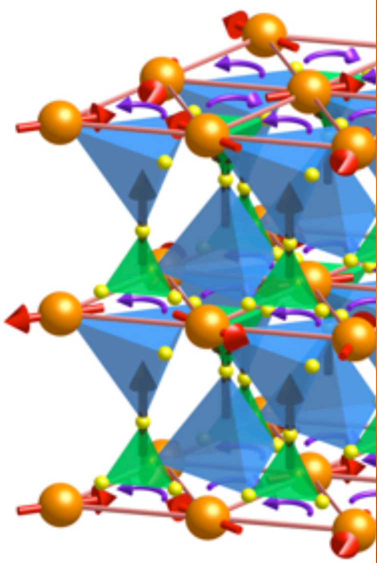
Correlations and Topology: a Review

Leon Balents, KITP

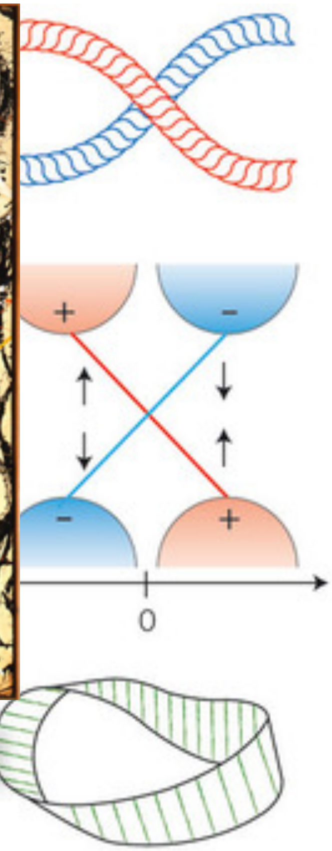
Quantum Criticality and Topology in Itinerant Electron Systems

August 2016

Seeking a convergence?



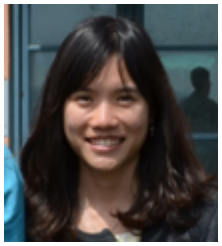
Jackson Pollack, Convergence, 1952



Correlations

Topology

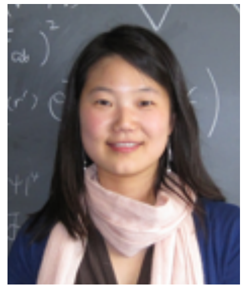
People



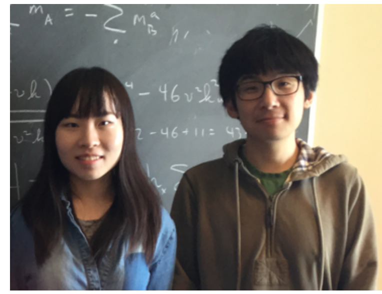
Ru Chen

Lucile Savary

Eun-Gook Moon



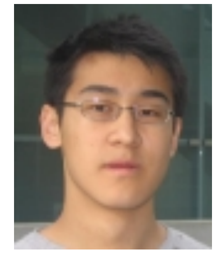
Sung-Bin Lee



宋雪洋 尤亦庄

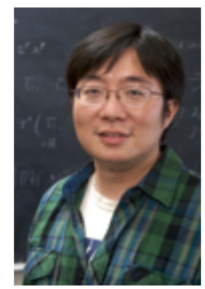
Xue-Yang Song

Yi-Zhuang You



T. Hsieh

H. Ishizuka



Cenke Xu

Jianpeng Liu

S. Nakatsuji

T. Kondo

Z. Tian

S. Shin

Yong-Baek Kim

Jay-Z

Outline

- Where can correlations enrich topology?
- Three types of topology: band topology, Berry phase topology, intrinsic topological order
- Correlations in these:
  - Bosonic SPT phases
  - Correlated Weyl fermions
  - Drumhead surface states
- If there's time, intrinsic topological order

Three types of topology

Topological Insulator

topology of filled bands

"symmetry protected topological order"

Topological Semimetal

topology of k-surfaces

"Berry phase topology"

Topological Spin Liquid

topology of entanglement

"intrinsic topological order"

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+ Correlations??

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- ✦Topological Kondo Insulator SmB_6?

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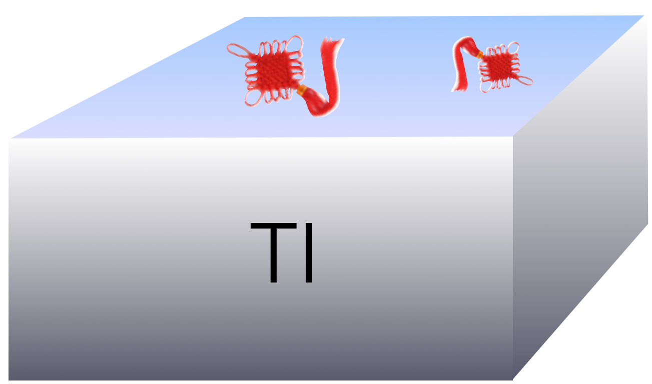
+ Correlations:

- ✦Topological Kondo Insulator SmB_6?
- ✦Surface states may be more correlated than bulk

C. Wang *et al*, 2013
L. Fidkowski *et al*, 2013
M.A. Metlitski *et al*, 2013
P. Bonderson *et al*, 2013

surface state with gapped non-abelian anyons



Three types of topology

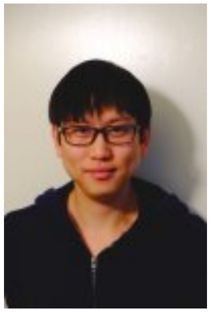
Topological Insulator

topology of filled bands

"symmetry protected topological order"

+ Correlations:

- ✦ Topological Kondo Insulator SmB_6?
- ✦ Surface states may be more correlated than bulk
- ✦ <u>Bosonic SPT states</u>



SPT phases

Symmetry protected topological order

From Wikipedia, the free encyclopedia

Symmetry Protected Topological order (**SPT** order)[1] is a kind of order in zero-temperature quantum-mechanical states of matter that have a symmetry and a finite energy gap.

To derive the results in a most-invariant way, renormalization group methods are used (leading to equivalence classes corresponding to certain fixed points).[1] The SPT order has the following defining properties:

(a) *distinct SPT states with a given symmetry cannot be smoothly deformed into each other without a phase transition, if the deformation preserves the symmetry.*
(b) *however, they all can be smoothly deformed into the same trivial product state without a phase transition, if the symmetry is broken during the deformation.*

Using the notion of quantum entanglement, we can say that SPT states are short-range entangled states *with a symmetry* (by contrast: for long-range entanglement see topological order, which is not related to the famous EPR paradox). Since short-range entangled states have only trivial topological orders we may also refer the SPT order as Symmetry Protected "Trivial" order.

Contents [hide]

1 Characteristic properties of SPT order
2 Relation between SPT order and (intrinsic) topological order
3 Examples of SPT order
4 Group cohomology theory for SPT phases
5 A complete classification of 1D gapped quantum phases (with interactions)
6 See also
7 References

A list of bosonic SPT states from group cohomology $H^{d+1}[G, U(1)] \oplus_{k=1}^{d} H^k[G, iTO^{d+1-k}]$ (Z_2^T = time-reversal-symmetry group)

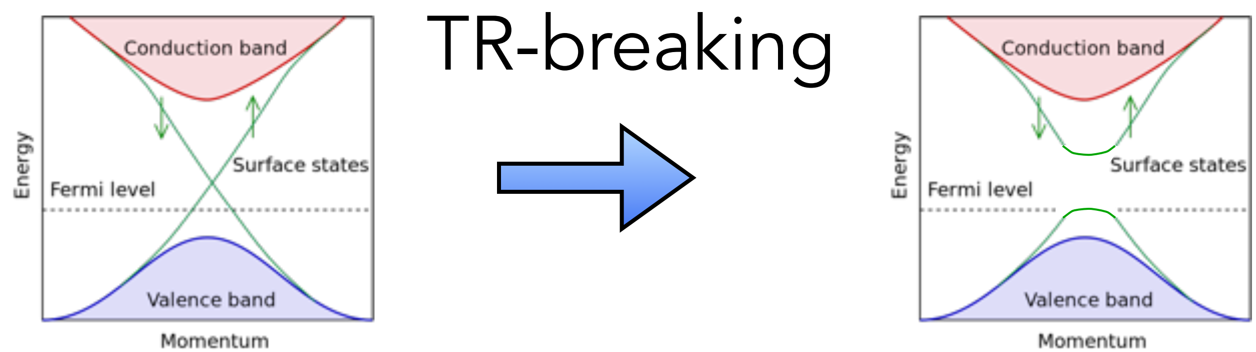
symm. group	1+1D	2+1D	3+1D	4+1D	comment
0	0	Z	0	Z_2	iTO phases with no symmetry: iTO^{d+1}
$U(1) \rtimes Z_2^T$	Z_2	Z_2	$2Z_2 + Z_2$	$Z \oplus Z_2 + Z$	bosonic topological insulator
Z_2^T	Z_2	0	$Z_2 + Z_2$	0	bosonic topological superconductor
Z_n	0	Z_n	0	$Z_n + Z_n$	
$U(1)$	0	Z	0	$Z + Z$	2+1D: quantum Hall effect
$SO(3)$	Z_2	Z	0	Z_2	1+1D:Haldane phase; 2+1D: spin Hall effect
$SO(3) \times Z_2^T$	$2Z_2$	Z_2	$3Z_2 + Z_2$	$2Z_2$	
$Z_2 \times Z_2 \times Z_2^T$	$4Z_2$	$6Z_2$	$9Z_2 + Z_2$	$12Z_2 + 2Z_2$	

SPT phases

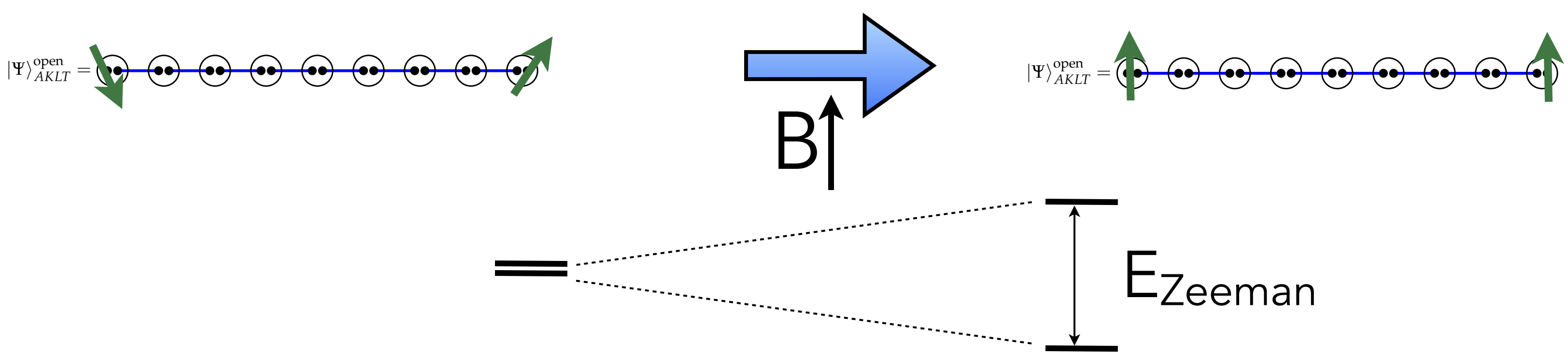
- An SPT phase is:
  - A gapped state which can be deformed to a product state if and only if a symmetry broken during the deformation
  - A state with usually gapless but always anomalous states at its boundary
  - A generalization of topological band insulators to interacting systems, spins, bosons etc.

The examples

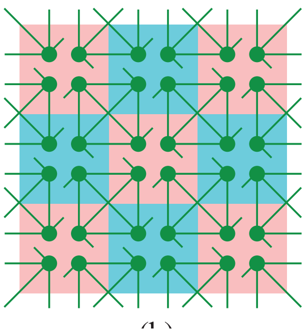
Topological insulator



Haldane/AKLT chain - a bosonic SPT



Bosonic SPTs in d>1?



Chen, Gu, Liu, Wen - *many* tensor network states

classified by $\mathcal{H}^{1+d}[G, U_T(1)]$

Symm. group	$d=0$	$d=1$	$d=2$	$d=3$
Z_2^T	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z}_2$
$Z_2^T \times \mathrm{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$
Z_n	$\mathbb{Z}_n$	$\mathbb{Z}_1$	$\mathbb{Z}_n$	$\mathbb{Z}_1$
$Z_n \times \mathrm{trn}$	$\mathbb{Z}_n$	$\mathbb{Z}_n$	$\mathbb{Z}_n^2$	$\mathbb{Z}_n^4$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$U(1) \times \mathrm{trn}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}^4$
$U(1) \rtimes Z_2^T$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$
$U(1) \rtimes Z_2^T \times \mathrm{trn}$	$\mathbb{Z}$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^3$	$\mathbb{Z} \times \mathbb{Z}_2^8$
$U(1) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_1$	$\mathbb{Z}_2^3$
$U(1) \times Z_2^T \times \mathrm{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^9$
$U(1) \rtimes Z_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_2$
$U(1) \times Z_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}_1$
$Z_n \rtimes Z_2^T$	$\mathbb{Z}_n$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \times Z_2^T$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \rtimes Z_2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_n \times \mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$
$Z_m \times Z_n$	$\mathbb{Z}_m \times \mathbb{Z}_n$	$\mathbb{Z}_{(m,n)}$	$\mathbb{Z}_m \times \mathbb{Z}_n \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(m,n)}^2$
$D_2 \times Z_2^T = D_{2h}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^6$	$\mathbb{Z}_2^9$
$Z_m \times Z_n \times Z_2^T$	$\mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)} \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(2,m,n)}^2 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m,n)}^4 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$
$SU(2)$	$\mathbb{Z}_1$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3)$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3) \times \mathrm{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}^3 \times \mathbb{Z}_2^3$
$SO(3) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$
$SO(3) \times Z_2^T \times \mathrm{trn}$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^{12}$

(c) Time reversal & $U(1)_{charge}$ Symmetry: $\mathbb{Z}_2$ classes. Non-chiral Edge.

YM Liu + Vishwanath - K-matrix theory in 2d

$$S = \frac{i}{4\pi} \int d^2x d\tau \; K_{IJ} \epsilon^{\mu\nu\lambda} a_\mu^I \partial_\nu a_\lambda^J$$

all these states have a c=1 Luttinger liquid edge

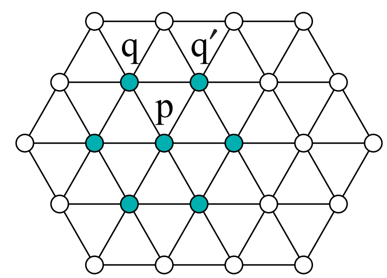
Any models?

- Tensor network constructions

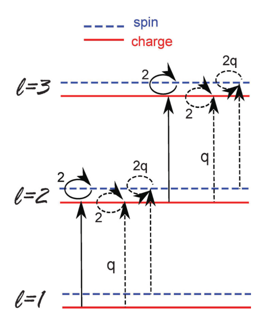
$$H_i = \sigma_i^+ \eta_{21}^+ \eta_{32}^+ \eta_{43}^+ \eta_{45}^+ \eta_{56}^+ \eta_{61}^+ + \sigma_i^- \eta_{21}^- \eta_{32}^- \eta_{43}^- \eta_{45}^- \eta_{56}^- \eta_{61}^-,$$

- Levin-Gu model

$$H_1 = -\sum_p B_p, \quad B_p = -\sigma_p^x \prod_{\langle pqq' \rangle} i^{\frac{1-\sigma_q^z \sigma_{q'}^z}{2}},$$



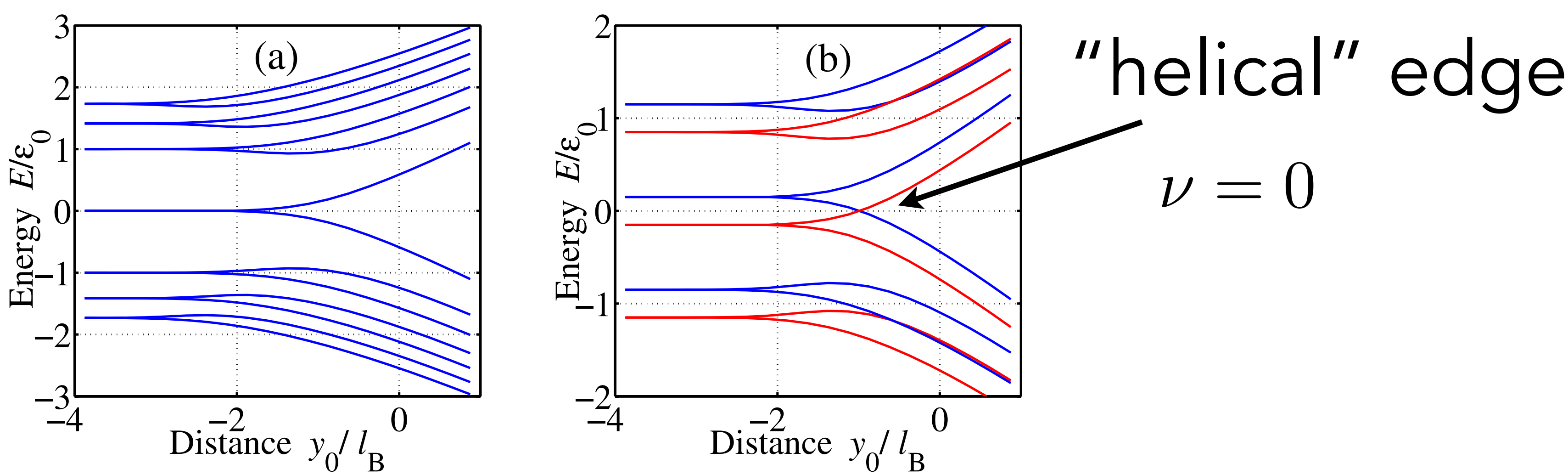
- Coupled wires



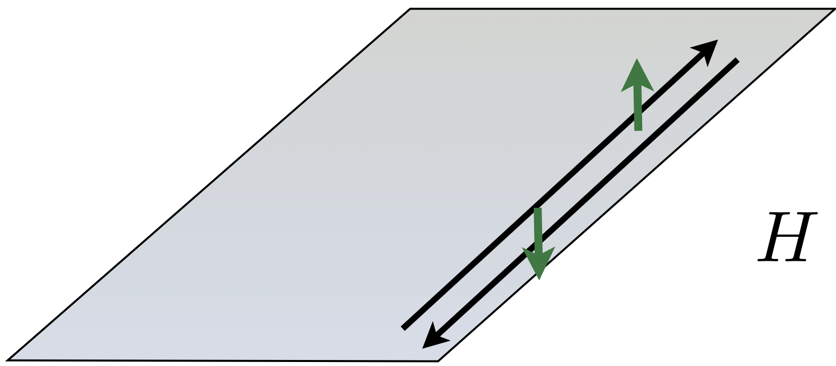
Pretty hard to realize

Graphene

- Kane+Mele: QSHE at zero field in graphene from SOC - but tiny effect
- Abanin, Lee, Levitov: “fake” QSHE in graphene due in quantum Hall regime



Graphene "QSHE"



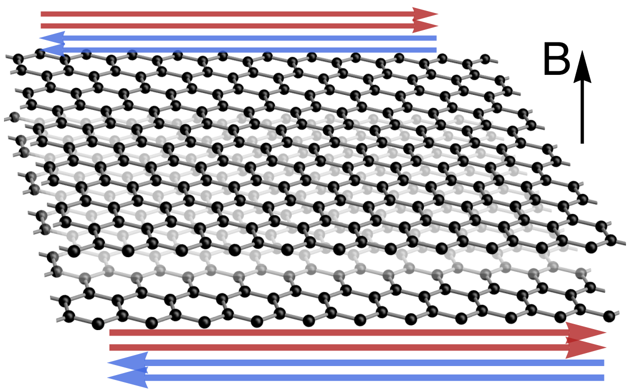
edge is spin-momentum locked

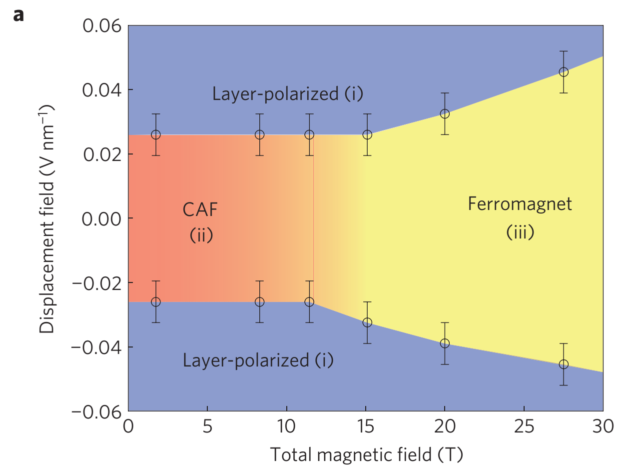
$$H = iv\int dx\,\left[\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L\right]$$
$$= iv\int dx\,\left[\psi_\uparrow^\dagger \partial_x \psi_\uparrow - \psi_\downarrow^\dagger \partial_x \psi_\downarrow\right]$$

This is a "Fermionic SPT"

- Backscattering is prohibited by spin-conservation symmetry (excellent approximation since SOC weak)

