Interplay of real space and momentum space topological defects

Leon Balents, KITP

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Collaborators

Theory

Jianpeng Liu

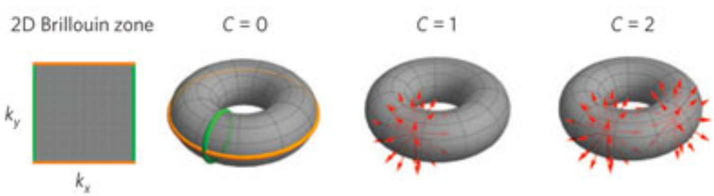
Inspiration

Satoru Nakatsuji

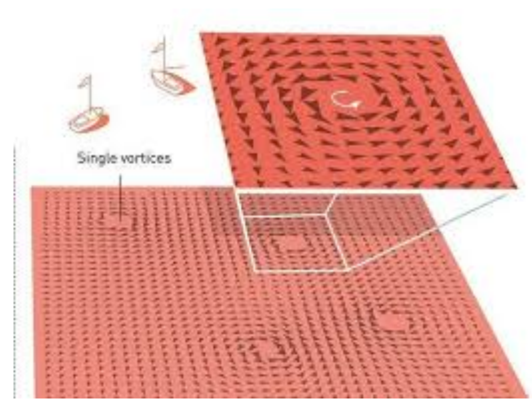
YoshiChika Otani

Thouless:
Chern number

$$q_n = \frac{1}{2\pi} \int d^2k\, \mathcal{B}_n^z$$

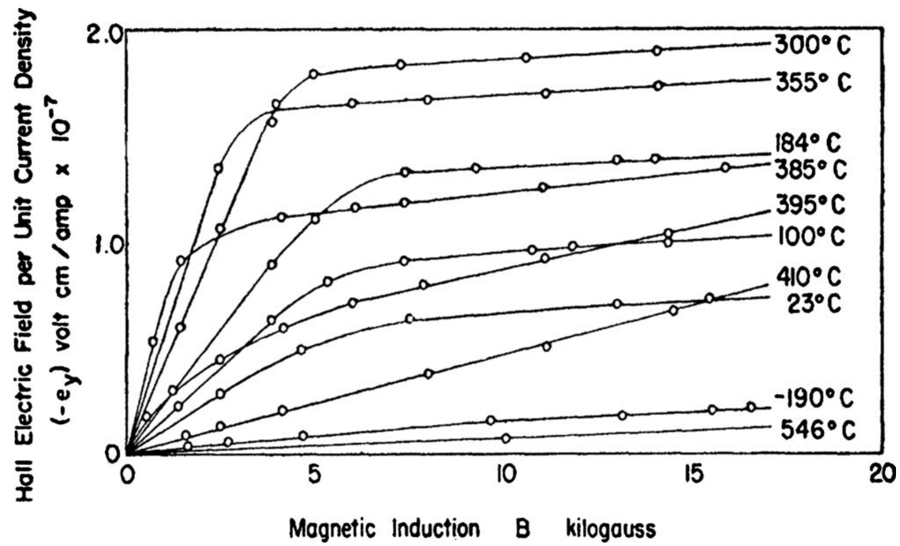


Kosterlitz+ Thouless:
Vortices



Anomalous Hall Effect

A famous effect in metallic ferromagnets, c.f. Ni



$$\sigma_{xy} = aB^z + bM^z$$

Karplus + Luttinger (1954!) related this to anomalous velocity due to Berry curvature

Anomalous velocity

$$\mathcal{A}_{\boldsymbol{k}} = i\langle n, \boldsymbol{k}|\boldsymbol{\nabla}_{\boldsymbol{k}}|n, \boldsymbol{k}\rangle$$

Berry curvature

$$\mathcal{B}_{\boldsymbol{k}} = \boldsymbol{\nabla}_{\boldsymbol{k}} \times \mathcal{A}_{\boldsymbol{k}}$$

Anomalous velocity
(Karplus+Luttinger, Niu)

$$\partial_t \boldsymbol{r} = \boldsymbol{\nabla}_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}} - e\boldsymbol{E} \times \mathcal{B}_{\boldsymbol{k}}$$

$$\Rightarrow \quad \boldsymbol{j}_A = e^2\boldsymbol{E} \times \int_{\boldsymbol{k}} f(\epsilon_{\boldsymbol{k}})\mathcal{B}_{\boldsymbol{k}}$$

Gives AHE

For a full band, it becomes the
quantized Hall conductivity = q e^2/h

Weyl semimetal

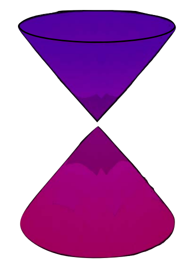
937 PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING
Princeton University, Princeton, New Jersey
(Received June 16, 1937)

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neigborhood of a point $\mathbf{k}$ where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa \rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

$$H = v\vec{\sigma} \cdot \vec{k}$$



A two-component spinor in three dimensions: “half” of a Dirac fermion. Weyl fermions have a chirality and *must* be massless

(Dirac semimetals also exist)

Weyl semimetal

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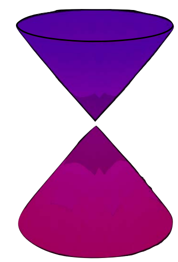
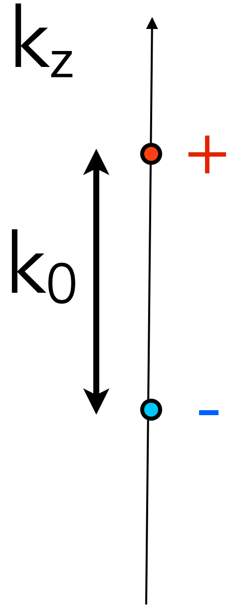
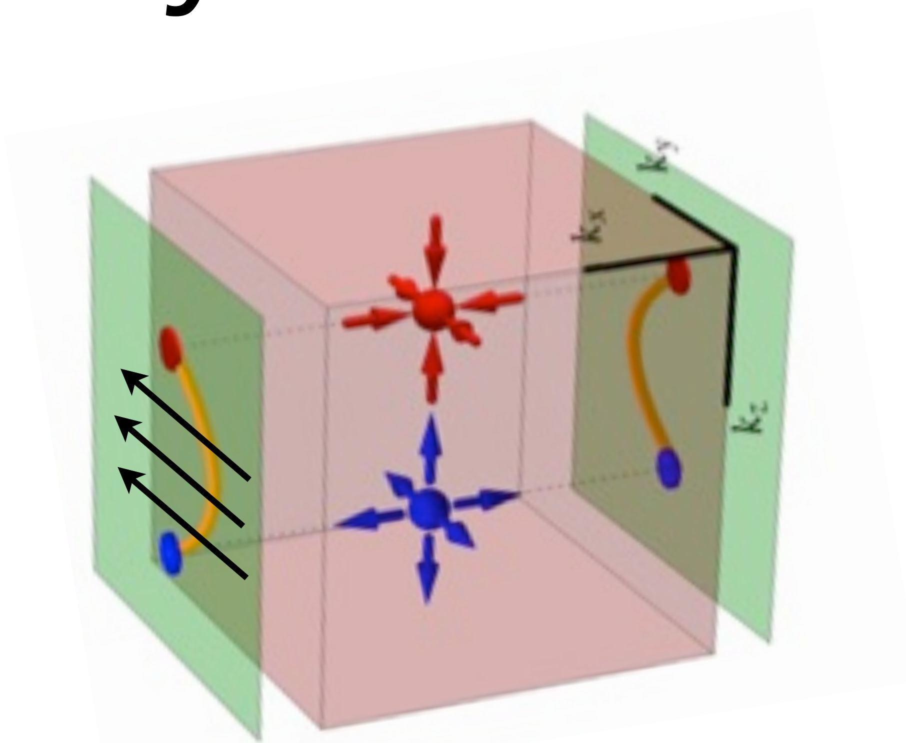


A Weyl point is a "topological defect" in momentum space: a monopole for the Berry curvature

$$\boldsymbol{\nabla}_{\boldsymbol{k}} \cdot \mathcal{B} = \pm 2\pi q$$

Weyl semimetal





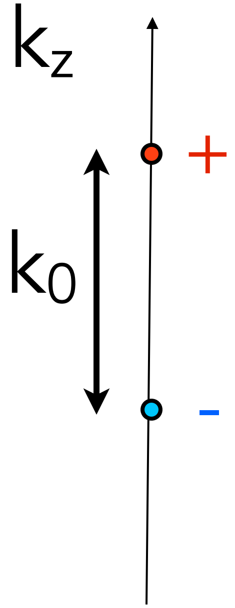
Fermi arc = chiral edge state

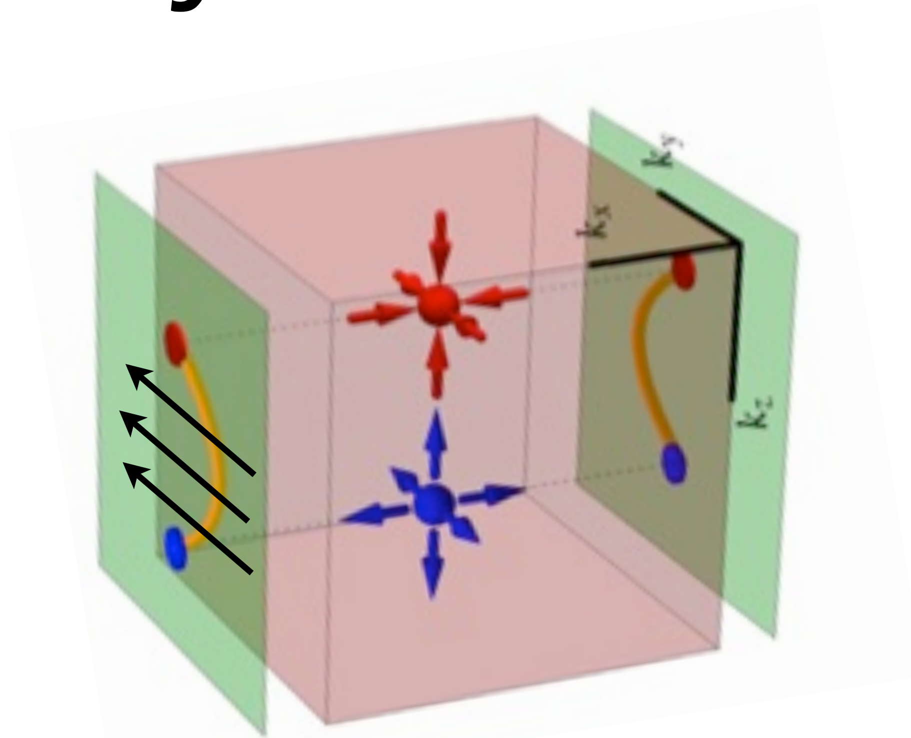
X. Wan *et al*, 2011

A.A. Burkov+LB, 2011

Expts: non-centrosymmetric materials TaAs, Na_3Bi, TaP, WTe_2,...

Weyl semimetal





Fermi arc = chiral edge state

X. Wan *et al*, 2011

A.A. Burkov+LB, 2011

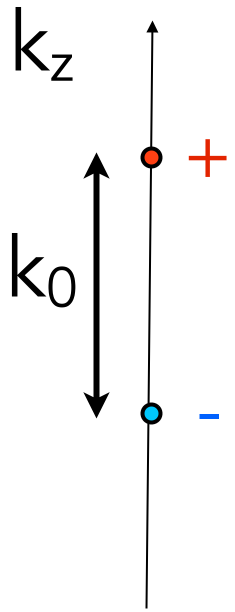
$$\sigma_{\mu\nu} = \frac{e^2}{2\pi h}\epsilon_{\mu\nu\lambda}Q_\lambda$$

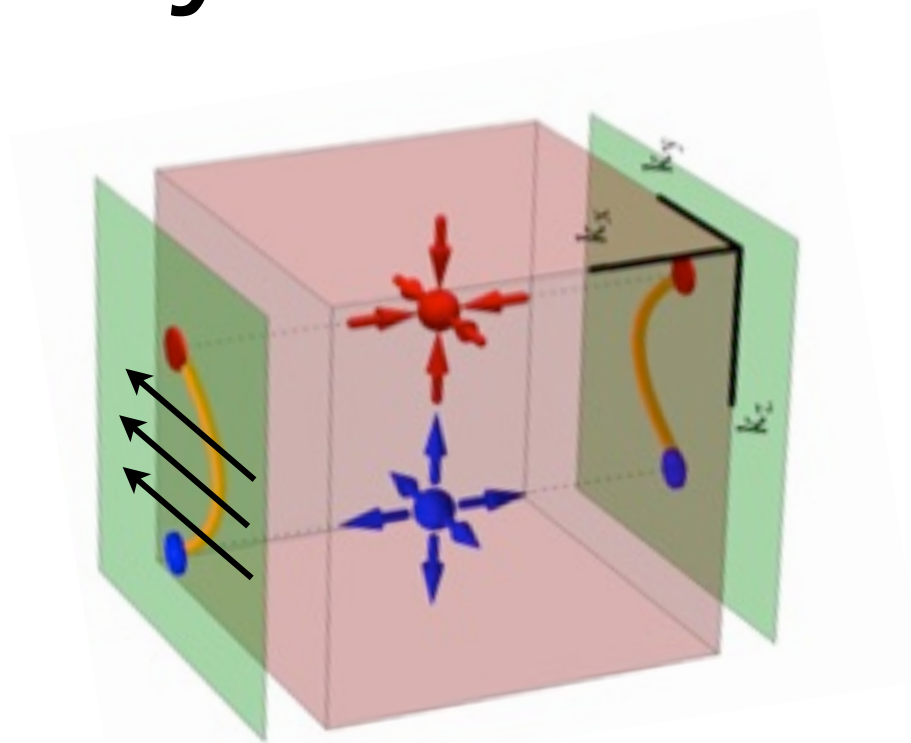
$$\vec{Q} = \sum_i \vec{k}_i q_i + \vec{Q}_{RLV}$$

