

Transport in strongly correlated systems

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Transport

$$\mathbf{j} = \underline{\sigma} \mathbf{E}$$

$$\mathbf{j}_e = -\underline{\kappa} \nabla T$$

- Arguably most important aspect of quantum materials: electrical and thermal conductivity (and crossed coefficients)
- Sensitive, versatile
- Probes extreme long wavelength, low frequency

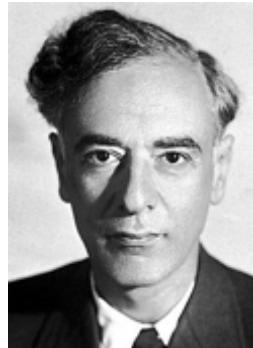
Theory

- *Understanding* of transport mainly through **quasiparticle** picture
- Boltzmann equation:

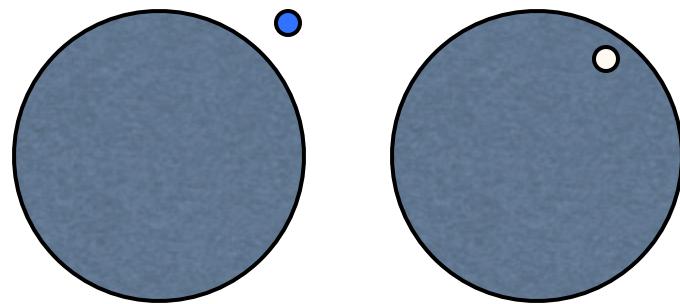
$$[\partial_t + \mathbf{v}_n(\mathbf{k}) \cdot \nabla_r - e\mathbf{E} \cdot \nabla_k] f_n = \frac{\partial f_n}{\partial t} \Big|_{\text{collision}}$$

Linearizing this around equilibrium gives conductivities in terms of band velocities and scattering rates

Fermi Liquid Theory



Landau provided justification for
quasiparticle picture in metals
when $T \ll E_F$



Low energy excitations act like
electrons and holes but with
wavefunction dressing ($Z < 1$), effective
mass, and Landau interactions

$$E = \sum_k \epsilon_k \delta n_k + \frac{1}{2V} \sum_{k,k'} U_{k,k'} \delta n_k \delta n_{k'}$$

scattering is weak because
not so many low energy qp
states to scatter to

This talk

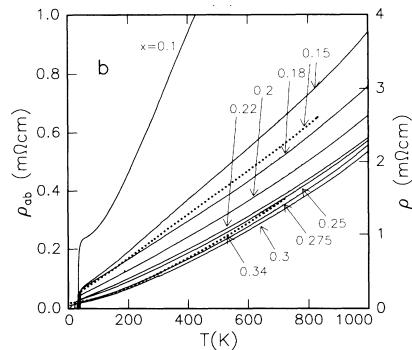
- I. **SYK model of a strongly correlated metal: FL to NFL crossover and a disordered strange metal**



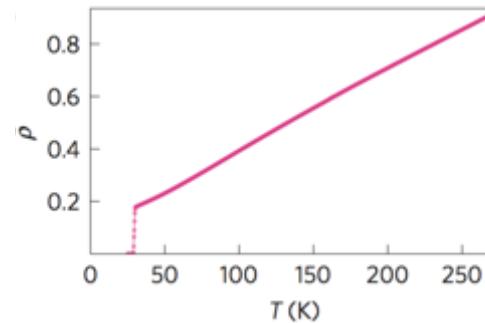
- II. Heat transport in spin systems - towards a non-quasiparticle description (progress report)



Non-Fermi Liquids



LSCO Takagi *et al*, 1992



BaFe₂(As_{1-x}P_x)₂, Hayes *et al*, 2016

T-linear resistivity is challenging theoretically:

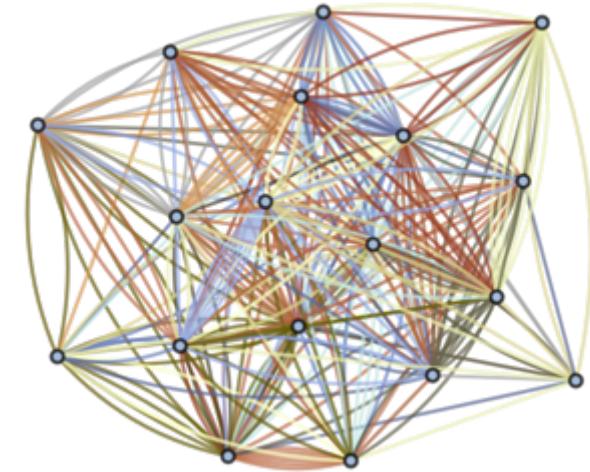
- If even “bad” quasiparticles exist, it is difficult to make them scatter strongly on the *entire* Fermi surface
- If no quasiparticles exist, what is the starting point?

Sachdev-Ye-Kitaev model

A toy *exactly soluble* model
of a non-Fermi liquid

$$H = \sum_{i < j, k < l} U_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3}$$



Like a strongly interacting quantum dot
or atom with complicated Kanamori
interactions between many “orbitals”

SYK Model

Sachdev-Ye, 1993: Model has a soluble large-N limit

$$\Sigma = \begin{array}{c} \text{---} \\ \text{---} \end{array} + O(1/N)$$

In equations: very similar to DMFT:

$$G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)} \quad \Sigma(\tau) = -U^2 G(\tau)^2 G(-\tau)$$

Solution:

