alpha\text{-RuCl}_3$



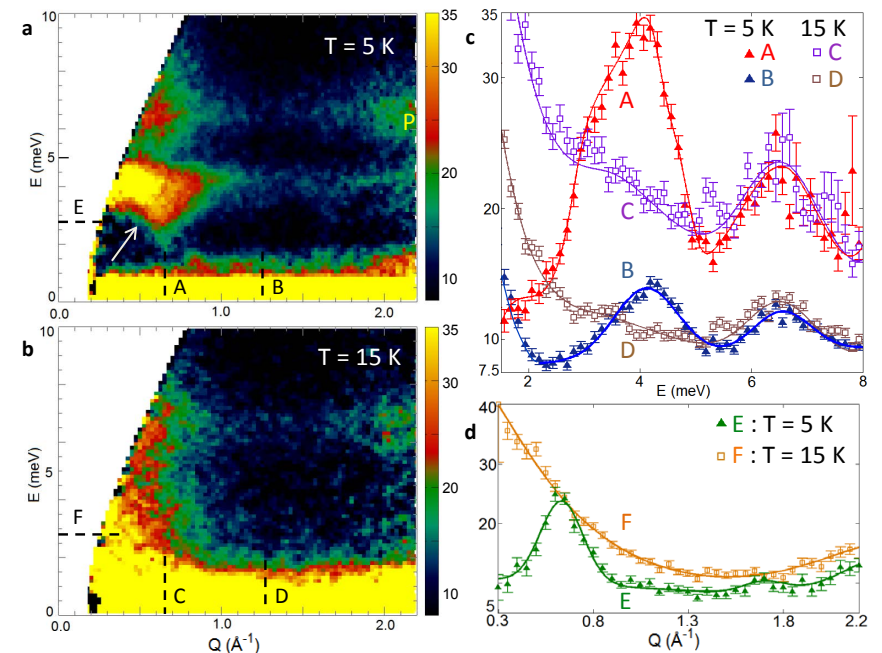
Honeycomb and hyper-honeycomb structures

Kitaev Materials



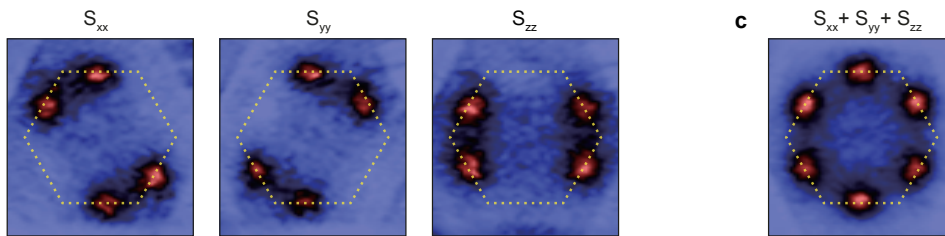
direct evidence for
direction-dependent
anisotropic exchange
from diffuse magnetic
x-ray scattering in
 Na_2IrO_3 (BJ Kim group)

there is pretty strong evidence
of substantial Kitaev exchange
in quite a few materials



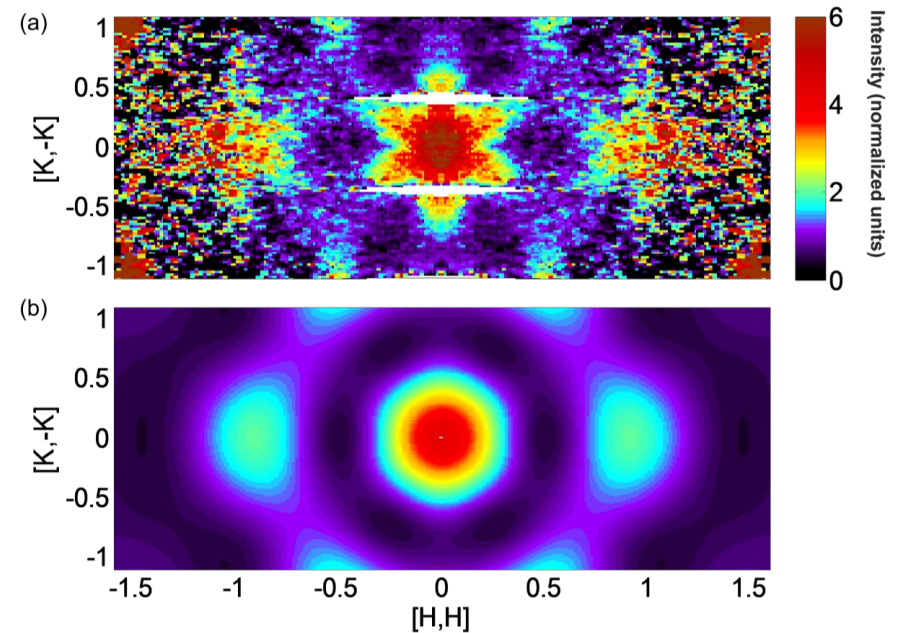
Observation of gapped
continuum mode persisting
above T_N in $\alpha\text{-RuCl}_3$
consistent with Majoranas
(A. Banerjee *et al*)

Kitaev Materials



direct evidence for
direction-dependent
anisotropic exchange
from diffuse magnetic
x-ray scattering in
 Na_2IrO_3 (BJ Kim group)

there is pretty strong evidence
of substantial Kitaev exchange
in quite a few materials



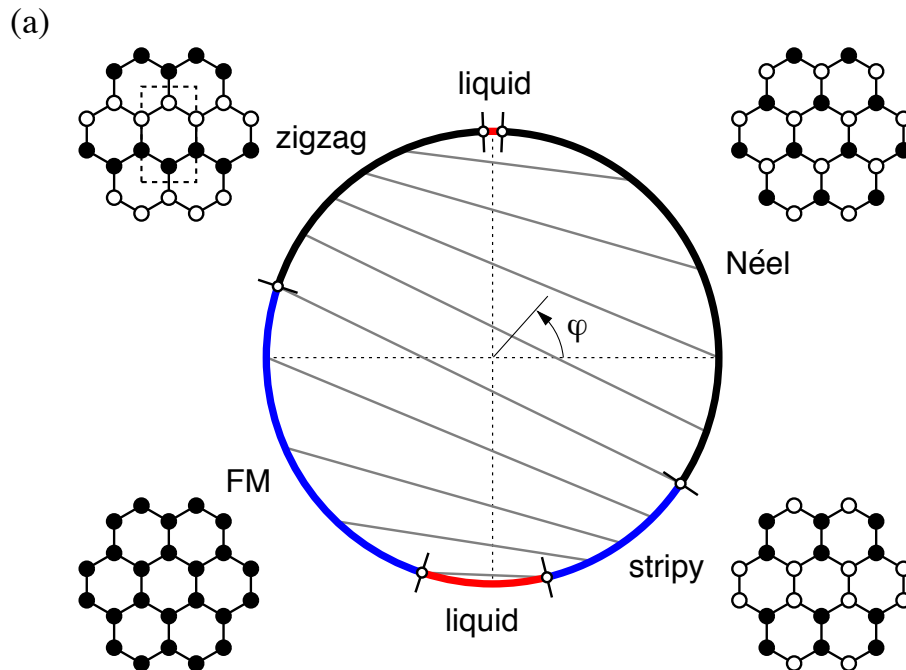
single-crystal data in $\alpha\text{-RuCl}_3$
compared to Kitaev's soluble
model (A. Banerjee *et al*)

Magnetism

- But...they all order so far

due to additional interactions,
e.g. Heisenberg

$$H = \sum_{i,\alpha} K S_i^\alpha S_{i+\alpha}^\alpha + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



zigzag ordered state
has been observed in
 Na_2IrO_3 and
incommensurate order
in Li_2IrO_3

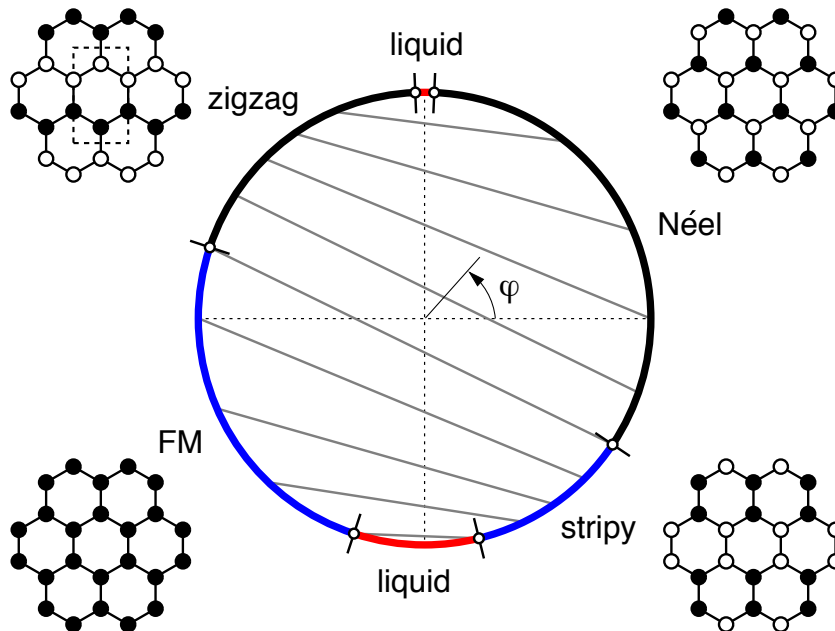
Magnetism

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$$H = \sum_{i,\alpha} K S_i^\alpha S_{i+\alpha}^\alpha + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