Hall vector **Q** ~ “dipole moment” of Weyl points

(when E_F away from Weyl points add FS contributions)

Weyl semimetal





Fermi arc = chiral edge state

Hall effect obviously breaks time-reversal symmetry

→ need a magnetic material

Anomalous Hall Effect

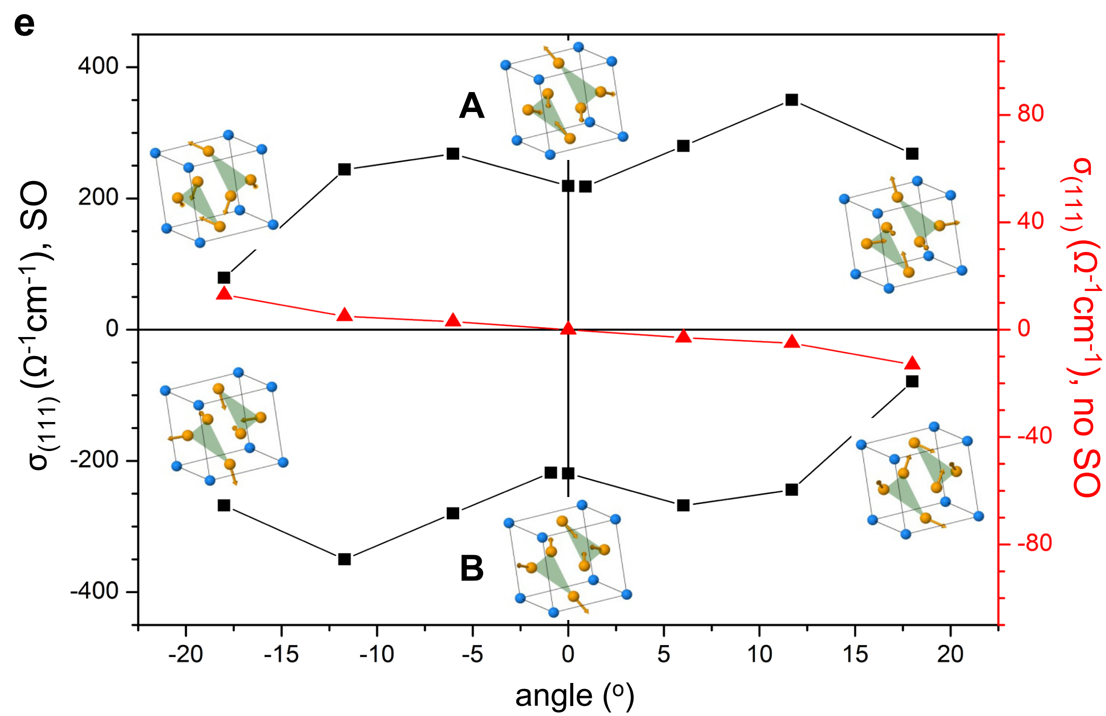
It should be generically present in time-reversal broken *Weyl semimetals*, with sufficiently low symmetry

How about AHE in an *antiferromagnet*?

Could be useful to provide switchable Hall effect w/o large stray fields

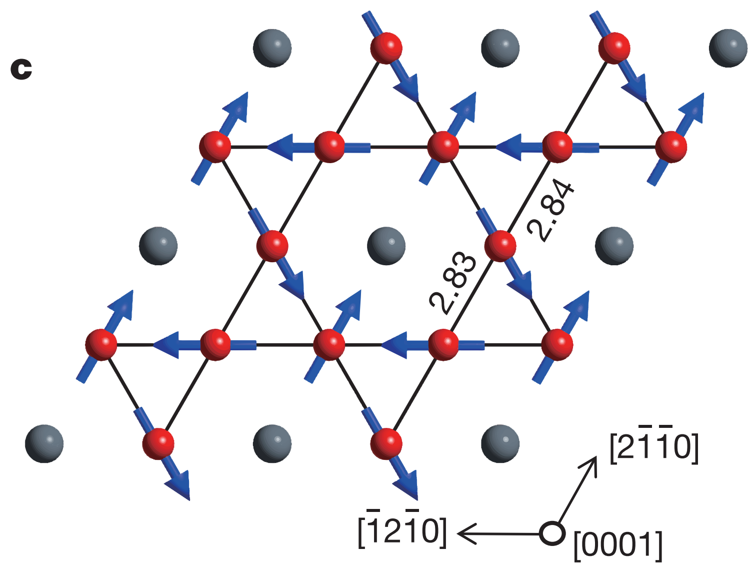
Theoretical proposal

Hua Chen, Q. Niu, A. MacDonald, 2013: AHE in the non-collinear antiferromagnet Mn_3Ir



I would like to discuss a related material family

Mn_3Sn family



two kagomé layers of
Mn, related by inversion

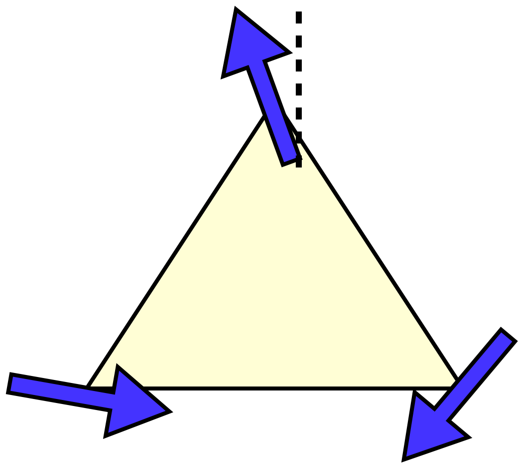
large ordered
antiferromagnetic
moment
~ 2 μ_B /Mn

tiny FM moment:
.002 μ_B/Mn

T_N ~ 420K

Nagamiya *et al*, 1982

Energetics: triangle

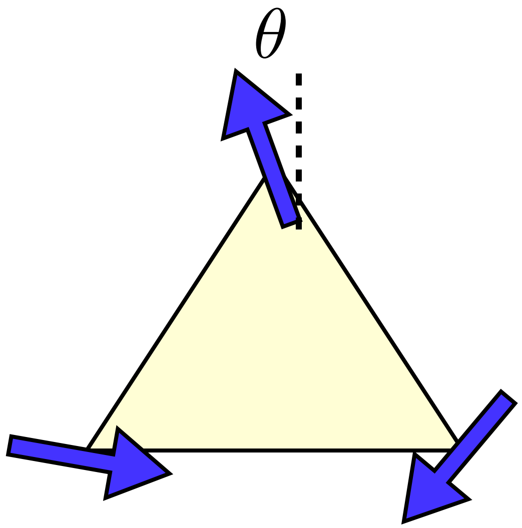


$$E = J\left(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + \boldsymbol{S}_2 \cdot \boldsymbol{S}_3 + \boldsymbol{S}_3 \cdot \boldsymbol{S}_1\right)$$
$$+D\hat{\boldsymbol{z}} \cdot \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2 + \boldsymbol{S}_2 \times \boldsymbol{S}_3 + \boldsymbol{S}_3 \times \boldsymbol{S}_1\right)$$
$$-K\sum_i \left(\hat{n}_i \cdot \boldsymbol{S}_i\right)^2$$

$J \gg D \gg K$ Hierarchy of interactions

- J: spins at 120° angles and M=0
- D: spins are "anti-chiral" in XY plane
- K: weak canting toward easy axes creates tiny moment and fixes in-plane angle

Energetics: triangle



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$$-K \sum_i (\hat{n}_i \cdot \boldsymbol{S}_i)^2$$

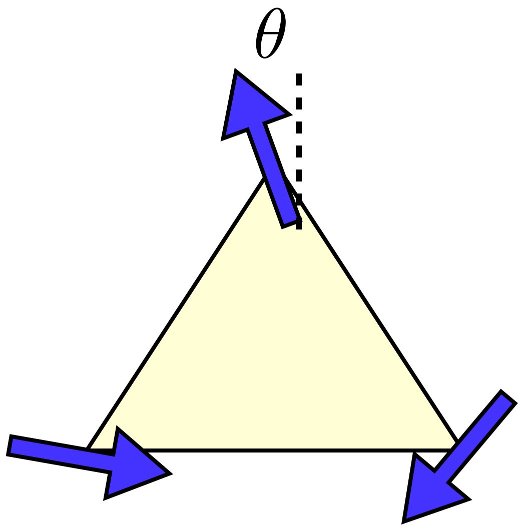
$J \gg D \gg K$ Hierarchy of interactions

$$\theta_1 = \theta + \phi_1$$
$$\theta_2 = \frac{4\pi}{3} + \theta + \phi_2$$
$$\theta_3 = \frac{2\pi}{3} + \theta - \phi_1 - \phi_2$$

θ is almost free

$\phi_{1,2}$ are tiny canting angles

Energetics: triangle



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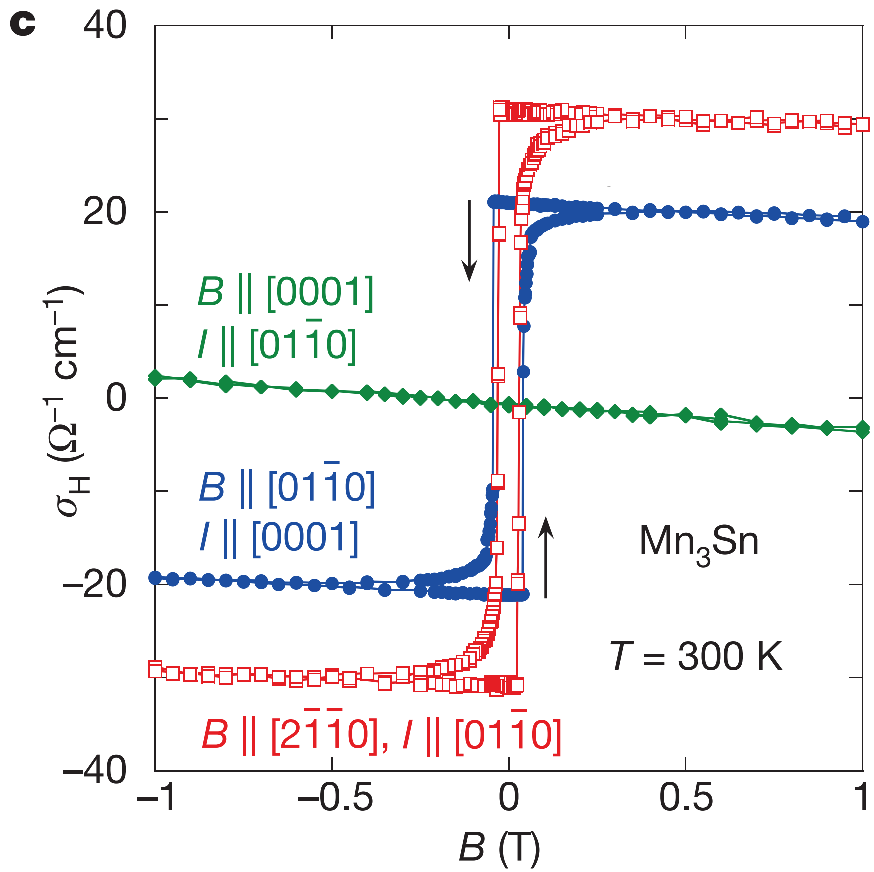
$$\theta_3 = \frac{2\pi}{3} + \theta - \phi_1 - \phi_2$$

$$\boldsymbol{m}_\triangle = \frac{K}{J} m_{Mn}(\cos\theta, \sin\theta, 0)$$
$$E_\triangle(\theta) = -\frac{K^3}{6J^2}\cos 6\theta$$

