

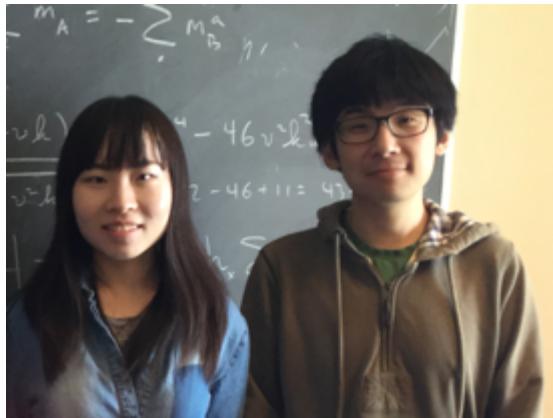


Quantum Spin Liquids

Leon Balents, KITP

ISCOM2017, Miyagi

Collaborators (whose work I'll mention)



Xue-Yang Song
Yi-Zhuang You



Gábor Halász



Chunxiao Liu



Lucile Savary



Jason Iaconis



Xiao Chen

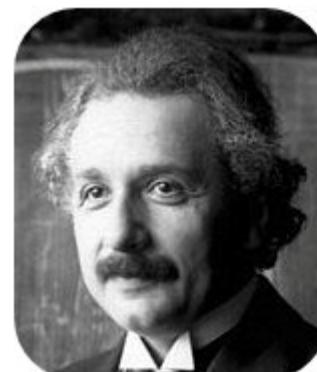
Quantum non-locality

EPR

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



??where is the information??



A. Einstein

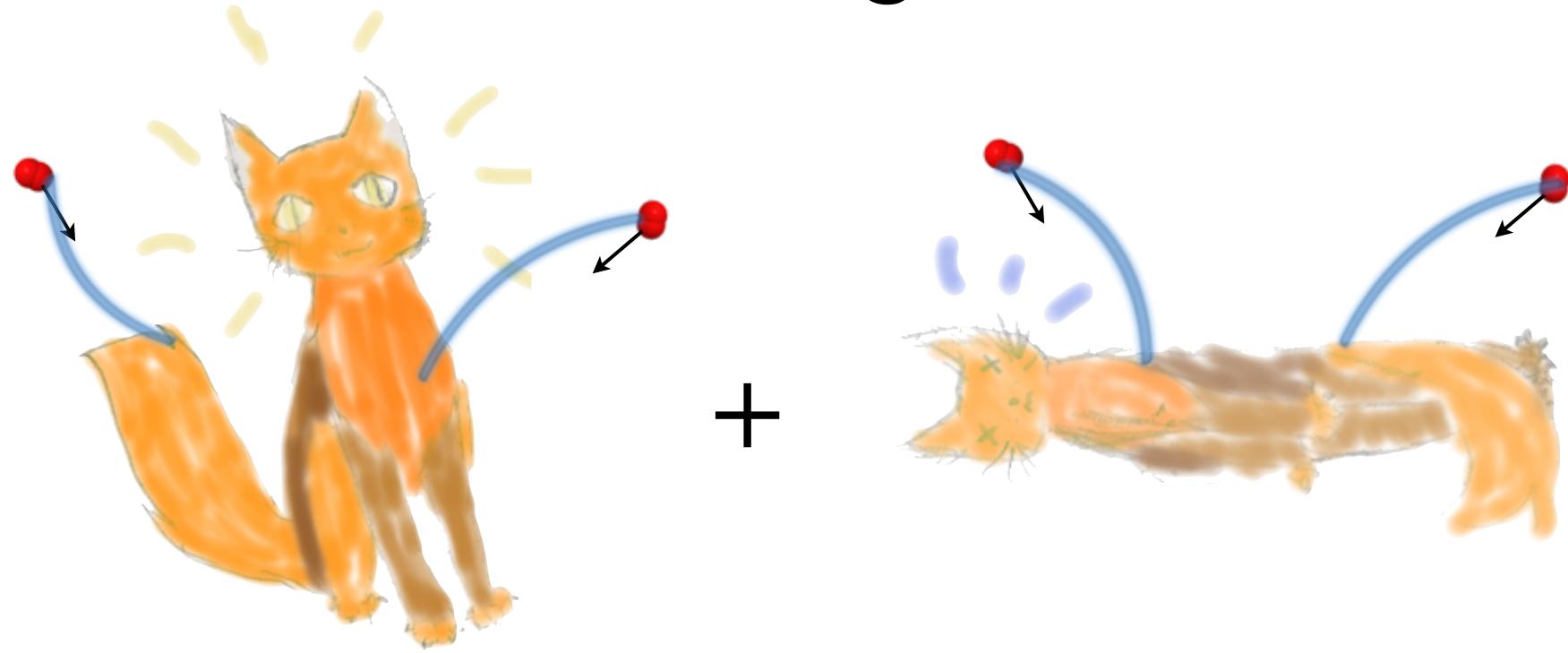


B. Podolsky



N. Rosen

Schrödinger's Cat



UNSTABLE to decoherence - uncontrolled entanglement with the environment



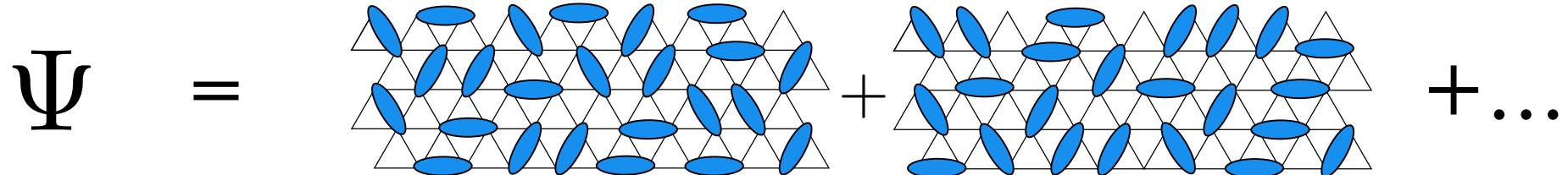
Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Resonating Valence Bond state

Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Psi = \text{Diagram 1} + \text{Diagram 2} + \dots$$

The diagrams show a triangular lattice of sites. Blue ovals representing spins are placed on some of the lattice sites. In Diagram 1, the ovals are on every other site along the horizontal rows. In Diagram 2, the ovals are on every third site along the horizontal rows. The ellipsis indicates that there are more diagrams in the sum.

Resonating Valence Bond state



Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties



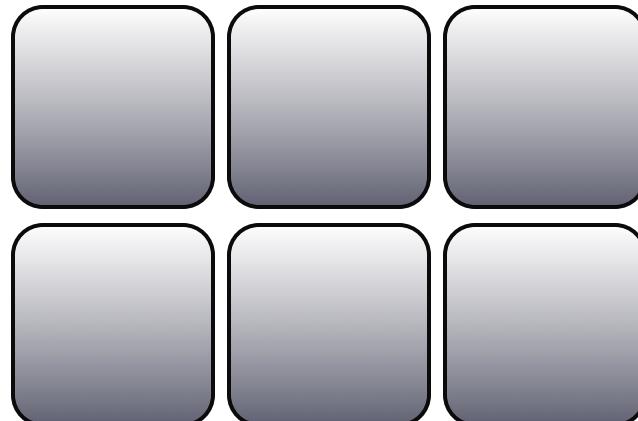
- Measurements far away do not affect one another
- From local measurements we can deduce the global state

Ordinary (local) Matter

Hamiltonian is local

$$H = \sum_x \mathcal{H}(x) \quad \mathcal{H}(x) \text{ has local support near } x$$

Ground state is “essentially”
a product state

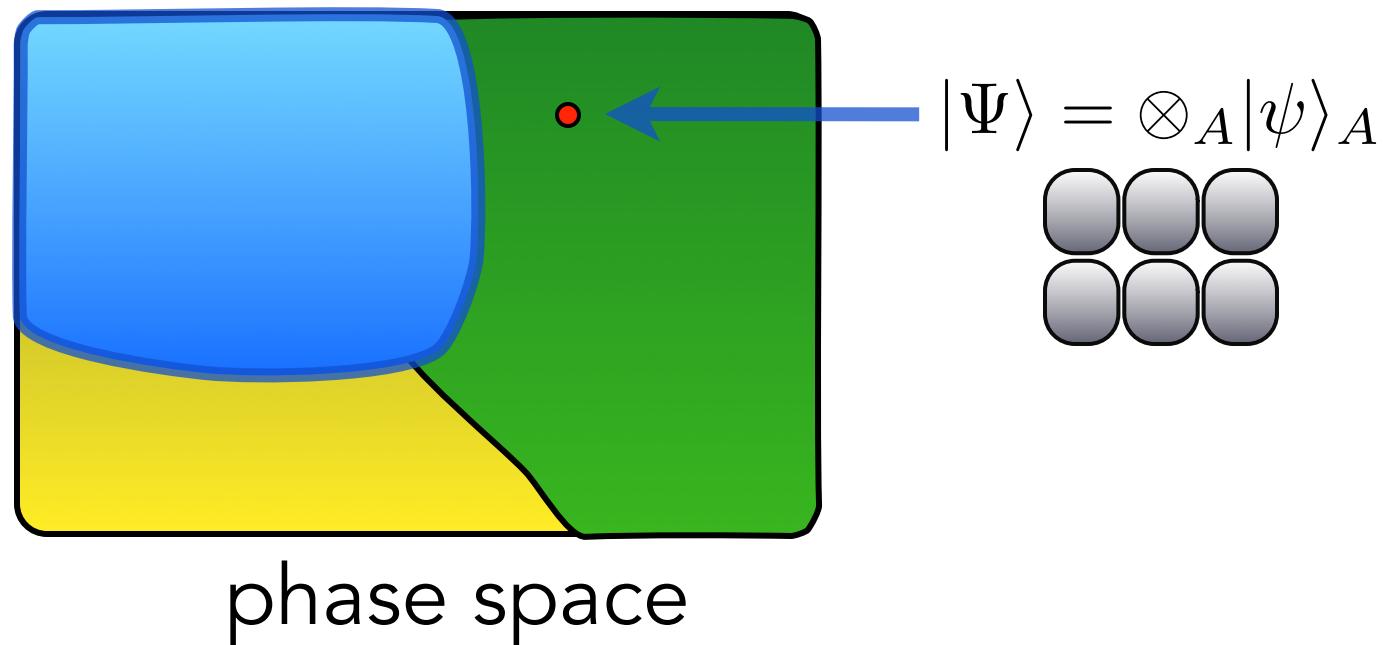


$$|\Psi\rangle = \otimes_A |\psi\rangle_A$$

no entanglement
between blocks

“Essentially” a product state?

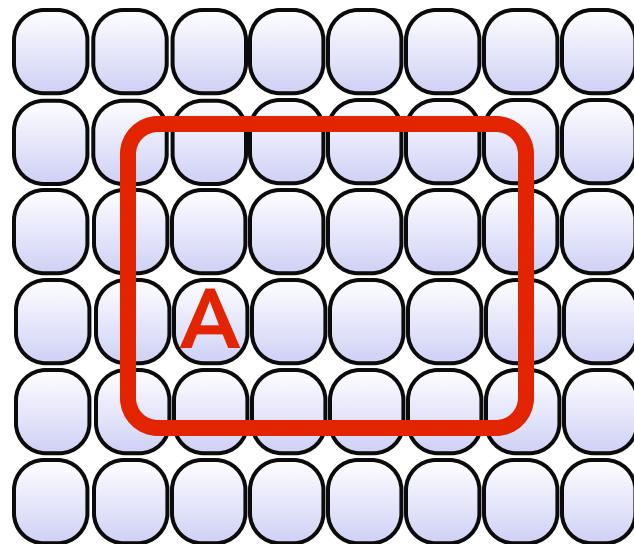
- Adiabatic continuity



n.b. This is not true for gapless fermi systems

“Essentially” a product state?

