Towards QSLs and SPTs in the real world

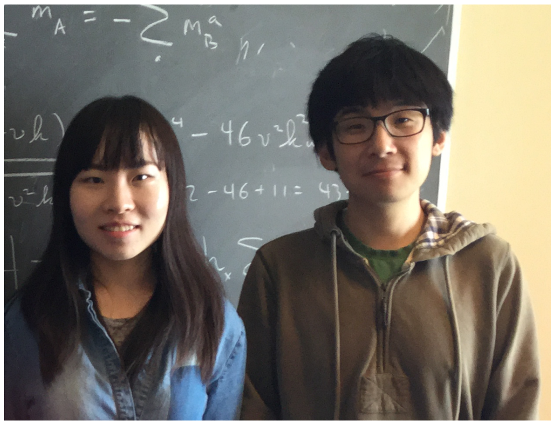
topmat16, Dresden

Leon Balents, KITP

Topics

- Low energy structure factor of Kitaev's gapless QSL
- A bosonic SPT phase in experiment

The young guns



Xueyang Song

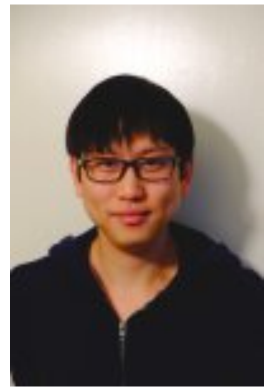
”A □ □

3rd yr ugrad
Peking U.

Yi-Zhuang You

“Yĺ ⌜

postdoc
UCSB



Zhen Bi

□ □

grad
UCSB

Cenke Xu

许岑珂

faculty
UCSB

\+ Ruixing Zhang, Penn State
Andrea Young, UCSB
Chao-Xing Liu, Penn State

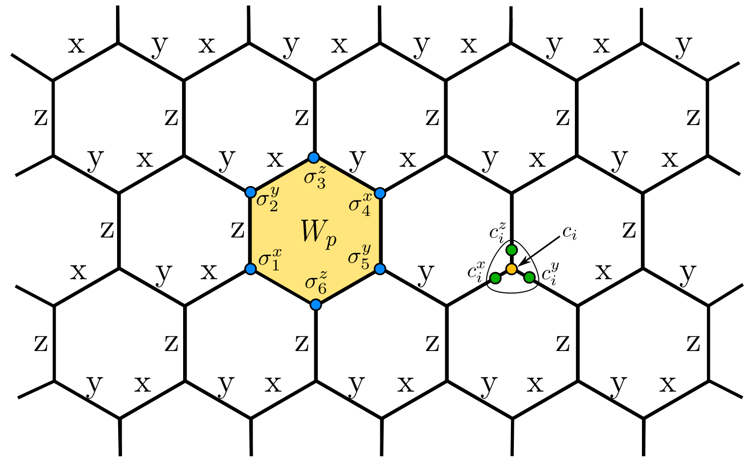
Part 1

Kitaev QSL

Kimchi, Jackeli, Lemmens, Nagler, Hermanns, Manna, Valenti

We heard a lot about this

$$H = K \sum_{\langle ij \rangle, \mu} \sigma_i^\mu \sigma_j^\mu$$



partons

$$\sigma_i^\mu = i c_i c_i^\mu \qquad c_i c_i^x c_i^y c_i^z = 1$$

fluxes $W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^6$ =+1 in ground state

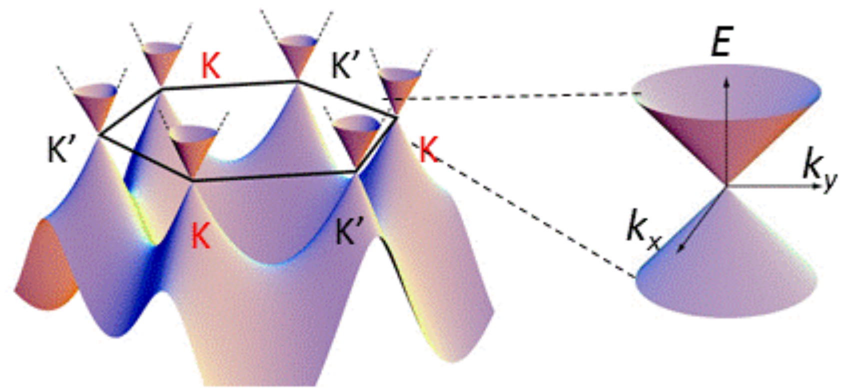
physical Majoranas

$$H_{\text{eff}} = K \sum_{\langle ij \rangle} i c_i c_j$$

Gapless Majoranas

$$H_c = K \sum_{\langle ij \rangle} i c_i c_j$$

Fourier $\longrightarrow$

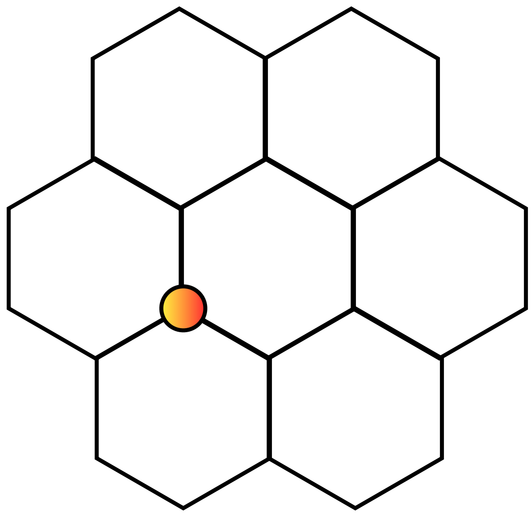


two Majorana cones = 1 Dirac cone

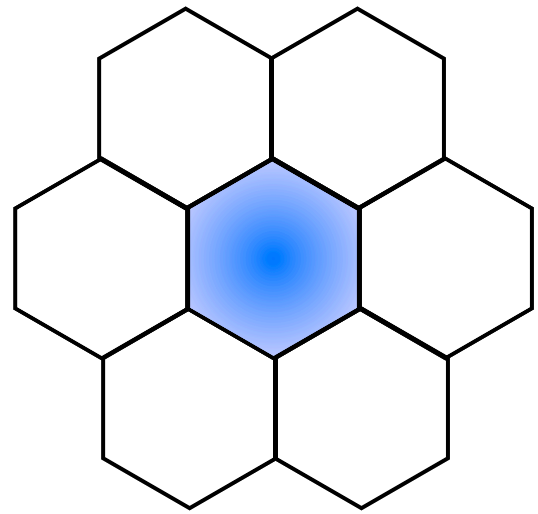
Low energy

$$H_{\text{eff}} = iv \int d^2x \, \psi^\dagger (\tau^x \partial_x + \tau^y \partial_y) \psi$$