Anomalous Hall effect

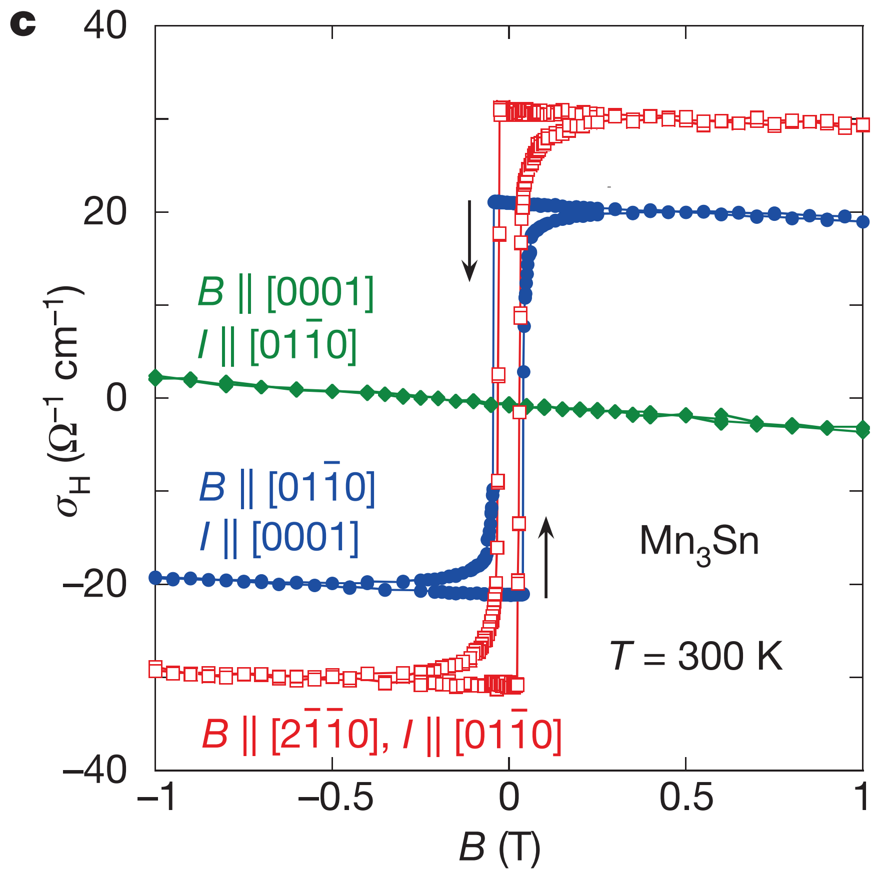


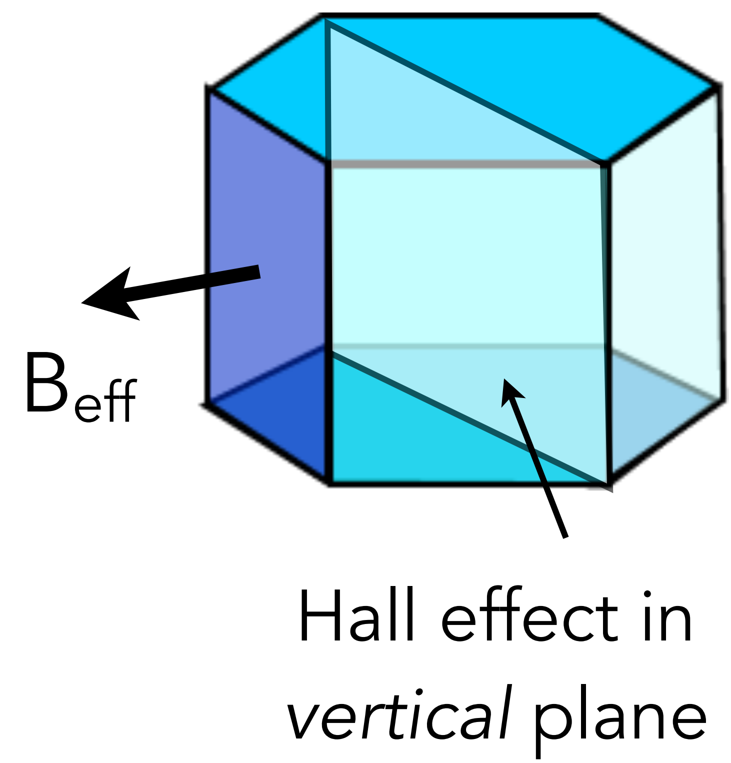
Nakatsuji *et al*, 2015

comparable to metallic FMs

switchable because of small magnetic moment and small anisotropy

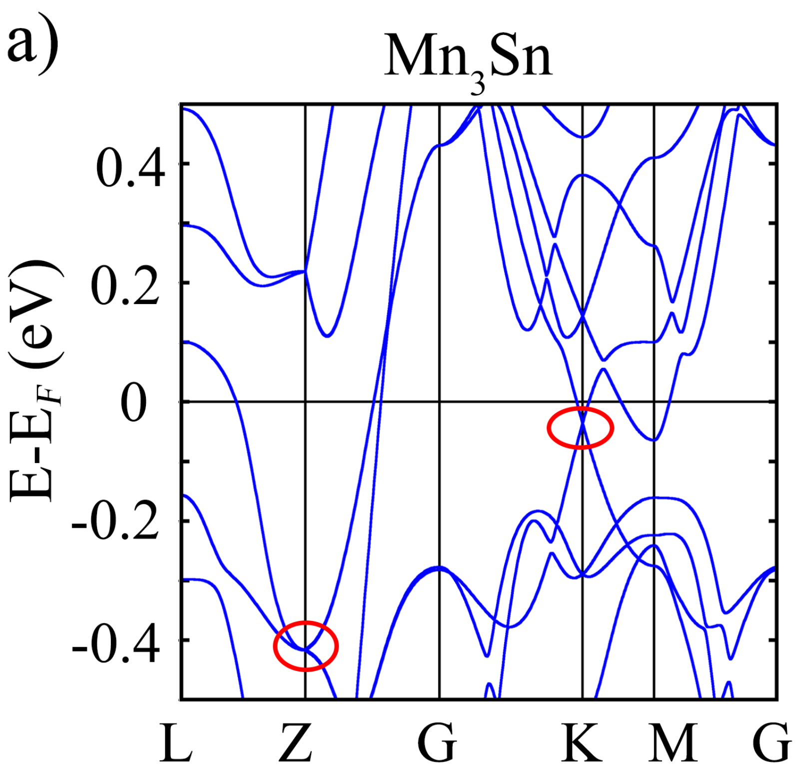
Anomalous Hall effect

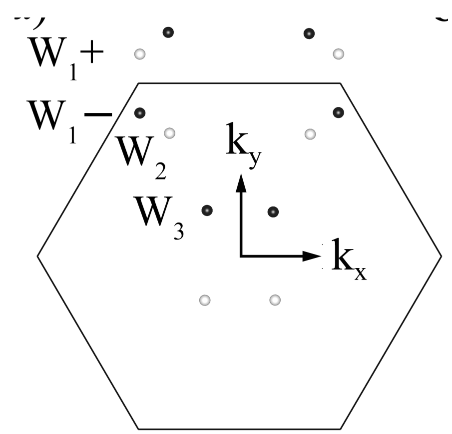




Weyl

ab initio finds Weyl points and surface Fermi arcs



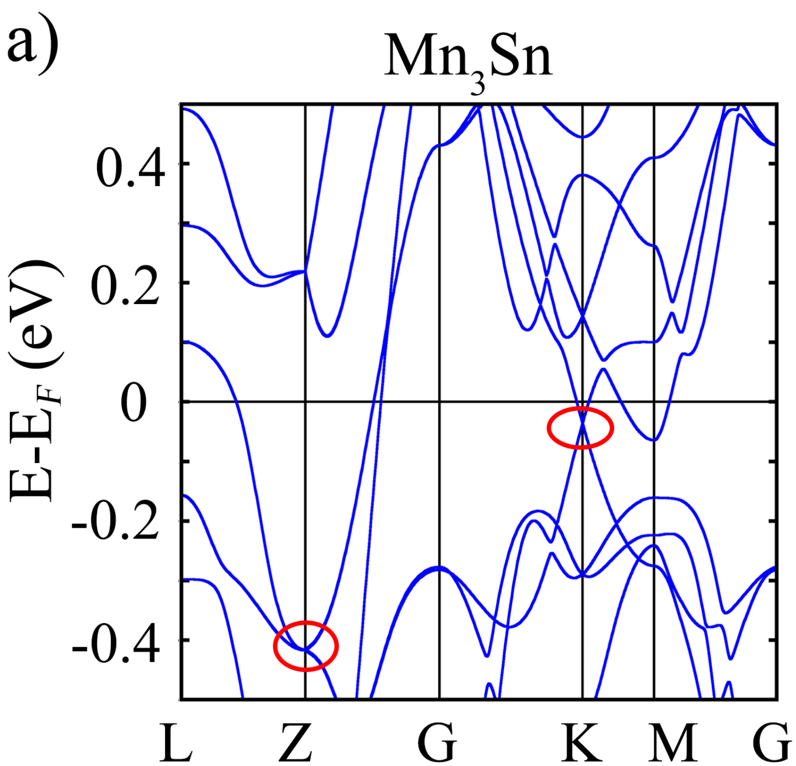


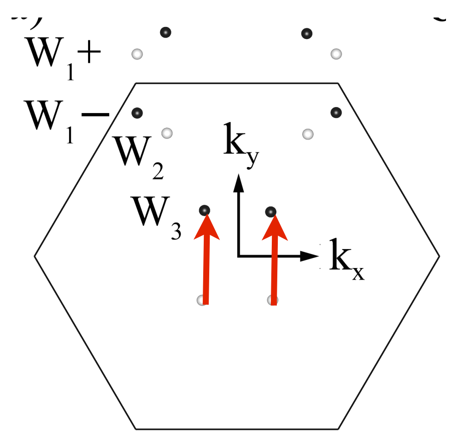
notice lack of 6-fold symmetry: due to direction of AF order

Hao Yang *et al*, arXiv:1608.03404

Weyl

ab initio finds Weyl points and surface Fermi arcs





Hall vector ~ "dipole moment"

Hao Yang *et al*, arXiv:1608.03404

Weyl

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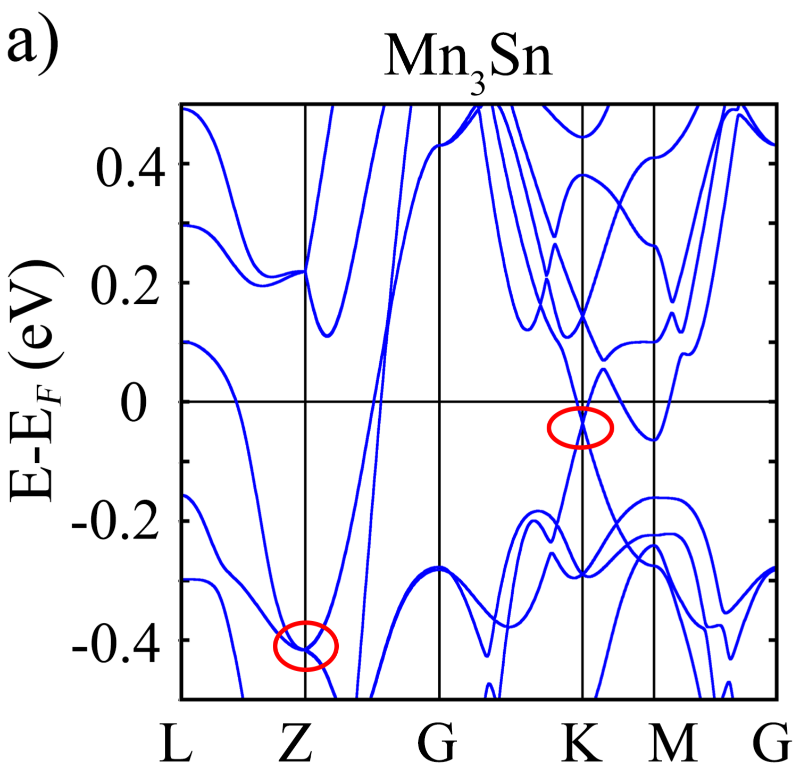
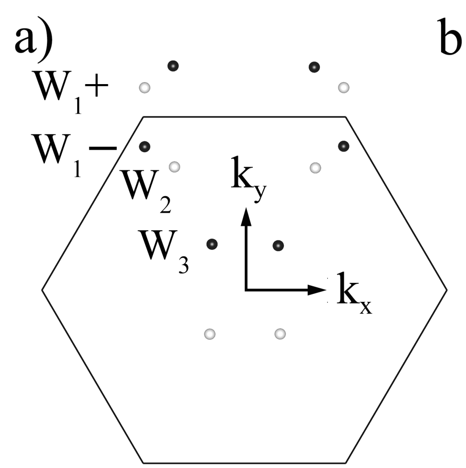
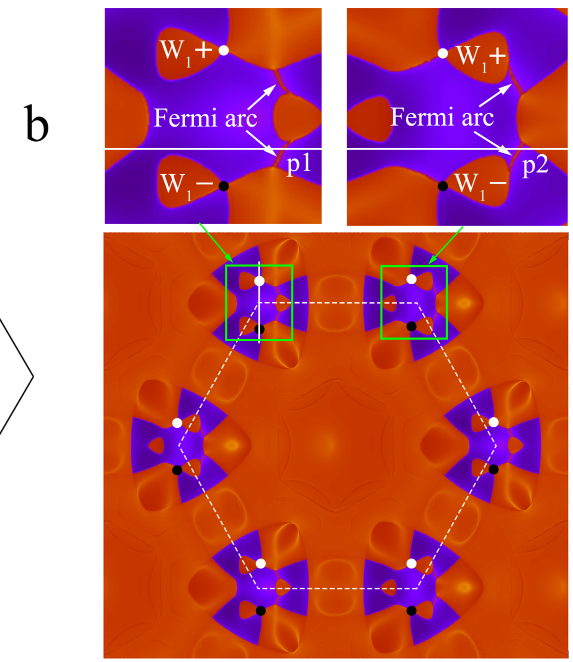