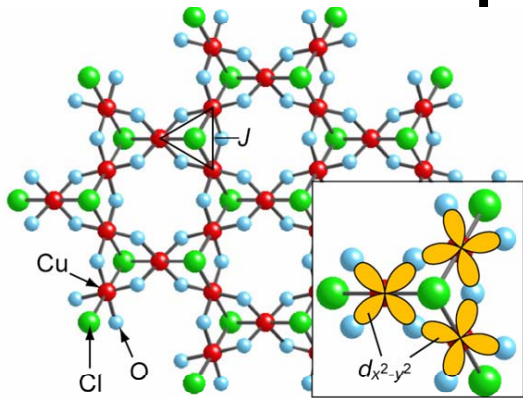
(a)



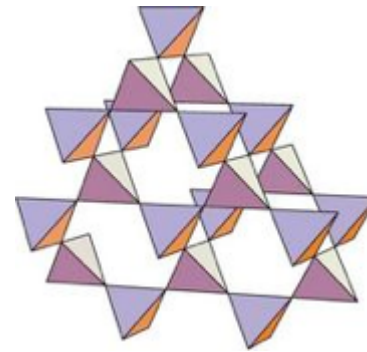
so far no QSL!



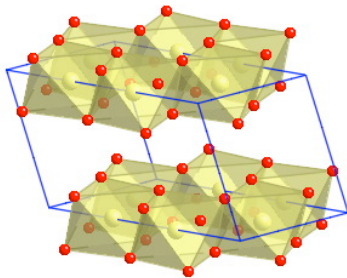
Top experimental platforms



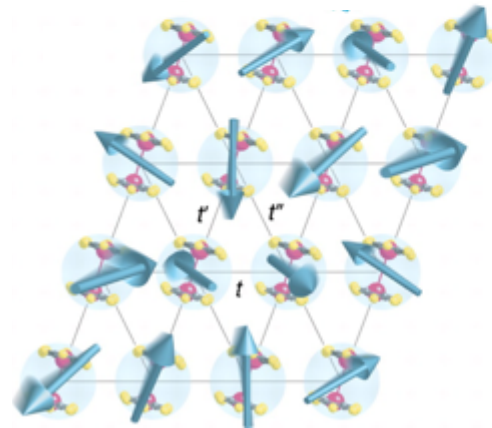
kagomé



Quantum spin ice



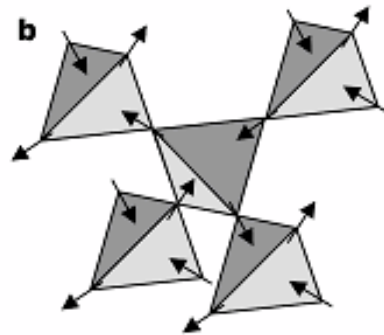
Kitaev materials



organics

Spin ice

- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$

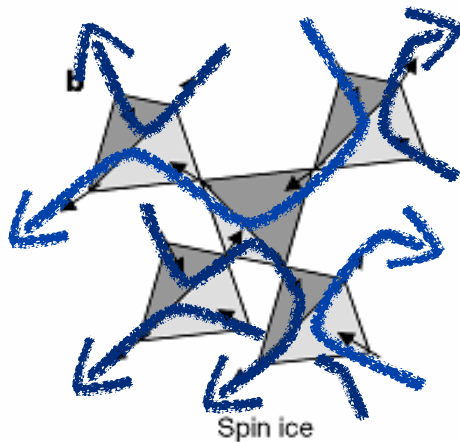


Spin ice

thermal fluctuations
through many
degenerate states

Spin ice

- Spins in $\text{Ho}_2\text{Ti}_2\text{O}_7$, $\text{Dy}_2\text{Ti}_2\text{O}_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out “ice rules” for $T < 1\text{K}$



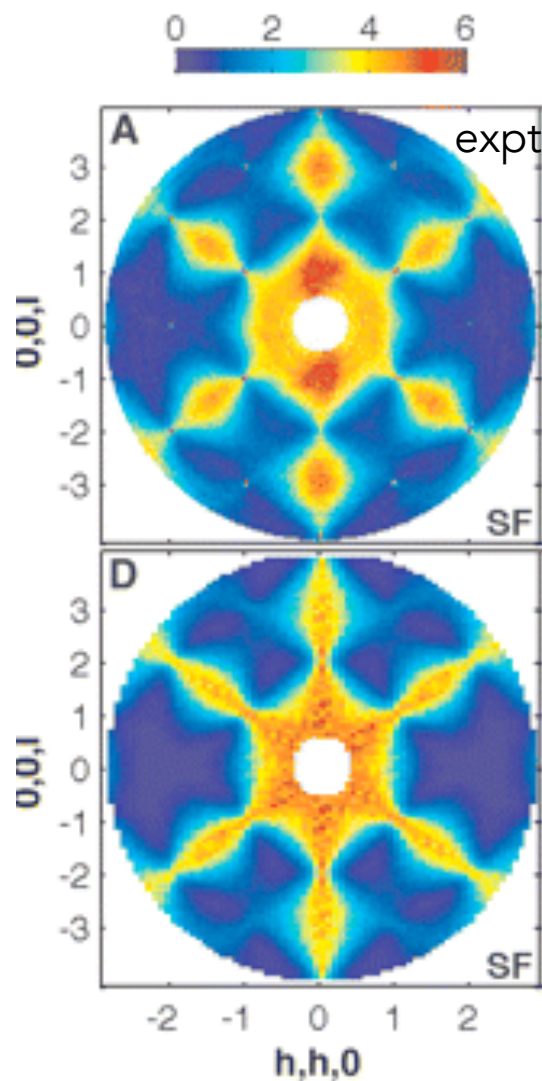
$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

artificial magnetostatics: spins map to field lines

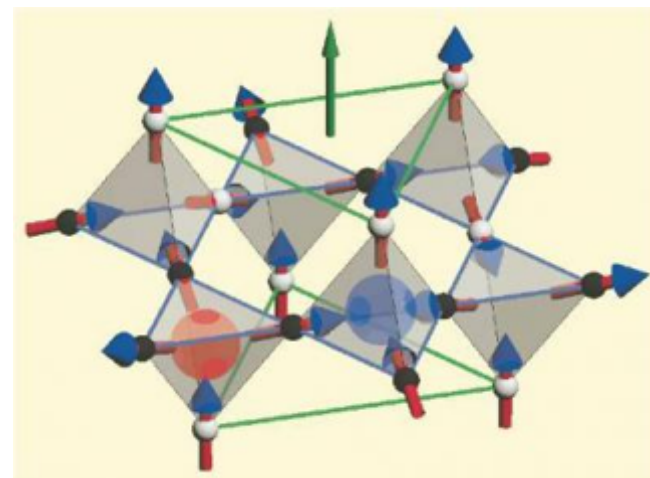
Classical spin liquid

T. Fennell *et al*, 2009

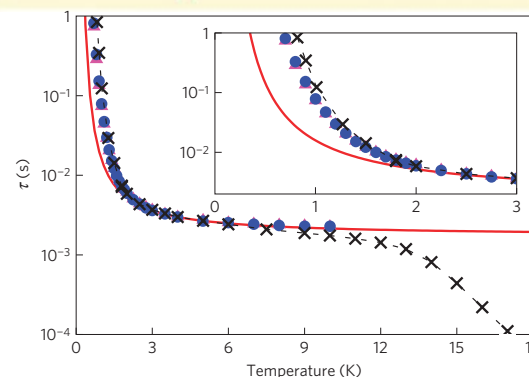


pinch
points

$$\vec{\nabla} \cdot \vec{b} = 0$$



Castelnovo
et al, 2008



Jaubert and
Holdsworth

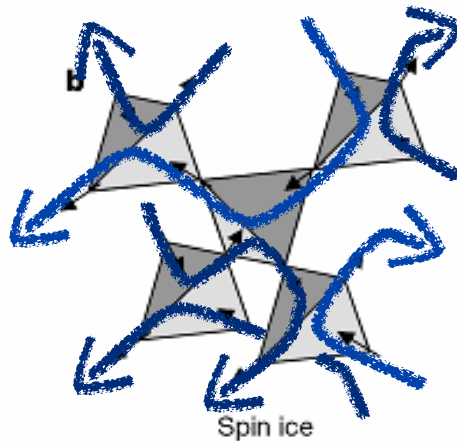
magnetic monopoles
behave like diffusing
ions in a polyelectrolyte