All the excitations



Majorana ε

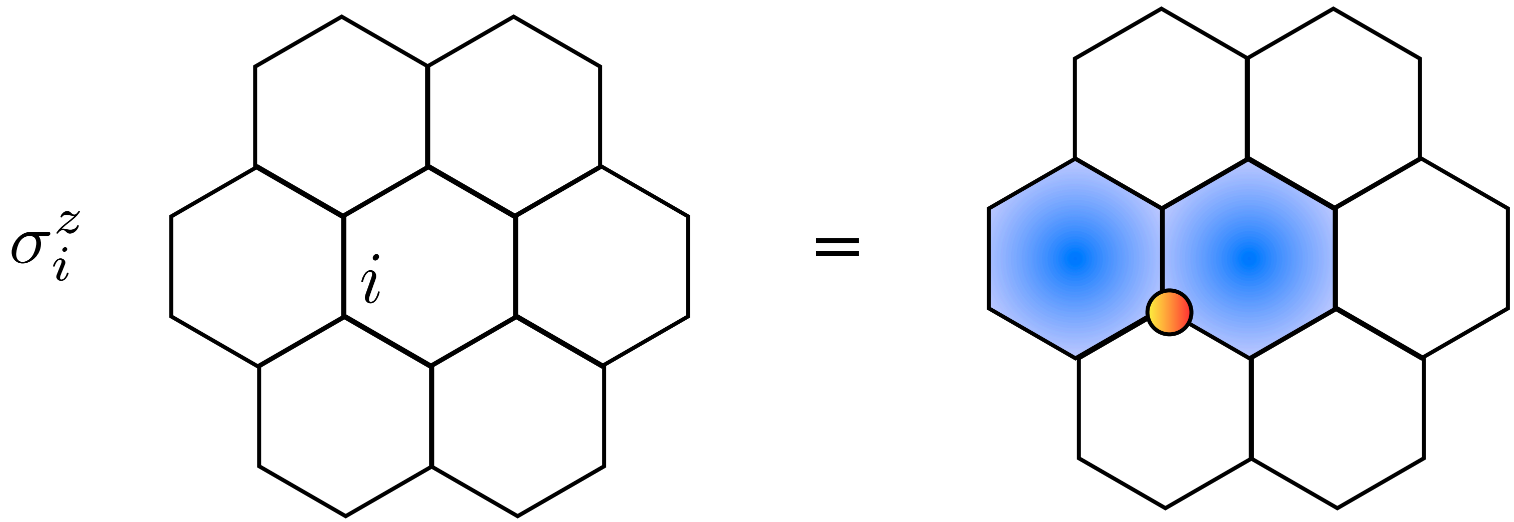


Flux e, m

In Kitaev's model:

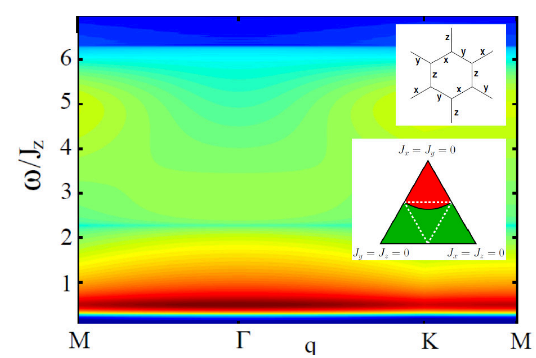
- Majorana's dispersion ~ K and gapless
- Fluxes are localized and gapped