- Entanglement scaling



$$\rho_A = \text{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\text{Tr}_A (\rho_A \ln \rho_A)$$

$$S(A) \sim \sigma L^{d-1} \quad \text{area law}$$

satisfied with exponentially small corrections

Best example: ordered magnet

Hamiltonian

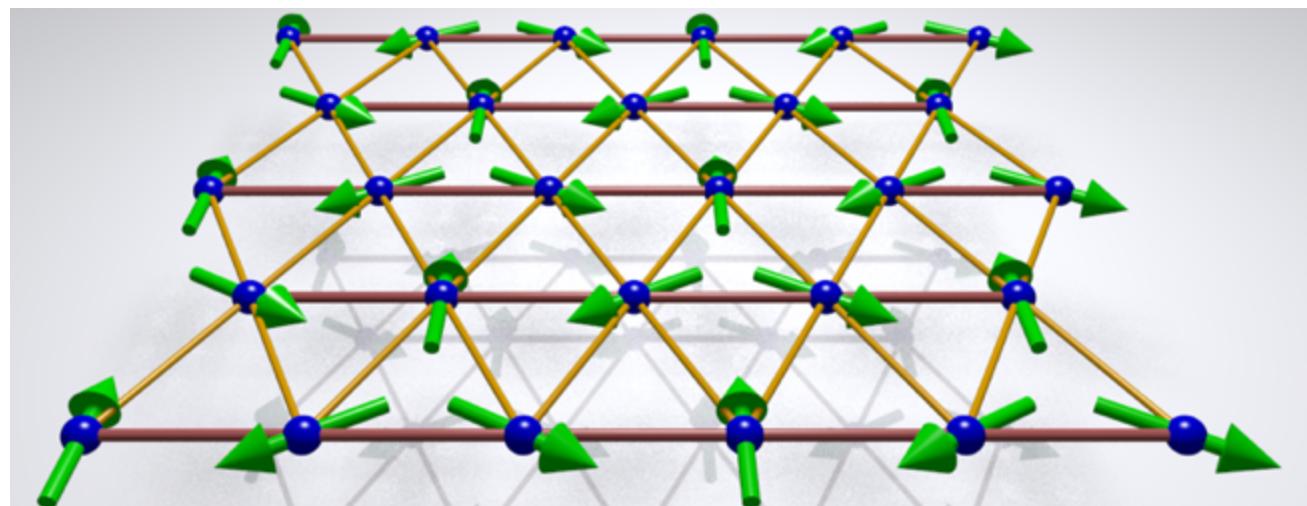
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-
range: local

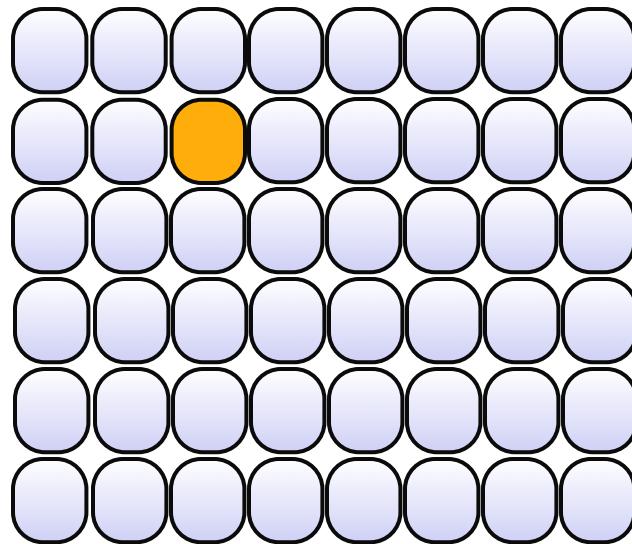
ordered state

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{\mathbf{n}}_i = +S\rangle$$

block is a single
spin



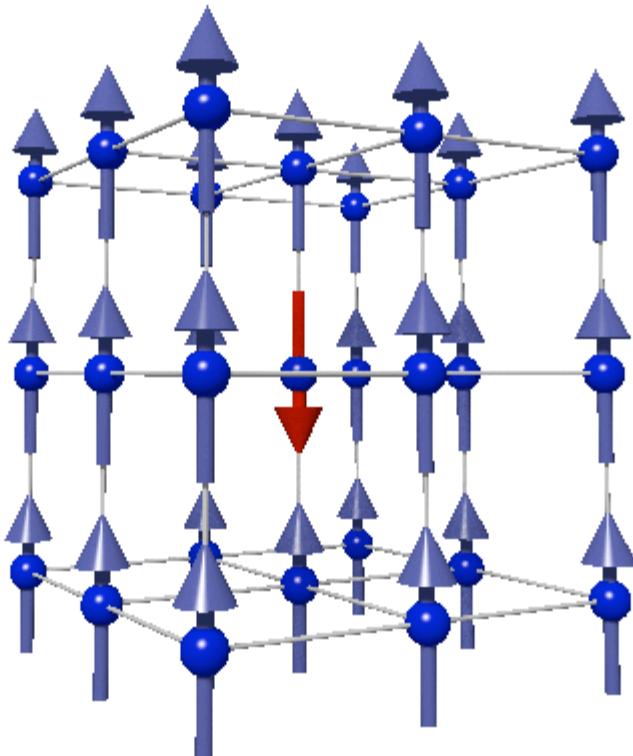
Quasiparticles



excited states \sim excited levels of one block

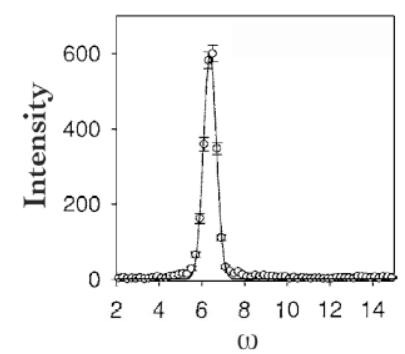
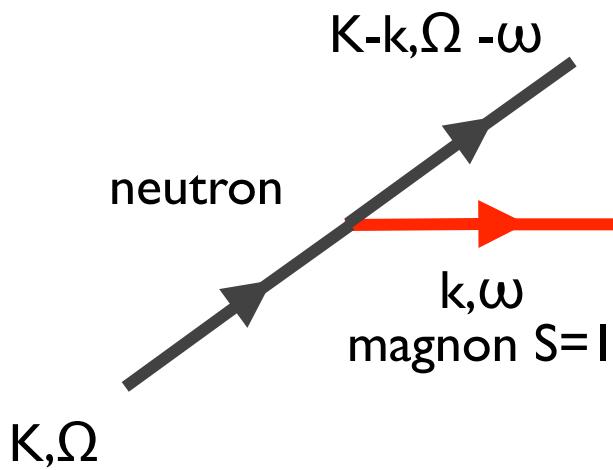
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