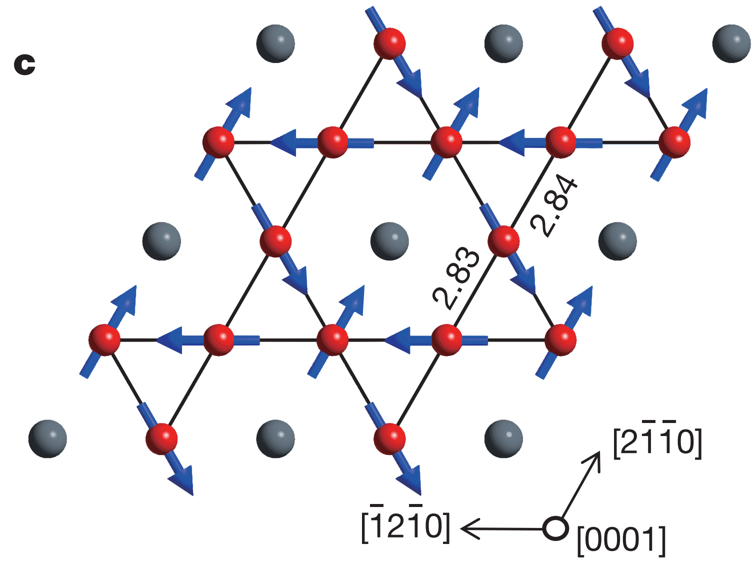




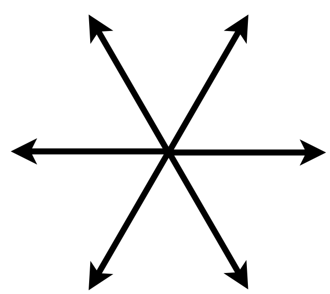
Hao Yang *et al*, arXiv:1608.03404

Textures

Magnetic order has Z_6 structure

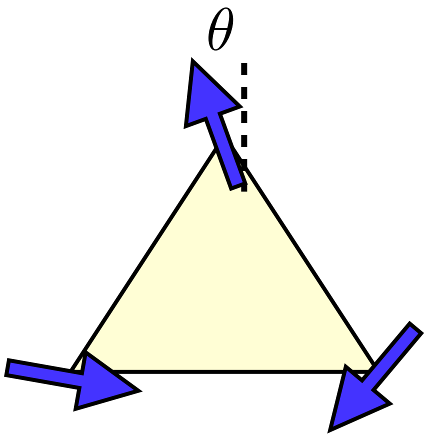


direction of inward-pointing spin



$$\psi = |\psi| e^{2\pi i n/6}$$

3 pairs of time-reversed domains

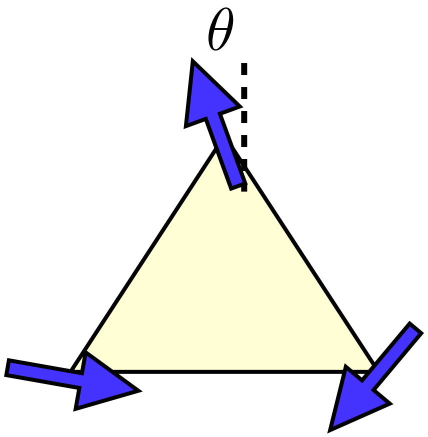


Textures

$$\psi = |\psi| e^{i\theta} \qquad F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla\theta)^2 - \lambda \cos 6\theta \right\}$$

sine-Gordon model with 6-fold anisotropy

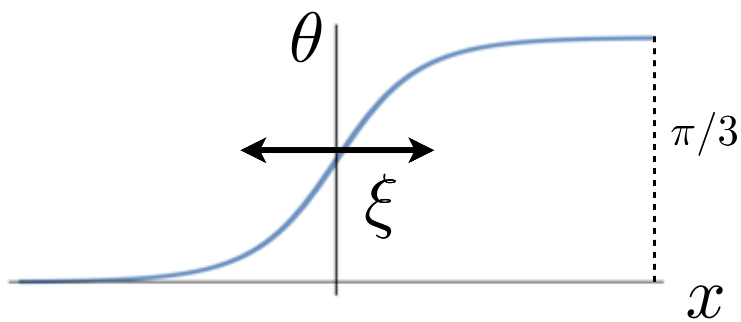
$$\rho \sim \frac{J}{a} \qquad \lambda \sim \frac{K^3}{J^2 a^3}$$



Textures

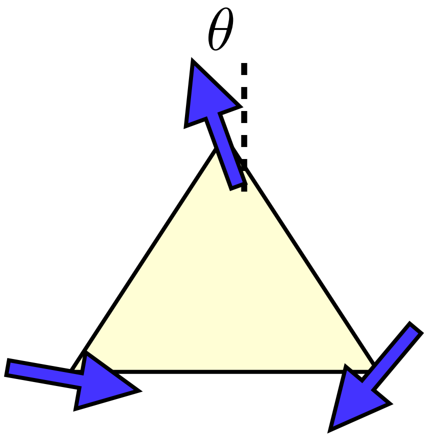
$$\psi = |\psi| e^{i\theta} \qquad F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla\theta)^2 - \lambda \cos 6\theta \right\}$$

soliton = domain wall connecting *neighboring* minima of cosine



$$\theta(x) = \frac{2}{3} \tan^{-1} \exp(x/\xi) \qquad \xi = \frac{1}{6} \sqrt{\frac{\rho}{\lambda}}$$

wide DWs



Textures

$$\psi = |\psi| e^{i\theta} \qquad F \sim \int d^3x \left\{ \frac{\rho}{2} (\nabla\theta)^2 - \lambda \cos 6\theta \right\}$$

sine-Gordon model with 6-fold anisotropy

- Minimal energy domain walls are *not* between time-reversed states
- Magnetization, Hall vector, location of Weyl points are all determined by domain choice, not by field in general
- Stable Z_6 vortices exist

Dynamics

Symmetry-allowed *hydrodynamic* equation of motion is *overdamped*

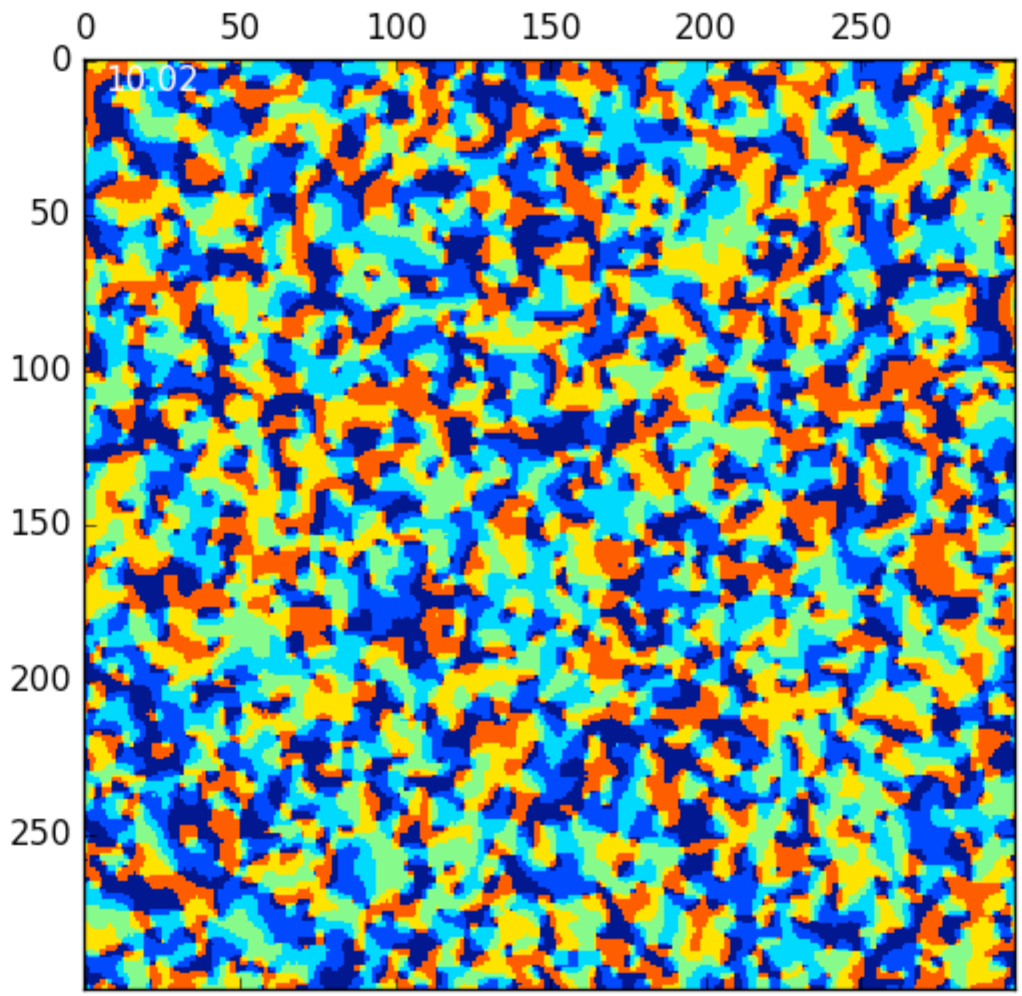
$$\gamma \partial_t \theta = \rho \nabla^2 \theta - 6\lambda \sin 6\theta - h\bar{m} \sin\theta + \eta(\boldsymbol{r}, t)$$

↑

damping

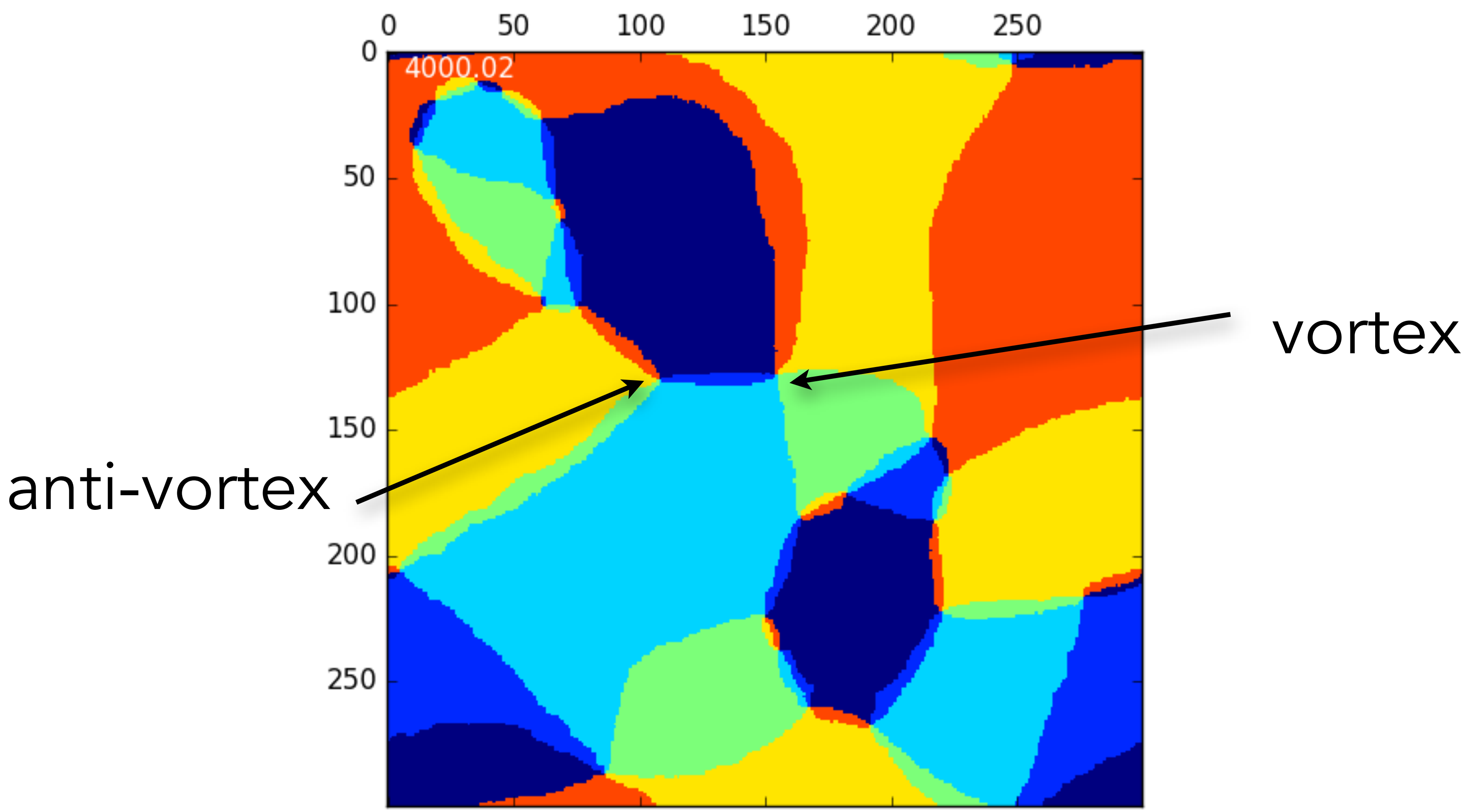
(uniquely determines long-time, low-frequency dynamics)