Bilayer graphene



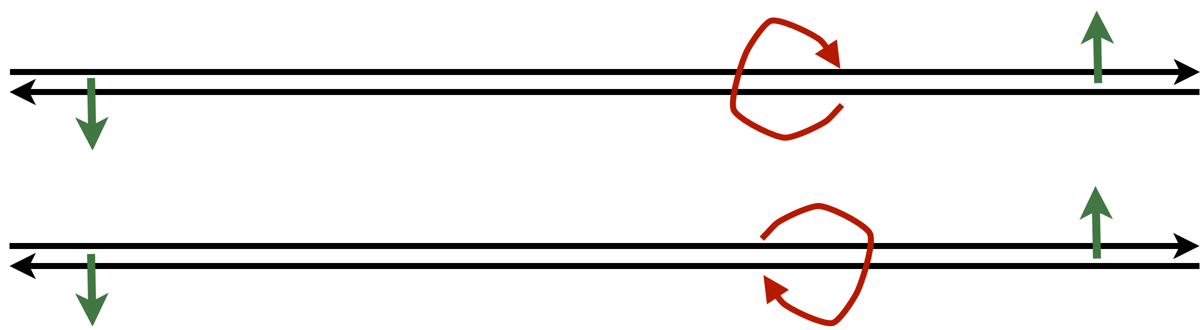


Maher *et al*, 2013

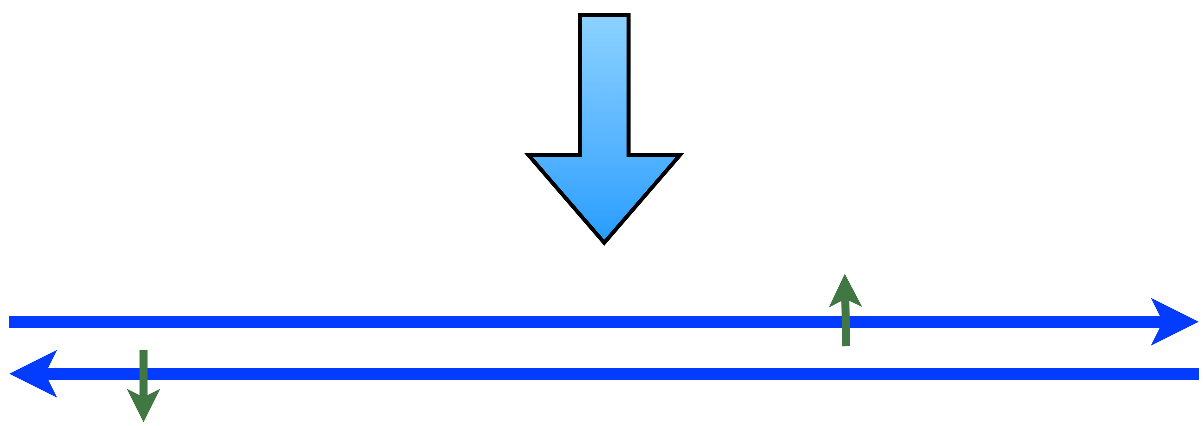
If spin is conserved, this is characterized by spin Chern number 2

Edge has two helical fermionic edge states

Interactions



backscattering $H_{\rm bs} = g \int dx \left[\psi^\dagger_{1R}\psi_{1L}\psi^\dagger_{2L}\psi_{2R} + \text{h.c.} \right]$



a single bosonic helical edge

How to get this

- Bosonization $\psi_{a,L/R}, \psi^\dagger_{a,L/R} \to \theta_a, \phi_a$
- Rotate $\theta_\pm = \theta_1 \pm \theta_2, \phi_\pm = \frac{1}{2}(\phi_1 \pm \phi_2)$
- Interaction induces gap for "-" sector

$$H_{bs} \sim g \int \cos 2\phi_-$$

- Only symmetric sector remains

$$H_{\mathrm{eff}} = \int dx \left[\frac{v}{2K} (\partial_x \theta)^2 + \frac{vK}{2} (\partial_x \phi)^2 \right]$$

SPT?

$$H_{\text{eff}} = \int dx \left[\frac{v}{2K}(\partial_x \theta)^2 + \frac{vK}{2}(\partial_x \phi)^2 \right]$$

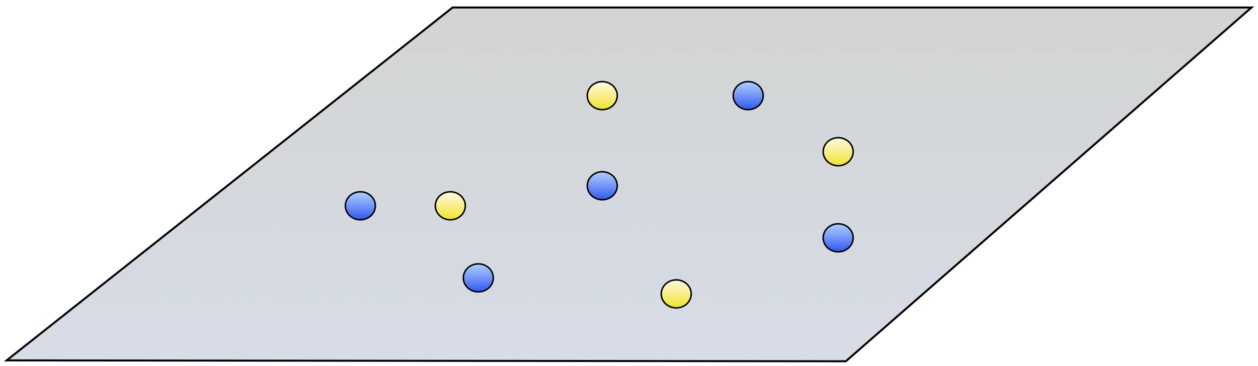
- How is it different from just a spin-polarized quantum wire (which has the same bosonized Hamiltonian)?
- Symmetry: $U(1)_c \times U(1)_s$
  - Charge conservation: $\theta \to \theta + \alpha$
  - Spin conservation: $\phi \to \phi + \alpha$

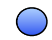
Bosonic?

$$H_{\rm eff} = \int dx \left[\frac{v}{2K}(\partial_x \theta)^2 + \frac{vK}{2}(\partial_x \phi)^2 \right]$$

- All fermionic excitations are *gapped*
- Excitations of even number of fermions are gapless. Primarily:
  - Singlet pairs $\epsilon_{\alpha\beta}\psi_{1\alpha}\psi_{2\alpha} \sim e^{i\theta}$
  - Neutral spins $\psi^\dagger_{1\uparrow}\psi_{1\downarrow} - \psi^\dagger_{2\uparrow}\psi_{2\downarrow} \sim e^{i\phi}$

Bosonic?



boson	Q	S^z
	2	0
	0	1

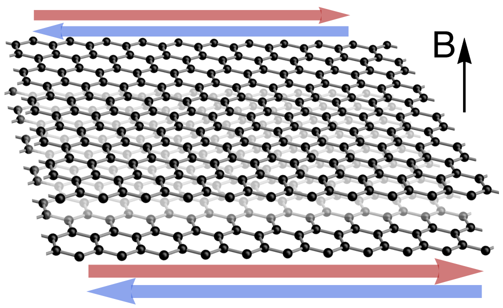
c.f. Senthil-Levin, 2012

$$S = \frac{i}{4\pi} \int d^2x d\tau \; K_{IJ} \epsilon^{\mu\nu\lambda} a^I_\mu \partial_\nu a^J_\lambda$$

$$K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad t_c = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad t_s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- |det K| = 1 : no anyons
- diag (K) = (0,0): bosonic quasiparticles

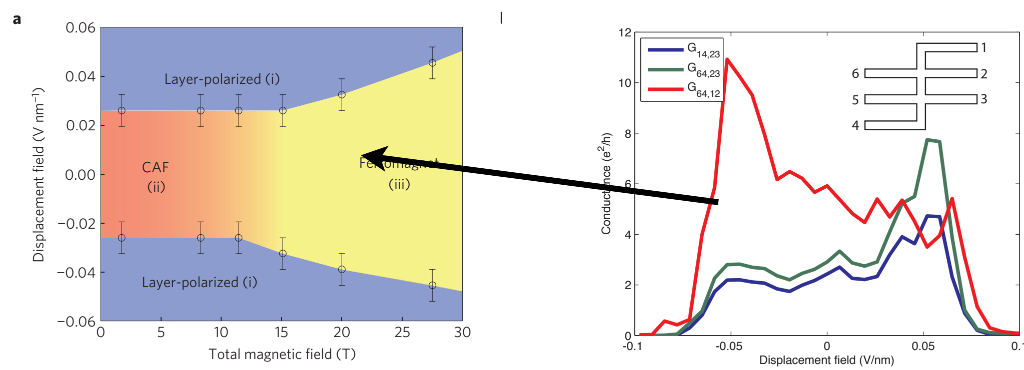
Potential experiments



Can one identify it?

Differentiate from fermionic state?

- Existing:
  - Zero Hall conductivity
  - Gapless edge
- New?
  - Tunnel into edge: single-e gap
  - But gapless charge 2e may be visible with SC tip or by shot noise



c.f. Nandishkore, Parameswaran

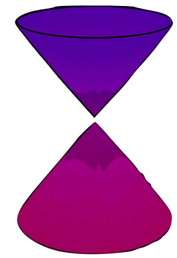
Weyl semimetal

Topological Semimetal

topology of k-surfaces

"Berry phase topology"

$$H = v\vec{\sigma}\cdot\vec{k}$$



937 PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

Conyers Herring
Princeton University, Princeton, New Jersey
(Received June 16, 1937)

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neigborhood of a point $\mathbf{k}$ where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa\rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

c.f. Nandishkore, Parameswaran

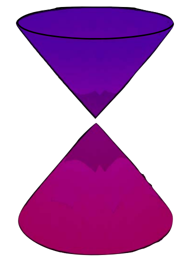
Weyl semimetal

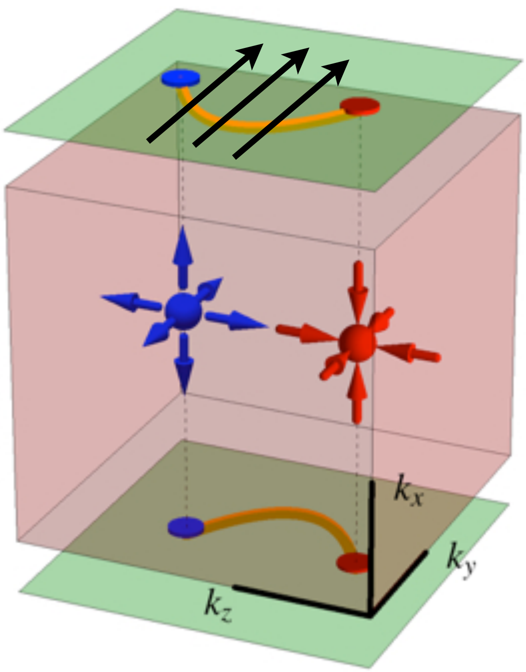
Topological Semimetal

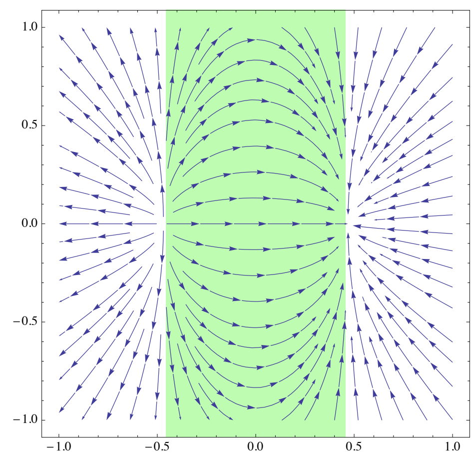
topology of k-surfaces

"Berry phase topology"

$$H = v\vec{\sigma} \cdot \vec{k}$$





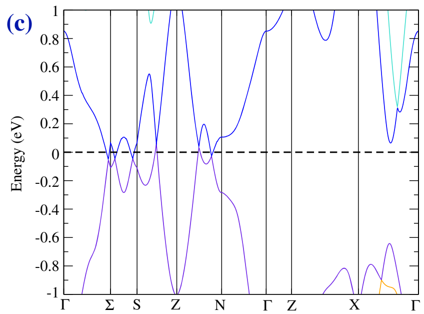


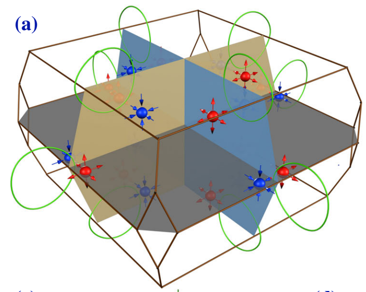
S. Murakami, 2007
X. Wan *et al*, 2011
A. Burkov+LB, 2011

Weyl points are "monopoles" of Berry curvature: topology in k-space!

Experiment

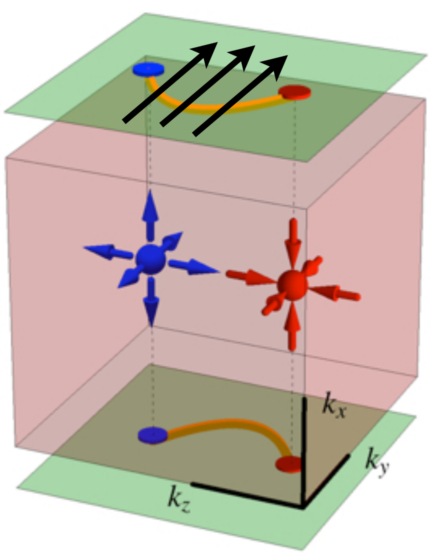
TaAs

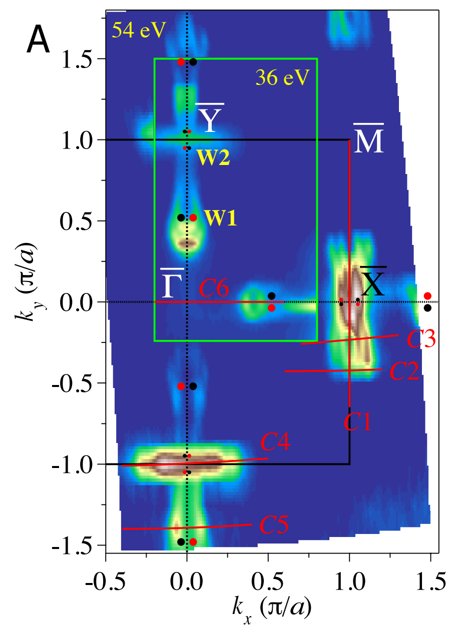


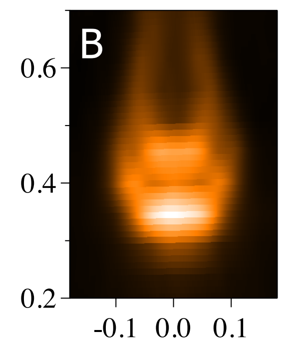


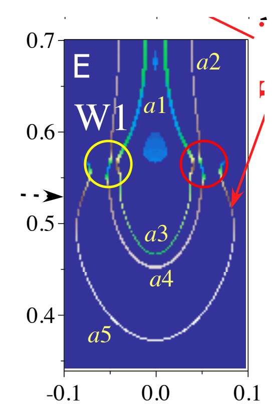
Prediction:
Hongmin Weng *et al*, 2015

- Striking properties:
  - Surface Fermi arcs





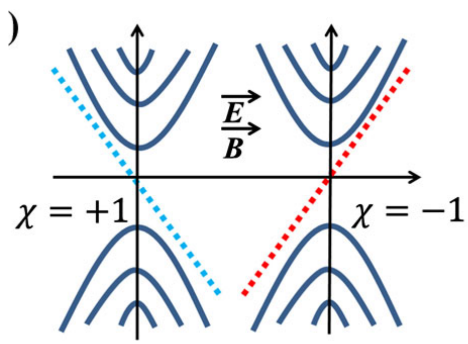




B.Q. Lv *et al*, 2015

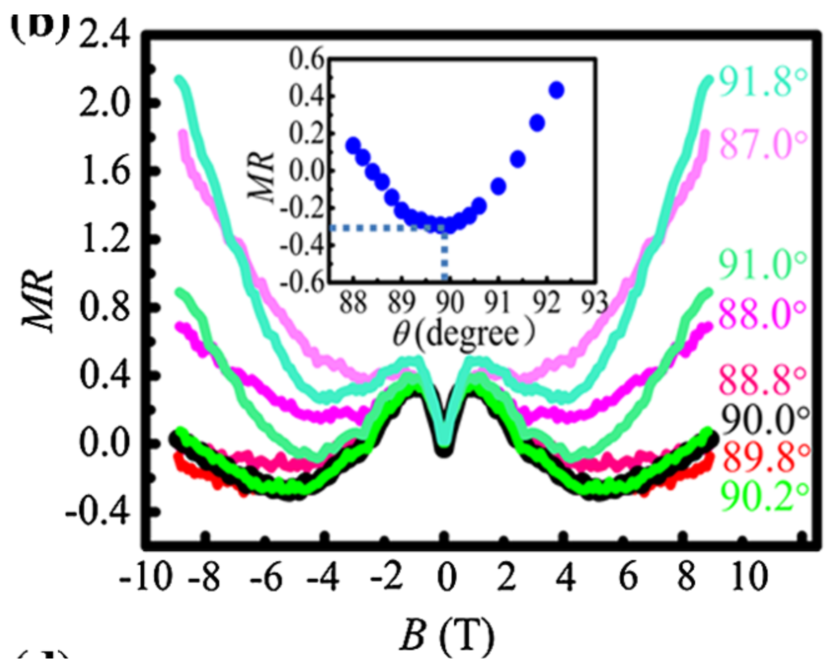
Experiment

- Striking properties:
  - Surface Fermi arcs
  - ABJ "anomaly": strong negative MR for I ∥ B



0th LL connects
Weyl points

Nielson + Ninomiya ,1983
Zyuzin+Burkov, 2012
Son+Spivak, 2013

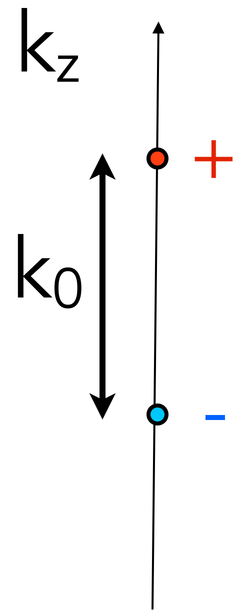


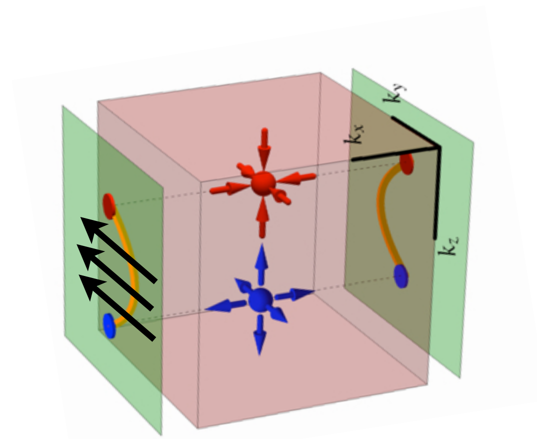
Xiaochun Huang *et al*, 2015

+ non-local conductivity, non-linear response...