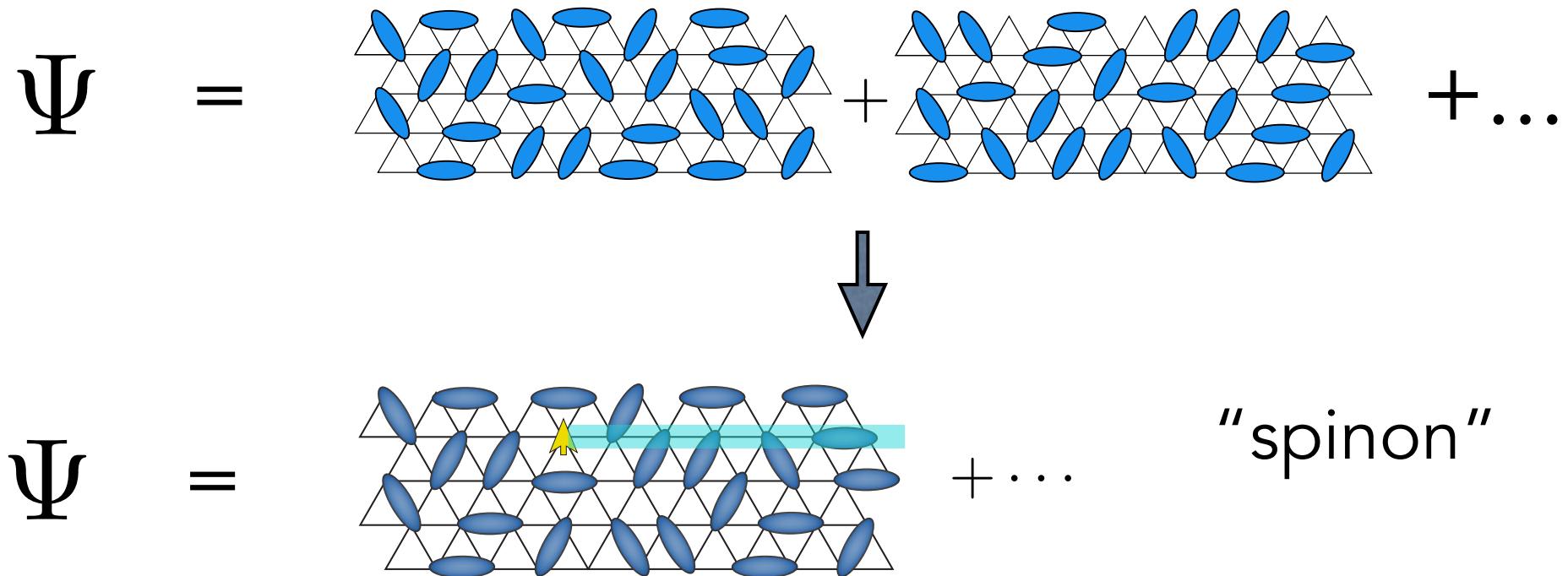
$$|f\rangle = S_k^+ |i\rangle$$



Line shape in Rb_2MnF_4

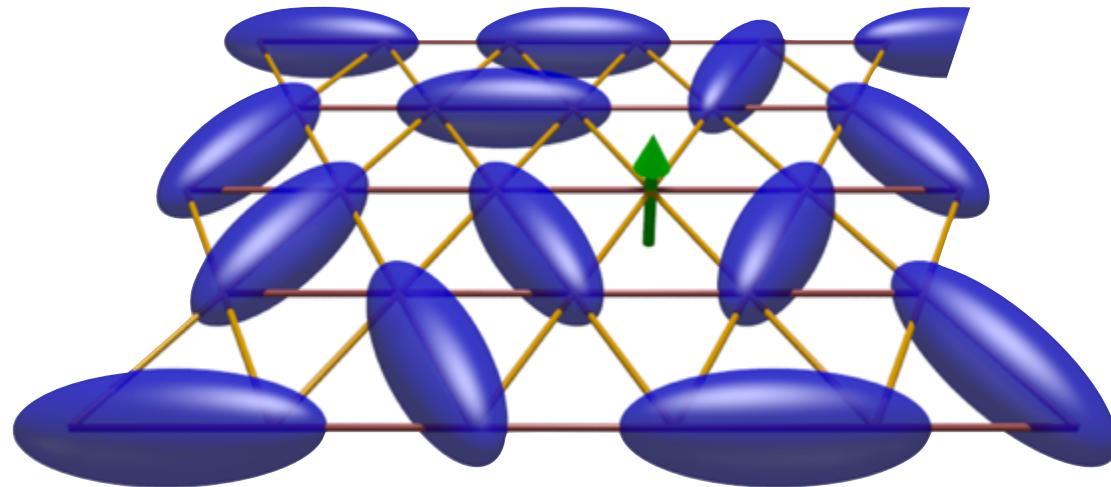
Quantum spin liquid

Entanglement \rightarrow non-local excitation



“quasiparticle” above a non-zero gap

Fractional quantum number

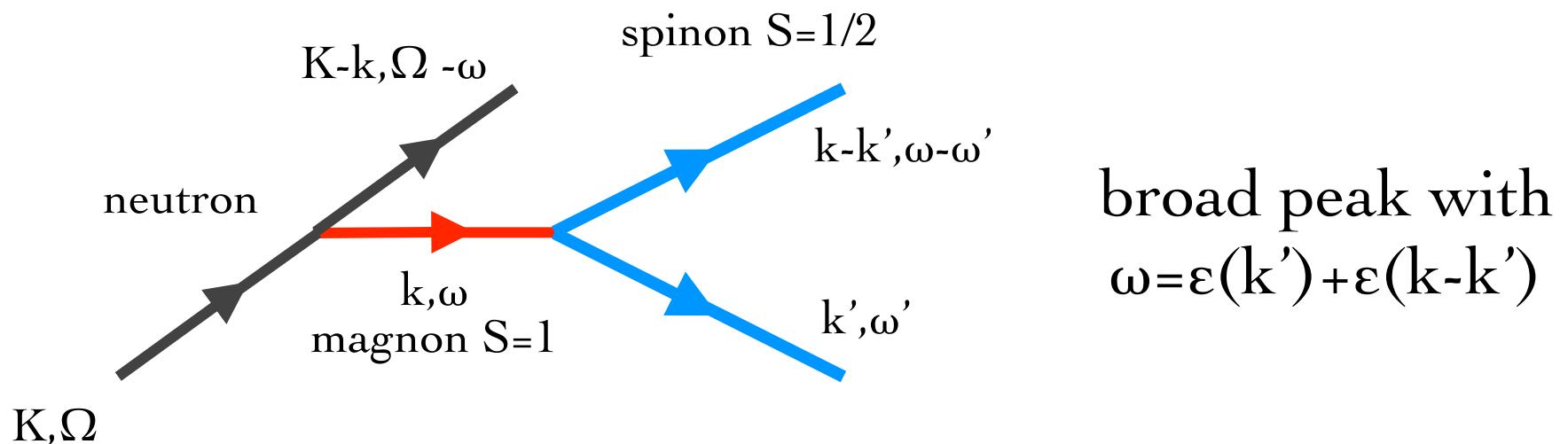


excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins

always created in pairs by any
local operator

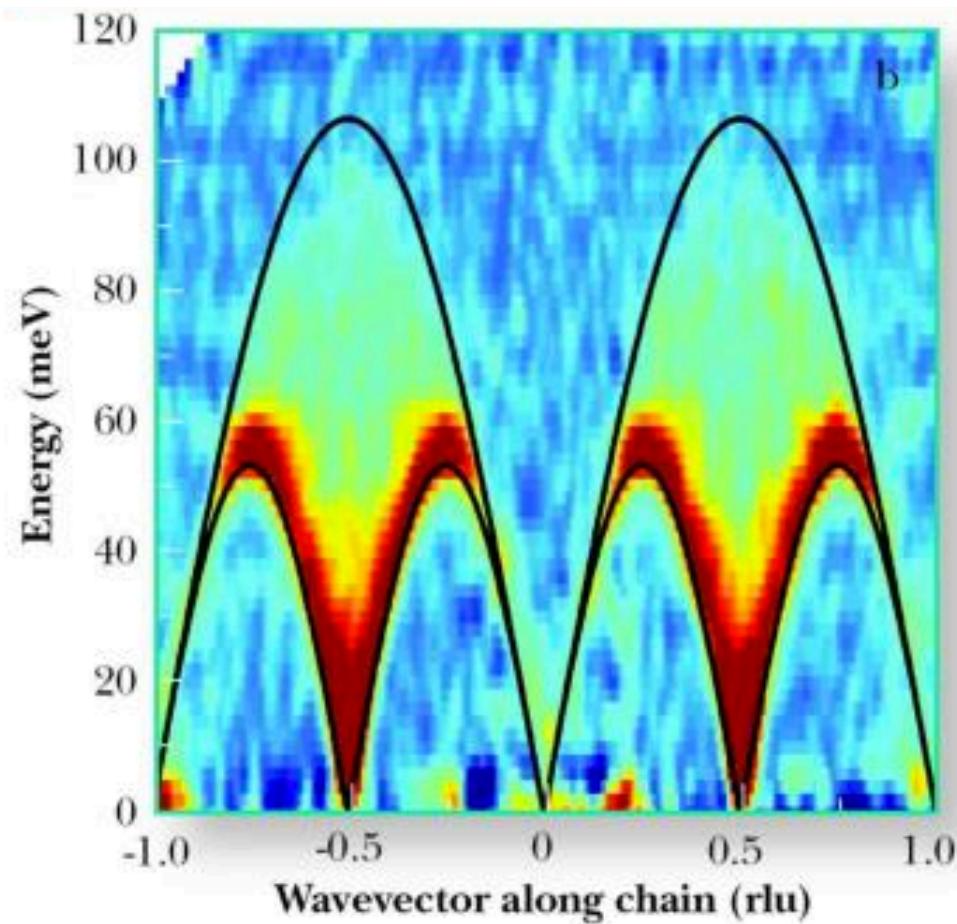
No spin waves

- Magnon is not elementary: decays into two spinons



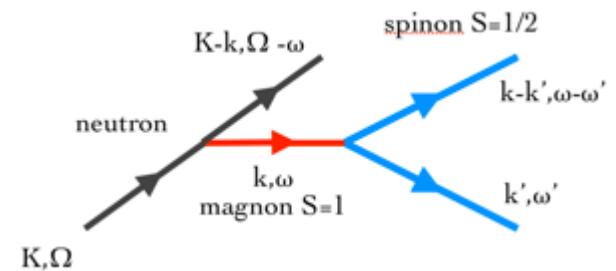
- Sharp peaks should be reduced or absent in the spin structure factor

c.f. One dimension

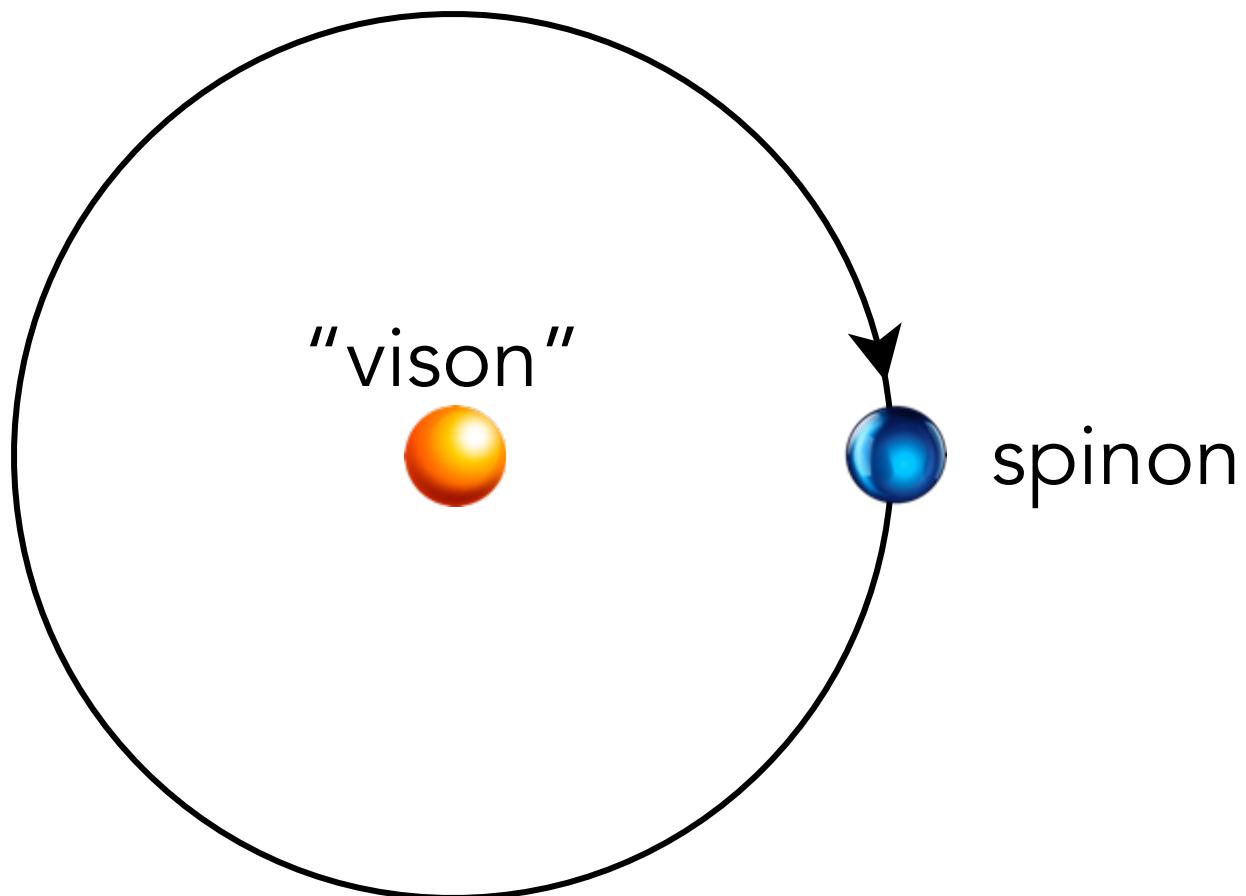


A. Tennant *et al*, 2001

KCuF₃



Anyons



$$\Psi \rightarrow -\Psi$$

"mutual semions"

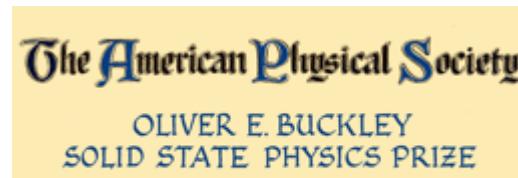


X.-G. Wen



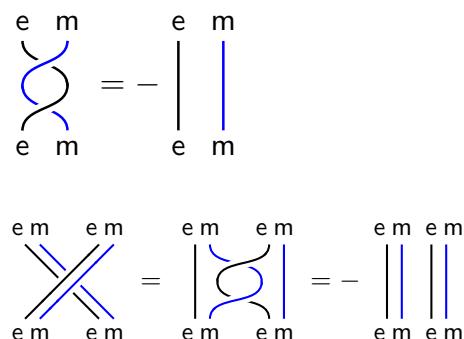
A. Kitaev

Topological phases



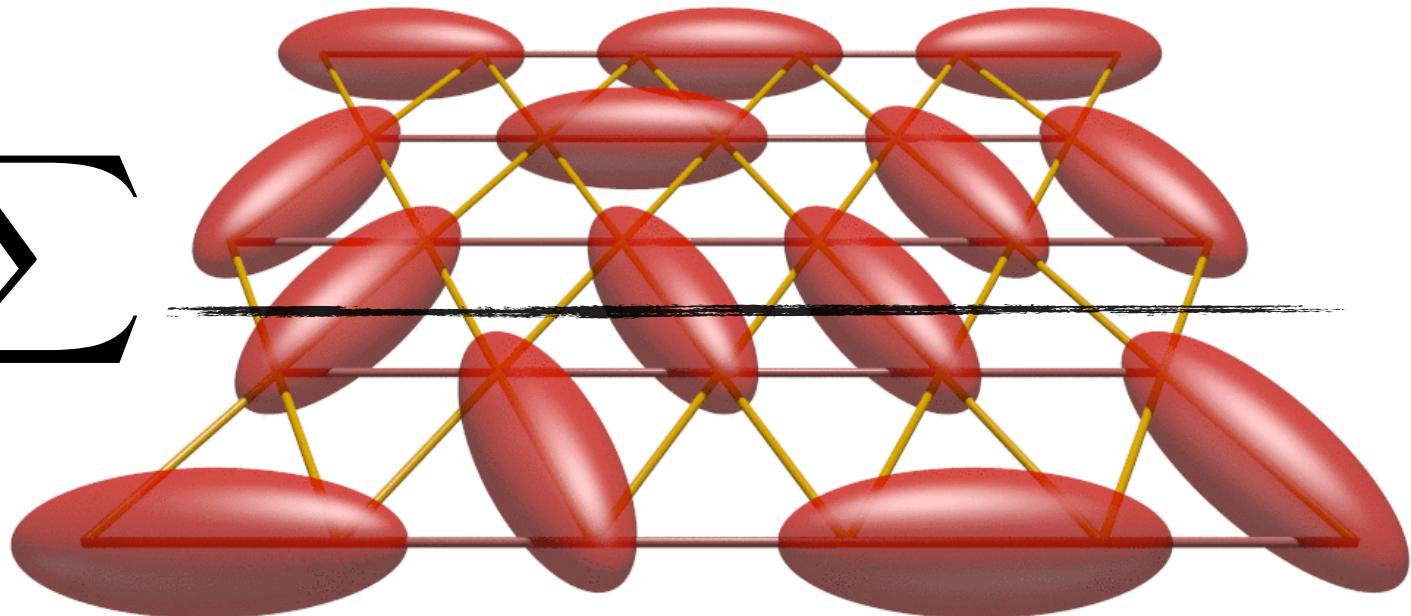
Anderson's RVB state is thus an example of a “topological phase” - the best understood sort of QSL

Understood and classified by anyons and their braiding rules in 2d



Stability

$$\Psi = \sum$$



Robustness arises from topology: a QSL is a stable phase of matter (at T=0)

Quantum spin liquid

$$\Psi = \begin{array}{c} \text{Diagram of a triangular lattice with blue ovals representing spins, showing two different local arrangements of spins.} \end{array} + \dots$$

For ~ 500 spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site ($S=0$)

$$|\Psi_0\rangle = \prod_{k \in FS} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$

