

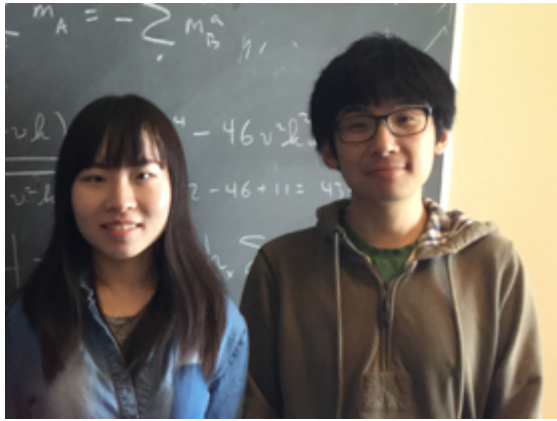
Quantum Spin Liquids

A photograph of a volcanic crater with a turquoise lake in the center. Three red arrows point towards the lake from the surrounding slopes. The text "Quantum Spin Liquids" is overlaid in white.

Leon Balents, KITP

ISCOM2017, Miyagi

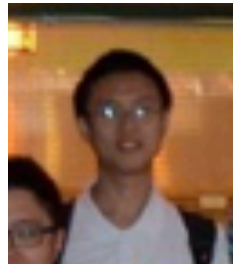
Collaborators (whose work I'll mention)



Xue-Yang Song
Yi-Zhuang You



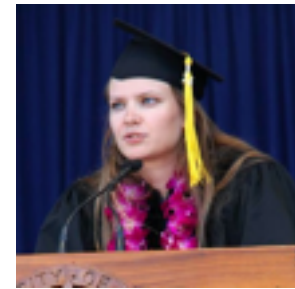
Gábor Halász



Chunxiao Liu



Jason Iaconis



Lucile Savary



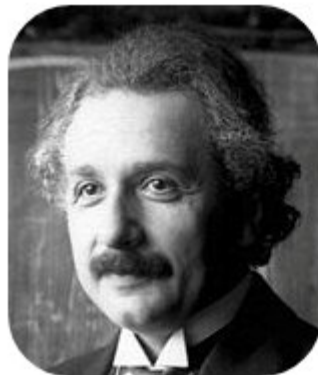
Xiao Chen

Quantum non-locality

EPR $|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$



??where is the information??



A. Einstein

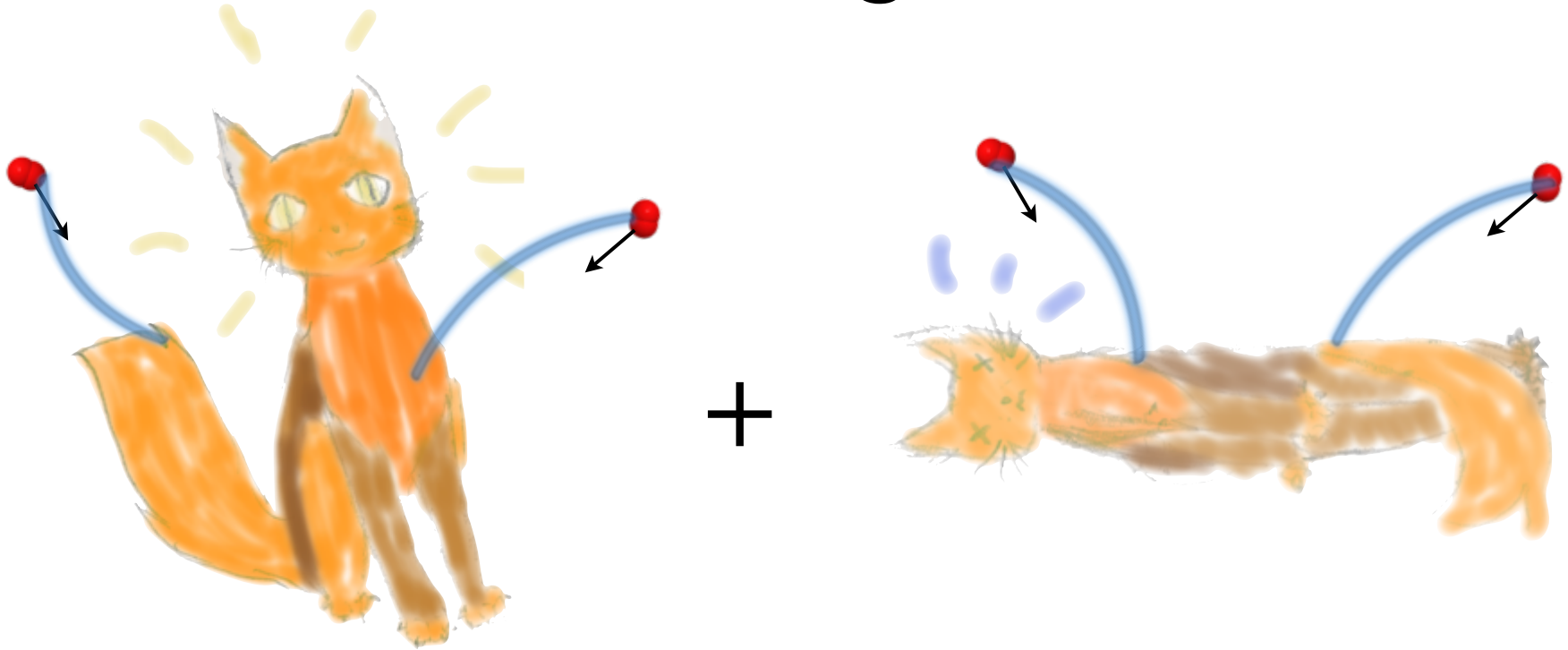


B. Podolsky



N. Rosen

Schrödinger's Cat



UNSTABLE to decoherence - uncontrolled entanglement with the environment



Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Psi = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$

The diagram shows two triangular lattices of blue ovals. The first lattice has ovals oriented in a regular, repeating pattern. The second lattice has ovals oriented in a different, also regular, repeating pattern. The two lattices are separated by a plus sign, and the entire expression is followed by a plus sign and an ellipsis.

Resonating **V**alence **B**ond state

Strange Stuff



Phil Anderson, 1973

a “quantum liquid” of spins

$$\text{blue oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Psi = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$

The diagrams show a triangular lattice of blue ovals representing spin states. In the first diagram, the ovals are arranged in a regular pattern. In the second diagram, the ovals are shifted, representing a different spin configuration. The ellipsis indicates that the wavefunction Ψ is a superposition of many such states.

Resonating **V**alence **B**ond state



Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties



- Measurements far away do not affect one another
- From local measurements we can deduce the global state

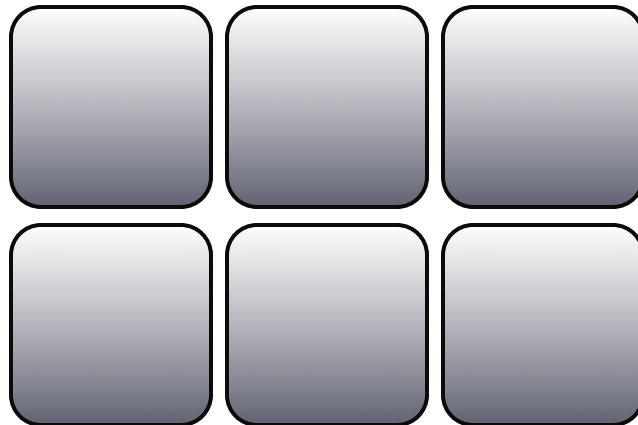
Ordinary (local) Matter

Hamiltonian is local

$$H = \sum_{\mathbf{x}} \mathcal{H}(x) \quad \mathcal{H}(x) \text{ has local support near } x$$

Ground state is "essentially"
a product state

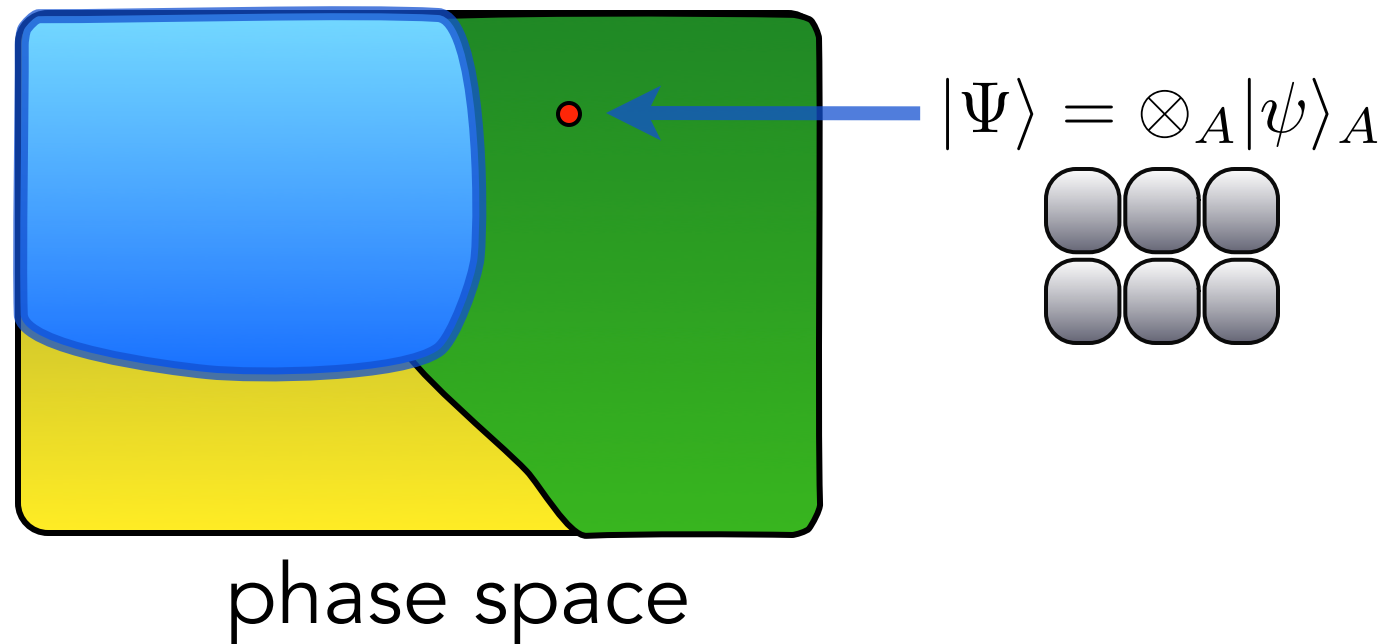
$$|\Psi\rangle = \otimes_A |\psi\rangle_A$$



no entanglement
between blocks

"Essentially" a product state?

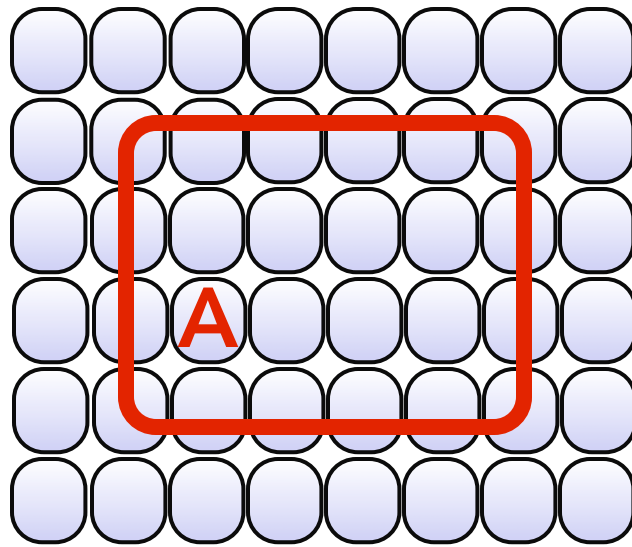
- Adiabatic continuity



n.b. This is not true for gapless fermi systems

“Essentially” a product state?