Spin correlations



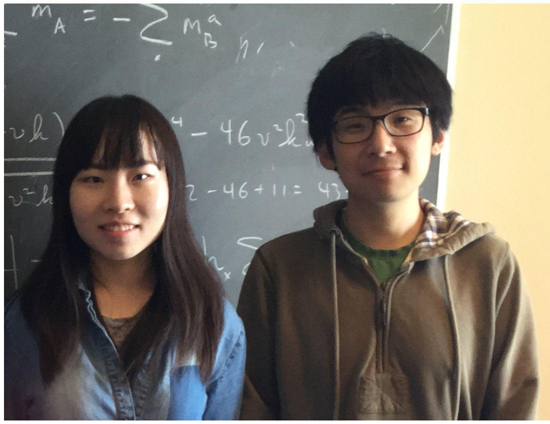
Because fluxes are created

- Spectral weight is zero below the flux gap
- Correlations vanish beyond NNs



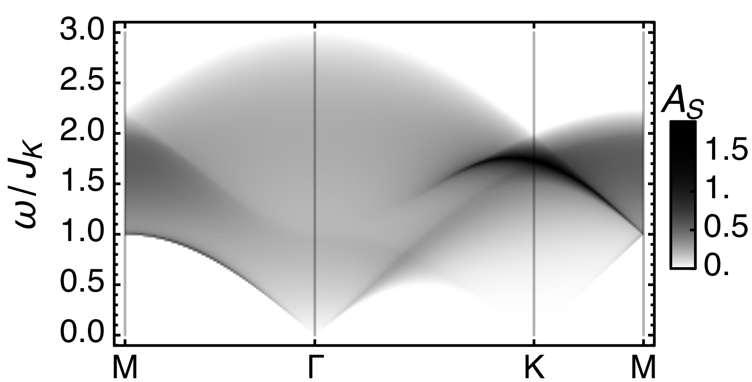
Universality

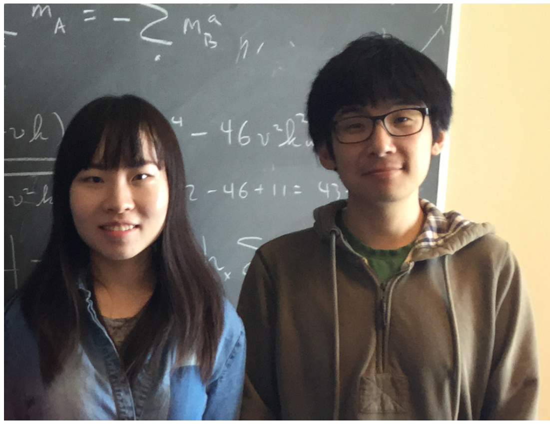
- We know the gapless QSL is locally stable provided time-reversal is maintained, *but* is this the generic behavior?
  - NN correlations? Obviously extended by perturbations.
  - Gap? This is less obvious. Is there a selection rule?



Answer

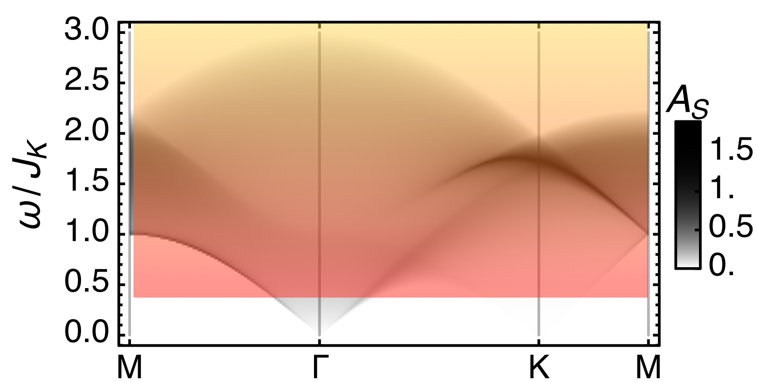
- Generically, there is *not* a gap in the structure factor
- Instead, power-law weight appears within two Dirac cones centered around k=0 and k=K





Answer

- Generically, there is *not* a gap in the structure factor
- Instead, power-law weight appears within two Dirac cones centered around k=0 and k=K



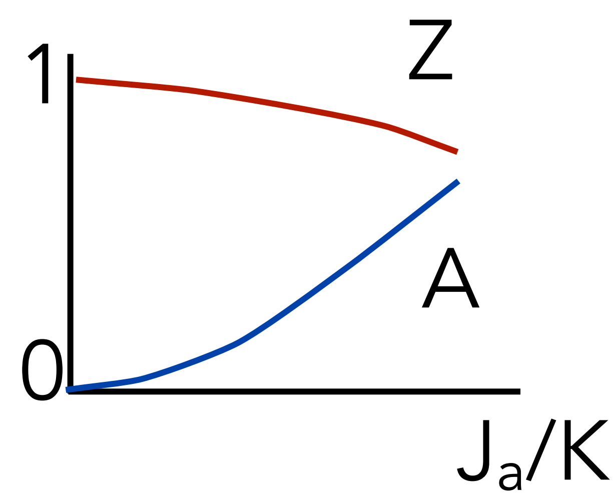
Why?

- Quasiparticles
  - A lattice operator can be expanded in a series of quasiparticle operators, which create exact eigenstates

$$\sigma_i^\mu = Z\, i c_i c_i^\mu + A\, i \epsilon^{\mu\nu\lambda} c_{i+\hat{\nu}} c_{i+\hat{\lambda}} + \cdots$$

above the gap below the gap

$$\sigma \sim \varepsilon\text{em} + \varepsilon\varepsilon + \cdots$$



Microscopic origin

- A simple view: perturbations to Kitaev mix virtual excitations into ground state, which can cancel the flux introduced by naive spin operator
- Surprisingly, this *does not* occur for the Heisenberg-Kitaev model due to "dihedral" symmetry

$$X, Y, Z = \prod_i \sigma_i^\mu$$

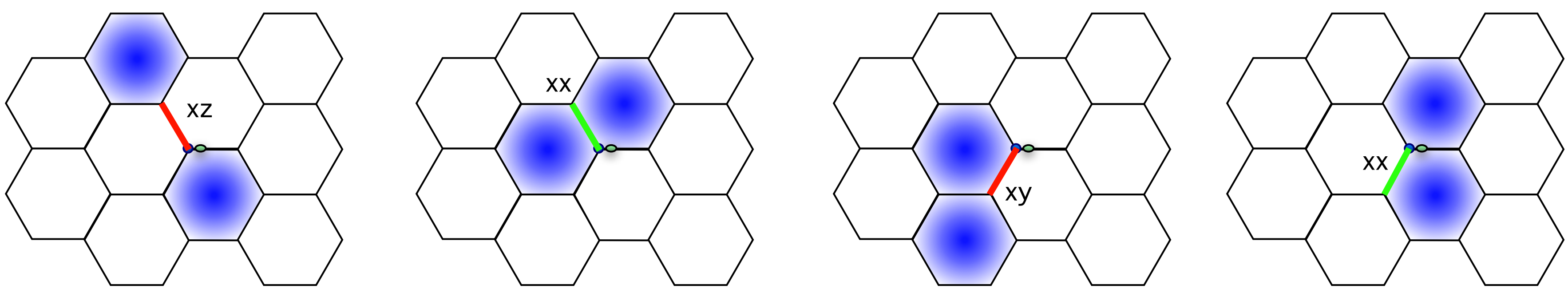
every spin is odd under 2 of these generators

Microscopic origin

- A simple view: perturbations to Kitaev mix virtual excitations into ground state, which can cancel the flux introduced by naive spin operator

$$H = \sum_{\langle ij \rangle \in \alpha\beta(\gamma)} [J\vec{S}_i \cdot \vec{S}_j + K S_i^\gamma S_j^\gamma + \Gamma(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha)]$$