“partons”
“spinons”

$$= c_1 \begin{array}{|c|c|c|c|c|c|} \hline & \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \downarrow & \uparrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|c|} \hline & \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \downarrow & \uparrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|c|} \hline & \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow \\ \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Gutzwiller Construction

- Project out any components with empty or doubly occupied sites

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

“partons”
“spinons”

$$= c_1 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + \dots$$

Gutzwiller Construction

- Can build many QSL states by choosing different free fermion states

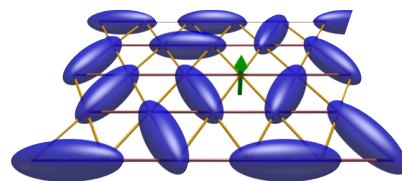
$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

“partons”
“spinons”

$$= c_1 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \cancel{\begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow & \downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array}} + \dots$$

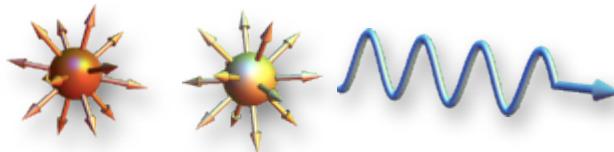
Classes of QSLs

- Topological QSLs



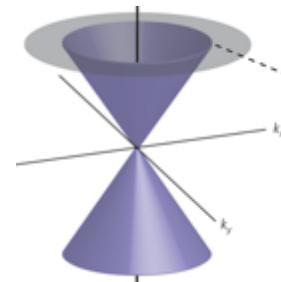
projected
superconductor

- $U(1)$ QSL



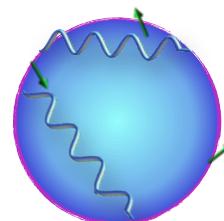
projected 3d band
insulator

- Dirac QSLs



projected
graphene

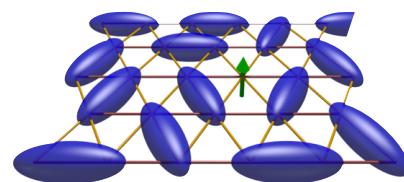
- Spinon Fermi surface



projected
metal

Classes of QSLs

- Topological QSLs



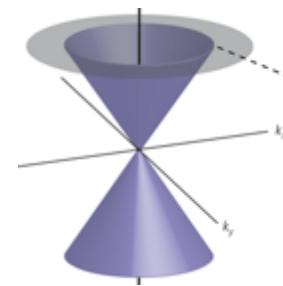
anyonic
spinons

- $U(1)$ QSL



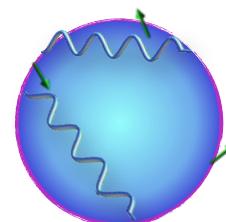
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



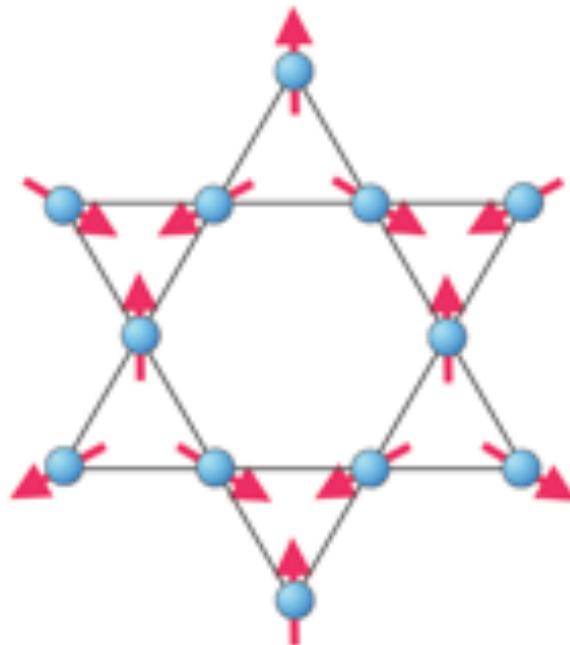
non-Fermi
liquid "spin
metal"

Strange stuff



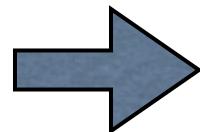
where do we find it?

Kagomé antiferromagnet



$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \dots$$

Very large classical
degeneracy

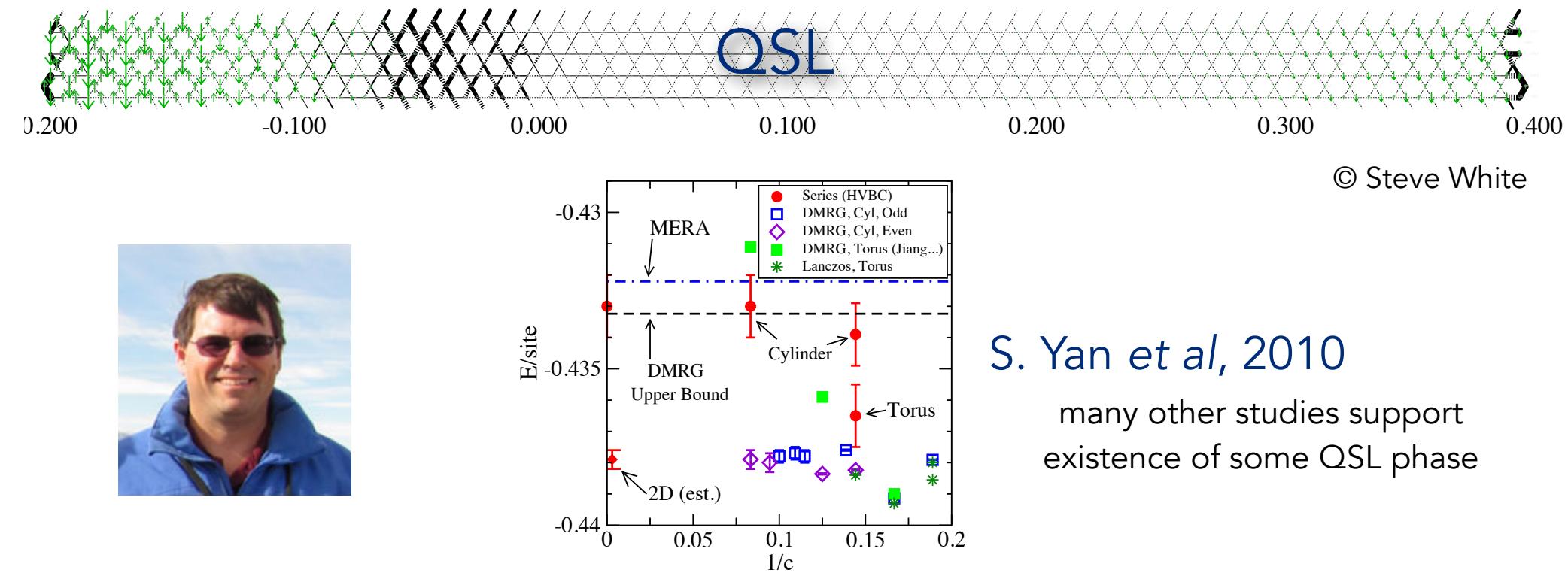


likely to be a QSL

V. Elser, 1989 + many many others

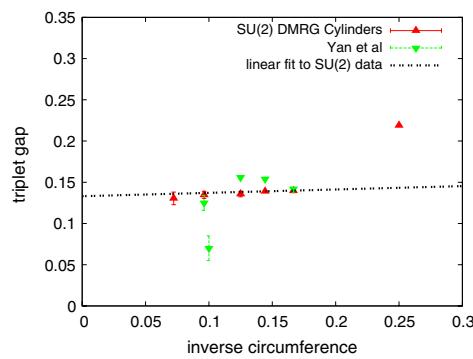
$S=1/2$ kagomé AF

- Rather definitive evidence for QSL by DMRG



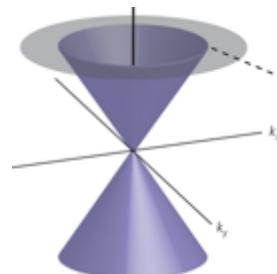
Theory

- What kind of QSL?