Domain formation



quench

Domain formation



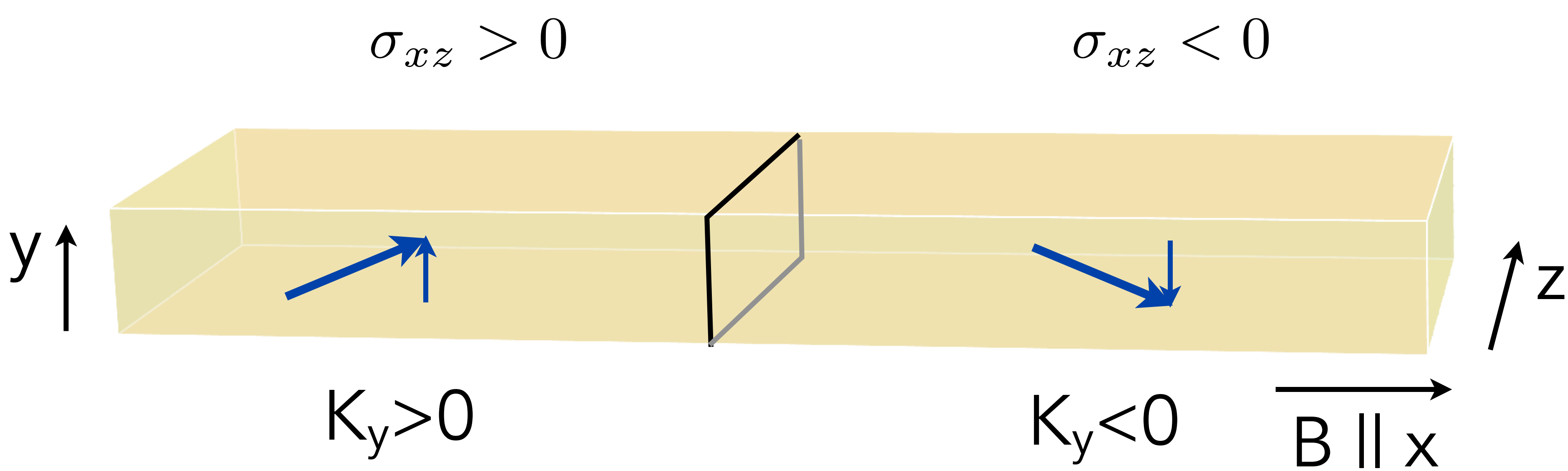
Could one use this?

Idea: apply field in "hard direction" below coercive value

two minimum energy domains with *opposite* component of Hall vector normal to field

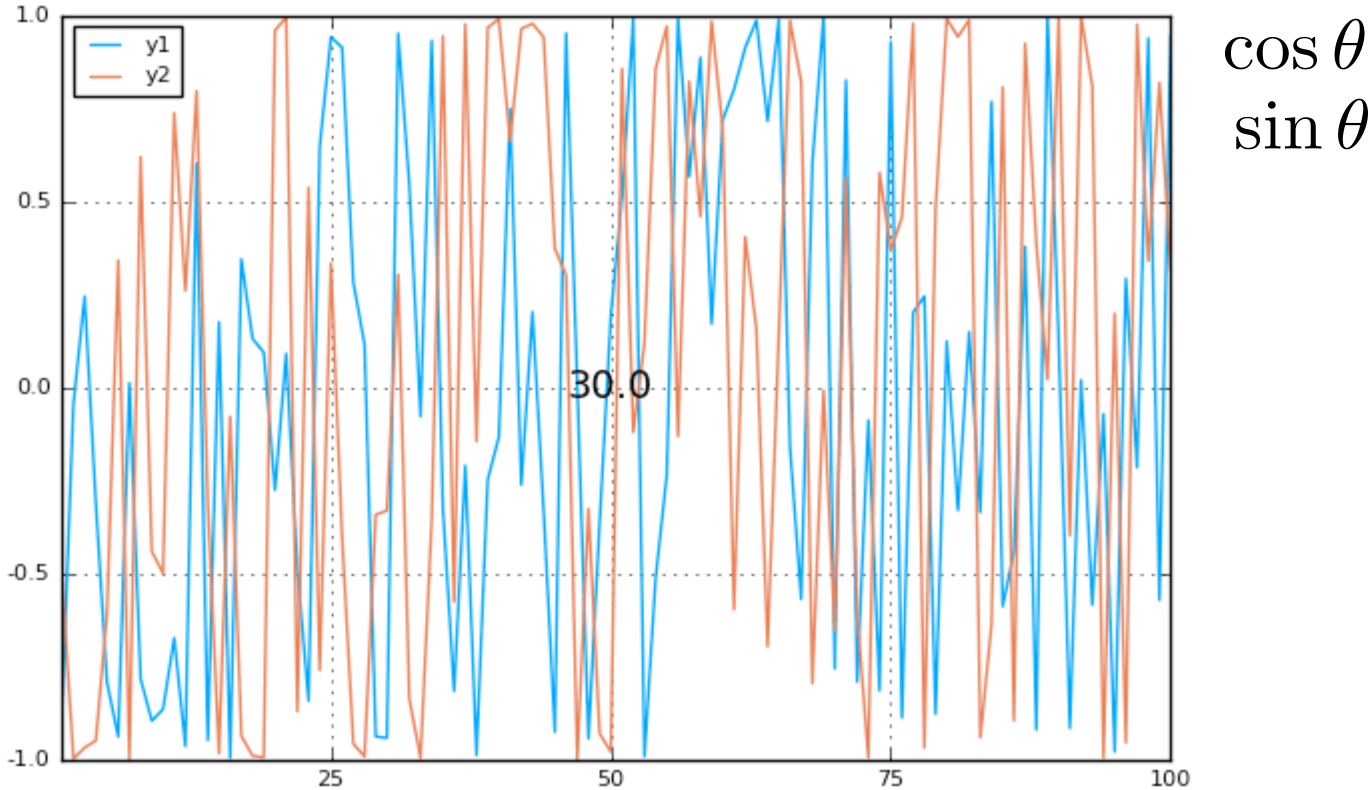
Could one use this?

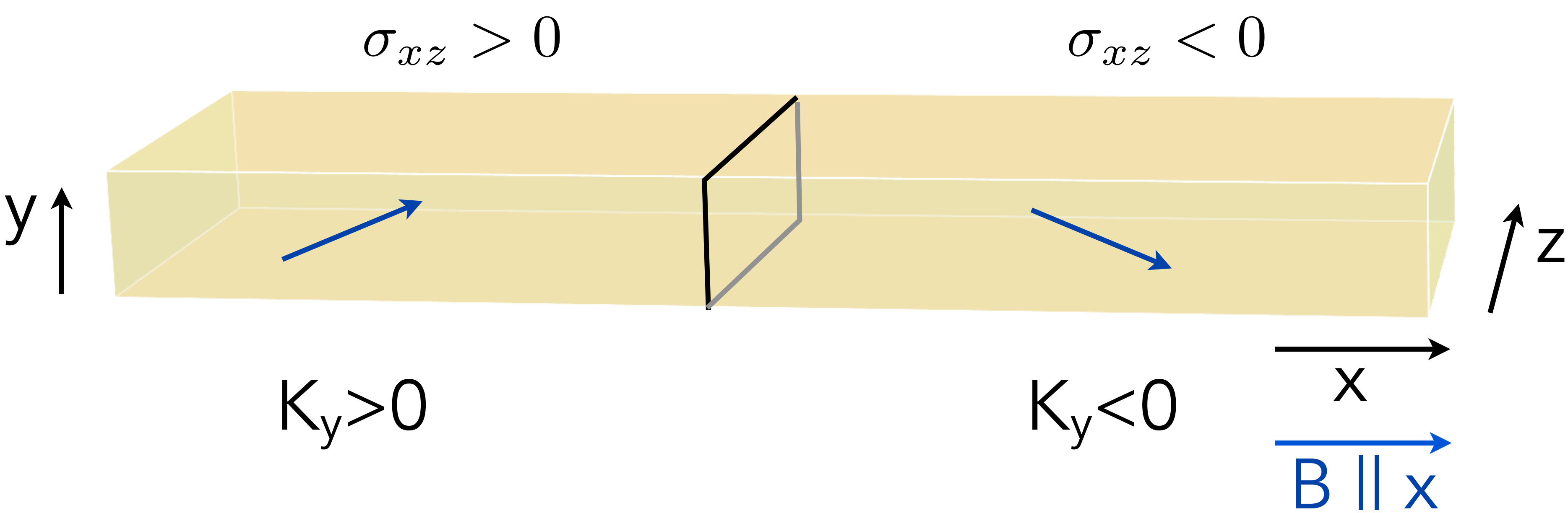
Idea: apply field in "hard direction" below coercive value

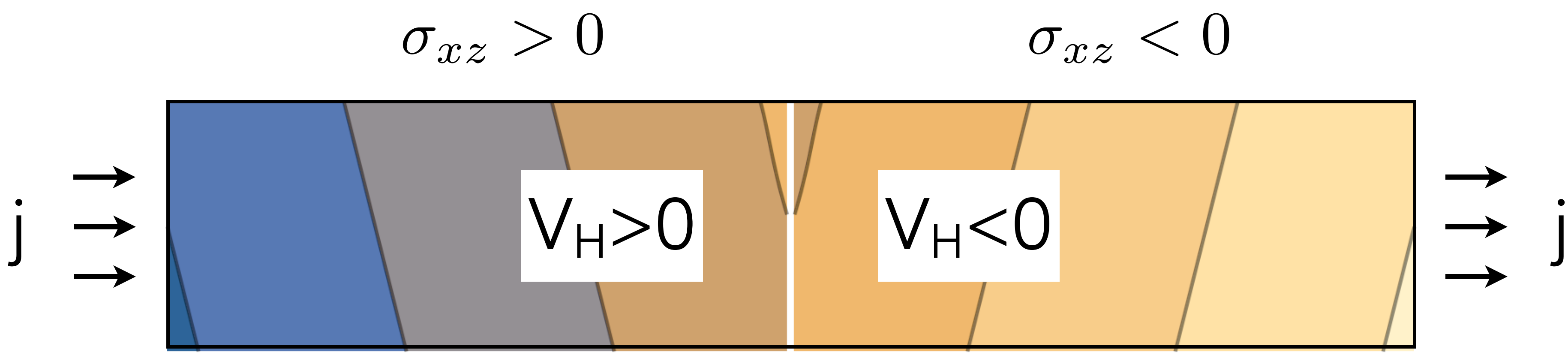


controllable domain walls in narrow Hall bar

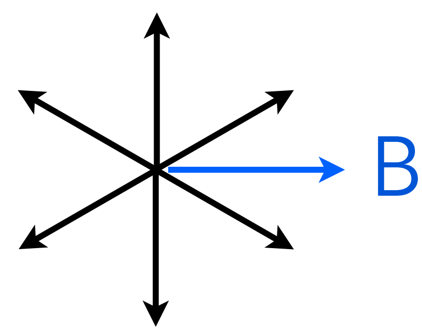
(triangle wave of $B_x(t)$)

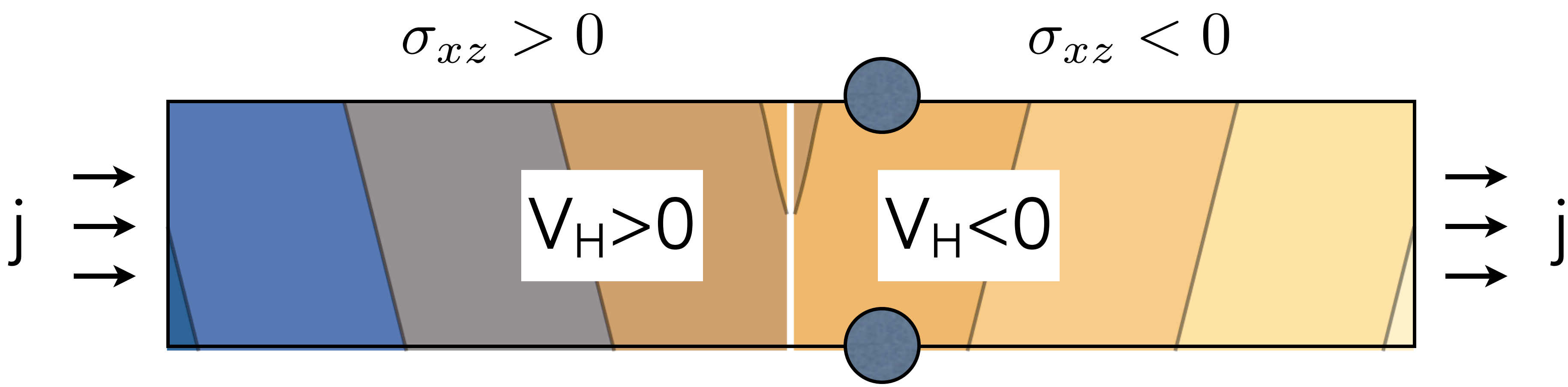






equipotentials from
solution of Laplace's
equation for a Hall bar
with two domains





One could imagine fixing the transverse contacts and *switching* Hall voltage by moving domain wall

a device?