Quantum spin ice

$$H = H_{CSI} + J_{\pm} \sum_{\langle ij \rangle} (S_i^+ S_j^- + \text{h.c.})$$

quantum dynamics creates
superposition state

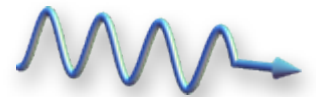
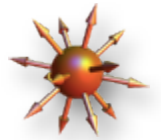
$$|\Psi\rangle = \sum_{\text{loops}}$$



QSL which "simulates"
quantum
electrodynamics -
vacuum fluctuations

Excitations

- fully coherent propagating monopoles
= "spinons"
- dual (electric) monopoles
- artificial photon: a gapless protected collective excitation which is *not* a Goldstone mode



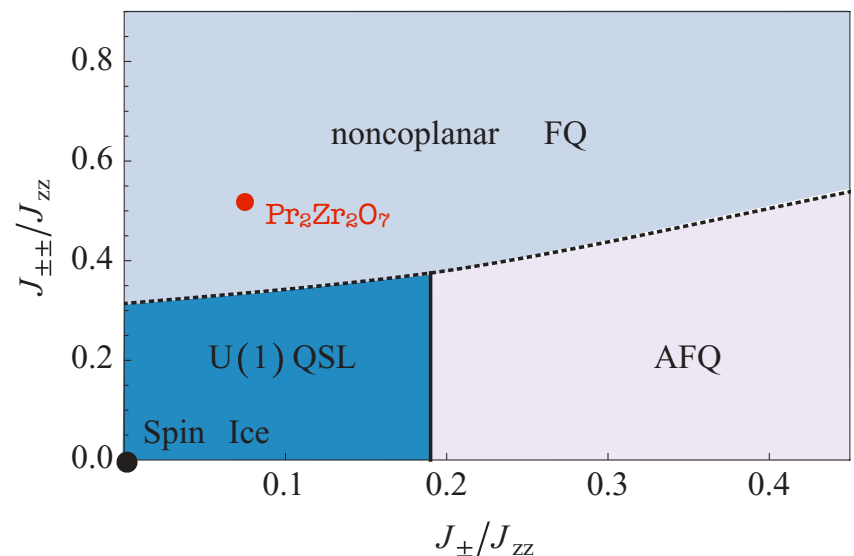
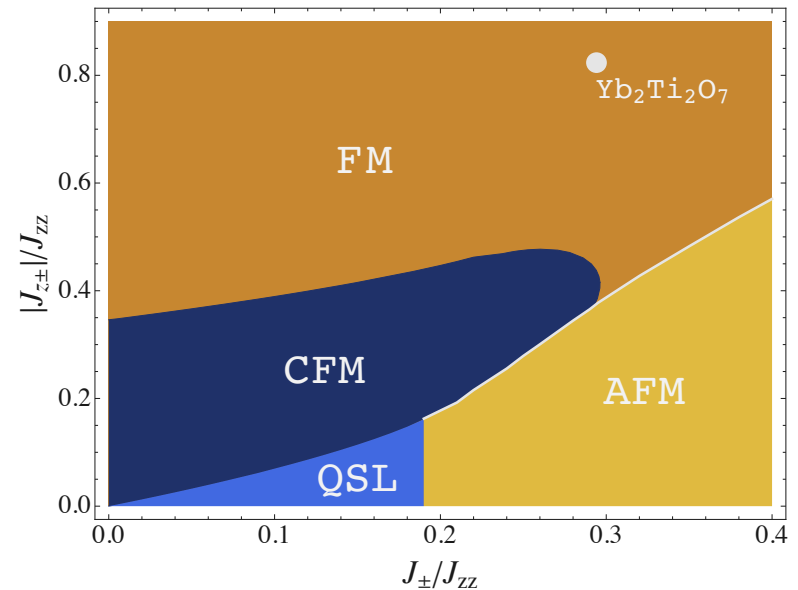
Quantum spin ice

