

The background of the slide features a photograph of the Toronto skyline, with the CN Tower prominently on the left. In the foreground, the Niagara Falls waterfall is visible, with several blue arrows pointing towards the water from the bottom of the slide.

Leon Balents

CIFAR school, Toronto, April 2016

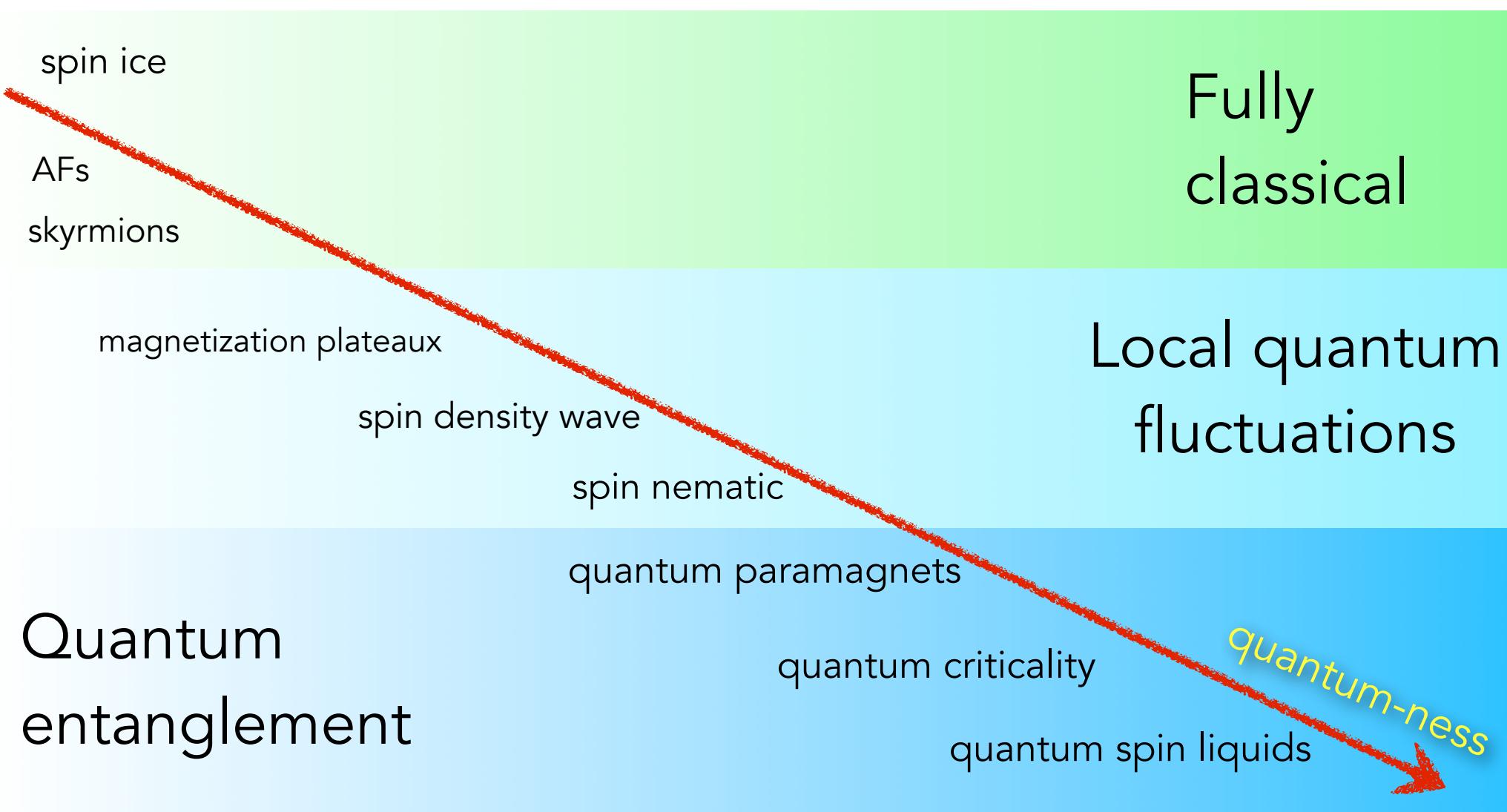
Quantum Spin Liquids

Plan

- The bird's eye view: QSLs as *ultra-quantum matter*
 - What is different from ordinary stuff?
- Review experimental status and recent developments

References here: <https://spinsandelectrons.com/pedagogy/>

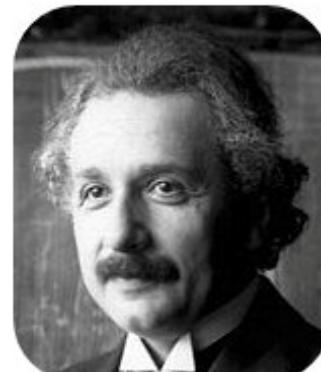
Quantum Magnetism



Quantum non-locality

EPR

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



A. Einstein

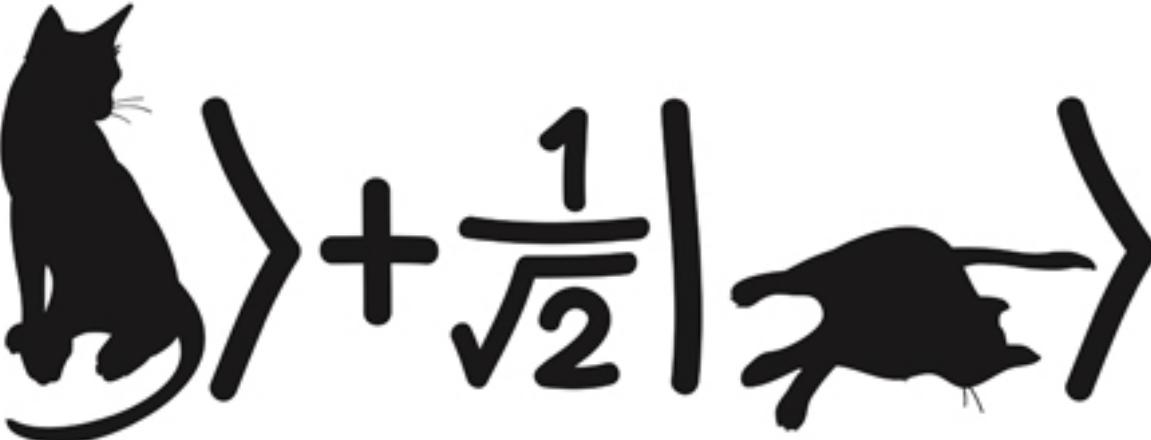


B. Podolsky



N. Rosen

Schrödinger Cat

$$\frac{1}{\sqrt{2}} \left| \begin{array}{c} \text{alive} \\ \text{cat} \end{array} \right\rangle + \frac{1}{\sqrt{2}} \left| \begin{array}{c} \text{dead} \\ \text{cat} \end{array} \right\rangle$$
Two black silhouettes of a cat are shown. The left silhouette shows a cat sitting upright with its tail curved over its back, representing the 'alive' state. The right silhouette shows a cat lying flat on its back with its legs and tail tucked under its body, representing the 'dead' state.

Cat states - superposition of a small number of macroscopically distinct components - are exponentially unstable: any local measurement collapses the superposition. They also require an exponentially long time to assemble with local unitary operators

How quantum can dense matter *stably* be?

Quantum spin liquids are ground states that retain long-distance entanglement and are robust to perturbations



“Ultra-quantum matter”: stable *phases* of matter that retain some degree of quantum non-locality

Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties



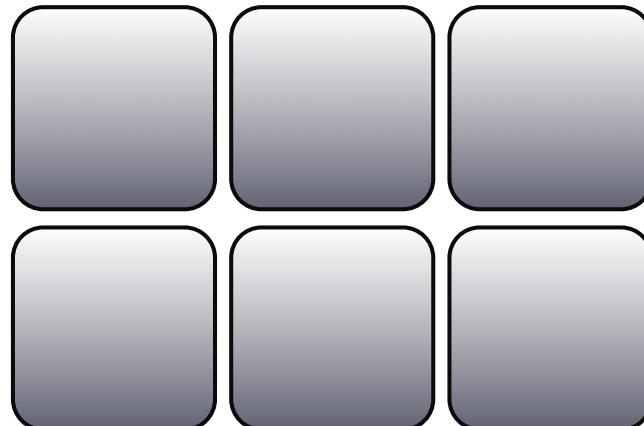
- Measurements far away do not affect one another
- From local measurements we can deduce the global state

Ordinary (local) Matter

Hamiltonian is local

$$H = \sum_x \mathcal{H}(x) \quad \mathcal{H}(x) \text{ has local support near } x$$

Ground state is “essentially”
a product state

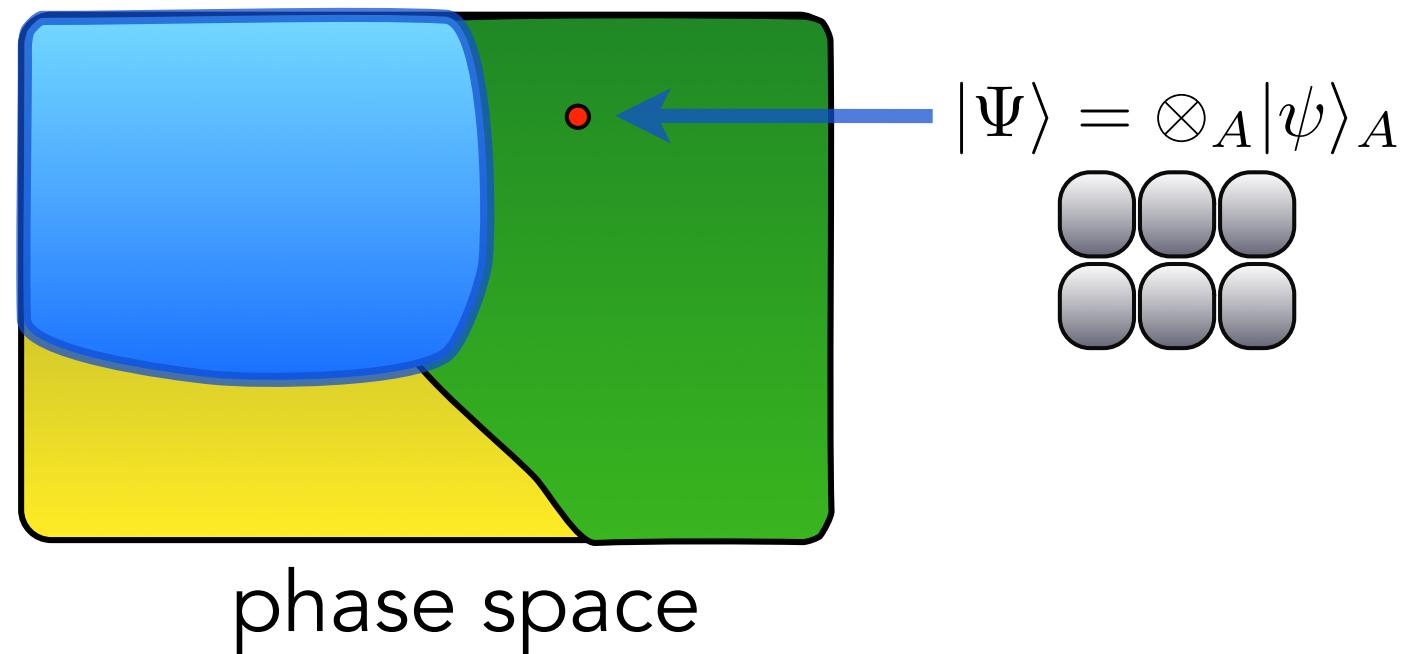


$$|\Psi\rangle = \otimes_A |\psi\rangle_A$$

no entanglement
between blocks

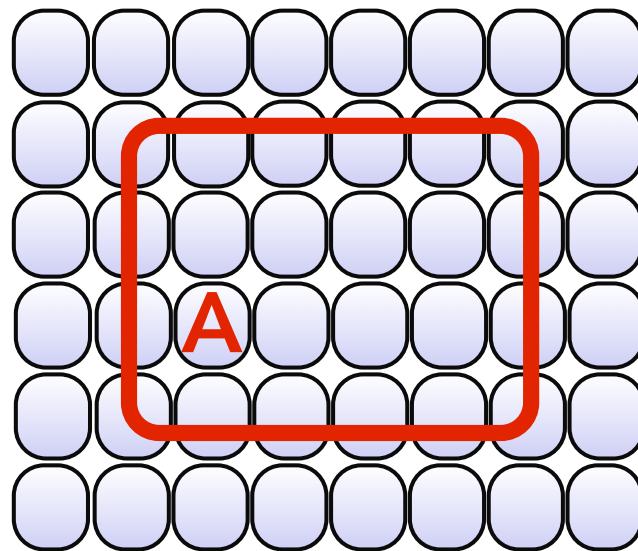
“Essentially” a product state?

- Adiabatic continuity



“Essentially” a product state?

- Entanglement scaling



$$\rho_A = \text{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\text{Tr}_A (\rho_A \ln \rho_A)$$

$$S(A) \sim \sigma L^{d-1} \quad \text{area law}$$

satisfied with exponentially small corrections

Best example: ordered magnet

Hamiltonian

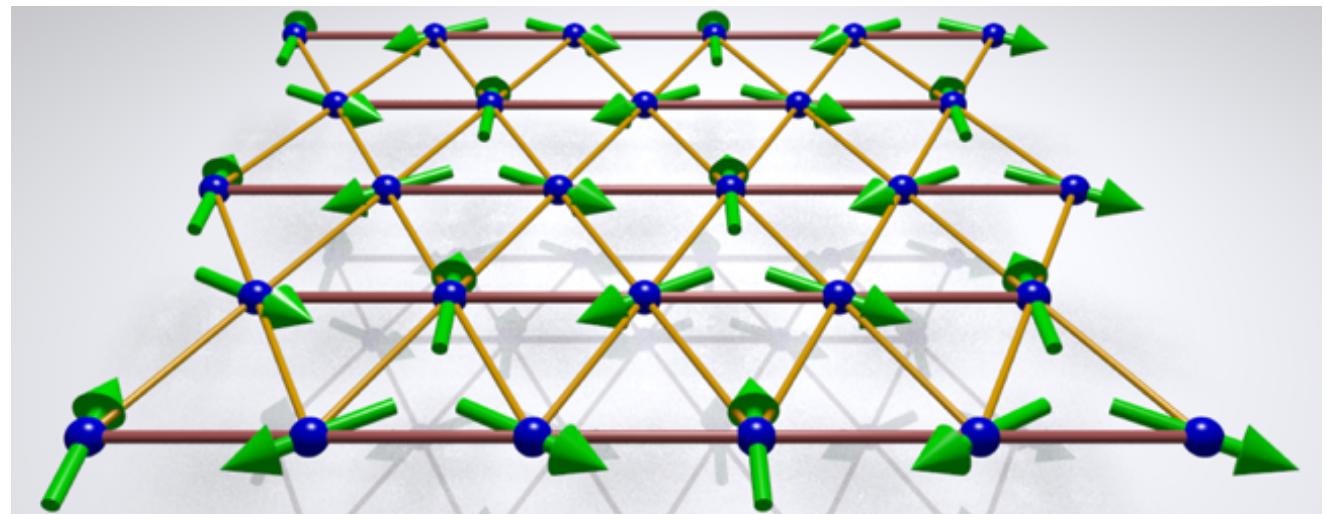
$$H = \sum_{(ij)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

exchange is short-range: local

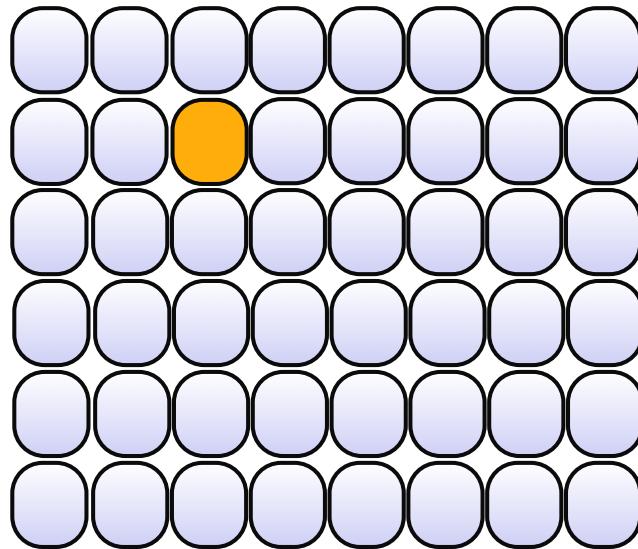
ordered state

$$|\Psi\rangle \approx \bigotimes_i |\mathbf{S}_i \cdot \hat{\mathbf{n}}_i = +S\rangle$$