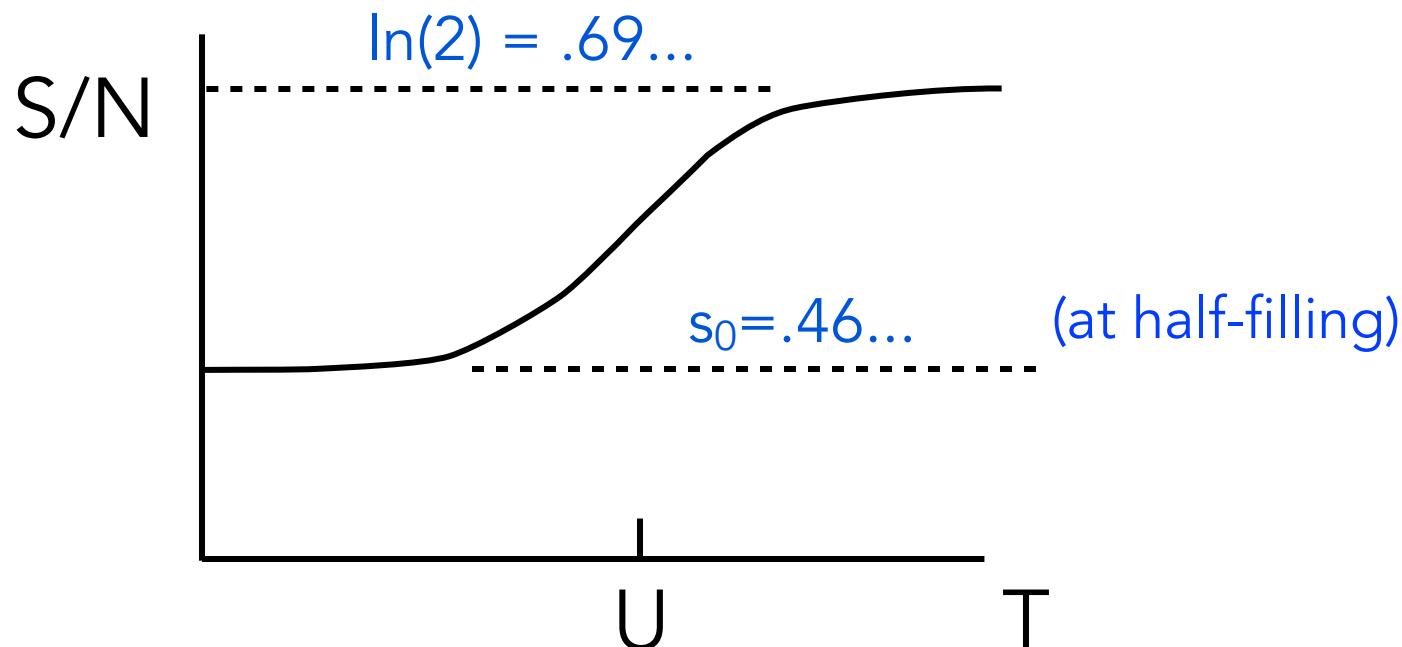
$$G(i\omega) \sim 1/\sqrt{\omega}$$

not a pole: **non-Fermi liquid**

SYK Model

Why not quasiparticles?

Georges, Parcollet, Sachdev, 2001: ground state entropy!



Many states available for scattering
“level spacing” $\sim U \exp(-Ns_0)$

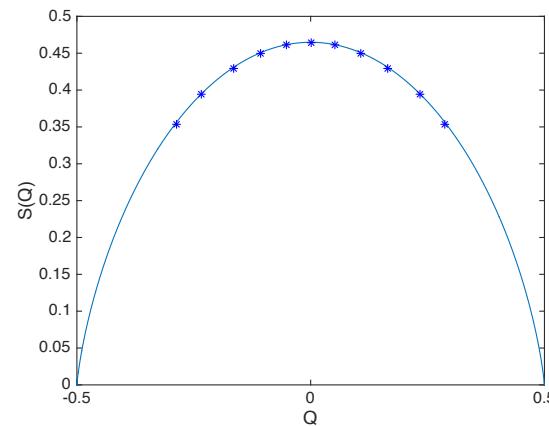
Density dependence

$$H \rightarrow H - \mu \mathcal{N}$$

$$\mathcal{N} = \sum_i c_i^\dagger c_i$$

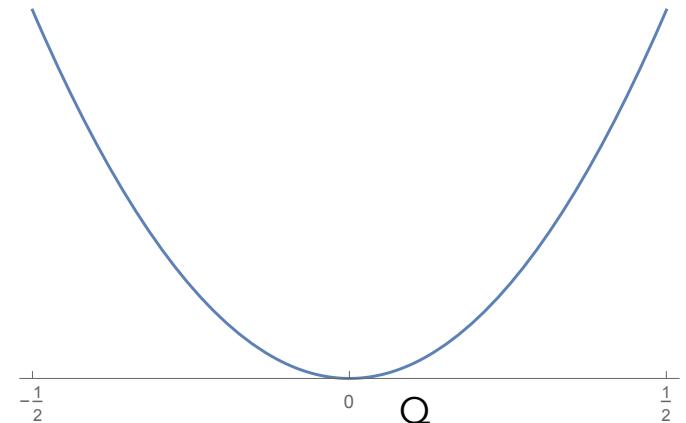
$$\mathcal{Q} = \frac{\mathcal{N}}{N} - \frac{1}{2}$$

Entropy



Davison et al, arXiv:1612.00849

Energy

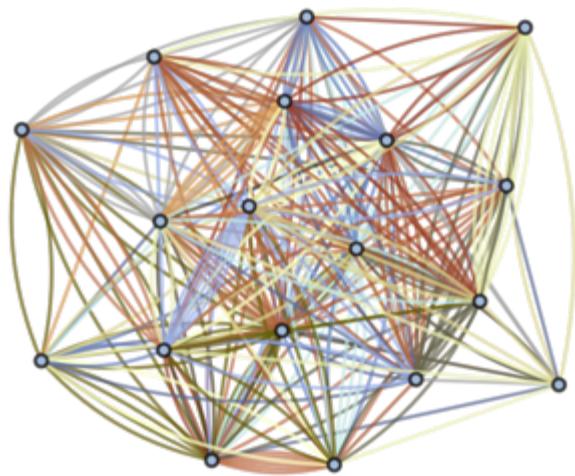


schematic

- Compressibility is constant at $T=0$

$$K = \left. \frac{\partial \mathcal{Q}}{\partial \mu} \right|_{\mu=0} = \frac{1.04}{U}$$

SYK Summary



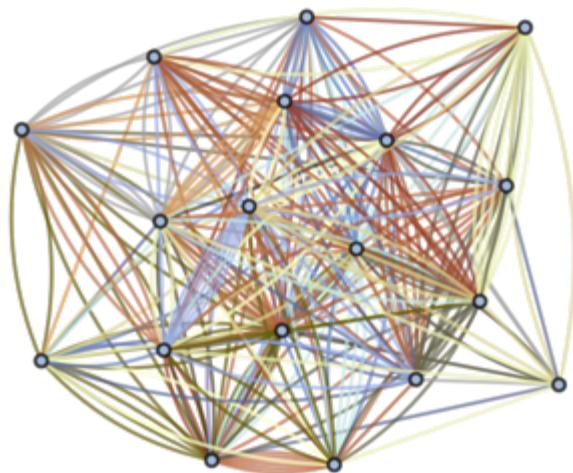
- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T = 0) = \frac{1.04}{U}$$

$$S(T = 0)/N = .46 \dots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

SYK Summary



- Compressible
- Ground state entropy
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$$K(T = 0) = \frac{1.04}{U}$$

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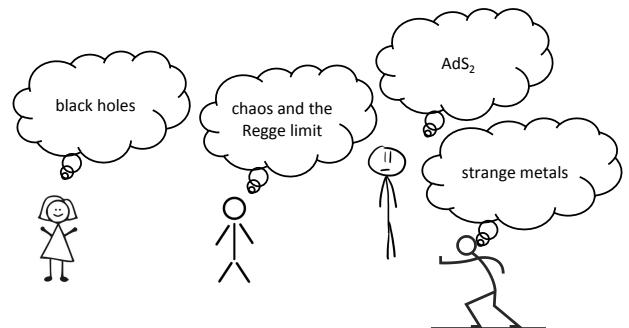
$$G(i\omega) \sim 1/\sqrt{\omega}$$

Chaos

$$\left\langle [\mathcal{O}(0), \tilde{\mathcal{O}}^\dagger(t)]^2 \right\rangle \sim \frac{1}{N} e^{\lambda_L t}$$