Realistic theory for
quantum rare earth
pyrochlores

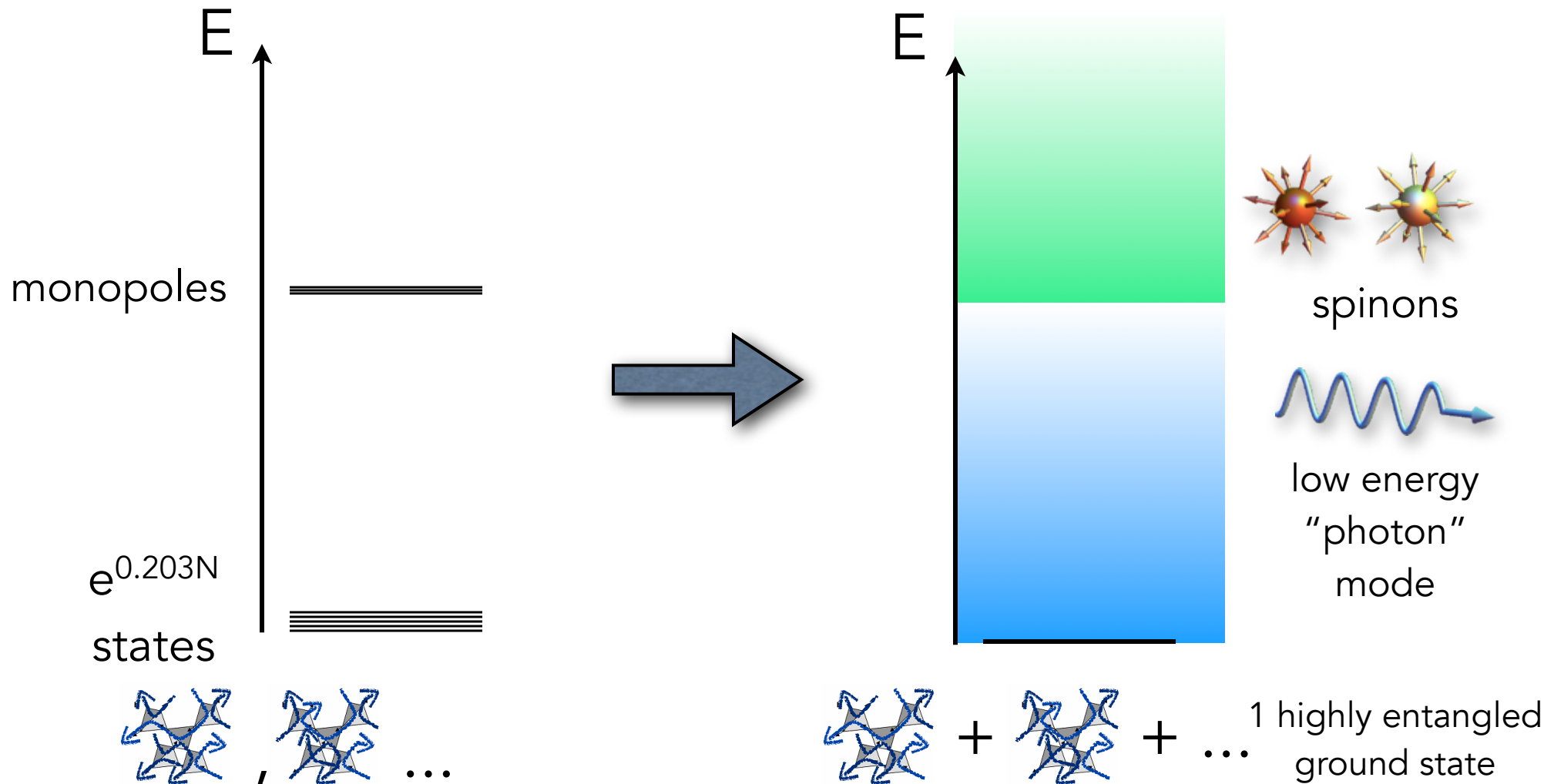
L. Savary + LB, 2012

S.B. Lee, S. Onoda + LB, 2012

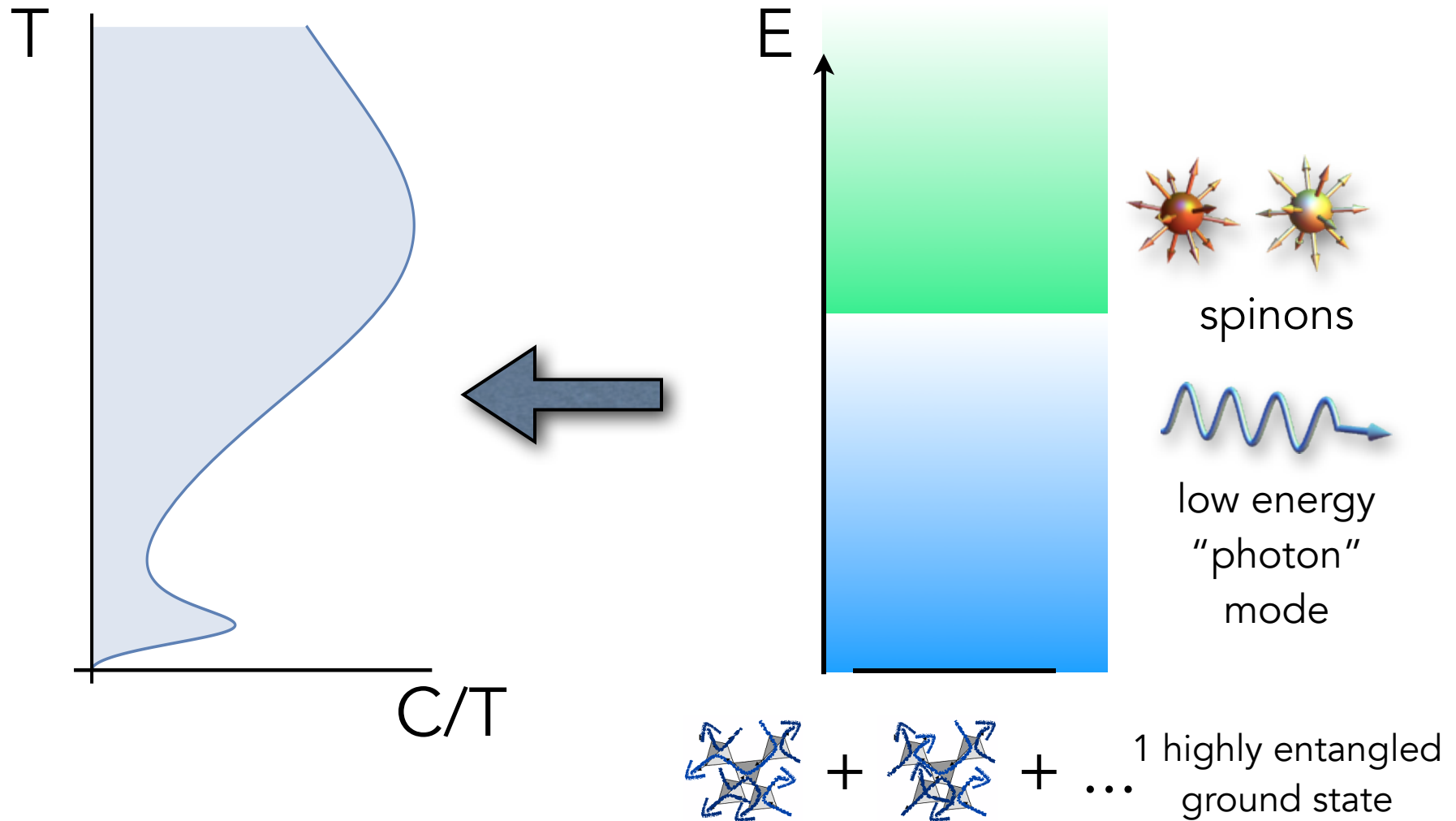
Possible application to
 $\text{Yb}_2\text{Ti}_2\text{O}_7$, $\text{Pr}_2\text{Zr}_2\text{O}_7$,
others...



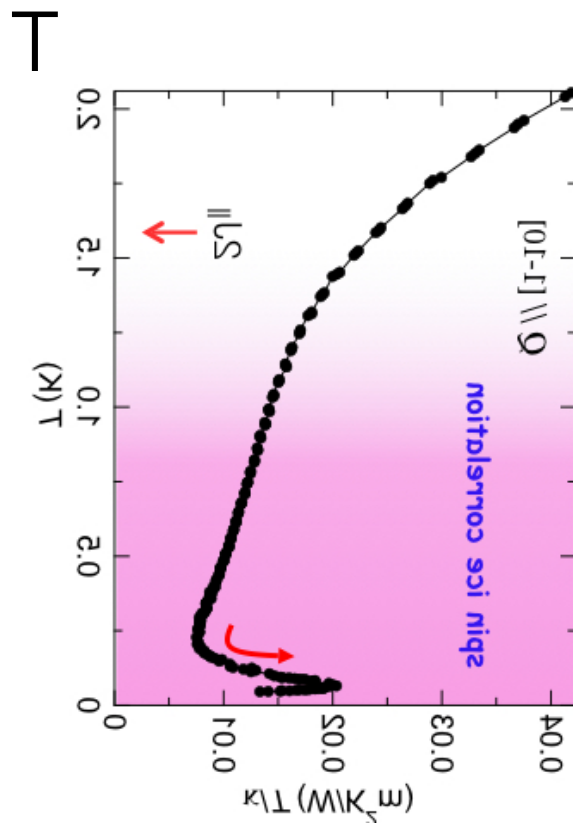
Quantum spin ice



Quantum spin ice



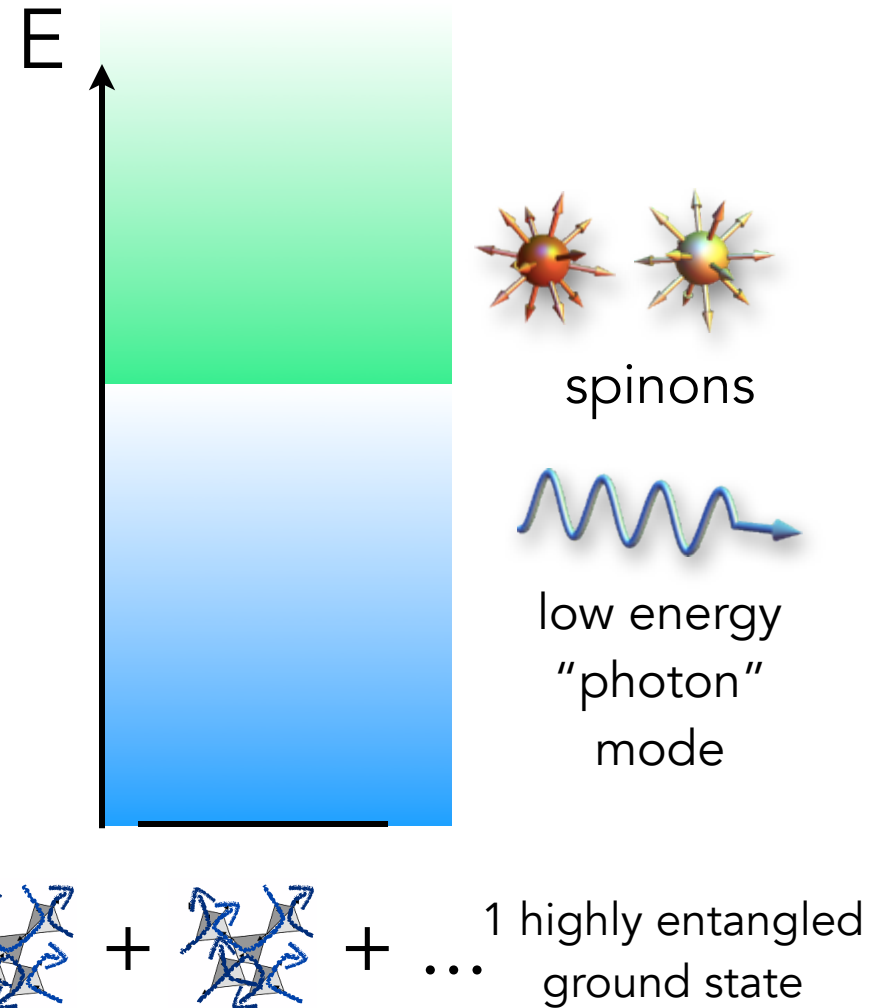
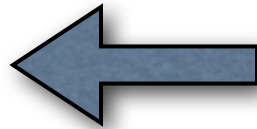
Quantum spin ice