Rau, Lee, Kee



$$A \sim J^2 \Gamma^2$$

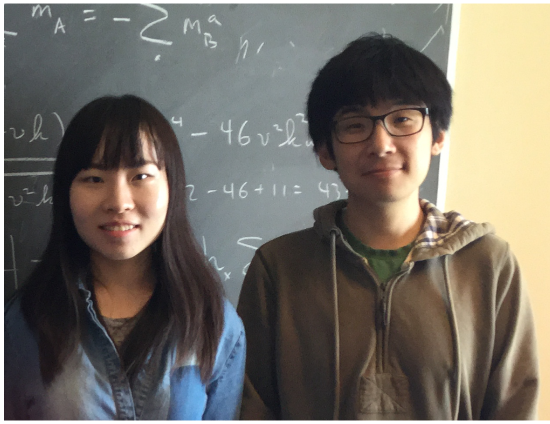
Field theory

- Highbrow picture: effective field theory
  - A lattice operator can be expanded at low energy in a series of "primary fields". The coefficient are constrained by symmetry and depend on microscopics

$$\sigma_i^\mu \sim M_{s(i)}^\mu(\boldsymbol{x}_i) + \mathrm{Re}\left[N_{s(i)}^\mu(\boldsymbol{x}_i) e^{i\boldsymbol{K}\cdot\boldsymbol{x}_i}\right]$$

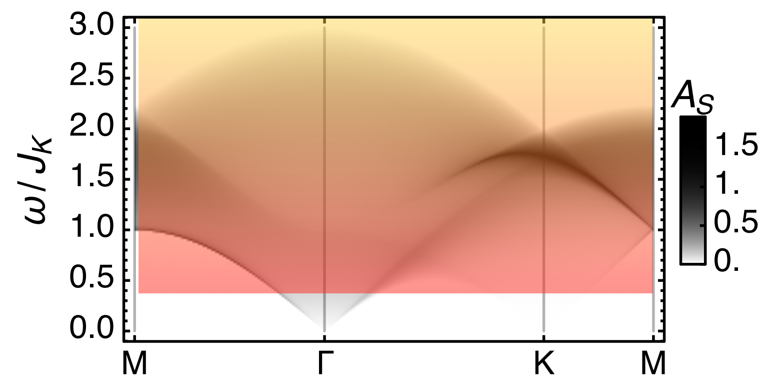
$$M_{s(i)}^\mu \sim \psi^\dagger \psi \qquad\qquad N_{s(i)}^\mu \sim \psi \partial \psi$$

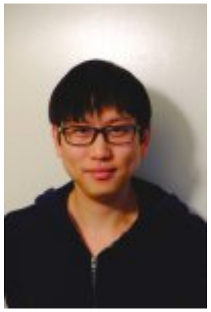
- Amusing similarity to 1d Heisenberg chain



Answer

- Generically, there is *not* a gap in the structure factor
- Instead, power-law weight appears within two Dirac cones centered around k=0 and k=2K





Part 2

SPT phases

Symmetry protected topological order

From Wikipedia, the free encyclopedia

Symmetry Protected Topological order (**SPT** order)[1] is a kind of order in zero-temperature quantum-mechanical states of matter that have a symmetry and a finite energy gap.

To derive the results in a most-invariant way, renormalization group methods are used (leading to equivalence classes corresponding to certain fixed points).[1] The SPT order has the following defining properties:

(a) *distinct SPT states with a given symmetry cannot be smoothly deformed into each other without a phase transition, if the deformation preserves the symmetry.*
(b) *however, they all can be smoothly deformed into the same trivial product state without a phase transition, if the symmetry is broken during the deformation.*

Using the notion of quantum entanglement, we can say that SPT states are short-range entangled states *with a symmetry* (by contrast: for long-range entanglement see topological order, which is not related to the famous EPR paradox). Since short-range entangled states have only trivial topological orders we may also refer the SPT order as Symmetry Protected "Trivial" order.

Contents [hide]

A list of bosonic SPT states from group cohomology $H^{d+1}[G,U(1)] \oplus_{k=1}^{d} H^k[G,iTO^{d+1-k}]$ (Z_2^T = time-reversal-symmetry group)

symm. group	1+1D	2+1D	3+1D	4+1D	comment
0	0	Z	0	Z_2	iTO phases with no symmetry: iTO^{d+1}
$U(1) \rtimes Z_2^T$	Z_2	Z_2	$2Z_2 + Z_2$	$Z \oplus Z_2 + Z$	bosonic topological insulator
Z_2^T	Z_2	0	$Z_2 + Z_2$	0	bosonic topological superconductor
Z_n	0	Z_n	0	$Z_n + Z_n$	
$U(1)$	0	Z	0	$Z + Z$	2+1D: quantum Hall effect
$SO(3)$	Z_2	Z	0	Z_2	1+1D:Haldane phase; 2+1D: spin Hall effect
$SO(3) \times Z_2^T$	$2Z_2$	Z_2	$3Z_2 + Z_2$	$2Z_2$	
$Z_2 \times Z_2 \times Z_2^T$	$4Z_2$	$6Z_2$	$9Z_2 + Z_2$	$12Z_2 + 2Z_2$	