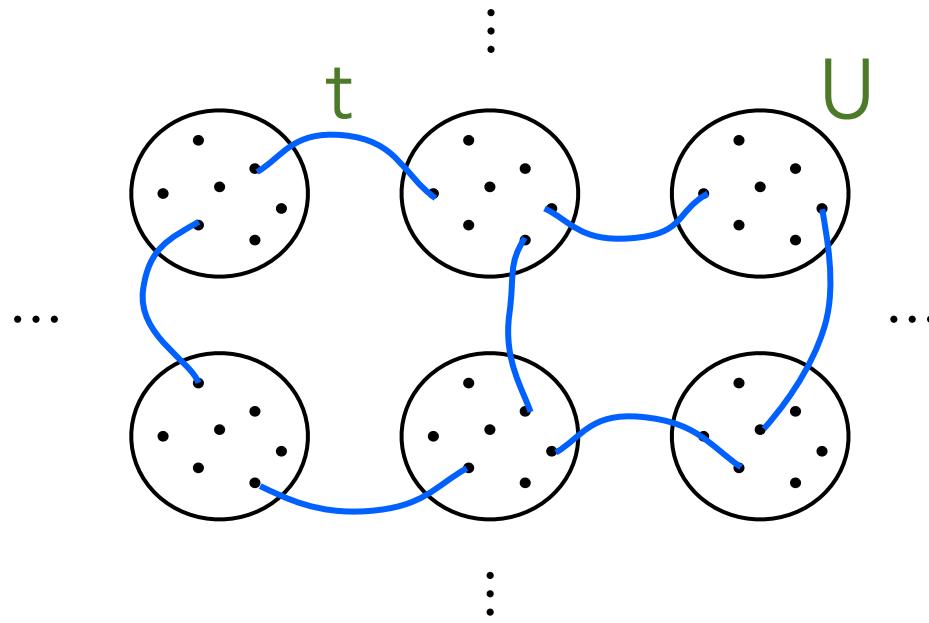
$$\lambda_L = \frac{2\pi k_B T}{\hbar}$$

Holography



slide from D. Stanford, IAS, 2017

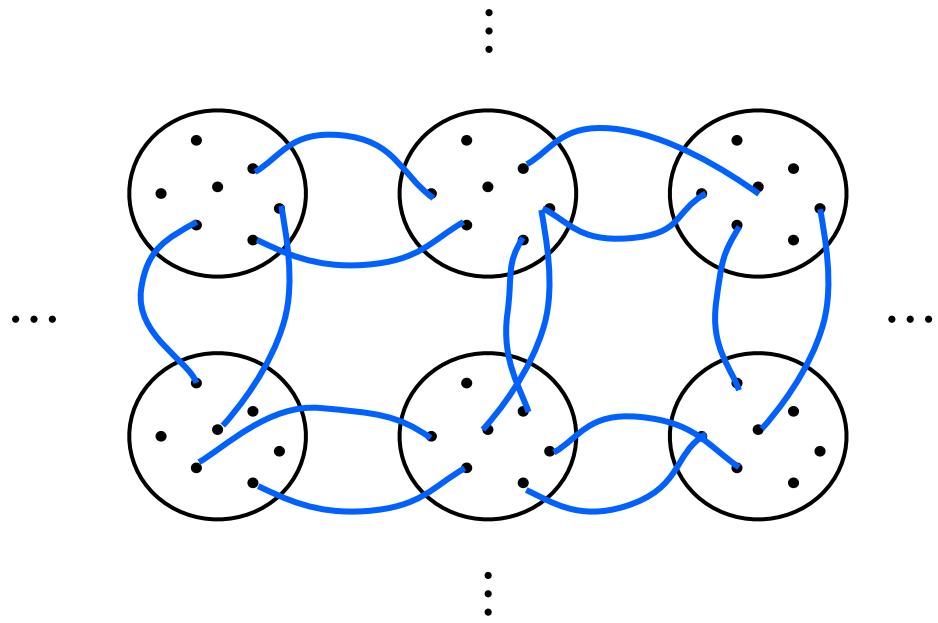
Building a metal



$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$

$$\overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Building a metal



Other work: 2-electron hopping

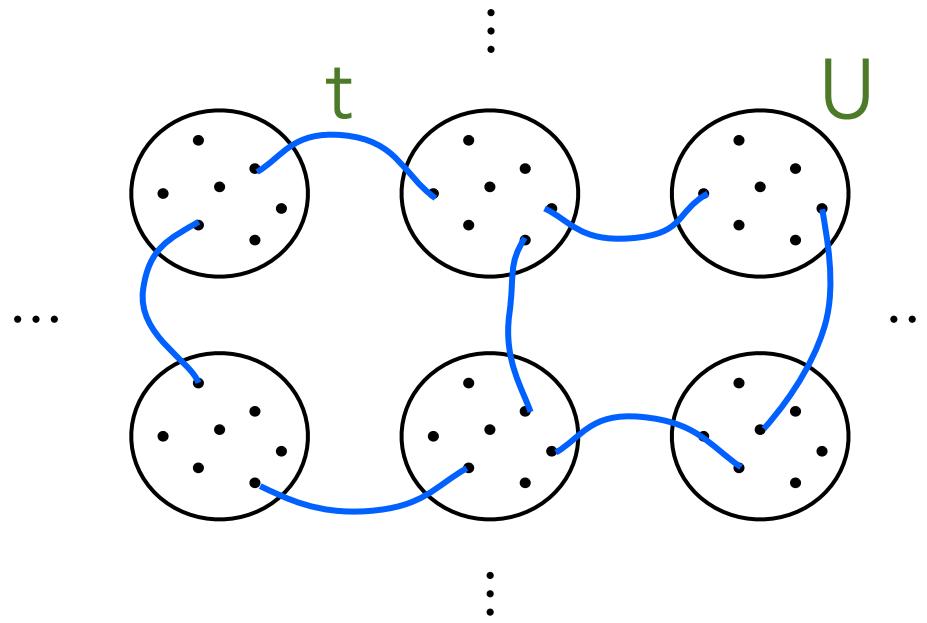
Y.Gu *et al*, arXiv:1609.07832

R. Davison *et al*, arXiv:1612.00849

$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ijkl,xx'} c_{i,x}^\dagger c_{j,x}^\dagger c_{k,x'} c_{l,x'} + \text{h.c.}$$

Omitting *relevant* 1-electron hopping leaves system NFL even at T=0

Building a metal



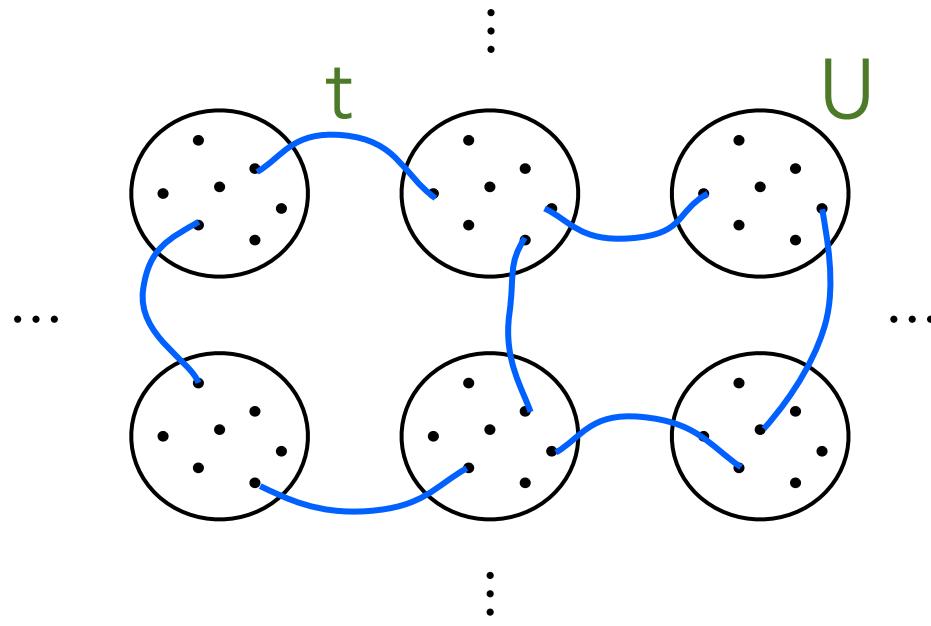
$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$