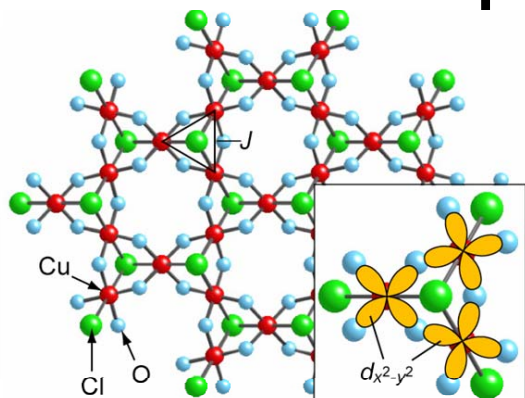
Y. Tokiwa et al,
unpublished



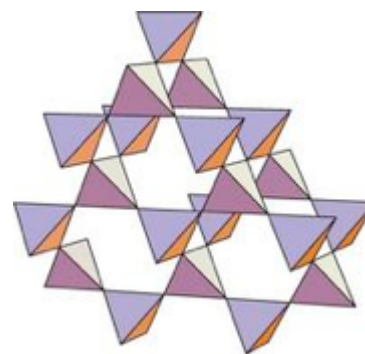
K/T



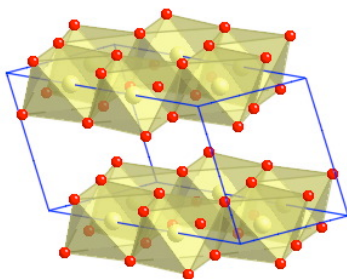
Top experimental platforms



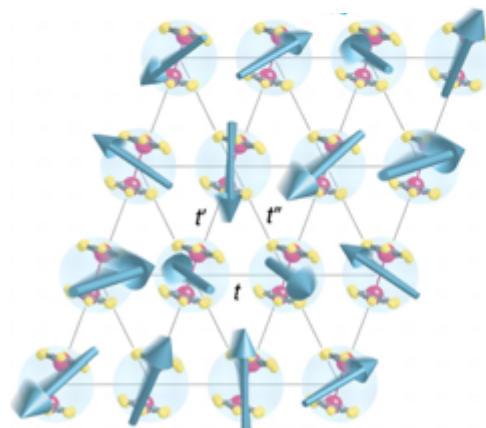
kagomé



Quantum spin ice

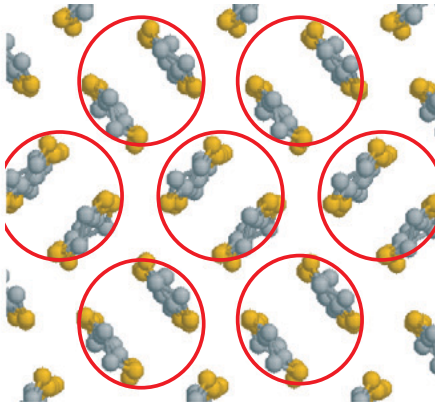


Kitaev materials

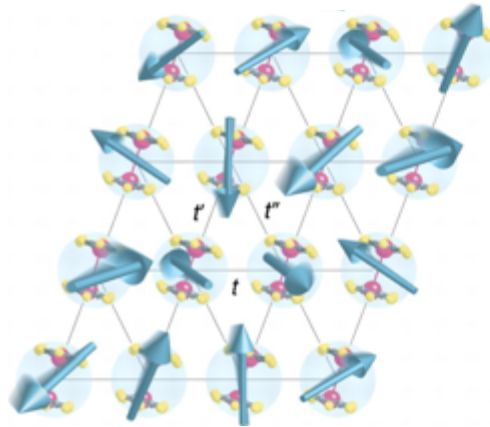


organics

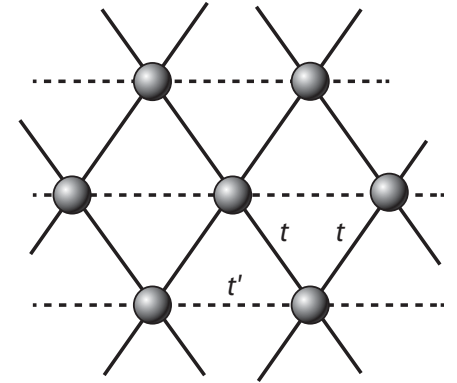
Organics



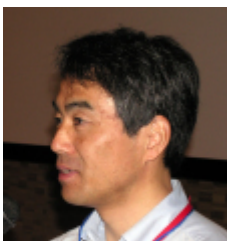
$\kappa\text{-(ET)}_2\text{X}$



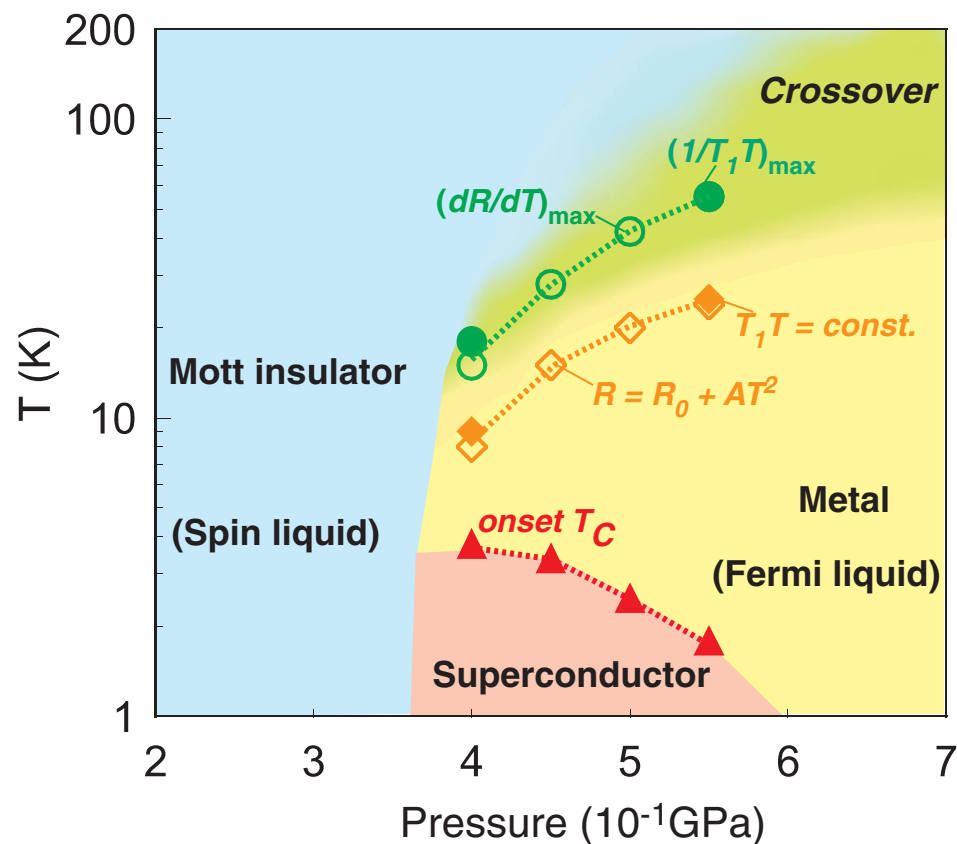
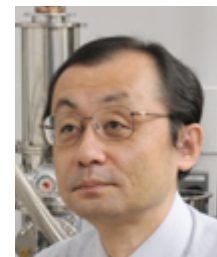
$\beta'\text{-Pd(dmit)}_2$



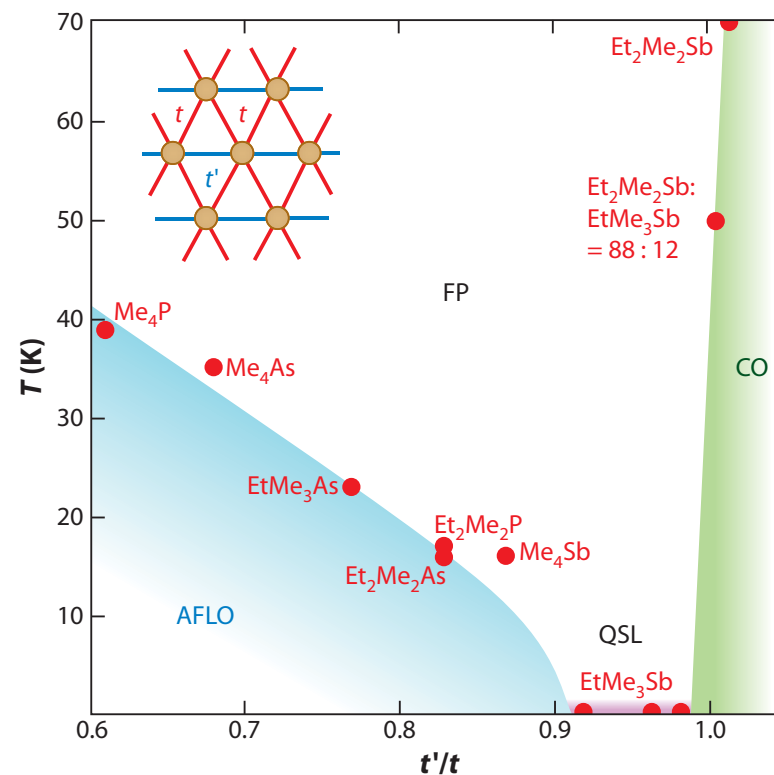
- Molecular materials which behave as effective triangular lattice $S=1/2$ antiferromagnets with $J \sim 250\text{K}$
- significant charge fluctuations