- Entanglement scaling



$$\rho_A = \text{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\text{Tr}_A (\rho_A \ln \rho_A)$$

$$S(A) \sim \sigma L^{d-1} \quad \text{area law}$$

satisfied with exponentially small corrections

Best example: ordered magnet

Hamiltonian

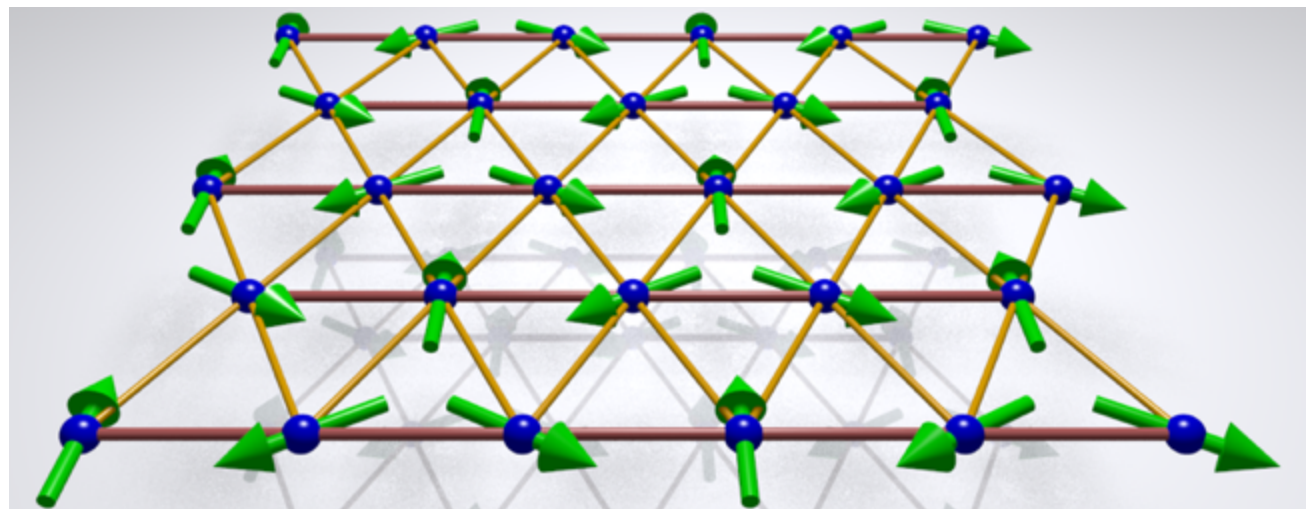
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-
range: local

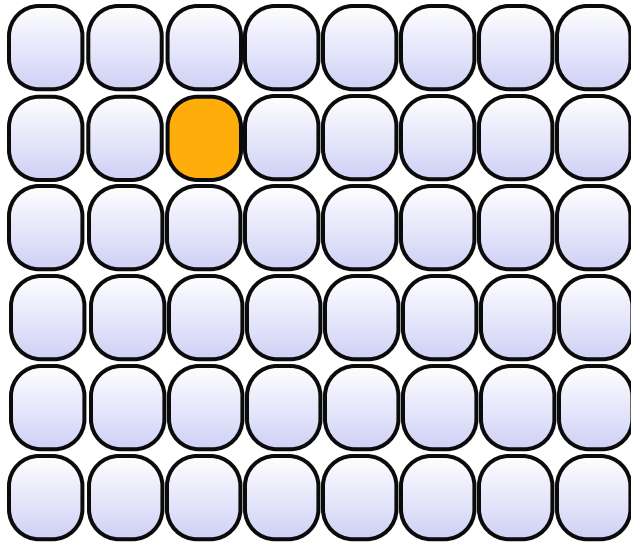
ordered state

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{n}_i = +S\rangle$$

block is a single
spin



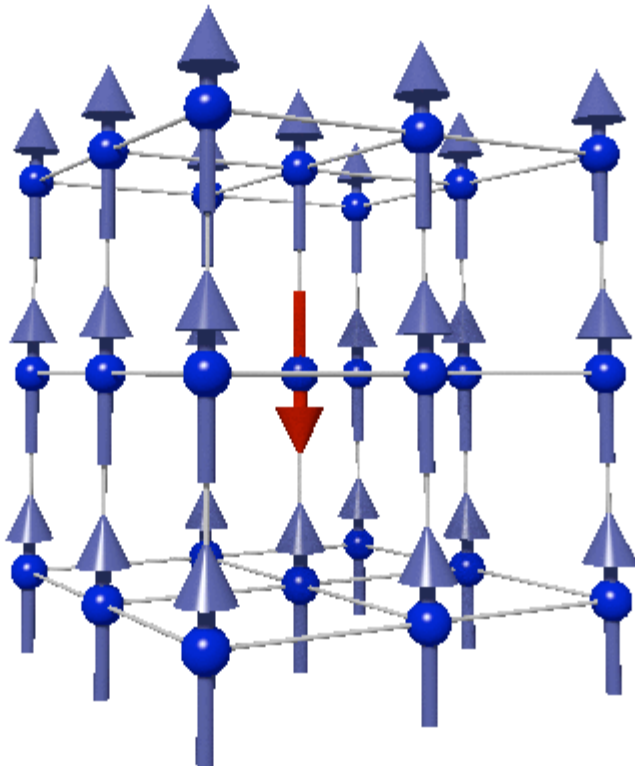
Quasiparticles



excited states \sim excited levels of one block

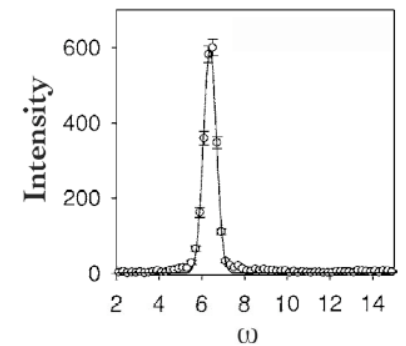
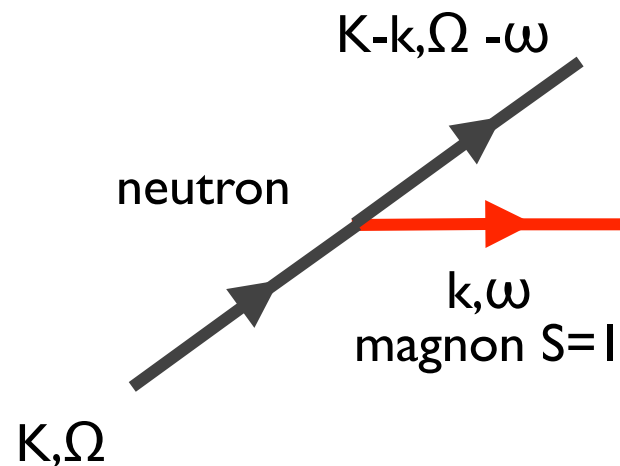
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

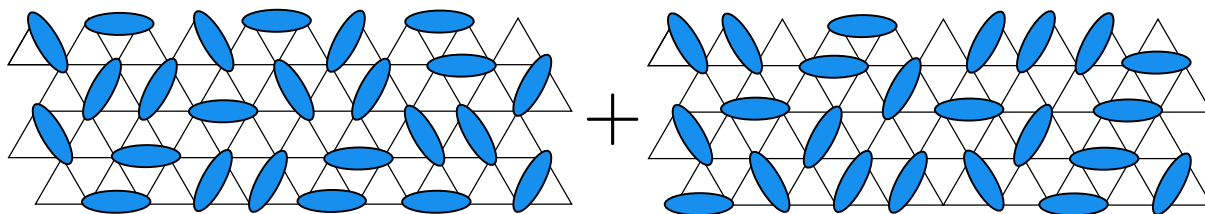
$$|f\rangle = S_k^+ |i\rangle$$



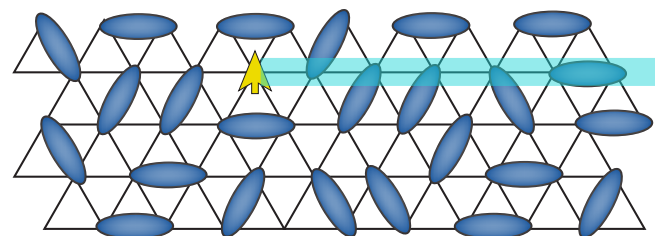
Line shape in Rb₂MnF₄

Quantum spin liquid

Entanglement \rightarrow non-local excitation

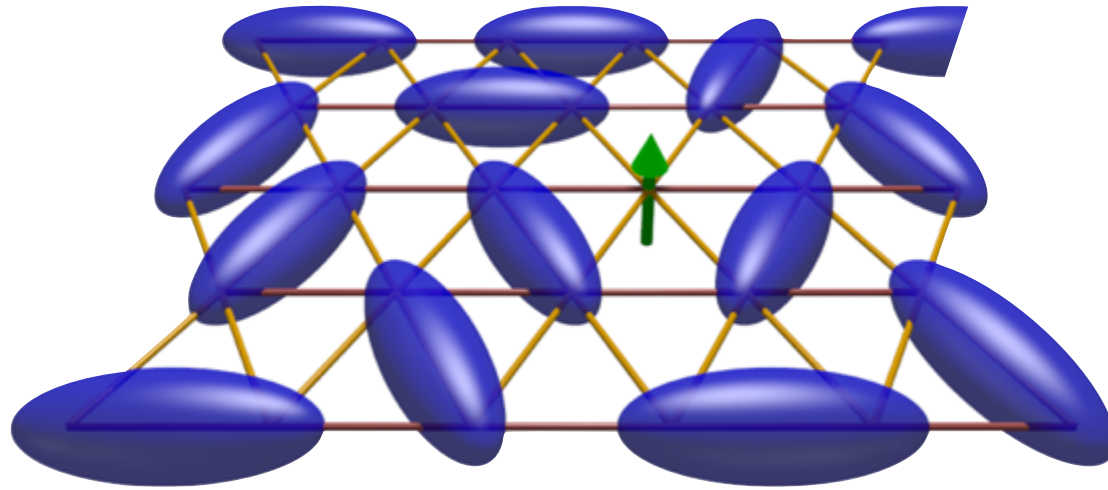
$$\Psi = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$




$$\Psi = \text{[Diagram 3]} + \dots \quad \text{"spinon"}$$


"quasiparticle" above a non-zero gap

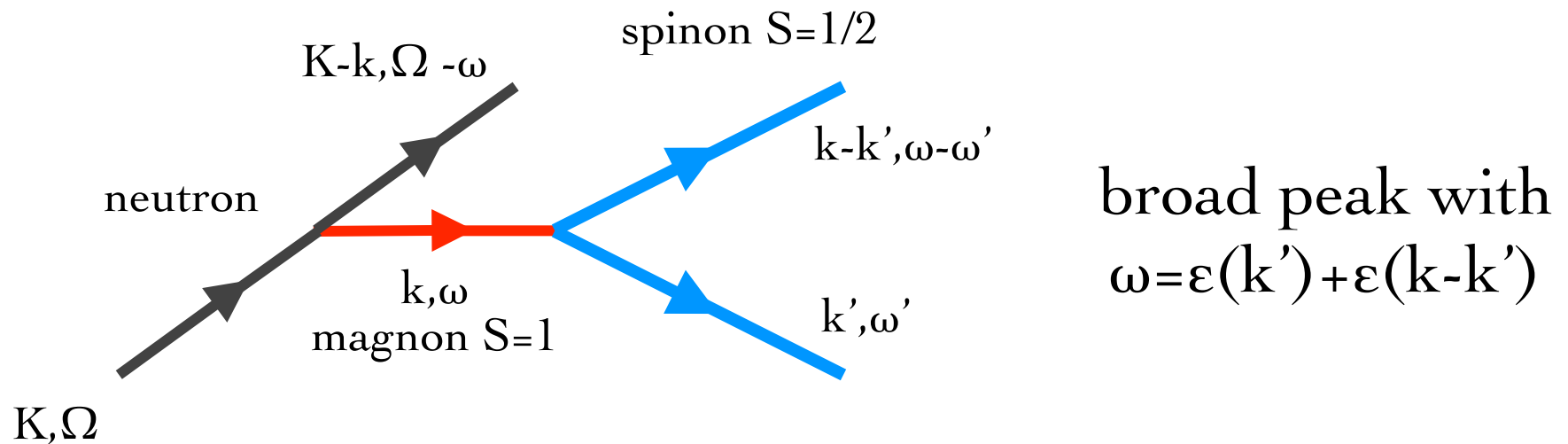
Fractional quantum number



excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins
always created in pairs by any
local operator