competition!
 $t/U \ll 1$ interesting



Self-consistent equations



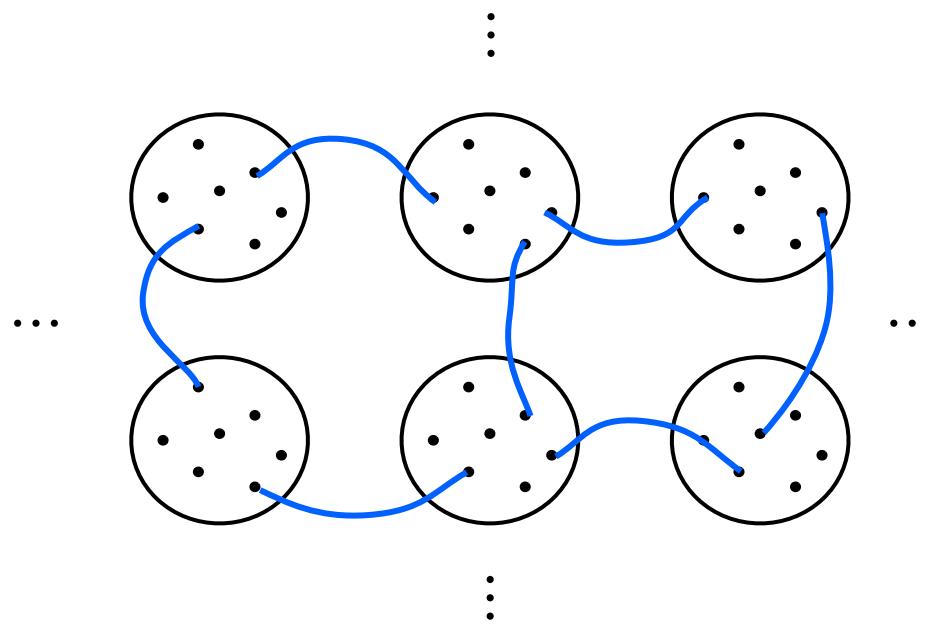
$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$
$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$
$$\bar{G}(i\bar{\omega}) = \tilde{t} G(i\omega)$$



strong similarities to DMFT equations

mathematical structure appeared in early study of doped t-J model with double large N and infinite dimension limits: O. Parcollet+A. Georges, 1999

Coherence scale



Rescaling

$$\bar{\omega} = \frac{\omega}{\tilde{E}_c}, \quad \bar{\tau} = \tau \tilde{E}_c,$$

$$\bar{G}(i\bar{\omega}) = \tilde{t}G(i\omega) \quad \bar{\Sigma}(i\bar{\omega}) = \Sigma(i\omega)/\tilde{t}$$

$$\tilde{t} = \left(\frac{z}{2}\right)^{\frac{1}{2}} t$$

$$\bar{G}(i\bar{\omega}) = \frac{\tilde{t}}{U} i\bar{\omega} - \bar{\Sigma}(i\bar{\omega})$$

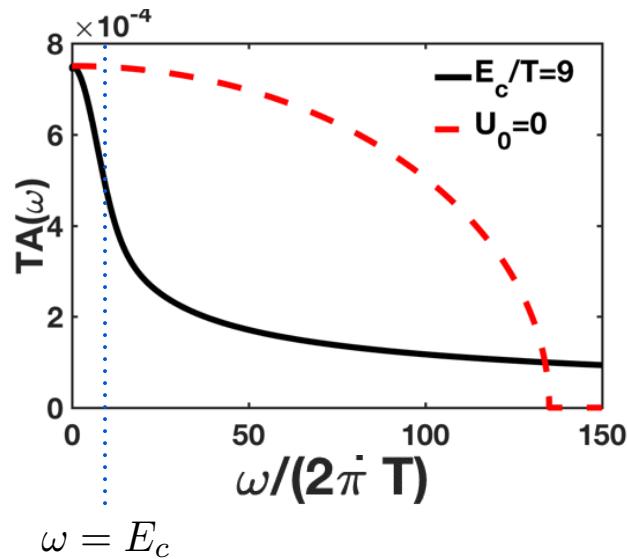
$$\bar{\Sigma}(\bar{\tau}) = -\bar{G}(\bar{\tau})^2 \bar{G}(-\bar{\tau}) + 2\bar{G}(\bar{\tau}),$$

For $t \ll U$, a single universal coherence scale appears

$$\tilde{E}_c = \frac{\tilde{t}^2}{U}$$

Coherence scale

We solve these equations in a real time Keldysh formulation numerically and determine asymptotics analytically.

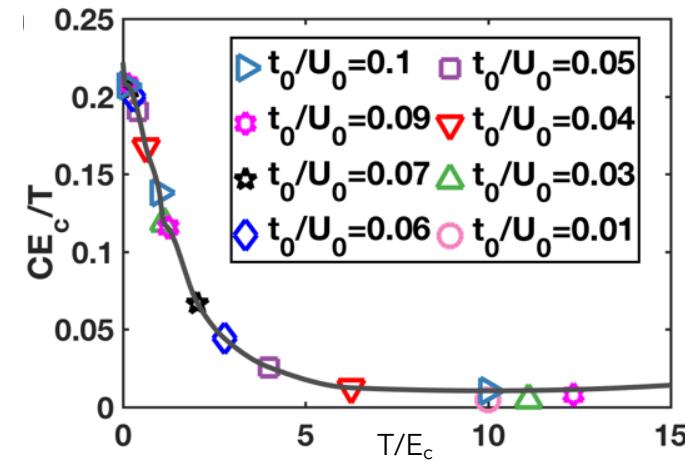
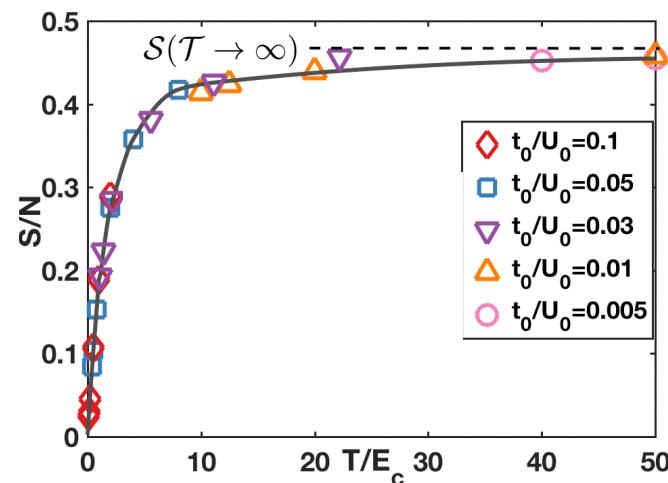


Narrow “coherence peak” appears in spectral function: heavy quasiparticles form for $T \ll E_c$

Quasiparticle weight $Z \sim t/U$

Entropy

Level repulsion: entropy is released for $T < E_c$!



Universal scaling forms

$$S/N = S(T/E_c)$$

$$C/N = T/E_c S'(T/E_c)$$

$$\gamma \equiv \lim_{T \rightarrow 0} \frac{C}{T} = \frac{S'(0)}{E_c}$$

Sommerfeld
enhancement

$$m^*/m \sim U/t$$

Compressibility

For $t \ll U$, compressibility is almost unaffected by hopping

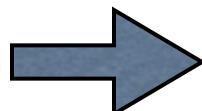
$$K = \left. \frac{\partial \mathcal{Q}}{\partial \mu} \right|_{\mu=0} = \frac{1.04}{U} \ll \gamma \sim \frac{U}{t^2}$$

??How to reconcile with Sommerfeld enhancement??