Anomalous Hall Effect

The third striking property of a Weyl semimetal





Fermi arc = chiral edge state

$$\sigma_{xy} = \frac{e^2}{h}\frac{k_0}{2\pi}$$

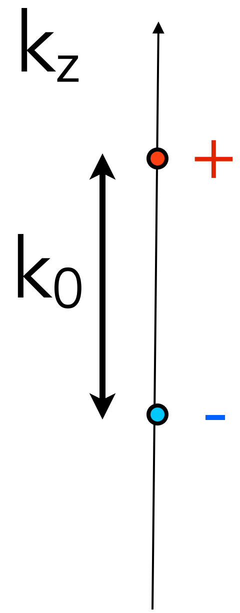
semi-quantum AHE

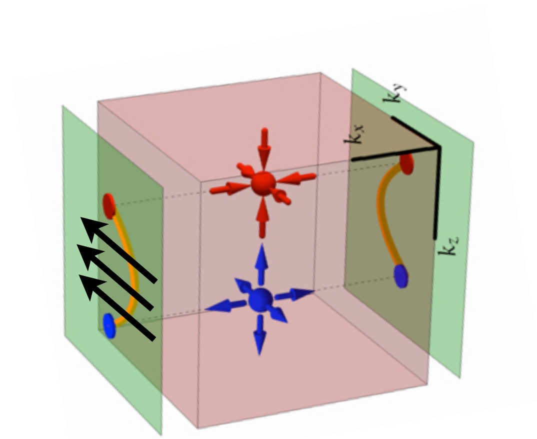
obviously breaks time-reversal symmetry

need a magnetic material

Anomalous Hall Effect

The third striking property of a Weyl semimetal





Fermi arc = chiral edge state

$$\sigma_{\mu\nu} = \frac{e^2}{2\pi h}\epsilon_{\mu\nu\lambda}Q_\lambda$$

$$\vec{Q} = \sum_i \vec{k}_i q_i + \vec{Q}_{RLV}$$

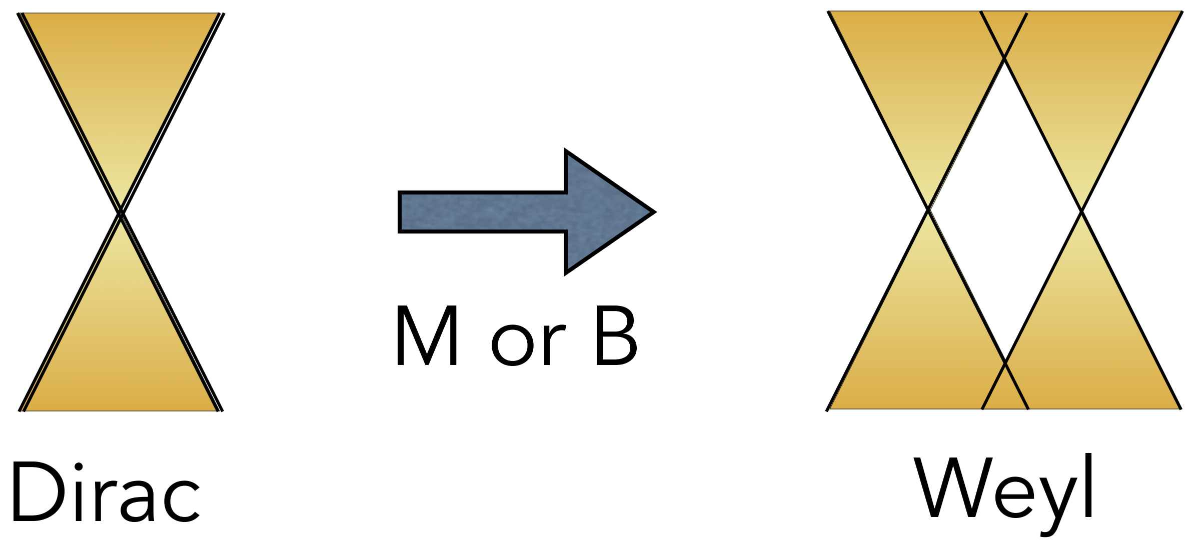
semi-quantum AHE

obviously breaks time-reversal symmetry

need a magnetic material

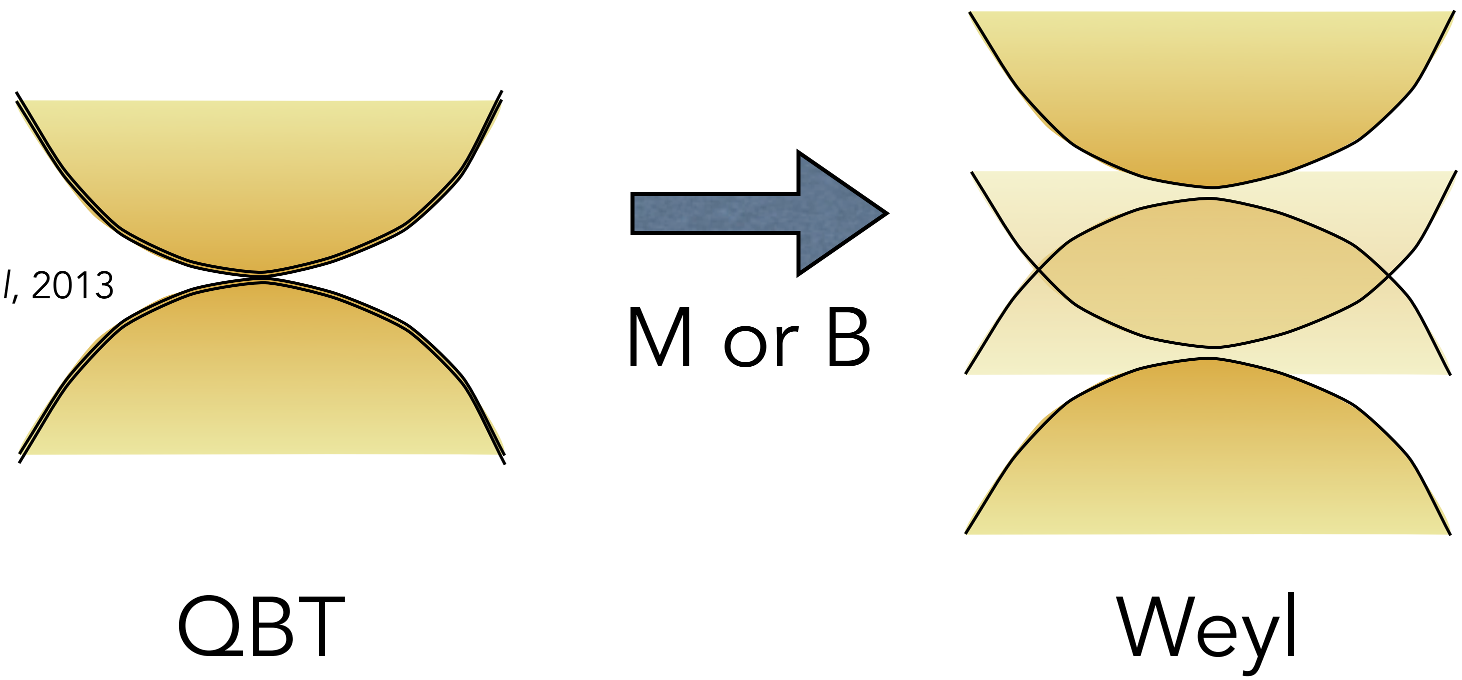
Magnetic Weyl semimetals

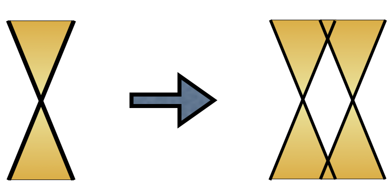
A. Burkov+LB, 2011



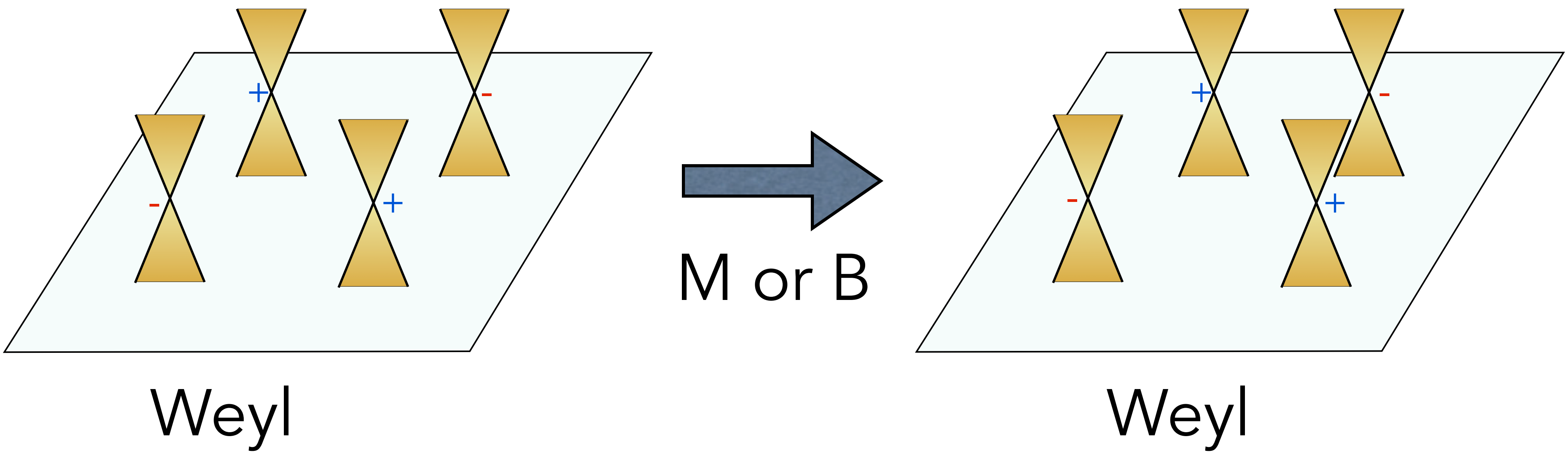
Magnetic Weyl semimetals

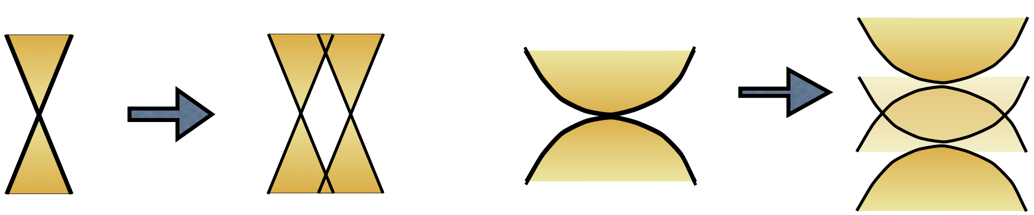
E.G. Moon *et al*, 2013





Magnetic Weyl semimetals



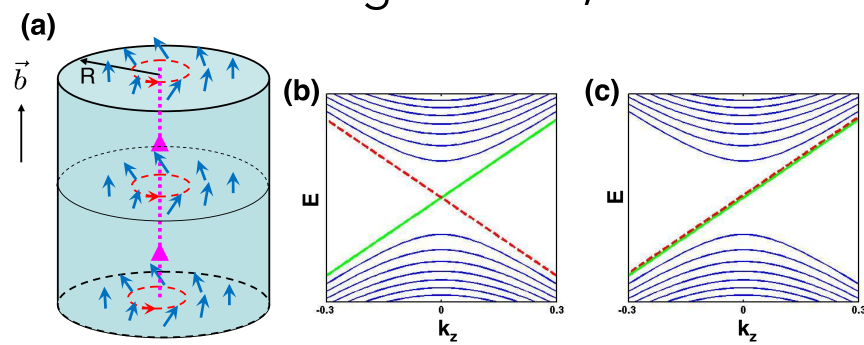


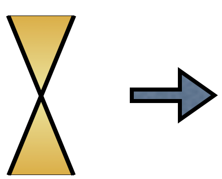
Magnetic Weyl semimetals

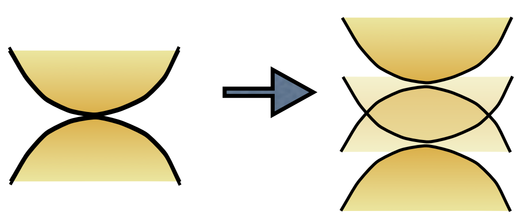
Movable Weyl points have their own interest

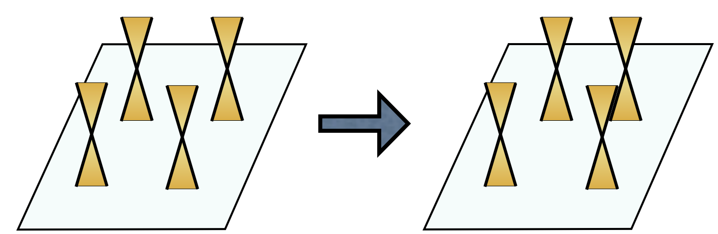
magnetic fluctuations generate a *dynamical chiral gauge field* for Weyl fermions, by shifting Weyl points

Chao-Xing Liu *et al*, 2012









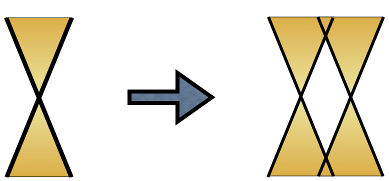
Magnetic Weyl semimetals

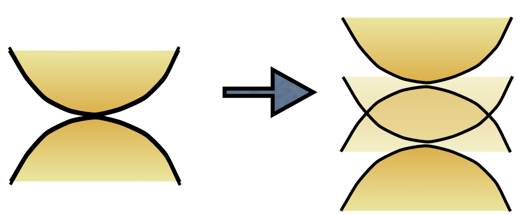
Exercise: how far can we move Weyl points?

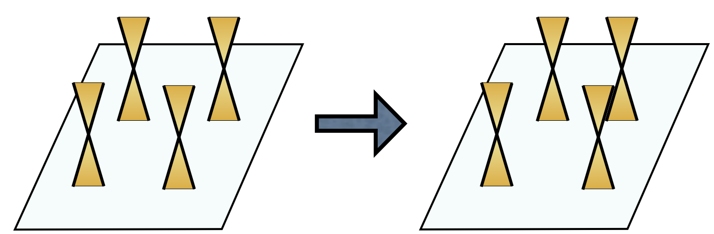
We'll see that this is why we need correlations

Dirac/Weyl: $\hbar v_F \Delta k \approx E_Z$

QBT: $$\frac{\hbar^2 (\Delta k)^2}{2m^*} \approx E_Z$$



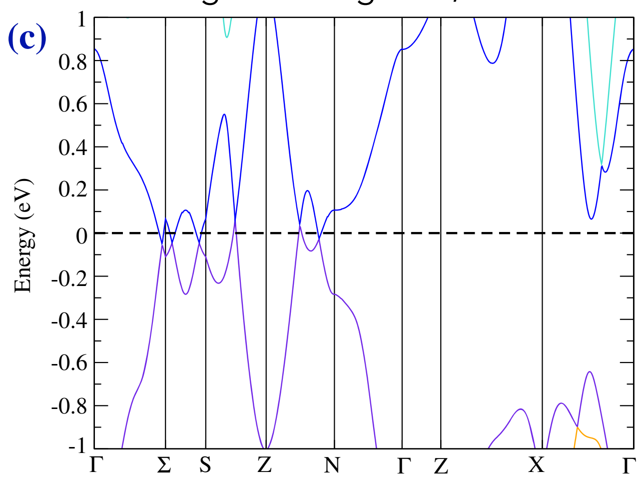




Magnetic Weyl semimetals

Suppose you modify, e.g. magnetically dope, TaAs

Hongmin Weng *et al*, 2015

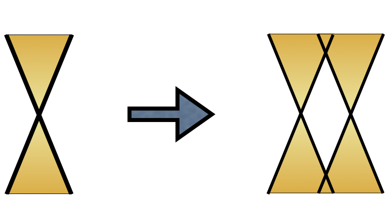


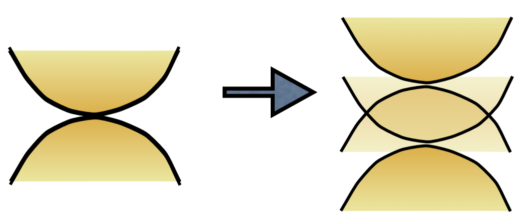
$$\Delta k \approx E_Z/(\hbar v_F)$$

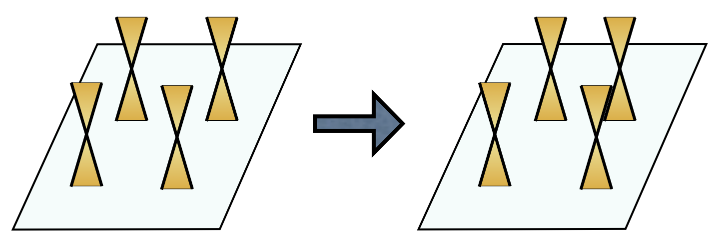
$$\hbar v_F \approx 2eV\mathring{A} \quad E_Z < 1000K$$

$$\Delta k < 0.04\mathring{A}^{-1}$$

Result: Weyl points move < 1/50th of the zone



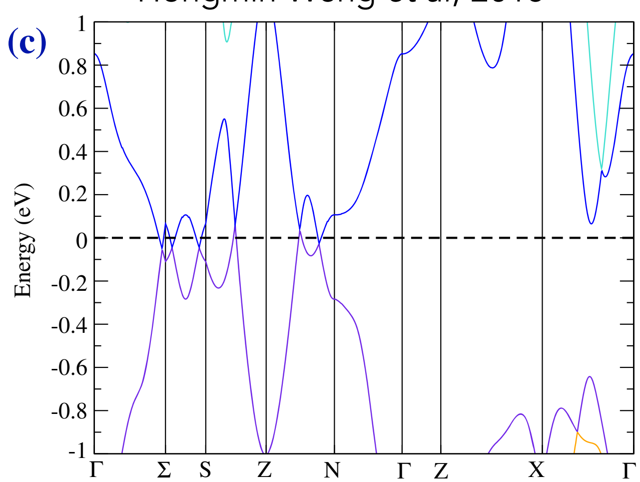




Magnetic Weyl semimetals

Suppose you modify, e.g. magnetically dope, TaAs

Hongmin Weng *et al*, 2015



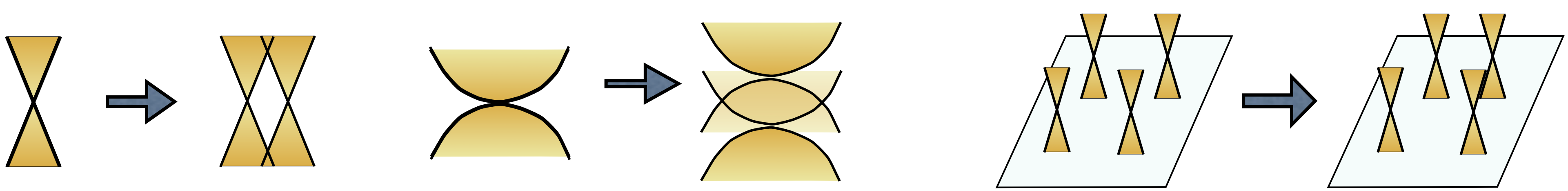
$$\Delta k \approx E_Z/(\hbar v_F)$$

$$\hbar v_F \approx 2eV\mathring{A} \quad E_Z < 1000K$$

$$\Delta k < 0.04\mathring{A}^{-1}$$

Result: Weyl points move < 1/50th of the zone

need a narrower band

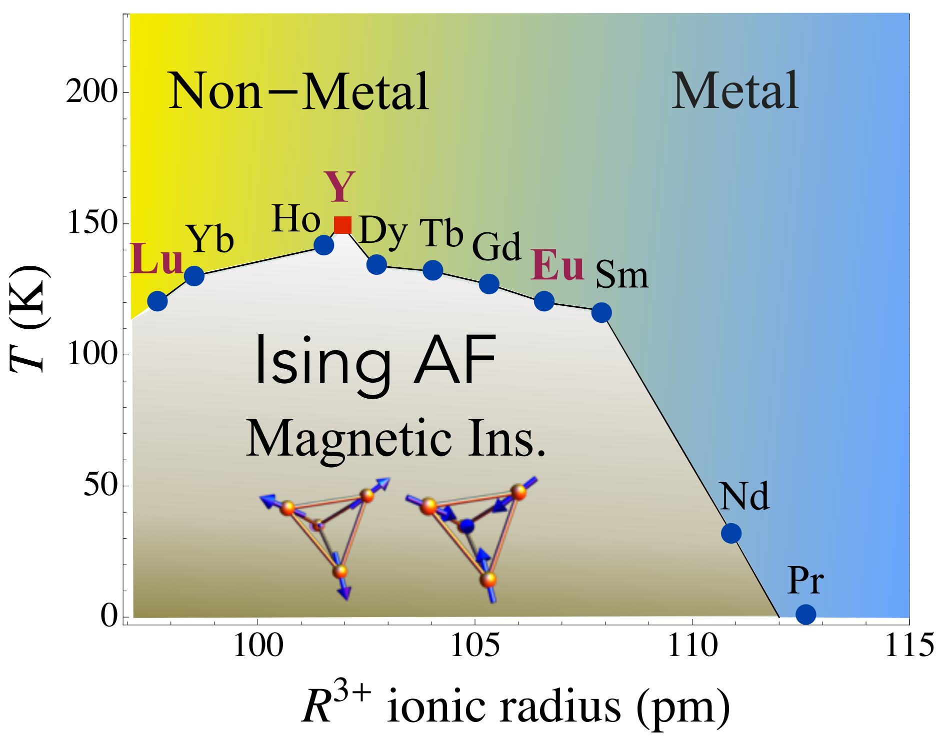


Nakatsuji, YB Kim

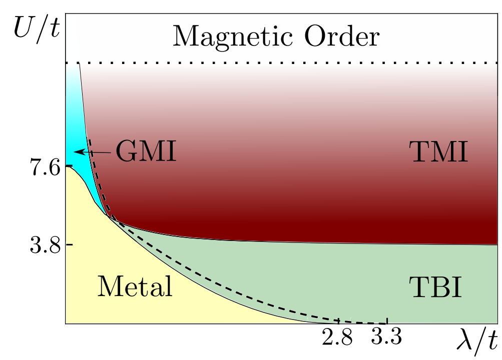
Pyrochlore iridates

$A_2Ir_2O_7$

- A good place to look for correlated Weyls



Yanagashima+Maeno, JPSJ 2001
K. Matsuhira et al, JPSJ 2011
W. Witczak-Krempa et al, ARCMP 2013



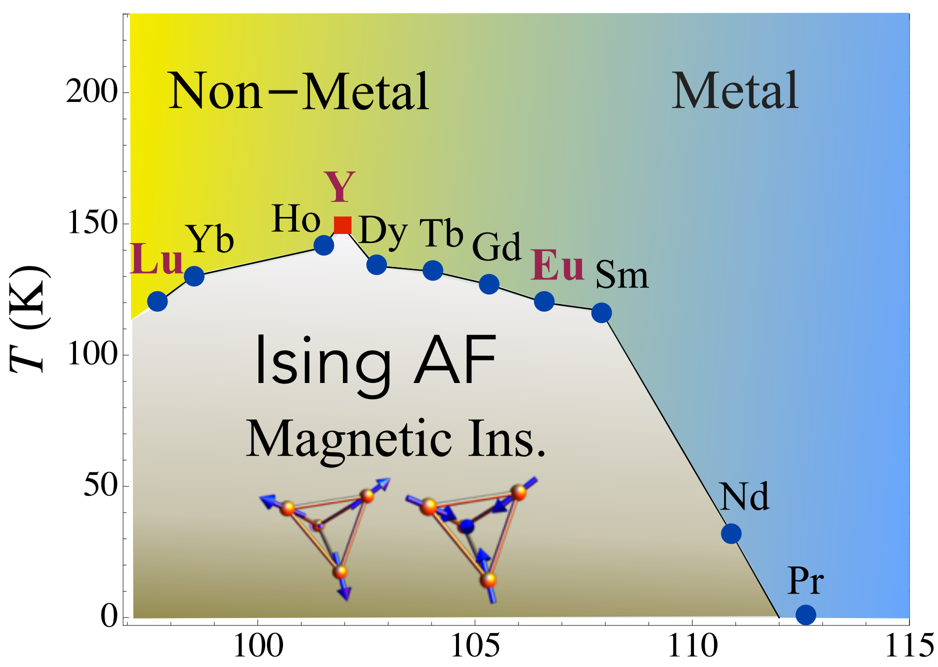
D. Pesin + LB, 2010
Topological MI?