block is a single spin



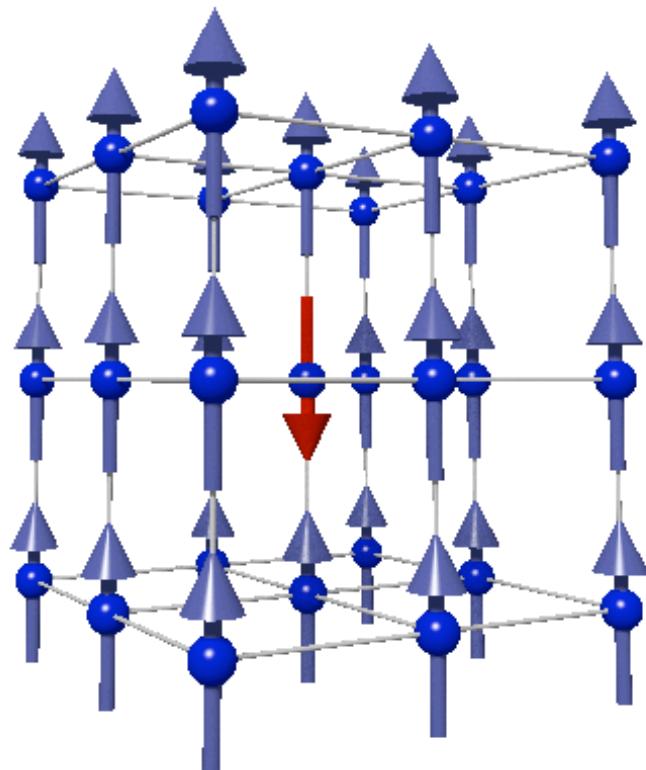
Quasiparticles



excited states \sim excited
levels of one block

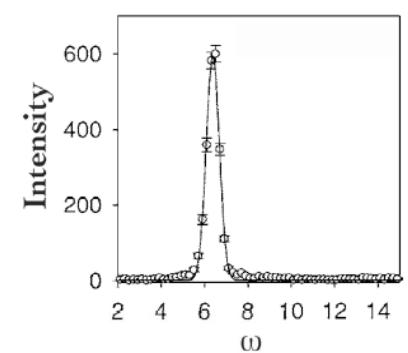
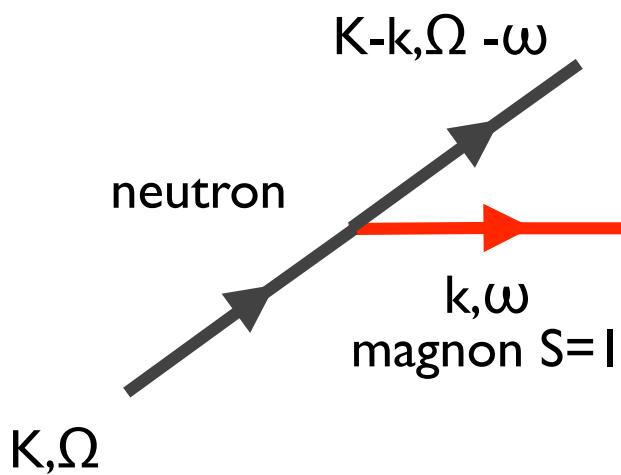
- local excitation can be created with operators in one block
- localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- quantum numbers consistent with finite system: no emergent or fractional quantum numbers

Spin wave



$$\omega(k) \approx \Delta - 2t \cos k_x a - \dots$$

$$|f\rangle = S_k^+ |i\rangle$$



Line shape in Rb_2MnF_4

How quantum can dense matter *stably* be?

Quantum spin liquids are ground states that retain long-distance entanglement and are robust to perturbations



©Bruce Gaulin

“Ultra-quantum matter”: stable *phases* of matter that retain some degree of quantum non-locality

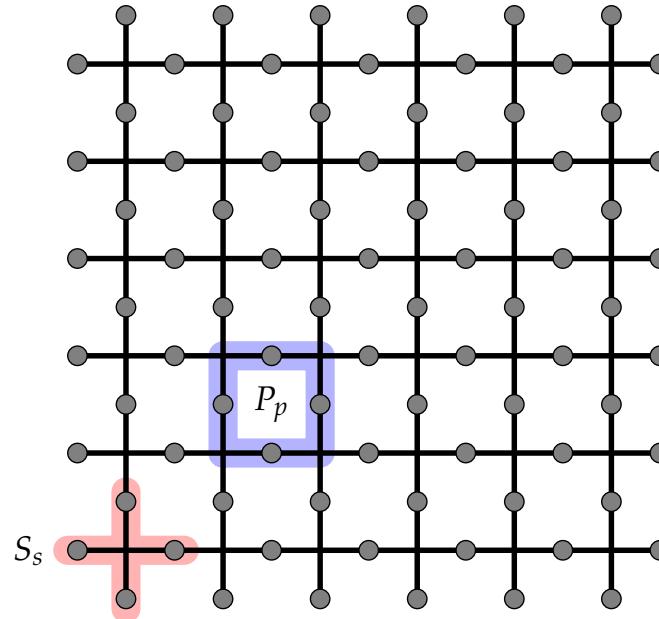
Example: toric code



A. Kitaev

$$H_{\text{tc}} = -K \sum_p P_p - K' \sum_s S_s,$$

$$P_p = \prod_{i \in p} \sigma_i^z \quad S_s = \prod_{i \in S} \sigma_i^x$$



Everything commutes!

→ In ground state simply $P_p = S_s = +1$

But what is the state?

Ground state

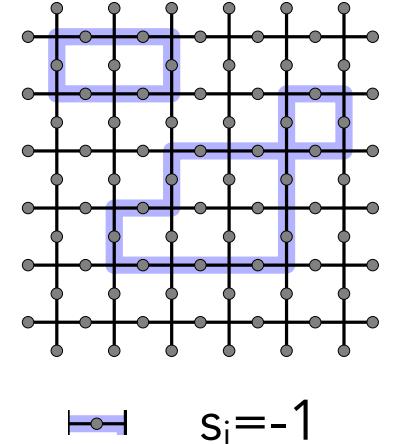
Product basis $|\{s_i\}\rangle = \bigotimes_i |\sigma_i^x = s_i\rangle$

Solve $S_s=+1$:

$$|\psi_0\rangle = \bigotimes_i |\sigma_i^x = +1\rangle$$

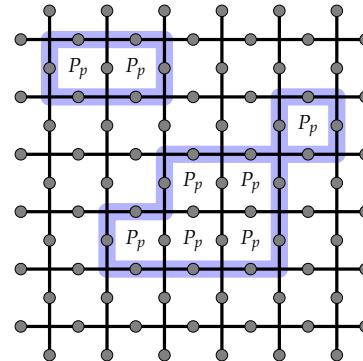
Now project to $P_p=+1$: $Q_p = \frac{1+P_p}{2} = \frac{1}{2} \sum_{q_p=0,1} P_p^{q_p}$,

$$|0\rangle = \prod_p Q_p |\psi_0\rangle = 2^{-N} \sum_{q_1 \dots q_N=0,1} \prod_p P_p^{q_p} |\psi_0\rangle$$



Ground state

$$|0\rangle = 2^{-N} \sum_{q_1 \dots q_N=0,1} \prod_p P_p^{q_p} |\psi_0\rangle = \sum_{\text{loops}}$$



massive
superposition
state

(similar form in Z variables)

All spins are $\langle 0 | \sigma_i^x | 0 \rangle = \langle 0 | S_s \sigma_i^x S_s | 0 \rangle = -\langle 0 | \sigma_i^x | 0 \rangle = 0$
uncertain $\langle 0 | \sigma_i^z | 0 \rangle = \langle 0 | P_s \sigma_i^z P_s | 0 \rangle = -\langle 0 | \sigma_i^z | 0 \rangle = 0$

And yet there is some structure...

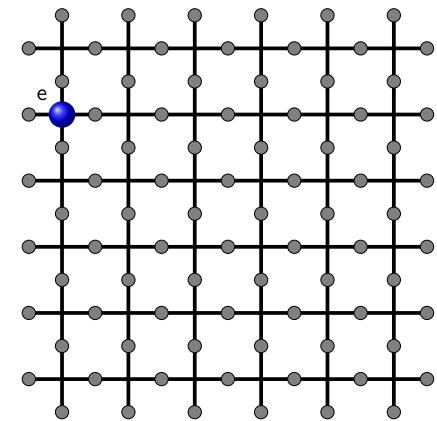
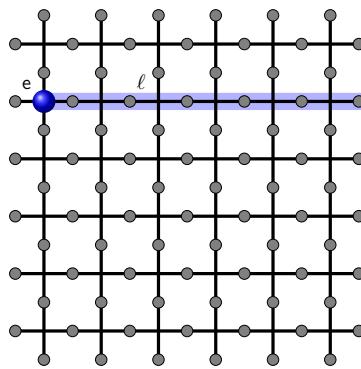
Excitations

$$H_{\text{tc}} = -K \sum_p P_p - K' \sum_s S_s,$$

Consider state with
just one $S_s = -1$

$$|e_s\rangle = \prod_{i \in \ell} \sigma_i^z |0\rangle$$

$$|e_s\rangle =$$

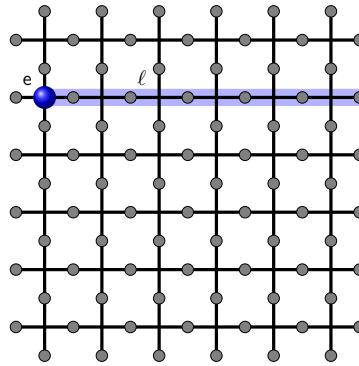


non-local excitation!

local operators, like finite
product of Z operators,
create **e** particles in pairs

Non-local excitations and entanglement

$$|e_s\rangle = \prod_{i \in \ell} \sigma_i^z |0\rangle$$



Why does this have finite energy? $\langle e_s | H | e_s \rangle \propto L_\ell ??$

$$\begin{aligned} & \left(\begin{array}{c} \text{grid with a red line} \\ \prod_{i \in \ell} \sigma_i^z \end{array} \right) \left(\begin{array}{c} \text{grid with a blue line} \\ + \end{array} \right. \left. \begin{array}{c} \text{grid with a blue line} \\ + \end{array} \right. \left. \begin{array}{c} \text{grid with a blue line} \\ + \end{array} \right. \left. \begin{array}{c} \text{grid with a blue line} \\ + \dots \end{array} \right) \\ &= \left(\begin{array}{c} \text{grid with a blue line} \\ + \end{array} \right. \left. \begin{array}{c} \text{grid with a blue line} \\ + \end{array} \right. \left. \begin{array}{c} \text{grid with a blue line} \\ + \end{array} \right. \left. \begin{array}{c} \text{grid with a blue line} \\ + \dots \end{array} \right) \end{aligned}$$

Two green arrows point from the top row of the first grid to the top row of the second grid, indicating a reshuffling of elements in the superposition.

away from the ends, the string just reshuffles elements of the superposition

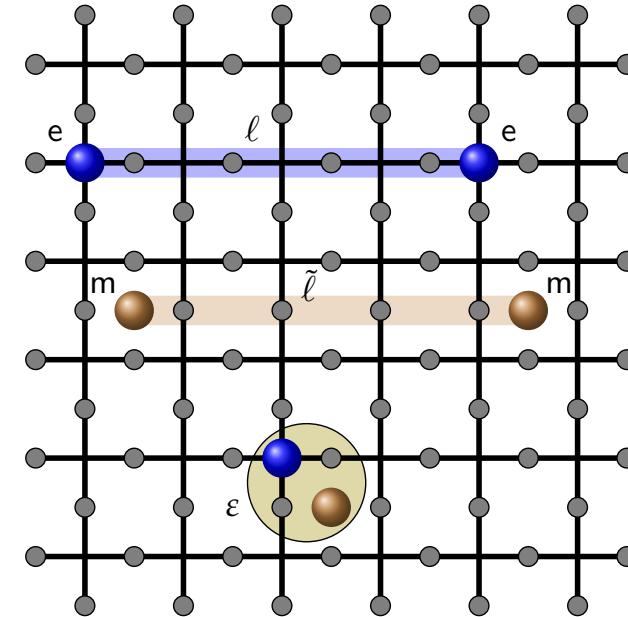
Excitations

There is also an **m** excitation

$$P_p = -1$$

And we can consider two of these together

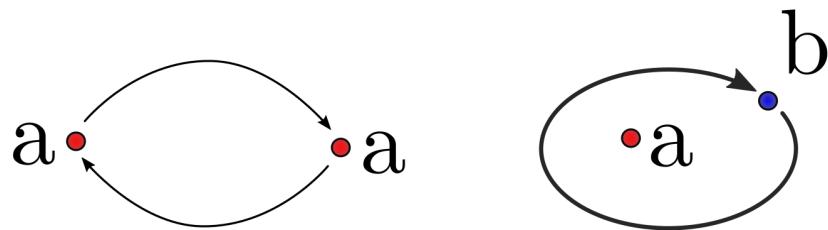
$$\varepsilon = \mathbf{e} \times \mathbf{m}$$



e,m,ε are the emergent *non-local* quasiparticles of the toric code

Non-locality

For the toric code, and in general for *topological phases* (i.e. gapped QSLs), the non-locality of excitations manifests as *statistics*



the excitations are *anyons*, i.e. the state acquires a unitary rotation (e.g. phase) when one excitation is taken around another

$$\begin{array}{c} e \\ \diagup \\ m \\ \diagdown \\ e \\ m \end{array} = - \begin{array}{c} e \\ m \\ | \\ e \\ m \end{array}$$

e and **m** see each other as “pi flux”

$$\begin{array}{c} e \\ m \\ \diagup \\ e \\ m \end{array} \times \begin{array}{c} e \\ m \\ \diagup \\ e \\ m \end{array} = \begin{array}{c} e \\ m \\ | \\ e \\ m \end{array} \times \begin{array}{c} e \\ m \\ \diagup \\ e \\ m \end{array} = - \begin{array}{c} e \\ m \\ | \\ e \\ m \\ | \\ e \\ m \end{array}$$

ϵ is a fermion

Stability

One can show that the toric code phase is *absolutely stable* to arbitrary (small) local perturbations, even those which break all symmetries.