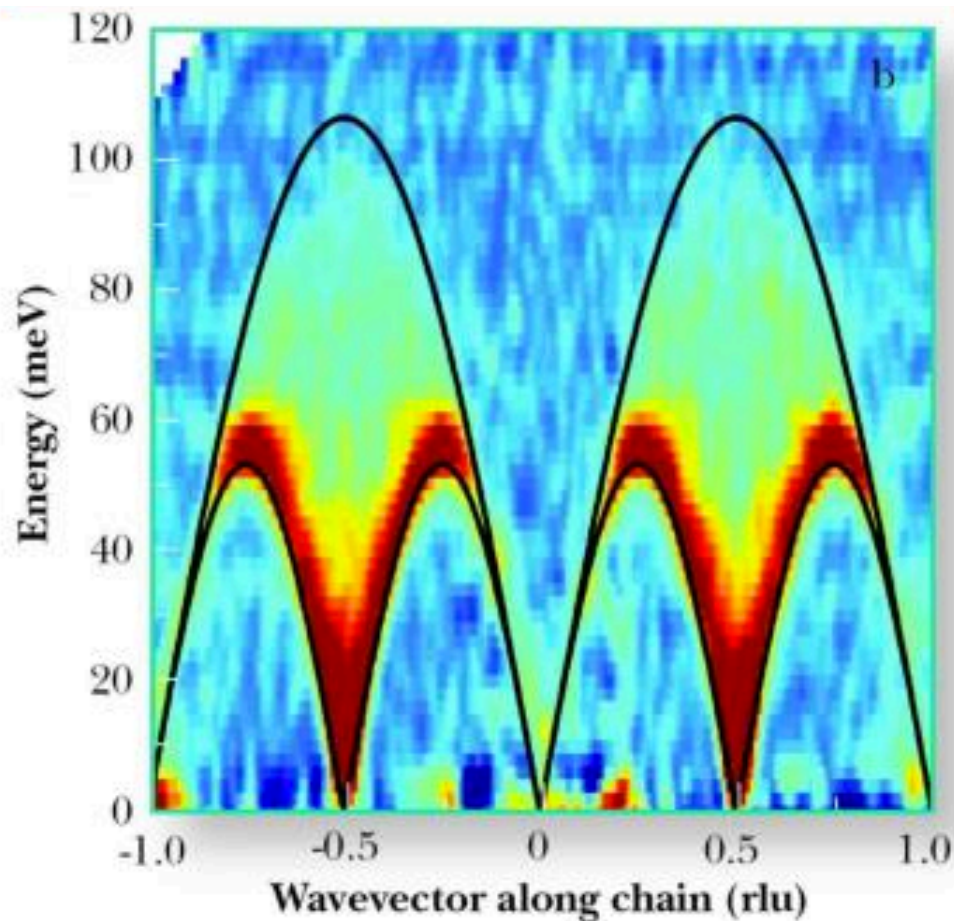
No spin waves

- Magnon is not elementary: decays into two spinons



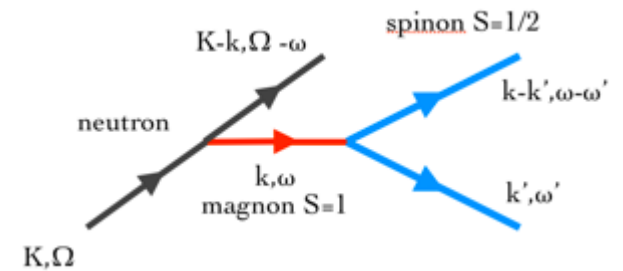
- Sharp peaks should be reduced or absent in the spin structure factor

c.f. One dimension

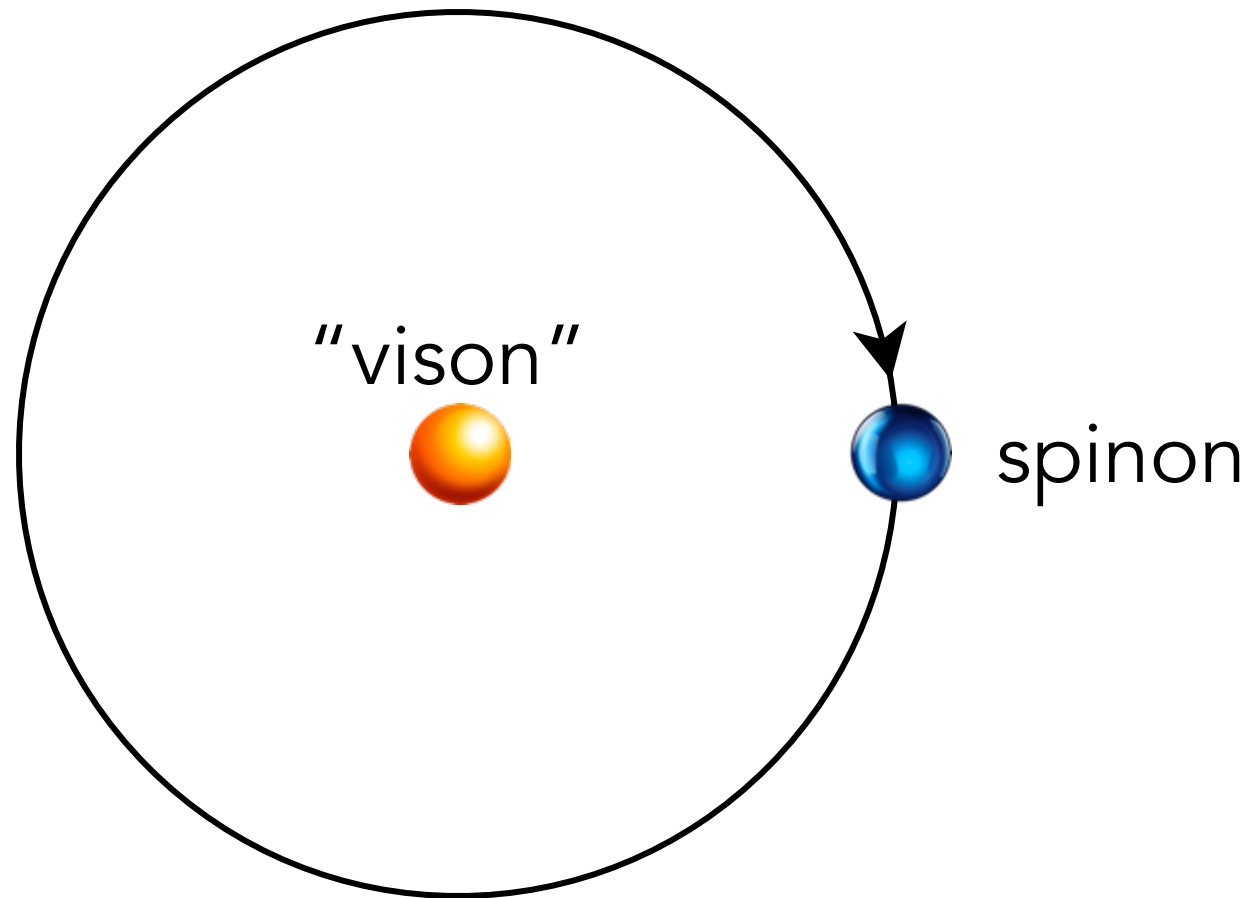


A. Tennant *et al*, 2001

KCuF_3



Anyons



$$\Psi \rightarrow -\Psi$$

"mutual semions"



X.-G. Wen

Topological phases



A. Kitaev



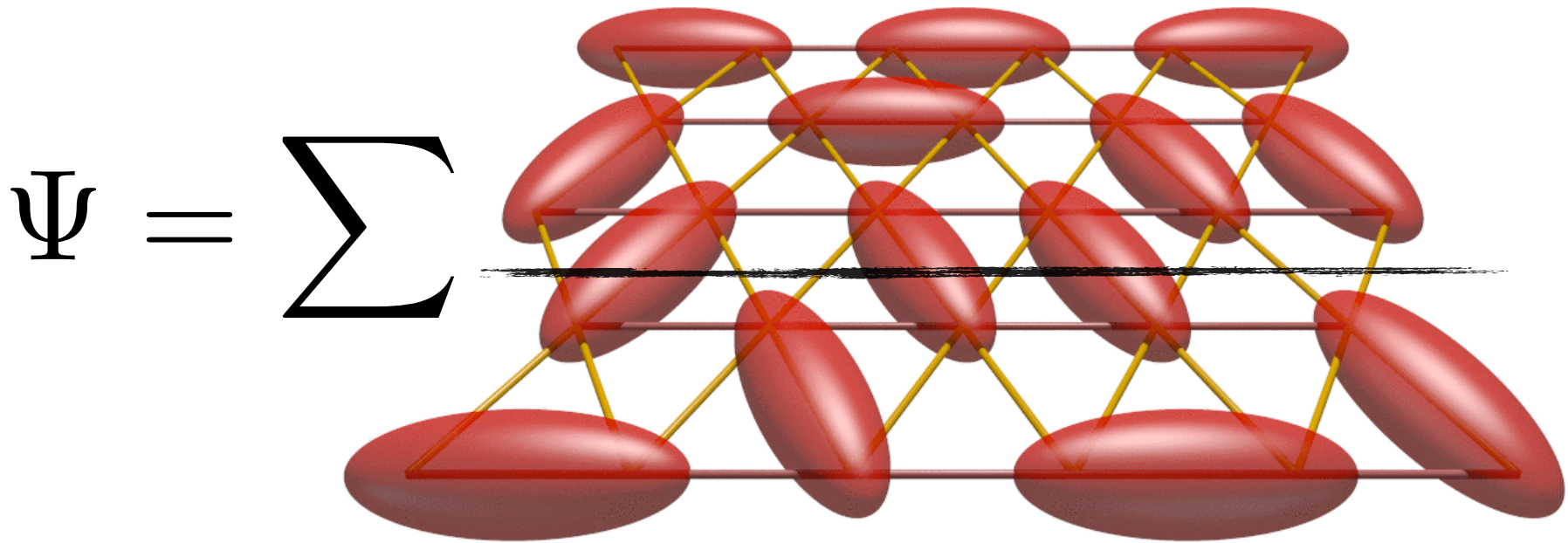
Anderson's RVB state is thus an example of a "topological phase" - the best understood sort of QSL

Understood and classified by anyons and their braiding rules in 2d

$$\begin{array}{c} e \quad m \\ \diagdown \quad \diagup \\ \text{---} \\ \diagup \quad \diagdown \\ e \quad m \end{array} = - \begin{array}{cc} e & m \\ | & | \\ e & m \end{array}$$

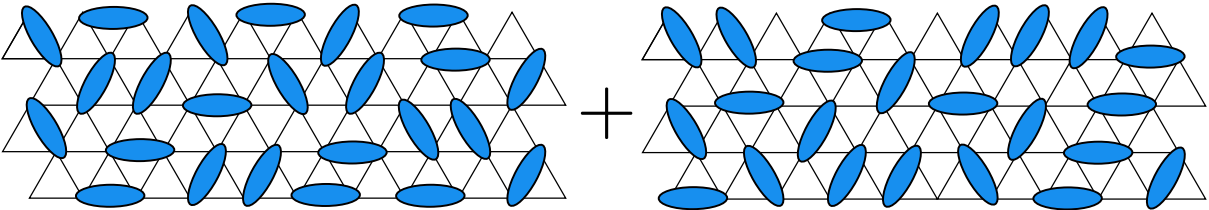
$$\begin{array}{cc} e m & e m \\ \diagdown & \diagup \\ \text{---} & \text{---} \\ \diagup & \diagdown \\ e m & e m \end{array} = \begin{array}{cc} e m & e m \\ | & | \\ \text{---} & \text{---} \\ | & | \\ e m & e m \end{array} = - \begin{array}{ccc} e m & e m & e m \\ | & | & | \\ e m & e m & e m \end{array}$$

Stability



Robustness arises from topology: a QSL is a stable *phase* of matter (at $T=0$)

Quantum spin liquid

$$\Psi = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$


For ~ 500 spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site ($S=0$)

$$|\Psi_0\rangle = \prod_{k \in FS} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle$$

"partons"
"spinons"

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow & \updownarrow & \downarrow \\ \hline \downarrow & \downarrow & \updownarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \updownarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & & \downarrow \\ \hline \downarrow & & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \updownarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Gutzwiller Construction

- Project out any components with empty or doubly occupied sites

$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

"partons"
"spinons"

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow\downarrow & \downarrow & \\ \hline \downarrow & \downarrow & \uparrow\downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow\downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow\downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Gutzwiller Construction