Nakatsuji, YB Kim

Pyrochlore iridates

$A_2Ir_2O_7$

- A good place to look for correlated Weyls



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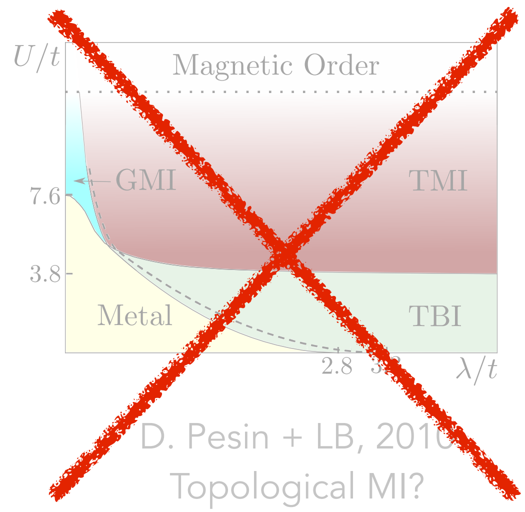
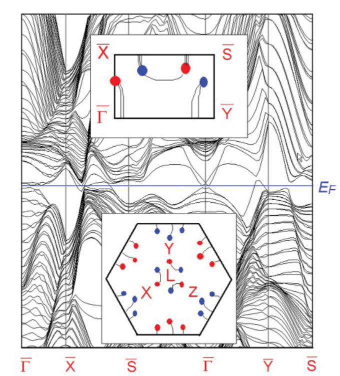


actually our bilayer graphene SPT is a 2d analog of this



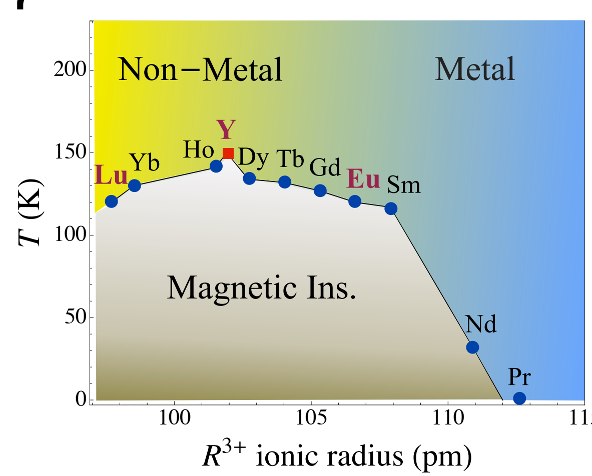
X. Wan *et al*, 2011
AF Weyl semimetal?

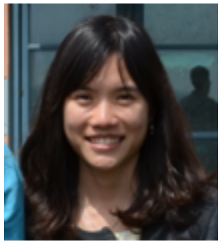
Nakatsuji, YB Kim

Ingredients

- Semimetallic electronic structure ✓
- Magnetism via Ir e-e Hubbard interactions ✓
- Rare earth moments?

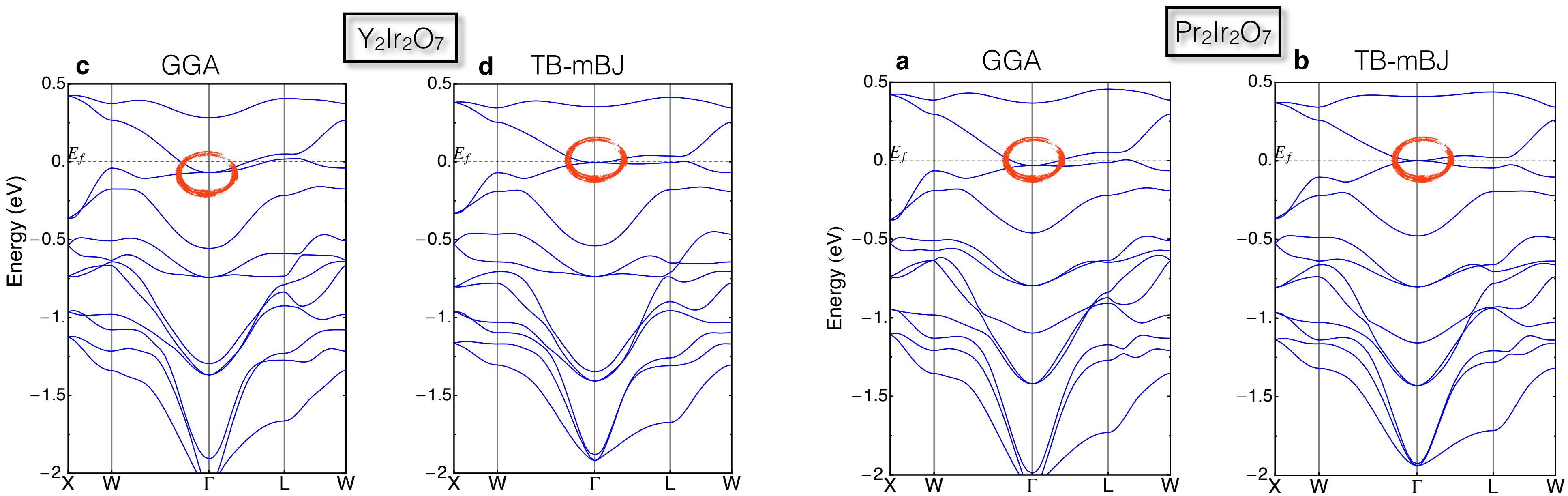
probably not important?

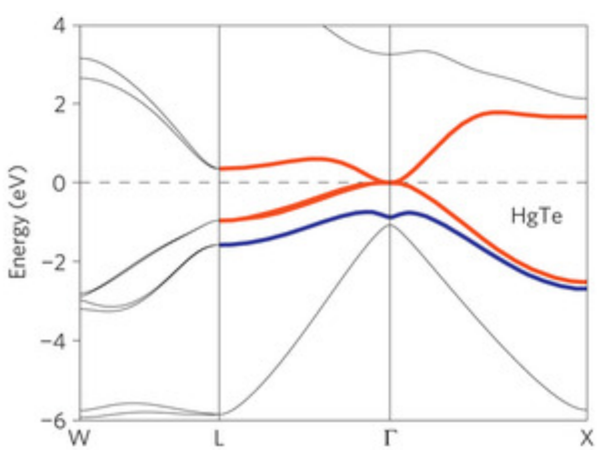




T. Kondo *et al*, Nat. Comm., 2015

Paramagnetic electronic structure





Γ_8 4-dimensional irrep guaranteed by cubic symmetry and time reversal

$Pr_2Ir_2O_7$

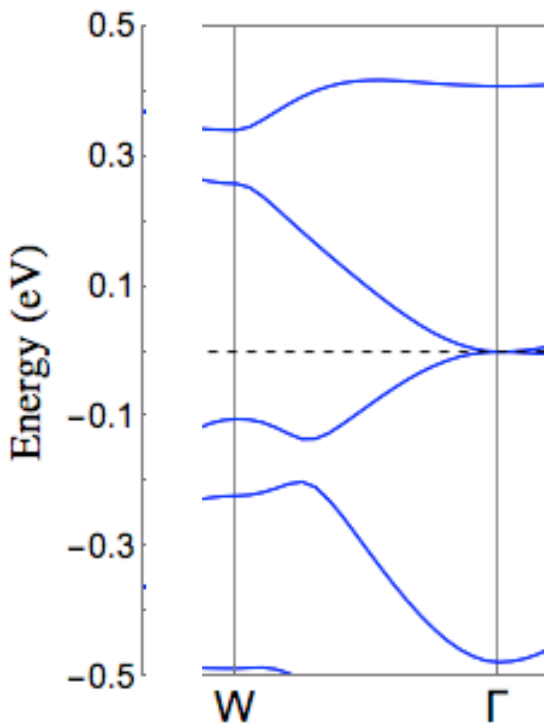
S. Nakatsuji

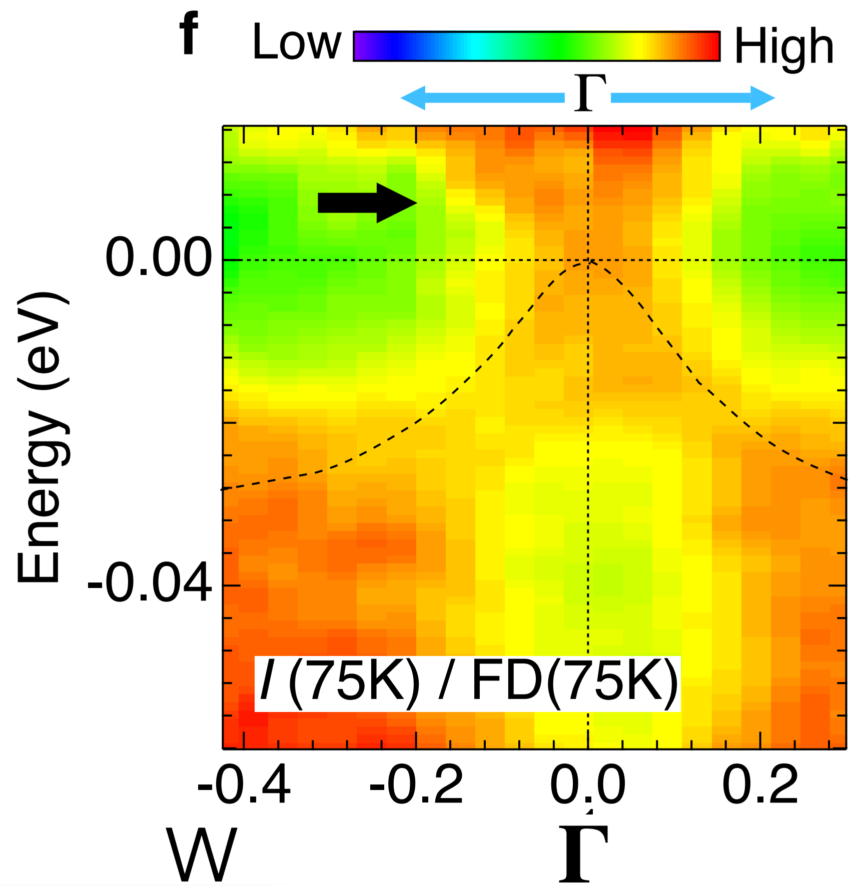
ARPES

T. Kondo *et al*, Nat. Comm., 2015

T. Kondo

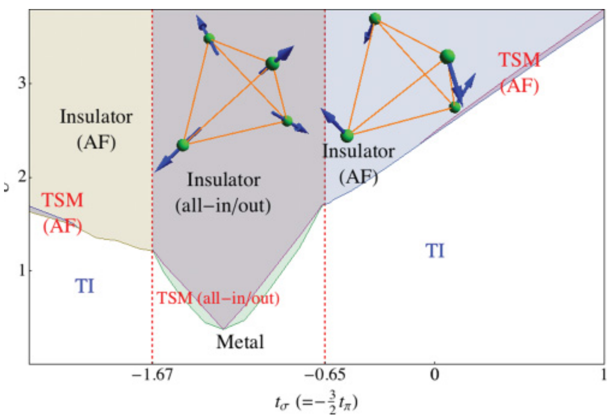
S. Shin





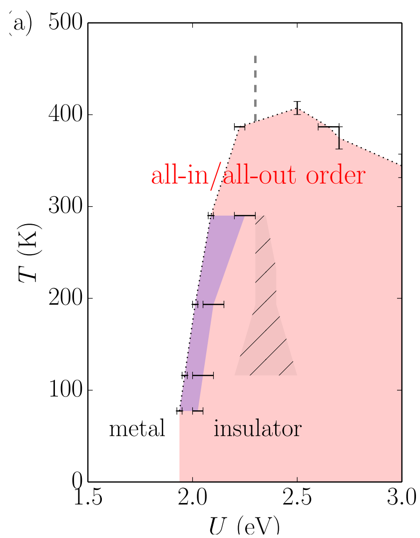
✓Bandwidth reduced by 3-5 from DFT correlations

Magnetism: theory



W. Witzak-Krempa + YB Kim, 2012

Hartree-Fock

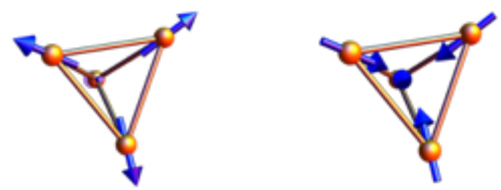


H. Shinaoka *et al*, 2015

DMFT

Jay-Z model

$$H = J_z \sum_{\langle i,j \rangle} S_i^z S_j^z$$

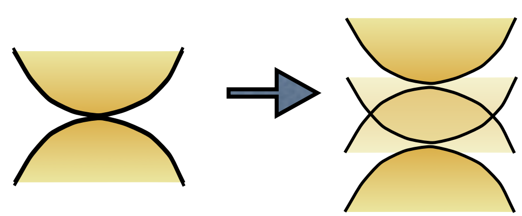


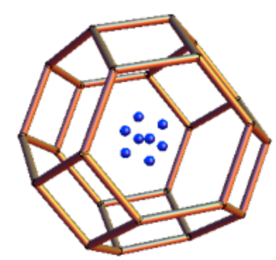
$J_z < 0$: all-in/all-out order

superexchange

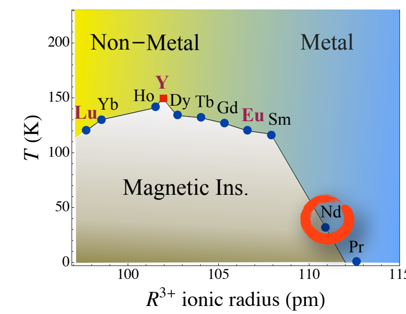
- General agreement: transition to AIAO Ising AF order

We expect this to lead to Weyl points

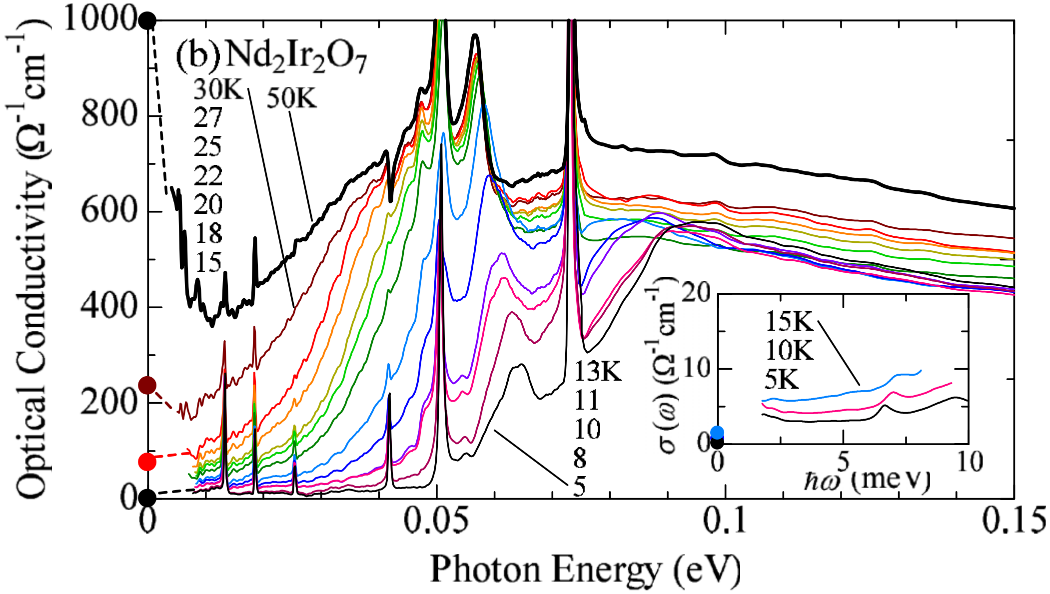




Weyl not?

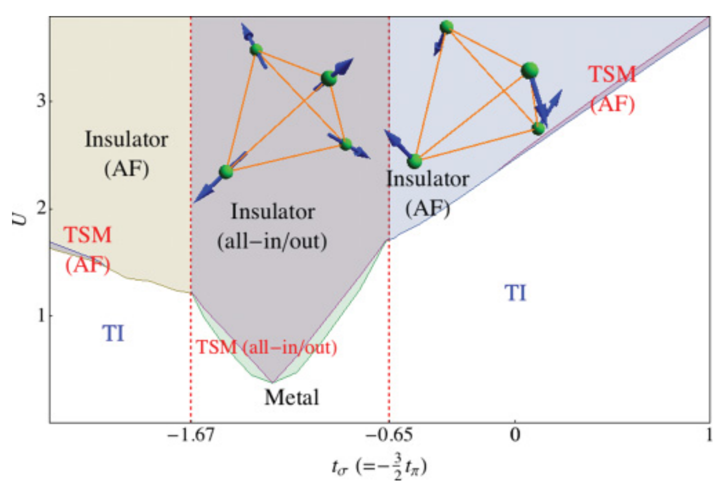


K. Ueda *et al*, 2012



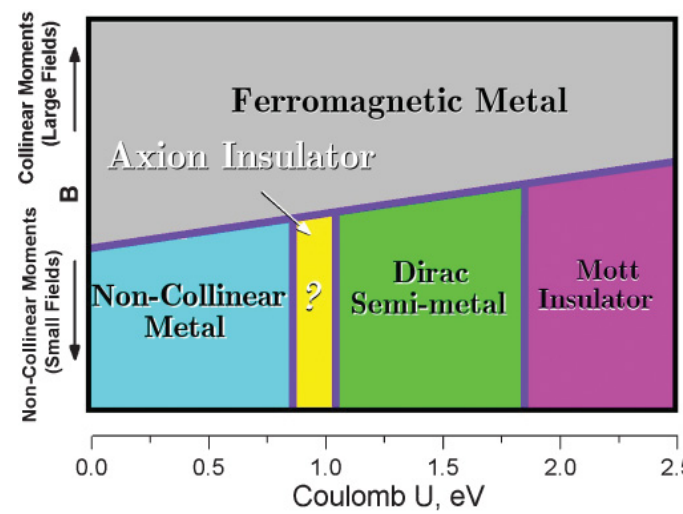
charge gap ~ 45meV

Moving Weyl Points



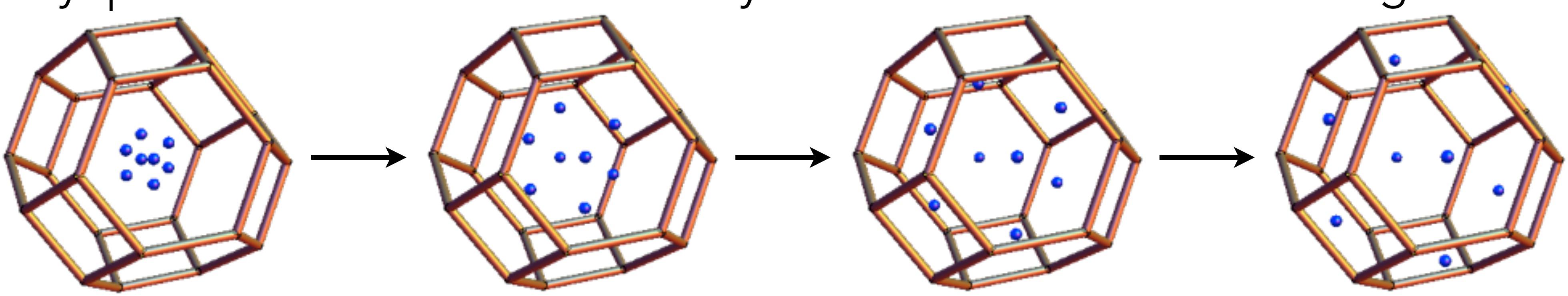
W. Witzak-Krempa + YB Kim, 2012

vs



X. Wan *et al*, 2011

Weyl points move to zone boundary and annihilate with increasing order?

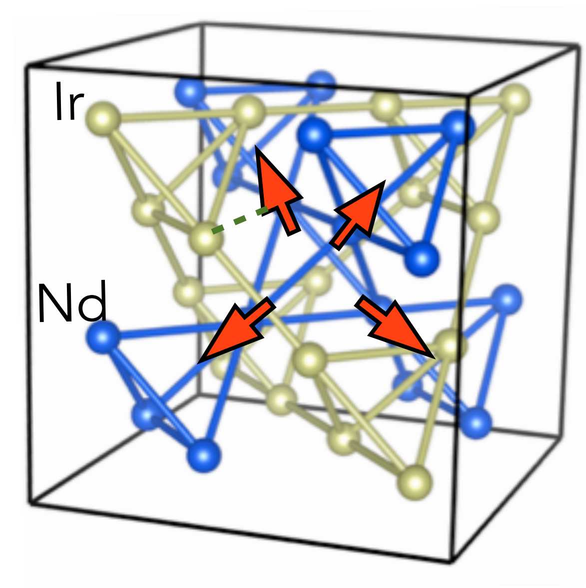


If so, Weyl points are *too* mobile!