This is because the “order” of the toric code is purely a type of entanglement, not any symmetry breaking. Only by bringing the gap of a quasiparticle to zero can one “unwind” the entangled ground state.

rigorous proofs by Hastings, Bravyi



RVB states

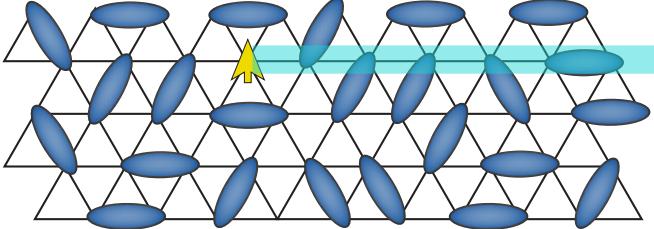
Historically the first proposal of a QSL by Anderson in 1973

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|\Psi\rangle = \text{Diagram 1} + \text{Diagram 2} + \dots$$
<img alt="Diagram showing the superposition of two singlet states on a triangular lattice. Each state is represented by a grid of blue ovals (singlets) on a triangular lattice. The first state has ovals at (i,j) = (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12), (13,13), (14,14), (15,15), (16,16), (17,17), (18,18), (19,19), (20,20), (21,21), (22,22), (23,23), (24,24), (25,25), (26,26), (27,27), (28,28), (29,29), (30,30), (31,31), (32,32), (33,33), (34,34), (35,35), (36,36), (37,37), (38,38), (39,39), (40,40), (41,41), (42,42), (43,43), (44,44), (45,45), (46,46), (47,47), (48,48), (49,49), (50,50), (51,51), (52,52), (53,53), (54,54), (55,55), (56,56), (57,57), (58,58), (59,59), (60,60), (61,61), (62,62), (63,63), (64,64), (65,65), (66,66), (67,67), (68,68), (69,69), (70,70), (71,71), (72,72), (73,73), (74,74), (75,75), (76,76), (77,77), (78,78), (79,79), (80,80), (81,81), (82,82), (83,83), (84,84), (85,85), (86,86), (87,87), (88,88), (89,89), (90,90), (91,91), (92,92), (93,93), (94,94), (95,95), (96,96), (97,97), (98,98), (99,99), (100,100), (101,101), (102,102), (103,103), (104,104), (105,105), (106,106), (107,107), (108,108), (109,109), (110,110), (111,111), (112,112), (113,113), (114,114), (115,115), (116,116), (117,117), (118,118), (119,119), (120,120), (121,121), (122,122), (123,123), (124,124), (125,125), (126,126), (127,127), (128,128), (129,129), (130,130), (131,131), (132,132), (133,133), (134,134), (135,135), (136,136), (137,137), (138,138), (139,139), (140,140), (141,141), (142,142), (143,143), (144,144), (145,145), (146,146), (147,147), (148,148), (149,149), (150,150), (151,151), (152,152), (153,153), (154,154), (155,155), (156,156), (157,157), (158,158), (159,159), (160,160), (161,161), (162,162), (163,163), (164,164), (165,165), (166,166), (167,167), (168,168), (169,169), (170,170), (171,171), (172,172), (173,173), (174,174), (175,175), (176,176), (177,177), (178,178), (179,179), (180,180), (181,181), (182,182), (183,183), (184,184), (185,185), (186,186), (187,187), (188,188), (189,189), (190,190), (191,191), (192,192), (193,193), (194,194), (195,195), (196,196), (197,197), (198,198), (199,199), (200,200), (201,201), (202,202), (203,203), (204,204), (205,205), (206,206), (207,207), (208,208), (209,209), (210,210), (211,211), (212,212), (213,213), (214,214), (215,215), (216,216), (217,217), (218,218), (219,219), (220,220), (221,221), (222,222), (223,223), (224,224), (225,225), (226,226), (227,227), (228,228), (229,229), (230,230), (231,231), (232,232), (233,233), (234,234), (235,235), (236,236), (237,237), (238,238), (239,239), (240,240), (241,241), (242,242), (243,243), (244,244), (245,245), (246,246), (247,247), (248,248), (249,249), (250,250), (251,251), (252,252), (253,253), (254,254), (255,255), (256,256), (257,257), (258,258), (259,259), (260,260), (261,261), (262,262), (263,263), (264,264), (265,265), (266,266), (267,267), (268,268), (269,269), (270,270), (271,271), (272,272), (273,273), (274,274), (275,275), (276,276), (277,277), (278,278), (279,279), (280,280), (281,281), (282,282), (283,283), (284,284), (285,285), (286,286), (287,287), (288,288), (289,289), (290,290), (291,291), (292,292), (293,293), (294,294), (295,295), (296,296), (297,297), (298,298), (299,299), (300,300), (301,301), (302,302), (303,303), (304,304), (305,305), (306,306), (307,307), (308,308), (309,309), (310,310), (311,311), (312,312), (313,313), (314,314), (315,315), (316,316), (317,317), (318,318), (319,319), (320,320), (321,321), (322,322), (323,323), (324,324), (325,325), (326,326), (327,327), (328,328), (329,329), (330,330), (331,331), (332,332), (333,333), (334,334), (335,335), (336,336), (337,337), (338,338), (339,339), (340,340), (341,341), (342,342), (343,343), (344,344), (345,345), (346,346), (347,347), (348,348), (349,349), (350,350), (351,351), (352,352), (353,353), (354,354), (355,355), (356,356), (357,357), (358,358), (359,359), (360,360), (361,361), (362,362), (363,363), (364,364), (365,365), (366,366), (367,367), (368,368), (369,369), (370,370), (371,371), (372,372), (373,373), (374,374), (375,375), (376,376), (377,377), (378,378), (379,379), (380,380), (381,381), (382,382), (383,383), (384,384), (385,385), (386,386), (387,387), (388,388), (389,389), (390,390), (391,391), (392,392), (393,393), (394,394), (395,395), (396,396), (397,397), (398,398), (399,399), (400,400), (401,401), (402,402), (403,403), (404,404), (405,405), (406,406), (407,407), (408,408), (409,409), (410,410), (411,411), (412,412), (413,413), (414,414), (415,415), (416,416), (417,417), (418,418), (419,419), (420,420), (421,421), (422,422), (423,423), (424,424), (425,425), (426,426), (427,427), (428,428), (429,429), (430,430), (431,431), (432,432), (433,433), (434,434), (435,435), (436,436), (437,437), (438,438), (439,439), (440,440), (441,441), (442,442), (443,443), (444,444), (445,445), (446,446), (447,447), (448,448), (449,449), (450,450), (451,451), (452,452), (453,453), (454,454), (455,455), (456,456), (457,457), (458,458), (459,459), (460,460), (461,461), (462,462), (463,463), (464,464), (465,465), (466,466), (467,467), (468,468), (469,469), (470,470), (471,471), (472,472), (473,473), (474,474), (475,475), (476,476), (477,477), (478,478), (479,479), (480,480), (481,481), (482,482), (483,483), (484,484), (485,485), (486,486), (487,487), (488,488), (489,489), (490,490), (491,491), (492,492), (493,493), (494,494), (495,495), (496,496), (497,497), (498,498), (499,499), (500,500), (501,501), (502,502), (503,503), (504,504), (505,505), (506,506), (507,507), (508,508), (509,509), (510,510), (511,511), (512,512), (513,513), (514,514), (515,515), (516,516), (517,517), (518,518), (519,519), (520,520), (521,521), (522,522), (523,523), (524,524), (525,525), (526,526), (527,527), (528,528), (529,529), (530,530), (531,531), (532,532), (533,533), (534,534), (535,535), (536,536), (537,537), (538,538), (539,539), (540,540), (541,541), (542,542), (543,543), (544,544), (545,545), (546,546), (547,547), (548,548), (549,549), (550,550), (551,551), (552,552), (553,553), (554,554), (555,555), (556,556), (557,557), (558,558), (559,559), (560,560), (561,561), (562,562), (563,563), (564,564), (565,565), (566,566), (567,567), (568,568), (569,569), (570,570), (571,571), (572,572), (573,573), (574,574), (575,575), (576,576), (577,577), (578,578), (579,579), (580,580), (581,581), (582,582), (583,583), (584,584), (585,585), (586,586), (587,587), (588,588), (589,589), (590,590), (591,591), (592,592), (593,593), (594,594), (595,595), (596,596), (597,597), (598,598), (599,599), (600,600), (601,601), (602,602), (603,603), (604,604), (605,605), (606,606), (607,607), (608,608), (609,609), (610,610), (611,611), (612,612), (613,613), (614,614), (615,615), (616,616), (617,617), (618,618), (619,619), (620,620), (621,621), (622,622), (623,623), (624,624), (625,625), (626,626), (627,627), (628,628), (629,629), (630,630), (631,631), (632,632), (633,633), (634,634), (635,635), (636,636), (637,637), (638,638), (639,639), (640,640), (641,641), (642,642), (643,643), (644,644), (645,645), (646,646), (647,647), (648,648), (649,649), (650,650), (651,651), (652,652), (653,653), (654,654), (655,655), (656,656), (657,657), (658,658), (659,659), (660,660), (661,661), (662,662), (663,663), (664,664), (665,665), (666,666), (667,667), (668,668), (669,669), (670,670), (671,671), (672,672), (673,673), (674,674), (675,675), (676,676), (677,677), (678,678), (679,679), (680,680), (681,681), (682,682), (683,683), (684,684), (685,685), (686,686), (687,687), (688,688), (689,689), (690,690), (691,691), (692,692), (693,693), (694,694), (695,695), (696,696), (697,697), (698,698), (699,699), (700,700), (701,701), (702,702), (703,703), (704,704), (705,705), (706,706), (707,707), (708,708), (709,709), (710,710), (711,711), (712,712), (713,713), (714,714), (715,715), (716,716), (717,717), (718,718), (719,719), (720,720), (721,721), (722,722), (723,723), (724,724), (725,725), (726,726), (727,727), (728,728), (729,729), (730,730), (731,731), (732,732), (733,733), (734,734), (735,735), (736,736), (737,737), (738,738), (739,739), (740,740), (741,741), (742,742), (743,743), (744,744), (745,745), (746,746), (747,747), (748,748), (749,749), (750,750), (751,751), (752,752), (753,753), (754,754), (755,755), (756,756), (757,757), (758,758), (759,759), (760,760), (761,761), (762,762), (763,763), (764,764), (765,765), (766,766), (767,767), (768,768), (769,769), (770,770), (771,771), (772,772), (773,773), (774,774), (775,775), (776,776), (777,777), (778,778), (779,779), (780,780), (781,781), (782,782), (783,783), (784,784), (785,785), (786,786), (787,787), (788,788), (789,789), (790,790), (791,791), (792,792), (793,793), (794,794), (795,795), (796,796), (797,797), (798,798), (799,799), (800,800), (801,801), (802,802), (803,803), (804,804), (805,805), (806,806), (807,807), (808,808), (809,809), (810,810), (811,811), (812,812), (813,813), (814,814), (815,815), (816,816), (817,817), (818,818), (819,819), (820,820), (821,821), (822,822), (823,823), (824,824), (825,825), (826,826), (827,827), (828,828), (829,829), (830,830), (831,831), (832,832), (833,833), (834,834), (835,835), (836,836), (837,837), (838,838), (839,839), (840,840), (841,841), (842,842), (843,843), (844,844), (845,845), (846,846), (847,847), (848,848), (849,849), (850,850), (851,851), (852,852), (853,853), (854,854), (855,855), (856,856), (857,857), (858,858), (859,859), (860,860), (861,861), (862,862), (863,863), (864,864), (865,865), (866,866), (867,867), (868,868), (869,869), (870,870), (871,871), (872,872), (873,873), (874,874), (875,875), (876,876), (877,877), (878,878), (879,879), (880,880), (881,881), (882,882), (883,883), (884,884), (885,885), (886,886), (887,887), (888,888), (889,889), (890,890), (891,891), (892,892), (893,893), (894,894), (895,895), (896,896), (897,897), (898,898), (899,899), (900,900), (901,901), (902,902), (903,903), (904,904), (905,905), (906,906), (907,907), (908,908), (909,909), (910,910), (911,911), (912,912), (913,913), (914,914), (915,915), (916,916), (917,917), (918,918), (919,919), (920,920), (921,921), (922,922), (923,923), (924,924), (925,925), (926,926), (927,927), (928,928), (929,929), (930,930), (931,931), (932,932), (933,933), (934,934), (935,935), (936,936), (937,937), (938,938), (939,939), (940,940), (941,941), (942,942), (943,943), (944,944), (945,945), (946,946), (947,947), (948,948), (949,949), (950,950), (951,951), (952,952), (953,953), (954,954), (955,955), (956,956), (957,957), (958,958), (959,959), (960,960), (961,961), (962,962), (963,963), (964,964), (965,965), (966,966), (967,967), (968,968), (969,969), (970,970), (971,971), (972,972), (973,973), (974,974), (975,975), (976,976), (977,977), (978,978), (979,979), (980,980), (981,981), (982,982), (983,983), (984,984), (985,985), (986,986), (987,987), (988,988), (989,989), (990,990), (991,991), (992,992), (993,993), (994,994), (995,995), (996,996), (997,997), (998,998), (999,999), (1000,1000), (1001,1001), (1002,1002), (1003,1003), (1004,1004), (1005,1005), (1006,1006), (1007,1007), (1008,1008), (1009,1009), (1010,1010), (1011,1011), (1012,1012), (1013,1013), (1014,1014), (1015,1015), (1016,1016), (1017,1017), (1018,1018), (1019,1019), (1020,1020), (1021,1021), (1022,1022), (1023,1023), (1024,1024), (1025,1025), (1026,1026), (1027,1027), (1028,1028), (1029,1029), (1030,1030), (1031,1031), (1032,1032), (1033,1033), (1034,1034), (1035,1035), (1036,1036), (1037,1037), (1038,1038), (1039,1039), (1040,1040), (1041,1041), (1042,1042), (1043,1043), (1044,1044), (1045,1045), (1046,1046), (1047,1047), (1048,1048), (1049,1049), (1050,1050), (1051,1051), (1052,1052), (1053,1053), (1054,1054), (1055,1055), (1056,1056), (1057,1057), (1058,1058), (1059,1059), (1060,1060), (1061,1061), (1062,1062), (1063,1063), (1064,1064), (1065,1065), (1066,1066), (1067,1067), (1068,1068), (1069,1069), (1070,1070), (1071,1071), (1072,1072), (1073,1073), (1074,1074), (1075,1075), (1076,1076), (1077,1077), (1078,1078), (1079,1079), (1080,1080), (1081,1081), (1082,1082), (1083,1083), (1084,1084), (1085,1085), (1086,1086), (1087,1087), (1088,1088), (1089,1089), (1090,1090), (1091,1091), (1092,1092), (1093,1093), (1094,1094), (1095,1095), (1096,1096), (1097,1097), (1098,1098), (1099,1099), (1100,1100), (1101,1101), (1102,1102), (1103,1103), (1104,1104), (1105,1105), (1106,1106), (1107,1107), (1108,1108), (1109,1109), (1110,1110), (1111,1111), (1112,1112), (1113,1113), (1114,1114), (1115,1115), (1116,1116), (1117,1117), (1118,1118), (1119,1119), (1120,1120), (1121,1121), (1122,1122), (1123,1123), (1124,1124), (1125,1125), (1126,1126), (1127,1127), (1128,1128), (1129,1129), (1130,1130), (1131,1131), (1132,1132), (1133,1133), (1134,1134), (1135,1135), (1136,1136), (1137,1137), (1138,1138), (1139,1139), (1140,1140), (1141,1141), (1142,1142), (1143,1143), (1144,1144), (1145,1145), (1146,1146), (1147,1147), (1148,1148), (1149,1149), (1150,1150), (1151,1151), (1152,1152), (1153,1153), (1154,1154), (1155,1155), (1156,1156), (1157,1157), (1158,1158), (1159,1159), (1160,1160), (1161,1161), (1162,1162), (1163,1163), (1164,1164), (1165,1165), (1166,1166), (1167,1167), (1168,1168), (1169,1169), (1170,1170), (1171,1171), (1172,1172), (1173,1173), (1174,1174), (1175,1175), (1176,1176), (1177,1177), (1178,1178), (1179,1179), (1180,1180), (1181,1181), (1182,1182), (1183,1183), (1184,1184), (1185,1185), (1186,1186), (1187,1187), (1188,1188), (1189,1189), (1190,1190), (1191,1191), (1192,1192), (1193,1193), (1194,1194), (1195,1195), (1196,1196), (1197,1197), (1198,1198), (1199,1199), (1200,1200), (1201,1201), (1202

Spinon

$$|\Psi\rangle = \text{Diagram} + \dots$$


create a spinon by rearranging valence bonds to expose a single free spin

New feature: SU(2) spin symmetry

- spinon excitation has $S=1/2$, a *fractional quantum number* (spin flips are $S=1$)
- this “enriches” the topological label **e,m,ε**

many efforts to understand Symmetry Enriched Topological order

Quantum spin ice

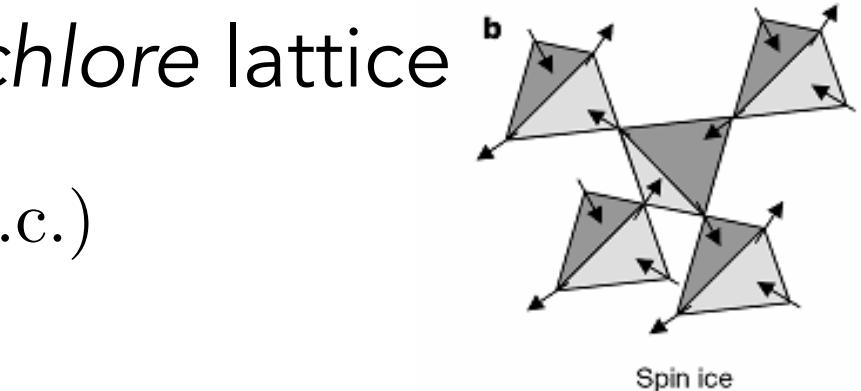
Minimal XXZ model on pyrochlore lattice

$$H = \underbrace{\sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z}_{J_{zz}} - J_{\pm} (S_i^+ S_j^- + \text{h.c.})$$

$$\frac{J_{zz}}{2} \sum_t \left(\sum_{i \in t} S_i^z \right)^2 \rightarrow J_{\pm} \ll J_{zz} \text{ is the spin ice limit}$$

$$\sum_{i \in t} S_i^z = 0 \text{ in the classical ground state: "2in-2out"}$$

With a lot more work one can show that J_{\pm} selects a massive superposition of these states