S. Depenbrock *et al*, 2012

gapped,
topological QSL

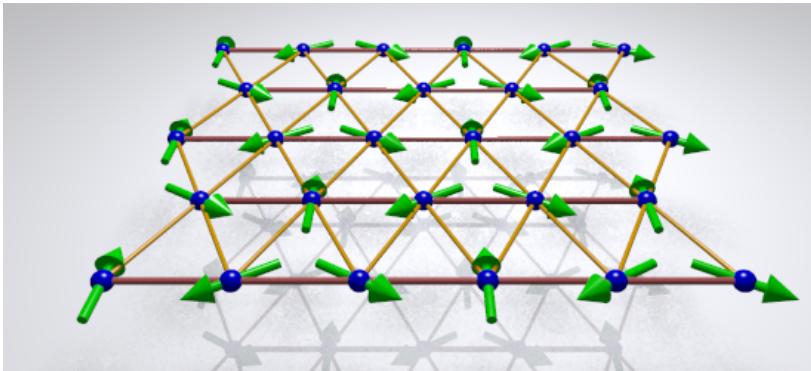


Y. Ran *et al*, 2007
F. Becca...
Y.C. He *et al*, 2016

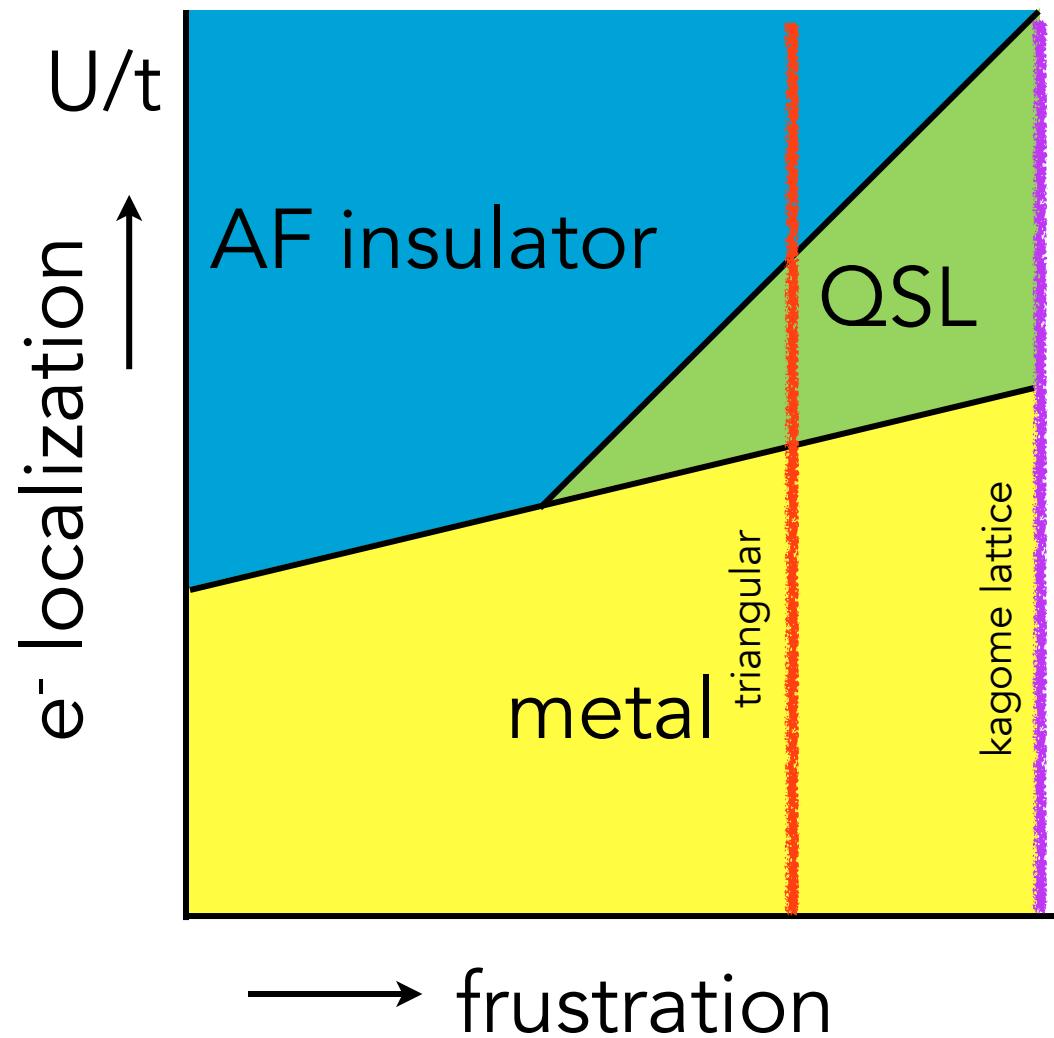
gapless
Dirac QSL

+ various other
proposals with
weaker
quantitative
support

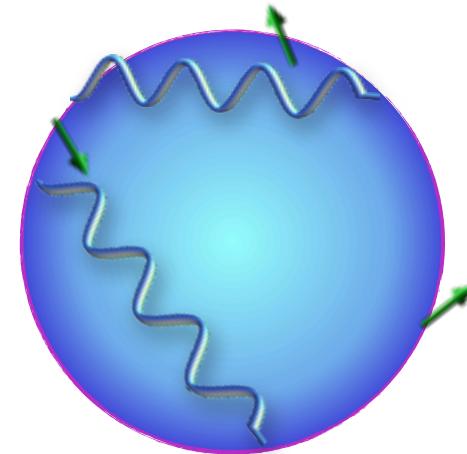
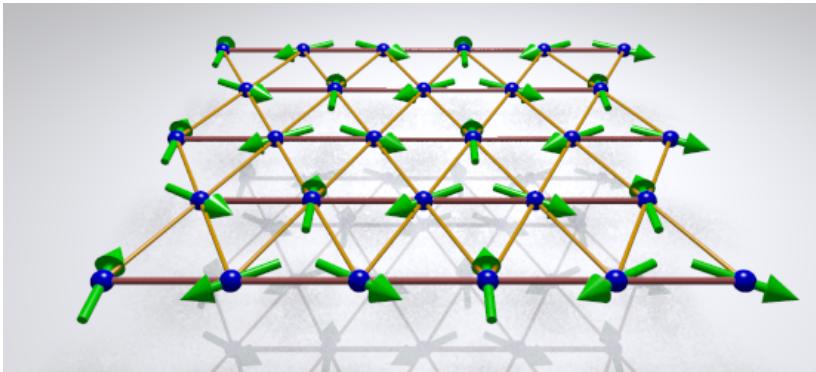
Triangular lattice w/ ring exchange



- Motrunich (2005): ring exchange stabilizes a spin liquid



Triangular lattice w/ ring exchange



- Motrunich (2005): ring exchange stabilizes a spin liquid

- Motrunich, Lee/Lee: spin liquid state favored by ring exchange is the “spinon Fermi sea” state

SOC triangular

Heavy elements:

highly localized electrons, strong
spin-orbit coupling

$$H = \sum_{\langle ij \rangle} \left[J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_z S_i^z S_j^z \right. \\ \left. + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \right. \\ \left. + i J_{\pm z} (\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j)) \right]$$

XXZ

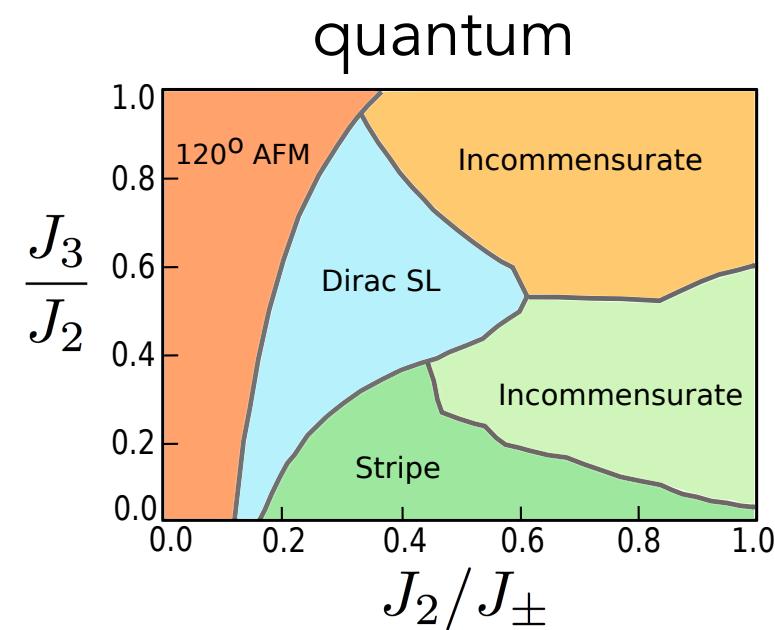
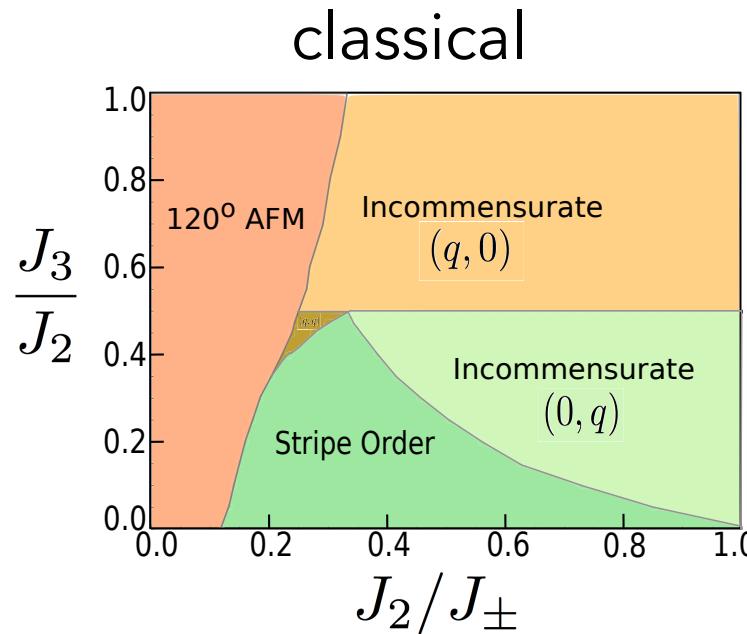
bond-dependent
couplings

Y. Li *et al*, 2015

SOC triangular

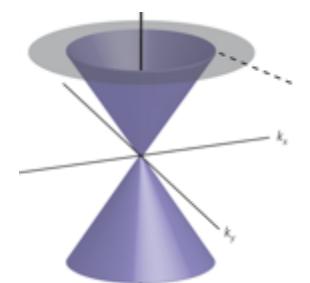


QSLs versus magnetic order



Some window exists for Dirac QSL

J. Iaconis, C. Liu, G. Halász, LB, 2017





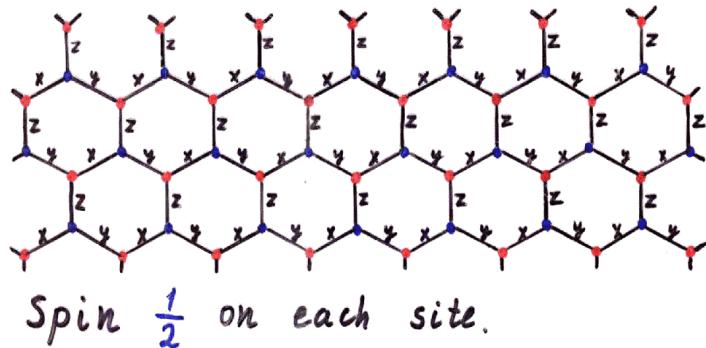
Kitaev model

Kitaev's honeycomb model

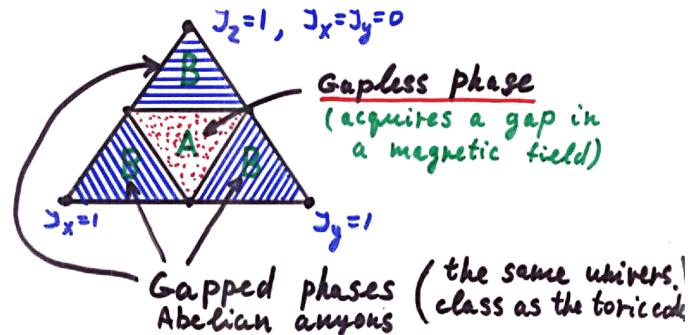
$$H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$$

KITP, 2003

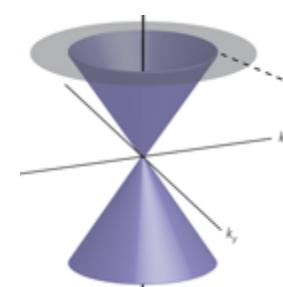
1. The model



Phase diagram



exact parton construction
- spinon is a gapless
Majorana fermion



How to probe QSLs?

Two main characteristics:

- Massive entanglement
 - Almost no experiments known to probe this.
- Fractional/non-local excitations
 - Probed by most low energy response measurements. Challenge is to distinguish the fractional/non-local nature.

A rough guide to experiments on QSLs

Does it order?

- NMR line splitting
- muSR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/x-ray

Delocalized excitations?

- thermal conductivity
- INS

Is there a gap?

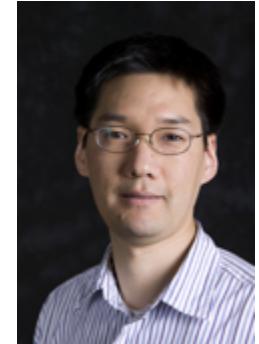
- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of χ

Structure of excitations?

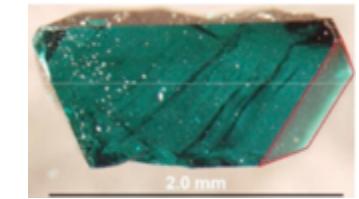
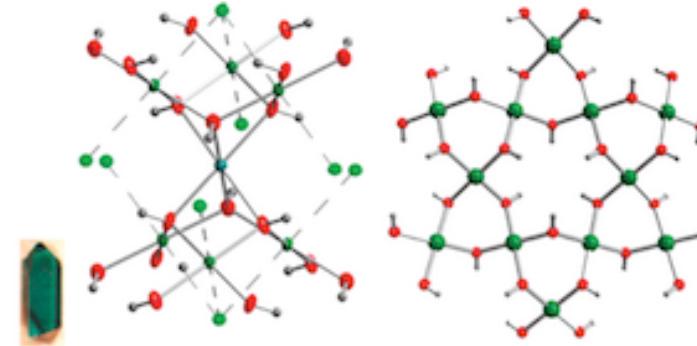
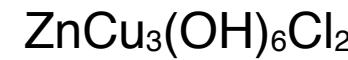
- $E(k)$ from INS, RIXS
- optics, Raman