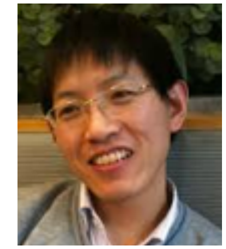
Nd physics

Nd spins provide anisotropic "exchange enhancement"

- Large moment couples strongly to field
- Polarized Nd act back on Ir via $J_{Ir\text{-}Nd}$ ~ 10 meV
- Maximum effect: B || (100) - aligns all Nd moments



Nd physics



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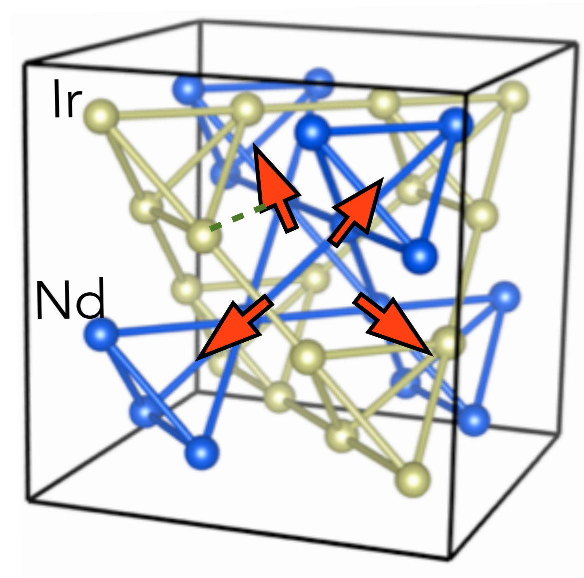
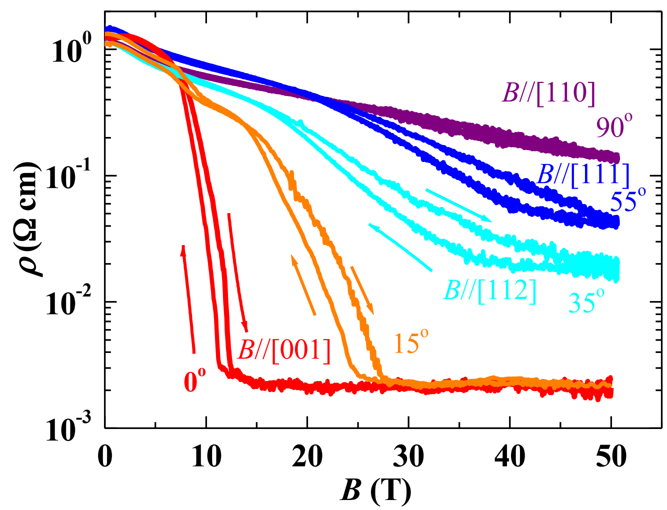
Result: MIT

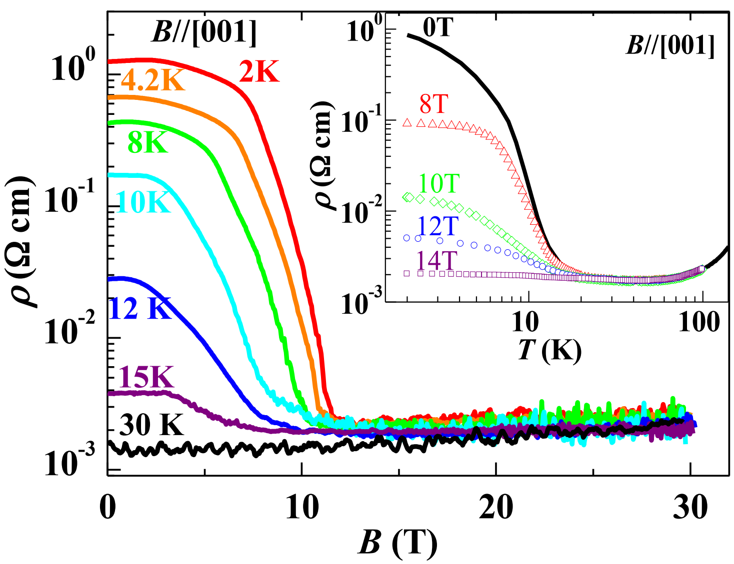


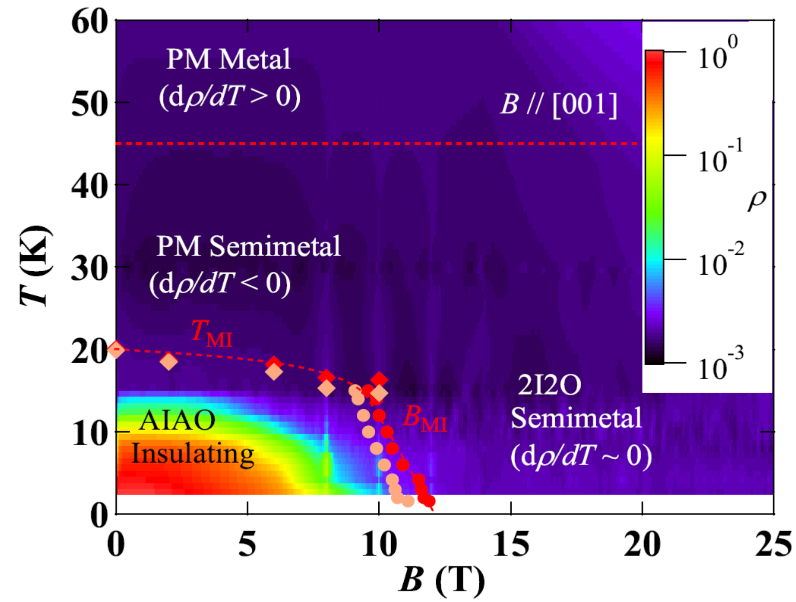
Zhaoming Tian *et al*, Nature Physics, 2015



also K. Ueda *et al*, 2015

Metal-Insulator Transition

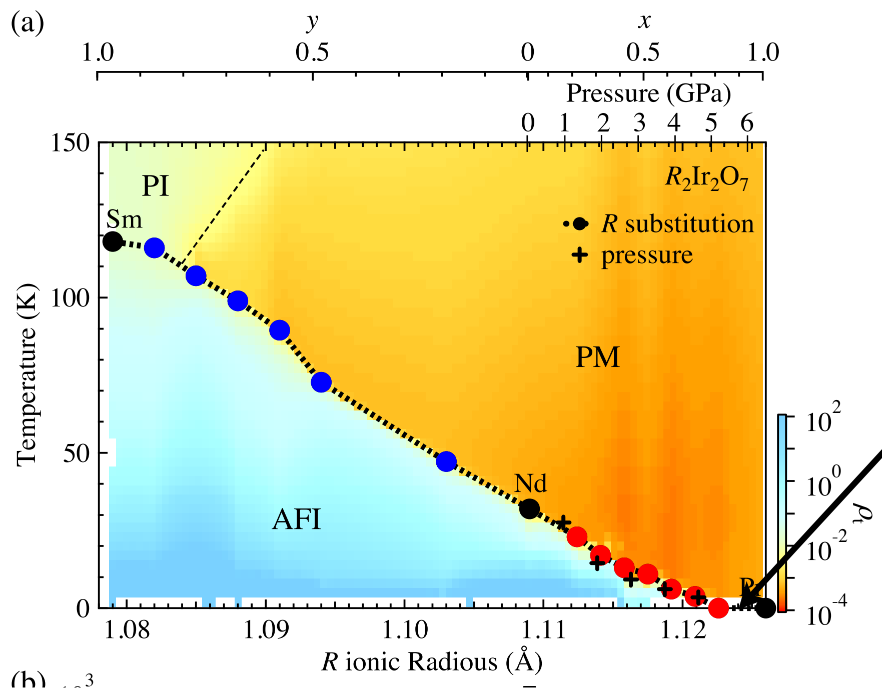




- Seems that the antiferromagnetic phase forms a closed region at small B and T.
- Not known: what is the nature of the high-field semimetal? Maybe a magnetic Weyl state?

Prospects

It may be possible to weaken the order sufficiently to expose the Weyl points, and perhaps also explore quantum criticality



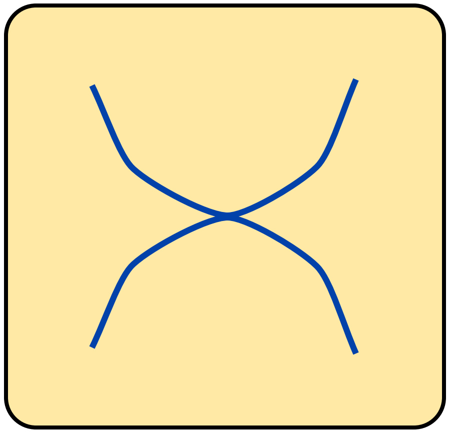
QCP?

K. Ueda *et al*, 2015

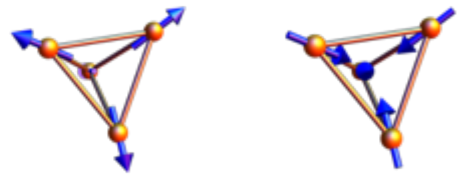
c.f. Herbut

Field theory

modification of Hertz-Millis-Moriya



+



= ??

ϕ

$$S = \int d\tau d^3 x \left[\overline{\psi}_a (\partial_\tau + \hat{\mathcal{H}}_0)\psi_a + \frac{g}{\sqrt{N}} \phi \, \overline{\psi}_a \mathsf{M} \, \psi_a + \frac{r}{2}\phi^2 \right]$$

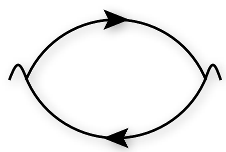
$$+ \frac{ie}{\sqrt{N}} \varphi \, \overline{\psi}_a \psi_a + \frac{1}{2}(\nabla\varphi)^2$$

poor screening

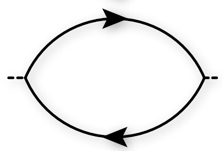
Renormalization Group

large N_{fermion} expansion & RG

all-in-all-out order parameter

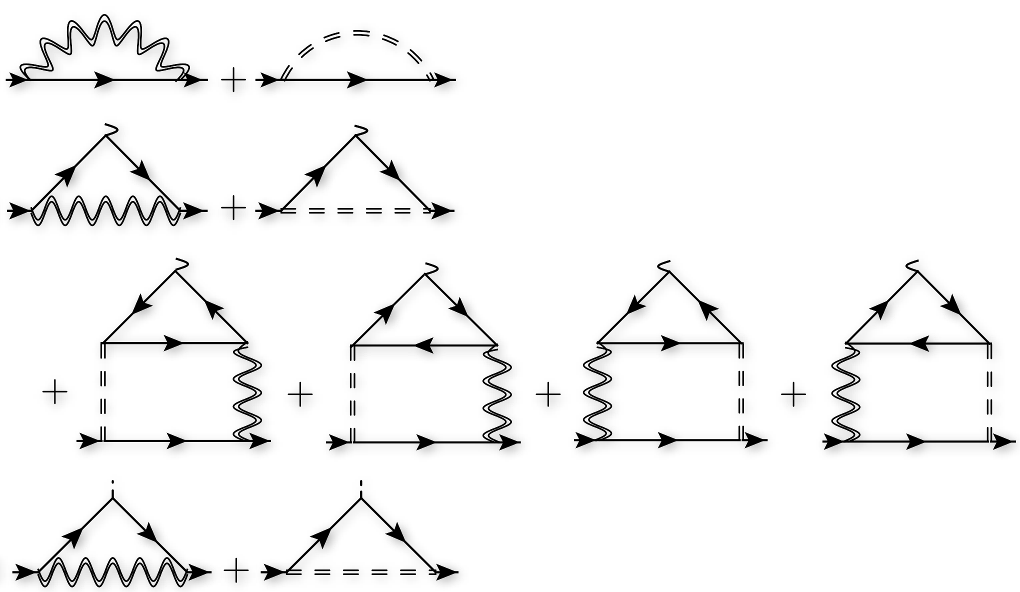
$\Sigma_\phi =$  $\sim |\mathbf{k}| |\ln c_1/c_2| f_\phi(\hat{\mathbf{k}}) + \sqrt{\omega_n}\, C \quad \gg \overbrace{\mathbf{k}^2 + \omega_n^2}^{\mathcal{G}_{0;b}^{-1}}$

Coulomb interactions

$\Sigma_\varphi =$  $\sim |\mathbf{k}| |\ln c_1/c_2| f_\varphi(\hat{\mathbf{k}}) \quad \gg \mathbf{k}^2$

bare

$\Sigma_f =$, $\Xi_\phi =$, $\Xi_\varphi =$



$\propto 1/N_{\text{fermion}} \times \ln\Lambda$

<u>RG:</u> calculate "only" the coefficient of lnΛ

Quantum criticality

- Stable fixed point with unusual exponents: extreme deviations from MFT

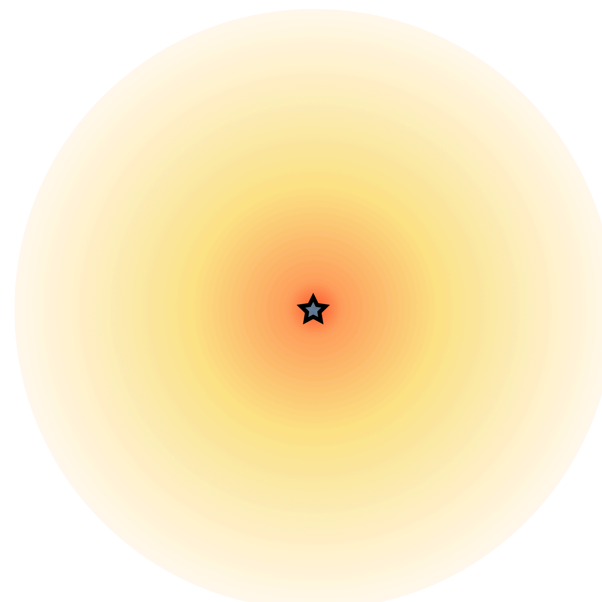
e.g. $\langle \phi \rangle \sim (r_c - r)^2$

(x logs)

wide QC regime

$z \approx 2$

$\nu \approx 1$



Semimetal

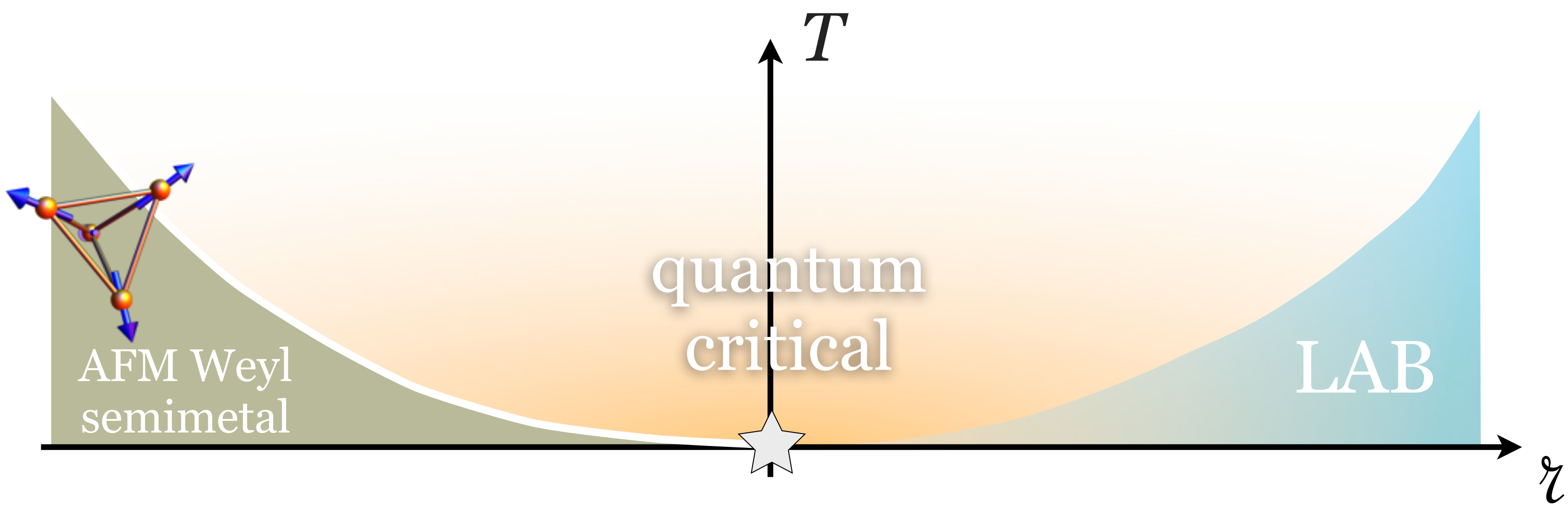
$\chi^{-1} \sim q + \sqrt{|\omega|}$

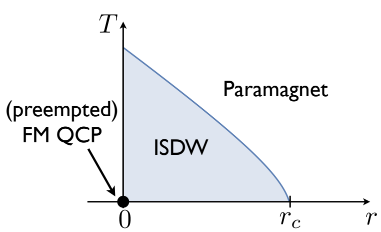
matched to electrons

electrons scatter strongly

Quantum criticality

- QBT leads to extremely non-classical QCP





Theory of analogous FM QCP: an incommensurate SDW is induced

J. Murray, O. Vafek +LB, PRL 2016

L. Savary, EG Moon, LB, PRX, 2014

Three types of topology

Topological Insulator

topology of filled bands

"symmetry protected topological order"

Topological Semimetal

topology of k-surfaces

"Berry phase topology"

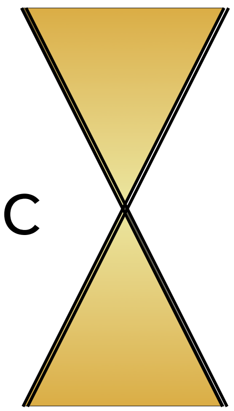
+ Correlations:

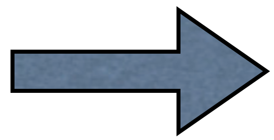
- ✦ Topological Kondo Insulator SmB_6?
- ✦ Surface states may be more correlated than bulk

What about surfaces states of gapless topological semimetals?