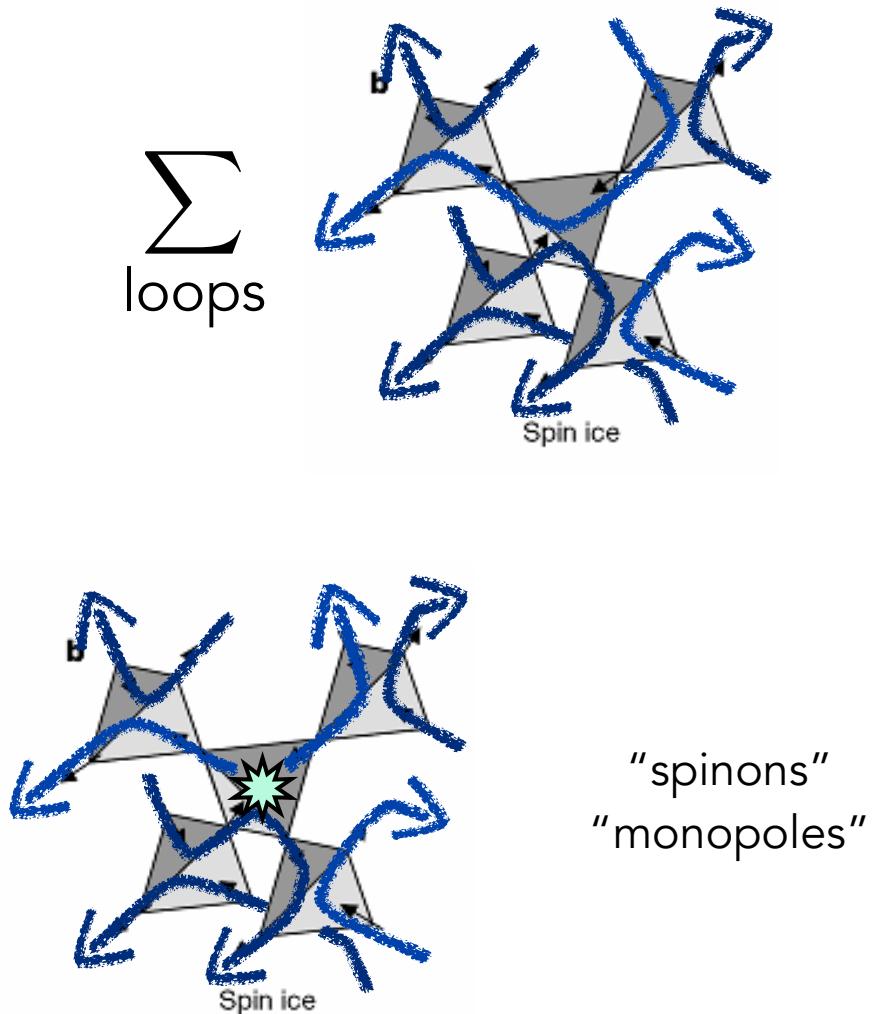
M. Hermele, MPA Fisher, L.B., 2004;
A. Banerjee et al, 2008

Quantum spin ice

like the toric code, the QSL state can be viewed as a sum of loops - follow the arrows!

There are also non-local excitations

$$\sum_{i \in t} S_i^z = \pm 1$$



Gauge theory

These QSL states are all conveniently described mathematically by *gauge theory*

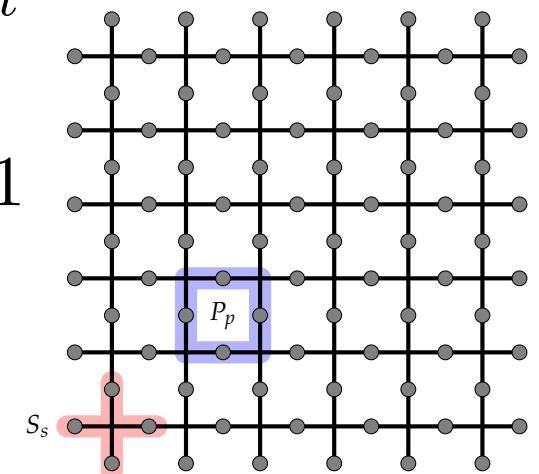
- Local constraints: $S_s = +1$ $\sum_{i \in t} S_i^z = 0$
- Generate local unitaries

$$U = U^\dagger = \prod_s S_s^{(1-q_s)/2} \quad q_s = \pm 1$$

$$\sigma_{ss'}^z \rightarrow U^\dagger \sigma_{ss'}^z U = q_s q_{s'} \sigma_{ss'}^z$$

$\sigma_{ss'}^z$ are \mathbb{Z}_2 gauge fields

toric code = “ \mathbb{Z}_2 QSL”



Gauge theory

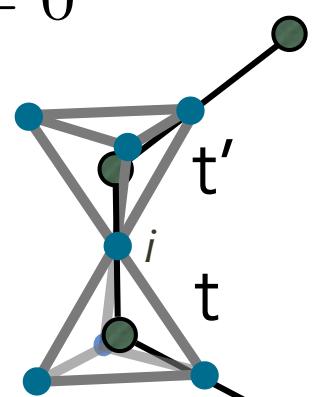
These QSL states are all conveniently described mathematically by *gauge theory*

- Local constraints: $S_s = +1$ $\sum_{i \in t} S_i^z = 0$
- Generate local unitaries

$$U = \prod_t e^{i\chi_t S_t^z} \quad \chi_t \in U(1)$$

$$S_{tt'}^\pm \rightarrow U^\dagger S_{tt'}^\pm U = e^{i(\chi_t - \chi_{t'})} S_{tt'}^\pm$$

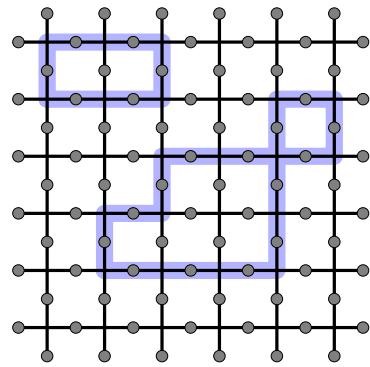
$S_{tt'}^\pm \sim e^{\pm iA_{tt'}}$ is a $U(1)$ gauge connection



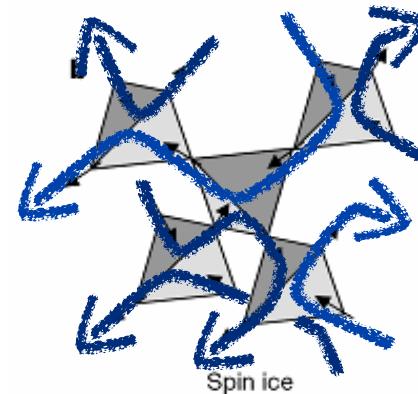
QSI hosts a $U(1)$ QSL

Gauge theory

- Loops ~ field lines describing “vacuum fluctuations”



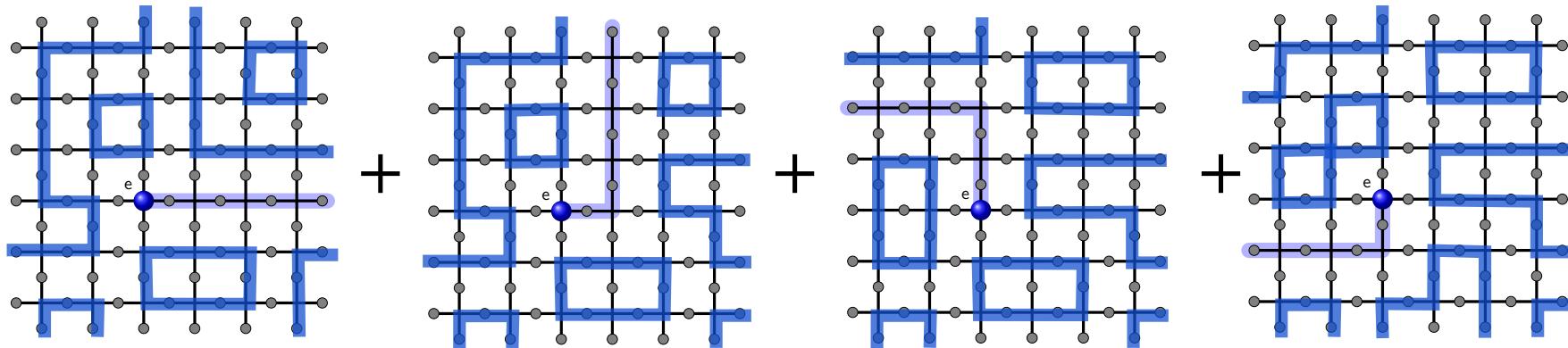
toric code: Z_2



QSI: $U(1)$

Deconfinement

$|e_s\rangle =$



The superposition of different string configurations “smears out” the flux emanating from **e**, so that it cannot be detected by any *local* measurement away from the quasiparticle

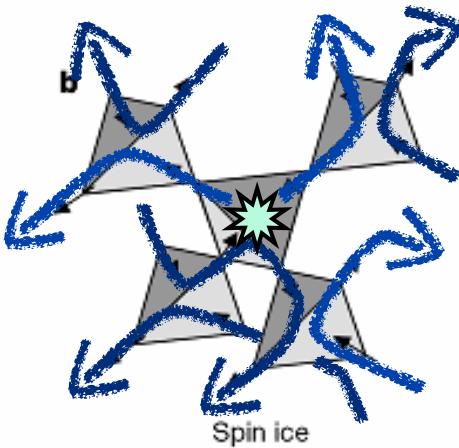
- consequently the string tension is zero = deconfinement
- the complete local undetectability of the flux is characteristic of a *topological* QSL

Deconfinement

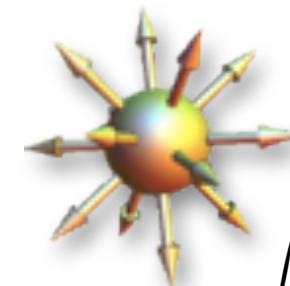
$U(1)$ QSL

$|\text{spinon}\rangle =$

$\sum_{\text{flux lines}}$



\sim



$$\int_{S \ni *} \mathbf{B} \cdot d\mathbf{A} = q$$

Here the flux is a number, not a parity, and can be added.

The superposition of many field lines smears the flux into a “uniform” dispersed magnetic/electric field

$$\mathbf{B} \sim \frac{\hat{r}}{r^2}$$

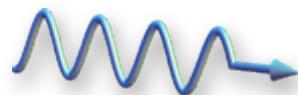
This leads to the usual $1/r$ Coulomb interaction between charges. It decays with distance so charges are “deconfined”, but the flux is locally detectable

Photon

Since there is a detectable average field, there is energy density associated with the field lines

Consequently, there are “pure gauge” excitations. These are not non-local particles but emergent collective excitations. They are exactly analogous to the photons of electromagnetism.

$$\omega \sim ck$$



U(1) QSL is a gapless, not topological example of ultra-quantum matter

Variational method

- Up to now I described results out of soluble or otherwise tractable models
- Variational wavefunctions are an attractive way to approach models when no other handle is available

RVB:

$$\Psi = c_1 \begin{array}{c} \text{Diagram 1:} \\ \text{Two vertical columns of four red dots each. Each dot is connected to a horizontal blue line. The top row has two ovals above the dots, and the bottom row has two ovals below the dots. There are two horizontal black lines connecting the dots in each row.} \end{array} + c_2 \begin{array}{c} \text{Diagram 2:} \\ \text{Two vertical columns of four red dots each. Each dot is connected to a horizontal blue line. The top row has one oval above the dots, and the bottom row has one oval below the dots. There are two horizontal black lines connecting the dots in each row.} \end{array} + \dots$$

but how to keep track of so many coefficients??

Free Fermions

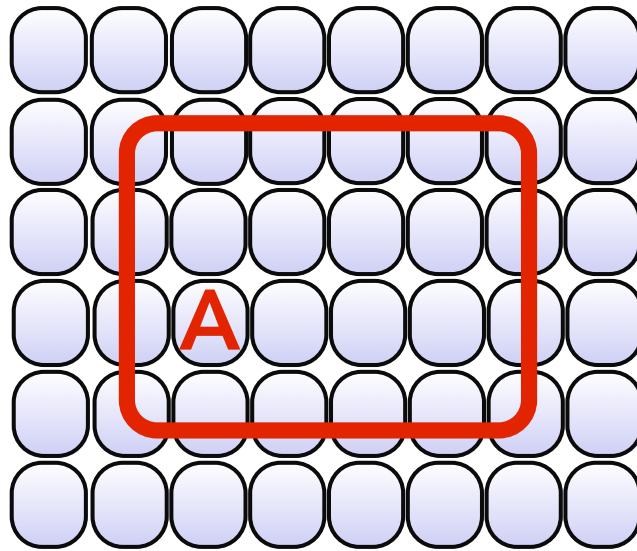
- One useful construction uses a Fermi gas: a product in momentum space rather than real space

$$\Psi = \prod_{k < k_F} c_k^\dagger |0\rangle$$

$$= c_1 \begin{array}{|c|c|c|c|c|} \hline \bullet & & \bullet & \bullet & \\ \hline & \bullet & & & \bullet \\ \hline & \bullet & \bullet & & \\ \hline & & \bullet & & \bullet \\ \hline & \bullet & & & \bullet \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \bullet & & \bullet & & \\ \hline & & & & \bullet \\ \hline & \bullet & \bullet & & \bullet \\ \hline & & \bullet & \bullet & \\ \hline & \bullet & & \bullet & \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \bullet & & \bullet & & \\ \hline & & & & \bullet \\ \hline & \bullet & & & \bullet \\ \hline & \bullet & \bullet & & \\ \hline & & & \bullet & \\ \hline \end{array} + \dots$$

Entanglement Entropy

- Free fermions $S \sim \sigma L^{d-1} \log L$



D. Gioev+I. Klich, 2006
M.M. Wolf, 2006

- Very large entanglement is generic. A metal is in this sense “ultraquantum”
however, it has local quasiparticles

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site ($S=0$)

$$\Psi_0 = c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & & \uparrow & \uparrow\downarrow & \downarrow \\ \hline \downarrow & \downarrow & \uparrow\downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow\downarrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \uparrow\downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site ($S=0$)

$$\Psi = c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \uparrow \\ \hline \downarrow & \downarrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

- Projection removes empty and doubly occupied sites $\Psi = P_G \Psi_0$

“like” a gauge constraint $n_i = 1$

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site ($S=0$)

$$\Psi = c_1 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_2 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \uparrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + c_3 \begin{array}{|c|c|c|c|c|} \hline \uparrow & \downarrow & \uparrow & \uparrow & \downarrow \\ \hline \downarrow & \uparrow & \uparrow & \downarrow & \uparrow \\ \hline \downarrow & \uparrow & \downarrow & \downarrow & \downarrow \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \hline \downarrow & \uparrow & \downarrow & \downarrow & \uparrow \\ \hline \uparrow & \downarrow & \downarrow & \downarrow & \uparrow \\ \hline \end{array} + \dots$$

The wavefunction Ψ is a superposition of three terms. Each term is a product of coefficients c_1, c_2, c_3 and a 5x5 grid of spin states. The grids are filled with up (\uparrow) and down (\downarrow) spins. The first grid has a red 'X' drawn through it, indicating it is not a valid configuration. The second grid has all up spins, and the third grid has a mix of up and down spins. The red 'X' on the first grid is very large, while on the second and third grids, it is smaller and only covers the first two columns.

- Such wavefunctions can be efficiently simulated using Monte Carlo methods

Partons

- Gutzwiller-type variational wavefunction uses a reference Hamiltonian

$$H_{ref} = \sum_{ij} \left[t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \text{h.c.} + \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + \text{h.c.} \right]$$

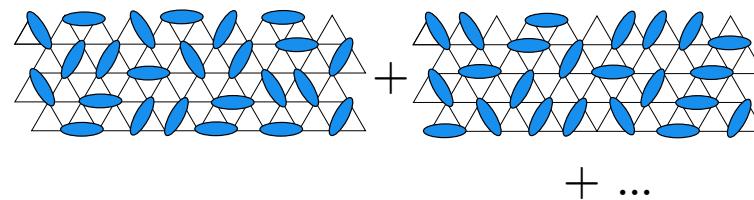
- Project

$$|\Psi_{var}\rangle = \prod_i \hat{P}_{n_i=1} |\Psi_{ref}\rangle$$

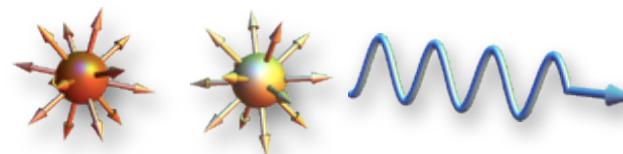
- The fermions are “partons” $\vec{S}_i = c_{i\alpha}^\dagger \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{i\beta}$
- Standard (MIT) belief: each such projected wavefunction represents a true QSL phase, in which the partons become the non-local quasiparticles - “spinons” - and they are coupled to an effective gauge field

Classes of QSLs

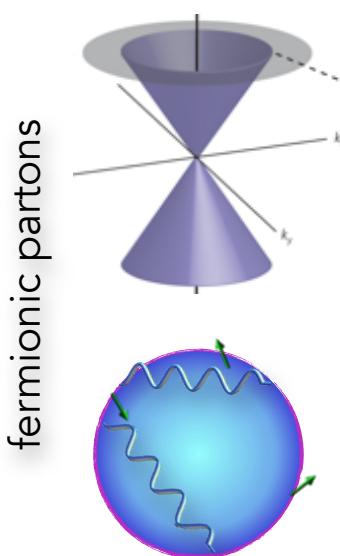
- Topological QSLs
 - full gap
- U(1) QSL
 - gapless emergent “photon”
- Algebraic QSLs
 - Relativistic CFT (power-laws)
- Spinon Fermi surface QSL



TQFT



compact
U(1)
gauge
theory



QED_3

QED_3
w/ $\mu > 0$

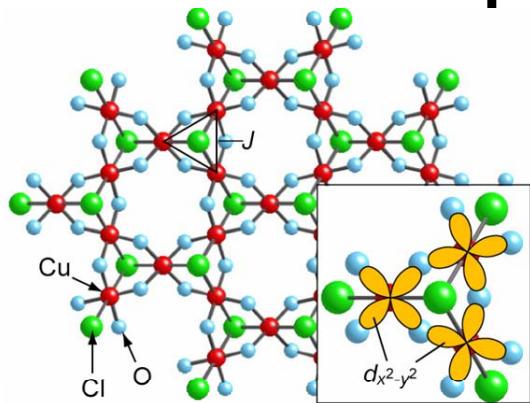
Summary so far

- QSLs are examples of “ultra-quantum matter” whose ground states are massive superpositions and exhibit long-range entanglement
- Characteristically they support *non-local* excitations, which might be anyons or other exotic particles
- The natural theoretical description of many QSLs is gauge theory
- Many QSLs are absolutely stable to all small perturbations, irrespective of symmetry. “Highly gapless” QSLs are less stable.