- Fermi liquid theory: compressibility is renormalized by Fermi liquid parameter $F = g(E_F) U_{FL}$

$$\gamma/K = \frac{\pi^2}{3}(1 + F)$$

$$\gamma/K \sim (U/t)^2$$

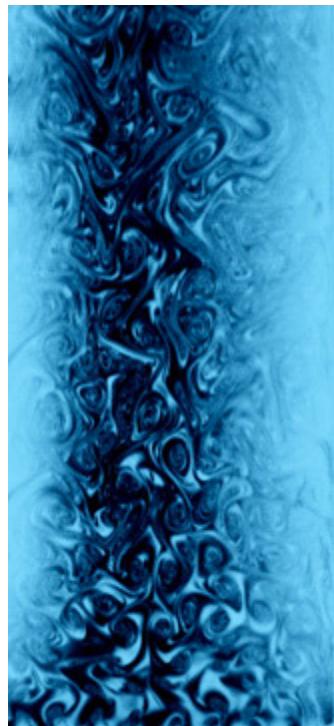


$$F \sim \left(\frac{U}{t} \right)^2 \gg 1$$

Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics



$$\sigma = \lim_{\omega \rightarrow 0} \lim_{p \rightarrow 0} \frac{-i\omega}{p^2} D_{Rn}(p, \omega)$$

- ◆ Calculate density response using Keldysh method.
- ◆ Do analogously for thermal conductivity

N.B. Because of randomness, momentum is not a hydrodynamic variable

Transport

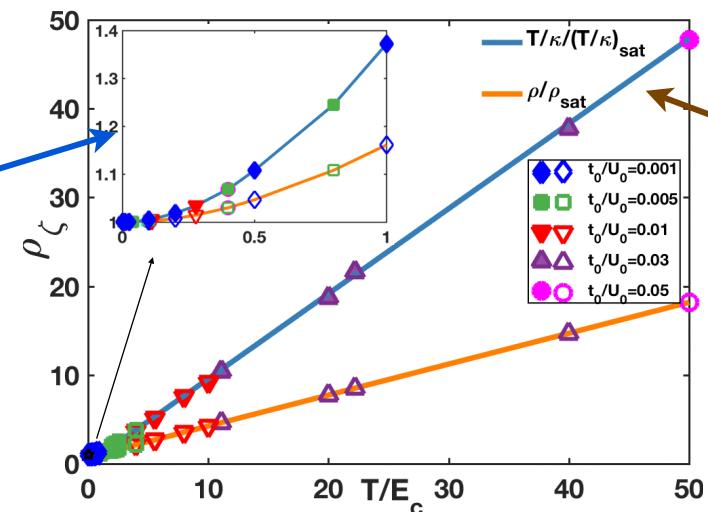
Generalized
resistivity

$$\rho_c = 1/\sigma \quad \rho_e = T/\kappa$$

scaling

$$\rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta \left(\frac{T}{E_c} \right)$$

Fermi liquid
 $R = R_0 + A T^2$
for $T \ll E_c$



Linear in T for
 $E_c \ll T \ll U$

Crossover from heavy FL to strange metal

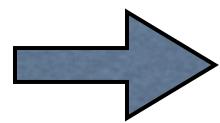
Transport ratios

Kadowaki-Woods

$$\rho_\zeta(T \ll E_c) \approx \rho_\zeta(0) + A_\zeta T^2,$$

$$A \sim 1/(NE_c^2)$$

recall $\gamma \sim 1/E_c$



$$KW = A/(N\gamma)^2 \sim 1/N^3$$

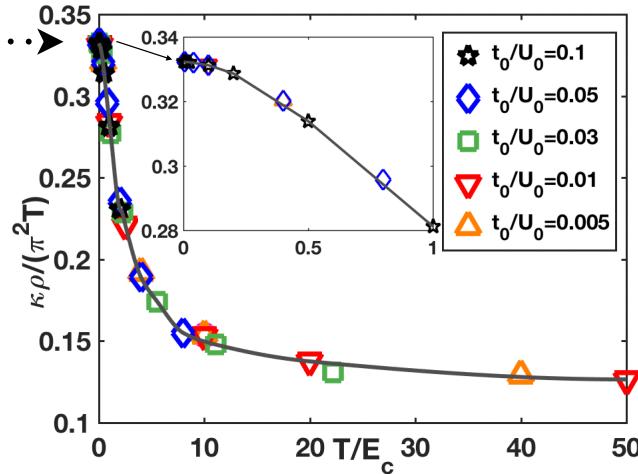
independent of
t, U!

Lorenz

FL

$$\frac{\pi^2}{3}$$

$$\begin{aligned} L &= \frac{\kappa}{\sigma T} \\ &= \mathcal{L}(T/E_c) \end{aligned}$$



FL $\frac{\pi^2}{3}$ NFL $\frac{\pi^2}{8}$



SYK metal

- Small coherence scale $E_c = t^2/U$
- Heavy mass $\gamma \sim m^*/m \sim U/t$
- Small QP weight $Z \sim t/U$
- Kadowaki-Woods $A/\gamma^2 = \text{constant}$
- Linear in T resistivity and T/κ
- Lorenz ratio crosses over from FL to NFL value

Where is this going?

Similarity with DMFT is encouraging: can we bootstrap this to a more realistic treatment of incoherent metals?

- Translational invariant SYK lattice model

$$H = \sum_x \sum_{i < j, k < l} U_{ijkl} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} - t \sum_{\langle xx' \rangle} \sum_i c_{i,x}^\dagger c_{i,x'}$$

- Large N equations

$$\begin{aligned} G(\mathbf{k}, i\omega_n)^{-1} &= i\omega_n - \epsilon(\mathbf{k}) - \Sigma(\mathbf{k}, i\omega_n), \\ \Sigma(\mathbf{x}, \tau) &= -U_0^2 G(\mathbf{x}, \tau)^2 G(-\mathbf{x}, -\tau), \end{aligned}$$

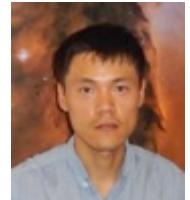
describes NFL to FL crossover.
Similarities to DCA?

- Apply to realistic band structures

This talk

I. SYK model of a strongly correlated metal: FL to NFL crossover and a disordered strange metal

II. Heat transport in spin systems - towards a non-quasiparticle description (progress report)





What's in there?
a QSL...or a chicken?