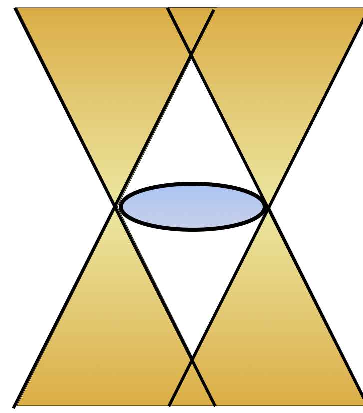
Nodal loop semimetal

Dirac



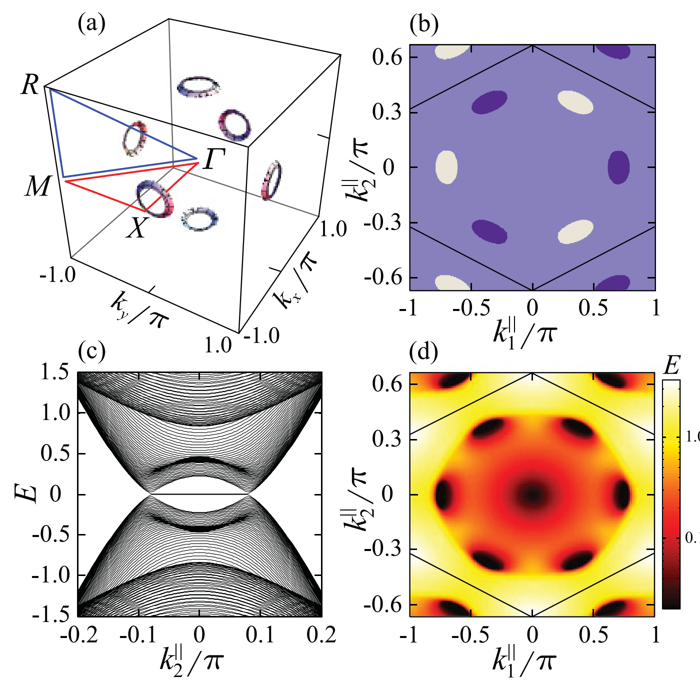


other T or I breaking perturbations



Nodal loop

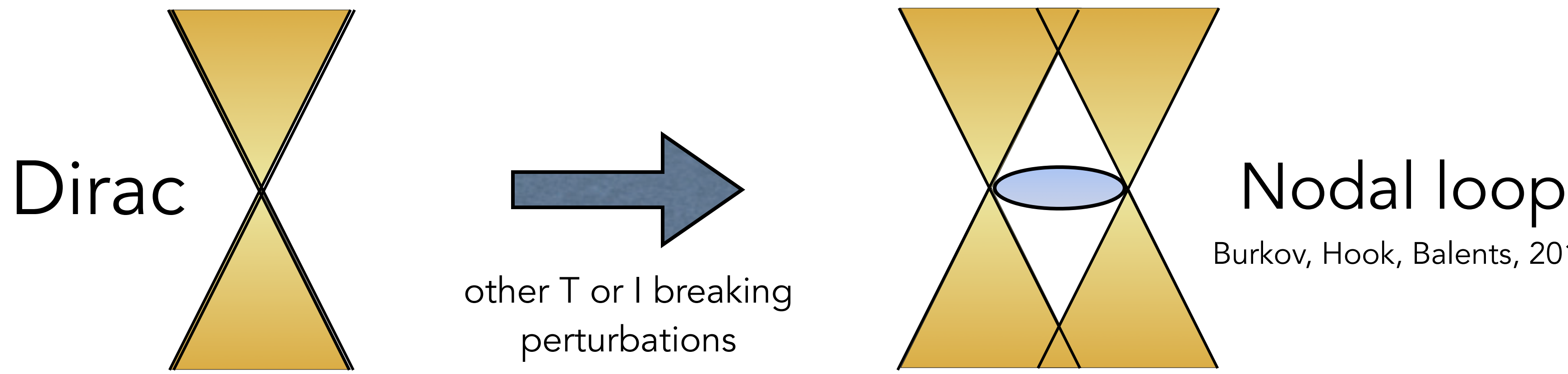
Burkov, Hook, Balents, 2011



"Dirac loop" states <u>w/o SOC</u>

Schnyder+Ryu, 2011

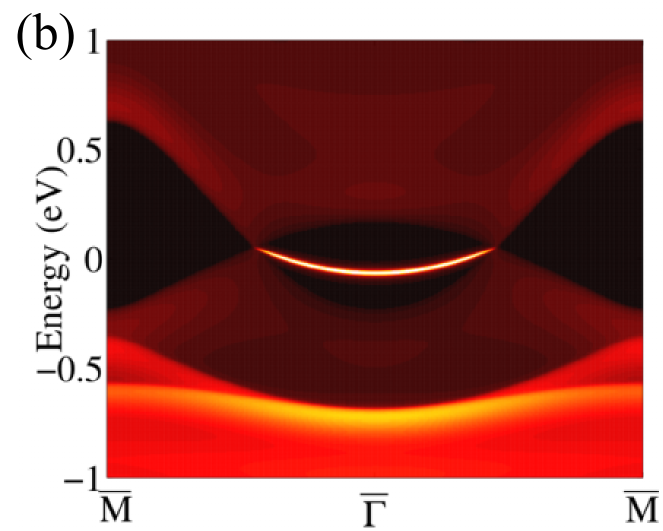
Nodal loop semimetal



Burkov, Hook, Balents, 2011

"Dirac loop" states w/o SOC

"drumhead" surface band

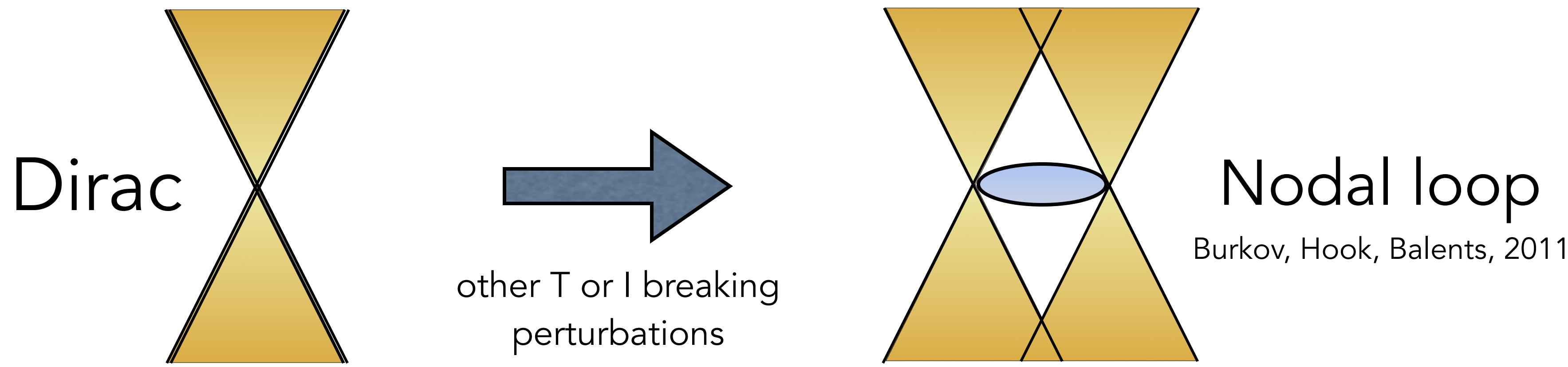


c.f. ZrSiS, $PbTaSe_2$

Ca_3P_2

Y.H. Chan *et al*, 2016

Nodal loop semimetal

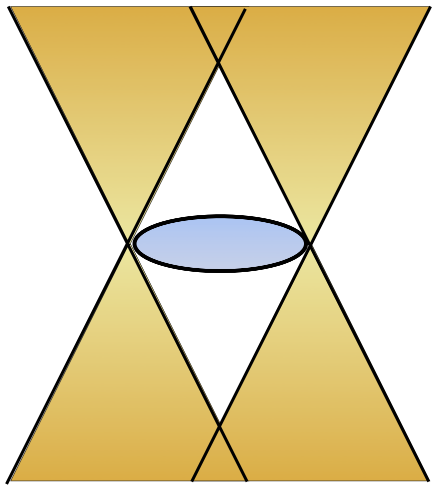


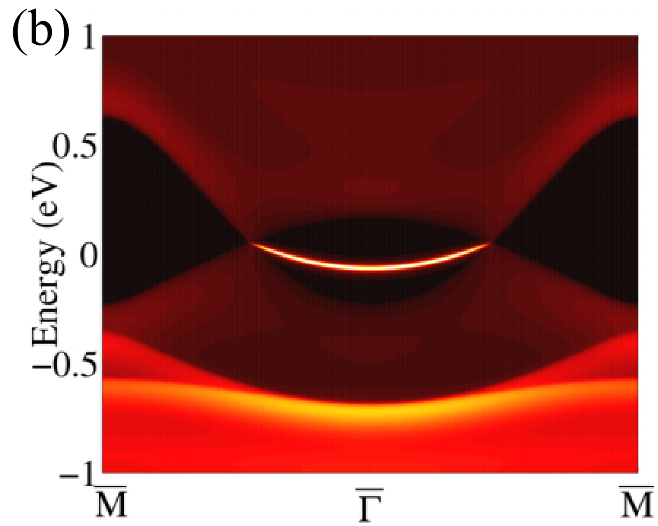
"Dirac loop" states w/o SOC

"drumhead" surface band

c.f. ZrSiS, $PbTaSe_2$

Nodal loop semimetal





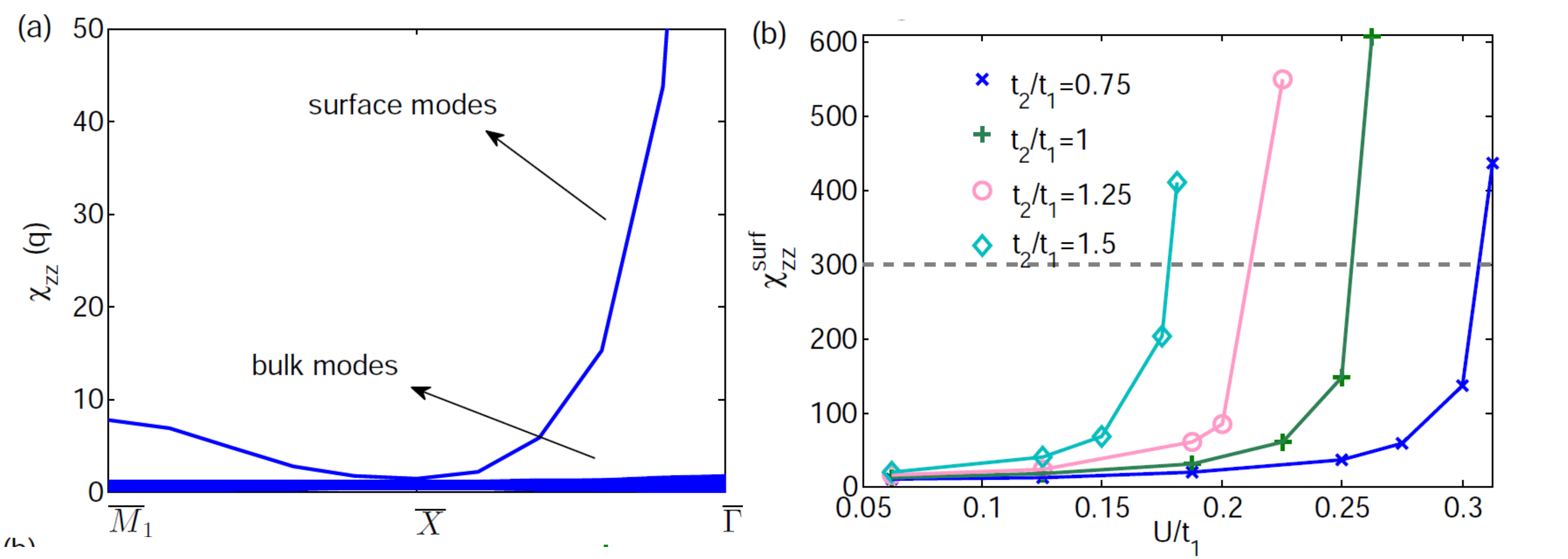
Quasi-flat surface band:

- ✦Enhanced correlation instabilities?
- ✦Is it important how the surface states merge with the gapless bulk nodes?
- ✦Quantum criticality?

Jianpeng Liu

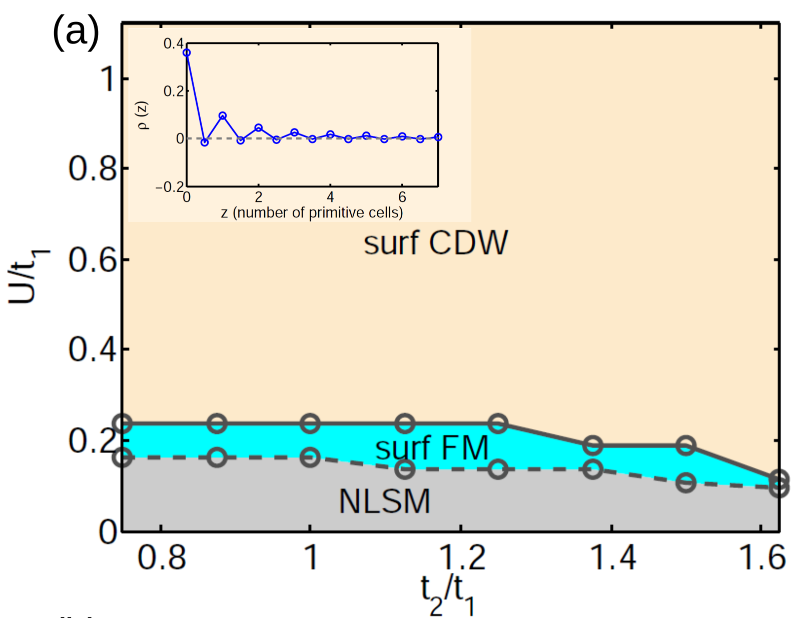
Nodal loop + correlations

Susceptibility enhanced at
surface Gamma point

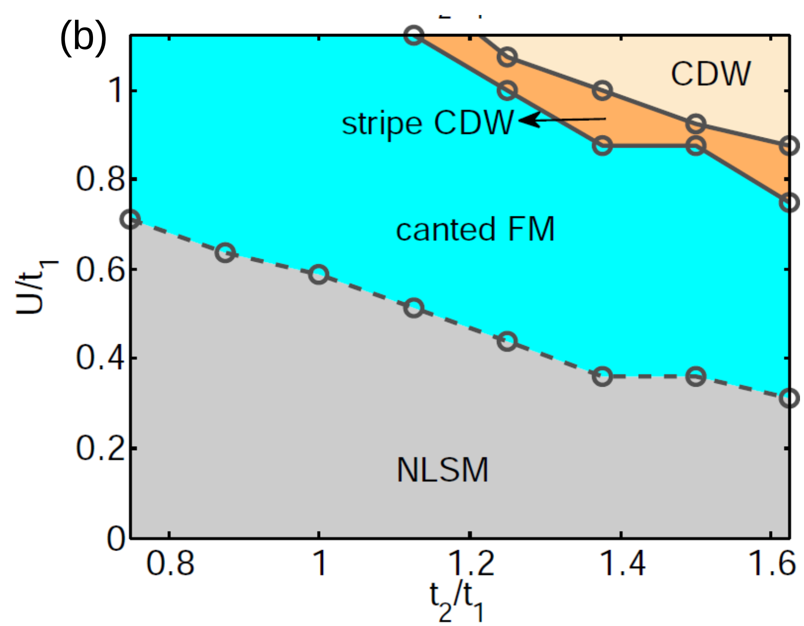


Nodal loop + correlations

Self-consistent Hartree-Fock
in p/h channels



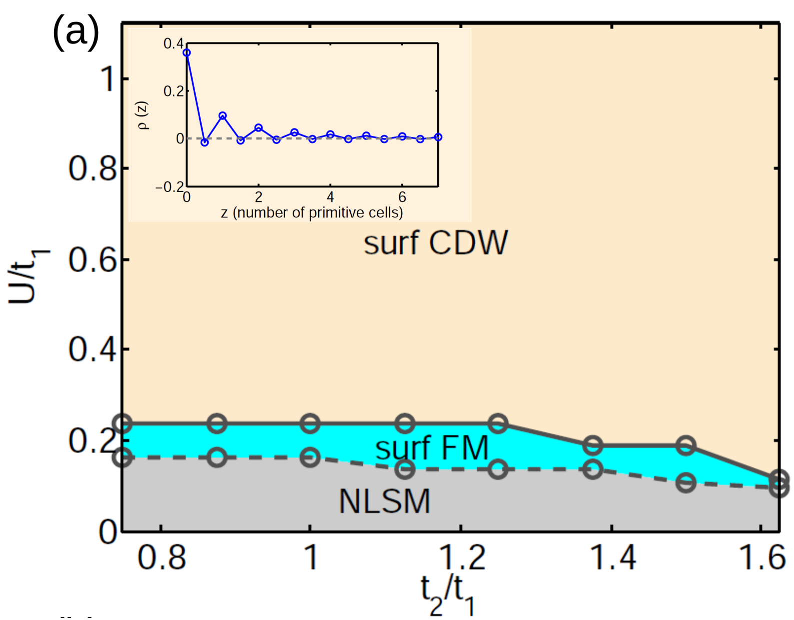
so SOC



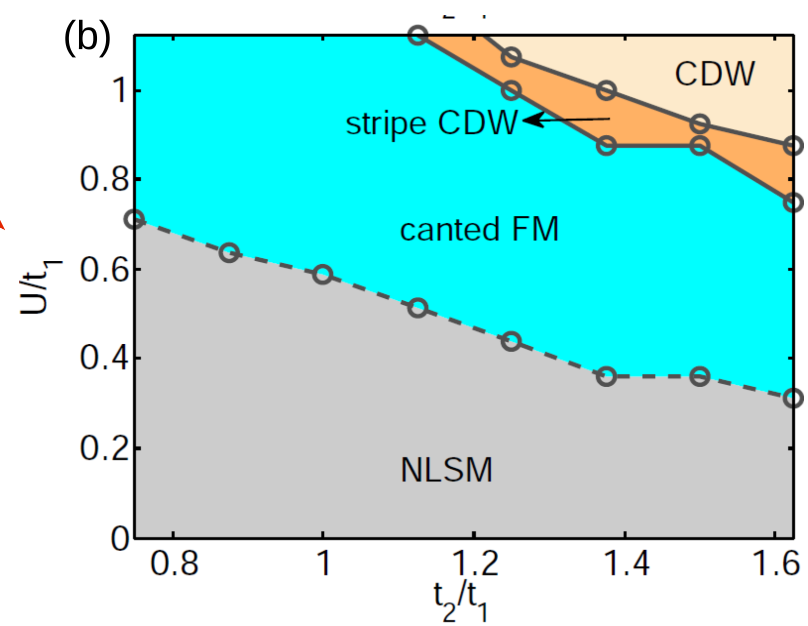
w/ SOC

Nodal loop + correlations

Self-consistent Hartree-Fock
in p/h channels



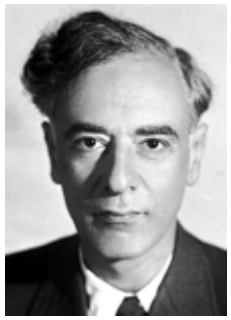
so SOC



quantum criticality?

w/ SOC

Hertz-Millis-Moriya

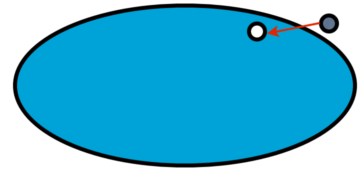


Landau theory

Landau damping

$$\mathcal{L}_0 = |\nabla\varphi|^2 + r\varphi^2 + u\varphi^4$$

+

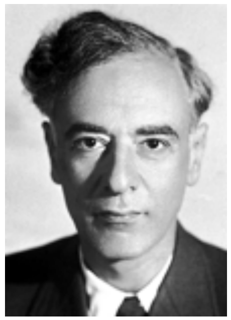


$$\mathcal{L}_\omega = c\frac{|\omega|}{q}|\varphi|^2$$ etc.

Sachdev, Chubukov, Raghu

n.b. We are going to neglect all the "strong coupling" subtleties that may be important in 2d at low enough energies

Hertz-Millis-Moriya

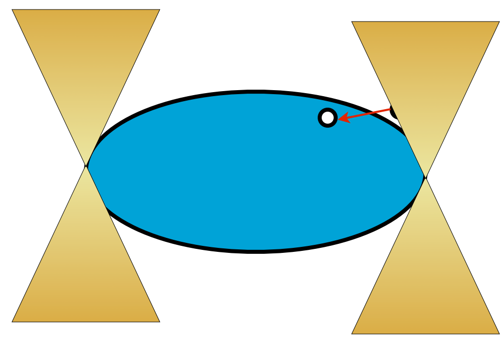


Landau theory

$$\mathcal{L}_0 = |\nabla\varphi|^2 + r\varphi^2 + u\varphi^4$$

Landau damping

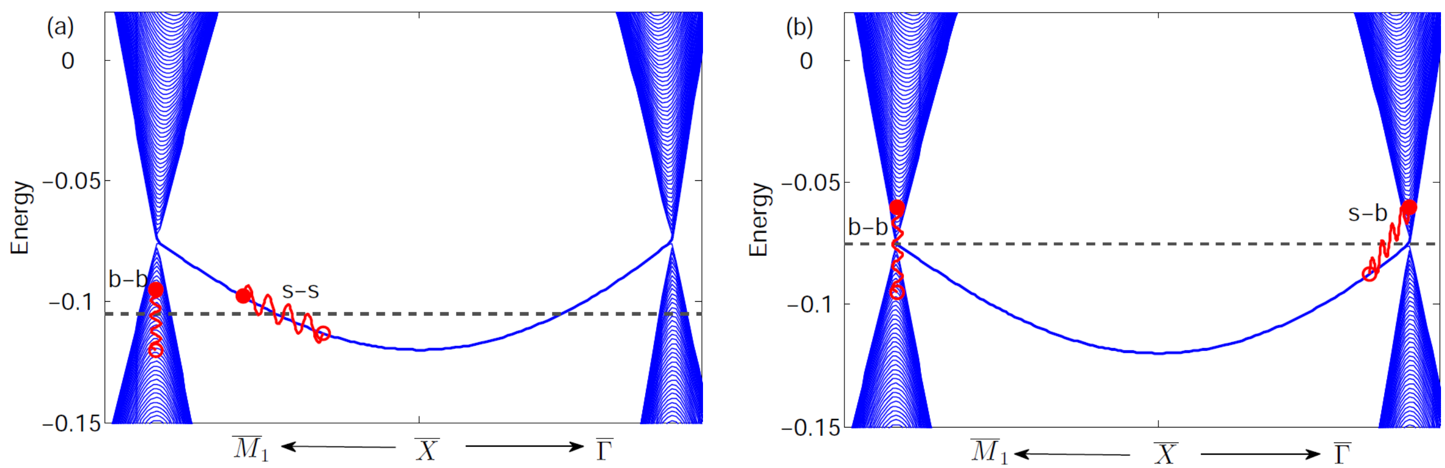
+



is it 2d or 3d?