- Can build many QSL states by choosing different free fermion states

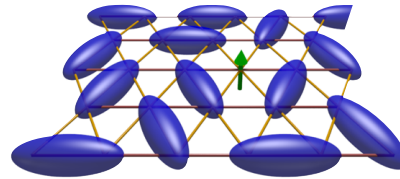
$$|\Psi\rangle = \hat{P}_G |\Psi_0\rangle$$

"partons"
"spinons"

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow\downarrow & \downarrow & \\ \hline \downarrow & \downarrow & \uparrow\downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow\downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow\downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

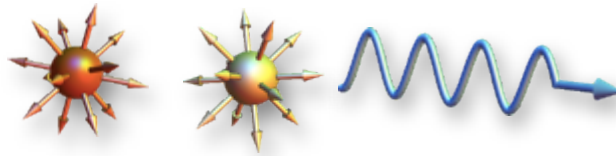
Classes of QSLs

- Topological QSLs



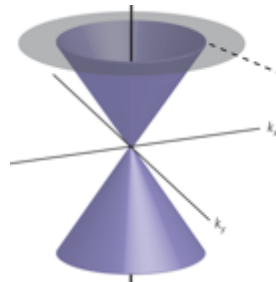
projected
superconductor

- $U(1)$ QSL



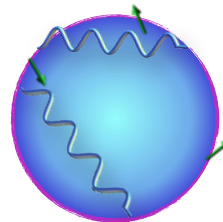
projected 3d band
insulator

- Dirac QSLs



projected
graphene

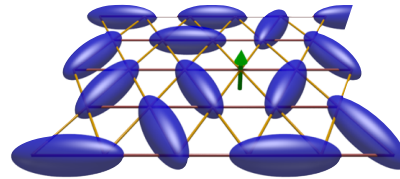
- Spinon Fermi surface



projected
metal

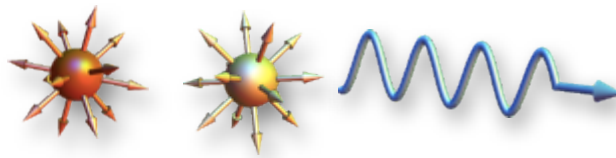
Classes of QSLs

- Topological QSLs



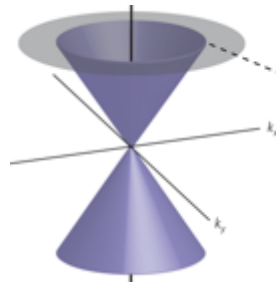
anyonic
spinons

- U(1) QSL



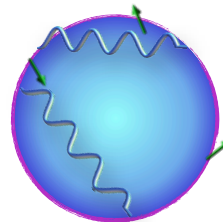
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



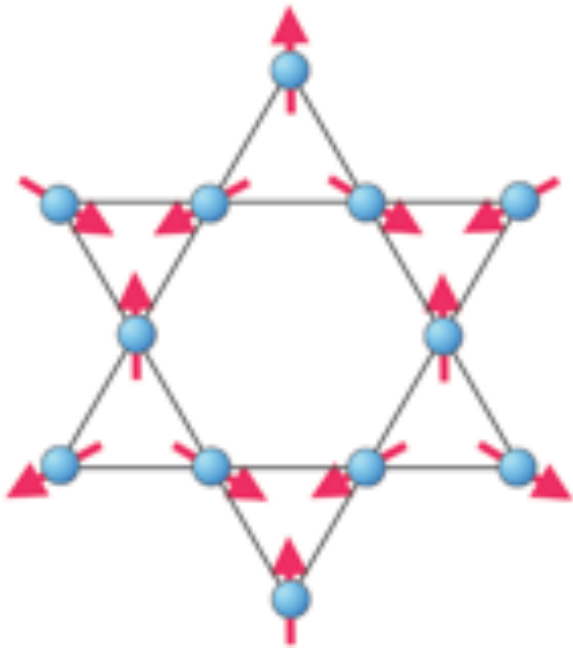
non-Fermi
liquid "spin
metal"

Strange stuff



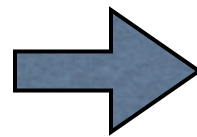
where do we find it?

Kagomé antiferromagnet



$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \dots$$

Very large classical
degeneracy

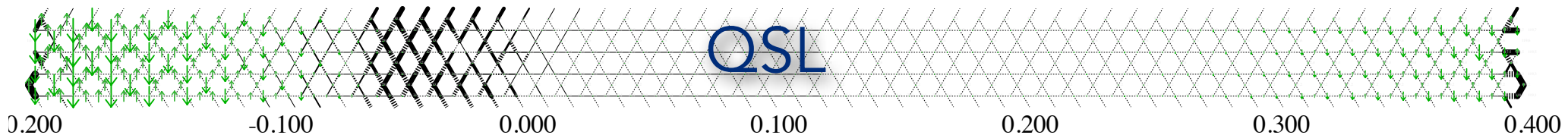


likely to be a QSL

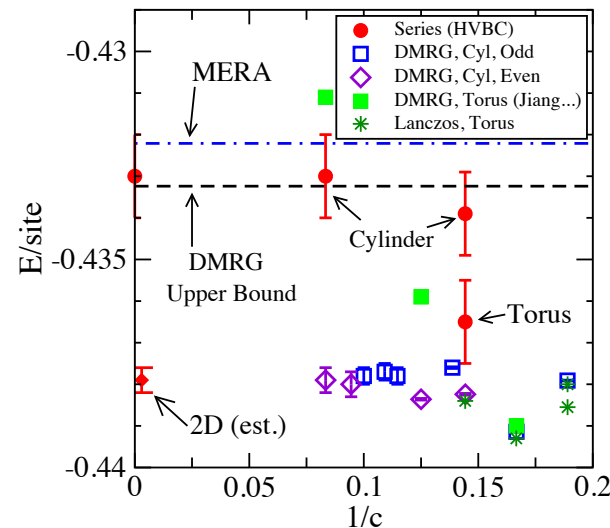
V. Elser, 1989 + many many others

$S=1/2$ kagomé AF

- Rather definitive evidence for QSL by DMRG



© Steve White

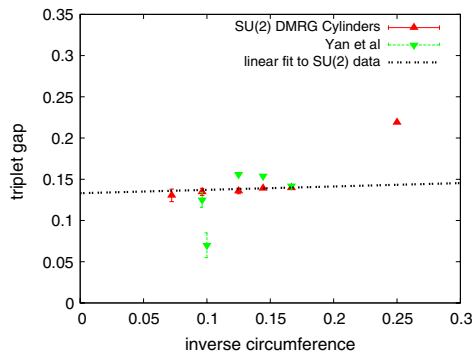


S. Yan et al, 2010

many other studies support
existence of some QSL phase

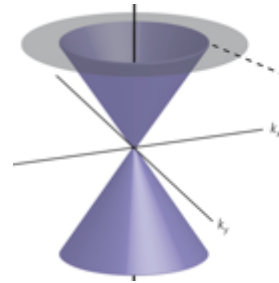
Theory

- What kind of QSL?



S. Depenbrock *et al*, 2012

gapped,
topological QSL



Y. Ran *et al*, 2007

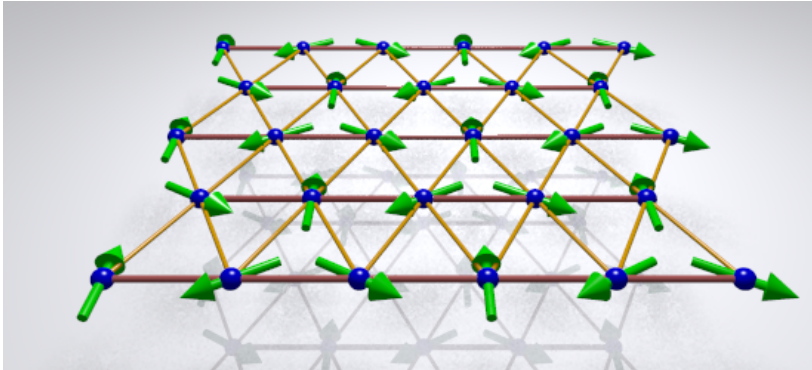
F. Becca...

Y.C.He *et al*, 2016

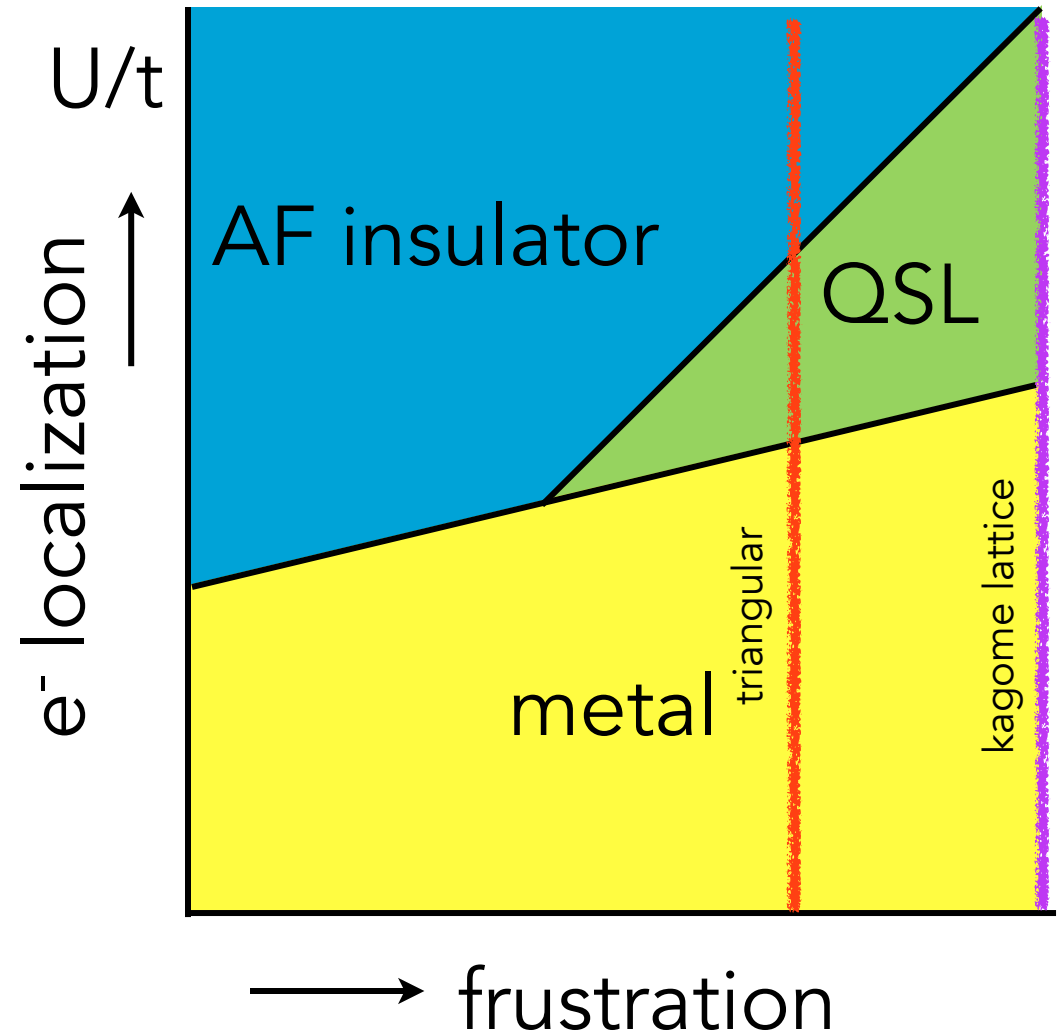
gapless
Dirac QSL

+ various other
proposals with
weaker
quantitative
support

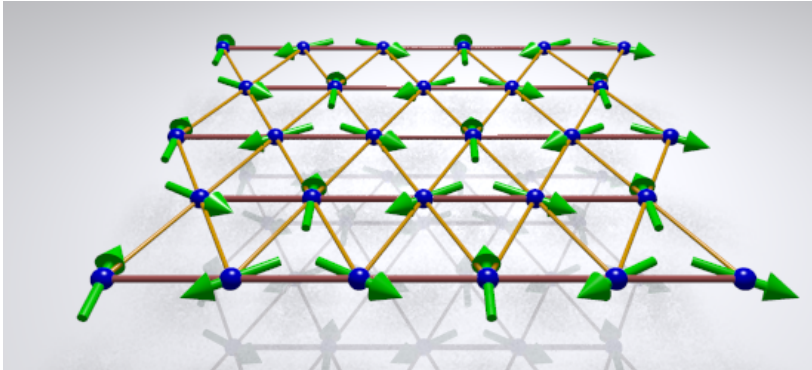
Triangular lattice w/ ring exchange



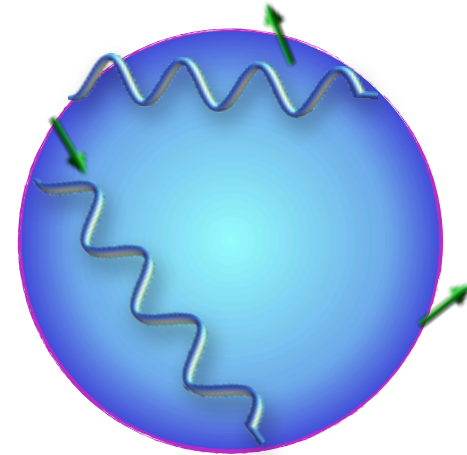
- Motrunich (2005): ring exchange stabilizes a spin liquid