Organics

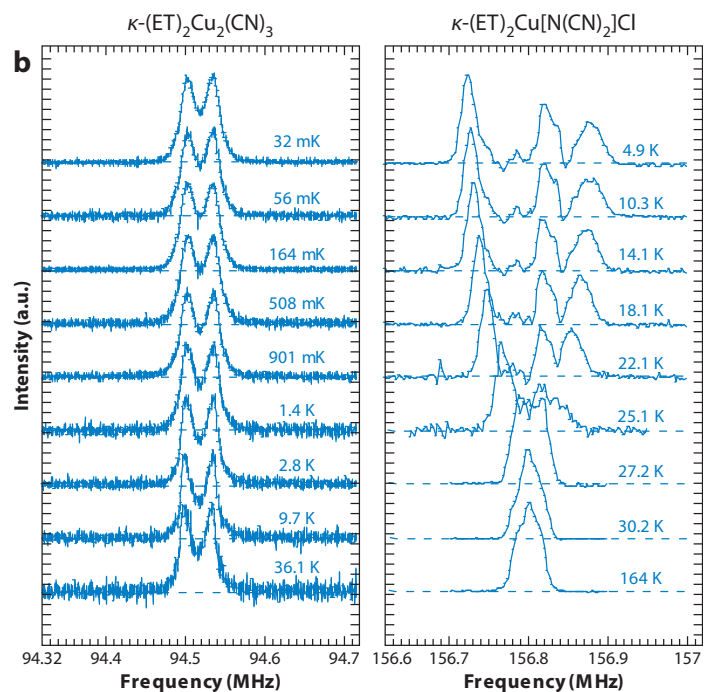


K. Kanoda group (2003-)

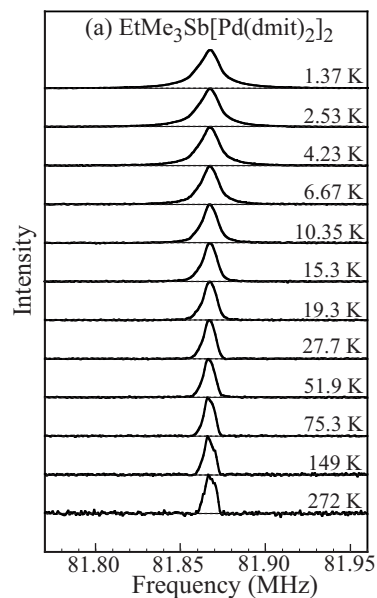


R. Kato group (2008-)

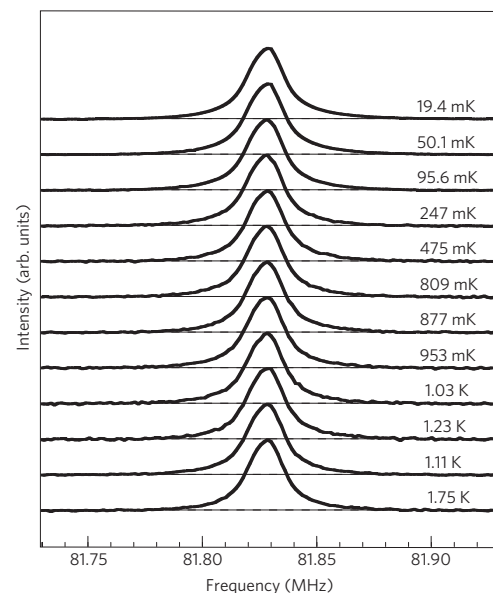
NMR lineshapes



Y. Shimizu
et al, 2003 ^1H NMR



T. Itou *et al*,
2008, 2010

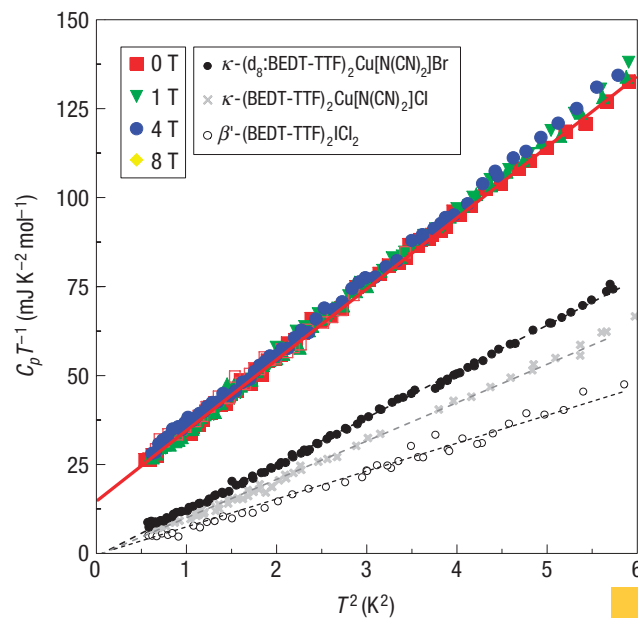


^{13}Cs NMR

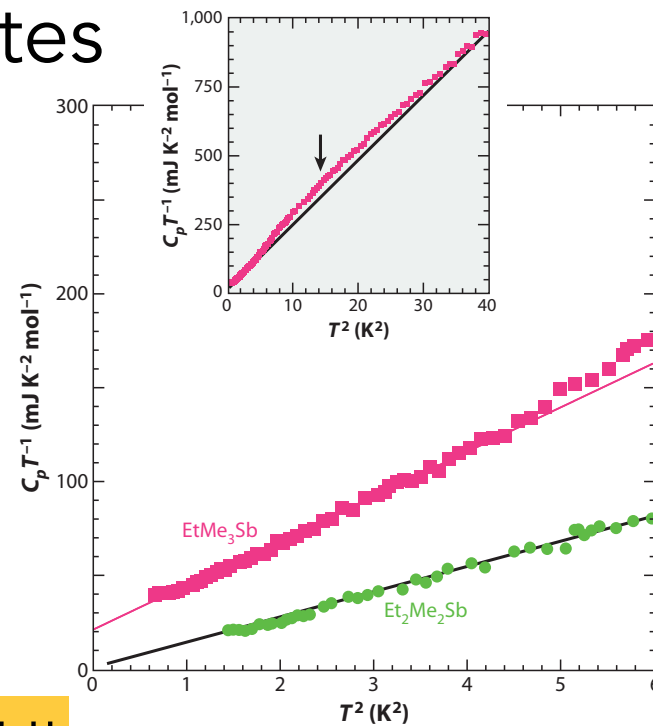
Evidence for lack of static moments: $f > 1000$!

Specific Heat

- $C \sim \gamma T$ indicates gapless behavior with large density of states



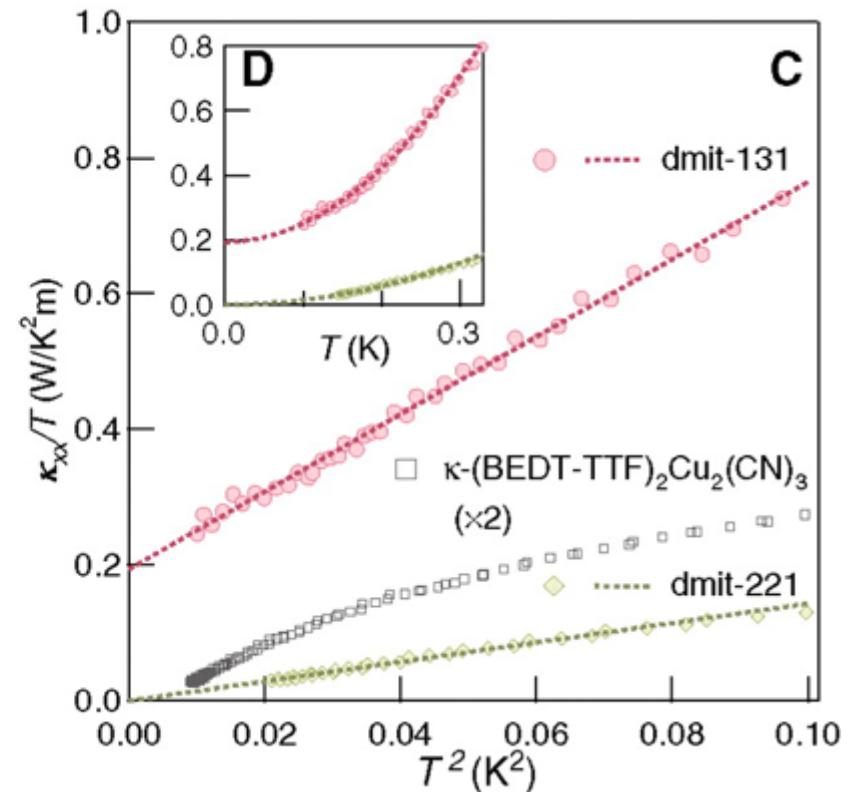
$\gamma_{Cu} \sim 0.7$!!