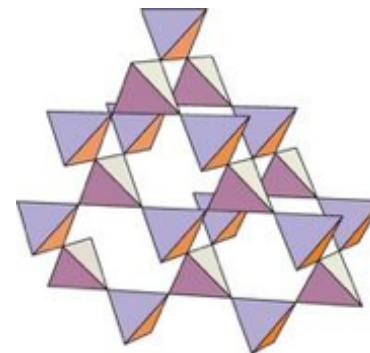
Spin liquid candidates



Top experimental platforms

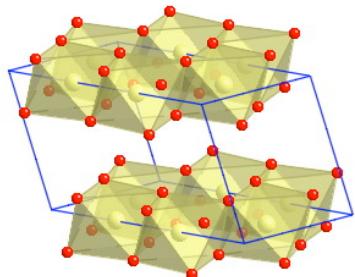


Herbertsmithite



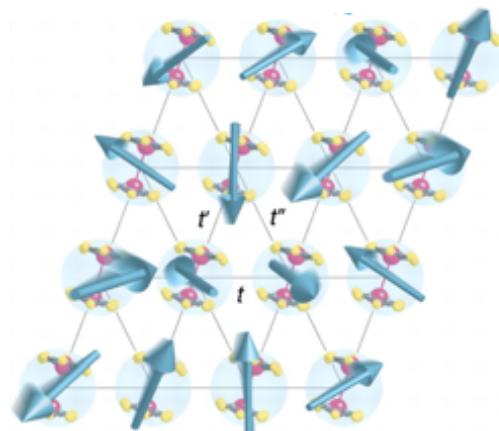
$\text{Yb}_2\text{Ti}_2\text{O}_7$
...

Quantum spin ice



Kitaev materials

Na_2IrO_3 ,
 (α, β, γ) -
 Li_2IrO_3
 $\alpha\text{-RuCl}_3$



organics

A rough guide to experiments on QSLs

Does it order?

- NMR line splitting
- muSR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/ x-ray

Delocalized excitations?

- thermal conductivity
- INS

Is there a gap?

- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of χ

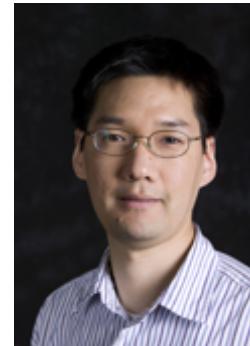
Structure of excitations?

- $E(k)$ from INS, RIXS
- optics, Raman

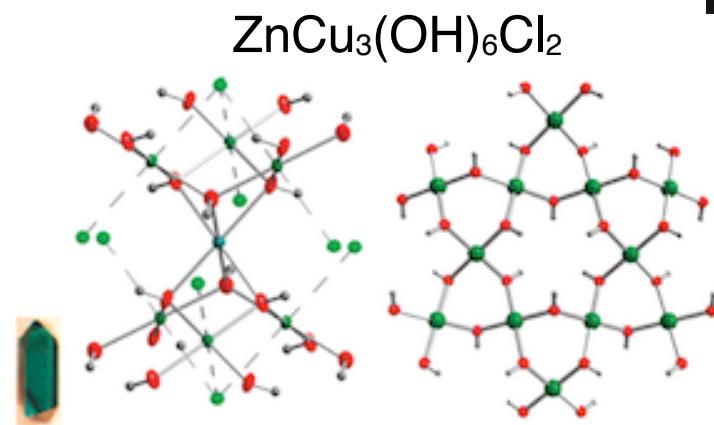
Exotica

- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects

Herbertsmithite

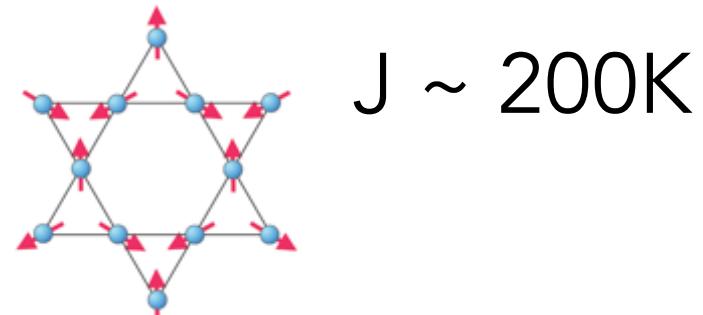


kagomé layers of Cu
S=1/2 spins, separated
by non-magnetic Zn



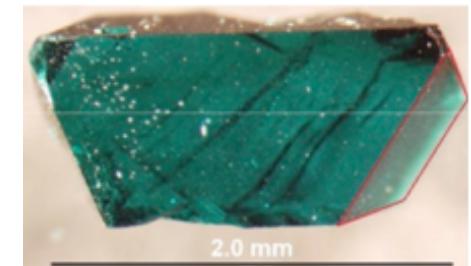
Hamiltonian

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \dots$$



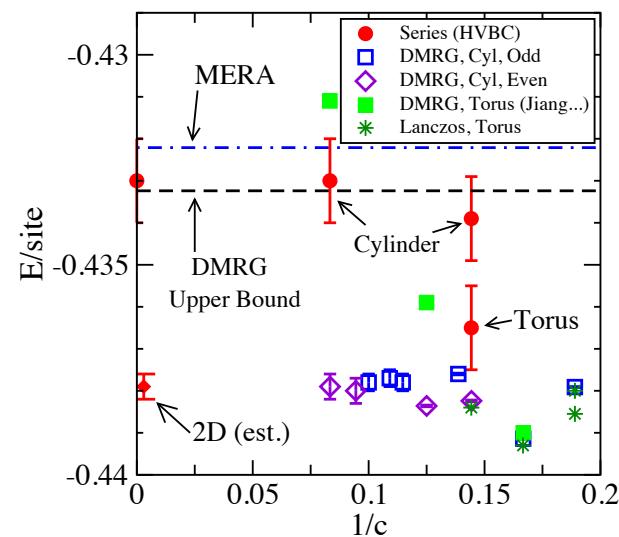
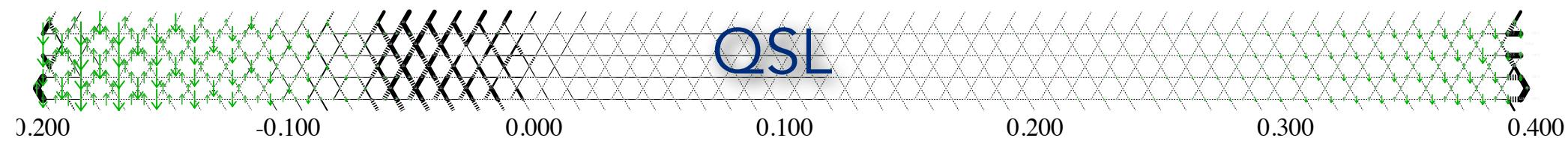
Long efforts by Nocera, Young Lee
groups produced crystals

beautiful material, but complicated by Cu/Zn site defects



$S=1/2$ kagomé AF

- Long history - but definitive evidence for QSL by DMRG



© Steve White

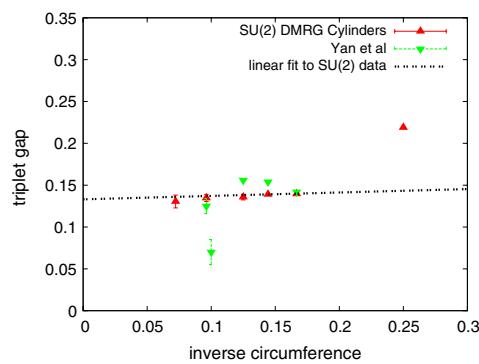
S. Yan *et al*, 2010

many other studies support
existence of some QSL phase

$S=1/2$ kagomé AF

- What kind of QSL?

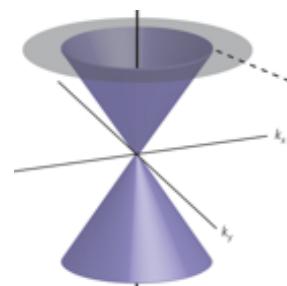
DMRG



S. Depenbrock *et al*, 2012

gapped,
presumably Z_2
topological QSL

partons



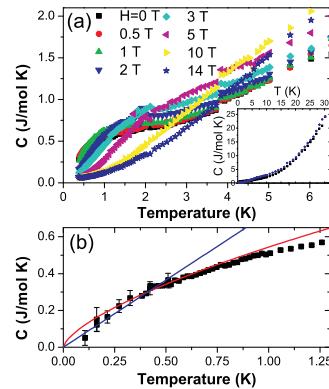
Y. Ran *et al*, 2007
F. Becca...

gapless U(1)
Dirac QSL

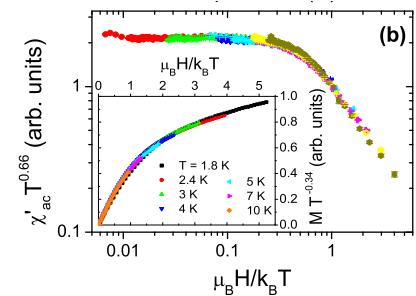
+ various other
proposals with
weaker
quantitative
support

Herbertsmithite

Lots of early evidence
for gaplessness



Helton *et al*, 2007

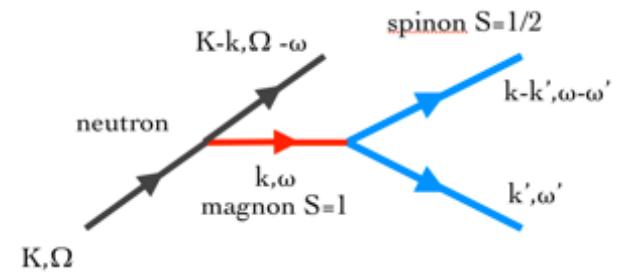
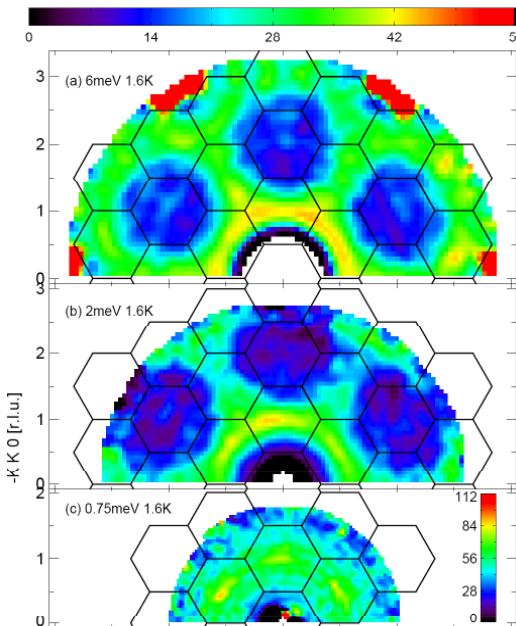


Helton *et al*, 2010

Single crystal INS

smooth continuum
scattering

T-H Han *et al*, 2012



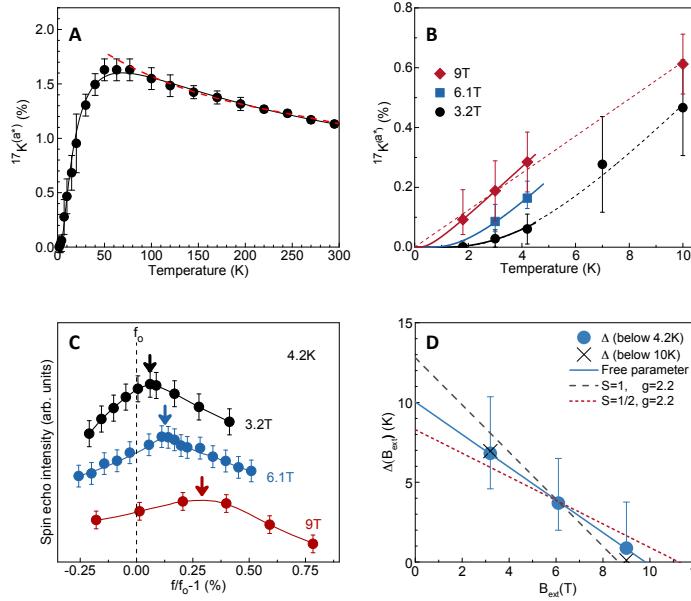
continuum scattering
expected
...but probably with more
structure?

Herbertsmithite

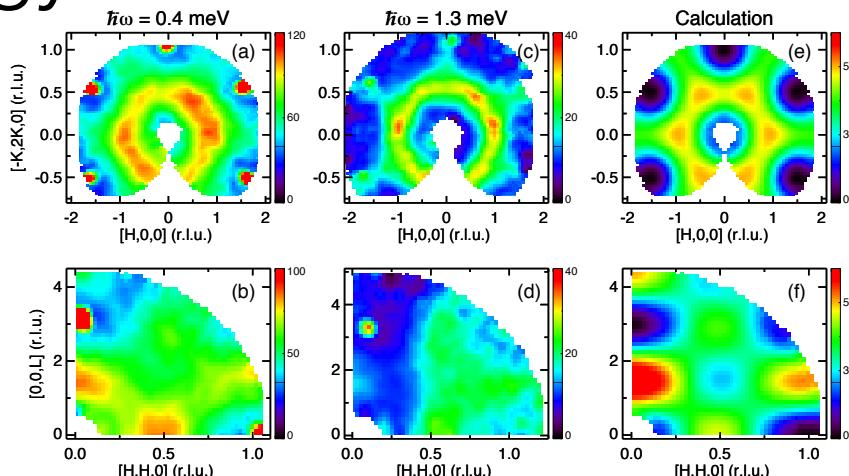
Single crystal NMR

M. Fu *et al*, 2015

c.f. T. Imai lecture



Low energy INS



T-H Han *et al*, 2015

estimate gap ~ 10K

claim to separate impurity signal below 0.7meV



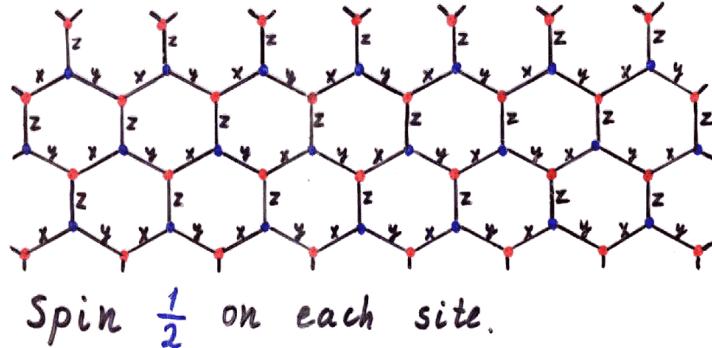
Kitaev model

Kitaev's honeycomb model

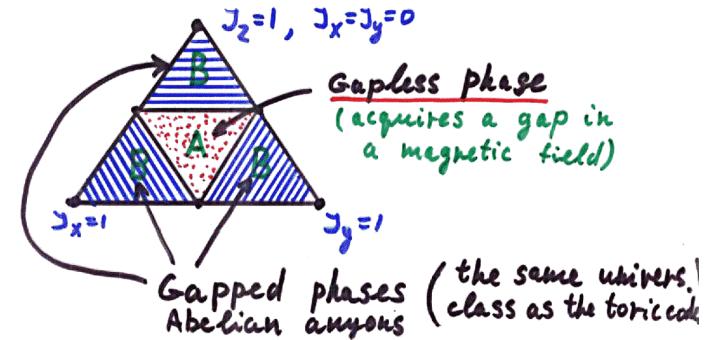
$$H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$$