Triangular lattice w/ ring exchange



- Motrunich (2005): ring exchange stabilizes a spin liquid



- Motrunich, Lee/Lee: spin liquid state favored by ring exchange is the "spinon Fermi sea" state

SOC triangular

Heavy elements:

highly localized electrons, strong
spin-orbit coupling

$$H = \sum_{\langle ij \rangle} \left[J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_z S_i^z S_j^z \right. \\ \left. + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \right. \\ \left. + i J_{\pm z} (\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j)) \right]$$

XXZ

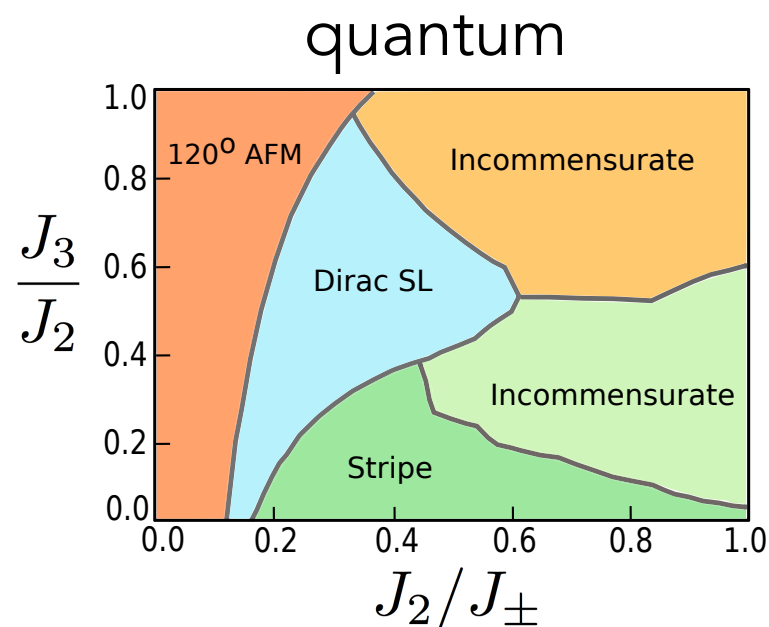
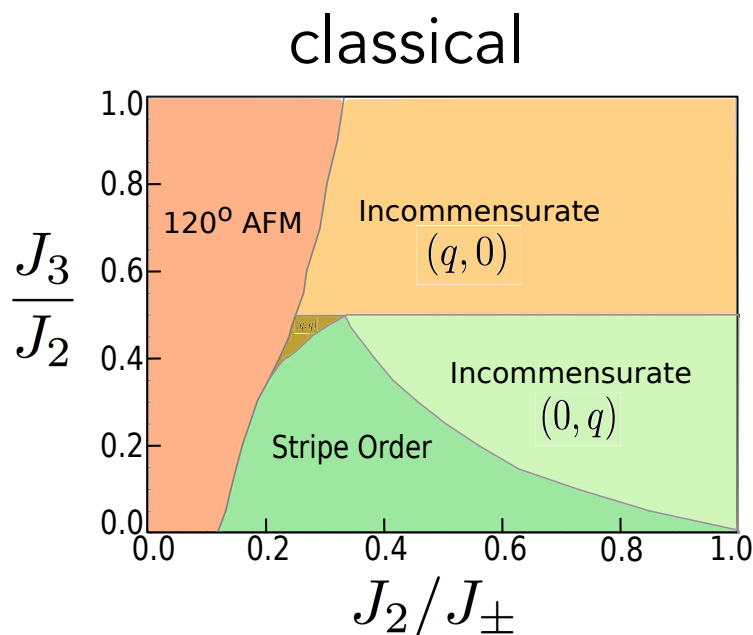
bond-dependent
couplings

Y. Li et al, 2015

SOC triangular

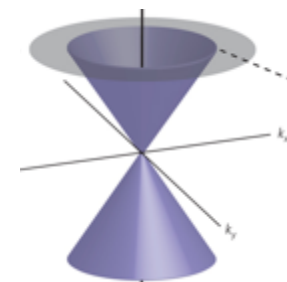


QSLs versus magnetic order



Some window exists for Dirac QSL

J. Iaconis, C. Liu, G. Halász, LB, 2017





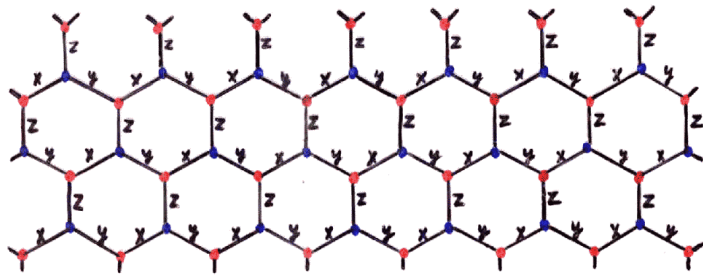
Kitaev model

Kitaev's honeycomb model

$$H = \sum_{i,\mu} K_{\mu} \sigma_i^{\mu} \sigma_{i+\mu}^{\mu}$$

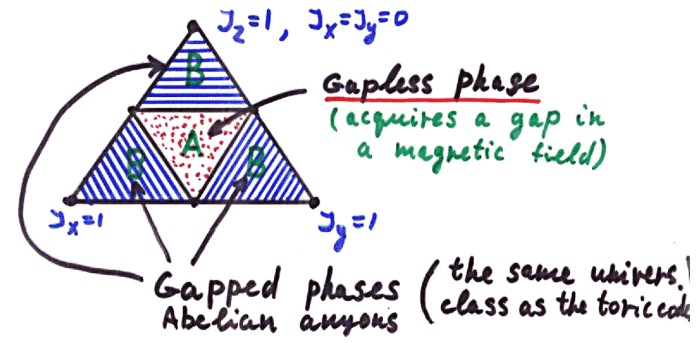
1. The model

KITP, 2003

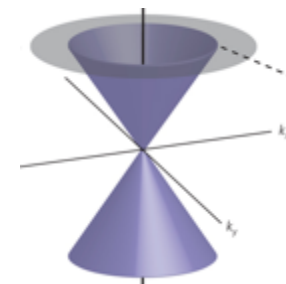


Spin $\frac{1}{2}$ on each site.

Phase diagram



exact parton construction
- spinon is a gapless Majorana fermion



How to probe QSLs?

Two main characteristics:

- Massive entanglement
 - Almost no experiments known to probe this.
- Fractional/non-local excitations
 - Probed by most low energy response measurements. Challenge is to distinguish the fractional/non-local nature.

A rough guide to experiments on QSLs

Does it order?

- NMR line splitting
- μ SR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/x-ray

Is there a gap?

- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of χ

Delocalized excitations?

- thermal conductivity
- INS

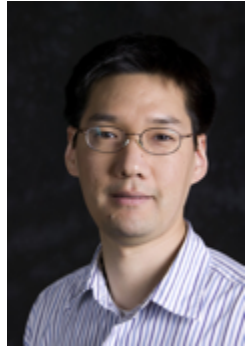
Structure of excitations?

- $E(k)$ from INS, RIXS
- optics, Raman

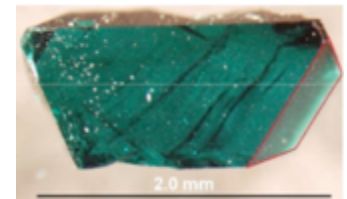
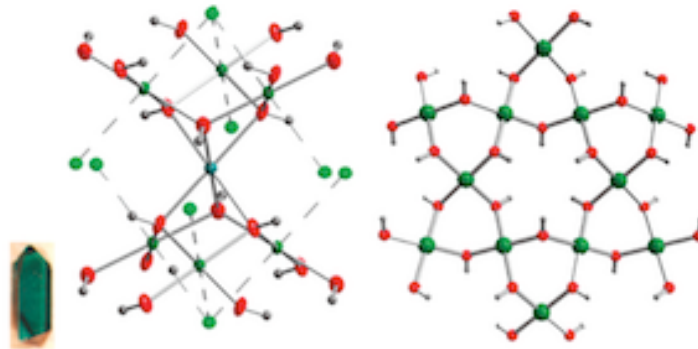
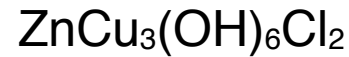
Exotica

- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects

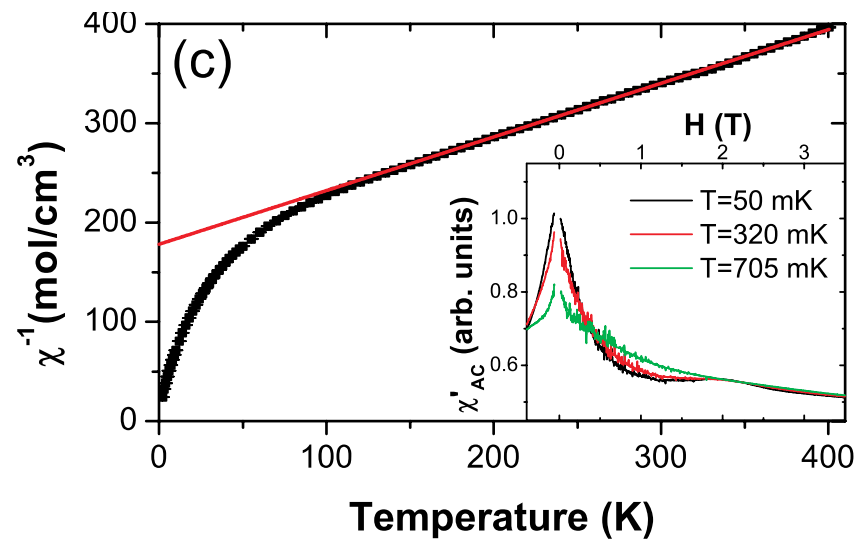
Scattering



Herbertsmithite

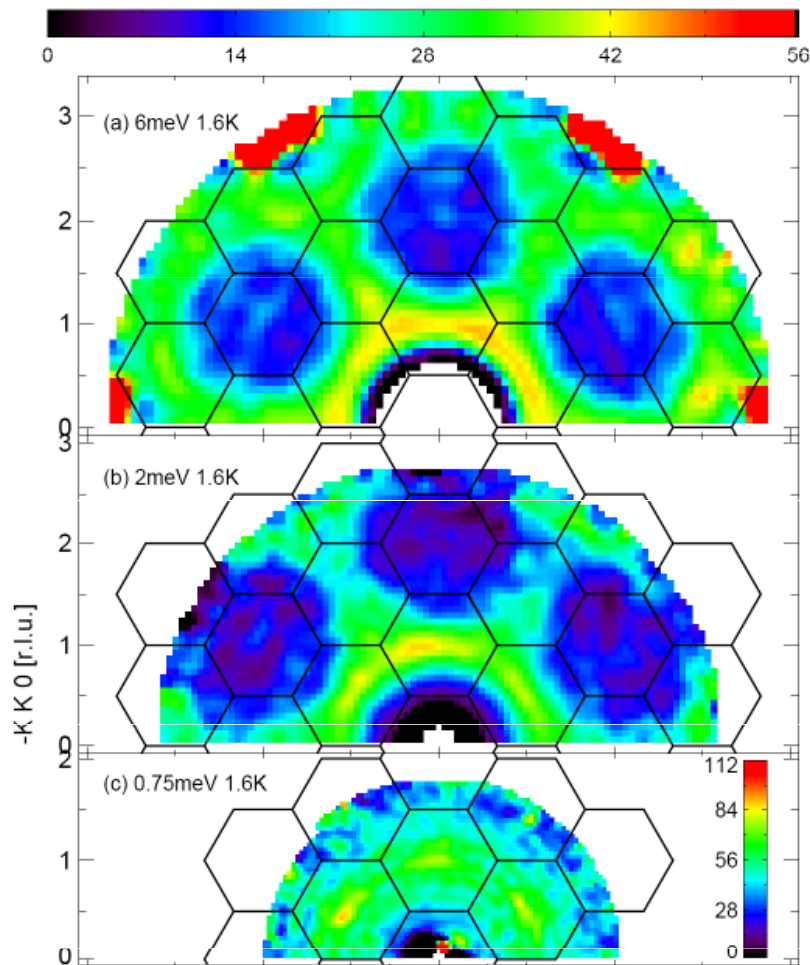


Heisenberg-like
with $J \sim 200\text{K}$
no order down to
50mK

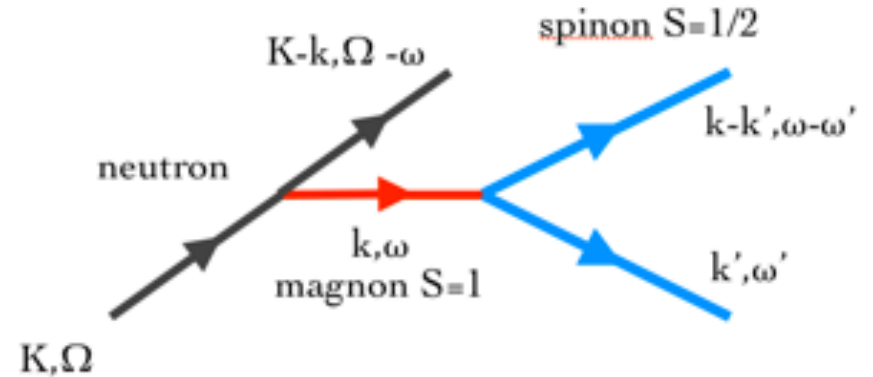


Helton et al, 2007

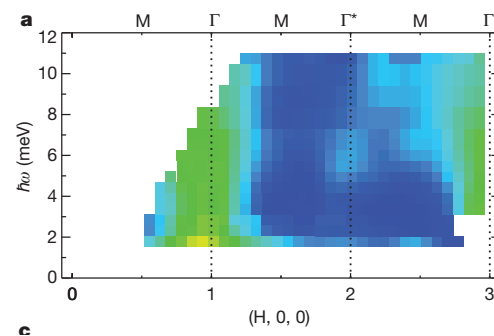
Scattering



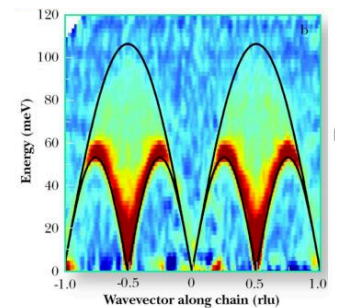
T-H Han *et al*, 2012



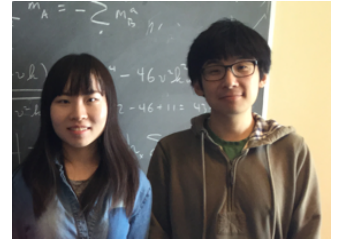
continuum scattering
expected
...but probably with more
structure?



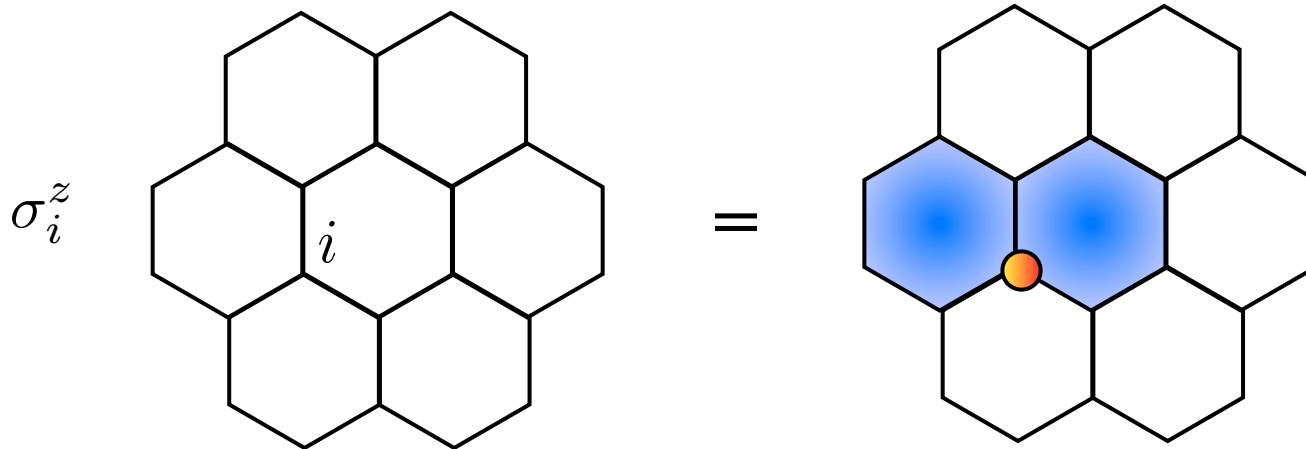
VS



Kitaev QSL



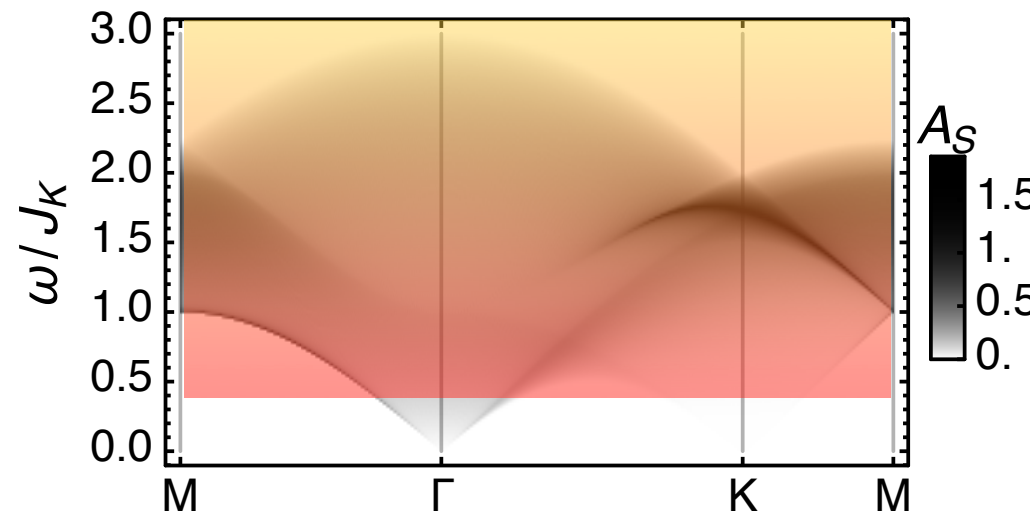
Spin flip produces a free Majorana fermion and two immobile fluxes



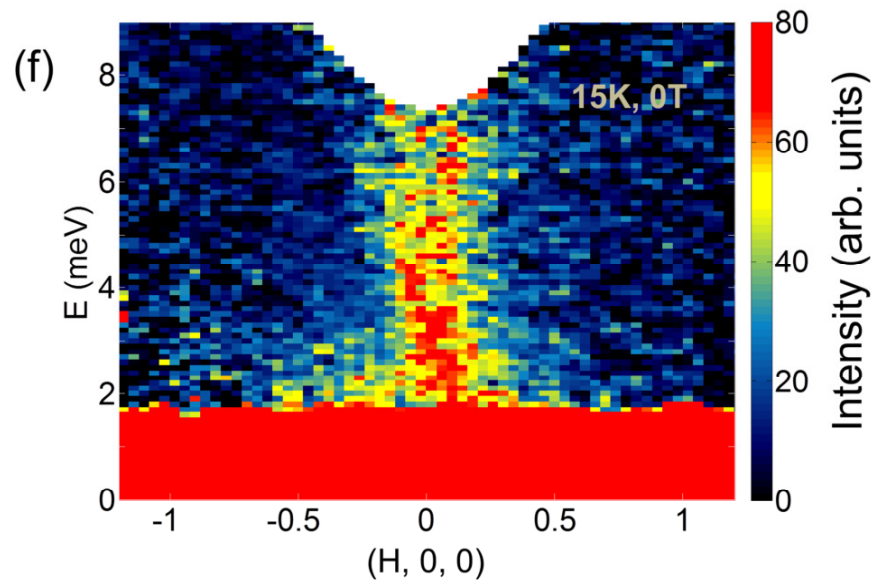
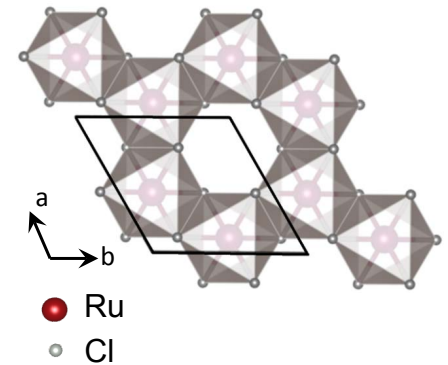
J. Knolle *et al*, 2014

Xueyang Song, Yi-Zhuang You + LB, 2016

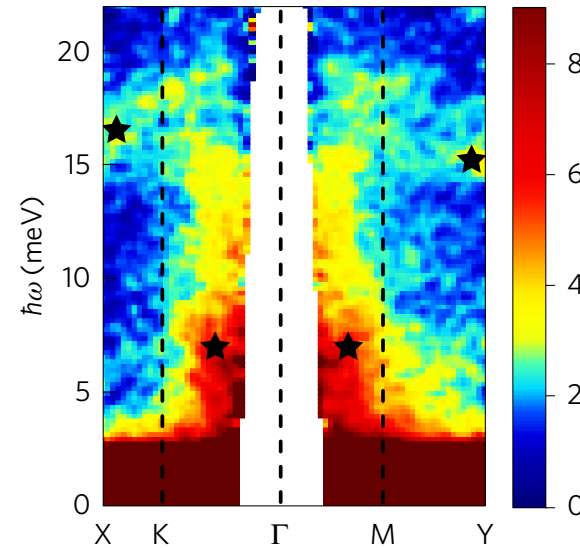
dynamical spin
correlations in the
Kitaev QSL



alpha-RuCl₃



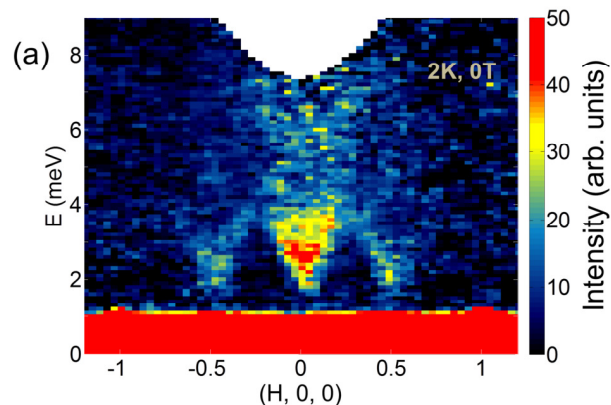
A. Banerjee *et al*, 2017



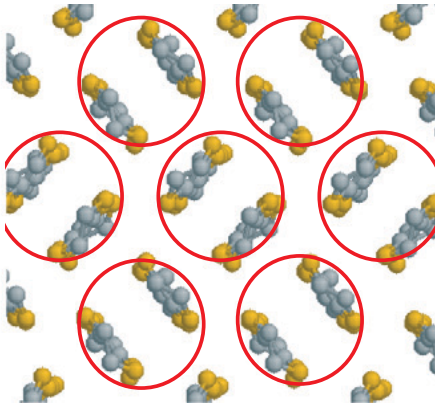
S.-H. Do *et al*, 2017

“column” of scattering suggested to be related to Majorana spinons

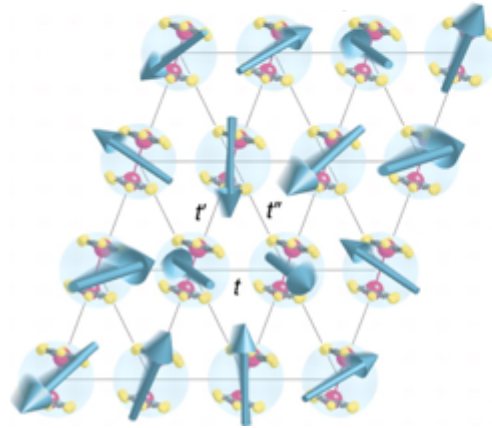
c.f.