S. Yamashita et al, 2008

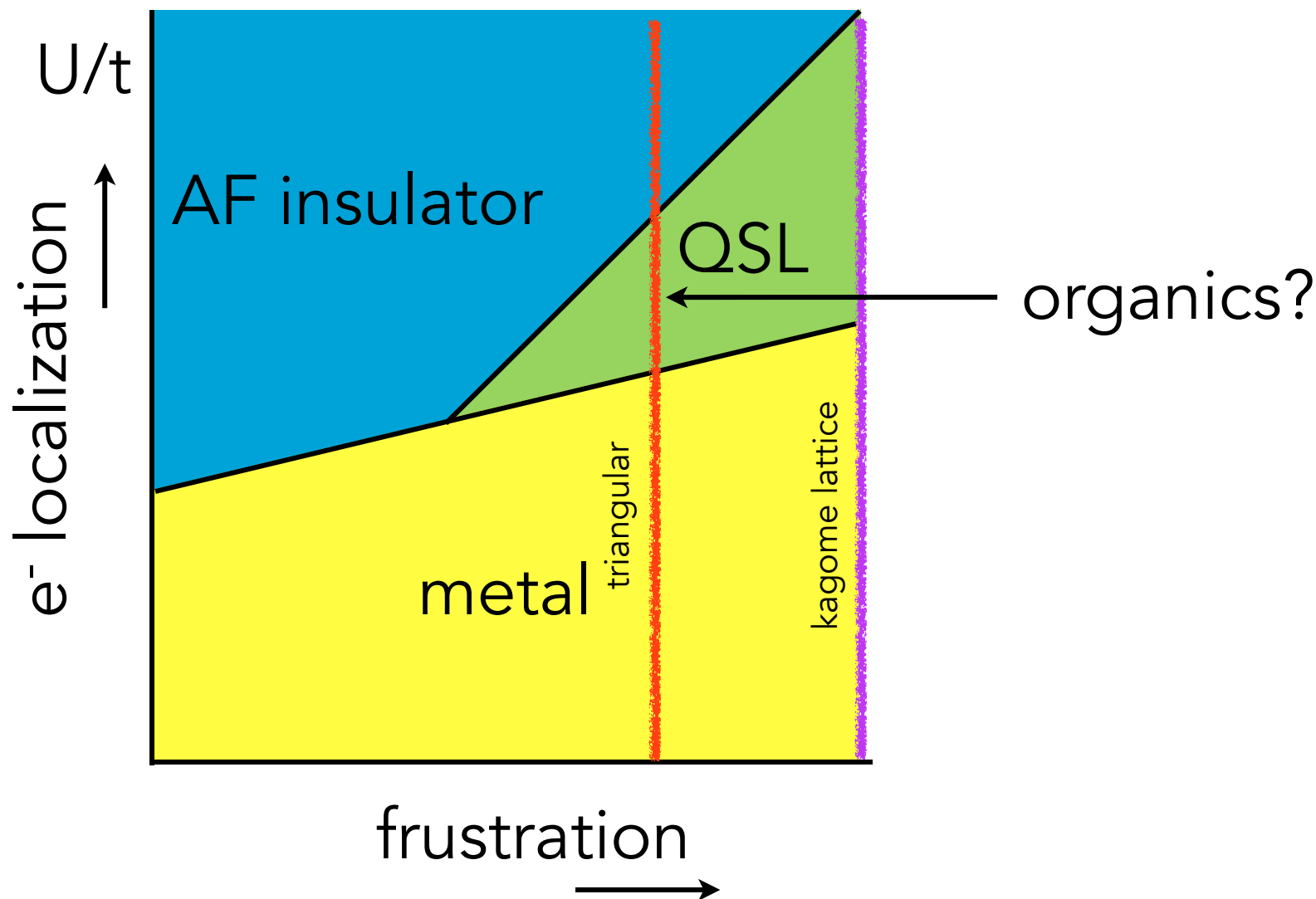
Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit
- Estimate for a *metal* would correspond to a mean free path $l \sim 1 \mu\text{m} \approx 1000 a$!

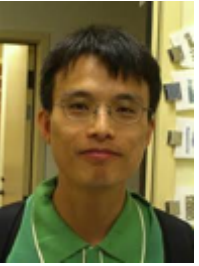


M. Yamashita et al, 2010

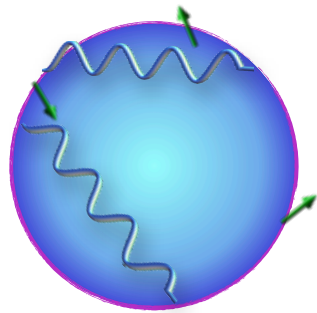
Charge fluctuations



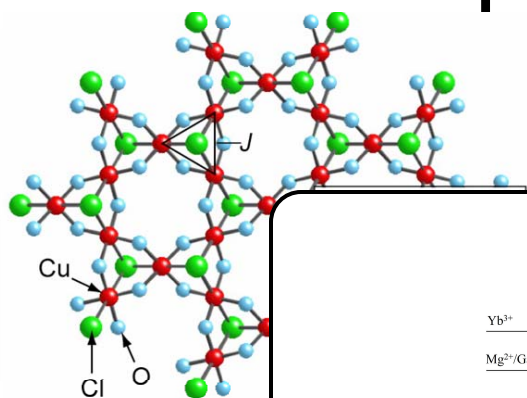
Organics - Theory



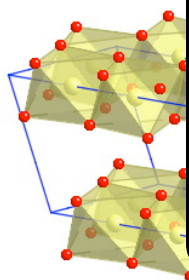
- RVB/QSL state:
 - Motrunich, Lee+Lee: (2005) "uniform RVB"
 - It is described by a "**Fermi sea**" of **spinons** coupled to a U(1) gauge field
- The anomalous thermal conductivity may be a window into an emergent fermi surface in an insulator!



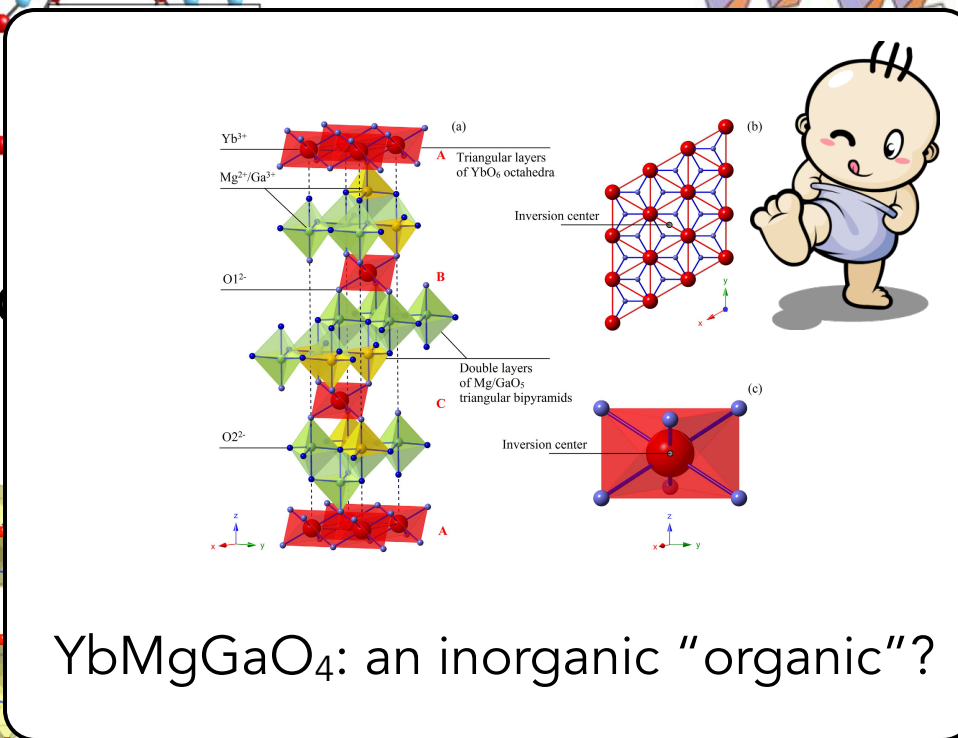
Top experimental platforms



kagome



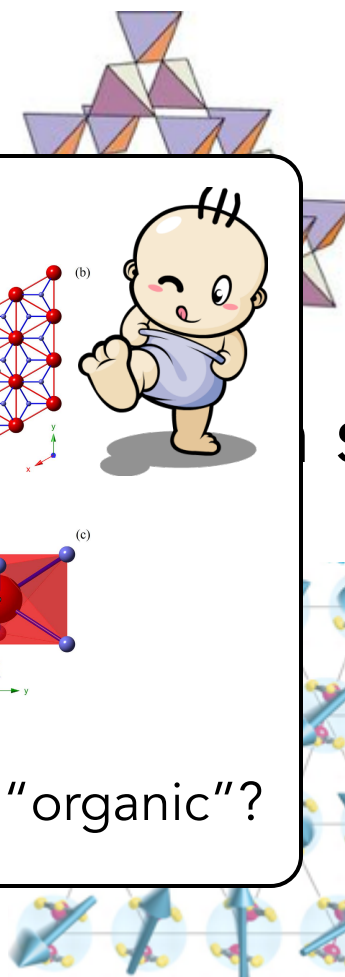
Kitaev materials



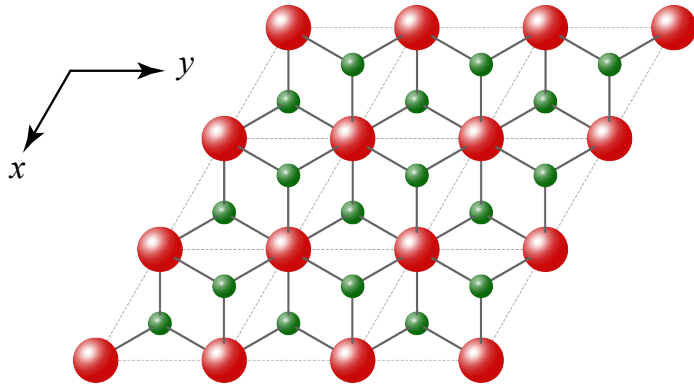
YbMgGaO_4 : an inorganic "organic"?