Domain wall drive

Recall dynamical equation

$$\begin{aligned}\gamma\partial_t\theta &= \rho\nabla^2\theta - 6\lambda\sin 6\theta - h\bar{m}\sin\theta + \eta(\boldsymbol{r},t)\\ &= -\frac{\delta F}{\delta\theta} + \eta(\boldsymbol{r},t)\end{aligned}$$

- This form satisfies FDT and eventually leads to equilibrium
- It describes forces on DWs from applied magnetic fields but not deviations from *electronic* equilibrium

Domain wall drive

electronic disequilibrium ⟹ additional forces

$$\gamma \partial_t \theta = -\frac{\delta F}{\delta \theta} + \eta(\boldsymbol{r}, t) + f(\boldsymbol{j})$$

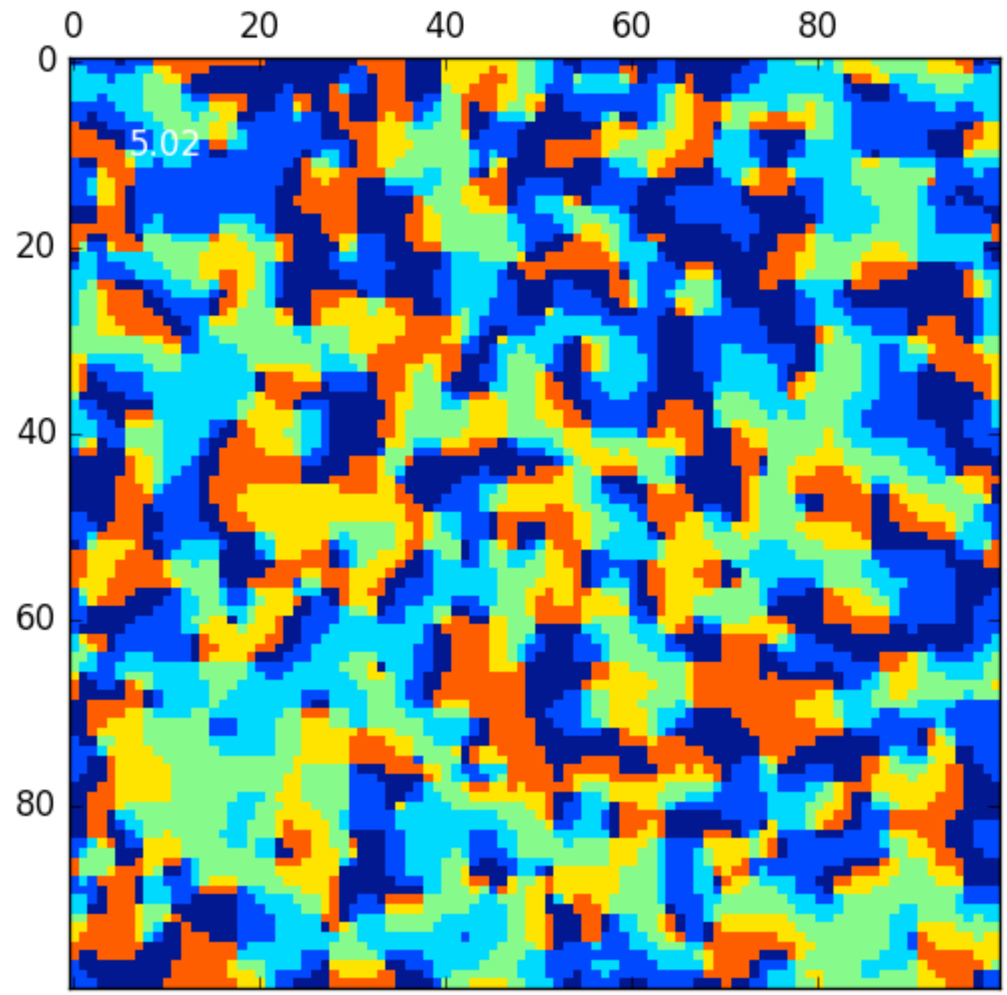
- Though similar to "spin transfer torque", the *antiferromagnetic* nature of the system makes angular momentum counting suspect
- Instead we rely on symmetry (for now!)

$$f(\boldsymbol{j}) = \gamma \boldsymbol{v} \cdot \boldsymbol{\nabla} \theta \qquad \boldsymbol{v} = (c_1 j_x, c_1 j_y, c_2 j_z)$$

▲ spin texture "convects" with velocity ***v*** proportional to the current

Current drive

J →

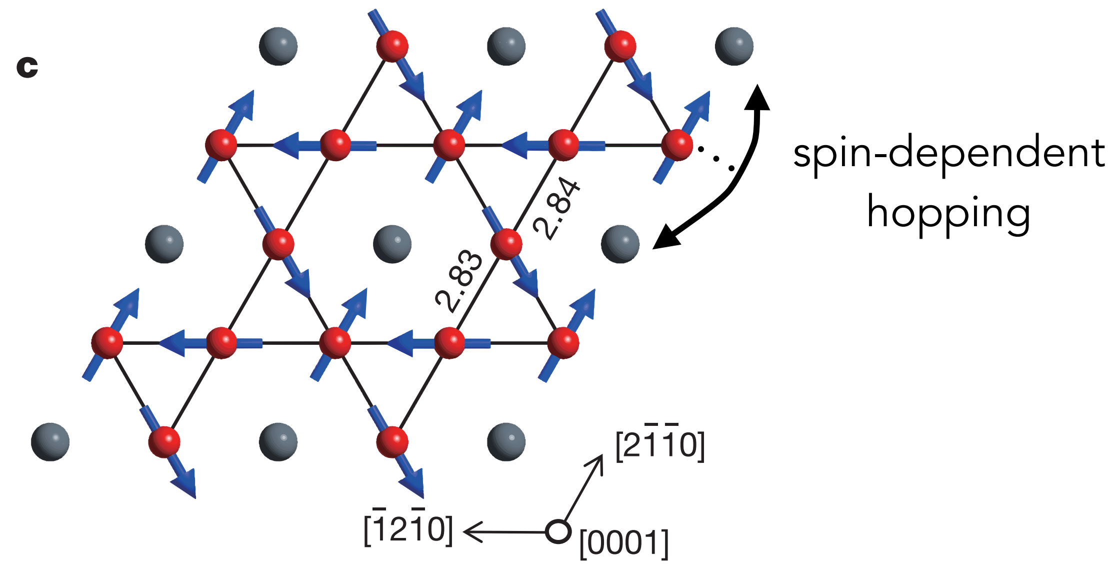


possible in principle.

practice???

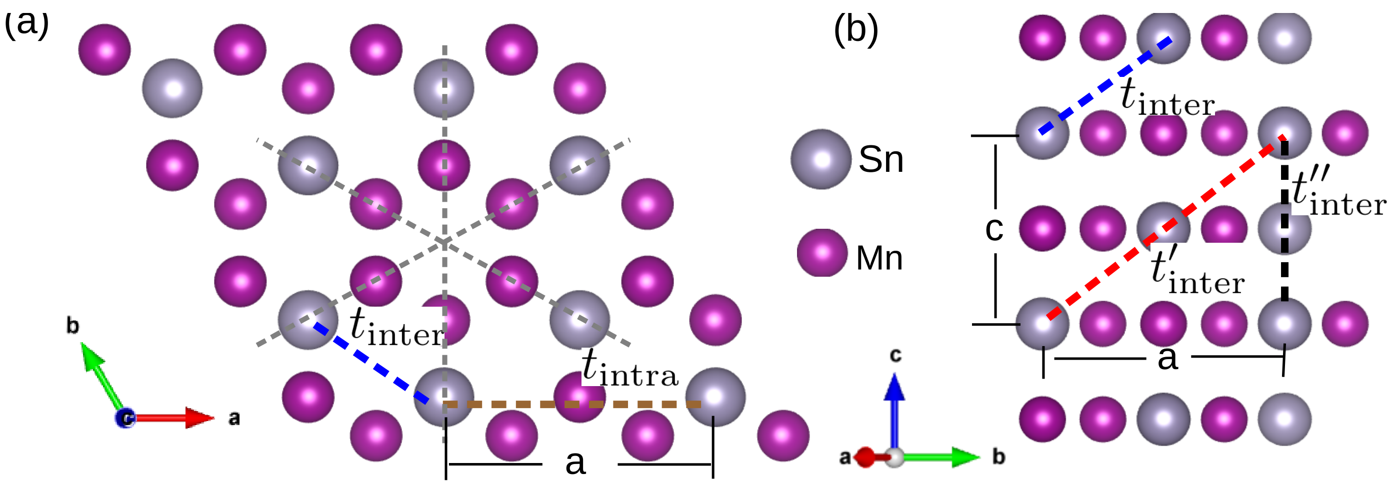
Electronic properties of textures?

tight-binding of single orbital on Sn sites: a 4 band model



Enables efficient study of domain walls, vortices etc.

TB Model



$$t_{\rm intra}(\mathbf{r}_{nm}) = t_0\,\mathbb{I}_{2\times 2} + t_J\,\sigma\cdot\mathbf{S}_{nm} + (-1)^{\xi_{mn}} i\lambda_z\,\sigma_z\ ,$$

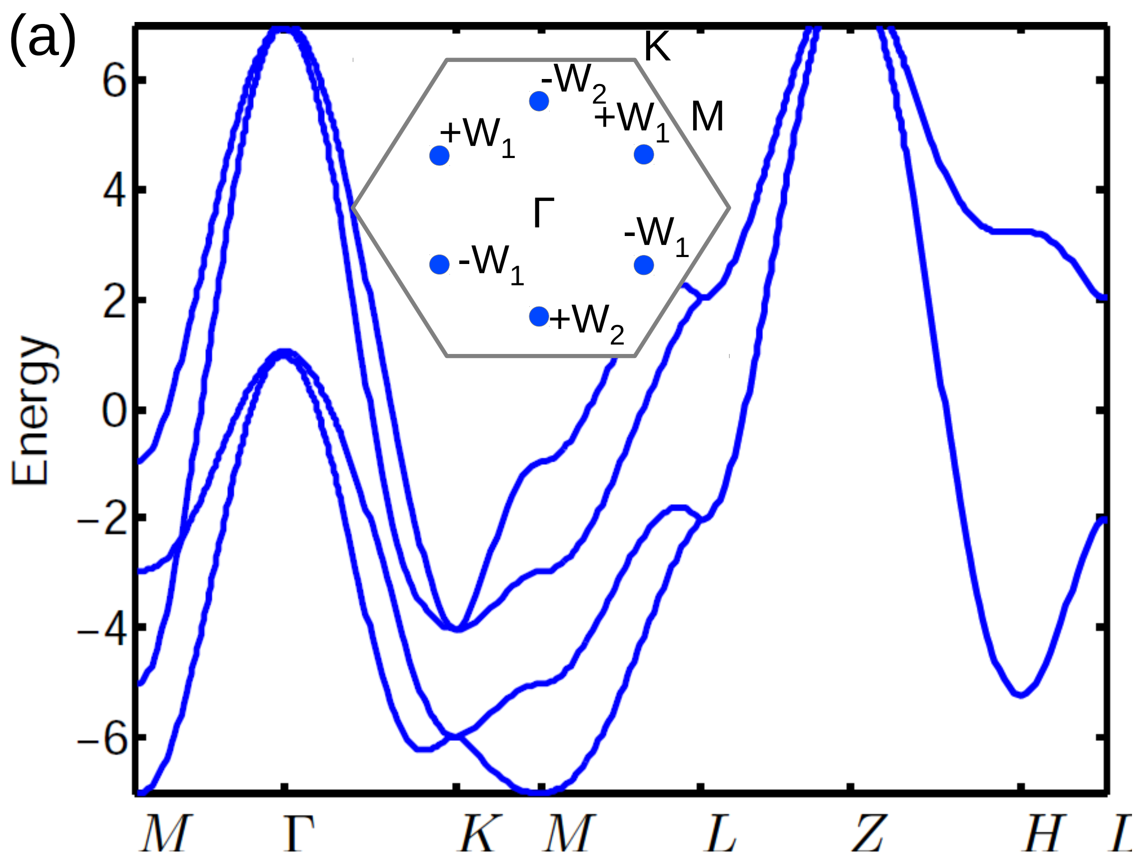
$$t_{\rm inter}(\mathbf{r}_{nm}) = t_1\,\mathbb{I}_{2\times 2}\ ,$$

$$t'_{\rm inter}(\mathbf{r}_{nm}) = i\lambda_{\rm R}\,\mathbf{e}_{\rm soc}^{\mathbf{r}_{nm}}\cdot\sigma\ ,$$

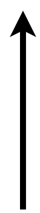
$$t''_{\rm inter}(\mathbf{r}_{nm}) = t_2\,\mathbb{I}_{2\times 2}\ ,$$