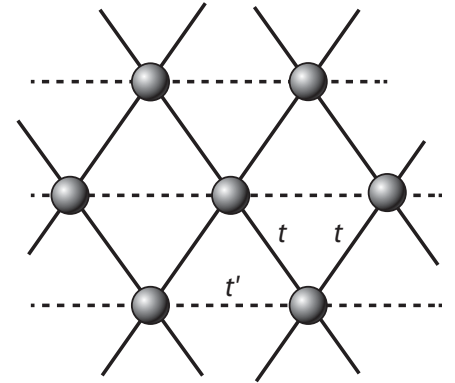
Low energy signatures: triangular organics



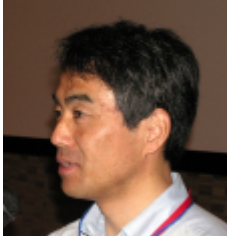
$\kappa\text{-(ET)}_2\text{X}$



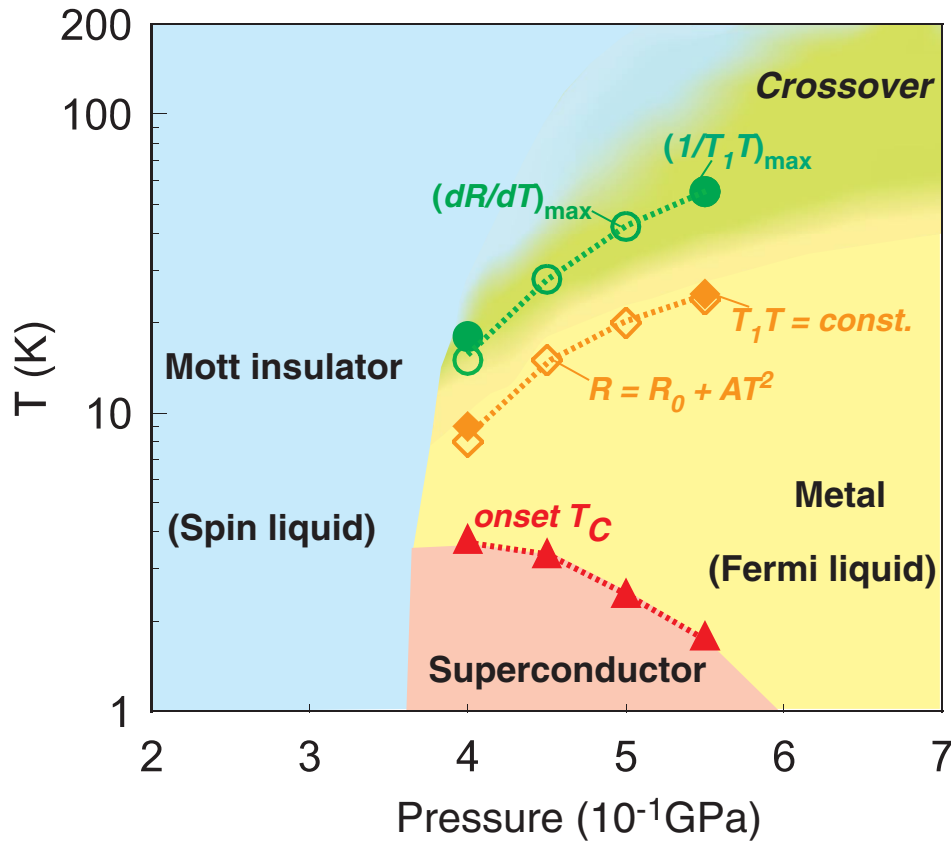
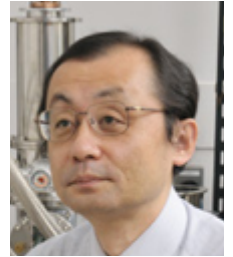
$\beta'\text{-Pd(dmit)}_2$



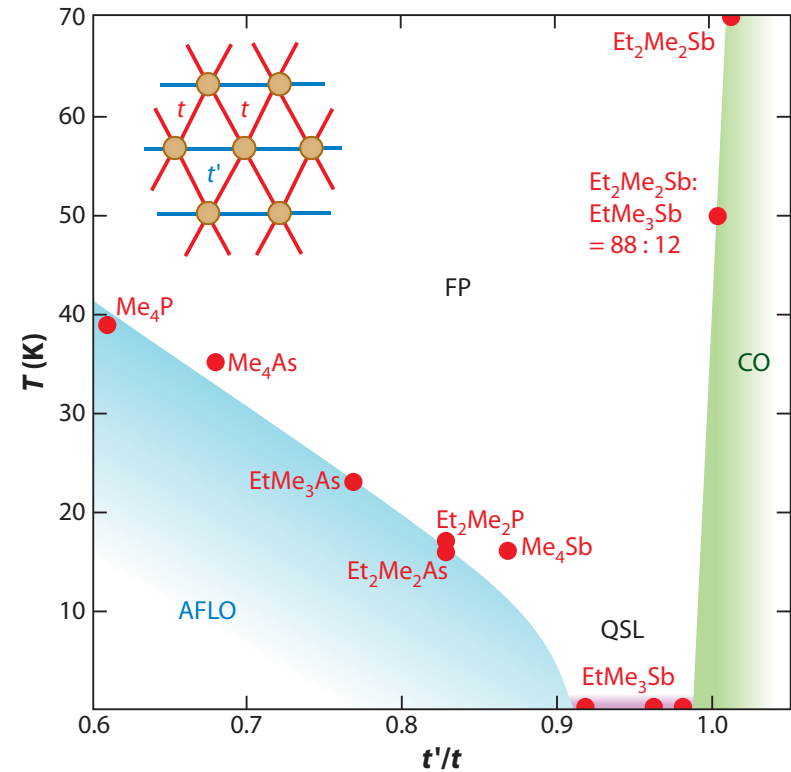
- Molecular materials which behave as effective triangular lattice $S=1/2$ antiferromagnets with $J \sim 250\text{K}$
- significant charge fluctuations



triangular organics

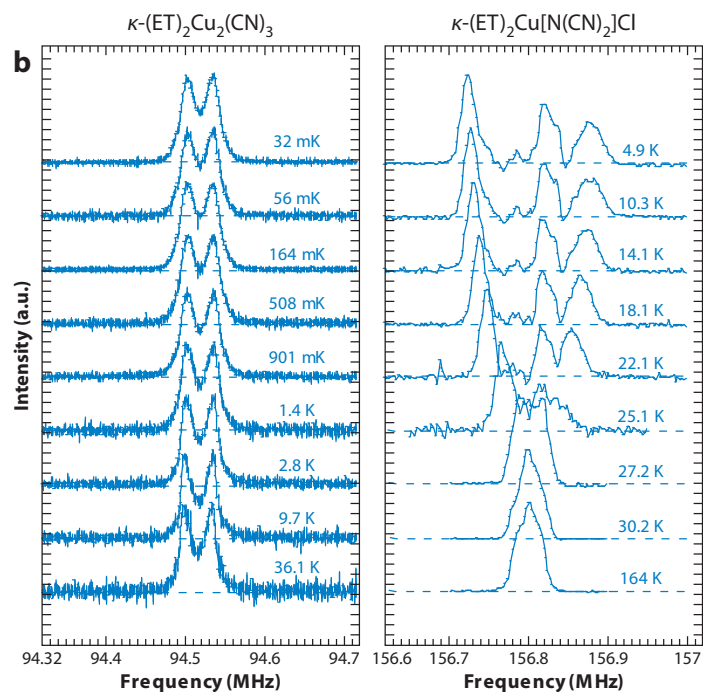


K. Kanoda group (2003-)

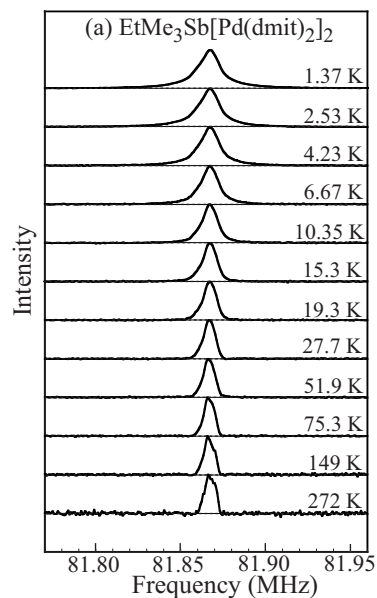


R. Kato group (2008-)

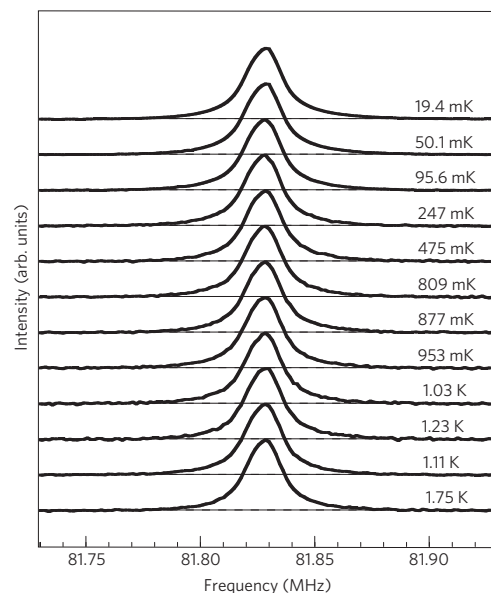
NMR lineshapes



Y. Shimizu ^1H NMR
et al, 2003



T. Itou et al,
2008,2010

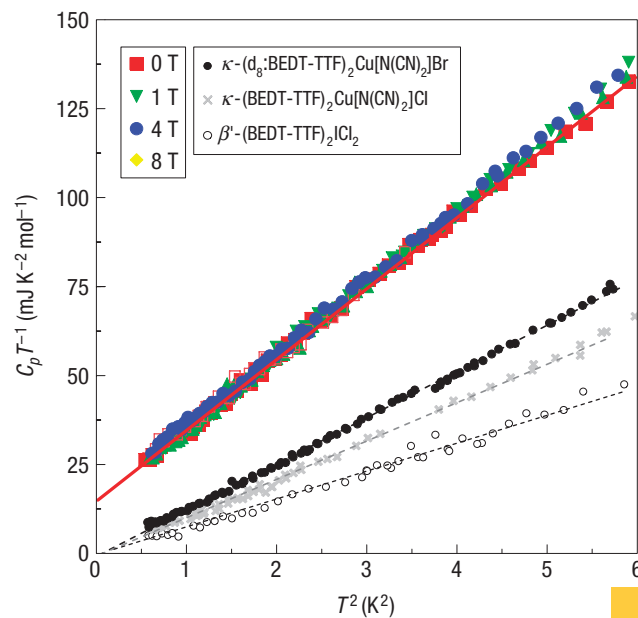


^{13}Cs NMR

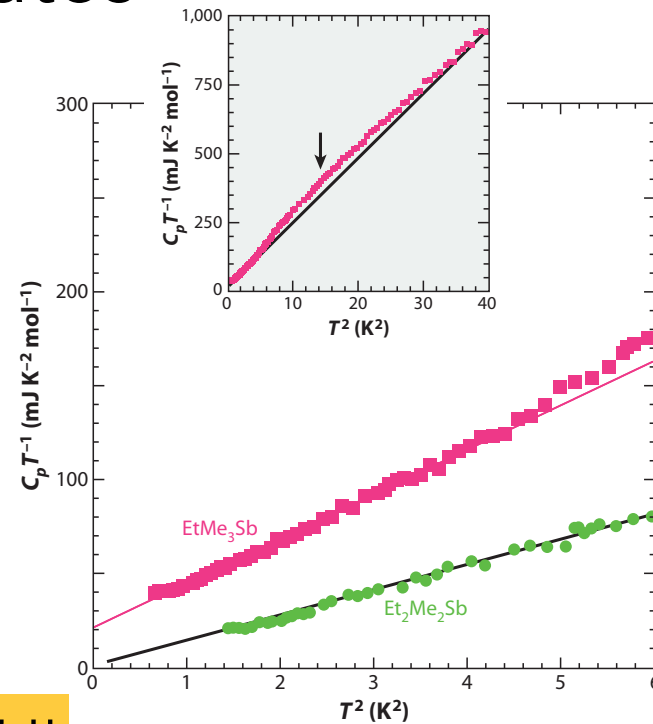
Evidence for lack of static moments: $f > 1000$!