Exotica

- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects



Scattering

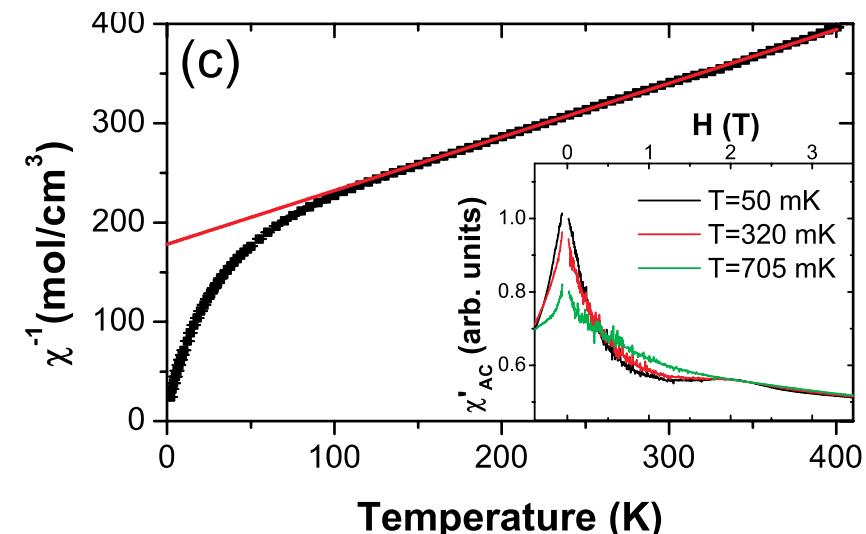


Herbertsmithite

Heisenberg-like
with $J \sim 200\text{K}$

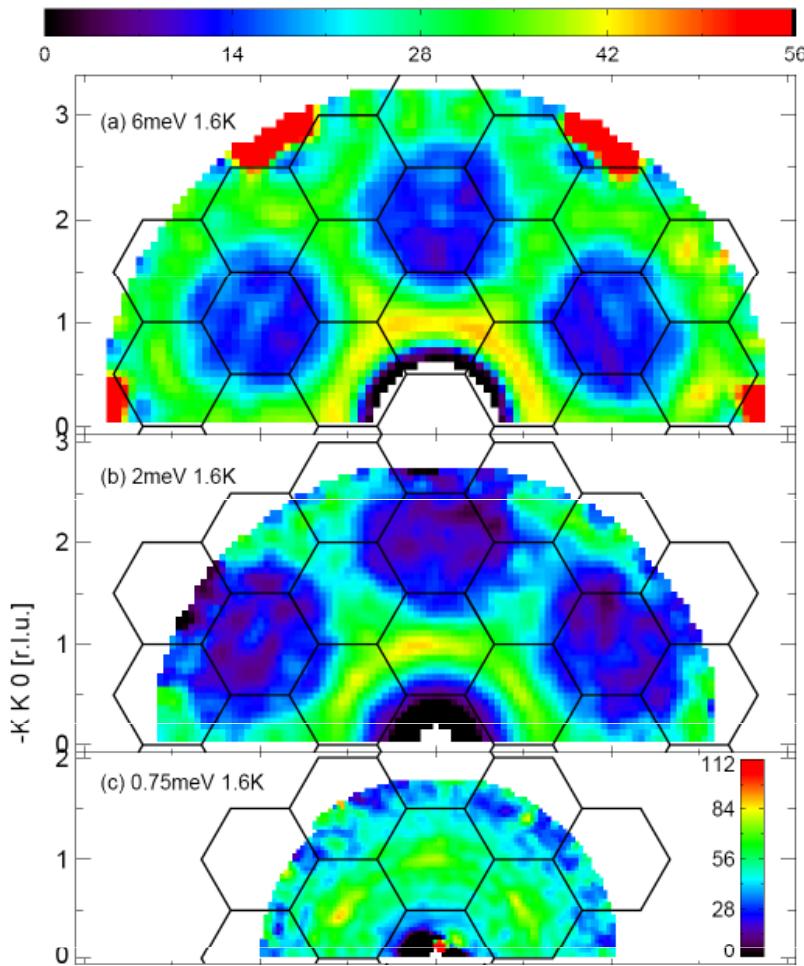
no order down to

50mK

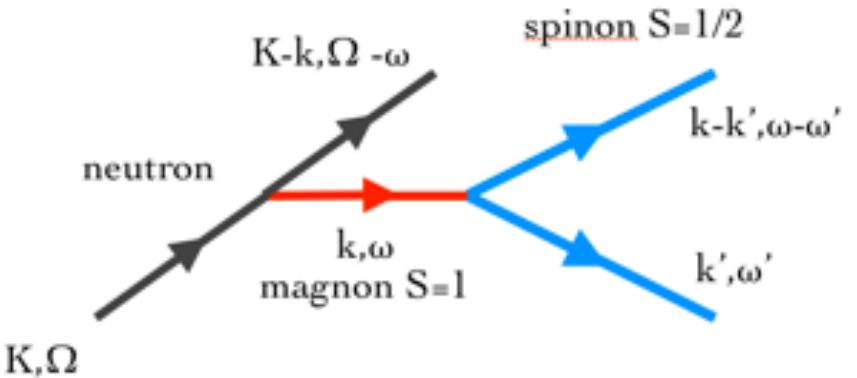


Helton et al, 2007

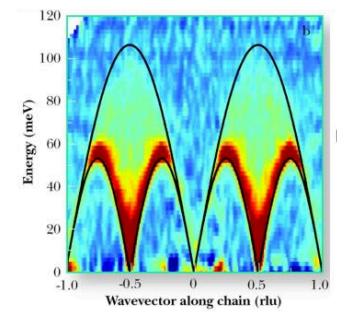
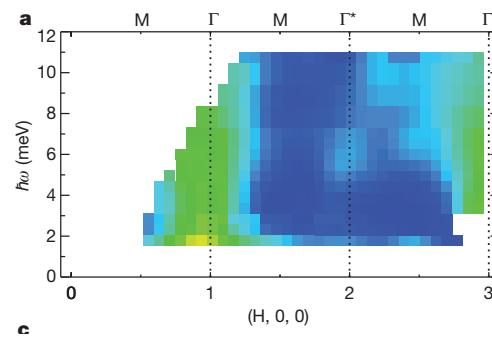
Scattering



T-H Han et al, 2012

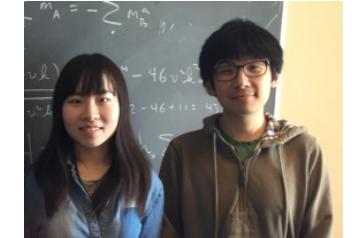


continuum scattering
expected
...but probably with more
structure?

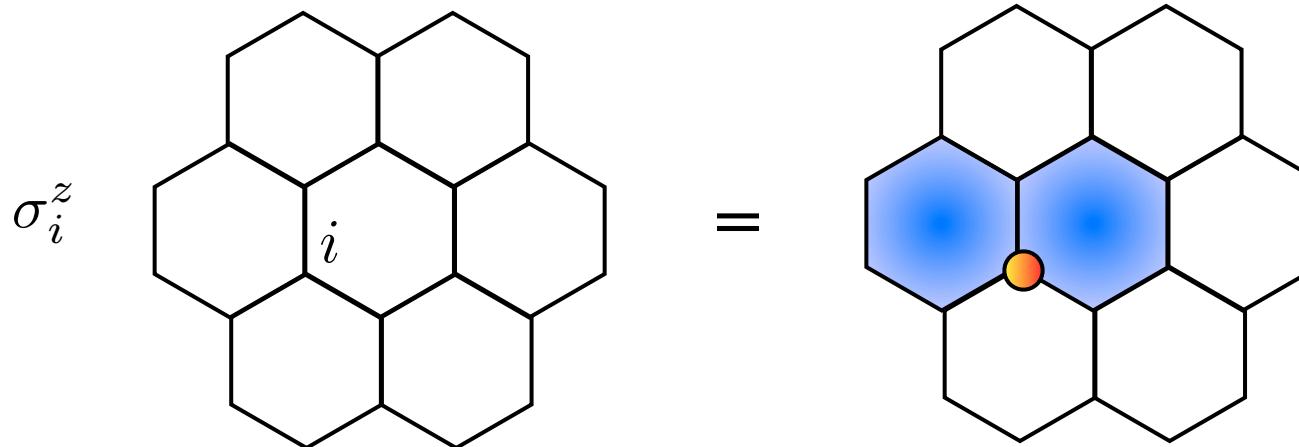


VS

Kitaev QSL



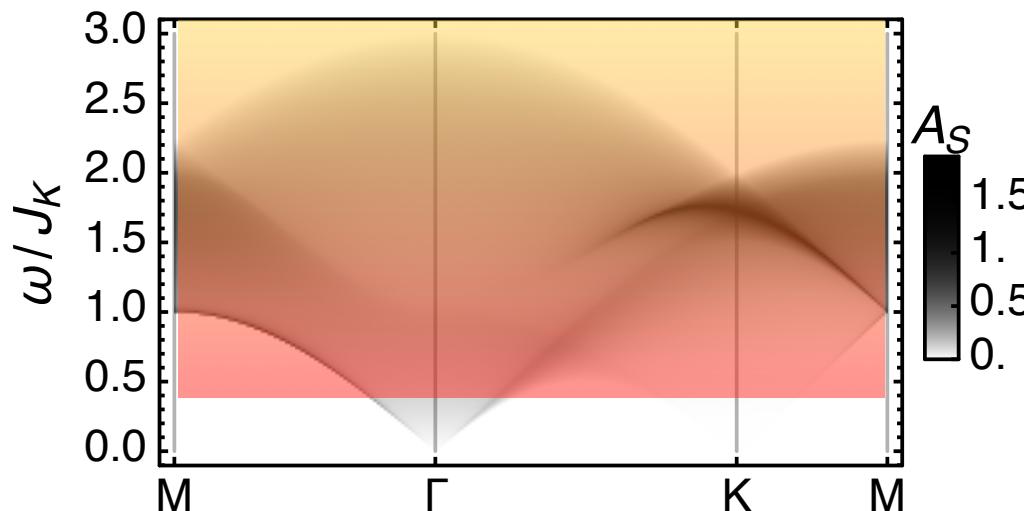
Spin flip produces a free Majorana fermion and two immobile fluxes



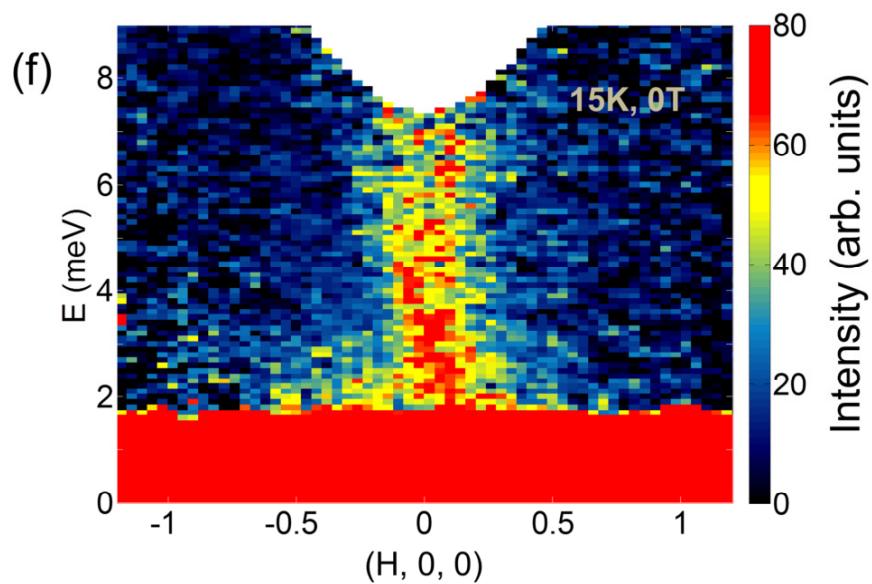
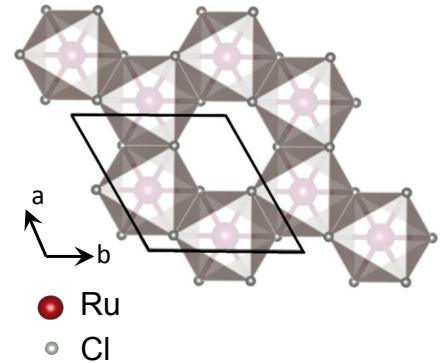
J. Knolle et al, 2014

Xueyang Song, Yi-Zhuang You + LB, 2016

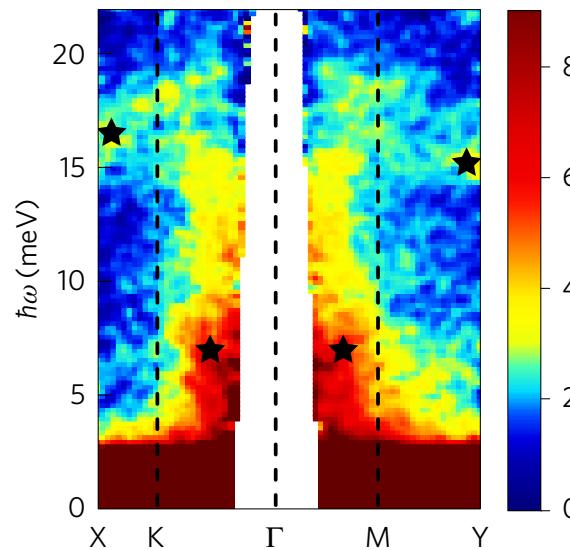
dynamical spin
correlations in the
Kitaev QSL



alpha-RuCl₃



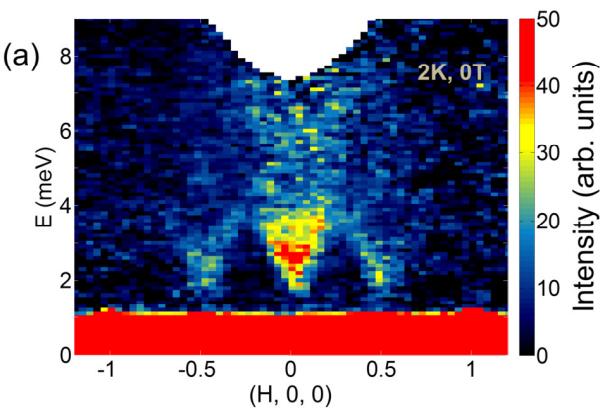
A. Banerjee *et al*, 2017



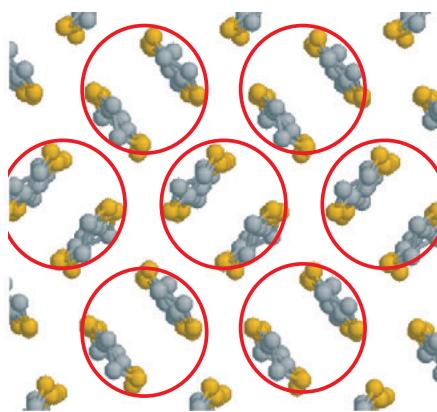
S.-H. Do *et al*, 2017

“column” of scattering suggested to be related to Majorana spinons

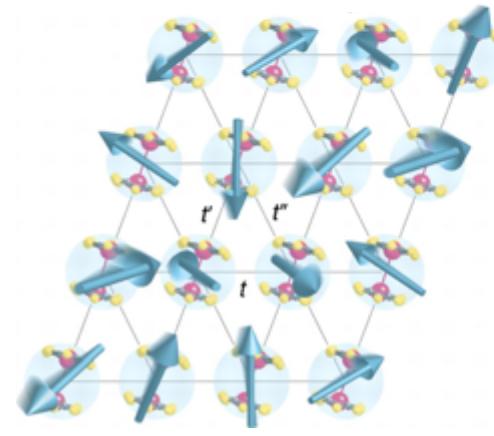
c.f.