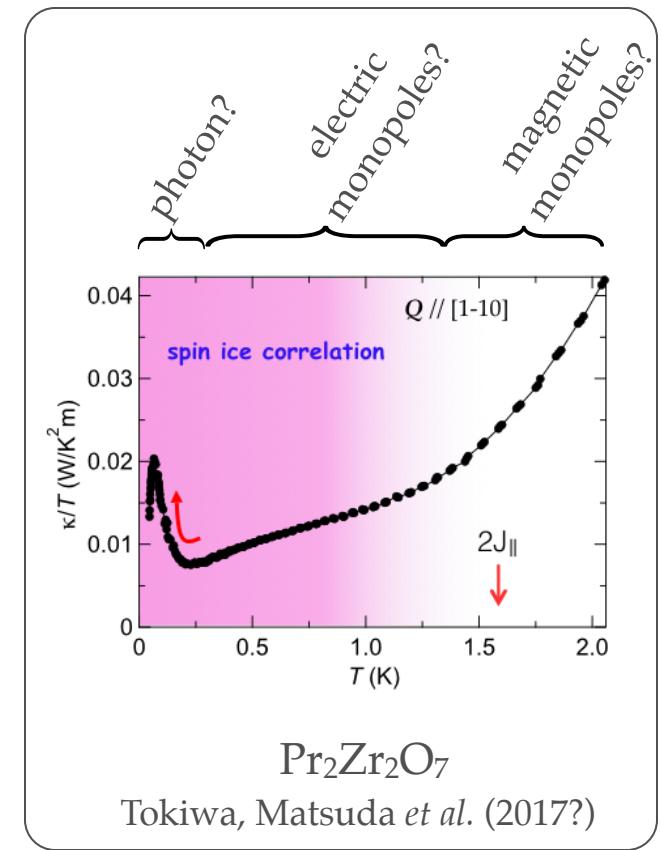
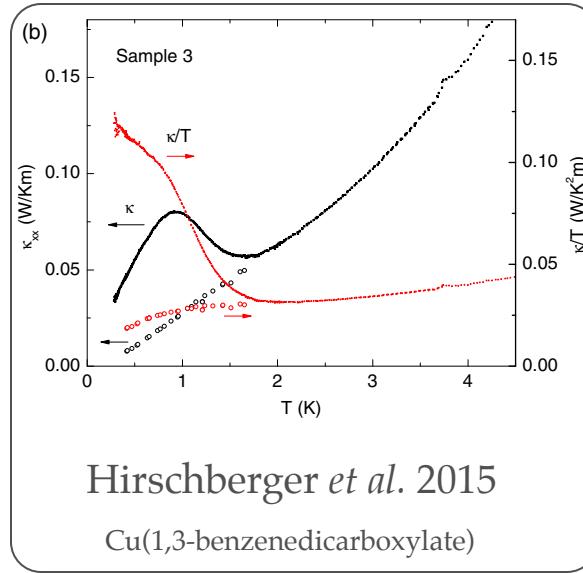
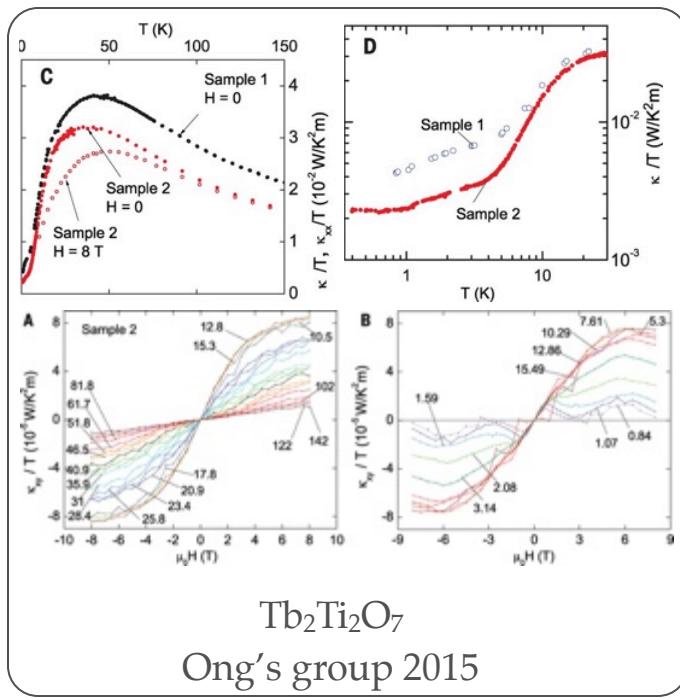
Heat transport in magnets

Insulating magnet: typically heat is the only possible transport measurement

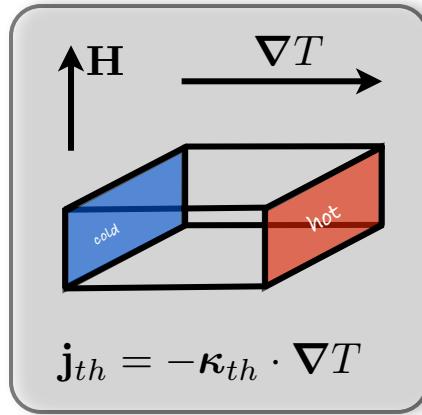
- Probes extended nature of excitations at ultra-low frequency
- Automatically removes contributions from localized nuclei and isolated defects, which may carry large entropy

Heat transport in magnets

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Heat transport in magnets



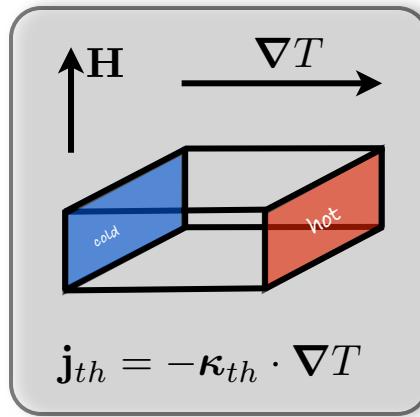
Two types of heat current

$$\mathbf{j} = \mathbf{j}_{\text{spins}} + \mathbf{j}_{\text{lattice}}$$

Two important effects of spins

1. Spin heat current $\mathbf{j}_{\text{spins}}$ induced by thermal gradient
2. Scattering of heat-carrying phonons off of spins

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Quasiparticles?

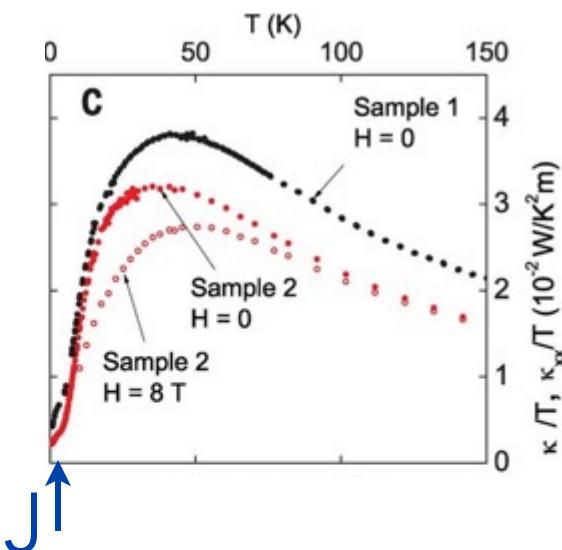
Spin Hamiltonian

$$H = \frac{1}{2} \sum_{\langle ij \rangle} \sum_{\mu\nu} J_{ij}^{\mu\nu} S_i^\mu S_j^\nu$$

- no obvious free particle starting point
- $E_F \rightarrow J$ much smaller

Many (most?) measurements have $T > J$
and are not in the QP regime

e.g. $Tb_2Ti_2O_7$



Luttinger theory

Luttinger (1964) formulated thermal conductivity as response to a fictional “gravitational field”

$$\kappa^{\mu\nu}(\omega) = \frac{-1}{N_{uc} \omega T} \int_0^{+\infty} dt e^{i\omega t} \langle [j^\mu(t), j^\nu(0)] \rangle$$

Challenging: even when equal-time correlations are fully short-range, e.g. $\beta J \ll 1$, the long-time dynamics enters, and this is still hard because $Ht \gg 1$