KITP, 2003

1. The model

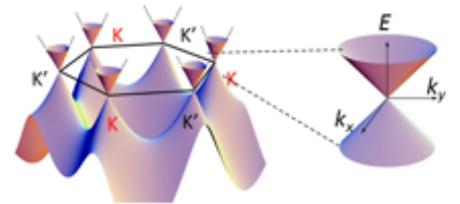


Phase diagram

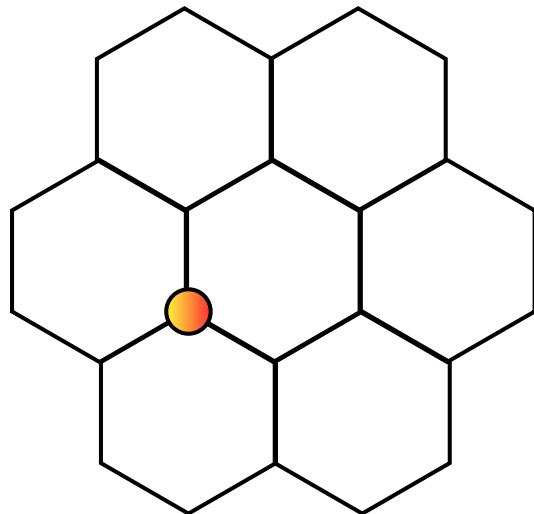


exact parton construction $\sigma_i^\mu = i c_i c_i^\mu$ $c_i c_i^x c_i^y c_i^z = 1$

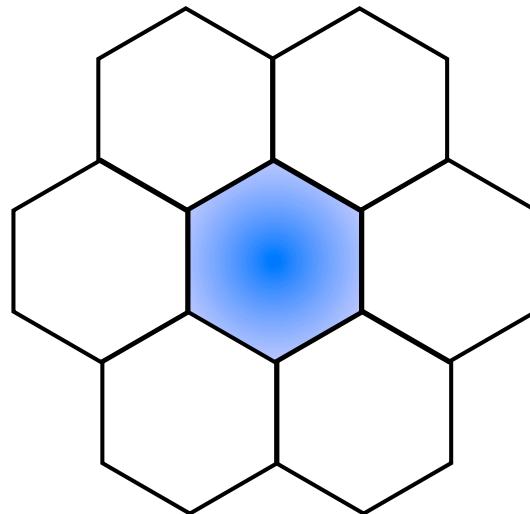
physical Majoranas $H_m = K \sum_{\langle ij \rangle} i c_i c_j$



Non-local excitations



Majorana ε



Flux e, m

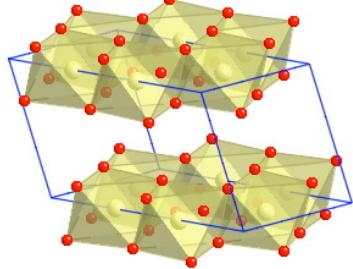
In Kitaev's model:

- Majorana's dispersion $\sim K$ and gapless
- Fluxes are localized and gapped

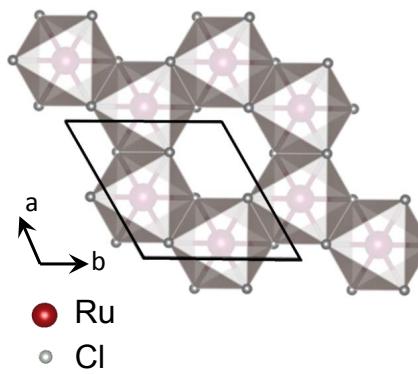
Kitaev Materials

Jackeli, Khaliullin

Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling

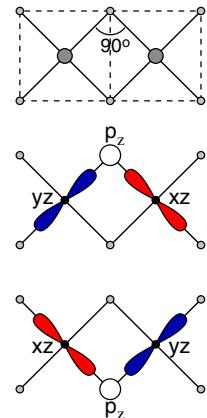


Na_2IrO_3 ,
 (α, β, γ) -
 Li_2IrO_3

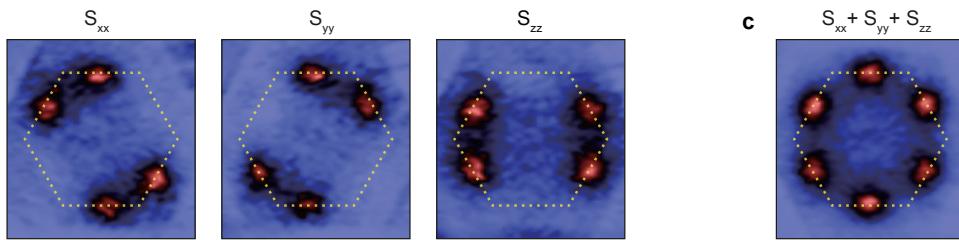


$\alpha\text{-RuCl}_3$

Honeycomb and hyper-honeycomb structures

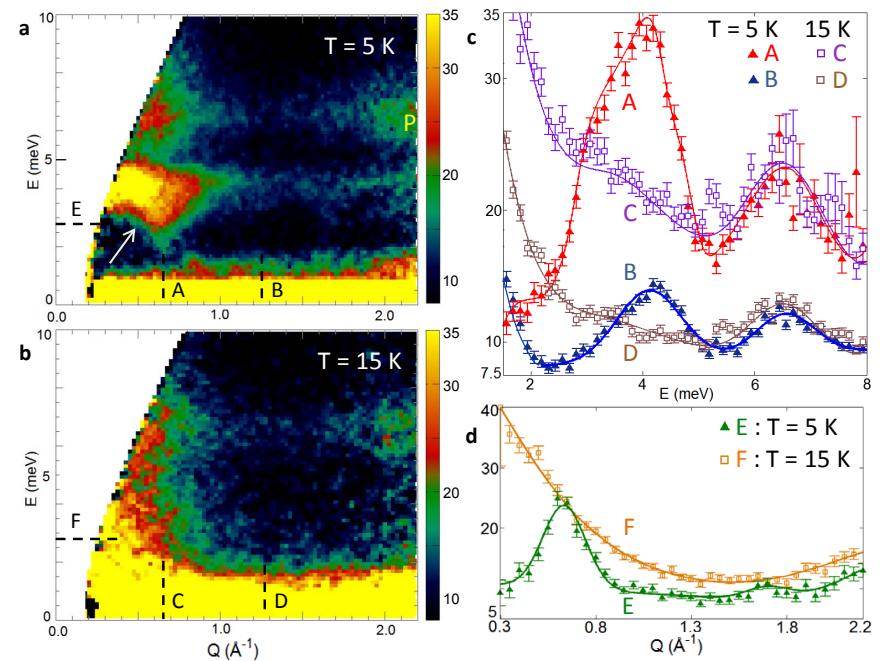


Kitaev Materials



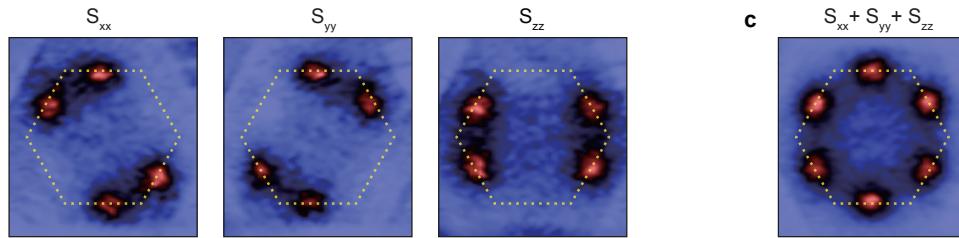
direct evidence for
direction-dependent
anisotropic exchange
from diffuse magnetic
x-ray scattering in
 Na_2IrO_3 (BJ Kim group)

there is pretty strong evidence
of substantial Kitaev exchange
in quite a few materials



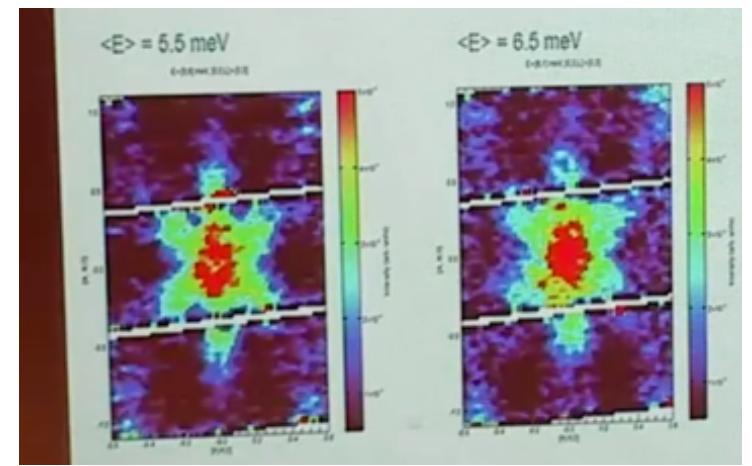
Observation of gapped
continuum mode persisting
above T_N in $\alpha\text{-RuCl}_3$
consistent with Majoranas
(A. Banerjee *et al*)

Kitaev Materials



direct evidence for
direction-dependent
anisotropic exchange
from diffuse magnetic
x-ray scattering in
 Na_2IrO_3 (BJ Kim group)

there is pretty strong evidence
of substantial Kitaev exchange
in quite a few materials



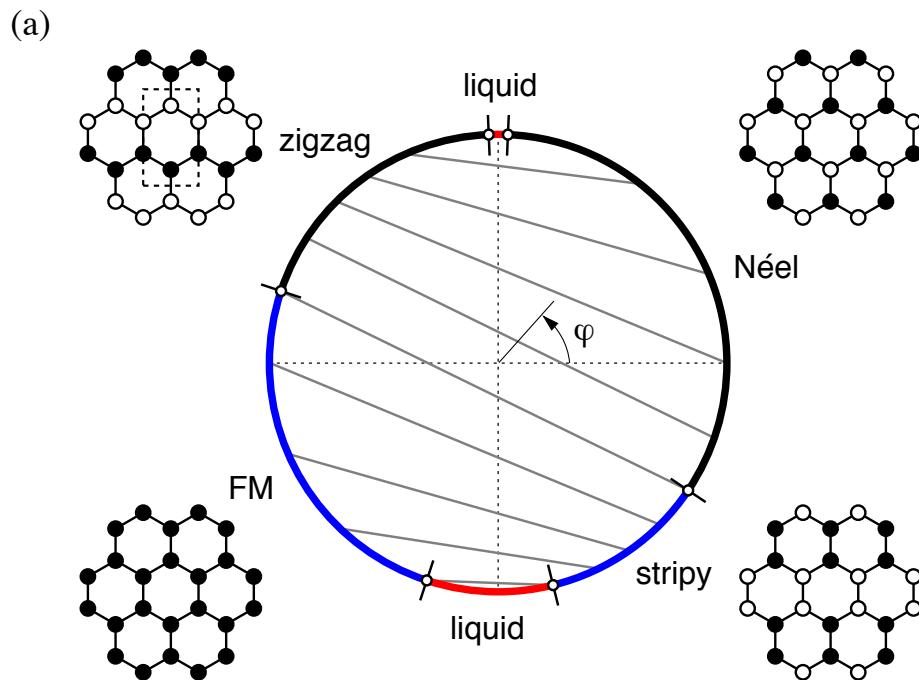
Unpublished single-crystal
data (A. Banerjee et al) in α -
 RuCl_3 has expected
momentum structure for
Kitaev QSL

Magnetism

- But...they all order so far

due to additional interactions,
e.g. Heisenberg

$$H = \sum_{i,\alpha} K S_i^\alpha S_{i+\alpha}^\alpha + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



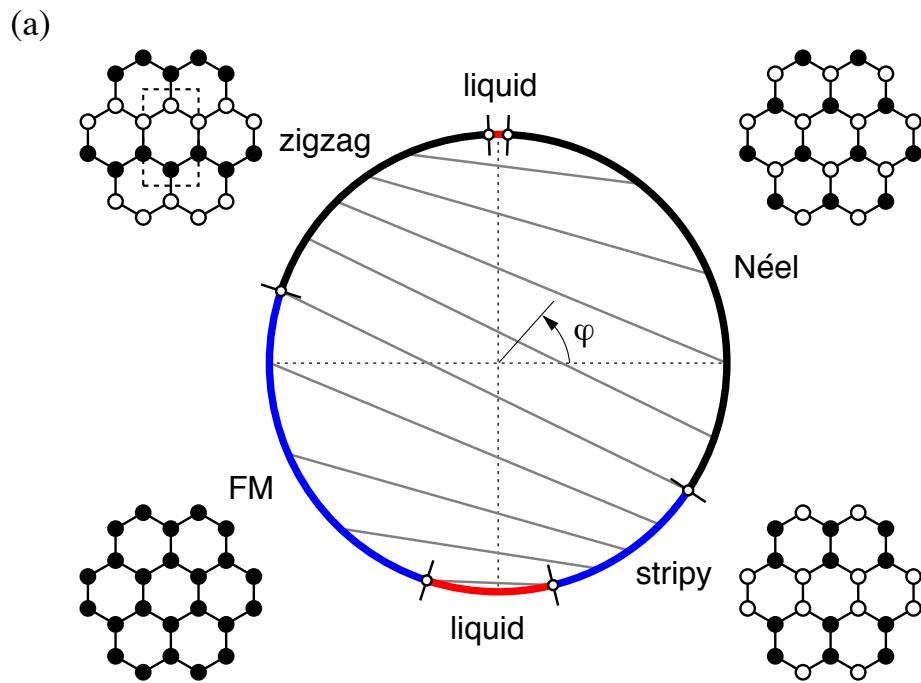
zigzag ordered state
has been observed in
 Na_2IrO_3 and
incommensurate order
in Li_2IrO_3

Magnetism

- But...they all order so far

due to additional interactions,
e.g. Heisenberg

$$H = \sum_{i,\alpha} K S_i^\alpha S_{i+\alpha}^\alpha + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



so far no QSL!