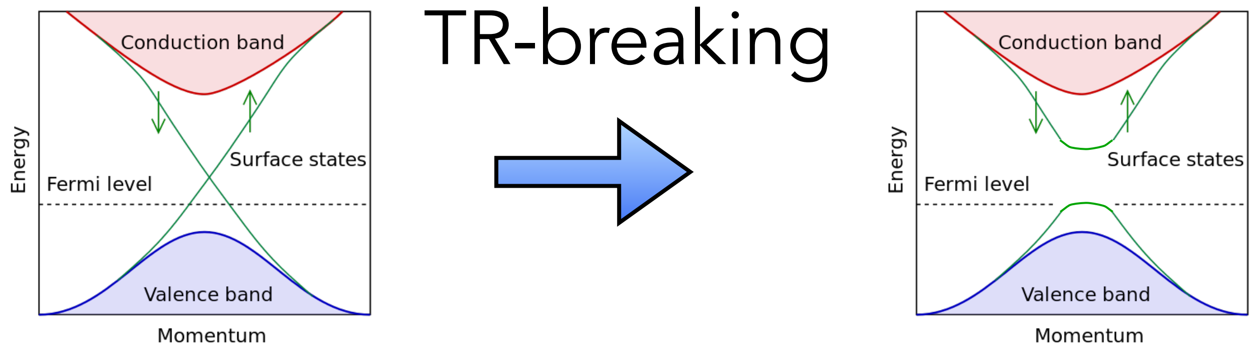
A big subject for theorists

SPT phases

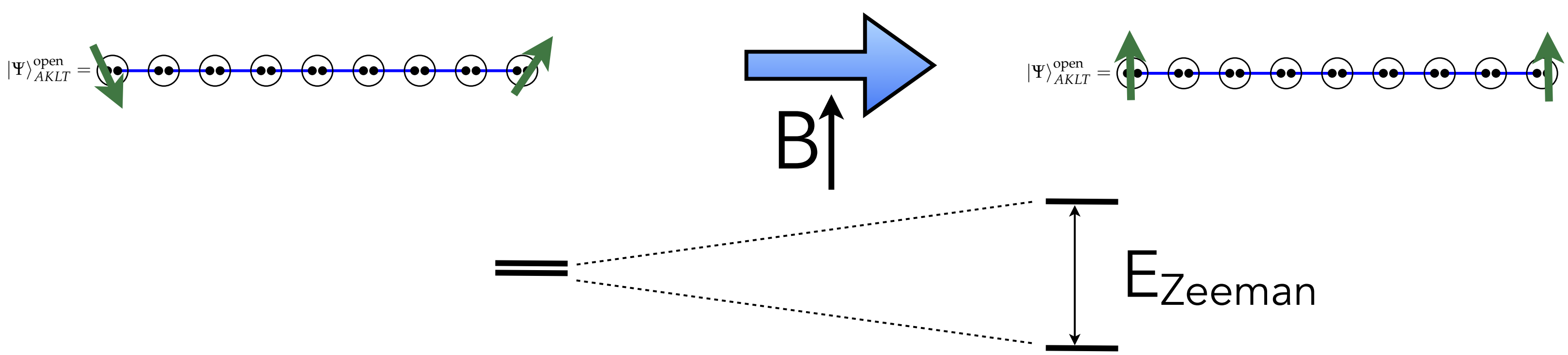
- An SPT phase is:
  - A gapped state which can be deformed to a product state if and only if a symmetry broken during the deformation
  - A state with usually gapless but always anomalous states at its boundary
  - A generalization of topological band insulators to interacting systems, spins, bosons etc.

The examples

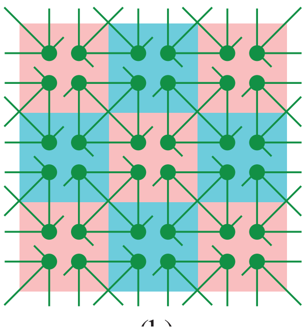
Topological insulator



Haldane/AKLT chain - a bosonic SPT



Bosonic SPTs in d>1?



Chen, Gu, Liu, Wen - *many* tensor network states

classified by $\mathcal{H}^{1+d}[G, U_T(1)]$

Symm. group	$d=0$	$d=1$	$d=2$	$d=3$
Z_2^T	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z}_2$
$Z_2^T \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$
Z_n	$\mathbb{Z}_n$	$\mathbb{Z}_1$	$\mathbb{Z}_n$	$\mathbb{Z}_1$
$Z_n \times \text{trn}$	$\mathbb{Z}_n$	$\mathbb{Z}_n$	$\mathbb{Z}_n^2$	$\mathbb{Z}_n^4$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$U(1) \times \text{trn}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}^4$
$U(1) \rtimes Z_2^T$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$
$U(1) \rtimes Z_2^T \times \text{trn}$	$\mathbb{Z}$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^3$	$\mathbb{Z} \times \mathbb{Z}_2^8$
$U(1) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_1$	$\mathbb{Z}_2^3$
$U(1) \times Z_2^T \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^9$
$U(1) \rtimes Z_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_2$
$U(1) \times Z_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}_1$
$Z_n \rtimes Z_2^T$	$\mathbb{Z}_n$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \times Z_2^T$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \rtimes Z_2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_n \times \mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$
$Z_m \times Z_n$	$\mathbb{Z}_m \times \mathbb{Z}_n$	$\mathbb{Z}_{(m,n)}$	$\mathbb{Z}_m \times \mathbb{Z}_n \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(m,n)}^2$
$D_2 \times Z_2^T = D_{2h}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^6$	$\mathbb{Z}_2^9$
$Z_m \times Z_n \times Z_2^T$	$\mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)} \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(2,m,n)}^2 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m,n)}^4 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$
$SU(2)$	$\mathbb{Z}_1$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3)$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3) \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}^3 \times \mathbb{Z}_2^3$
$SO(3) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$
$SO(3) \times Z_2^T \times \text{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^{12}$

(c) Time reversal & $U(1)_{charge}$ Symmetry: $\mathbf{Z}_2$ classes. Non-chiral Edge.

YM Liu + Vishwanath - K-matrix theory in 2d

$$S = \frac{i}{4\pi} \int d^2x d\tau \; K_{IJ} \epsilon^{\mu\nu\lambda} a_\mu^I \partial_\nu a_\lambda^J$$

all these states have a c=1 Luttinger liquid edge

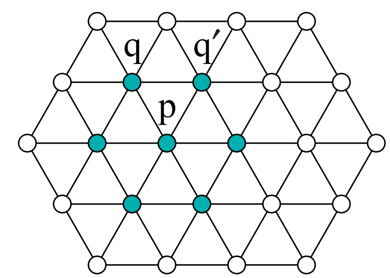
Any models?