Surface susceptibility

particles and holes can be drawn from either bulk or surface



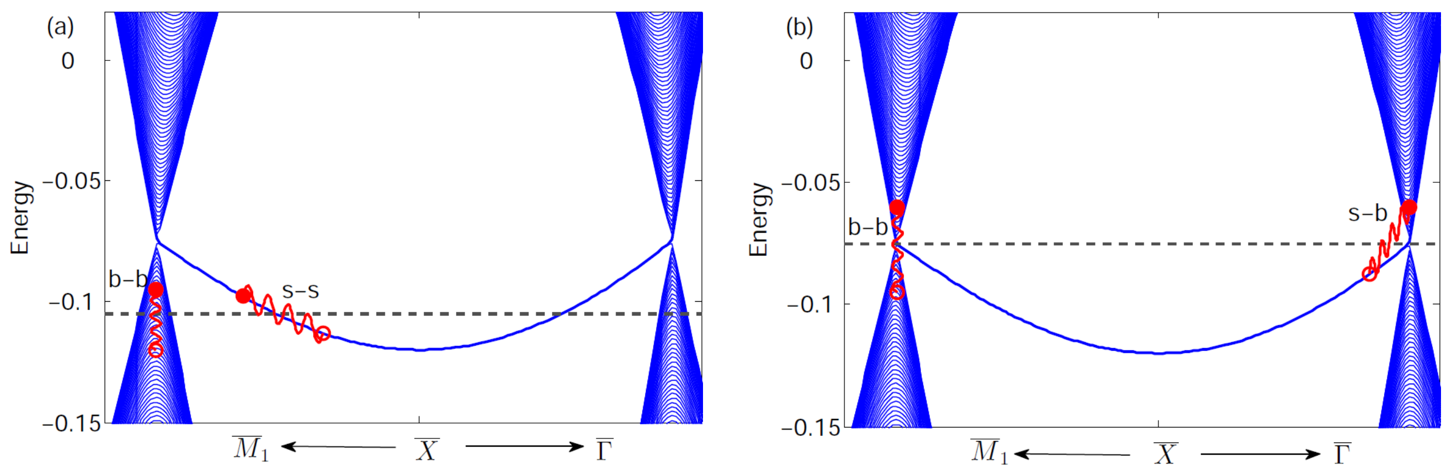
Tricky:

- ✦ Extended states behave non-trivially near the surface
- ✦ Surface states become “dilute” near the nodes

safe approach: calculate full Green’s functions near real-space surface, and from that the surface susceptibility

Surface susceptibility

particles and holes can be drawn from either bulk or surface



$$\chi_{ss} \sim \frac{|\omega|}{q} \gg \chi_{bb} \sim \frac{|\omega|}{\sqrt{q}|}$$

$$\chi_{sb} \sim |\omega|\sqrt{q} \gg \chi_{bb} \sim \frac{\omega^2}{\sqrt{q}}$$

looks like isolated 2d

different from 2d

HMM z = 3/2

would be interesting to explore in more detail

Three types of topology

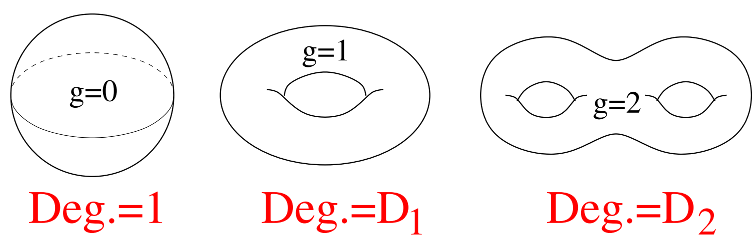
Topological Spin Liquid

topology of entanglement

"intrinsic topological order"

This type of topological phase can *only* exist with strong correlations. It reflects extreme entanglement of the many-body states

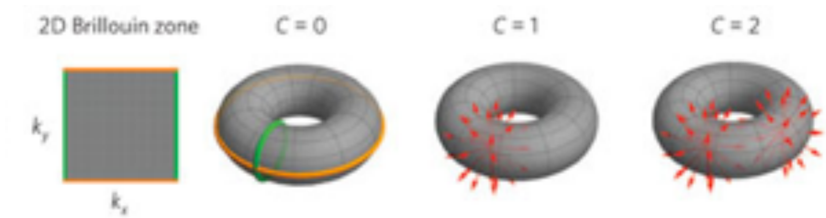
- Wen (1989): some many-body systems exhibit an "order" which is sensitive to the topology of the *spatial* manifold

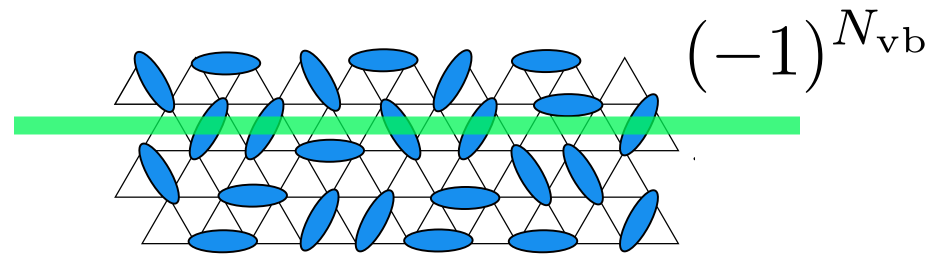


- This type of order is *completely robust*: does not need any symmetry

TI versus iTO

Topological invariants: a non-local integral over an extended manifold



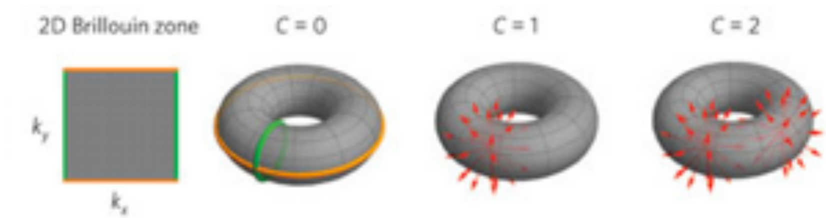


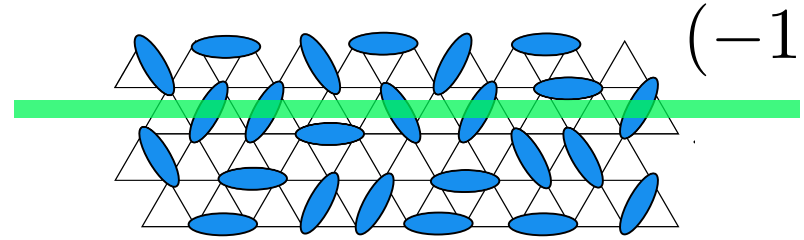
Chern number: integral over **2d k space** whose value differentiates **phases**

Wilson loop: integral over a **1d real space** curve whose value differentiates **states** in the same phase

TI versus iTO

Topological invariants: a non-local integral over an extended manifold





$(-1)^{N_{\mathrm{vb}}}$

Chern number: integral over **2d k space** whose value differentiates **phases**

Wilson loop: integral over a **1d real space** curve whose value differentiates **states** in the same phase

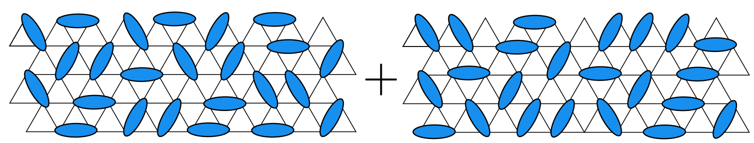
break the 2d space: forms a **gapless edge**

break the 1d curve: forms a **gapped exotic quasiparticle**

Where is iTO?

- Fractional quantum Hall effect is *both* an iTO state *and* a TI (Chern insulator)
- Other main candidates are *quantum spin liquids*

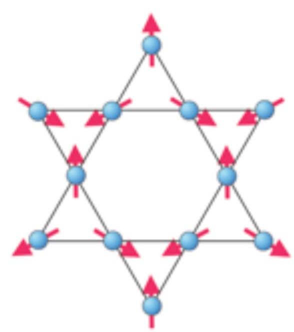
$$|\Psi\rangle = $$



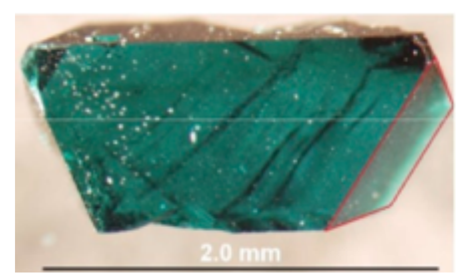
$+ \ldots$

"RVB" state on kagomé lattice?

still seeking definitive id

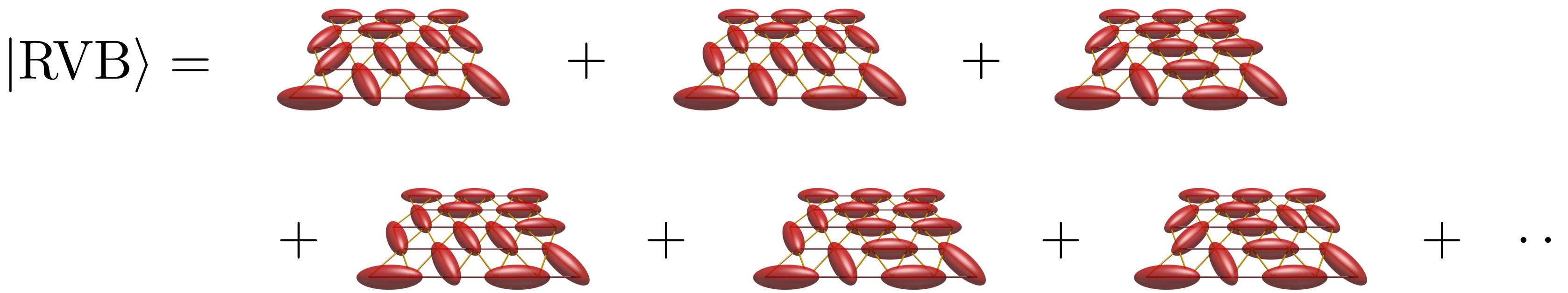


$ZnCu_3(OH)_6Cl_2$



Young Lee, Takashi Imai,...

Quantum Spin Liquids



Quantum Spin Liquids: a Review

Lucile Savary[1], **Leon Balents**[2]

[1]Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

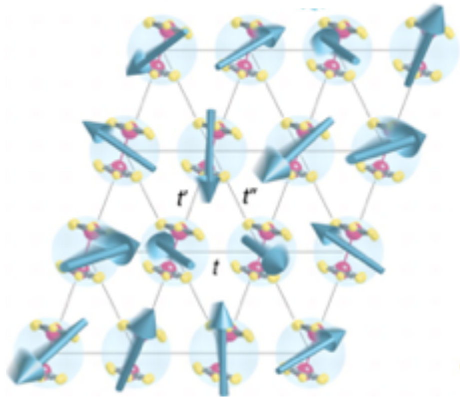
[2]Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, U.S.A.

Abstract. Quantum spin liquids may be considered "quantum disordered" ground states of spin systems, in which zero point fluctuations are so strong that they prevent conventional magnetic long range order. More interestingly, quantum spin liquids are prototypical examples of ground states with massive many-body entanglement, of a degree sufficient to render these states distinct *phases* of matter. Their highly entangled nature imbues quantum spin liquids with unique physical aspects, such as non-local excitations, topological properties, and more. In this review, we discuss the nature of such phases and their properties based on paradigmatic models and general arguments, and introduce theoretical technology such as gauge theory and partons that are conveniently used in the study of quantum spin liquids. An overview is given of the different types of quantum spin liquids and the models and theories used to describe them. We also provide a guide to the current status of experiments to study quantum spin liquids, and to the diverse probes used therein.

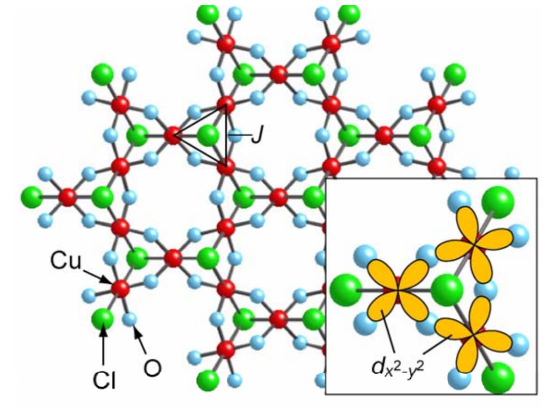
New authoritative review: to be published in ROPP

Spin liquid candidates?

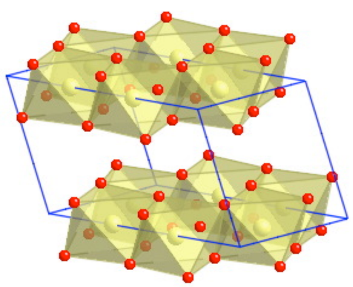
Top experimental platforms



Organics

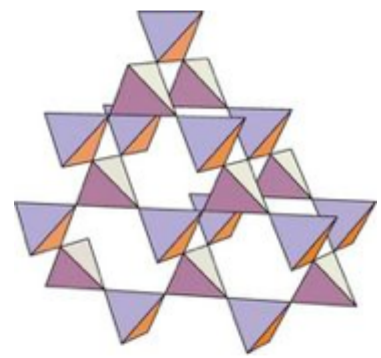


Herbertsmithite



Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3
α-$RuCl_3$

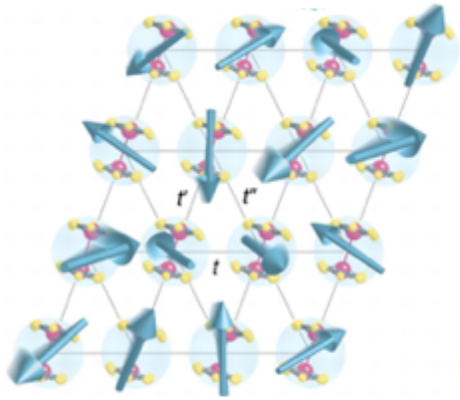
Kitaev materials



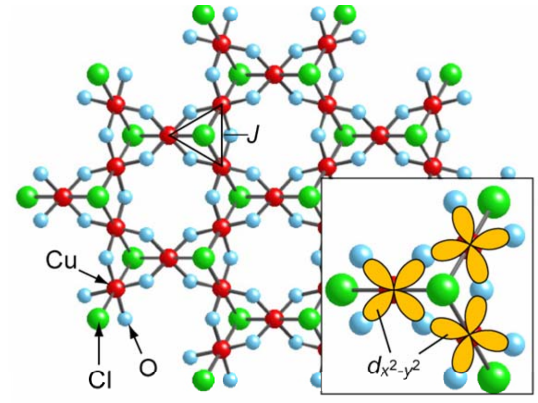
$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$
...

Quantum spin ice

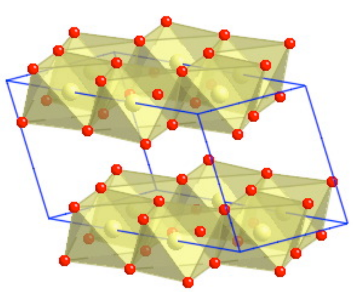
Top experimental platforms



Organics



Herbertsmithite



Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3
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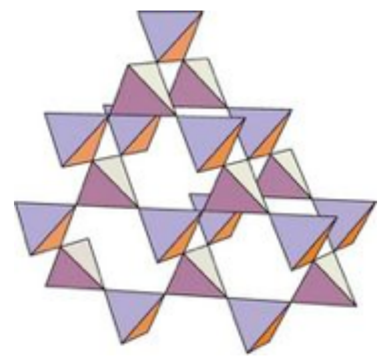
Kitaev materials



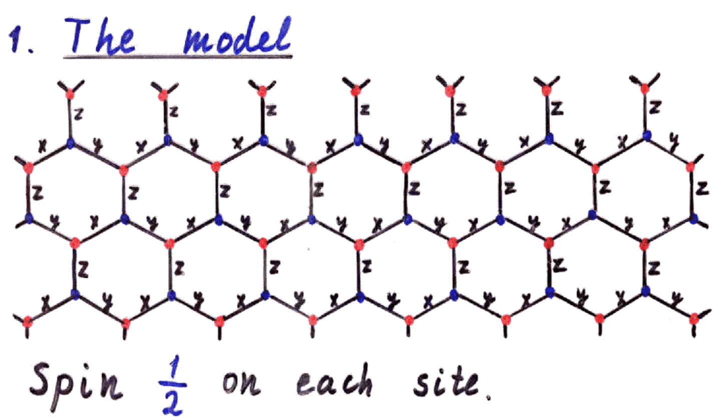
$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$
...

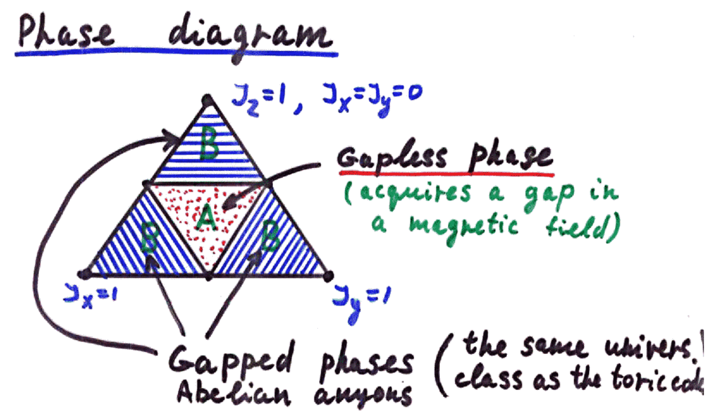
Quantum spin ice

Kitaev model

Kitaev's honeycomb model $$H = \sum_{i,\mu} K_\mu \sigma_i^\mu \sigma_{i+\mu}^\mu$$

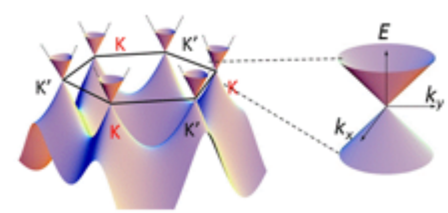
KITP, 2003



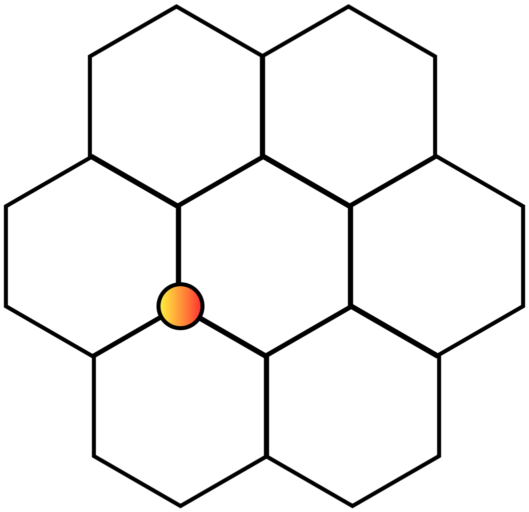


exact parton construction $\sigma_i^\mu = i c_i c_i^\mu \quad c_i c_i^x c_i^y c_i^z = 1$

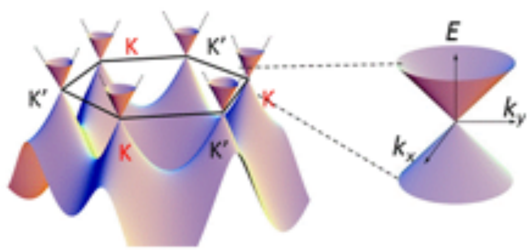
physical Majoranas $$H_\mathrm{m} = K \sum_{\langle ij \rangle} i c_i c_j$$



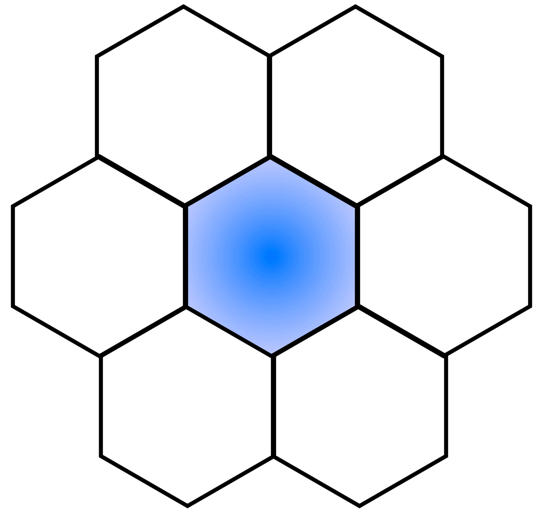
Non-local excitations



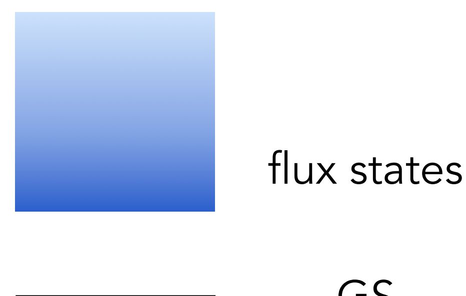
Majorana ε



gapless Dirac



Flux e, m

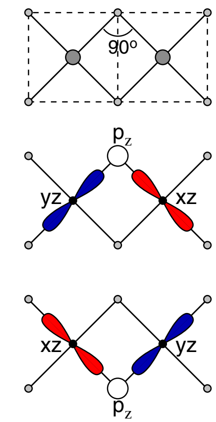


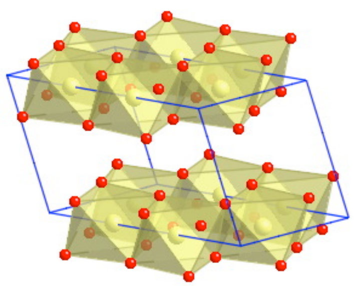
gapped

Kitaev Materials

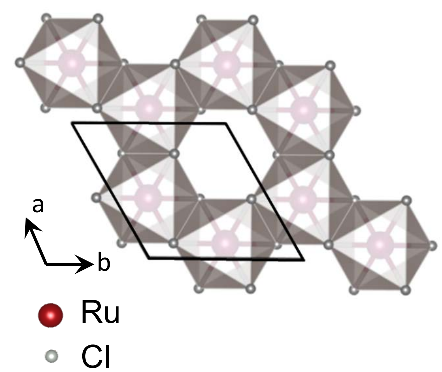
Jackeli, Khaliullin

Showed that Kitaev interaction can be large in edge-sharing octahedra with large spin-orbit-coupling