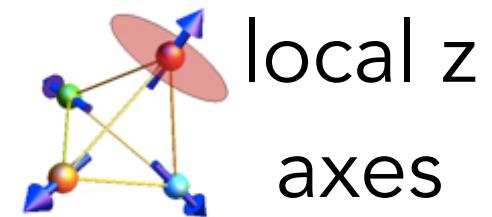


Real quantum spin ice

- Symmetry constrains form of generic Hamiltonian for

s. Curnoe, 2008 Kramer's doublets

$$H = \left. \begin{aligned} & J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \\ & - J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) \end{aligned} \right\} \quad \begin{aligned} & \text{these terms give the} \\ & \text{earlier model} \end{aligned}$$
$$+ J_{z\pm} \sum_{\langle i,j \rangle} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j]$$
$$+ J_{\pm\pm} \sum_{\langle i,j \rangle} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)$$

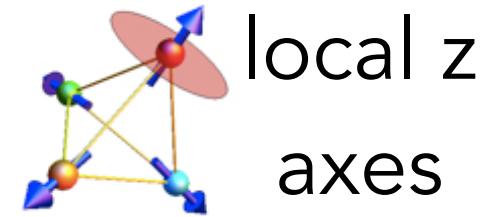


Hamiltonian

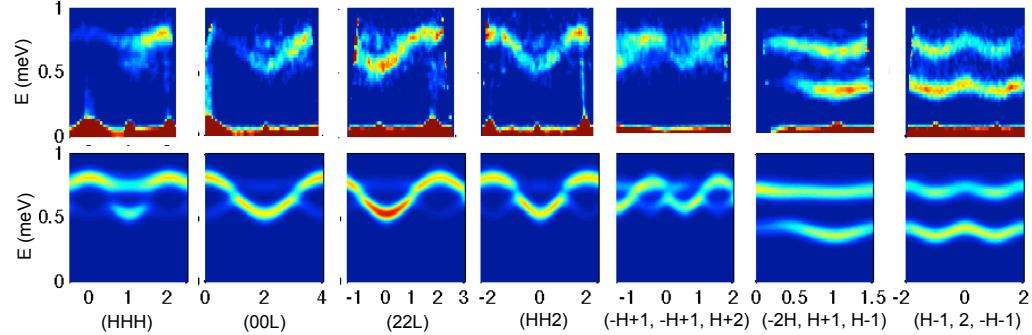


- Symmetry constrains form of generic Hamiltonian for Kramer's doublets

$$\begin{aligned}
 H = & J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \\
 & - J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) \\
 & + J_{z\pm} \sum_{\langle i,j \rangle} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j] \\
 & + J_{\pm\pm} \sum_{\langle i,j \rangle} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)
 \end{aligned}$$



L. Savary et al, 2012



$\text{Er}_2\text{Ti}_2\text{O}_7$

XY-like

$$J_{zz} = -2.5 \pm 1.8 \times 10^{-2} \text{ meV}$$

$$J_{z\pm} = -0.88 \pm 1.5 \times 10^{-2} \text{ meV}$$

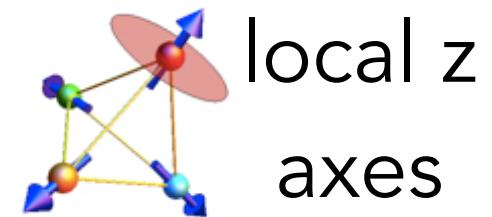
$$J_{\pm} = 6.5 \pm 0.75 \times 10^{-2} \text{ meV}$$

$$J_{\pm\pm} = 4.2 \pm 0.5 \times 10^{-2} \text{ meV}$$

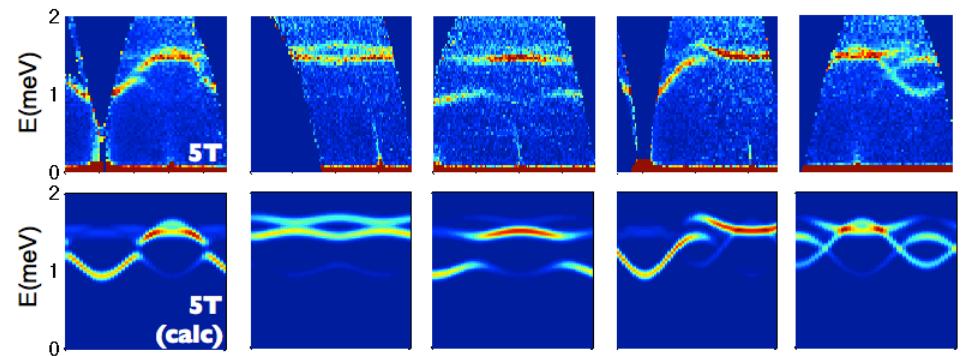
Hamiltonian

- Symmetry constrains form of generic Hamiltonian for Kramer's doublets

$$\begin{aligned}
 H = & J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \\
 & - J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) \\
 & + J_{z\pm} \sum_{\langle i,j \rangle} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j] \\
 & + J_{\pm\pm} \sum_{\langle i,j \rangle} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)
 \end{aligned}$$



local z
axes



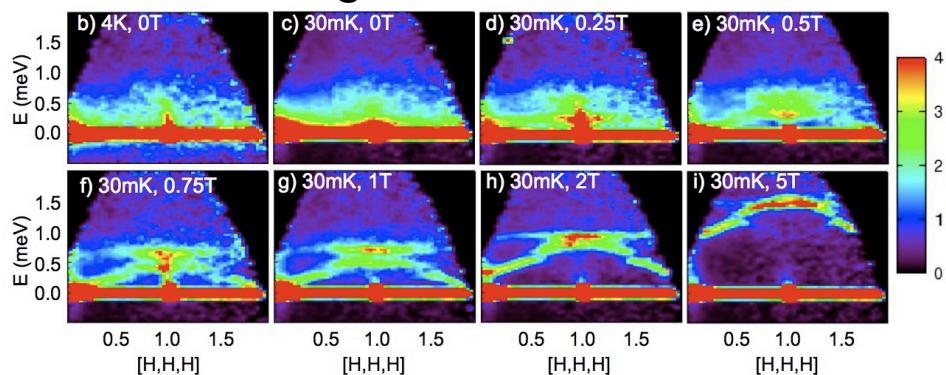
K. Ross et al, 2011

$\text{Yb}_2\text{Ti}_2\text{O}_7$
QSI

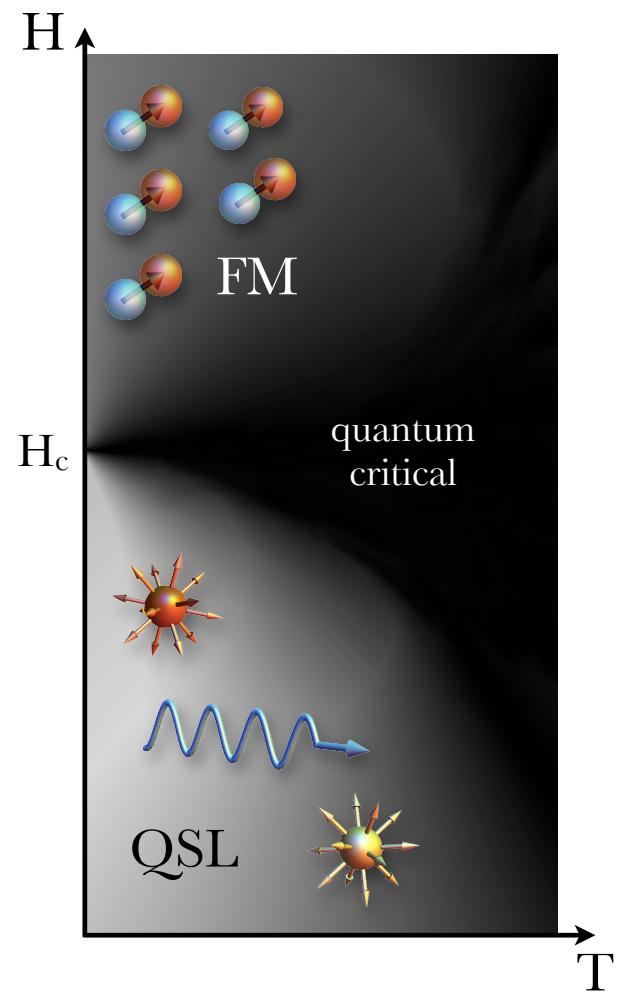
$J_{zz} = 0.17 \pm 0.04 \text{ meV}$
 $J_{z\pm} = 0.14 \pm 0.01 \text{ meV}$
 $J_{\pm} = 0.05 \pm 0.01 \text{ meV}$
 $J_{\pm\pm} = 0.05 \pm 0.01 \text{ meV}$

$\text{Yb}_2\text{Ti}_2\text{O}_7$?

missing
magnon?

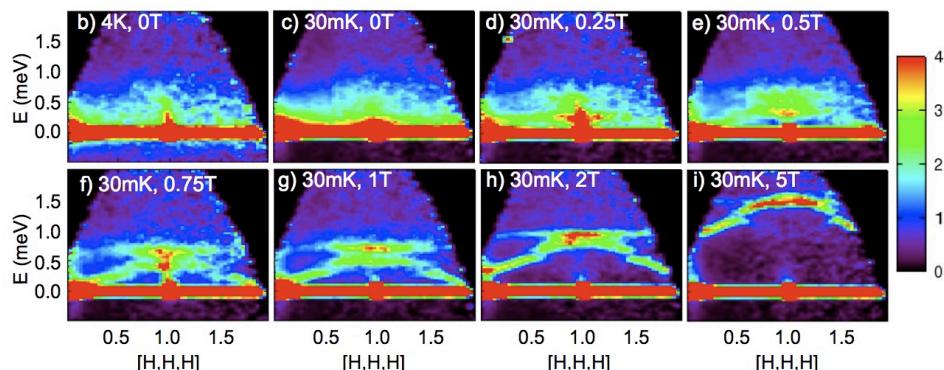


K.A. Ross *et al* (2009)



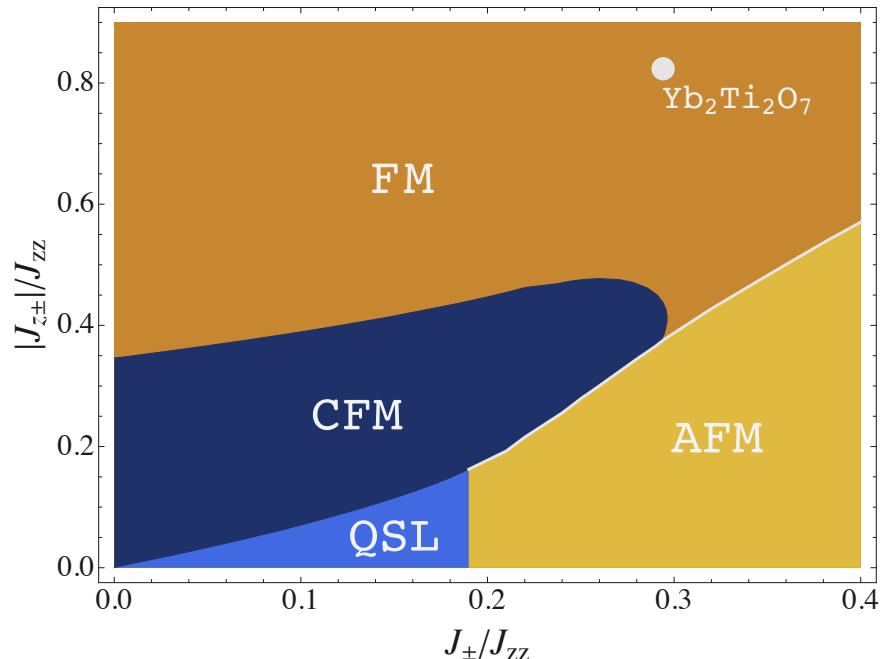
$\text{Yb}_2\text{Ti}_2\text{O}_7$?

missing
magnon?



K.A. Ross *et al* (2009)

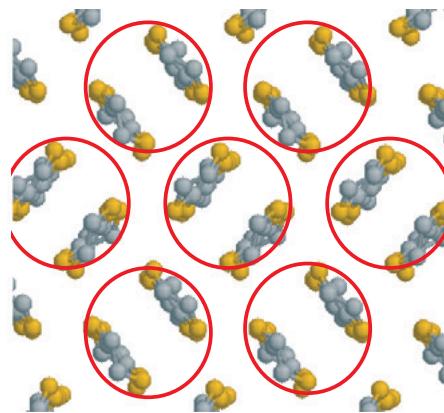
L. Savary + LB, 2012



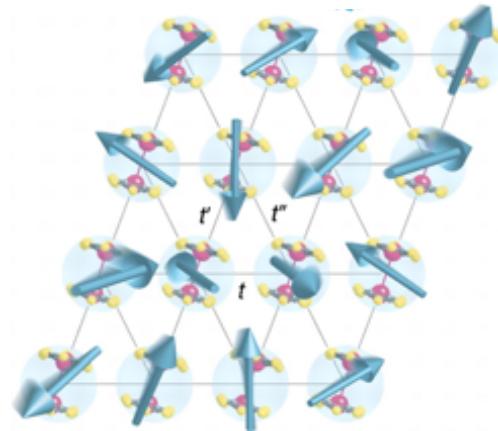
I tend to think recent evidence points away from the U(1) QSL and toward an unconventional FM. But this is still an unfinished story.

$J_{\pm\pm}$ might help?
disorder effects?
other materials?

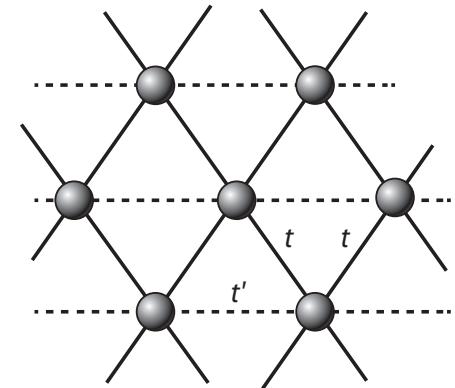
Organics



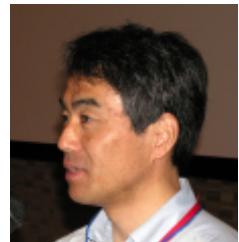
κ -(ET)₂X



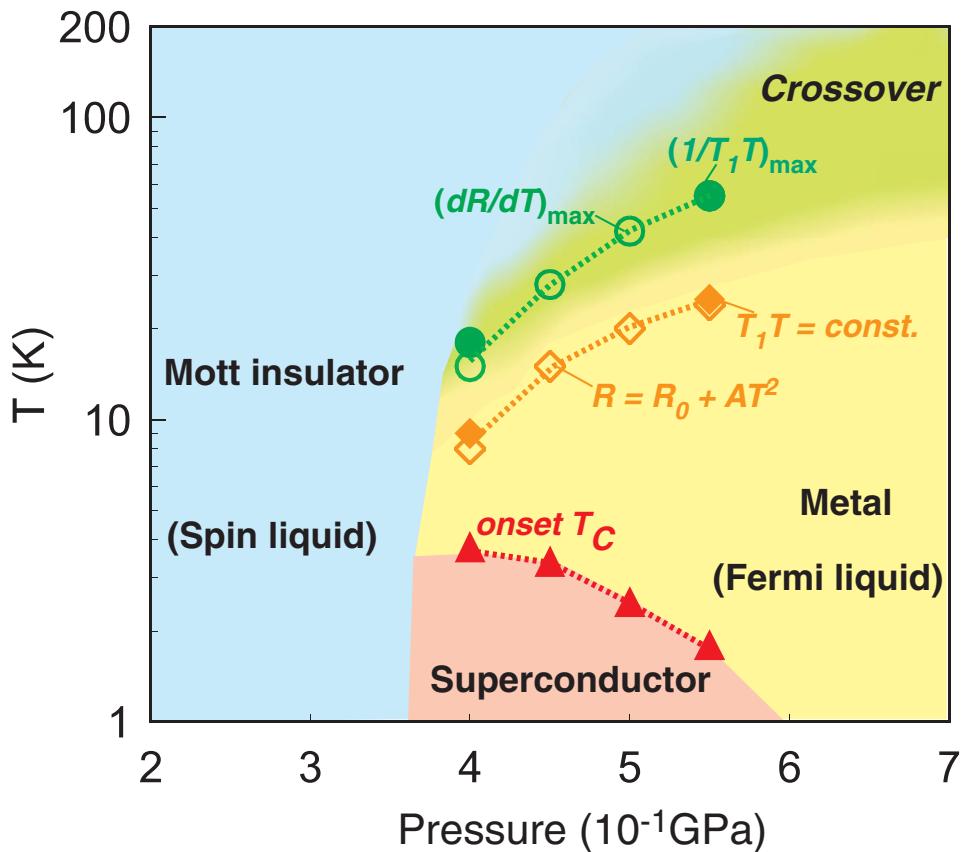
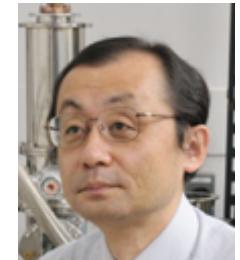
β' -Pd(dmit)₂



- Molecular materials which behave as effective triangular lattice $S=1/2$ antiferromagnets with $J \sim 250\text{K}$
- significant charge fluctuations