- Tensor network constructions

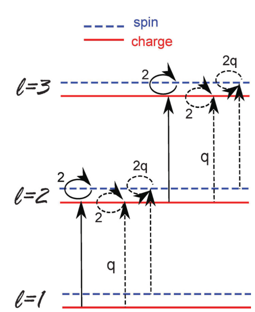
$$H_i = \sigma_i^{+}\eta_{21}^{+}\eta_{32}^{+}\eta_{43}^{+}\eta_{45}^{+}\eta_{56}^{+}\eta_{61}^{+} + \sigma_i^{-}\eta_{21}^{-}\eta_{32}^{-}\eta_{43}^{-}\eta_{45}^{-}\eta_{56}^{-}\eta_{61}^{-},$$

- Levin-Gu model

$$H_1 = -\sum_p B_p, \quad B_p = -\sigma_p^x \prod_{\langle pqq'\rangle} i^{\frac{1-\sigma_q^z\sigma_{q'}^z}{2}},$$



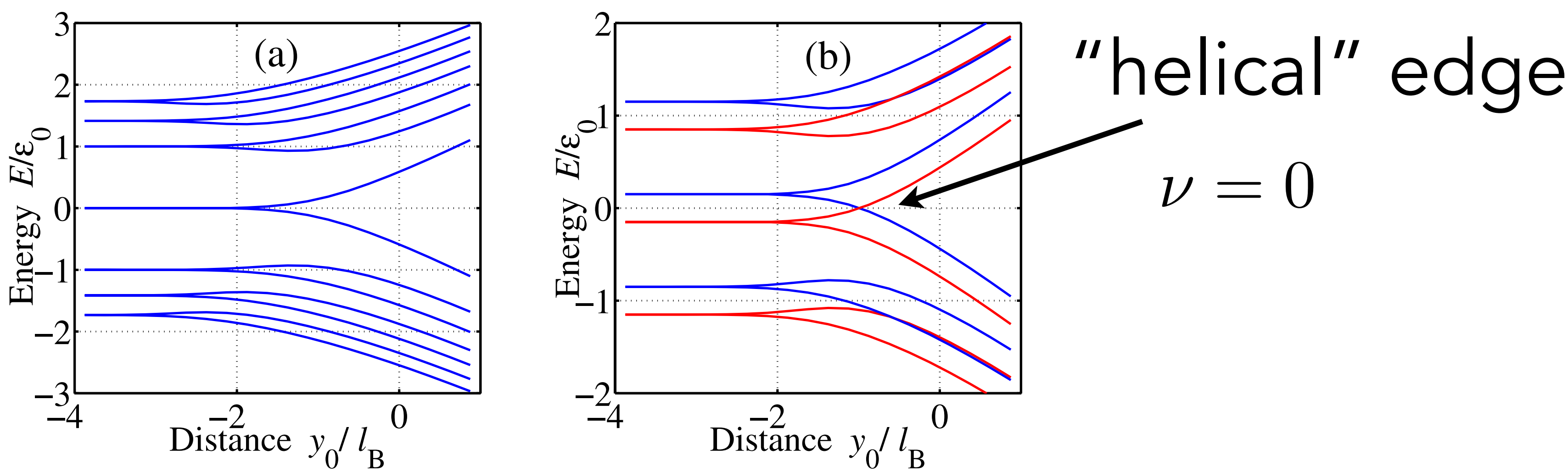
- Coupled wires



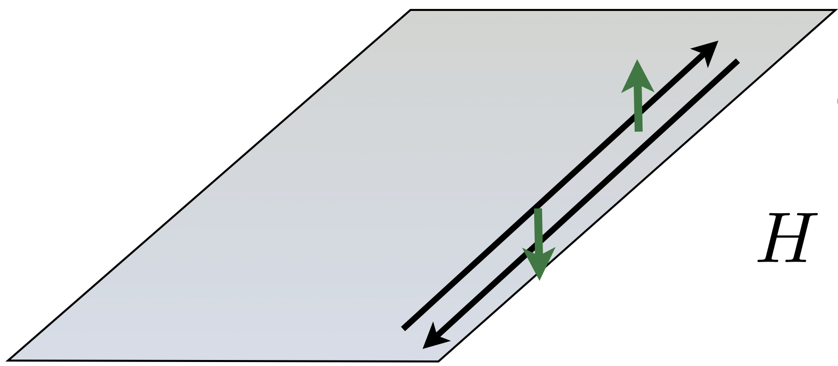
Pretty hard to realize

Graphene

- Kane+Mele: QSHE at zero field in graphene from SOC - but tiny effect
- Abanin, Lee, Levitov: “fake” QSHE in graphene due in quantum Hall regime



Graphene "QSHE"