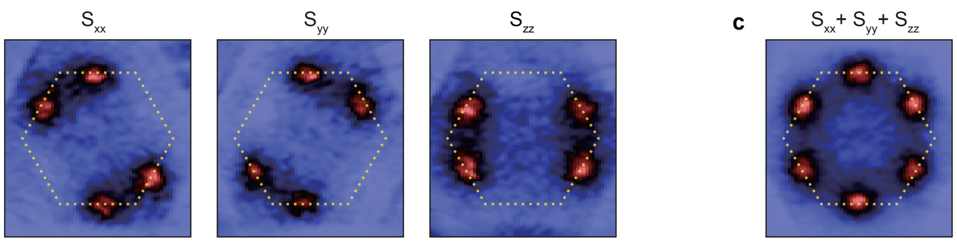
Na_2IrO_3, (α,β,γ)-Li_2IrO_3



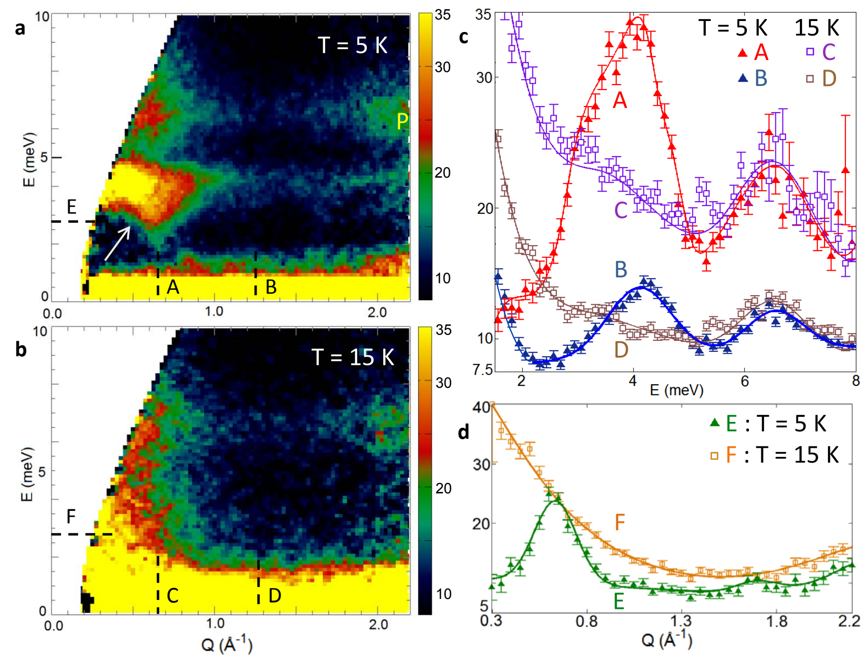
α-$RuCl_3$

Honeycomb and hyper-honeycomb structures

Kitaev Materials



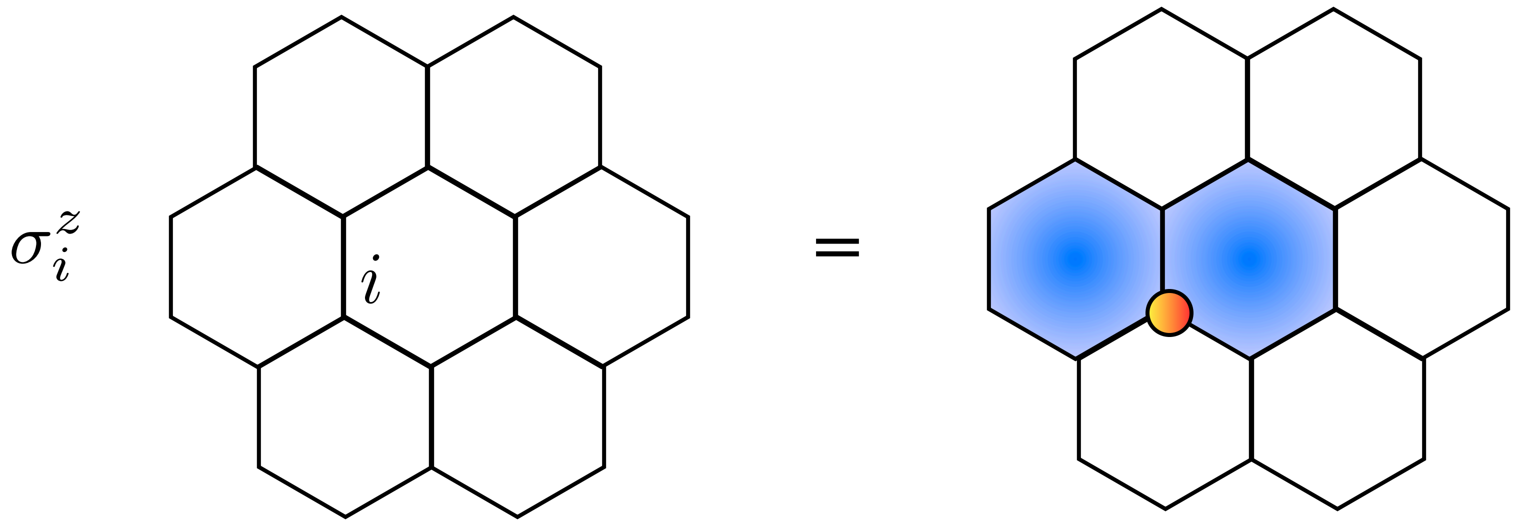
direct evidence for direction-dependent anisotropic exchange from diffuse magnetic x-ray scattering in Na_2IrO_3 (BJ Kim group)



Observation of gapped continuum mode persisting above T_N in α-$RuCl_3$ consistent with Majoranas (A. Banerjee *et al*)

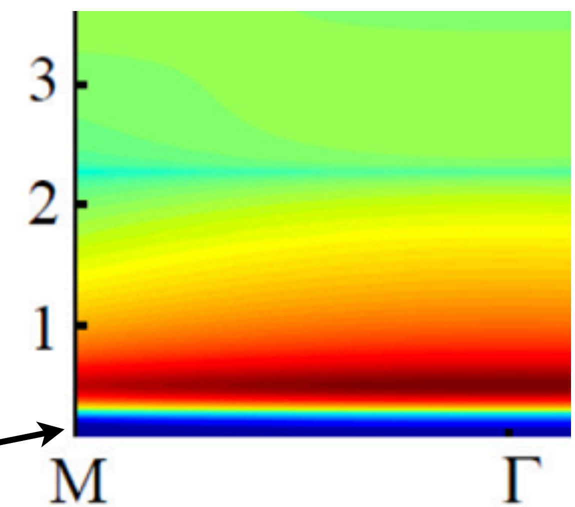
there is pretty strong evidence of substantial Kitaev exchange in quite a few materials

Exact spin correlations

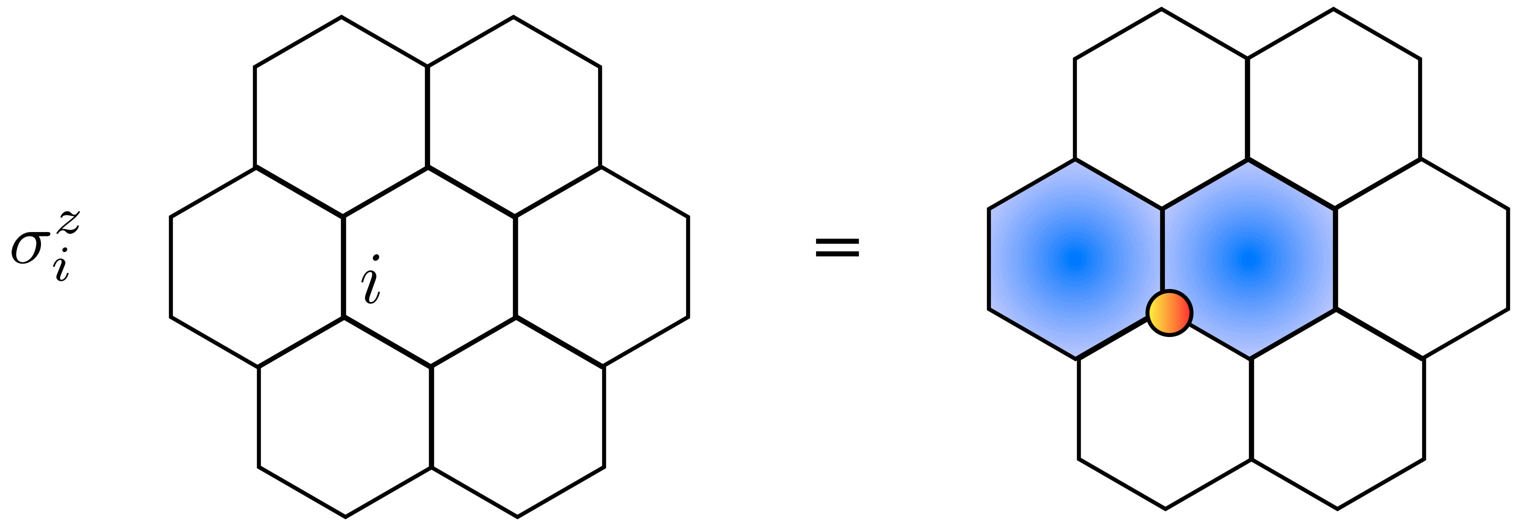


In the soluble model:

- The spin creates two fluxes
- Spectral weight is zero below the flux gap
- Correlations vanish beyond NNs



Exact spin correlations

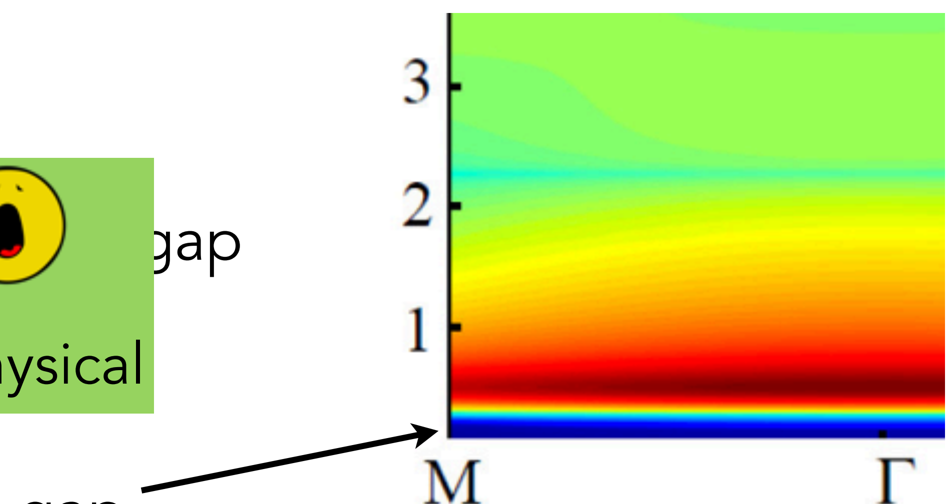


In the soluble model:

- The spi
- Spectra … gap
- Correla

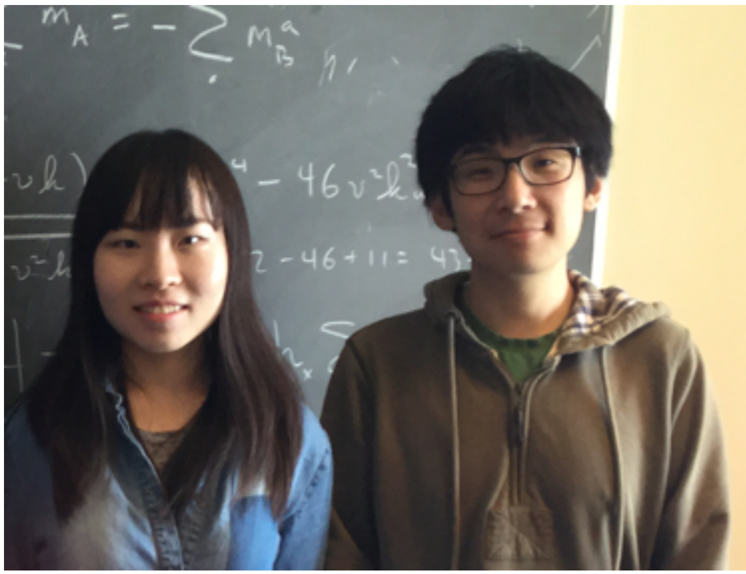
very boring

But fortunately it is not physical



J. Knolle *et al*, 2014

Inexact but correct (universal) answer



宋雪洋
Xue-Yang Song

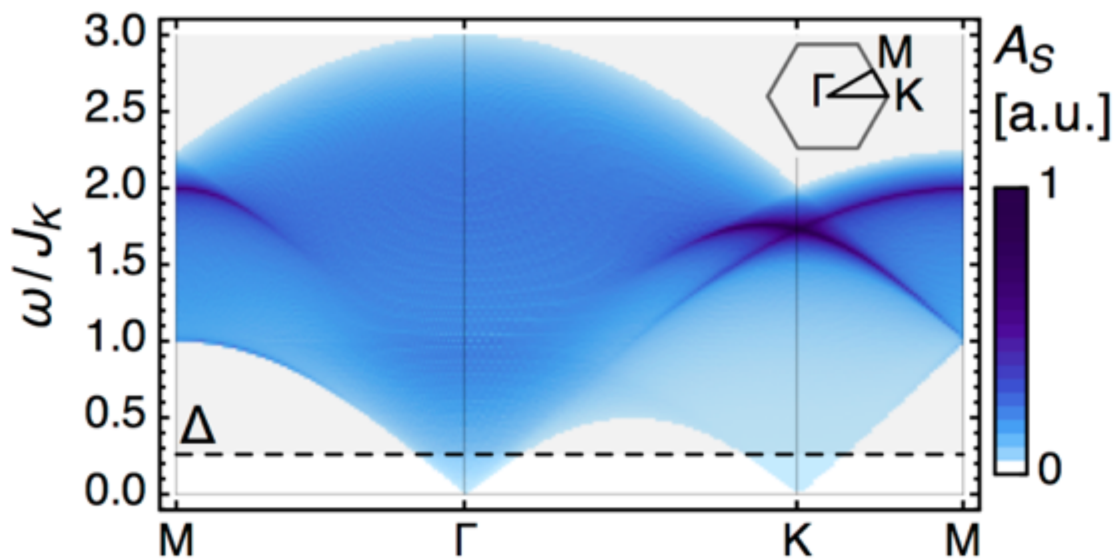
3rd yr ugrad
Peking U.

尤亦庄
Yi-Zhuang You

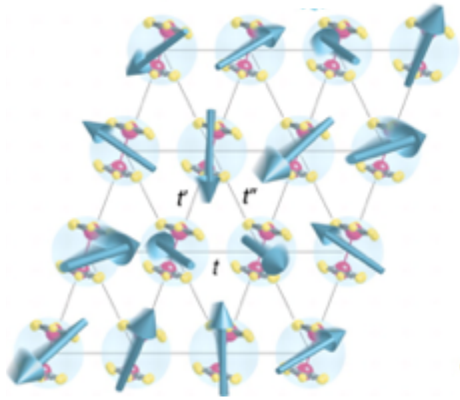
postdoc
UCSB

Generically: spin correlations are *gapless* and structured

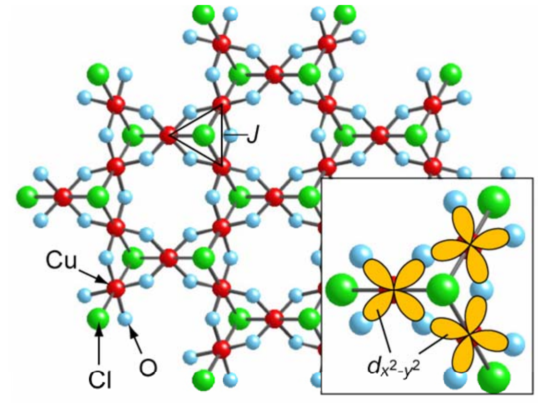
(gapless contribution should be added to the other one, which is like the “incoherent” part in a Fermi liquid)



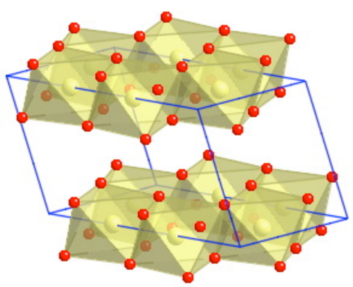
Top experimental platforms



Organics

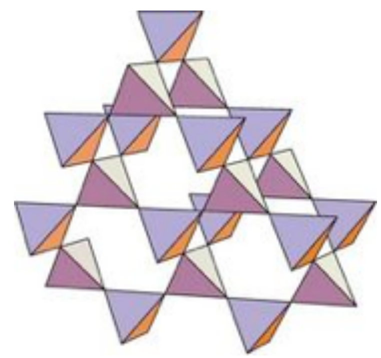


Herbertsmithite



Na_2IrO_3,
(α,β,γ)-
Li_2IrO_3
α-$RuCl_3$

Kitaev materials

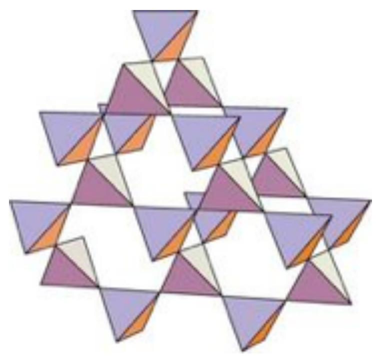


$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$
...

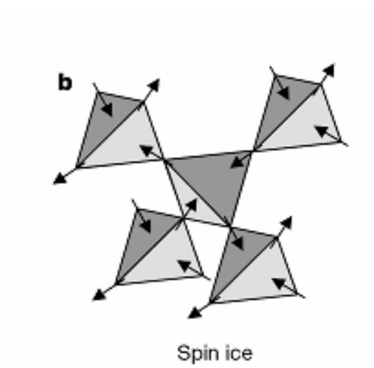
Quantum spin ice

Quantum spin ice

- Quantum S=1/2 spins on pyrochlore lattice



$$H = J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z + H_Q$$



+ Quantum fluctuations

2in-2out states "classical spin ice"

Quantum-Izing spin ice

- For *non-Kramer's ions* (Ho^{3+},Pr^{3+}), non-magnetic disorder acts like a *transverse field*

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$$H_Q = -\sum_i h_i S_i^+ + \mathrm{h.c.}$$

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(quantum transverse field Icing model)

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(quantum transverse field Icing model)

<u>Prediction</u>: quantum dynamics can be induced in diverse classical Ising systems by controlled disorder

Quantum-Ising spin ice

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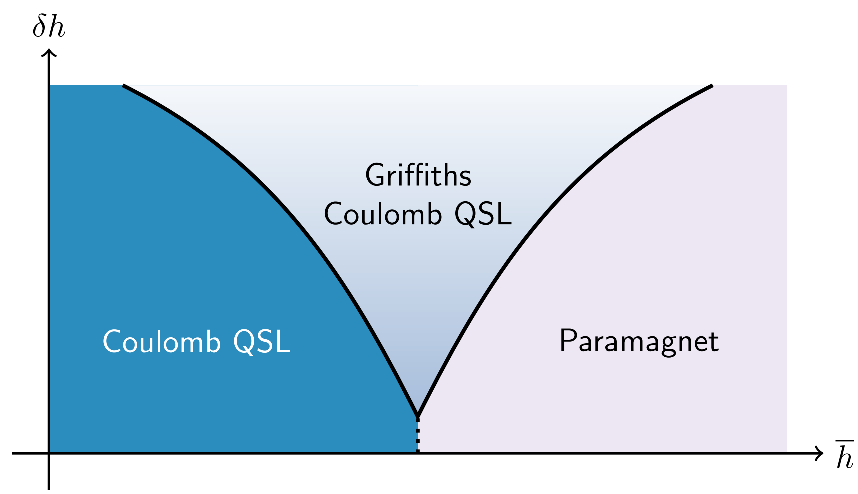
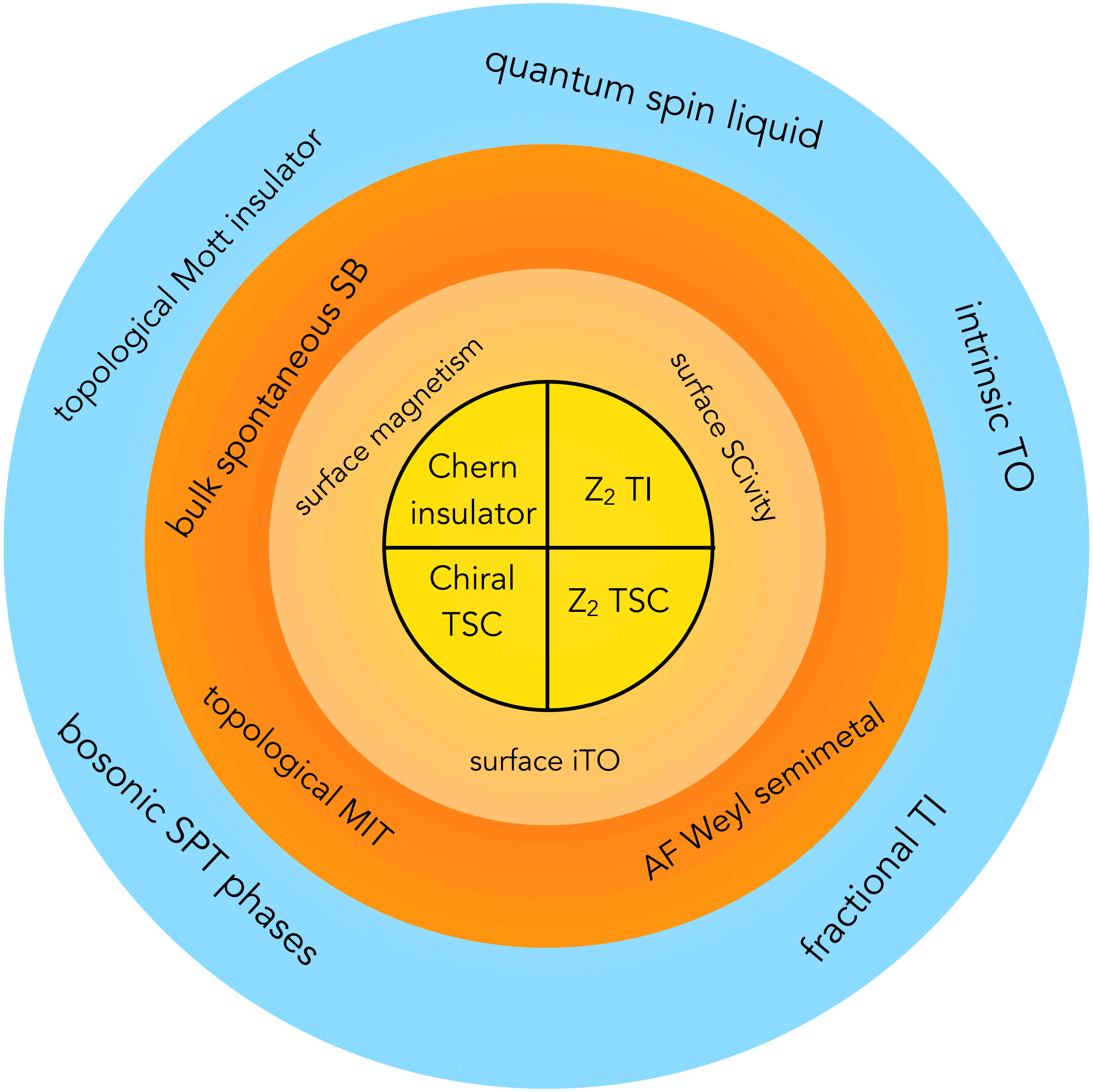
$$H_Q = -\sum_i h_i S_i^+ + \text{h.c.}$$

disorder alone can generate the "Coulomb" QSL

L. Savary + LB, arXiv:1604.04630



Thanks


quantum spin liquid
topological Mott insulator
intrinsic TO
bosonic SPT phases
fractional TI
bulk spontaneous SB
topological MIT
AF Weyl semimetal
surface magnetism
surface SCivity
surface iTO
Chern insulator
Z_2 TI
Chiral TSC
Z_2 TSC