TB bands

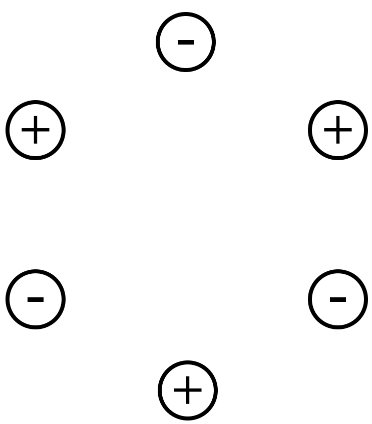
$\boldsymbol{K} \parallel \hat{\boldsymbol{y}}$

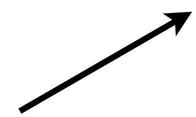


"Weyl math"

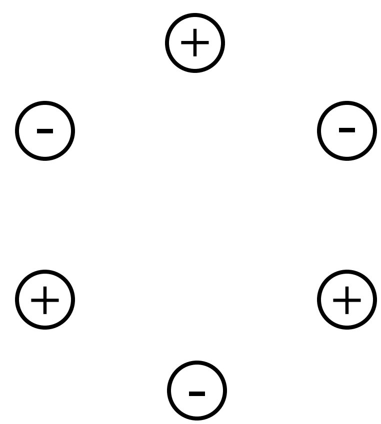


$$\boldsymbol{K} \parallel \hat{\boldsymbol{y}}$$

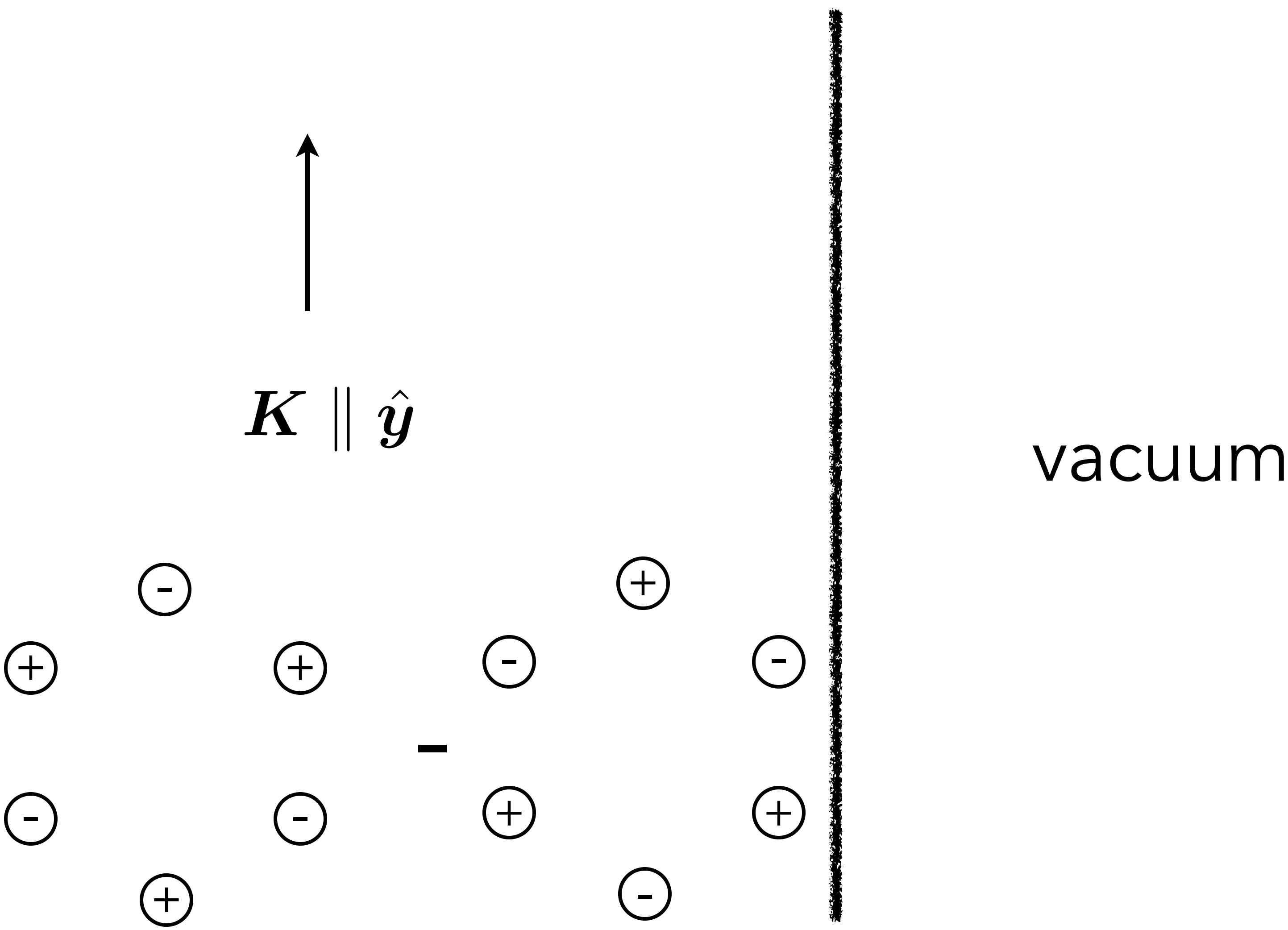




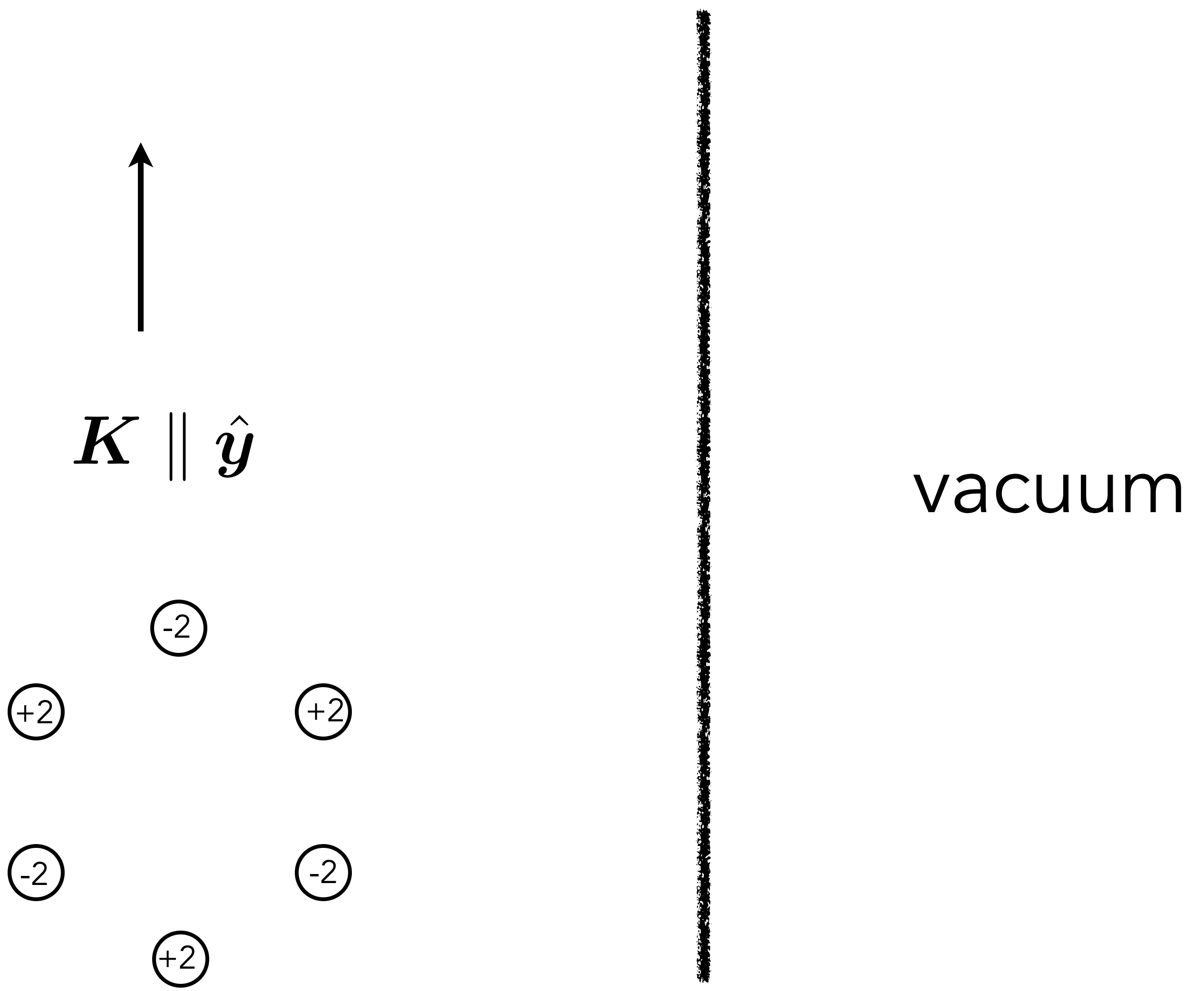
$$\boldsymbol{K} \parallel \frac{\sqrt{3}}{2}\hat{\boldsymbol{x}} + \frac{1}{2}\hat{\boldsymbol{y}}$$



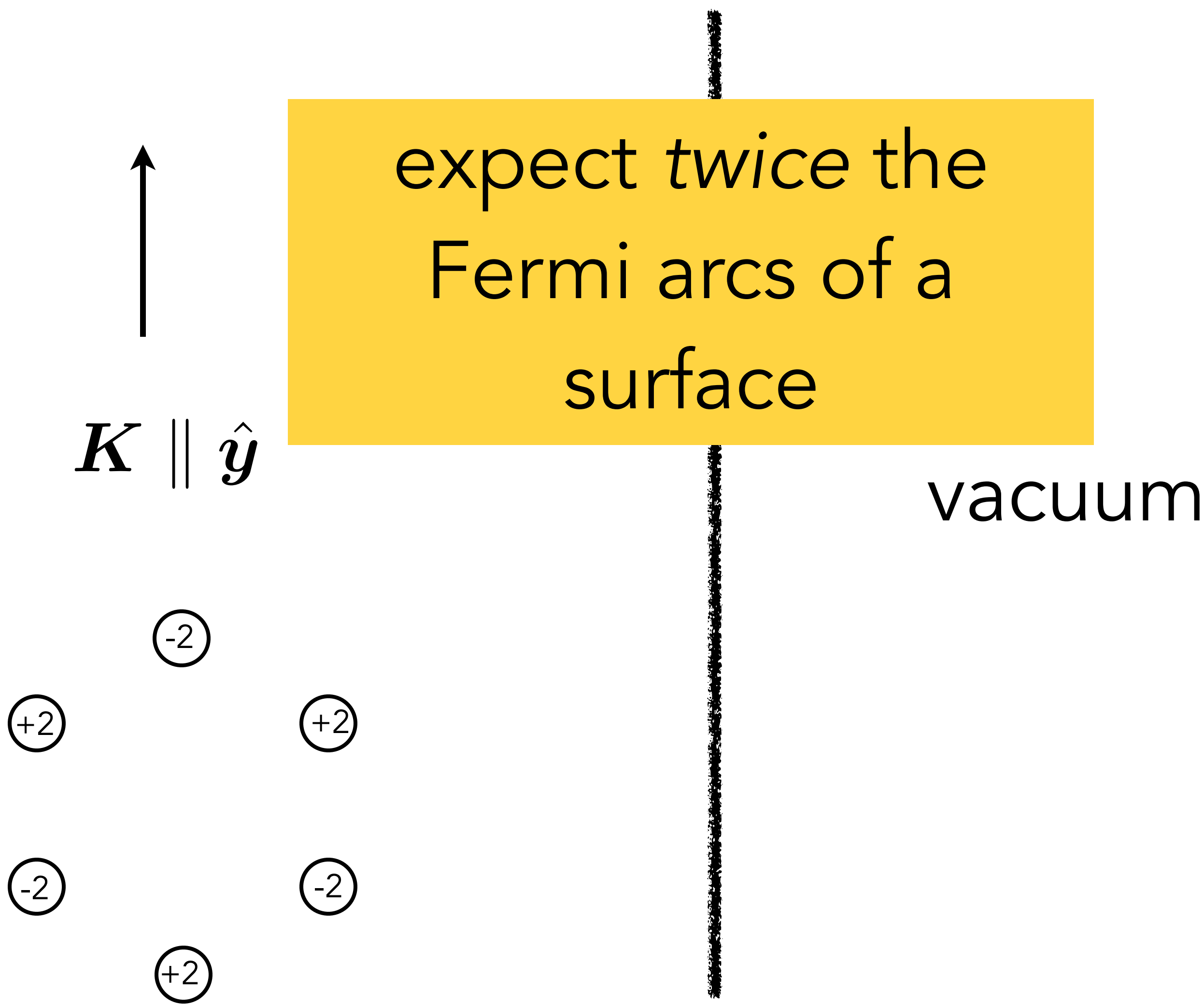
“Weyl math”