Spectral representation

longitudinal

$$\frac{1}{2}(\kappa^{\mu\nu} + \kappa^{\nu\mu}) = \frac{\pi}{ZN_{uc} k_B T^2} \sum_{m,n} \langle m | j^\mu | n \rangle \langle n | j^\nu | m \rangle e^{-\beta E_n} \delta(E_n - E_m)$$

Hall

$$\frac{1}{2}(\kappa^{\mu\nu} - \kappa^{\nu\mu}) = \frac{-i}{ZN_{uc} T} \sum_{m,n} \langle m | j^\mu | n \rangle \langle n | j^\nu | m \rangle \frac{e^{-\beta E_n} - e^{-\beta E_m}}{(E_n - E_m)^2}$$

- Useful for ED calculations
- Can prove thermal Hall vanishes unless TRS is broken and SOC present
- Can derive scaling relations

Definition of energy current

see also: H. Lee, J.H. Han, P.A. Lee 2015

similar setup based on Luttinger energy density defined on a bond

Break Hamiltonian into sum of local terms

$$\hat{H} = \sum_{\mathbf{r}} \hat{\rho}_{\mathbf{r}}$$

Continuity equation defines the current

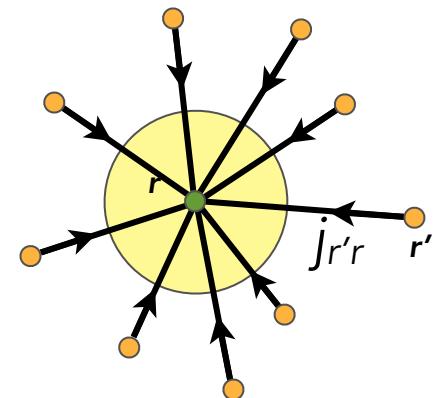
Noether current
(energy conservation):

$$\partial_{\mu} \hat{j}^{\mu} = 0 \quad \text{i.e.} \quad \text{div} \hat{\mathbf{j}} + \partial_t \hat{\rho} = 0$$

Heisenberg equation
of motion:

$$\partial_t \hat{\rho}_{\mathbf{r}} = -i[\hat{H}, \hat{\rho}_{\mathbf{r}}] = -i \sum_{\mathbf{r}'} [\hat{\rho}_{\mathbf{r}'}, \hat{\rho}_{\mathbf{r}}] \equiv \sum_{\mathbf{r}'} j_{\mathbf{r}' \mathbf{r}}$$

in the Kubo formulas, $j = j_{\text{tot}}$



Definition of energy current

$$j_{r' \rightarrow r} = -i [\hat{\rho}_{r'}, \hat{\rho}_r]$$

What does this look like?

$$\hat{\rho} \sim JSS + BS \quad \rightarrow$$

$$j \sim JS \times (JSS + BS)$$

"velocity"
"energy"

explicit forms depend heavily on lattice structure and interactions

Scaling

- Knowing form of current and Luttinger response formulae, we find

$$C^{jj}(t) = \langle j(t)j(0) \rangle \sim J^4/T f(Jt, T/J, B/T) = BJ^3/T \tilde{f}(Jt, T/J)$$

$$\kappa_H \sim BJ^2/T^2 \int dt f(t, T/J)$$

$$\kappa_{\text{long}} \sim J^3/T^2 \int dt f(t, T/J)$$

Scaling

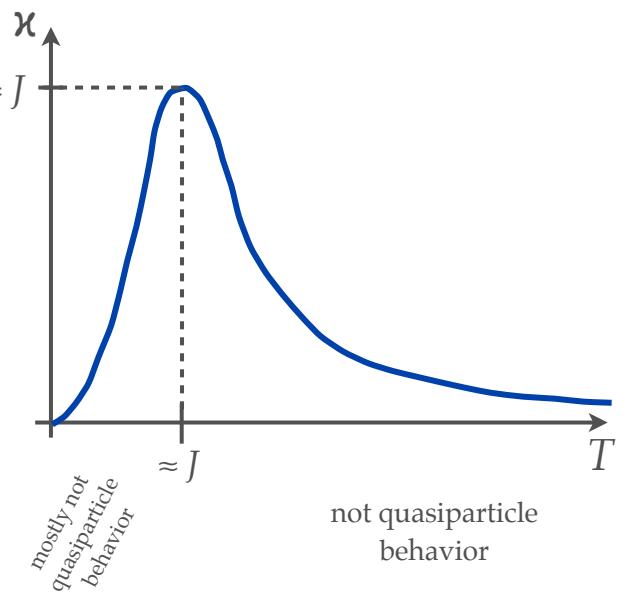
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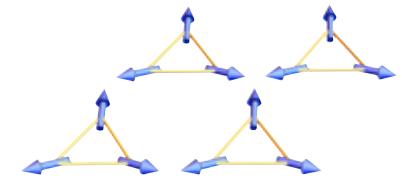
$$\kappa_H \sim BJ^2/T^2$$

$$\kappa_{\text{long}} \sim J^3/T^2$$

expect
constant at
high T

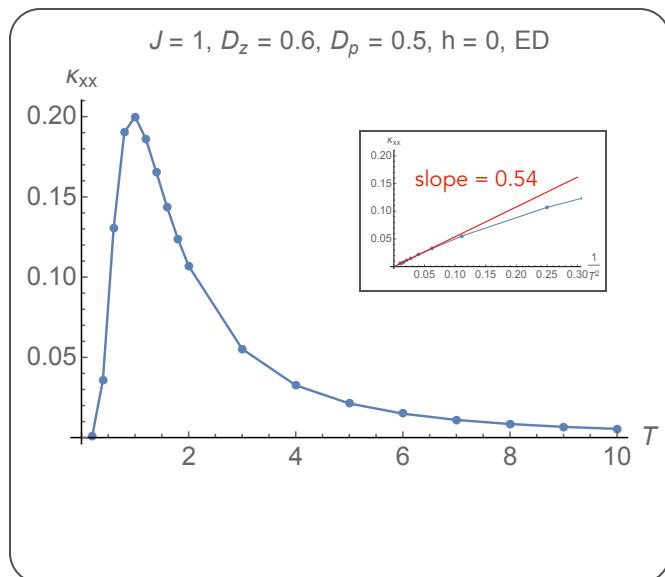


Calculations

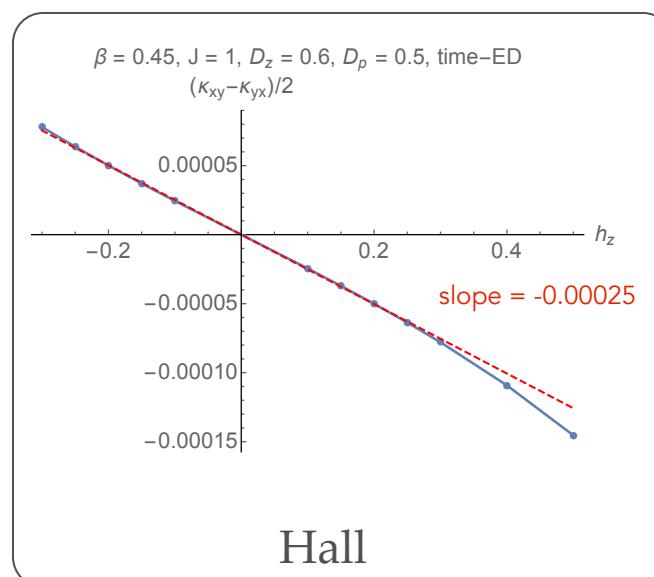


- 2x2 system (12 spins) set $J = 1, D_z = 0.6, D_{\perp} = 0.5$

- First ED calculations for the kagomé lattice



longitudinal $H=0$



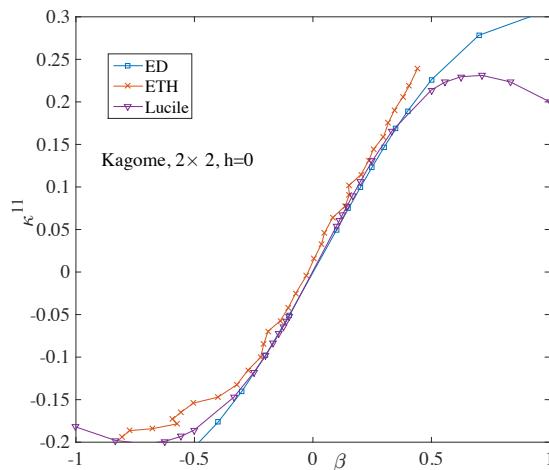
linear in H at small H

Features
promising
but *finite*
size effects??