Specific Heat

- $C \sim \gamma T$ indicates gapless behavior with constant density of states



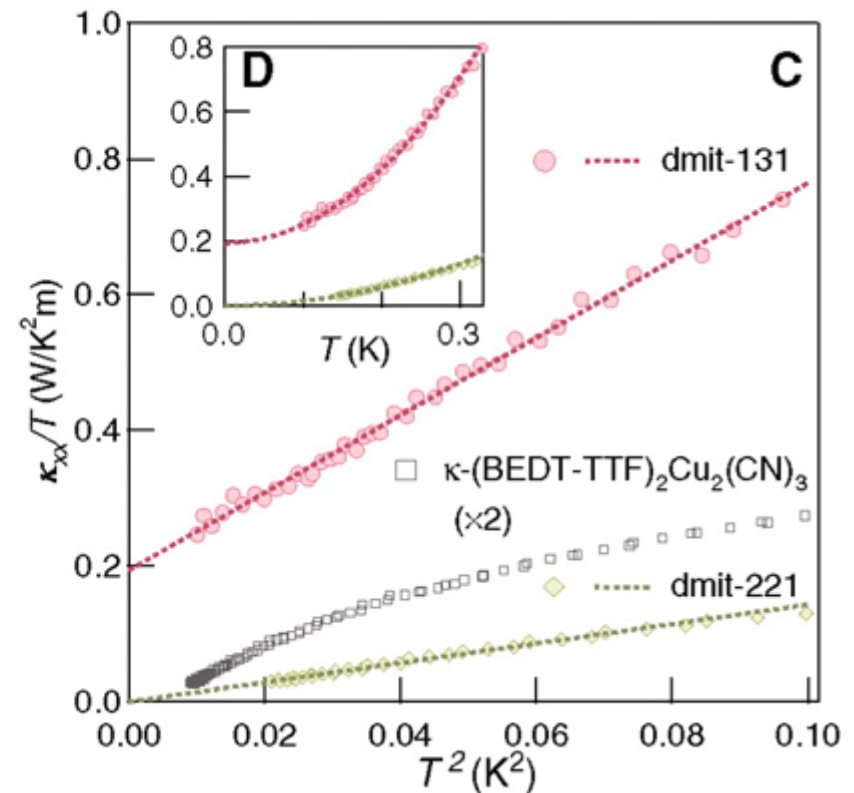
$\gamma_{Cu} \sim 0.7$!!



S. Yamashita et al, 2008

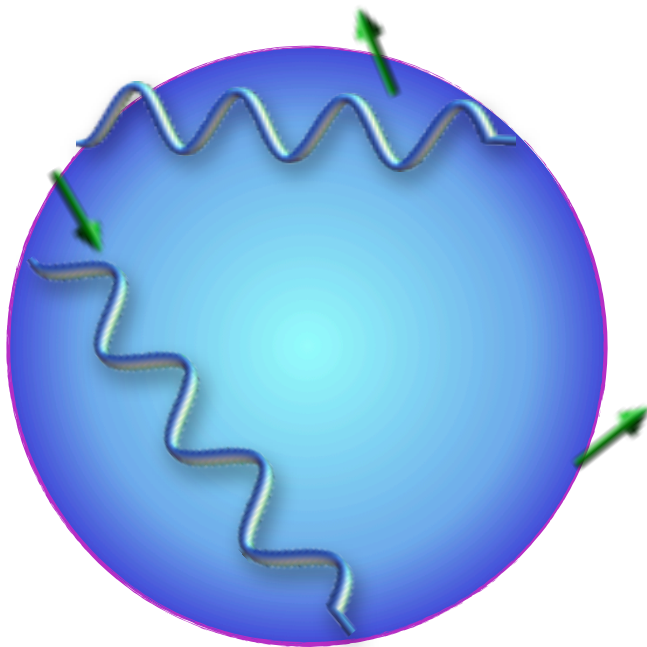
Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit
- Estimate for a *metal* would correspond to a mean free path $l \sim 1 \mu\text{m} \approx 1000 a$!

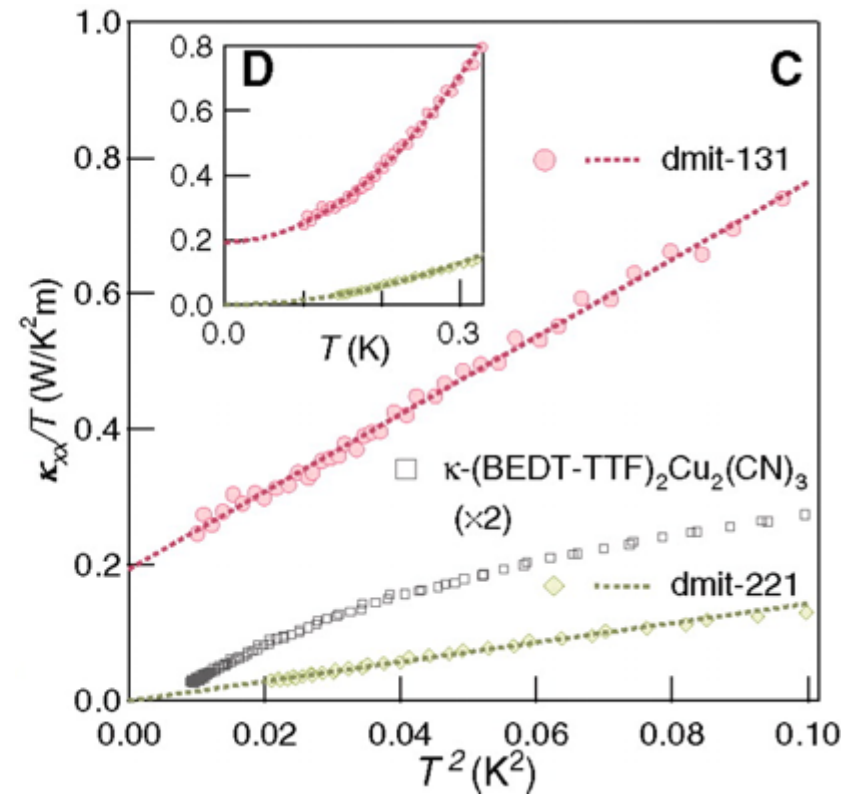


M. Yamashita et al, 2010

Thermal conductivity



?

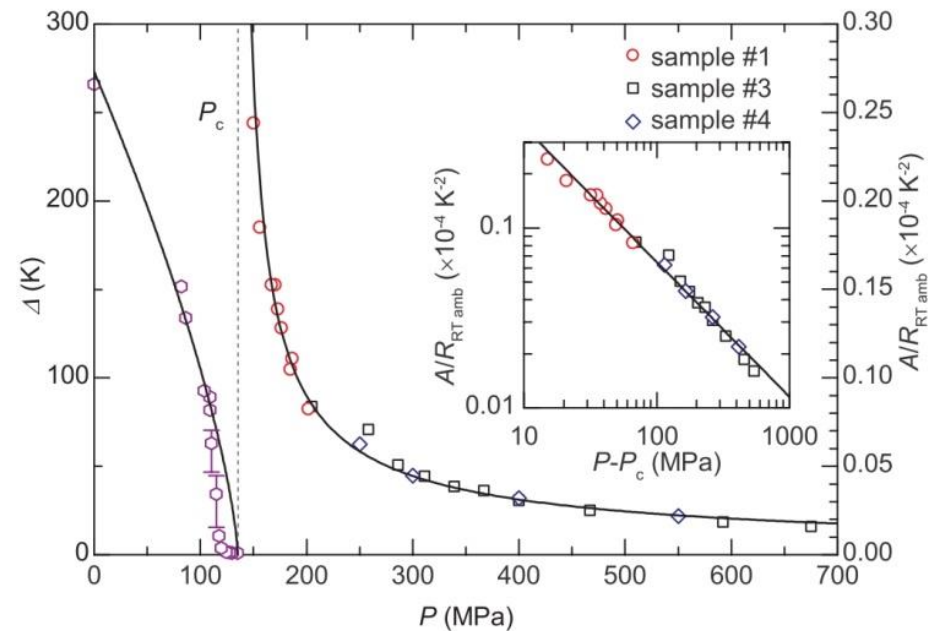
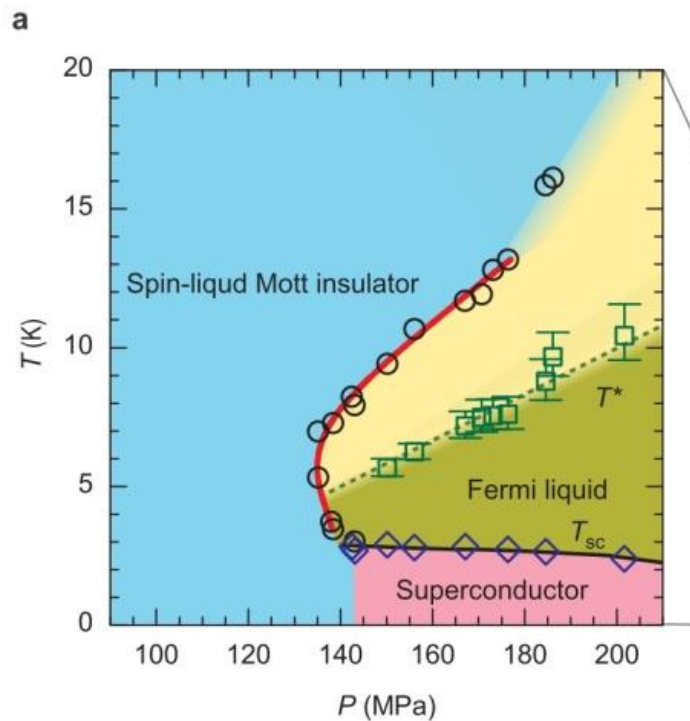


M. Yamashita *et al*, 2010

Spinon Fermi surface

- How could we firm this up?
 - Spinons should be *confined* to 2d. Can we see evidence of this?
 - e.g. $\kappa_c \ll \kappa_{ab}$ (c.f. Y. Werman *et al*, 2017)
 - See signs of \mathbf{k}_F ?
 - quantum oscillations, RKKY
 - Possible quantization effects in small systems
 - Observe *conversion* of spinons to electrons in adjacent metal (c.f. T. Senthil, 2008)

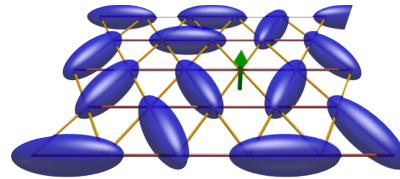
Spinons \rightarrow Electrons?



T. Furukawa *et al*, 2017

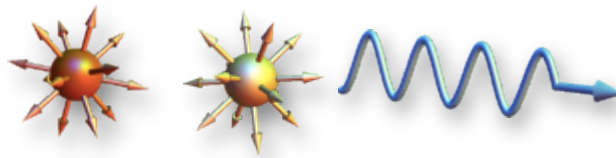
Other QSLs in organics?

- Topological QSLs



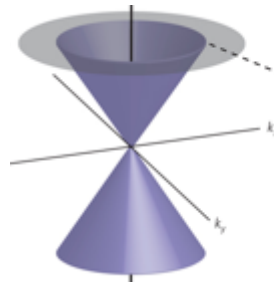
anyonic
spinons

- U(1) QSL



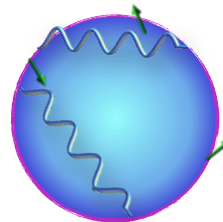
electric+magnetic
monopoles, photon

- Dirac QSLs



strongly
interacting
Dirac fermions

- Spinon Fermi surface



non-Fermi
liquid "spin
metal"

Thanks for your attention

References here:

<https://spinsandelectrons.com/>

<https://spinsandelectrons.com/pedagogy/>