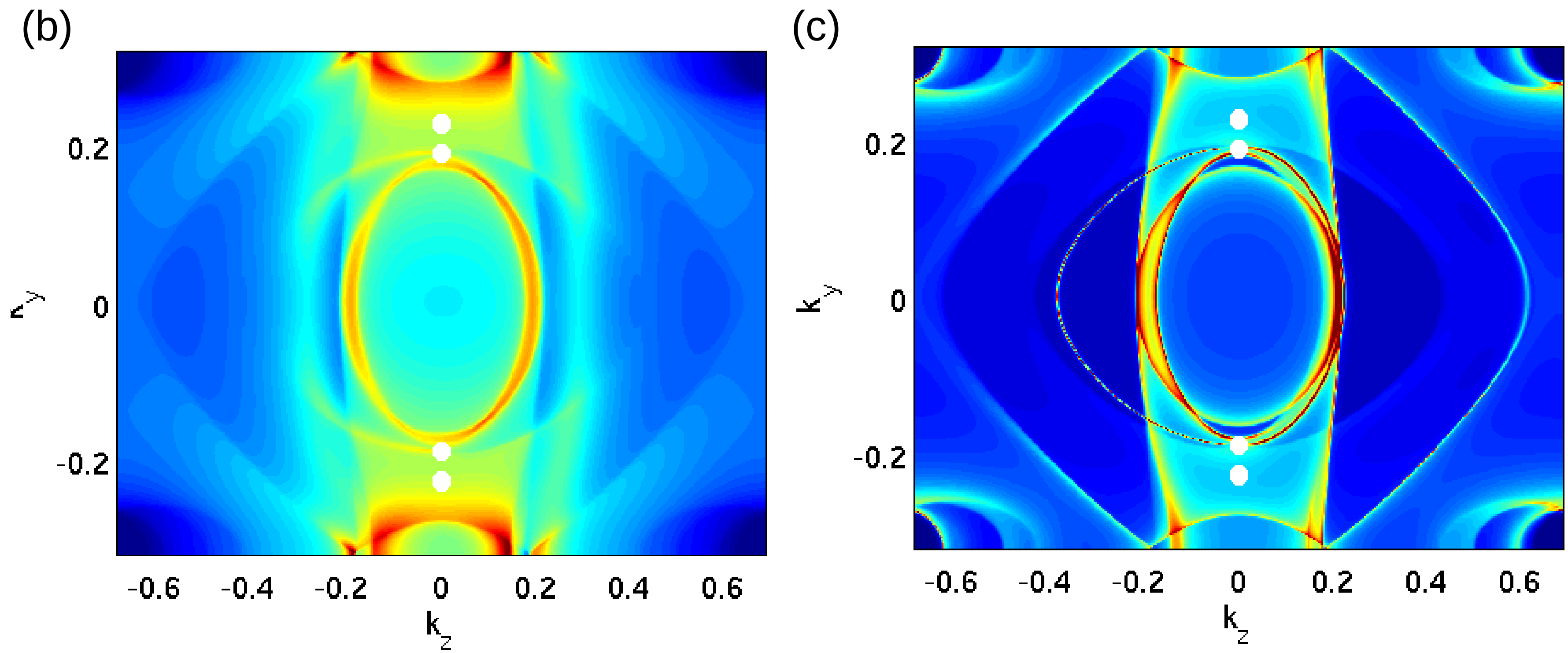
"Weyl math"



“Weyl math”


expect *twice* the Fermi arcs of a surface
$\boldsymbol{K} \parallel \hat{\boldsymbol{y}}$
vacuum
-2
+2
+2
-2
-2
+2

Solution

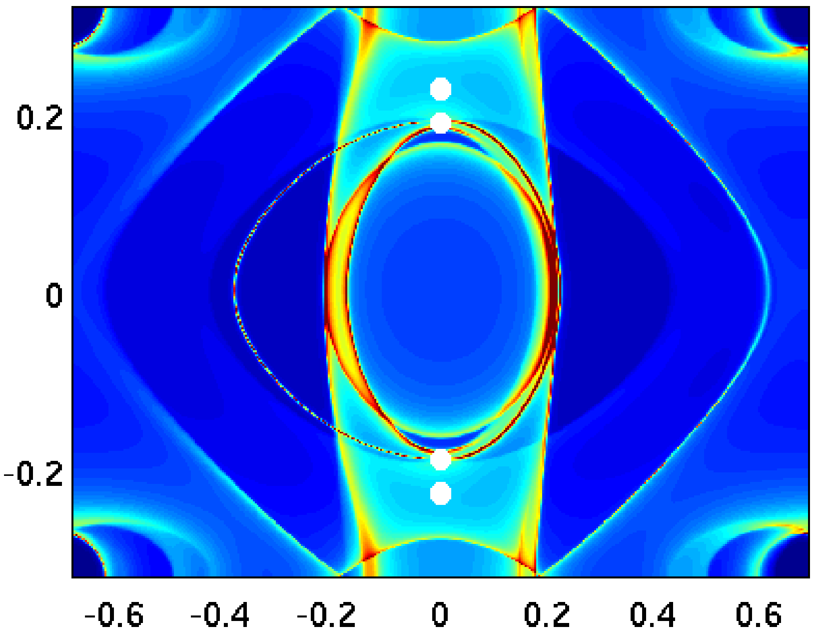


surface

domain wall

twice as many Fermi arcs as a surface

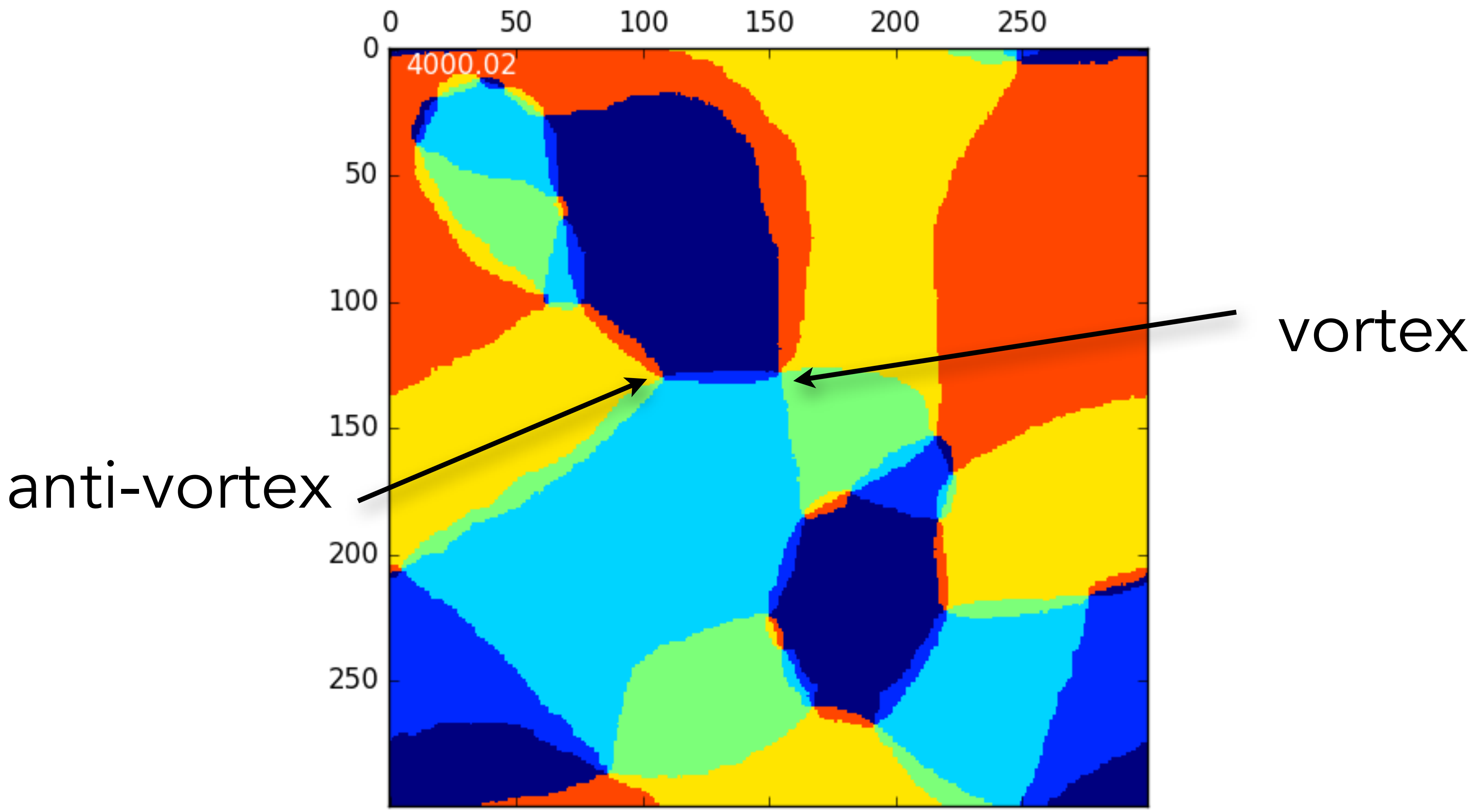
How to detect?



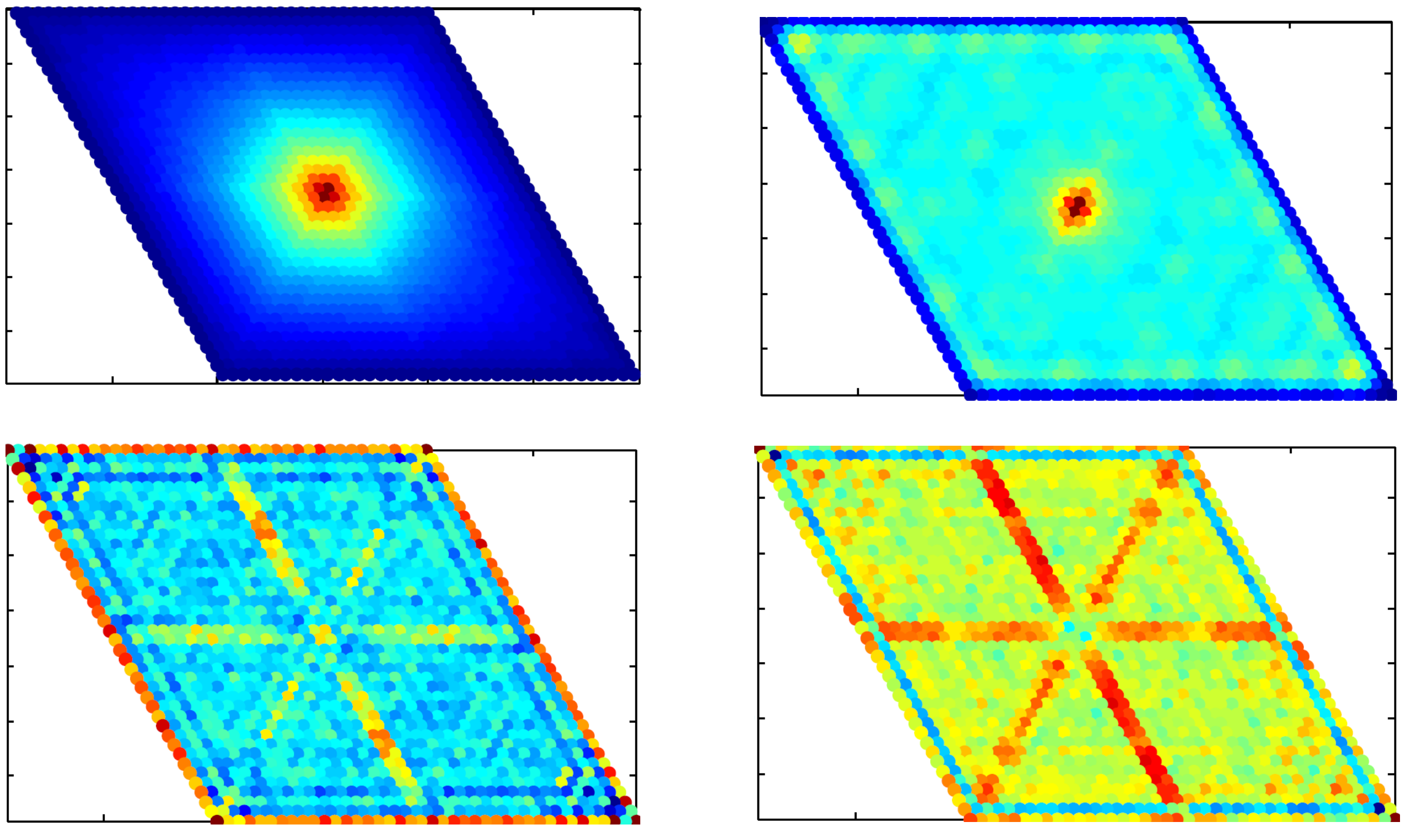
ARPES of domain wall seems challenging to say the least!

- Transport: enhanced intrinsic Hall conductivity within a DW?
- STM: signatures of bound states in LDOS?

Vortices?



Z_6 vortex



Quasi-bound states may appear. Origin?

"Chiral gauge field and axial anomaly in a Weyl semimetal", Lui, Ye, Qi (2013): suggest a 1d chiral mode at a FM vortex?

Conclusions

- Soft antiferromagnet provides a rich platform to explore electronic physics of topological textures
- I presented an order parameter description and minimal electronic model for Mn_3Sn and related materials
- For the future:
    - Theory of electronic mechanisms of damping, current drive, etc.
    - Strong coupling of order parameter to electrons: does chiral anomaly play a role?
    - Effect of disorder on textures

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