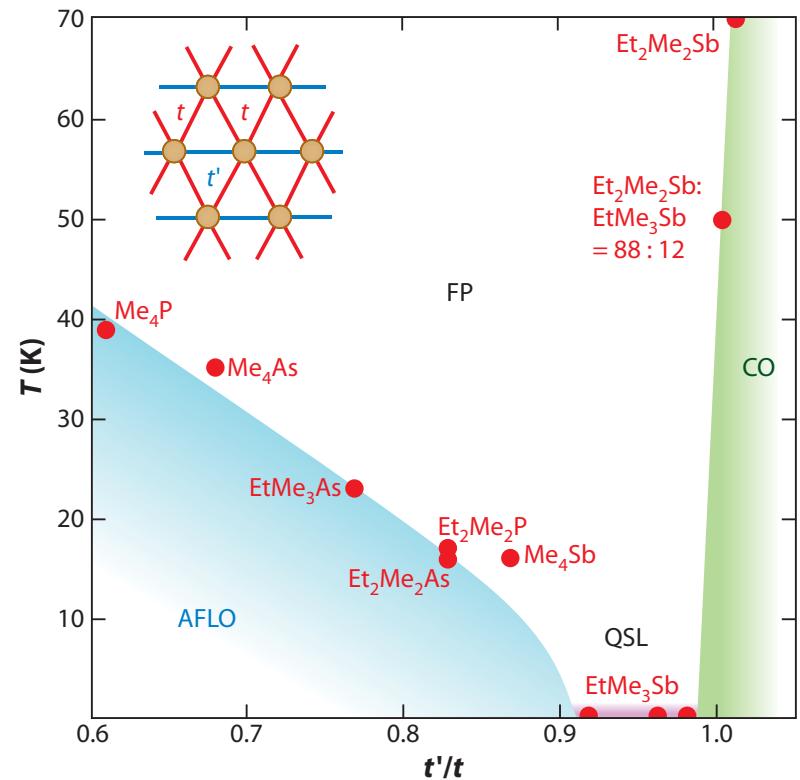


Organics



κ -(ET)₂Cu₂(CN)₃

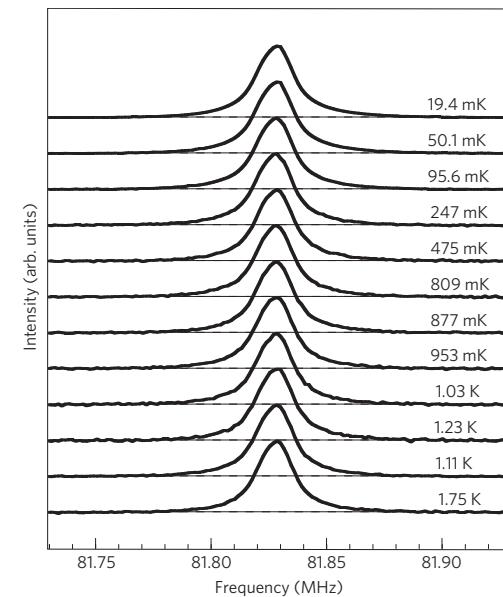
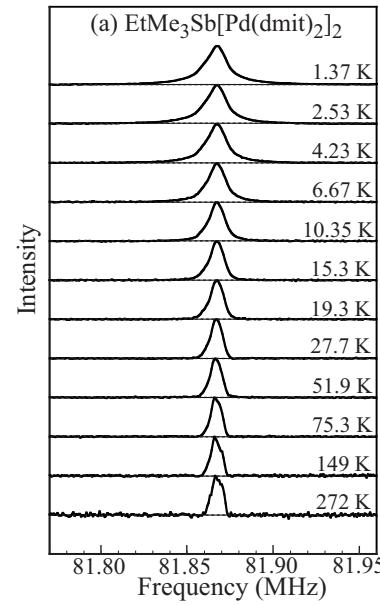
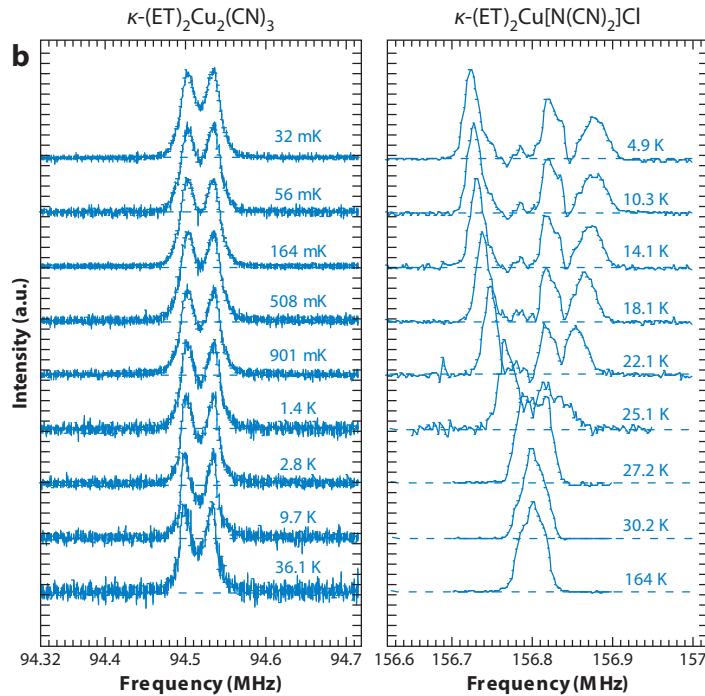
K. Kanoda group (2003-)



β' -Pd(dmit)₂

R. Kato group (2008-)

NMR lineshapes



κ -(ET)₂Cu₂(CN)₃

Y. Shimizu
et al, 2003 ^1H NMR

β' -Pd(dmit)₂

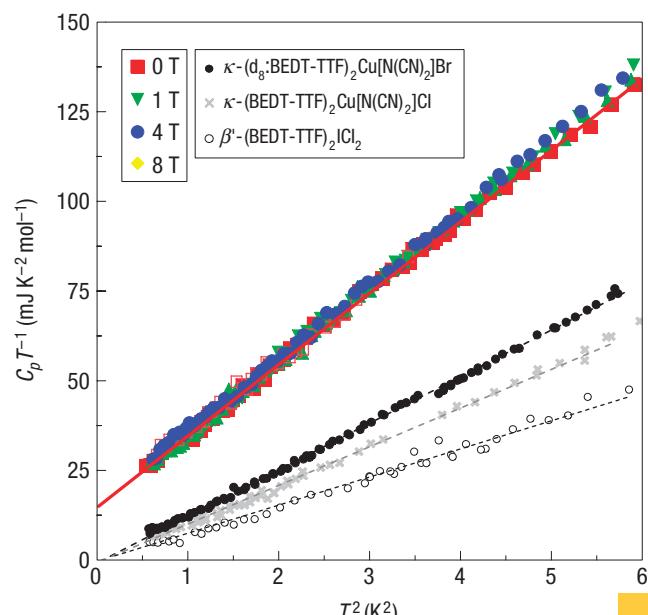
T. Itou et
al,
2008, 2010

^{13}Cs NMR

Evidence for lack of static moments: $f > 1000!$

Specific Heat

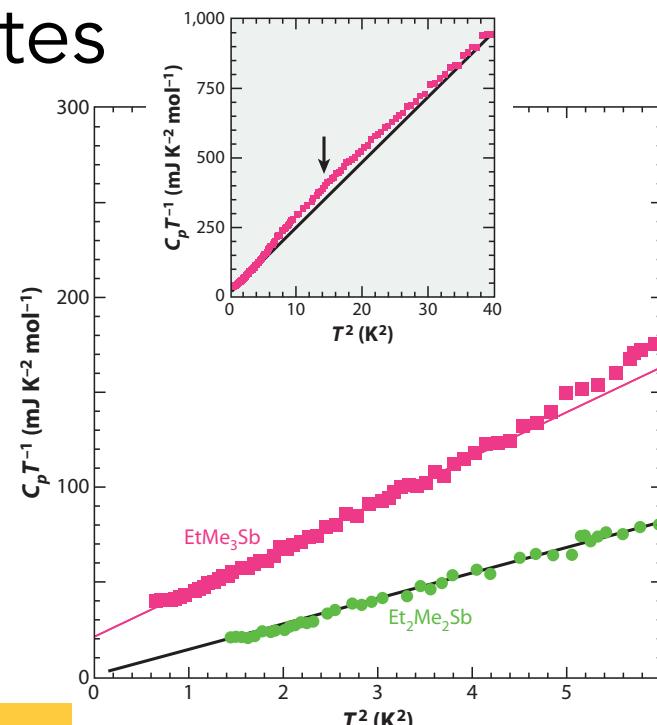
- $C \sim \gamma T$ indicates gapless behavior with large density of states



$$\gamma_{\text{Cu}} \sim 0.7 !!$$

κ - $(\text{ET})_2\text{Cu}_2(\text{CN})_3$

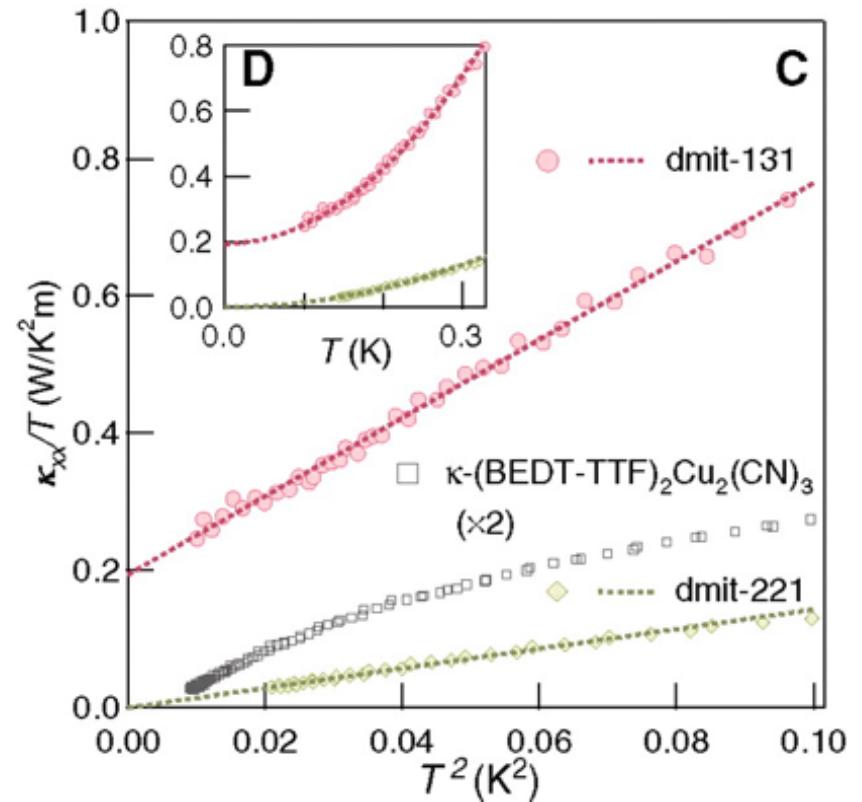
S. Yamashita *et al*, 2008



β' - $\text{Pd}(\text{dmit})_2$

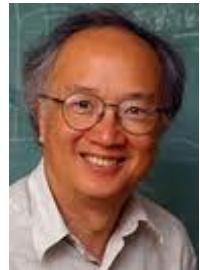
Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit
- Estimate for a *metal* would correspond to a mean free path $l \sim 1 \mu\text{m} \approx 1000 \text{ a} !$

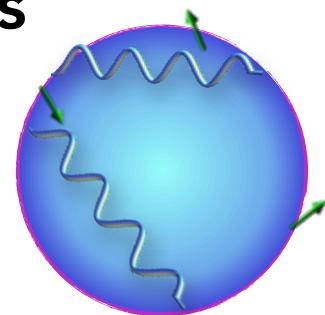


M. Yamashita *et al*, 2010

Organics - Theory



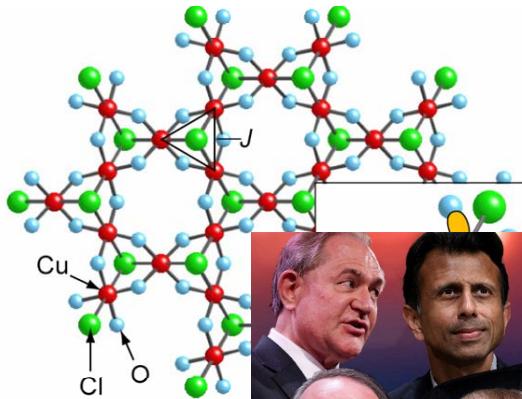
- RVB/QSL state:
 - Motrunich, Lee+Lee: (2005) “uniform RVB”
 - this is a kind of RVB state with very many (maybe a maximal number of?) long-range VBs
 - It is described by a **“Fermi sea” of spinons** coupled to a $U(1)$ gauge field
 - Good variational energy for triangular lattice Hubbard model



Organics: issues

- Why these very small set of materials?
- Spatial homogeneity?
- Indications of phase transitions. Gaps opening?
Charge ordering?
- Quantitative inconsistencies with expected scaling behavior from theory (c.f. $C \sim T^{2/3}$ etc.)
- Large isotope effects. Role of molecular rotations?
- Almost all experimental checks of QSL are limited to $T < 5K$, and many are not directly tied to spins, while $J \sim 200-300K$. Hampered by nature of materials.

New candidates are desirable



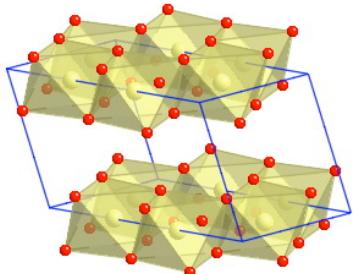
Herbert



$\text{Yb}_2\text{Ti}_2\text{O}_7$

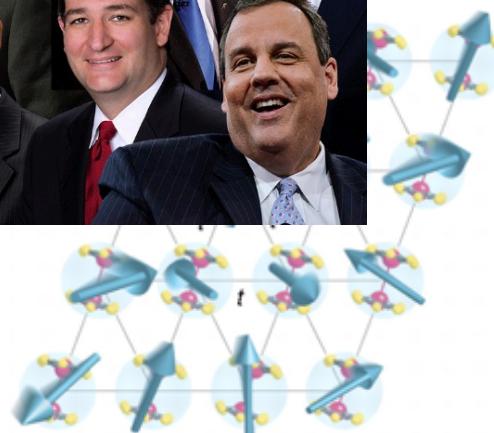
...

spin ice



Li_2IrO_3
 $\alpha\text{-RuCl}_3$

Kitaev materials



organics

New candidates are desirable

$\omega^1(k_1, k_2, k_3) = 6A^2 f_{NL}^{cg^1} \times \left\{ -\frac{1}{k_1^{4-n_2} k_2^{4-n_3}} - \frac{1}{k_2^{4-n_3} k_3^{4-n_1}} - \frac{1}{k_3^{4-n_1} k_1^{4-n_2}} \right.$

$\omega^0(k_1, k_2, k_3) = 6A^2 f_{NL}^{ortho} \times \left[-\frac{3}{k_1^{4-n_2} k_2^{4-n_3}} - \frac{3}{k_2^{4-n_3} k_3^{4-n_1}} - \frac{3}{k_3^{4-n_1} k_1^{4-n_2}} \right]$

bispectrum is:

$= 2 f_{NL}^{ijkl} [P_\Phi(k_1) P_\Phi(k_2) P_\Phi(k_3) + P_\Phi(k_1) P_\Phi(k_2) P_{\Phi^*}(k_3) + P_\Phi(k_1) P_{\Phi^*}(k_2) P_\Phi(k_3) + P_{\Phi^*}(k_1) P_\Phi(k_2) P_\Phi(k_3)] = 2 f_{NL}^{ijkl} [P_\Phi(k_1) P_\Phi(k_2) P_\Phi(k_3) + 2 f_{NL}^{ijkl} P_\Phi(k_1) P_\Phi(k_2) P_{\Phi^*}(k_3) + 2 f_{NL}^{ijkl} P_\Phi(k_1) P_{\Phi^*}(k_2) P_\Phi(k_3) + 2 f_{NL}^{ijkl} P_{\Phi^*}(k_1) P_\Phi(k_2) P_\Phi(k_3)]$

$\omega(k_1, k_2, k_3) = 2 f_{NL}^{ijkl} P_\Phi(k_1) P_\Phi(k_2) P_\Phi(k_3) + 2 f_{NL}^{ijkl} P_\Phi(k_1) P_\Phi(k_2) P_{\Phi^*}(k_3) + 2 f_{NL}^{ijkl} P_\Phi(k_1) P_{\Phi^*}(k_2) P_\Phi(k_3) + 2 f_{NL}^{ijkl} P_{\Phi^*}(k_1) P_\Phi(k_2) P_\Phi(k_3)$

$C_{l_1, l_2, l_3}^{x_1, x_2, x_3, l_4, l_5} = C_{l_1}^{x_1, x_2} \tilde{C}_{l_2, l_3}^{x_2, x_3} - C_{l_1}^{x_1, x_3} \tilde{C}_{l_2, l_3}^{x_2, x_3} + C_{l_1}^{x_1, x_2} \tilde{C}_{l_3, l_2}^{x_2, x_3} - C_{l_1}^{x_1, x_3} \tilde{C}_{l_3, l_2}^{x_2, x_3}$

and $C_{l_1, l_2, l_3}^{x_1, x_2, x_3}$ are cross power spectra

$C_{l_1, l_2, l_3}^{x_1, x_2, x_3} = \frac{1}{2} \sum_{l_4, l_5} C_{l_1, l_2, l_3, l_4, l_5}^{x_1, x_2, x_3, l_4, l_5} \delta(l_4, l_5)$

CIFAR is uniquely positioned

Kitaev materials

organics

Theory: Frontiers

New phases

- Fractal spin liquids (Haah++) in 3d
- SPT phases
- Quenched disorder

Fundamental problems

- QSLs with strongly coupled matter-gauge theory
- QCPs to/from QSL phases
- Out of equilibrium
- Doping

Reality

- Devise experimental protocols to reveal quantum non-locality of QSLs
- Computational methods: less bias, reliability of variational methods, beyond ground states

Thanks for your attention

References here: <https://spinsandelectrons.com/pedagogy/>