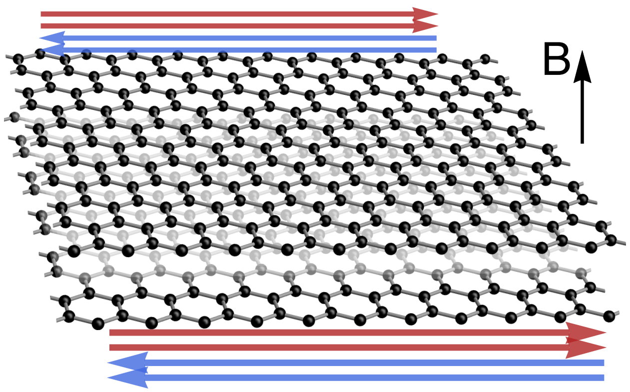
edge is spin-momentum locked

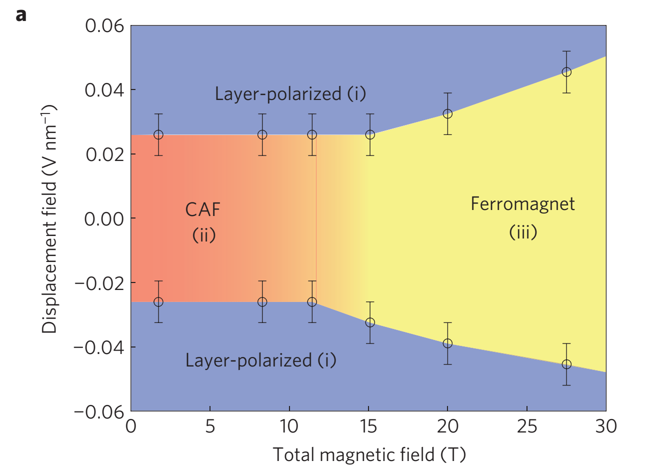
$$H = iv \int dx \left[\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L\right]$$
$$= iv \int dx \left[\psi_\uparrow^\dagger \partial_x \psi_\uparrow - \psi_\downarrow^\dagger \partial_x \psi_\downarrow\right]$$

This is a "Fermionic SPT"

- Backscattering is prohibited by spin-conservation symmetry (excellent approximation since SOC weak)

Bilayer graphene



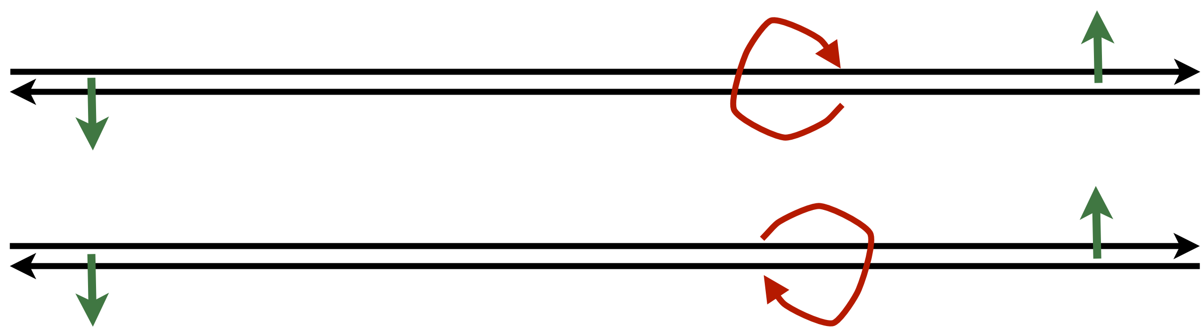


Maher *et al*, 2013

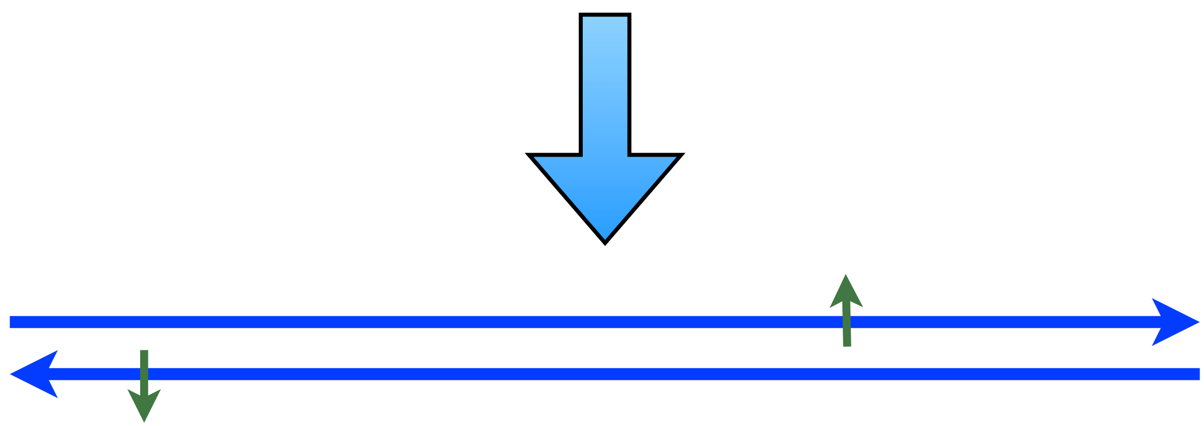
If spin is conserved, this is characterized by spin Chern number 2

Edge has two helical fermionic edge states

Interactions



backscattering $H_{\rm bs} = g \int dx \left[\psi^\dagger_{1R}\psi_{1L}\psi^\dagger_{2L}\psi_{2R} + \text{h.c.} \right]$



a single bosonic helical edge

How to get this

- Bosonization $\psi_{a,L/R}, \psi^{\dagger}_{a,L/R} \to \theta_a, \phi_a$
- Rotate $\theta_{\pm} = \theta_1 \pm \theta_2, \phi_{\pm} = \frac{1}{2}(\phi_1 \pm \phi_2)$
- Interaction induces gap for "-" sector

$$H_{bs} \sim g \int \cos 2\phi_{-}$$

- Only symmetric sector remains

$$H_{\text{eff}} = \int dx \left[\frac{v}{2K} (\partial_x \theta)^2 + \frac{vK}{2} (\partial_x \phi)^2 \right]$$

SPT?

$$H_{\text{eff}} = \int dx \left[\frac{v}{2K} (\partial_x \theta)^2 + \frac{vK}{2} (\partial_x \phi)^2 \right]$$