spin ice

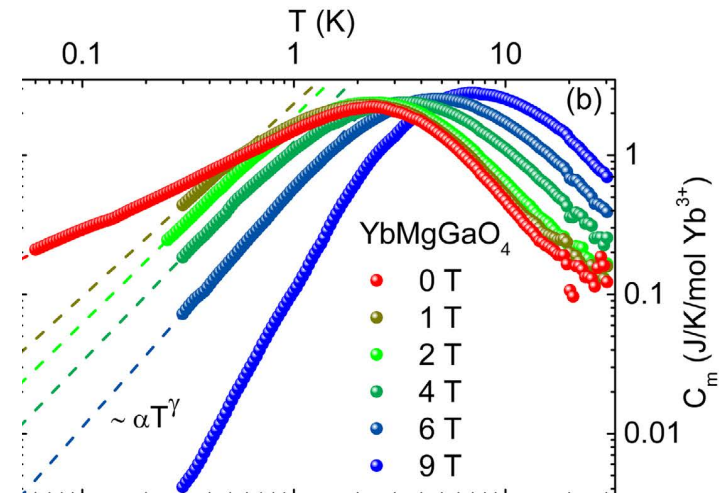
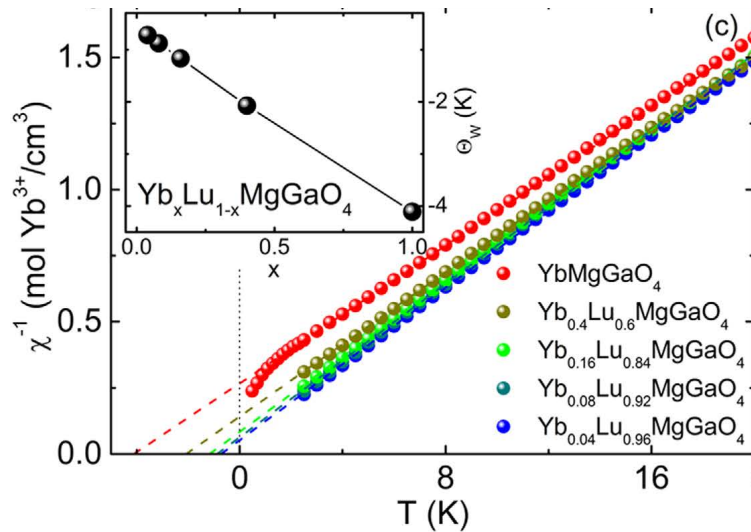
organics



YbMgGaO₄



effective $S=1/2$ triangular
lattice

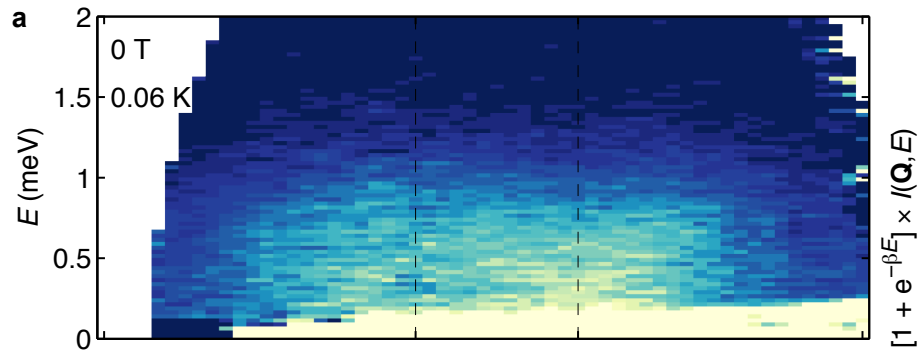


$$f \approx 4K/50mK \geq 80$$

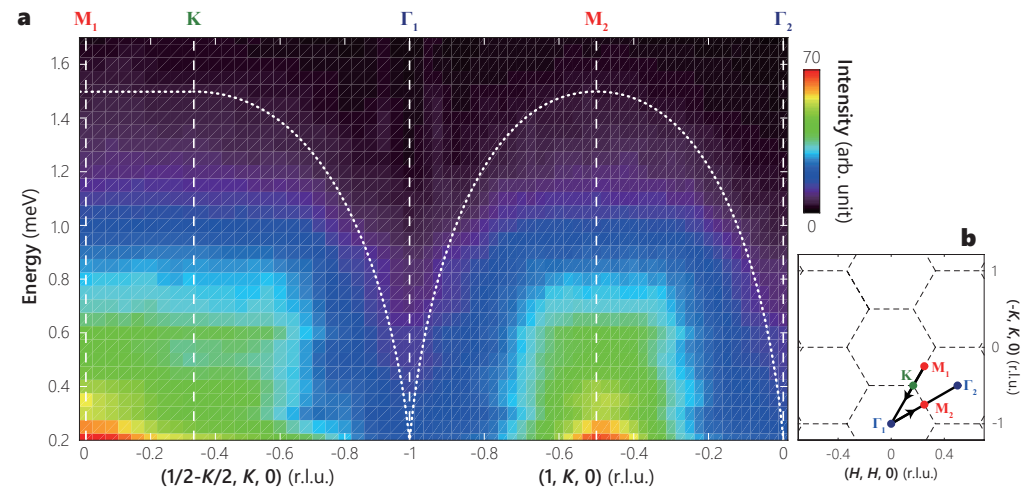
$$C \sim T^{0.7}$$

YbMgGaO₄

Neutron scattering studies



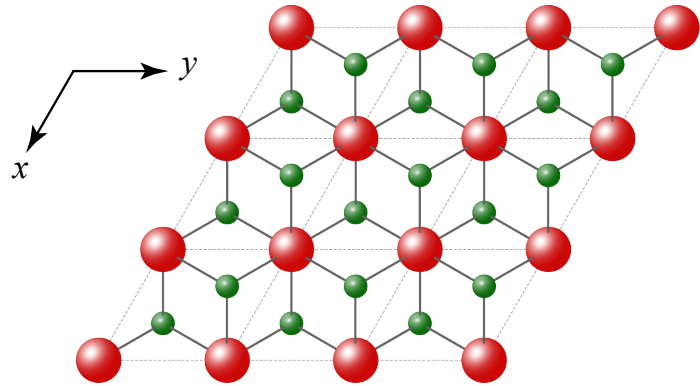
J. Paddison *et al*, arXiv:1607.3231



Y. Shen *et al*, arXiv:1607.02615

Shen *et al* suggest this is
the structure factor of a
spinon Fermi surface

YbMgGaO₄



Interesting difference from
organics:
strong spin-orbit coupling

Is the spinon Fermi sea a
Rashba system?

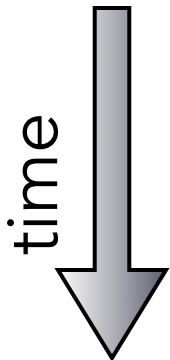
We may expect a strong
thermal Hall effect

A future history of magnetism (from 2014)

~500BC: Ferromagnetism
documented in Greece,
India, used in China



sinan, ~200BC



1949AD: Antiferromagnetism
proven experimentally

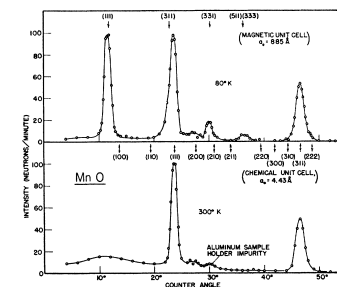


FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

~2016AD: Conclusive experiments on
quantum spin liquids?



Everything we call real is
made of things that cannot
be regarded as real.

If quantum mechanics hasn't
profoundly shocked you, you
haven't understood it yet.

References here: <https://spinsandelectrons.com/pedagogy/>