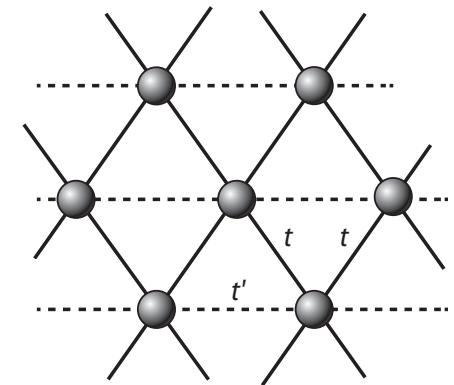
Low energy signatures: triangular organics



κ -(ET)₂X



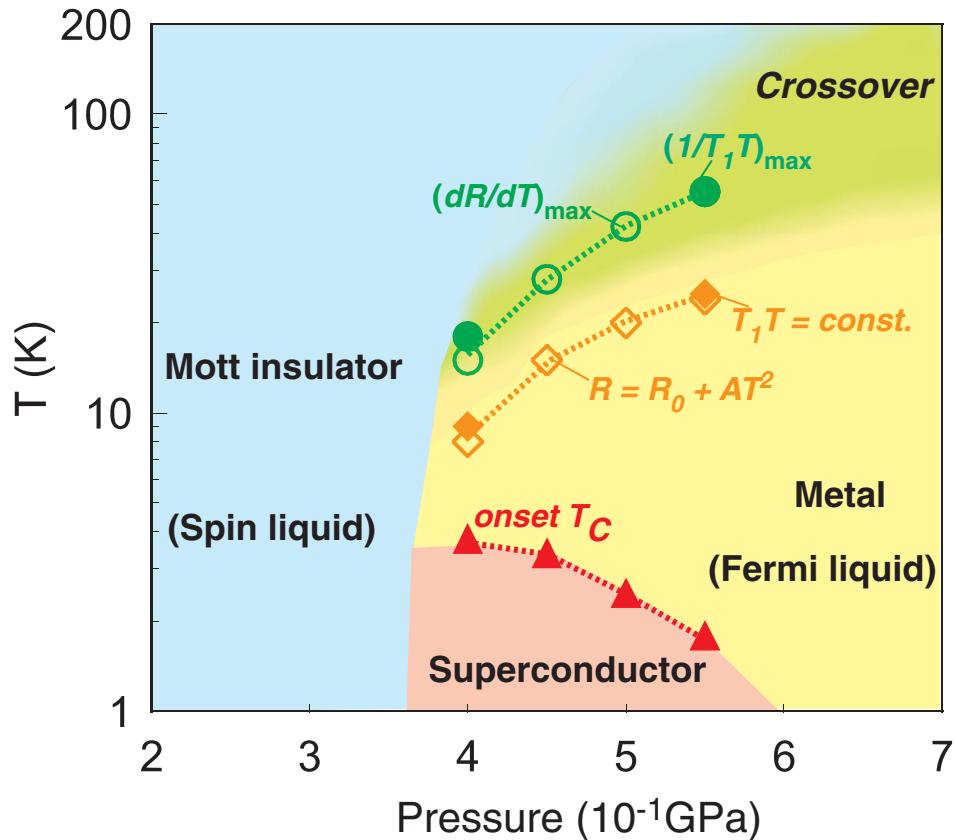
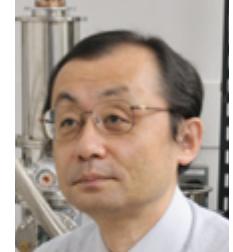
β' -Pd(dmit)₂



- Molecular materials which behave as effective triangular lattice $S=1/2$ antiferromagnets with $J \sim 250\text{K}$
- significant charge fluctuations

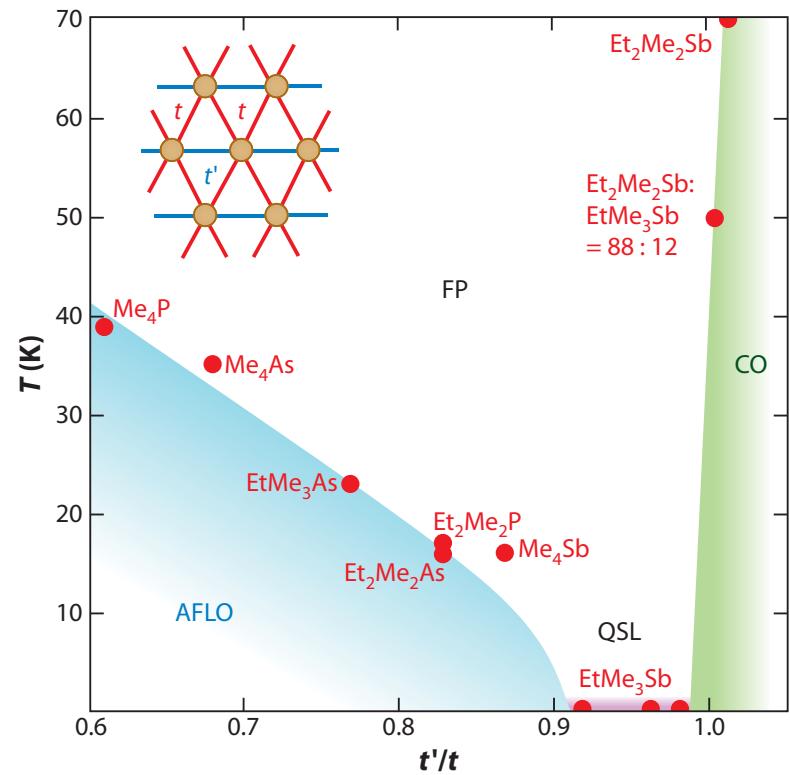


triangular organics



κ -(ET)₂Cu₂(CN)₃

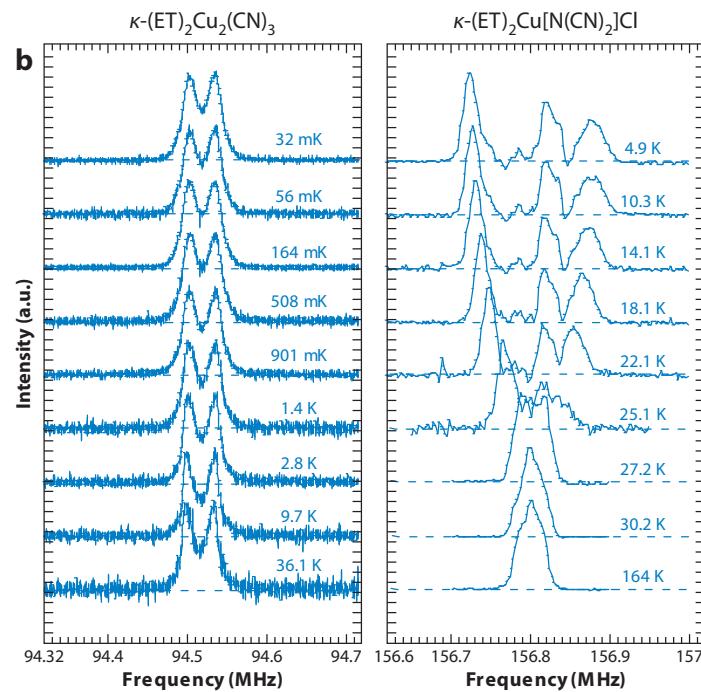
K. Kanoda group (2003-)



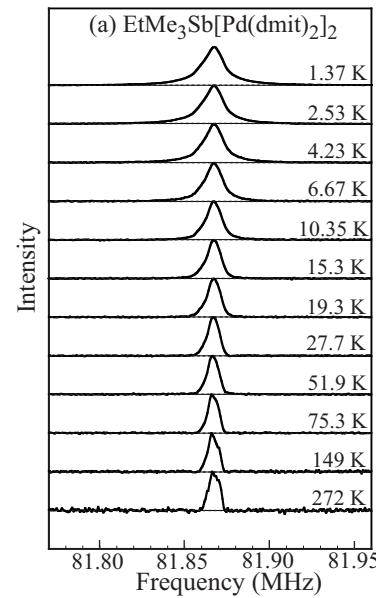
β' -Pd(dmit)₂

R. Kato group (2008-)

NMR lineshapes



κ -(ET)₂Cu₂(CN)₃



β' -Pd(dmit)₂

Y. Shimizu ^1H NMR
et al., 2003

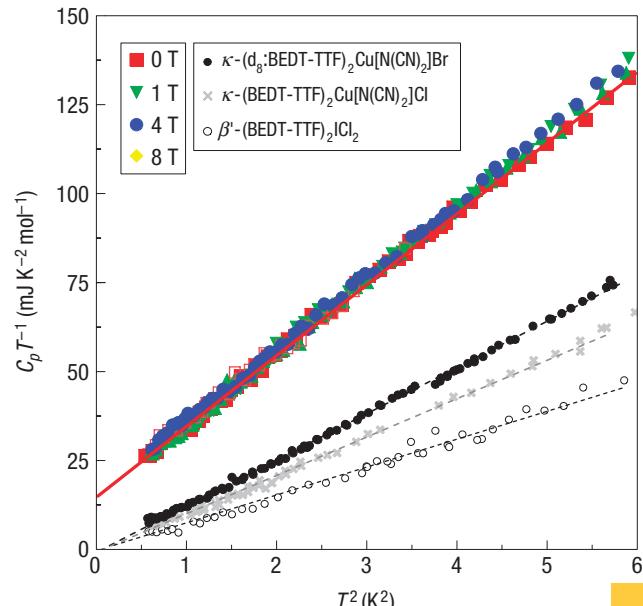
T. Itou *et al.*,
2008, 2010

¹³Cs NMR

Evidence for lack of static moments: $f > 1000!$

Specific Heat

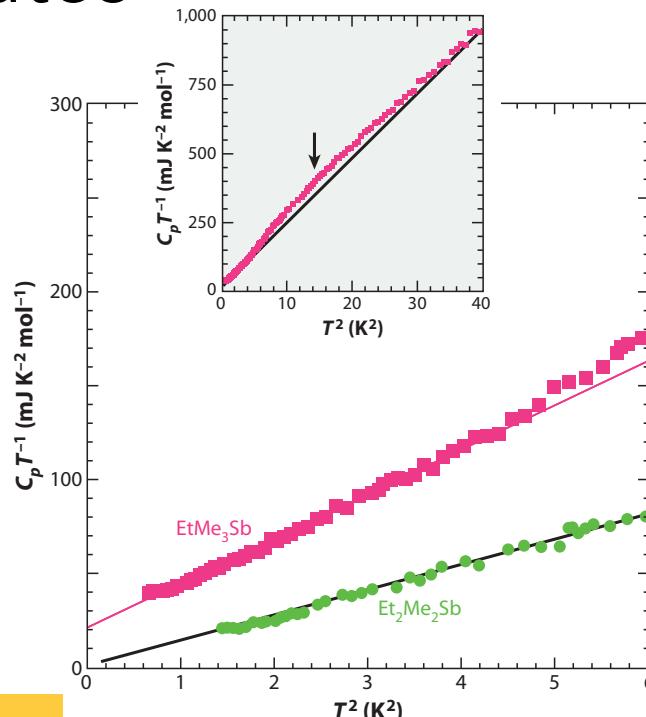
- $C \sim \gamma T$ indicates gapless behavior with constant density of states