Typical states method

$$\kappa^{\mu\nu}(\omega) = \frac{-1}{N_{uc} \omega T} \int_0^{+\infty} dt e^{i\omega t} \langle [j^\mu(t), j^\nu(0)] \rangle$$

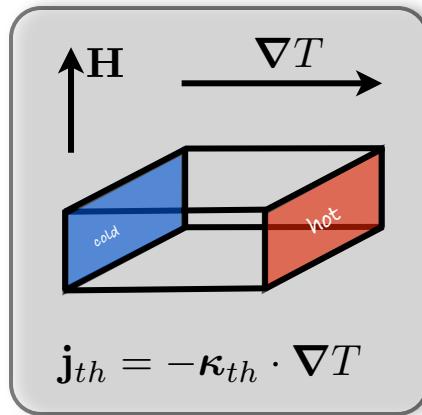
- Based on ETH, we expect that the thermodynamic average can be replaced by an average over a small number of “typical states” within some energy window
- Time evolve these states directly
- This method does not require full diagonalization.



Benchmark: reproduces ED results for upper temperature range

will allow up to 18-21 spins

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Phonon heat transport

Can use Boltzmann equation for phonons

$$\partial_t \bar{N}_{n,\mathbf{k}} + \mathbf{v}_{n,\mathbf{k}} \cdot \nabla_{\mathbf{r}} \bar{N}_{n,\mathbf{k}} = \left. \frac{\partial \bar{N}_{n,\mathbf{k}}}{\partial t} \right|_{\text{coll.}} \quad \text{scattering from spins}$$

Spin-lattice interactions

$$H_{\text{int}} \sim a^\dagger (S^\mu + S^\mu S^\nu + \dots) + a^\dagger a (S^\mu + S^\mu S^\nu + \dots) + a^\dagger a^\dagger (S^\mu + S^\mu S^\nu + \dots) + \text{h.c.} + \dots$$



Jahn-Teller coupling: non-Kramers ions

Phonon heat transport

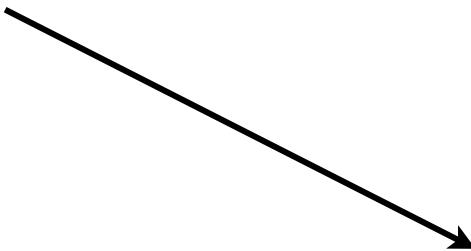
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scattering from spins

Spin-lattice interactions

$$H_{\text{int}} \sim [a^\dagger (S^\mu + S^\mu S^\nu + \dots)] + a^\dagger a (S^\mu + S^\mu S^\nu + \dots) + a^\dagger a^\dagger (S^\mu + S^\mu S^\nu + \dots) + \text{h.c.} + \dots$$



$$H_{\text{int}} = \sum_{\mathbf{k}, n} \sum_{\mathbf{r}, \alpha} \lambda_{n\alpha}(\mathbf{k}, \mathbf{r}) a_{\mathbf{k}, n}^\dagger \mathcal{O}_{\mathbf{r}, \alpha} + \text{h.c.}$$

Born approximation

transition probability

$$\Gamma_{\text{if}} = \frac{2\pi}{\hbar} |T_{\text{if}}|^2 \delta(E_{\text{f}} - E_{\text{i}}) p_{\text{i}}$$

spin-phonon product states

transition amplitude

transition amplitude

$$T_{\text{if}} = \langle \text{f} | H_{\text{int}} | \text{i} \rangle + \sum_{\text{n}} \frac{\langle \text{f} | H_{\text{int}} | \text{n} \rangle \langle \text{n} | H_{\text{int}} | \text{i} \rangle}{E_{\text{n}} - E_{\text{i}}}$$

first Born approximation



second Born approximation



trace over spins

$$\Gamma_{i_p, f_p} = \sum_{i_s, f_s} p(E_{i_s}) \Gamma_{i_p, i_s; f_p, f_s}$$

does not lead to a Hall effect

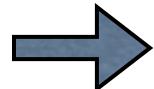
First Born Approximation

$$\frac{\partial \bar{N}_{n,\mathbf{k}}}{\partial t} \Big|_{\text{coll.}} = -\Gamma_{n,\mathbf{k}} \left(\frac{\bar{N}_{n,k}}{N_{n,k}^{\text{eq}}} - 1 \right)$$

$$\Gamma_{n,\mathbf{k}} = 2\pi \sum_{\alpha\alpha'} \lambda_{n\alpha}(\mathbf{k}) \lambda_{n\alpha'}^*(\mathbf{k}) G_{\alpha\alpha'}(\mathbf{k}, \omega_{n,k})$$

$$G_{\alpha\alpha'}(\mathbf{k}, \omega) = \frac{1}{2\pi} \sum_{rr'} e^{ik \cdot (r-r')} \int_{-\infty}^{\infty} dt e^{-i\omega t} \left\langle \mathcal{O}_{r',\alpha'}^\dagger(t) \mathcal{O}_{r,\alpha}(0) \right\rangle_\beta$$

Single-phonon emission/absorption rate measures dynamical structure factor of spin-lattice operator along the phonon dispersion relation



$$\kappa_{\mu\nu}^{\text{ph}} = \frac{1}{V} \sum_{n,\mathbf{k}} \Gamma_{n,\mathbf{k}}^{-1} v_{n\mathbf{k}}^\mu v_{n\mathbf{k}}^\nu \frac{\epsilon_{n\mathbf{k}}^2}{4T^2} \frac{1}{\sinh^2(\beta\epsilon_{n\mathbf{k}}/2)}$$

symmetric: no
Hall effect

2nd Born Approximation

Second order scattering violates *detailed balance*: allows “skew-scattering” processes that induce Hall effect.

c.f. Mori, Spencer-Smith, Sushkov, Maekawa 2014

General expression is complicated, but

$$\Gamma_{nk,n'k'} \sim FT \left[\langle \mathcal{O}_{r_4}(t_4) \mathcal{O}_{r_3}^\dagger(t_3) \mathcal{O}_{r_2}(t_2) \mathcal{O}_{r_1}^\dagger(t_1) \rangle \right]$$

intriguing “out of time ordered” correlator of spin system controls phonon skew scattering

challenging to calculate but in progress...

Summary

- I. SYK model of a strongly correlated metal: FL to NFL crossover and a disordered strange metal
- II. Heat transport in spin systems - towards a non-quasiparticle description (progress report)



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