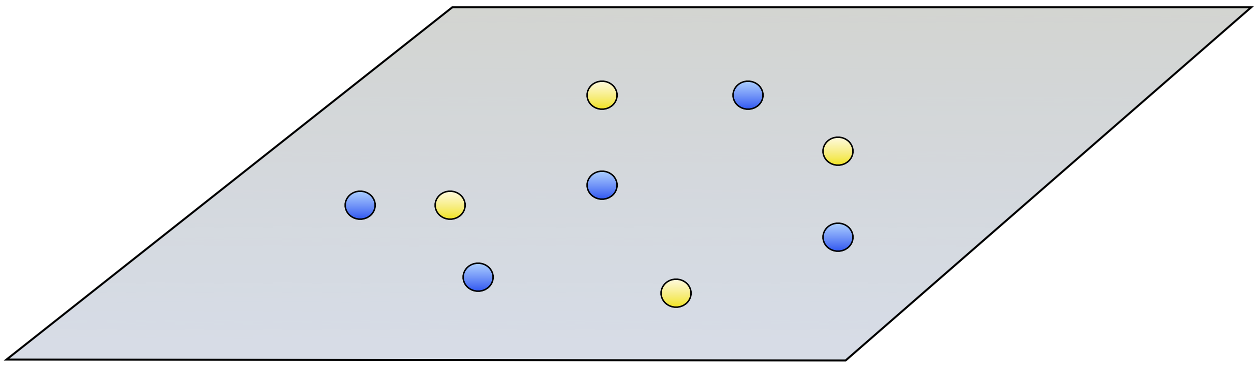
- How is it different from just a spin-polarized quantum wire (which has the same bosonized Hamiltonian)?
- Symmetry: $U(1)_c \times U(1)_s$
  - Charge conservation: $\theta \to \theta + \alpha$
  - Spin conservation: $\phi \to \phi + \alpha$

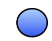
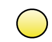
Bosonic?

$$H_{\text{eff}} = \int dx \left[\frac{v}{2K} (\partial_x \theta)^2 + \frac{vK}{2} (\partial_x \phi)^2 \right]$$

- All fermionic excitations are *gapped*
- Excitations of even number of fermions are gapless. Primarily:
  - Singlet pairs $\epsilon_{\alpha\beta}\psi_{1\alpha}\psi_{2\alpha} \sim e^{i\theta}$
  - Neutral spins $\psi^\dagger_{1\uparrow}\psi_{1\downarrow} - \psi^\dagger_{2\uparrow}\psi_{2\downarrow} \sim e^{i\phi}$

Bosonic?



boson	Q	S^z
	2	0
	0	1

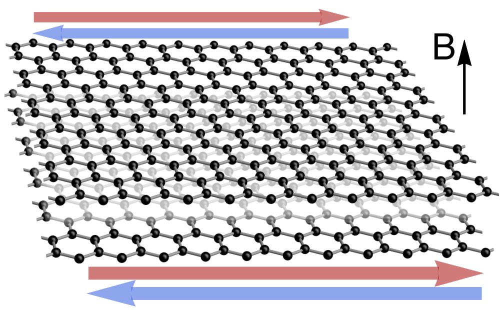
c.f. Senthil-Levin, 2012

$$S = \frac{i}{4\pi}\int d^2x d\tau \; K_{IJ}\epsilon^{\mu\nu\lambda} a^I_\mu \partial_\nu a^J_\lambda$$

$$K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad t_c = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad t_s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- |det K| = 1 : no anyons
- diag (K) = (0,0): bosonic quasiparticles

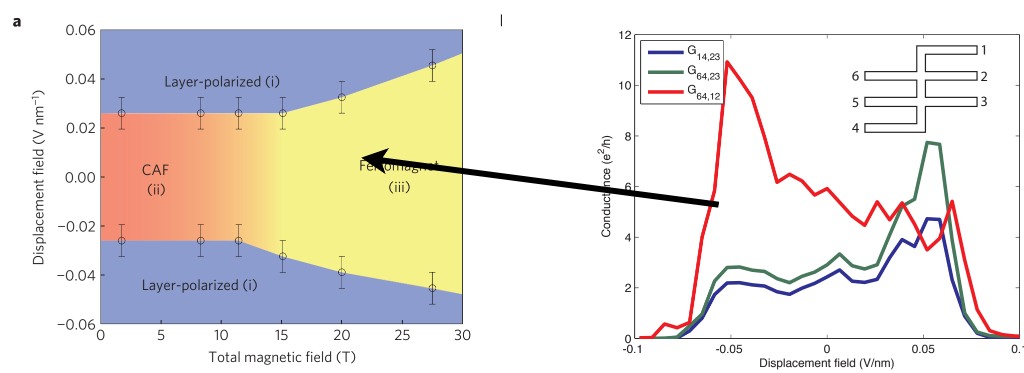
Potential experiments



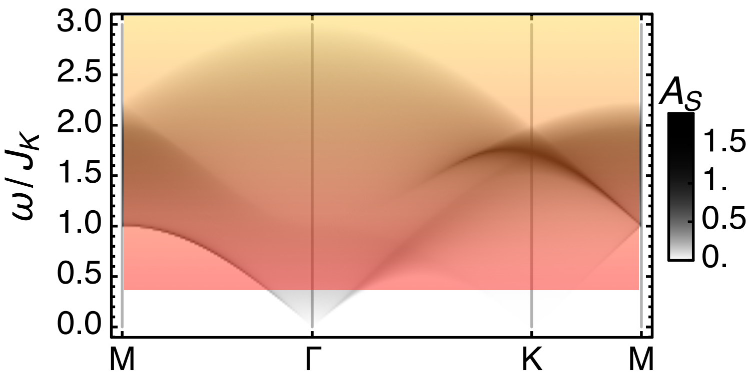
Can one identify it?

Differentiate from fermionic state?

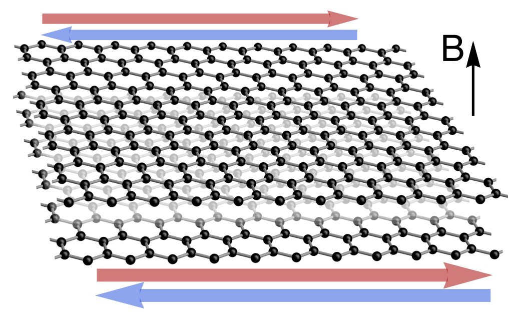
- Existing:
  - Zero Hall conductivity
  - Gapless edge
- New?
  - Tunnel into edge: single-e gap
  - Shot noise: charge 2e



Summary



Generic structure factor of Kitaev QSL



Bosonic SPT probably already exists in graphene

Thanks for a great conference!