$$\gamma_{\text{Cu}} \sim 0.7 !!$$

κ - $(\text{ET})_2\text{Cu}_2(\text{CN})_3$

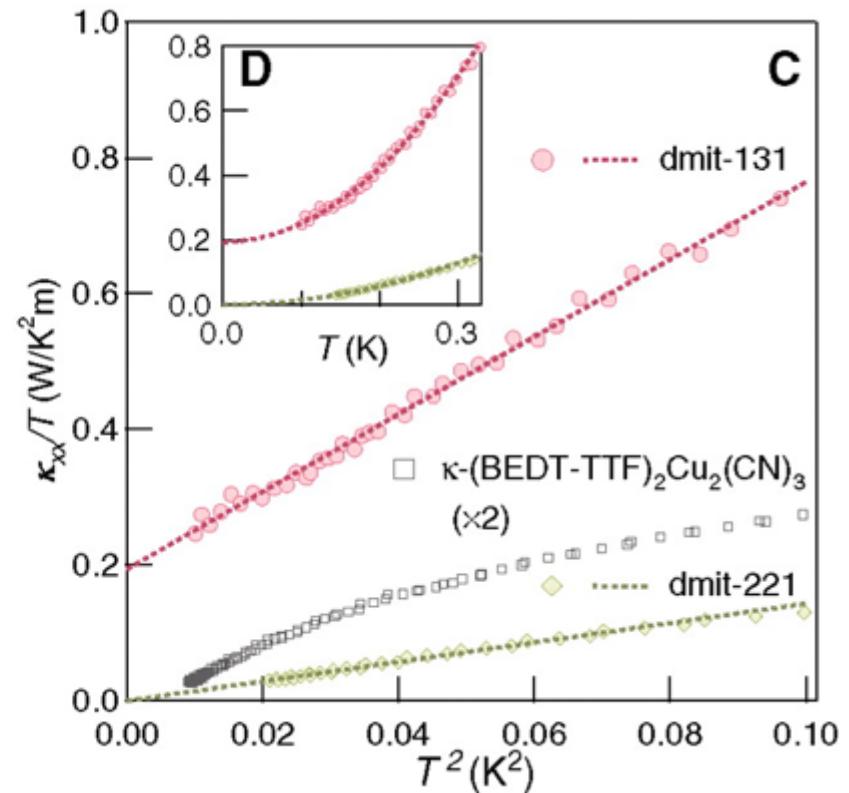
S. Yamashita *et al*, 2008



β' - $\text{Pd}(\text{dmit})_2$

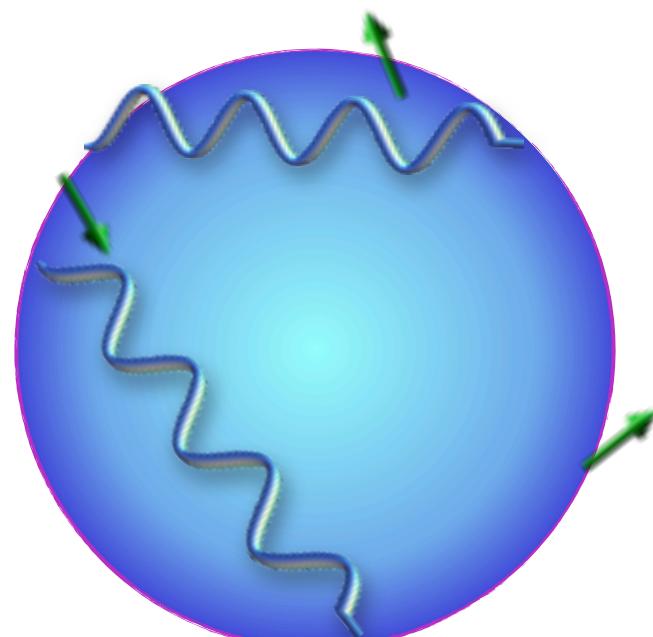
Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit
- Estimate for a *metal* would correspond to a mean free path $l \sim 1 \mu\text{m} \approx 1000 \text{ \AA}$!

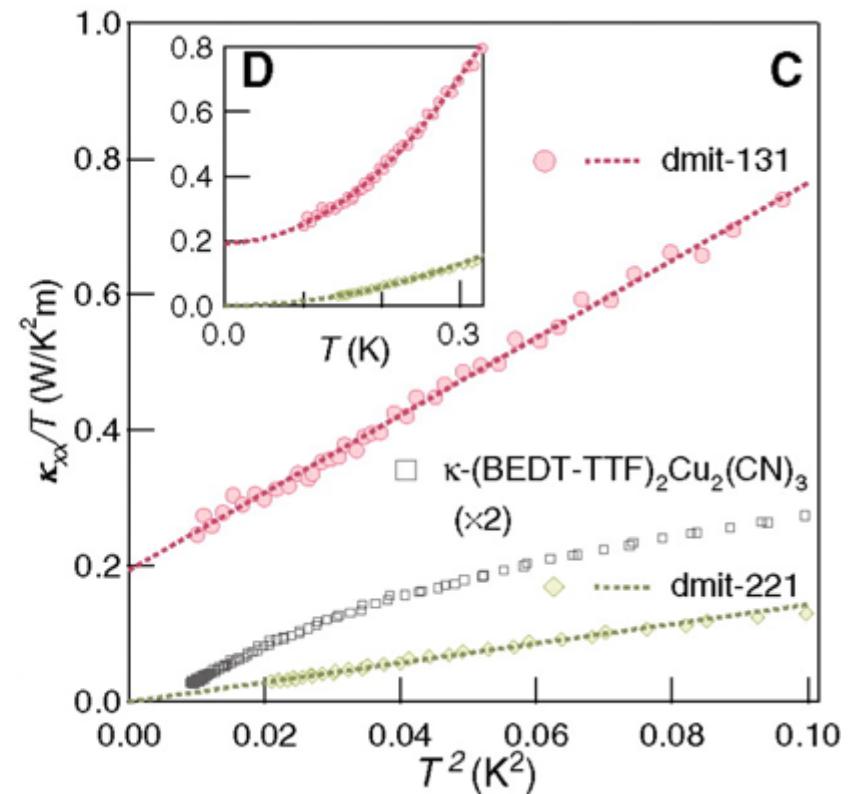


M. Yamashita *et al*, 2010

Thermal conductivity



?



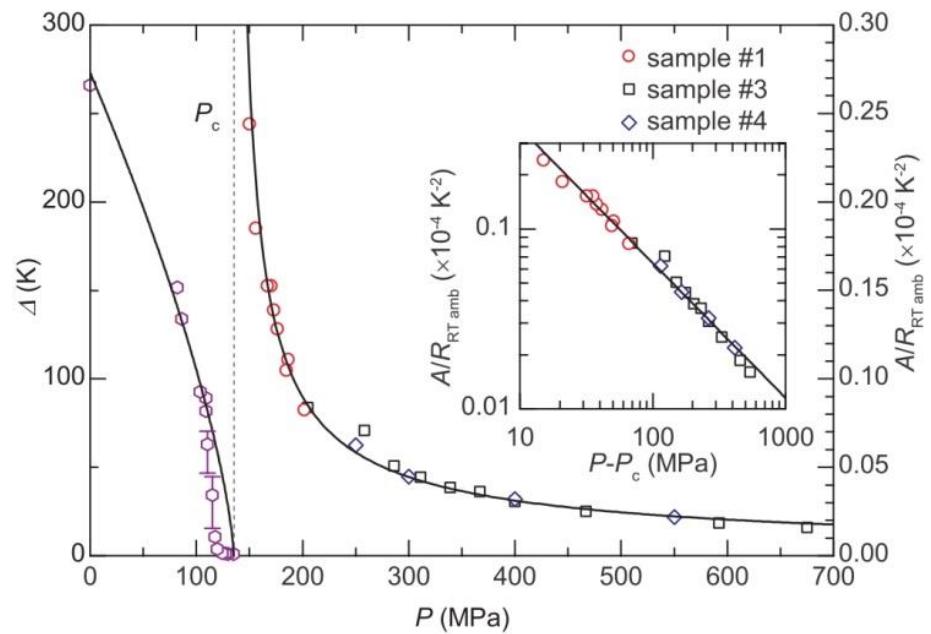
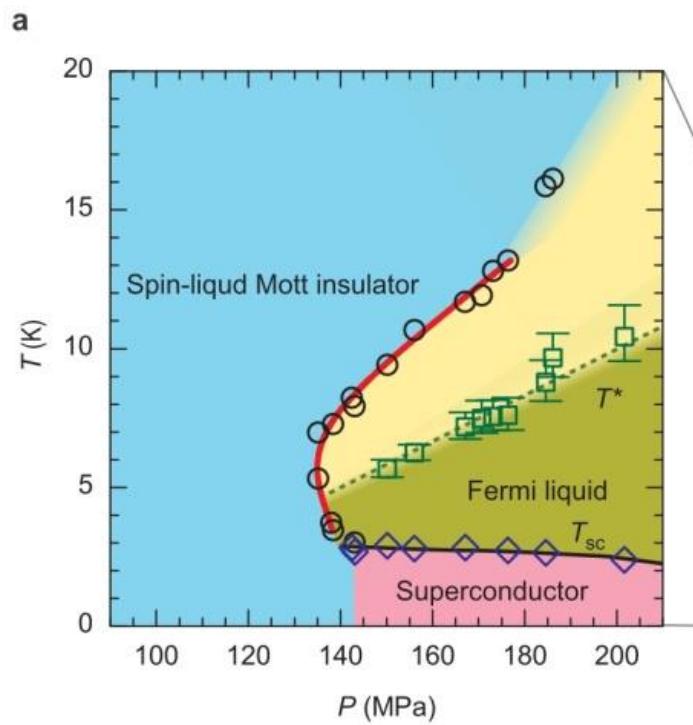
M. Yamashita *et al.*, 2010

Spinon Fermi surface

- How could we firm this up?

- Spinons should be *confined* to 2d. Can we see evidence of this?
 - e.g. $\kappa_c \ll \kappa_{ab}$ (c.f. Y. Werman *et al*, 2017)
- See signs of \mathbf{k}_F ?
 - quantum oscillations, RKKY
- Possible quantization effects in small systems
- Observe *conversion* of spinons to electrons in adjacent metal (c.f. T. Senthil, 2008)

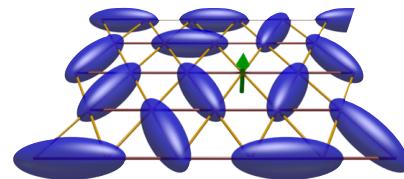
Spinons → Electrons?



T. Furukawa *et al*, 2017

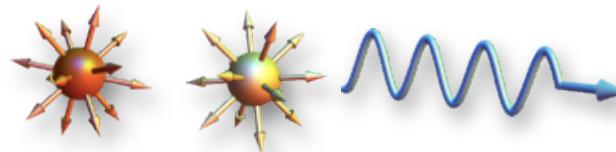
Other QSLs in organics?

- Topological QSLs



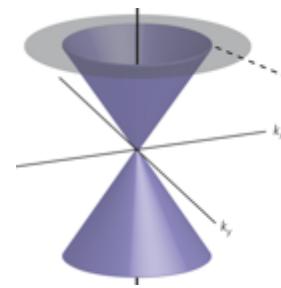
anyonic
spinons

- $U(1)$ QSL



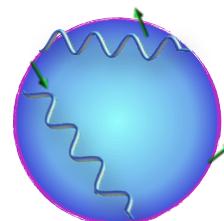
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



non-Fermi
liquid “spin
metal”

Thanks for your attention



References here:

<https://spinsandelectrons.com/>

<https://spinsandelectrons.com/pedagogy/>

